Workshop V

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Calculus II (01:640:152, section C2)

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a) Compute $\int_1^2 \frac{\mathrm{d}x}{x}$

- Being as the power is "negative," add one to x and "flip the sign"

$$-\frac{1}{x}\Big|_1^2$$

- Evaluate using Fundamental Theorem of Calculus:

$$\frac{1}{2}$$

b) Compute $\int_1^2 \frac{\mathrm{d}x}{x(x-m)}$ if m is a small positive number. What happens as m approaches 0 from the right?

- What we are considering is:

$$\lim_{m \to 0^+} \int_1^2 \frac{\mathrm{d}x}{(x(x-m))}$$

- Intuitively, it makes sense that if m is getting really small and is multiplied by x, as it approaches zero from the right, it will become less and less significant and our answer will take the form of problem (a).

- Consider the integral separately using partial fractions:

$$\int_1^2 \frac{1}{mx - m^2}\mathrm{d}x - \int_1^2 \frac{1}{mx}\mathrm{d}x$$

- Factor out constants:

$$\frac{1}{m}\int_1^2 \frac{\mathrm{d}x}{x-m} - \frac{1}{m}\int_1^2 \frac{\mathrm{d}x}{x}$$

- Integrate:

$$\frac{1}{m}\log(x-m)\Big|_1^2 - \frac{1}{m}\log(x)\Big|_1^2$$

- Now use the Fundamental Theorem of Calculus to evaluate, and bring back the limit from the beginning:

$$\lim_{m\to 0^+}\left(\frac{\log(2-m)-\log(2)}{m}-\frac{\log(1-m)}{m}\right)$$

- Evaluate the limit. Intuitively, consider that that $\frac{\log(2-.001)-\log(2)}{.001}$ is approximately $-\frac{1}{2}$ and $\frac{\log(1-.001)}{.001}$ is approximately negative one, our answer is, thankfully:

$$\frac{1}{2}$$

c) Compute $\int_1^2 \frac{\mathrm{d}x}{x^2+n}$ if n is a small positive number. What happens as n approaches 0 from the right?

- Again, intuitively it makes a lot of sense that if the n in the denominator becomes of less and less significance, then our integral becomes the same form as (a).
- The integral takes the form of the inverse tangent function. Consider:

$$\int \frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{u}{a}+C$$

- Where $a=\sqrt{n}$:

$$\int_1^2 \frac{\mathrm{d}x}{x^2+n}=\left.\frac{\tan^{-1}\frac{x}{\sqrt{n}}}{\sqrt{n}}\right|_1^2$$

- By Fundamental Theorem of Calculus, and insert the pertinent limit:

$$\lim_{n\to 0^+}\left(\frac{\tan^{-1}\frac{2}{\sqrt{n}}}{\sqrt{n}}-\frac{\tan^{-1}\frac{1}{\sqrt{n}}}{\sqrt{n}}\right)$$

- I can't evaluate that. I'm going to try something else. Consider where $x=\sqrt{n}\tan\Theta$ and $\mathrm{d}x=\sqrt{n}\sec^2\theta\mathrm{d}\theta$

$$\int_1^2 \frac{\mathrm{d}x}{x^2+n}=\int_1^2\frac{\sqrt{n}\sec^2\theta\mathrm{d}\theta}{\sqrt{n}\tan^2\theta+n}$$

d) Sketch the graphs: