Workshop IV

Paul Jones

Instructor Mariya Naumova

Calculus II (01:640:152, section C2)

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$$\text{a)} \int \sqrt{4-x^2}\,\mathrm{d}x$$

- Let $x = 2\sin\theta$, which makes $dx = 2\cos\theta \mathrm{d}\theta$ and $\theta = \sin^{-1}\frac{x}{2}$

$$\int 2\sqrt{1-\sin\theta}\, 2\cos\theta \mathrm{d}\theta = 4\int cos^2\theta \mathrm{d}u$$

- Apply the "double angle formula":

$$4\int \frac{cos(2\theta)+1}{2}\mathrm{d}\theta = 2\int cos(2\theta)\mathrm{d}\theta + 2\int \mathrm{d}\theta$$

- Integrate and substitute back:

$$\sin(2\theta) + 2\theta = \frac{1}{2}\sqrt{4-x^2}x + 2\sin^{-1}(\frac{x}{2}) + C$$

$$\text{b)} \int \sqrt{4+x^2}\,\mathrm{d}x$$

- Let $u = \frac{x}{2}$ which makes $2\mathrm{d}u = \mathrm{d}x$:

$$\int 4\sqrt{1+u^2}\,\mathrm{d}u$$

- Let $u = \tan\theta$, which makes $\mathrm{d}u = \sec^2\theta \mathrm{d}\theta$

$$4\int \sqrt{1+\tan^2\theta}\, \sec^2\theta \mathrm{d}\theta = 4\int \sec^3\theta \mathrm{d}\theta$$

- Apply reduction formula, integrate, and simplify (where $\theta = \tan^{-1}\frac{x}{2}$):

$$2\tan\theta\sec\theta + 2\log(\sec\theta + \tan\theta) = x\sec\tan^{-1}\frac{x}{2} + 2\log(\sec\tan^{-1}\frac{x}{2} + \frac{x}{2}) + C$$

$$\text{c)} \int \sqrt{5+2x+x^2}\,\mathrm{d}x$$

- Complete the square! Then, make $u = x+1$ and find $\mathrm{d}u$.

$$\int \sqrt{(x+1)^2+4}\,\mathrm{d}x = \int \sqrt{u^2+4}\,\mathrm{d}u$$

- Now use trigonometric substitution, specifically where $u = 2\tan\theta$:

$$4\int \sqrt{\tan\theta+1}\ \sec\theta \mathrm{d}\theta = 4\int \sec^3\theta \mathrm{d}\theta$$

- Now integrate with reduction formula:

$$2\tan\theta\sec\theta + 2\log(\tan\theta+\sec\theta)$$

- Substitute back in:

$$2\tan(2\tan^{-1}(x+1))\sec(2\tan^{-1}(x+1)) + 2\log(\tan(2\tan^{-1}(x+1)) + \sec(2\tan^{-1}(x+1)))$$