Video Game Pathfinding and Improvements to Discrete Search on Grid-based Maps

by

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Abstract

The most basic requirement for any computer controlled game agent in a video game is to be able to successfully navigate the game environment. Pathfinding is an essential component of any agent navigation system. Pathfinding is, at the simplest level, a search technique for finding a route between two points in an environment. The real-time multi-agent nature of video games places extremely tight constraints on the pathfinding problem. This study aims to provide the first complete review of the current state of video game pathfinding both in regards to the graph search algorithms employed as well as the implications of pathfinding within dynamic game environments. Furthermore this thesis presents novel work in the form of a domain specific search algorithm for use on grid-based game maps: the spatial grid A* algorithm which is shown to offer significant improvements over A* within the intended domain.

Keywords: Games, Video Games, Artificial Intelligence, Pathfinding, Navigation, Graph Search, Hierarchical Search Algorithms, Hierarchical Pathfinding, A*.

"In theory, theory and practice are the same.

In practice, they are not."

- Albert Einstein

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This work would not have been completed without the following people:

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Chapter 1

Introduction

1.1 Problem Statement and Overview

The most basic requirement for any computer controlled game agent in a video game is to be able to successfully navigate the game environment. The navigation problem is broken down into two problems: locomotion and pathfinding [1]. Locomotion is concerned with the physical motion of a game agent while pathfinding is concerned with finding a valid route to a game agent's desired future position. The pathfinding problem is, at the simplest level, a search technique for finding a route between two points in an environment.

The pathfinding problem is not exclusive to video games and is extensively used in both the networking and robotics fields [2]. While the pathfinding problem is fundamentally similar with regards to the environmental representations and search techniques employed, the requirements and constraints placed upon the pathfinding problem within the various fields differ greatly. The real-time multi-agent nature of video games results in extremely tight constraints being placed on the video pathfinding problem. In addition to the tight constraints modern video game environments often feature a degree of dynamicity and so further increase the complexity of the overall pathfinding problem.

To the author's knowledge, to date there does not exists a complete review of the video game pathfinding field with regards to environmental representations, search algorithms employed and considerations for dynamic environments. Several works [3], [4], [5] have attempted to provide either brief introductions or overviews of the field, but in all cases are lacking in either scope or depth. This study aims to provide a complete overview of the current state of video game pathfinding as well as produce novel work on the topic.

1.2 Thesis Objectives

The main objective of this thesis is to present an in-depth review of the video game pathfinding field, with an additional focus on the various graph search algorithms employed. During this study, a domain specific problem was identified and a novel discrete search algorithm was developed. In reaching the overall thesis goals the following sub-objectives are identified:

- To provide an introduction to video game AI and the video game pathfinding.

- To discuss the representation of complex 3D game worlds into searchable graphs.

- To investigate the overall video game pathfinding problem and to identify all the requirements and constraints of video game pathfinding.

- To discuss the additional implication of pathfinding within dynamic video game environments.

- To provide an overview of the three main families of pathfinding search algorithms, namely discrete, continuous, and hierarchical search algorithms.

- To identify a solution to a specific pathfinding problem that can be improved upon, and to develop an improved solution to that problem.

1.3 Thesis Contributions

The main contributions of this thesis can be divided into two sections: the contributions to knowledge in the field as well as the products resulting from this work.

The contributions to knowledge are:

- A detailed and complete review of the current state of the video game pathfinding field.

- A review of the various environmental representations in use as well as the requirements and constraints of the video game pathfinding problem.

- A detailed overview and discussion of the three main families of pathfinding search algorithm.

- The observation that the entire continuous search algorithm family is inherently unsuitability for use within video game pathfinding.

- A discussion about the selection of a pathfinding search algorithm and environmental representation within a specific problem domain.

The products of this thesis are:

- The development of a novel, domain specific, grid-based navigational graph search algorithm: the spatial grid A*. Three variants of the spatial grid A* algorithm are presented and evaluated across five game map test sets. The spatial grid A* algorithm is shown to offer improvements of up to 90% to both search times and memory usage of the A* search algorithm within the intended domain.

1.4 Thesis Outline

Chapter 2 introduces the field of video game artificial intelligence (AI). The history of video game AI and the difference between video game AI and academic AI is discussed. An introduction to game engine architecture is presented covering the operation of, and constraints placed upon current generation video games. The role of the video game AI system within the overall game engine hierarchy is discussed. The chapter concludes with a discussion of the game agent paradigm and the role of pathfinding within a video game AI system.

Chapter 3 discusses the various techniques available to convert complex 3D environments into simple searchable navigational graphs. The three most common representations are discussed namely waypoint based, navigation mesh based, and grid based navigational graphs.

