Batch Normalization:

Accelerating Deep Network Training by Reducing Internal Covariate Shift

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Stochastic gradient optimization in deep models

- Minimize loss over the training data

$$\Theta = \arg\min_{\Theta} \mathop{\mathrm{E}}_{\mathrm{x}\sim\mathcal{D}} [\ell(\mathrm{x}, \Theta)]$$

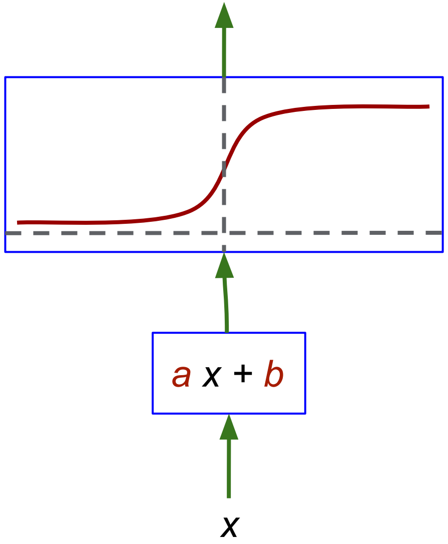
- Follow gradient for mini-batches

$$\Theta \leftarrow \Theta - \frac{\alpha}{m} \sum_{i=1}^{m} \frac{\partial \ell(\mathrm{x}_i, \Theta)}{\partial \Theta}$$

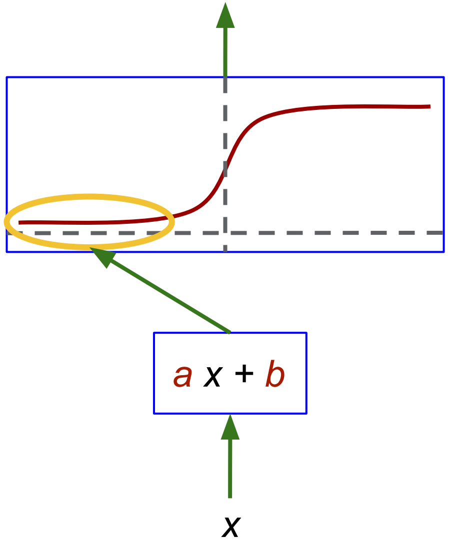
Outline

- Internal covariate shift
  - Distributions of activations in deep models change during training
  - Eliminating these changes speeds up training
- Batch Normalization
  - Normalize values using mini-batch mean and variance
  - Backprop through the transform enables gradient optimization
- Speedup >10x in ImageNet training
- Beats state of the art in ImageNet classification

Care and training of deep models



Care and training of deep models



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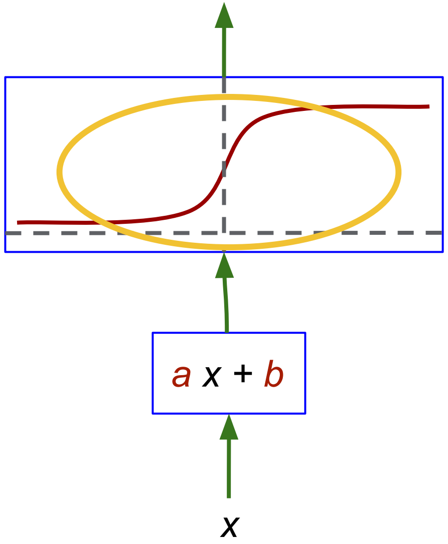
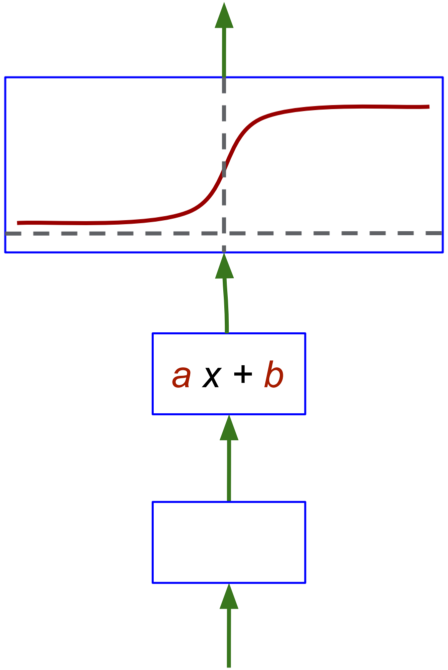
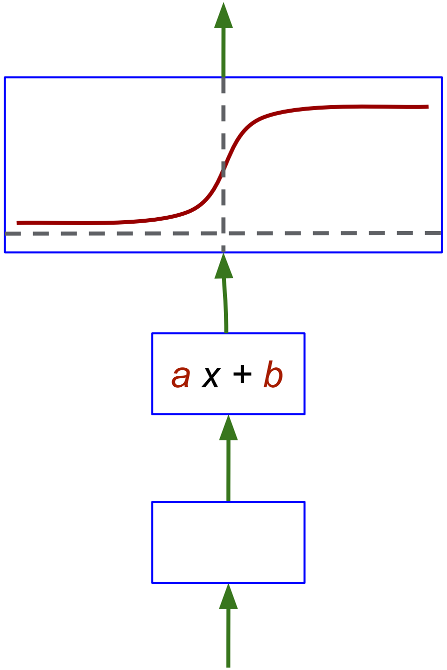


