Workshop XI

Paul Jones

Instructor Mariya Naumova

Calculus II (01:640:152, section C2)

December 15, 2012

The thread length for a simple spool of cotton thread is 25 yards. To celebrate Valentine's Day, purchase a spool of red thread and send it to your beloved with these instructions:

> Unwind the thread and arrange it in the shape of a cardioid, $r = A(1 - \sin\theta)$. The area of that cardioid represents how much I love you compared to the ordinary Valentine's Day card!

Compute the arc length of $r = A(1 - \sin\theta)$ and find A so that the length is 25 yards. Then compute the area inside that cardioid. Sketch the result.

a) Arc length

- The formula for arc length in polar coordinate:

$$L = \int_a^b \sqrt{[r(\theta)]^2 + \left[\frac{dr(\theta)}{d\theta}\right]^2}\, d\theta$$

- Consider $[r(\theta)]^2$:

$$r^2 = A^2(1 - \sin\theta)^2 = A^2(1 - \sin^2\theta - 2\sin\theta)$$

- Consider $\left[\frac{dr(\theta)}{d\theta}\right]^2$:

$$r' = -A\cos\theta,\ (r')^2 = A^2\cos^2\theta$$

- Based on my rough sketch, function is symmetrical after $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Plug into formula:

$$L = 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{A^2(1 - \sin^2\theta - 2\sin\theta) + A^2\cos^2\theta}\, d\theta$$

- Simplify and evalutate:

$$L = 2\sqrt{2}A\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 - \sin\theta}\, d\theta = 8A$$

b) Surface area

- If we want an A such that $L = 25$, A must equal $\frac{25}{8}$. The equation for SA:

$$SA = \frac{1}{2}\int_a^b [r(\theta)]^2 \, d\theta$$

- Plugin for our values:

$$SA = \left[\frac{25}{8}\right]^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 - \sin\theta)^2 \, d\theta$$

- Integrate and simplify:

$$SA = \left[\frac{25}{8}\right]^2 \cdot \frac{3\pi}{2} \approx 46$$

c) Sketch