Chapter 4 discusses the video game pathfinding problem in detail. The requirements and constraints of the video game pathfinding search problem are discussed. The pathfinding search algorithm is presented as the solution to the path finding search problem, and a set of metrics used in the evaluation and comparisons of pathfinding search algorithms is presented. Advice on the selection of an appropriate pathfinding search algorithm for specific pathfinding search problems concludes the chapter.

Chapter 5 presents an overview of various discrete graph pathfinding search algorithms. The chapter presents discussions on the following search algorithms: Dijkstra's Algorithm [6], the A* algorithm [7], the Iterative deepening A* algorithm [8], the memory enhanced iterative deepening A* algorithm [9], the simplified memory-bounded algorithm [10] and the fringe search algorithm [11].

Chapter 6 presents an overview of various continuous graph pathfinding search algorithms. Continuous search algorithms are divided into three families: the incremental, the anytime, and the

real-time continuous search algorithms. The chapter presents discussions on the following search algorithms: the dynamic A* algorithm [12], the focused dynamic A* algorithm [13], the lifelong planning algorithm [14], the D* lite algorithm [15], the adaptive A* algorithm [16], the generalized adaptive A* algorithm [17], the anytime repair A* algorithm [18], the anytime dynamic A* [19], the learning real-time A* algorithm [20], and the real-time dynamic A* algorithm [21]. A discussion on the unsuitability of the entire continuous search algorithms family for use within video games concludes the chapter.

Chapter 7 presents an overview of various hierarchical graph pathfinding search algorithms. Discussions are presented on the following hierarchical search algorithms: the hierarchical pathfinding A* [22], the dynamic hierarchical pathfinding A* [23], the partial refinement A* [24], and the minimal memory abstraction algorithm [25].

Chapter 8 presents the spatial grid A* algorithm, a set of techniques for the improvement of the A* search algorithm's performance and memory usage when used to search grid-based navigational graphs. The spatial grid A* algorithm performance was evaluated on five distinct game map test sets. A detailed discussion of the performance results of the spatial grid A* concludes the chapter.

This thesis concludes in Chapter 9, with a summary of the video game pathfinding field as well as the conclusions made from the spatial grid A* algorithm's performance evaluation presented in chapter 8. Additionally, suggestions for possible future research are provided.

Chapter 2

Video Games and Artificial Intelligence

To better understand the requirements and constraints of video game pathfinding, it is beneficial to provide some background on both video games and video game artificial intelligence (AI) systems. This chapter first provides a brief introduction to video games and the video game AI. A discussion on game engine architecture follows, in order to better highlight the performance and memory constraints placed upon video game AI systems. Finally, a discussion into game AI systems, game agents, and the role of pathfinding is presented.

2.1 Introduction to Video Games

Video games are one of the most popular forms of modern entertainment, so much so that video games can be found on practically every digital platform available, ranging from mobile phones to specialized game consoles. PC gaming alone accounted for over 13 billion Dollars revenue for 2009 [26]. Mobile and social network gaming is also on the increase with over 80 million people playing Zynga's Farmville on Facebook in 2010 [27]. Video gaming is no longer a niche activity for the tech savvy and is enjoyed by people from all walks of life and technical aptitudes [28].

Video games have changed dramatically over the last 2 decades. As computing technology has grown, video game developers have taken advantage of the increased processing power available and create ever more realistic representations of virtual game worlds. For example, "Need for Speed", an arcade racing title released in 1994 featured no hardware acceleration for the visuals (commonly referred to as graphics), a very simplistic physics engine and no vehicle damage resulting from collisions. A screen shot of "Need for Speed" is shown in Figure 2.1. Since the release of "Need for Speed" in 1994, the improvement in computing technology as well as rendering technology has allowed for hyper-realistic gaming experience.

"Gran Turismo 5" is a simulation racing game released in 2010 for the PlayStation 3 and offers near-photo realistic visuals as well as highly accurate physics and damage models for the in-game vehicles. Compare a screenshot form "Gran Turismo 5" (Figure 2.2) to that of the earlier "Need for Speed" (Figure 2.1) and the advancements in video game technology over the last two decades is evident.

The video game era was kicked off by three MIT students in 1961 with their game "SpaceWar" [29]. Even though the two player spaceship battle game they created was rudimentary, it showed the potential of the computer to be more than a simple work machine. Over the next three decades, the video game industry grew at a rapid pace. Video game technology progressed hand in hand with advancements in central processing unit (CPU) and graphical rendering technology. Due to hardware limitations on storage, processing and memory of the time, early games offered simplistic pixel-art 2D graphics, pattern based computer opponents and small self-contained game levels. With the increase in computational power of each new generation of computing hardware, video games grew both in complexity and scale. As interesting as it is, a full history of video games is beyond the scope of this thesis, and interested readers are referred to [30], [31] and [32].