Care and training of deep models



Mitigating the effect of changing input distributions

- Careful initialization
- Small learning rates
- Rectifiers



Covariate shift

- Change in input distribution requires domain adaptation

$$\ell = F(\mathrm{x},\ \Theta)$$

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Internal covariate shift

- Layer input distributions change during training

$$\ell = F_2(\ F_1(\mathrm{u}, \Theta_1),\ \Theta_2)$$

- Change in internal activation distribution requires domain adaptation

Reducing internal covariate shift to speed up training

- Normalize each activation:

$$x \mapsto \frac{x - \mathrm{E}[x]}{\sqrt{\mathrm{Var}[x]}}$$

Normalization must participate in gradient optimization

- Mean and variance of an activation depend on model parameters
- Need $\frac{\partial \mathrm{E}[x]}{\partial \Theta}$ and $\frac{\partial \mathrm{Var}[x]}{\partial \Theta}$
- Cannot use population means and variances in mini-batch gradient optimization

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Batch Normalization

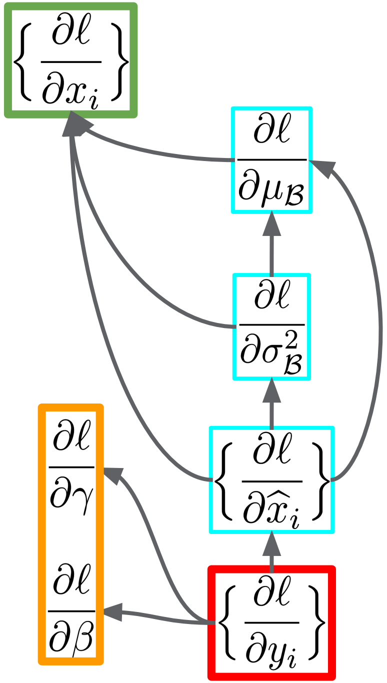
Mini-batch mean: $$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i$$

Mini-batch variance: $$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2$$

Normalize: $$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}}$$

Scale and shift: $$y_i \leftarrow \gamma\widehat{x}_i + \beta$$

Backprop with Batch Normalization



$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i$$

$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}}$$

$$y_i \leftarrow \gamma\widehat{x}_i + \beta$$

Inference with Batch Normalization

- Replace batch statistics with population statistics

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \quad \Rightarrow \quad \widehat{x} \leftarrow \frac{x - \mathrm{E}[x]}{\sqrt{\mathrm{Var}[x] + \epsilon}}$$

Batch Normalization in convolutional layers

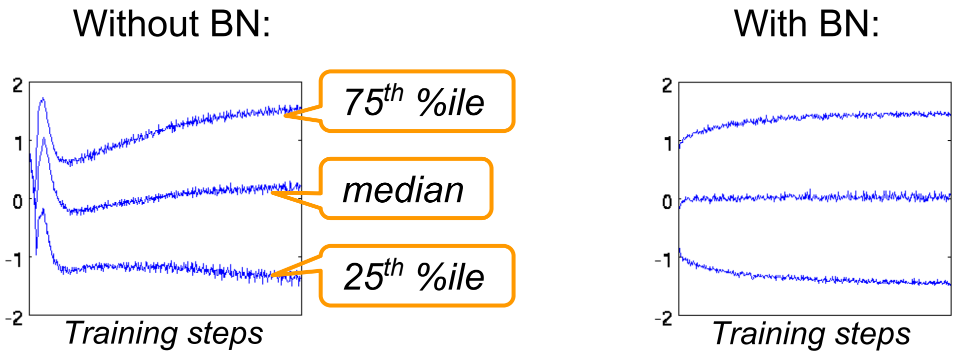
- Normalize over mini-batch examples *and* nodes
- Normalization before nonlinearity: $\mathrm{y} = g(\mathrm{BN}(W\mathrm{x}))$
    - Invariant to the scale of W

