Workshop I

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Calculus II (01:640:152, section C2)

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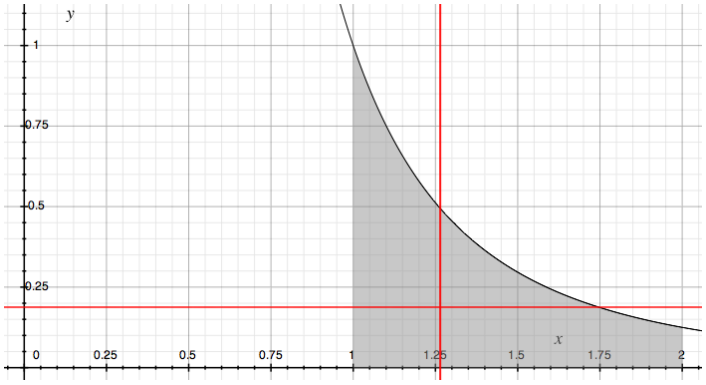
Sketch the region R by $1 \leq x \leq 2$ and $0 \leq y \leq \frac{1}{x^3}$.



1. Find (exactly) the number α such that the line $x = \alpha$ divides R into two parts of equal area.

   - To split the area found under a curve of a definite integral, define α as some point between the closed interval of [a, b].
   - Define two integrals between a and α, and α and b and set them equal to each other (as they are of equal area).
   - Solve for α.

$$\int_1^{\alpha} \frac{1}{x^3}\,\mathrm{d}x = \int_{\alpha}^{2} \frac{1}{x^3}\,\mathrm{d}x$$

$$a = \sqrt{\frac{8}{5}}$$

2. Then find (to three places) the number b such that the line $y = b$ divides R into two parts of equal area.

   First, find the total area under the curve

$$\int_1^{2} \frac{1}{x^3}\,\mathrm{d}x = \frac{3}{8} = A$$

   What we're looking for is what line that creates a rectangle with the x-axis whose area is equal to half this value, so we need only divide by half!

$$\frac{1}{2}A = \frac{3}{16} = 0.1875 = b$$