Visually, video games remained largely unchanged until the mid-90s. Prior to 1994, game graphics were software rendered, meaning that the CPU was responsible for drawing all the game visuals. Software rendering was expensive and consumed the bulk of the CPU time available to a game. This meant that very little time remained for other gameplay systems such as physics or AI. With the processing power available at the time, it was impossible to render visually appealing 3D graphics at any sort of acceptable animation frame rate so games remained primarily 2D sprite-based affairs. 2D graphics accelerator cards such as S3's Virge series were developed and released but while these cards improved the speed of 2D rendering, 3D rendering was still unpractical. The greatest impact in video game technology came in 1996 with the release of 3DFX's Voodoo Graphic Chipset. This add-in card was designed with a single purpose in mind: 3D graphics rendering. The Voodoo graphics card allowed for all 3D graphical calculations to be moved off of the CPU, thereby freeing the CPU for other gameplay related tasks. In addition to allowing game developers the ability to offer 3D visuals, 3D acceleration allowed for more complex gameplay mechanics and AI due to the reduced CPU time needed for the graphics rendering systems. The Voodoo graphics card ushered in the age of 3D acceleration and 3D graphics.

Figure 2.1: A screenshot from "Need for Speed" (1994)

Figure 2.2: A screenshot from "Gran Turismo 5" (2010)

Since the release of that first 3D graphics card, graphics card technology has improved exponentially. Modern graphics card chipsets now contain multi-core processing units, offering significantly more processing power than even the fastest desktop CPU available [33]. These graphics card chipsets are termed graphical processor units (GPU). Graphics cards are no longer simply restricted to graphical calculations and are now used for general purpose computing, referred to as general-purpose computing on graphics processing units (GPGPU) [33]. With all this extra processing power available to the game developer, the CPU is further freed, allowing for more complex gameplay and AI systems.

As an industry, game development is a highly innovative and competitive one, with game developers often working on the cutting edge of both 3D graphics and human level AI. This high level of competition and innovation has bred a high level of expectance from the consumers. With each new generation of games released, game players (gamers) expect, if not demand, ever higher degrees of realism from the visuals, sound and most importantly the AI of the in-game characters and opponents; in-game characters and opponents are referred as non-playable characters (NPCs). As a result of the high level of technology usage and player expectations, today's video games offer complex game mechanics, realistic Newtonian physics engines, detailed 3D environments, and hordes of computer controlled opponents for players to battle against.

Video games are no longer the simplistic kids' games of yesteryear and it has become more accurate to describe modern video games not just as a form of entertainment but rather as a highly detailed real-time simulation. This paradigm shift has fueled the recent academic trend of using commercial game engines as readily available, easy to use tools for the rapid visualization of real-time simulations [34], [35]. In addition, there have been calls for the increased use of interactive video games as test beds for the research, testing, and development of human-level AI systems [36].

2.2 Video Game Artificial Intelligence

Video game AI had humble beginnings. Prior to the 1970s, video games were mainly two-player games serving as a platform where two human players compete against one another. It was only in the 1970s that games began to include computer controlled opponents, for the first time allowing only a single player to play the game. These early opponents were very simple and their movement was pattern-based, meaning that they moved in a predefined hardcoded pattern [3] [37]. A famous

example of such pattern-based opponents is the game "Space Invaders". In "Space Invaders", the player controlled a space ship and was required to destroy all the alien invaders on the screen. The aliens were controlled by a simple pattern that moved them back and forth across the scene and nothing more. In 1980, Pac-Man was released by Namco and was one of the first games to feature computer controlled opponents with distinct personalities.

A screenshot from Pac-Man is shown in Figure 2.3. The game was rather strangely, an eating game in which an oddly shaped hero (Pac-Man) was stuck in a two dimensional maze. The game had a number of levels, each of which contained a different maze. To complete a level, Pac-Man was required to consume the various dots and fruit that lay scattered about the level. To make things interesting, the game also included four "ghosts" that chased Pac-Man around the maze. If a ghost caught Pac-Man, the game ended.

Figure 2.3: A screenshot from Namco's PACMAN (1980)

The game developer realized that if each ghost simply follows the player, the player would constantly be followed by a single-file line of ghosts. By having the "ghosts" cooperate with one another to try and trap the player, the game became more difficult and more interesting. This inter-ghost cooperation was implemented by giving each "ghost" its own distinct AI controlled personality [38]. In doing so, Pac-Man became one of the first and most well-known examples of video game AI. Pac-man is also notable for being one of the first video games to make use of a finite state machine (FSM) in controlling a game character's behavior [3] and since then FSMs have ended up becoming ubiquitous in the field of video game AI. The Pac-Man AI was a very popular example for the use of FSMs and has gone on to become a recommended test problem for AI courses teaching FSM systems [39].

