Workshop VIII

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Calculus II (01:640:152, section C2)

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1. Use the formula $\frac{a}{1-r} = a + ar + ar^2 + ar^3 + ...$ valid for $|r| < 1$ to express each of the following functions as a power series $a_0 + a_1x + a_2x^2 + ... + a_nx^n + ...$

   Give the formula for the coefficient a_n in each case.

$$f(x) = \frac{x}{1-x};\ g(x) = \frac{2}{3x^4+16}$$

- For $f(x)$, set $r = x$ and $a = x$.

$$x + x^2 + x^3 + ... + x^n + ...$$

- Therefore, $a_n = \begin{cases} 0 & \text{if a} = 0 \\ 1 & \text{otherwise} \end{cases}$
- Determine a and r for g(x):

$$\frac{2}{3x^4+16} = \frac{1}{8(1-(-\frac{3}{16}x^4))}$$

- Where $a = \frac{1}{8}$, $r = -\frac{3}{16}x^4$, multiplied by a constant of $\frac{1}{8}$:

$$\frac{1}{8}\left(-\frac{3}{16}x^4 + \left(\frac{3}{16}\right)^2 x^8 - \left(\frac{3}{16}\right)^3 x^{12} + ...\right) = \frac{1}{8}\sum_{n=0}^{\infty}\left(-\frac{3}{16}x^4\right)^n$$

- Therefore, $a_n = \begin{cases} \frac{1}{8} \cdot (\frac{3}{16})^k \cdot (-1)^k & \text{if } n = 4k \\ 0 & \text{otherwise} \end{cases}$

2. Determine the interval of x values in which each series in part 1 converge.

- $f(x)$ converges nowhere as it is a standard geometric series.
- $g(x)$ has a radius of convergence that can be found using this method:

$$\lim_{n\to\infty}\left|\frac{\left(\frac{1}{8}\right)\left(-\frac{3}{16}\right)^{n+1}(-1)^{n+1}}{\left(\frac{1}{8}\right)\left(-\frac{3}{16}\right)^{n}(-1)^{n}}\right| = \left|\frac{3x^4}{16}\right|$$

- The radius of convergence, according to this test, is therefore:

$$I = \left(-\frac{1}{\sqrt[4]{3}}, \frac{1}{\sqrt[4]{3}}\right)$$

3. Use you answer in part 1 to express $\int_0^1 \frac{2}{3x^4+16}$ as the sum of an infinite series.

$$\frac{1}{8}\sum_{n=0}^{\infty}\int_0^1 \left(-\frac{3}{16}x^4\right)^n = \int_0^1 \frac{1x^0}{8} - \frac{3x^4}{128} + \frac{9x^8}{2048} - \frac{27x^{12}}{32768} + ...$$

$$= -\frac{x}{8}\left(1 - \frac{3x^4}{2^4 \cdot 5} + \frac{x^8}{2^8} + ...\right)$$