Experiments

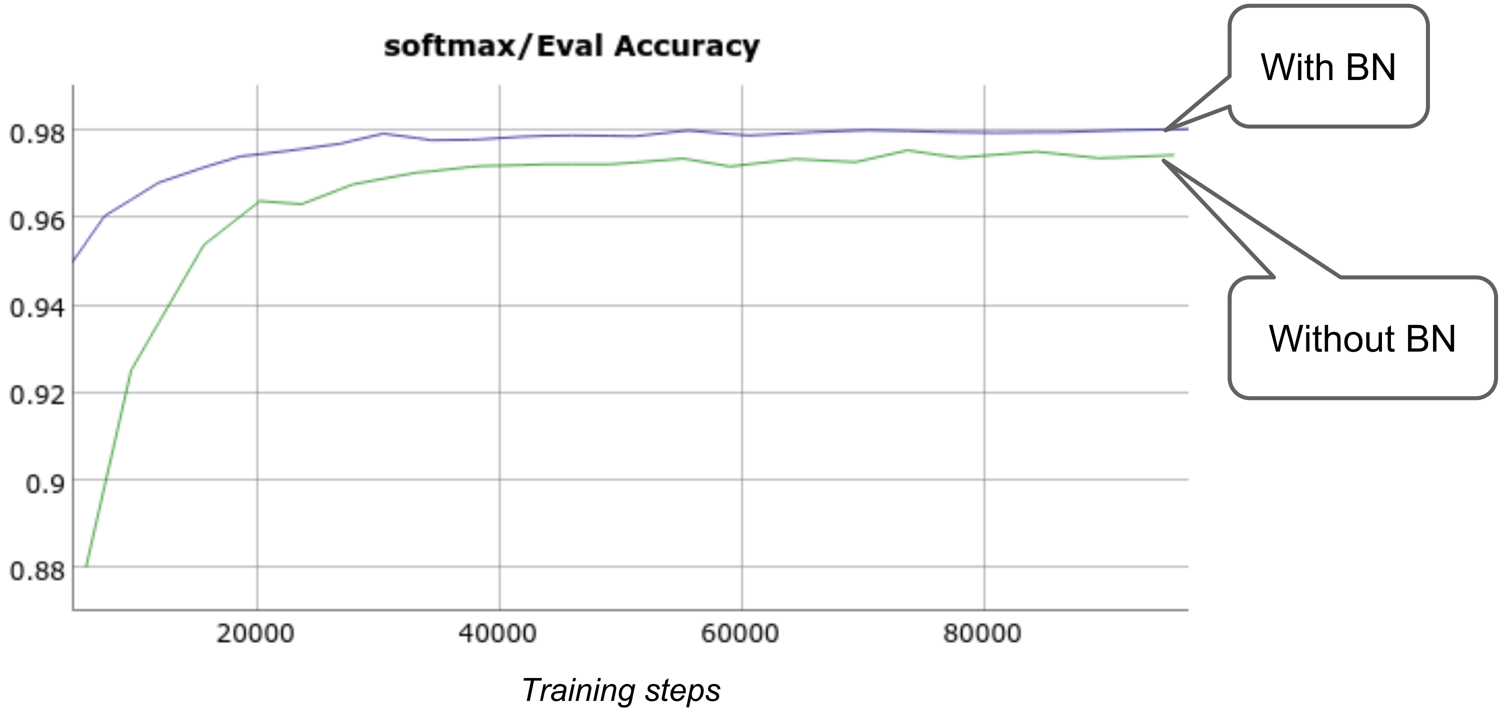
- Batch Normalization
  - reduces internal covariate shift
  - speeds up training of deep networks
  - sets state of the art in large-scale image recognition