2.2.1 Human Level AI

To date, there is no agreed upon definition of what artificial intelligence is. The definition provided in [40] states that "AI is the study of systems that act in a way, that to any observer would appear intelligent". This definition describes the video game AI field perfectly. The main objective of game AI is to create game opponents that seem as human as possible, leading to the term: human level AI. As there is no metric to measure the "humanness" of an AI, it is necessary, in order to create a more concrete list of requirements for a game AI, to briefly delve into the psychology of a game player as well as the act of playing a game.

There are various reasons why people enjoy playing games, but the simplest reason and the most obvious is that games are fun [28], [41], [42]. The enjoyment of a game (or fun) is derived from the challenge offered to the player and the sense of achievement the player gets from completing the game objectives or defeating the in-game opponents. Most video games feature some type of AI controlled NPCs which can take many forms, both friend and foe. Gameplay mechanics are often centered on defeating all the opponents in a game level before being allowed to proceed to the next. The gameplay of such games is entirely reliant on the high degree of interaction between the player and the various AI controlled opponents; the overall quality of the game becomes dependent on the quality of these human-AI interactions. There is a correlation between the level of difficulty of a game and the level of enjoyment of that game by players [43]. The vast majority of games offer user-adjustable difficulty levels so as to be able to cater for a large audience of people with different levels of skill. Adjusting the difficulty level usually means adjusting the behavior of the AI opponents; often decreasing their reaction times and allowing them to make more mistakes.

In games featuring a high level of player-AI interaction, a successful NPC is expected to respond to game events in a human-like manner. While humanness is a difficult quality to quantify, the idiom "to err is human" is apt and so the simplest method by which to simulate human behavior is to have an imperfect (or purposely flawed) AI [44]. This requires that the decision making process as well as other fundamental game AI systems be capable of returning sub-optimal results.

It is important to note that even though players expect intelligent AI systems, the players are aware that the actions of AI controlled NPCs will not be perfect (or in some cases will be perfect, resulting in inhuman actions) and as such will allow a certain degree of error on the AI's part. As it stands, it is impossible to fully simulate a human opponent, and while developers are not expected to succeed in that task, it is critical to ensure that any highly noticeable AI behavioral errors, such as navigation mistakes, are kept to a minimum. Players may not notice that an NPC's aim is consistently inaccurate and sometimes may even consider that a feature. However as soon as the NPC does something obviously incorrect, for example attempting to navigate through a minefield even though there is a safe route available, then the human-like illusion is shattered.

2.2.2 Comparison to Academic and Industrial AI

Video game AI differs greatly in both requirements and constraints to other prominent industrial and academic AI fields such as computational intelligence (CI) and robotics [3]. In most AI fields the goal is to create a "perfect" and specialized AI for a specific problem or operation, an AI that is faster, more efficient and more accurate than a human could ever be. As these industrial and academic fields do not require the versatility and adaptability of humans, these specialized AIs are usually easier to develop [36] [2]. In stark contrast, a video game AI does not need to solve a problem perfectly but rather needs to solve it to the satisfaction of the player, entailing that the AI often needs to be both intelligent and yet purposely flawed.

An intelligent and yet purposely flawed AI can present the player with an entertaining challenge, and yet still be defeatable through the mistakes and inaccuracies of its actions [44]. For example, for the famous chess match between IBM's Deep Blue Chess Computer and Gary Kasparov in May 1997 [45], Deep Blue's only goal was to defeat the chess grandmaster without care of whether the opponent was enjoying the game or not. Its only concern was calculating the optimal next move for the current situation. Game players want to enjoy playing a game and playing against an opponent that never makes a mistake isn't much fun.

A good example of the necessity of imperfect opponents is a commercial chess game. Such a game features flawed opponents of various difficulty levels and various personality traits: some opponents being more aggressive than others, some will sacrifice pieces to achieve victory while others will do everything possible to avoid losing a piece, and so on. It is these personality traits that result in imperfect decisions, making playing human players interesting and fun. Players can still enjoy a game even if they lose, as long as they feel that they lost due to their own mistakes and not due to an infallible opponent. In the case of a 'perfect' game AI, such as "Deep Blue", there is no incentive for players to play the game since they realize that they can never win. Game developers realize this and go to great lengths in attempting to simulate human behavior (including flaws) as closely as possible [46].

A further distinction between video game AI and other AI fields exists in both the type and the quality of environmental data available to the AI, as well as the processing and memory constraints within which the AI is required to make decisions. Due to the virtual self-contained nature of the game world, video game AIs have perfect knowledge of their environment, whereas in other fields AIs are designed specifically to deal with and adapt to unknown, incomplete or even incorrect data. The fact that complete knowledge about the environment is always available to a game AI makes it all too easy to write a perfect AI, therefore much care must be taken to allow the AI the ability to make mistakes in such circumstances. Furthermore, in having perfect environmental knowledge at any given time, NPCs need not have full sets of sensors (discussed in more detail in Section 2.4.1).

