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CS206: Introduction to Discrete Structures II, Spring 2013
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Homework 7

Instructions: Point values for each problem are listed. Write your solutions neatly or type them up. Typed solutions will also be accepted via Sakai.

1. (1.5 points each) Find the generating function for the sequence $a_0, a_1, a_2, \ldots$, where a_k is each of the following. Your solution does not need to be closed form.

   (a) a_k = the number of solutions to $e_1 + e_2 + e_3 = k$, where $0 \leq e_i \leq 4$ for each i.

   $$(1 + x^1 + x^2 + x^3 + x^4)^3$$

   (b) a_k = the number of solutions to $e_1 + e_2 + e_3 + e_4 = k$, where $0 \leq e_i < 4$ for each i, e_1 is odd, and e_2 is even.

   $$(x^1 + x^3)(x^0 + x^2)(1 + x^1 + x^2 + x^3 + x^4)^2$$

   (c) a_k = the number of solutions to $e_1 + e_2 + e_3 + e_4 = k$, where $0 \leq e_i$ for each i.

   $$\left(\sum_{i=0}^{\infty} x^i\right)^4$$

   (d) a_k = the number of solutions to $e_1 + e_2 + e_3 + e_4 + e_5 = k$, where $0 \leq e_i$ for each i, e_1 and e_3 are odd, and e_2 is even.

   $$\left(\sum_{i=0}^{\infty} x^{2i+1}\right)^2 \times \left(\sum_{i=0}^{\infty} x^{2i}\right) \times \left(\sum_{i=0}^{\infty} x^i\right)^2$$

2. (3 points each) Model the following problems using a generation function, which does not need to be in closed form:

   (a) Count the number of outcomes of rolling 6 dice that sum to r.

   $$(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)^6,\ 1 \leq x_i \leq 6$$

   (b) Count the number of outcomes of rolling 6 dice that sum to r, where the first three dice are odd and the last three are even.

   $$(x_1 + x_3 + x_5)^3 \times (x_2 + x_4 + x_6)^3,\ 1 \leq x_{2i+1} \leq 3, 0 \leq x_{2i} \leq 2$$

(c) Count the number of outcomes of rolling 6 dice that sum to r, where for each i the i-th dice is not equal to i (so the first die is not 1, the second is not 2, and so on).

$$= (x^2 + x^3 + x^4 + x^5 + x^6) \times (x^1 + x^3 + x^4 + x^5 + x^6) \times$$
$$(x^1 + x^2 + x^4 + x^5 + x^6) \times (x^1 + x^2 + x^3 + x^5 + x^6) \times$$
$$(x^1 + x^2 + x^3 + x^4 + x^6) \times (x^1 + x^2 + x^3 + x^4 + x^5)$$

3. (1.5 points each) Find the following coefficients. Show your work.

(a) The coefficient of x^{10} in the series expansion of $(x^5 + x^6 + x^7 + \cdots)^8$.

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(b) The coefficient of x^{20} in the series expansion of $(x+x^2+x^3+x^4+x^5)(x+x^2+x^3+x^4+\cdots)^5$.

$$\binom{18}{14}$$

(c) The coefficient of x^{12} in the series expansion of $x^2/(1+x)^8$.

$$\binom{8+10-1}{10}$$

(d) The coefficient of x^{12} in the series expansion of $1/(1+x^3)^2$.