Batch Normalization reduces internal covariate shift

- MNIST: 3 FC layers + softmax, 100 logistic units per hidden layer
- Distribution of inputs to a typical sigmoid, evolving over 100k steps:

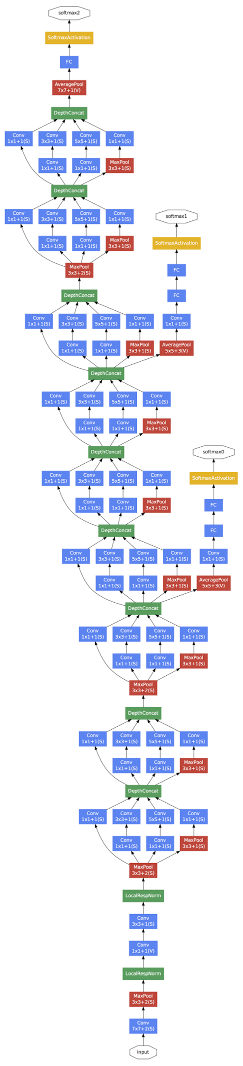


Batch Normalization reduces internal covariate shift

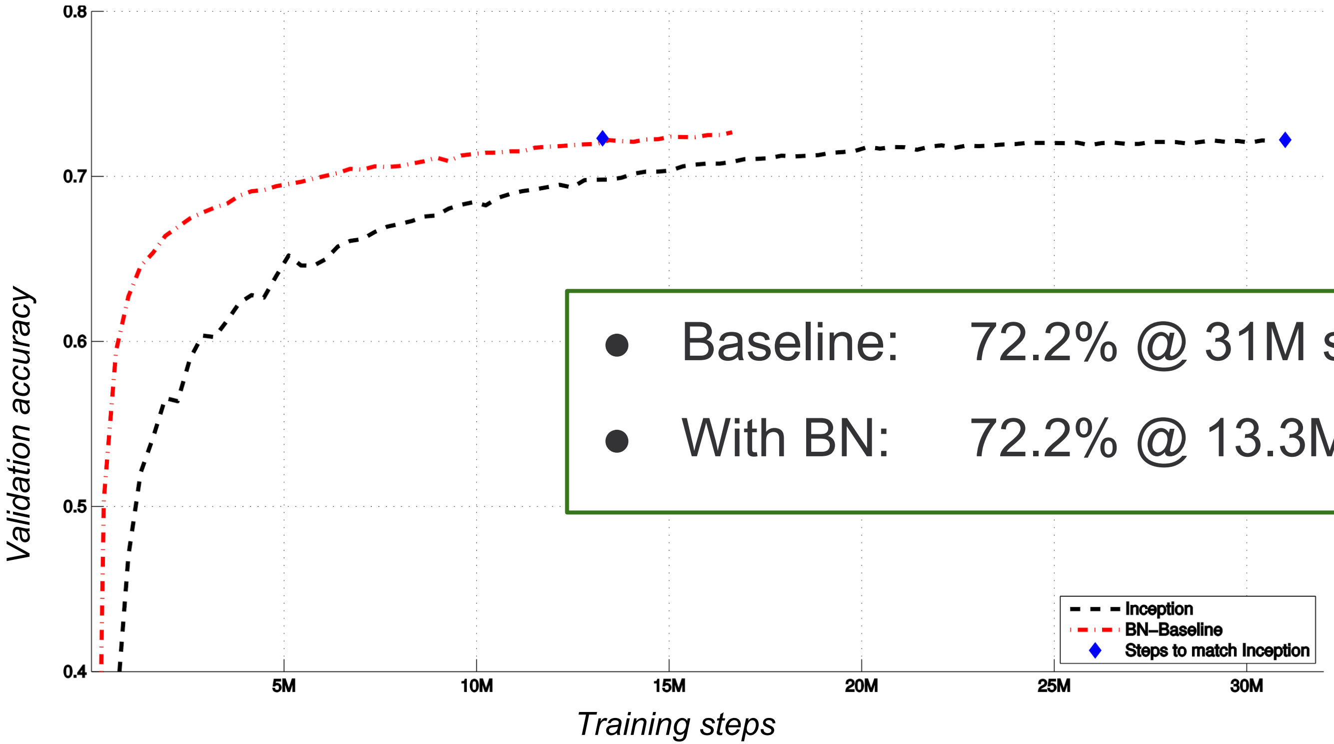


Experiment: ImageNet classification

- Inception: deep convolutional ReLU model
- Distributed SGD with momentum
- Batch Normalization applied at every convolutional layer
    - Extra cost (~30%) per training step