In regards to processing and memory constraints, video games are executed on a single CPU. The CPU therefore needs to be shared amongst the various processor intensive systems that make up the game engine (discussed in Section 2.3). In sharing the CPU across the various game engine systems, the game AI is only allocated a small fraction of the total processing time [3]. In comparison, in an academic or industrial setting, AIs are usually run in isolation, on dedicated hardware having a much larger allocation (if not all) of the CPU time. Furthermore, game AI systems are often tasked with controlling numerous NPCs simultaneously, unlike academic and industrial AI systems which are usually tasked with controlling only a single agent. All these factors combined tend to result in much looser performance and time constraints being placed on academic/industrial AI systems than on game AI systems [3]. The performance and memory constraints placed upon game AI systems are further discussed in Sections 2.3.4 and 2.3.5. These tighter processing and memory constraints limit the applicability of certain academic techniques and algorithms to game AI systems.

2.3 Game Engine Architecture

The architecture and design of a game engine is, in essence, a software engineering problem and even though this thesis is focused on game AI systems, specifically pathfinding, it is necessary to go into detail regarding some of the low-level components, architecture and basic operation of a game engine to better highlight the various design requirements and performance constraints introduced by the game engine environment. The following subsections discuss the need for a game engine, basic game engine architecture, as well as the constraints and requirements introduced by the game engine.

2.3.1 The Game Engine

As CPU and GPU technologies have grown, so has the complexity of video games. In the past, video games were written by a single engineer with a small budget, over the course of a few months. Today's video games involve hundred man teams, multimillion dollar budgets and take several years to develop [47]. Modern games are complex software engineering projects with codebases in excess of a million lines of code [48] [47]. The massive scale of today's games makes writing a game from scratch infeasible from both an engineering and budgetary standpoint. Game developers will often reuse key components from one game to the next, saving developers from having to rewrite large parts of the common game systems. The reuse of existing codebases in the development of a game will save both time and money, and so is an absolute necessity.

A game engine can be thought of as a framework upon which a game developer builds a game. A game engine will have built-in libraries for common tasks such as math or physics as well as for rendering. Game engines allow developers to focus on creating the game and not the technology behind the game. Game engines allow developers to quickly prototype game features without spending months re-engineering the wheel, as well as offering the developers prebuilt editing and art asset tools for the engine [49]. These prebuilt tools further reduce the total engineering time spent during development. The licensing of third-party game engines (and other game components such as physics and animation packages) has become big business. As a result, several leading game development studios such as "Epic Games", "Crytek" and "Valve Corporation" have shifted their focus into the development of game engines as a critical part of developing the actual games themselves. This presents such studios with an additional revenue stream, after the release of their games, by licensing their game engines to other third-party developers. Certain companies only focus

on the development of game engines and game libraries (termed middleware) with the goal of licensing their product to third-party clients. These companies' entire income is dependent on the licensing of their code bases. Popular middleware companies include "Havok", "Emergent Game Technologies", and "RAD Game Tools" [50].

Another popular development strategy is to build a custom in-house game engine with the intent of reusing it for several upcoming titles [49]. While this is large investment in terms of both time and money, game studios usually produce games of a single genre. Having a game engine that is custom built for a specific genre ends up more efficient in the long run due to in-house engine familiarity and specialization of engine features.

Game engines only appeared in the game development scene in the mid-1990s. Prior to this, games were simple enough to be engineered by a handful of developers in a relatively short period of time [47]. This all changed in 1993 with id Software's groundbreaking first person shooter: "DOOM". A screenshot of DOOM is shown in Figure 2.4.

Figure 2.4: A screenshot of id Software's "DOOM" (1993)

DOOM was one of the first games that had a well-defined separation of its core software components and the game assets (art, levels, etc.) [47]. This allowed the developers to work on the core game systems without the need for game assets from the art department. Another major benefit of the content separation of id Software's DOOM engine was that it allowed the studio to reuse the engine in the creation of the game's sequel DOOM II.

The prebuilt engine meant that the creation of DOOM II required relatively little effort, resulting in DOOM II being released less than a year after the original DOOM. DOOM is considered to be one of the first examples of a true game engine [47]. id Software went on to further pioneer game engine technology with their "id Tech" series of game engines (the DOOM engine being the first, termed "id Tech 1").

2.3.2 Basic Engine Architecture and Components

Game engines are built in layers with a high level of interconnection between the layers [47]. A simplified model of the layered architecture of a modern game engine is given in Figure 2.5. The lowest three layers of a game engine are concerned with the hardware interfaces between the game engine and the target platform's hardware.

