Workshop II

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Calculus II (01:640:152, section C2)

March 31st, 2012

Question:

The curve $y = f(x)$ is called a tractrix, and has the following property: the derivative at any point x satisfies the formula:

$$f'(x) = \frac{-f(x)}{\sqrt{1 - f(x)^2}}$$

Consider the region under the tractrix, within the limits $0 \leq x \leq a$. Find the volume of the solid object obtained by revolving this region around the x-axis in therms of the constant $c = f(a)$.

Solution:

1. To find the volume, use the "disk method," because the changing values are perpendicular to the x-axis, making the bounds are from 0 to a along the x-axis.

$$V_{tractrix} = \pi \int_0^a f^2(x)\,\mathrm{d}x$$

2. Being as the formula for the tractrix is not given, utilize "u-substitution" where $u = f(x)$ and $du = f'(x)dx$. Therefore, $dx = \frac{du}{f'(x)}$. Recalculate bounds for u, use given formula for $f'(x)$, plugin u for every $f(x)$, and simplify.

$$V_{tractrix} = -\pi \int_1^c u\sqrt{1 - u^2}du$$

3. Substitute $1 - u^2$ with t, making $udu = \frac{dt}{-2}$, recalculate bounds according to t:

$$V_{tractrix} = \frac{\pi}{2} \int_0^{1-c^2} \sqrt{t}\,dt$$

4. Evaluate for solution

$$V_{tractrix} = \frac{\pi}{3}(1 - c^2)^{\frac{3}{2}}$$