Inception with vs without Batch Normalization

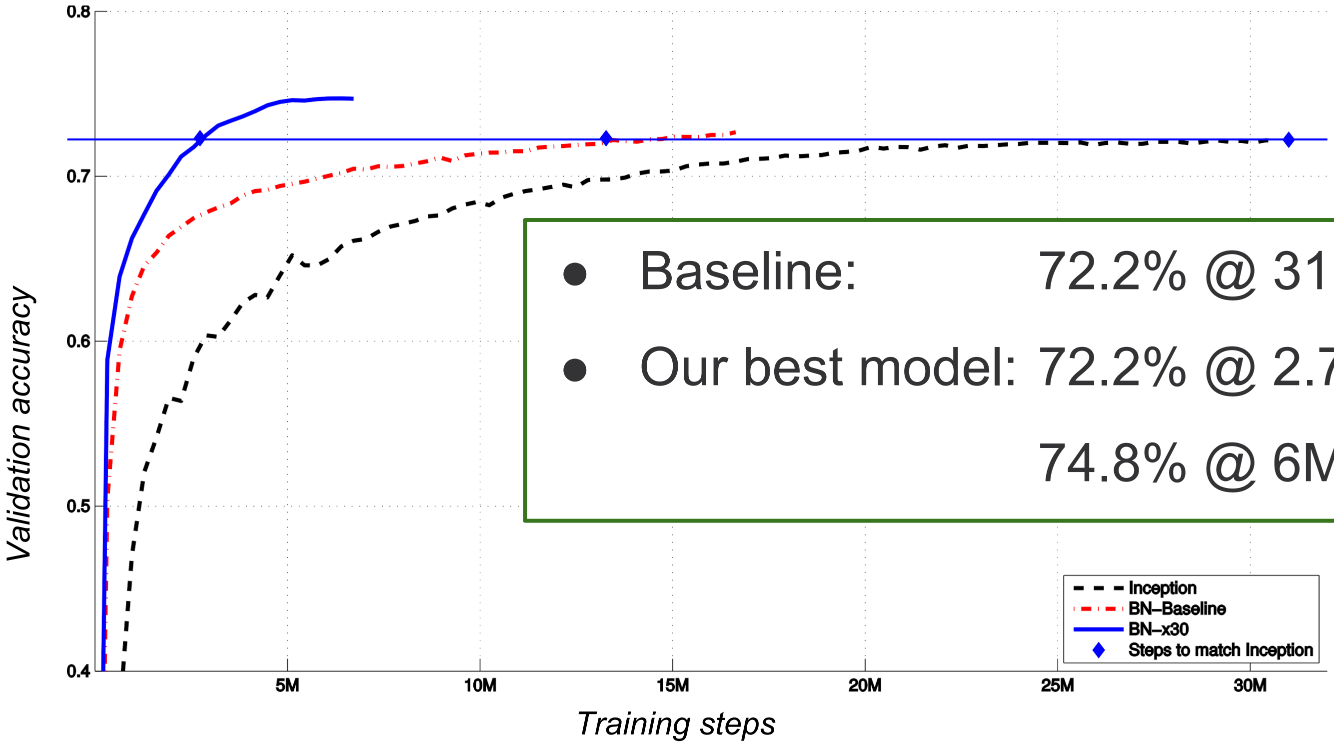


- Baseline: 72.2% @ 31M steps
- With BN: 72.2% @ 13.3M steps

Further acceleration with Batch Normalization

- Batch Normalization enables higher learning rate
    - Increased 30x
- Removing dropout improves validation accuracy
    - Batch Normalization as a regularizer?