These bottom three layers grouped together are referred to as the hardware abstraction layer (HAL). The developer has no direct means of interaction with the HAL. All interaction with the HAL occurs through the use of third party platform software development kits (SDK) such as the Microsoft Windows SDK and DirectX SDK. These platform SDKs interface directly with the HAL and allow the developer standardized access to the base platform hardware.

Game developers are often required to target multiple platforms for their games and a further requirement of a modern game engine is cross-platform compatibility. Each platform the game engine is required to run on will feature its own SDK. This means that the third party SDKs will often change across platforms, requiring that the game engine includes a layer to wrap the underlying SDKs into a standardized interface that is used by the higher level systems across all platforms. This wrapper layer is referred to as the platform independence layer. The platform independence layer allows developers to simply exchange the layer with an appropriate one when porting the game across platforms without the need to modify any of the higher level systems.

The next layer in the game engine hierarchy is the core systems layer. This layer contains all the low level utility libraries used by the game engine such as memory allocation and math libraries. Most of the game engine systems are built around these core libraries.

The game engine systems layer is the core of the engine and determines the overall feature set of the game engine. The game engine layer is responsible for all the resource management and scene management systems as well as all collision detection, physics, user interface and rendering systems. The game engine systems layer contains interfaces to the various components contained within it and these interfaces are exposed to the game specific systems layer.

The game engine systems layer can be thought of as nothing more than a collection of black boxes awaiting instructions.

Overall, the game engine is a basic framework, where the developer simply fills in the gaps. This "gap-filling" occurs within the game specific systems layer. The game specific systems layer contains and manages all the actual game rules and mechanics, as well as the game AI system. Although the game AI system's overall architecture is similar across games and can be thought of as a core system, the AI system is highly data driven and the system's behaviors and functions are determined by both the game type and the game data available. The game AI system is specific to the actual game being developed and cannot be as easily generalized as other core systems, such as scene management or physics.

2.3.3 Game Engine Operation and the Game Loop

When a game is executed, the first thing that needs to occur is the initialization of the game engine. The engine therefore enters an initialization stage. The game engine initialization stage is responsible for initializing all the core game engine systems as well as loading all constant game resources (found across all the game levels).

Normally, after the game engine initialization completes, the player is presented with a menu from which various game options can be selected, e.g. to begin a new game, continue a saved one, or to change game settings. Once the player has made a selection that triggers the start of the game, the game engine enters the level initialization stage. The level initialization stage is responsible for loading all the relevant level data and resources as well as resetting any engine systems that require re-initialization per game level.
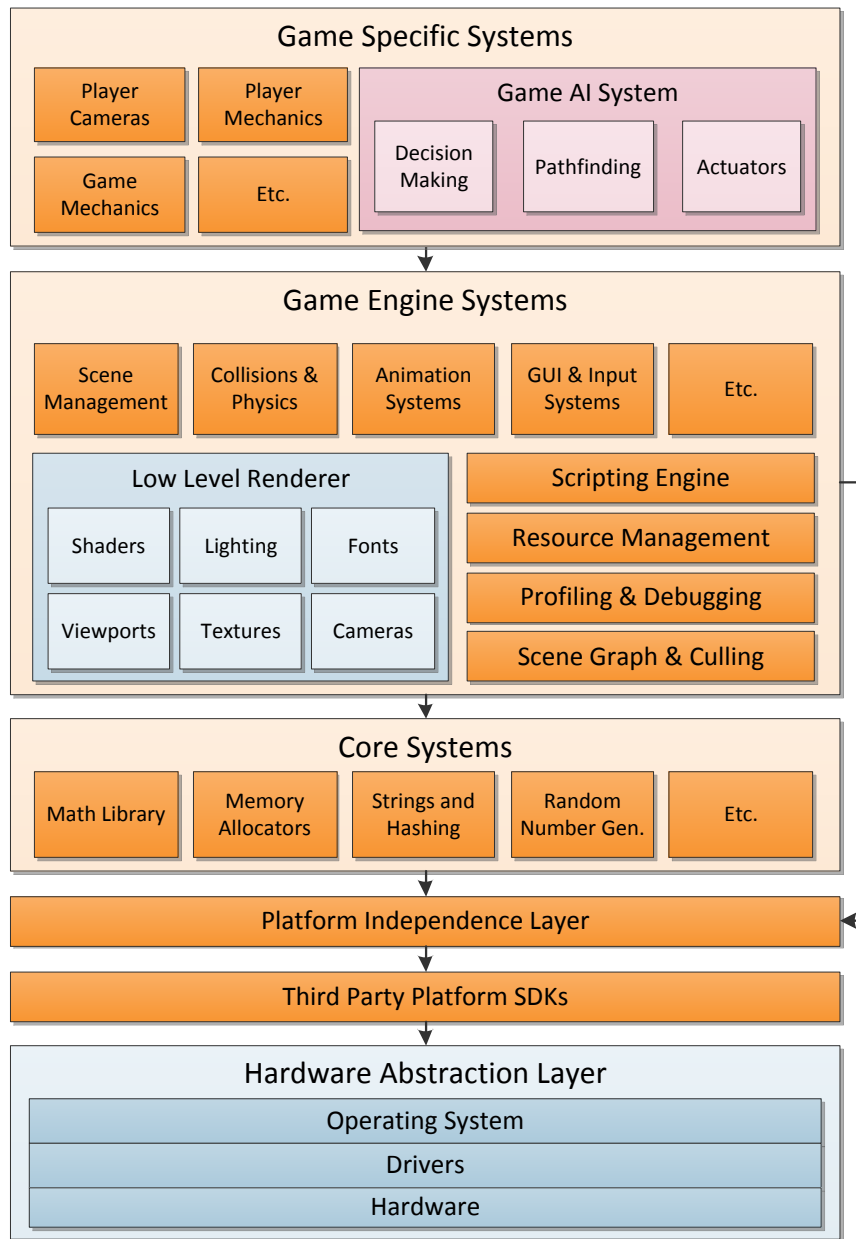
Figure 2.5: A high level overview of the various game Engine Systems

