Rutgers University
CS206: Introduction to Discrete Structures II, Spring 2013
Professor David Cash

Homework 4

Due at the beginning of class on Wednesday, March 13

Instructions: Point values for each problem are listed. Write your solutions neatly or type them up. Typed solutions will also be accepted via Sakai.

1. (3 points) In the card game bridge we deal 13-card hands to 4 players named North, South, East, and West (all 52 cards are dealt). What is the probability that East and West have no spades?

2. (4 points) An urn contains $n > 0$ white balls and $m > 0$ black balls. Suppose we draw two balls without replacement. What is the probability that the balls are of the same color? What if we draw them with replacement? Show you work. Which of these probabilities is larger? Briefly explain some intuition for why one should be larger.

3. (4 points) Again consider an urn with $n > 0$ white balls and $m > 0$ black balls. Suppose we draw $r \geq 1$ balls from the urn without replacement. What is the probability that we draw exactly k white balls?

4. (4 points) A box contains a mixture of cubes and spheres and any of these objects can be either white or black. Suppose the box contains 4 black cubes, 6 black spheres, 6 white cubes, and x white spheres. Consider the experiment of drawing a random object from the box, and let A be the event that a cube is drawn and B be the event that a black object is drawn. If A and B are independent, what is x?

5. (10 points total) Two fair dice are rolled. Define the random variables $X =$ the sum of the two rolls, $Y =$ the maximum of the two rolls, $Z =$ the absolute value of the difference of the two rolls and $W = XY$ (i.e., the product of X and Y).

   (a) (2 points) What are Range(X), Range(Y), Range(Z) and Range(W)?

   (b) (2 points) What are the partitions $\mathcal{A}_X$ and $\mathcal{A}_Z$?

   (c) (3 points) Give tables showing the values of f_X, f_Y, f_Z, and f_W.

   (d) (3 points) Are the events $X = 7$ and $Z = 1$ independent?

6. (6 points total) Consider the experiment of randomly shuffling n students amongst n desks, as we did in studying de Montmort's problem. Let A be the event that "neither student 1 nor student 2 gets put back in their own desk". Let X be the indicator function for A and let $Y =$ the number of students do *not* get put back in their own desk.

   - (4 points) Define a sample space for the experiment and find Range(X), $\mathcal{A}_X, f_X$.
   - (2 points) Are X and Y independent? Explain.

7. **Extra credit (4 points)** A true-false question has been posed to a husband-and-wife team on a quiz show. Each of the husband and wife will independently give the correct answer with probability $p > 0$ (and be wrong the rest of the time). Which of the following is a better strategy for answering the question correctly? (Explain your answer.)

   - Let one person (say the wife) answer the question, and ignore the other.
   - Have the husband and wife consult on their answers, and if they agree then use their answer. Otherwise guess true of false at random.