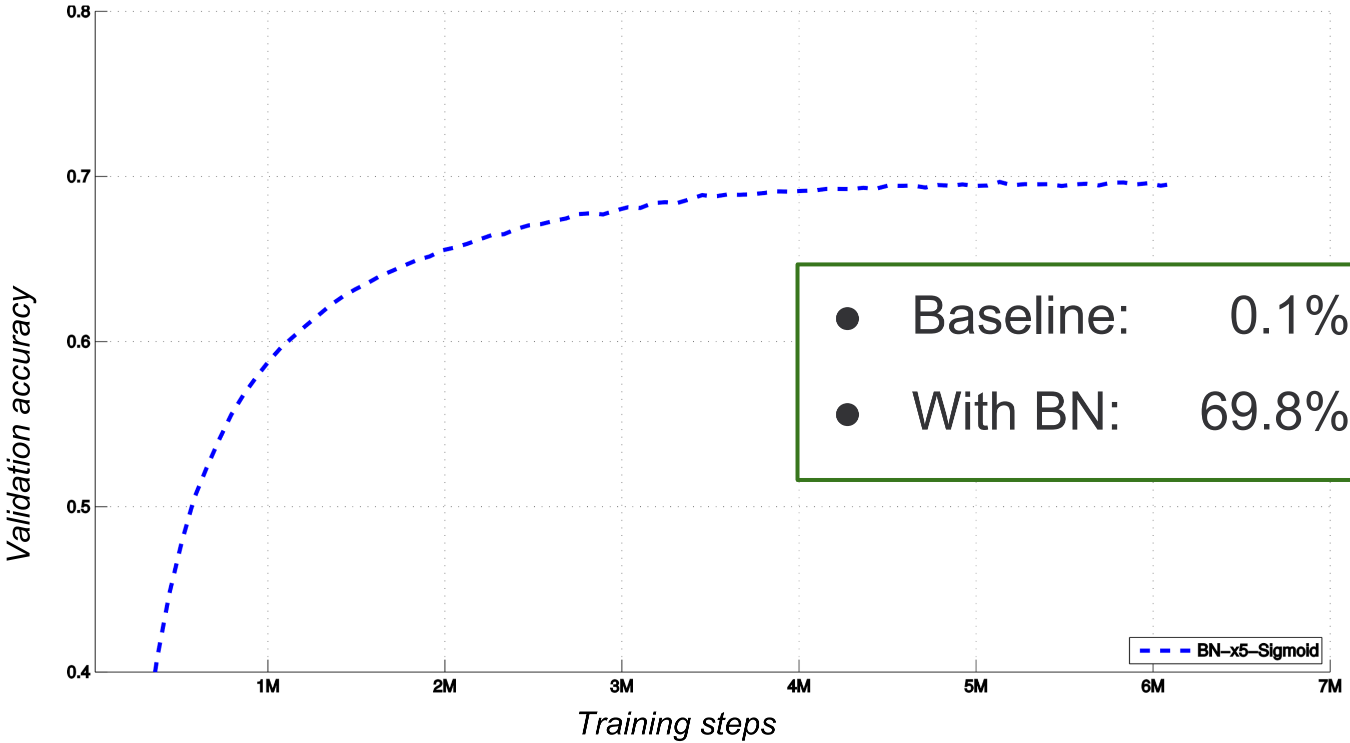
Higher learning rate, no dropout



- Baseline: 72.2% @ 31M steps
- Our best model: 72.2% @ 2.7M steps

  74.8% @ 6M steps

Saturating nonlinearities: Inception with logistic + BN



- Baseline: 0.1%
- With BN: 69.8%

Improving ImageNet classification

- Ensemble classifier
- Six Batch-Normalized Inception models
- Multi-crop, averaging over models and crops

ImageNet classification: state of the art

Model	Resolution	Crops	Models	Top-1 error	Top-5 error
GoogLeNet ensemble	224	144	7	-	6.67%
Deep Image low-res	256	-	1	-	7.96%
Deep Image high-res	512	-	1	24.88	7.42%
Deep Image ensemble	up to 512	-	-	-	5.98%
MSRA multicrop	up to 480	-	-	-	5.71%
MSRA ensemble*	up to 480	-	-	-	4.94%
BN-Inception single crop	224	1	1	25.2%	7.82%
BN-Inception multicrop	224	144	1	21.99%	5.82%
BN-Inception ensemble*	224	144	6	20.1%	**4.82%**

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Summary

- Reducing internal covariate shift speeds up training
- Batch Normalization using mini-batch mean and variance
- Preserve model expressivity
- Allows higher learning rates
- Reduces the need for dropout or careful parameter initialization
- Beats state of the art, and human accuracy, in ImageNet classification