Rutgers University
CS206: Introduction to Discrete Structures II, Spring 2013
Professor David Cash

Homework 5

Due at the beginning of class on Wednesday, March 27

Instructions: Point values for each problem are listed. Write your solutions neatly or type them up. Typed solutions will also be accepted via Sakai.

1. (10 points total) Two teams x and y are playing each other in the World Series, which is a best-of-seven-game match that ends when one team wins 4 games. Assume that team x wins each game with probability p, and that the outcome of each game constitutes an independent trial.

   (a) (0.5 points) What is the probability that x wins the first four games?

   (b) (2 points) What is the probability that x wins four games after at most five game have been played?

   (c) (2 points) What is the probability that x will win four games before y wins four games? (i.e., What is the probability that x wins the Series?)

   (d) (0.5 points) Calculate and simplify your answer in part (c) when $p = 1/2$ and when $p = 2/3$.

   (e) (1 point) Let X be the random variable that counts the number of games that are played. What is Range(X)?

   (f) (2 points) What is $P(X = 7)$?

   (g) (2 points) What is $P(X \geq 6)$?

2. (4 points) Suppose we roll two fair dice. Let the random variable $X =$ "the minimum of the two dice" and $Y =$ "the absolute value of the difference of the two dice". Find $E(X)$ and $E(Y)$.

3. (4 points) Suppose boxes of cereal are filled with a random prize, each drawn from independently and uniformly from 6 possible prizes. If we buy N boxes of cereal, what is the expected number of distinct prizes we will collect? Hint: Consider the indicator random variables I_{E_i} for the event $E_i =$ "the i-th price was in some box".

4. (4 points) A group of m men and w randomly sit in a single row at a theater. If a man and woman are seated next to each other we say they form a couple. (Couples can overlap, meaning that one person can be a member of two couples.) What is the expected number of couples? Hint: Use indicator random variables for each possible couple forming.

5. (3 points) Suppose an experiment tosses a fair coin twice; the experiment "succeeds" if both tosses were Heads. We repeat this experiment for 12 independent trials. Let N be the random variable that counts the fraction of trials that are successful (so $N = S/12$, where S is the number of successful trials). Find $E(N)$.

6. **Extra Credit: (4 points)** Consider the experiment where n balls are to be placed randomly into n boxes. Let N_1 count the number of boxes with exactly one ball, and let N_2 count the number of boxes with exactly two balls. Find the probability of the events "$N_1 = n$" and "$N_1 = n - 1$". Use the indicator technique to find $E(N_1)$ and $E(N_2)$.