Rutgers University
CS206: Introduction to Discrete Structures II, Spring 2013
Professor David Cash

Homework 1

Due at the beginning of class on Monday, Feb 4

Instructions: Point values for each problem are listed. Write your solutions neatly or type them up. Typed solutions will also be accepted via Sakai.

1. Let $n \geq 2$ and $A_1, \ldots, A_n$ be sets in some universe S. In this problem we will give a proof by induction of the identity
$$\left(\bigcap_{i=1}^{n} A_i\right)^c = \bigcup_{i=1}^{n} A_i^c.$$
   (a) (5 points) State and prove the base case for an inductive proof, meaning that the identity is true when $n = 2$.
   (b) (5 points) State and prove the inductive step, where one shows that the identity is true for general $n > 2$, assuming it is true for $n - 1$.

2. Give sample spaces that model the outcomes for the following experiments. You may use a regular expression or other formalisms that you find convenient. (2 points each)
   (a) Rolling 3 dice.
   (b) Rolling a die until an even result comes up, or the die is rolled three times.
   (c) Tossing a pair of coins until they both come up tails.
   (d) Draw 2 balls from an urn which contains 6 balls, each with a distinct label from $\{1, 2, 3, 4, 5, 6\}$.
   (e) Draw 1 ball from the same urn, then replace it and draw a ball again.

3. For each of the sample space, describe the events (as sets) $A \cup B$ and $A \cap B$, when A and B are as follows. (2 points each)
   (a) $A =$ "5 is rolled exactly twice" and $B =$ "dice values add to an odd number".
   (b) $A =$ "1 comes up exactly twice" and $B =$ "3 comes up exactly twice".
   (c) $A =$ "both coins come up heads at the same time at some point" and $B =$ "both coins come up tails at the same time at some point"
   (d) $A =$ "1 is drawn at least once" and $B =$ "1 is drawn twice".
   (e) $A =$ "1 is drawn at least once" and $B =$ "1 is drawn twice".