# Workshop III

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Calculus II (01:640:152, section C2)

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1. Suppose $m$ and $n$ integers. Compute $\int_0^{2\pi} cos(mx)cos(nx)\,\mathrm{d}x$.

    - Consider the trigonometric identity:

$$\frac{cos(a+b)+cos(a-b)}{2} = cos(a)cos(b)$$

    - Plug in $a = mx$ and $b = nx$:

$$\int_0^{2\pi} \cos(mx)\cos(nx)\,\mathrm{d}x = \frac{1}{2}\int_0^{2\pi} \cos(mx+nx)+\cos(mx-nx)\,\mathrm{d}x$$

    - This integrates to:

$$\frac{1}{2}\left(\frac{\sin(mx+nx)}{m+n}+\frac{\sin(mx-nx)}{m-n}\right)\Bigg|_0^{2\pi}$$

    - Evaluate between 0 and $2\pi$:

$$\frac{1}{2}\left(\frac{\sin(2\pi(m+n))}{m+n}+\frac{\sin(2\pi(m-n))}{m-n}\right) = 0$$

    - **Solution**: Any multiple of $2\pi$ within sin will yield zero.

    - But $m = n$ and $m = -n$ are different cases. $m = n$:

$$\int_0^{2\pi} \cos^2(mx)\,\mathrm{d}x = \frac{1}{2}\int_0^{2\pi} 1+\cos(2mx)\,\mathrm{d}x = \left(x+\frac{\sin(2mx)}{2m}\right)\Bigg|_0^{2\pi} = \pi$$

    - $m = -n$:

$$\int_0^{2\pi} \cos(-nx)\cos(nx)\,\mathrm{d}x = \frac{1}{2}\int_0^{2\pi} 1+\cos(-2nx)\,\mathrm{d}x = \pi+\frac{\sin(4n\pi))}{4n} = \pi$$

2. Suppose $f(x) = A\cos(x)+B\cos(2x)+C\cos(3x)$ and you know that

$\int_0^{2\pi} f(x)\cos(x)\,\mathrm{d}x = 5$ $\quad\int_0^{2\pi} f(x)\cos(2x)\,\mathrm{d}x = 6$ $\quad\int_0^{2\pi} f(x)\cos(3x)\,\mathrm{d}x = 7$

- Substitute in $f(x)$ into each integral:

$$\int_0^{2\pi} A\cos^2(x) + B\cos(x)\cos(2x) + C\cos(x)\cos(3x)\,\mathrm{d}x$$

- These can be considered separately.

$$A\int_0^{2\pi}\cos^2(x)\,\mathrm{d}x + B\int_0^{2\pi}\cos(x)\cos(2x)\,\mathrm{d}x + C\int_0^{2\pi}\cos(x)\cos(3x)\,\mathrm{d}x = 5$$

- By the proof in the previous section, the last two integrals equal zero, and the first equals pi, therefore:

$$A\pi = 5, A = \frac{5}{\pi}$$

- Now, substitute $f(x)$ into the next integral:

$$A\int_0^{2\pi}\cos(x)\cos(2x)\,\mathrm{d}x + B\int_0^{2\pi}\cos^2(2x)\,\mathrm{d}x + C\int_0^{2\pi}\cos(2x)\cos(3x)\,\mathrm{d}x = 6$$

- Using the same logic, the integrals multiplied by $A$ and $C$ are zero, and the integral beginning with $B$ equals pi, therefore:

$$B\pi = 6, B = \frac{6}{\pi}$$

- Finally, apply the same process to the last integral in the question:

$$A\int_0^{2\pi}\cos(x)\cos(3x)\,\mathrm{d}x + B\int_0^{2\pi}\cos(3x)\cos(2x)\,\mathrm{d}x + C\int_0^{2\pi}\cos^2(3x)\,\mathrm{d}x = 7$$

$$C\pi = 7, C = \frac{7}{\pi}$$