The game engine features two shutdown stages: the level shutdown stage and the game engine shutdown stage. The level shutdown stage is entered each time the game transitions from level to level, and is responsible for releasing all level data and resetting any level specific core systems. The game engine shutdown stage releases all game resources and shutdowns all game engine systems. A simplified flowchart of a game engine operation is shown in Figure 2.6. For simplicity's sake, all menu and interface updates have been left out of the flowchart.

Once the level initialization stage completes, the game engine enters a tight loop which is responsible for the runtime operation of the game: the game loop [48] [47]. The game loop consists of three stages:

- **The user input stage:** This stage handles all user input and modifies the game state accordingly.

- **The simulation stage:** The simulation stage is where the game is run. The simulation stage is responsible for all AI, physics and game rule updates. The simulation stage is often the most computationally expensive stage of the game loop [47] [48].

- **The rendering stage:** The rendering stage prepares all game objects for rendering, after which the game objects are sent to the renderer to be drawn to the frame buffer. After this stage completes, the frame is displayed on the monitor and the game loop restarts.

Each game loop iteration generates a single frame to be displayed. The time needed for a single iteration of the game loop to complete determines the number of frames that are generated per second. This is referred to as the frame rate of the game, and is measured in frames per second (FPS).

2.3.4 Performance Constraints

Video games are primarily visual entertainment, therefore a larger focus is given to the game's visual fidelity. With the improvements in central processing unit and graphical processing unit technology, developers have the ability to render near photo realistic images albeit at a higher processing cost. The bulk of the total processing time is allocated to the graphics rendering and scene management systems.

To maintain smooth animation and provide a good gaming experience, games are required to run at a worse case minimum of 30 frames per second (FPS) [51]. A minimum frame rate of 60 FPS is recommended to provide a truly smooth experience [52]. These frame rate limits allow a maximum processing time of 33ms per iteration of the game loop at the minimum playable frame rate of 30FPS; only 16ms per iteration of game loop is available at 60FPS.

Game Loop

Game Initialization

Level Initialization

Process User Input

Simulation
- Perform AI Updates
- Perform Physics Updates
- Perform Collision Detection
- Etc…

Rendering
- Prepare Game Objects for Rendering
- Send Rendering Data to Renderer
- Render GUI
- Etc…

