# Workshop X

Paul Jones

Instructor Mariya Naumova

Calculus II (01:640:152, section C2)

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a) Consider an isosceles triangle $\triangle OPQ$ where $OP = OQ = R$ and $\angle POQ = \theta$. Find $PQ$.

- If you were to split this isosceles triangle in half from $\theta$, one could say that:

$$\sin\frac{\theta}{2} = \frac{\frac{1}{2}PQ}{R}$$

- Solving for $PQ$ yields:

$$PQ = \frac{\sin\frac{\theta}{2}R}{2}$$

b) Consider a circle of radius $R$. Suppose that a regular $n$-gon is inscribed in this circle. Find the perimeter $P_n$ of this $n$-gon.

- In the previous exercise, the length of one side of this $n$-gon was found. The perimeter is $n$ times the value of this operation.
- The perimeter is equal to one side of an $n$-polygon multiplied by $n$:

$$2nR\sin\frac{\pi}{n}$$

c) Find the $\lim_{n\to\infty} P_n$.

- Factor out constants $(2R)$, "rearrange", apply L'Hôpital's rule to indeterminate form $\frac{0}{0}$, factor out $\pi$, and recognize that any multiple of $\pi$ within cosine yields zero:

$$2R\lim_{n\to\infty}\frac{\sin\frac{\pi}{n}}{n^{-1}} = 2\pi R\lim_{n\to\infty}\cos\pi t = 2\pi R$$

d) Using the arc-length integral, find the arc-length of a circle of radius $R$. Explain why this integral must give the same answer as the previous limit.

- When describing a circle, the $a$ and $b$ values must of the integral must yield an entire circle verses simply a portion of an arc. In an attempt to keep things simple, one can pick the values of $[0, 2\pi]$.

- Being as a circle is not a function, there is no simple function to input into the arc-integral's formula. Any "y as a function of x" will limit one to a portion of a circle, and a portion of an arc. But if one were to parametrically define the circle with radius $R$ as $x(t) = Rcos(t)$ and $y(t) = Rsin(t)$ where $0 \leq t \leq 2\pi$, then one would have a full circle.

- The arc-length integral for parametric equations, defined in section 11.2, is as follows:

$$s = \int_a^b \sqrt{x'(t)^2 + y'(t)^2} dt$$

- Therefore the integral for the arc length of a circle with radius R is as follows:

$$\int_0^{2\pi} \sqrt{(R\frac{d}{dt}\cos t)^2 + (R\frac{d}{dt}\sin t)^2}\ dt = \int_0^{2\pi} \sqrt{(-R\sin t)^2 + (R\cos t)^2}\ dt$$

$$= R\int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t}\ dt = R\int_0^{2\pi} dt = 2\pi R$$

- The "arc-length" of a circle with radius $R$ should equal the value of the perimeter of a polygon whose number of sides approach infinity because a circle's "arc-length" is its perimeter and a polygon with an infinite amount of sides is a circle. Every point on a circle is equidistant from the center, which could be thought as a polygon with an infinite amount of infinitesimally small sides. A circle's "arc-length" is the perimeter of a circle because arc-length is usually some subdivision of some function, and in the case of a circle that subdivision is the whole circle and the function is a circle.