# Workshop VI

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Calculus II (01:640:152, section C2)

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- Two students are sharing a loaf of bread. Student A, now hungrier and more ferocious, eats two thirds of the the loaf, then student B eats half of what remains, then A eats two-thirds of what remains, then B eats half of what remains, and so on. How much of the loaf will each student eat?

  - Begin working out the pattern:

Table 1: Beginning the pattern

| Student A | | Student B | |
|---|---|---|---|
| Eats | Remains | Eats | Remains |
| $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |
| $\frac{1}{9}$ | $\frac{1}{18}$ | $\frac{1}{36}$ | $\frac{1}{36}$ |
| $\frac{1}{9 \cdot 2 \cdot 3}$ | $\frac{1}{18 \cdot 2 \cdot 3}$ | $\frac{1}{18 \cdot 2 \cdot 3 \cdot 2}$ | $\frac{1}{18 \cdot 2 \cdot 3 \cdot 2}$ |

  - Consider Student B: She consumes bread according to this sum:

$$\sum_{i=1}^{\infty} \left(\frac{1}{6}\right)^i$$

  - Recall the geometric series:

$$\sum_{i=1}^{\infty} \left(\frac{1}{6}\right)^i = \frac{1}{1-\frac{1}{6}} - 1 = \frac{1}{5}$$

  - If Student B consumes this much, the other student must consume:

$$\frac{4}{5}$$

- For each sequence, stat exactly how large $n$ must be to ensure that the term $a_n$ of the sequence satisfies $|a_n| < 10^{-4}$. Then, use the information to explain which sequences approaches zero most rapidly and which approaches zero least rapidly.

  - Consider the sequence:

$$\left\{\frac{1}{\sqrt{n}}\right\}_{n=1}^{\infty}$$

– Now consider the definition of the limit of sequence:

$$\forall \epsilon > 0 \; \exists n_\epsilon \in N \; \forall n \geq n_\epsilon : |a_n - b| < \epsilon$$

– The limit of the sequence is zero. Therefore:

$$\left| \frac{1}{\sqrt{n}} - 0 \right| < \epsilon$$

– And therefore:

$$-\frac{1}{\sqrt{n}} < \epsilon \therefore n > 0$$

– And:

$$\frac{1}{\sqrt{n}} < \epsilon \therefore \left( \frac{1}{\epsilon} \right)^2 > n$$

– a) Thus, if the index is greater than $10^8$, the result will be smaller than $10^{-4}$.

– b) Thus, if the index is greater than $10^{16}$, the result will be smaller than $10^{-8}$.

– Consider the sequence:

$$\left\{ \frac{1}{10^n} \right\}_{n=1}^{\infty}$$

– Again, by definition of the limit of a sequence (our limit is zero):

$$\left| \frac{1}{10^n} \right| < \epsilon$$

– Therefore, there two cases are:

$$-\frac{1}{10^n} < \epsilon \therefore n > 0$$

– And:

$$\frac{1}{\sqrt{n}} < \epsilon \therefore 4 \log 10 < n \therefore 4 < n$$

- a) Thus, if the index is greater than 4, the result will be smaller than $10^{-4}$.
- b) Similarly, if the index is greater than 8, the result will be smaller than $10^{-8}$.
- c) Being as the index 4 brings the second sequence closer to a small number than the index $10^8$ brings our first sequence to the same small number, the second sequence must approach zero faster