Rutgers University
CS206: Introduction to Discrete Structures II, Spring 2013
Professor David Cash

Homework 3

Due at the beginning of class on Wednesday, Feb 27

Instructions: Point values for each problem are listed. Write your solutions neatly or type them up. Typed solutions will also be accepted via Sakai.

1. (6 points) What the probability that a 5 card hand contains exactly 3 spades? What if we condition on the hand containing at least 1 spade?

2. (7 points) Suppose n people each throw a six-sided die. Let A_n be the event that at least two distinct people roll the same number. Calculate $P(A_n)$ for $n = 1, 2, 3, 4, 5, 6, 7$.

3. (3 points) Suppose we draw 2 balls at random from an urn that contains 5 distinct balls, each with a different number from $\{1, 2, 3, 4, 5\}$, and define the events A and B as

   $$A = \text{“5 is drawn at least once”} \quad \text{and} \quad B = \text{“5 is drawn twice”}$$

   Compute $P(A)$ and $P(B)$.

4. (4 points) In the previous problem, suppose we place the first ball back in the urn before drawing the second. Compute $P(A), P(B), P(A|B), P(B|A)$ in this version of the experiment.

5. (4 points) Suppose 5 percent of cyclists cheat by using illegal doping. The blood test for doping returns positive 98 percent of the people doping and 12 percent who do not. If Lance's test comes back positive, what the probability that he is doping? (Ignoring all other evidence, of course...)

6. (6 points) If $A \subseteq B$, can A and B be independent? What we if require that $P(A)$ and $P(B)$ both not equal 0 or 1?

7. **Extra credit (5 points)** Consider the experiment where two dice are thrown. Let A be the event that the sum of the two dice is 7. For each $i \in \{1, 2, 3, 4, 5, 6\}$ let B_i be the event that at least one i is thrown.

   (a) Compute $P(A)$ and $P(A|B_1)$.

   (b) Prove that $P(A|B_i) = P(A|B_j)$ for all i and j.

   (c) Since you know that some B_i always occurs, does it make sense that $P(A) \neq P(A|B_i)$? (After all, if E is an event with $P(E) = 1$, then for any event F, $P(F|E) = P(F)$. What is going on? Does this seem paradoxical?)