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CS206: Introduction to Discrete Structures II, Spring 2013
Professor David Cash

Homework 8

1. (7 points each) Solve the following recurrences using generating functions. Show all of your work. Solutions without detailed explanations will receive little or no credit.

   (a) $a_0 = 1, a_1 = 1$, and for $n \geq 2$, $a_n = a_{n-1} + 2a_{n-2}$

$$a(x) = \sum a_n x^n$$

$$a_n = a_{n-1} + 2a_{n-2} = 0$$

$$\sum_{n=2}^{\infty} (x^n a_{n-1} + 2x^n a_{n-2}) = 0$$

$$\sum_{n=2}^{\infty} x^n a_{n-1} + 2\sum_{n=2}^{\infty} x^n a_{n-2} = 0$$

$$x\sum_{n=2}^{\infty} x^{n-1} a_{n-1} + 2x^2\sum_{n=2}^{\infty} x^{n-2} a_{n-2} = 0$$

$$x(a(x) - a(0)) + 2x^2 a(x) = 0$$

$$x(a(x) - 1) + 2x^2 a(x) = 0$$

$$xa(x) - x + 2x^2 a(x) = 0$$

$$xa(x) + 2x^2 a(x) = x$$

$$a(x)(x + 2x^2) = x$$

$$a(x) = \frac{x}{x + 2x^2}$$

$$a(x) = \frac{1}{1 + 2x}$$

(b) $a_0 = 1, a_1 = 1$, and for $n \geq 2$, $a_n = a_{n-1} + 2a_{n-2} + 4$

$$a(x) = \sum a_n x^n$$

$$a_n = a_{n-1} + 2a_{n-2} + 4 = 0$$

$$\sum_{n=2}^{\infty} (x^n a_{n-1} + 2x^n a_{n-2} + 4x^n) = 0$$

$$x \sum_{n=2}^{\infty} x^{n-1} a_{n-1} + 2x^2 \sum_{n=2}^{\infty} x^{n-2} a_{n-2} + 4 \sum_{n=2}^{\infty} x^n = 0$$

$$x(a(x) - a(0)) + 2x^2 a(x) + 4 \sum_{n=2}^{\infty} x^n = 0$$

$$x(a(x) - a(0)) + 2x^2 a(x) + \frac{4}{1-x} = 0$$

$$x(a(x) - 1) + 2x^2 a(x) + \frac{4}{1-x} = 0$$

(c) $a_0 = 1$, and for $n \geq 1$, $a_n = 3a_{n-1} + 4^{n-1}$

$$a(x) = \sum a_n x^n$$

$$a_n = 3a_{n-1} + 4^{n-1} = 0$$

$$\sum_{n=1}^{\infty} x^n a_{n-1} + \sum_{n=1}^{\infty} x^n 4^{n-1} = 0$$

$$x \sum_{n=1}^{\infty} x^{n-1} a_{n-1} + \frac{1}{4} \sum_{n=1}^{\infty} x^n 4^n = 0$$

$$x(a(x) - a(0)) + \frac{1}{4} \frac{1}{1-4x} = 0$$

$$x(a(x) - 1) + \frac{1}{4(1-4x)} = 0$$