Rutgers University
CS206: Introduction to Discrete Structures II, Spring 2013
Professor David Cash

# Homework 8

***Due at the beginning of class on Monday, April 22***

**Instructions:** Point values for each problem are listed. Write your solutions neatly or type them up. Typed solutions will also be accepted via Sakai.

1. (7 points each) Solve the following recurrences using generating functions. Show all of your work. Solutions without detailed explanations will receive little or no credit.

   (a) $a_0 = 1, a_1 = 1$, and for $n \geq 1$, $a_n = a_{n-1} + 2a_{n-2}$

   (b) $a_0 = 1, a_1 = 1$, and for $n \geq 2$, $a_n = a_{n-1} + 2a_{n-2} + 4$

   (c) $a_0 = 1$, and for $n \geq 1$, $a_n = 3a_{n-1} + 4^{n-1}$

2. **(Extra credit: 5 points)** Generating functions can also be used to prove some difficult identities. Prove that
$$\binom{a+b}{k} = \sum_{i=0}^{n} \binom{a}{i}\binom{b}{k-i}$$
by finding the generating function that has the left-hand side as its $k$-th coefficient, and then showing that it is equal to the product of two generating functions and applying the convolution formula. (See Example 5 in the scanned notes for a similar example.)