Level Shutdown

Game Shutdown
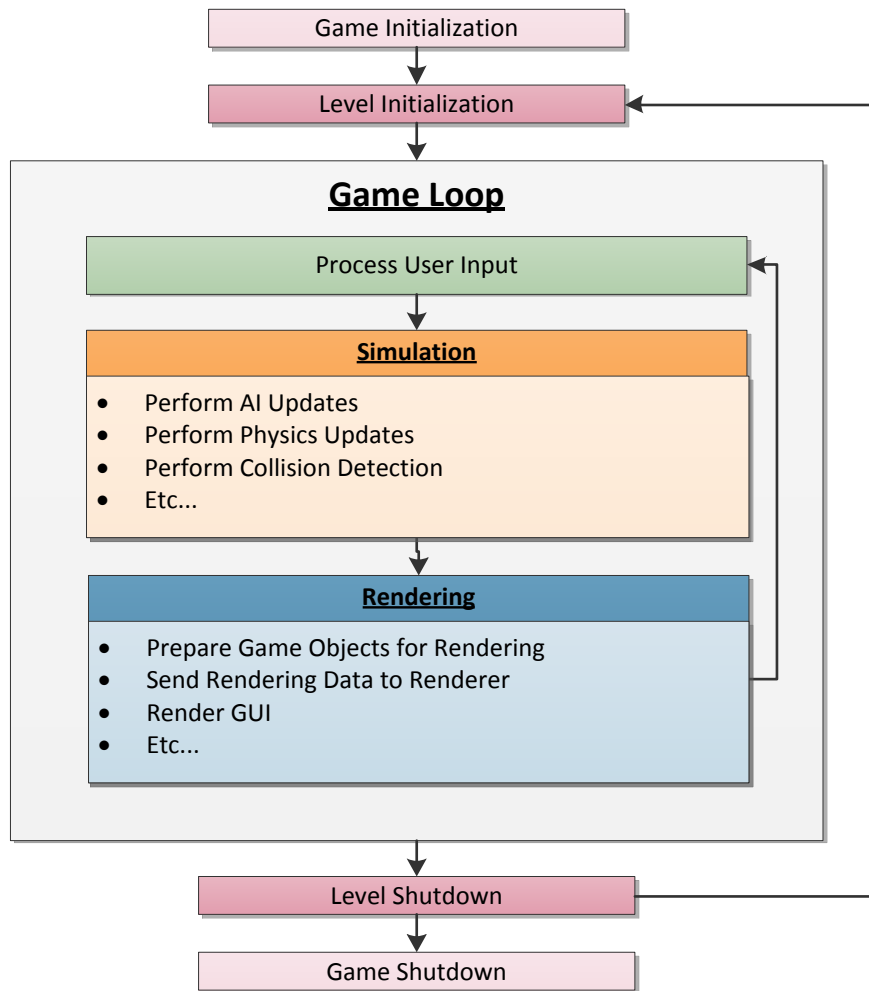
Figure 2.6: A generalized flowchart representing the runtime operation of a game engine.

Although the majority of the available processing time is allocated for the visual rendering of the scene, recent GPU developments offloaded much of the rendering computational workload of to the GPU resulting in more processing time available to AI developers than ever before [3]. Even with this increased processing time, AI systems are only allocated around 10% to 20% of the total processing time per frame [44].

The 20% of the processing time allocated is further divided amongst the various AI sub-systems such as perception, pathfinding, and steering. Considering a game with dozens, if not hundreds of AI controlled NPCs all requiring behavioral updates and/or pathfinding, the 6.7ms (at an optimistic 20% of the frame time) of processing time available at 30FPS puts some very tight performance constraints on the AI system.

2.3.5 Memory Constraints

Since video games are often targeted at multiple platforms ranging from PCs to mobile phones, cross platform compliance is an essential feature of a modern game engine. Each of these platforms has different hardware specifications and limitations.

Whereas on the PC platform large memories are ubiquitous, such is not the case for home consoles, and when developing for console platforms memory becomes a major concern. Valve Corporation's Monthly Steam Hardware Survey [53] shows that 90% of PC gamers have more than 2GB of RAM available, with 40% of gamers having more than 4GB.

In addition to primary system memory, the PC platform has separate memory dedicated for the graphical operations performed by the GPU (known as video memory); 70% of gamers have more than 512MB of such memory available. This is in stark contrast to the current generation consoles: Microsoft's XBOX360, Sony's PlayStation 3 and Nintendo's Wii which only have 512MB, 256MB and 88MB of shared memory (used in both primary and video memory roles) respectively [54] [55].

In discussing modern animation systems, [55] examines the memory implications of a simple animation system for a five character scene. The memory cost of the animation data alone accounts for over 10% of the total available memory on an XBOX360. Once 3D model data and bitmap textures are included into the memory usage, as well as the necessary game engine and physics data, very little memory remains for AI and pathfinding systems. Allocating 10MB on the PC platform may not be a problem, but that 10MB may simply not be available on a console platform. In catering for memory limited platforms such as the Wii, game engine developers often put hard limits on the memory usage for specific systems [56].

Due to the large memories available on PCs, the memory constraints of games only targeted at the PC platform can be loosened to a large degree [57]. Unfortunately, as mentioned, video game titles are often released for multiple platforms simultaneously, meaning that the platform with the lowest hardware specifications will set the processing and memory constraints for the game. Pathfinding memory budgets on consoles are on average around 2 to 3MB. Brutal Legend, a console RTS, uses around 6MB of memory for its entire pathfinding data storage while a console based FPS may use as little as 1MB or less [58] [57].

2.3.6 The Need for Concurrency

Even though CPU technology continues to grow at a rapid pace, single core performance has stagnated. In the past, developers were guaranteed that their game's performance would be automatically improved with each new generation of processors as a result of the increase in processor clock speeds. This increase in clock speed has saturated due to CPU manufacturing limitations and CPU manufacturers have opted to increase the number of processing cores instead of increasing the clock speed for new CPUs.

Multi-core processors are now prevalent in everything from low performance netbooks to high-end workstations, as well as in all current game consoles. Unfortunately, simply adding additional cores to a processor does not translate into an automatic performance improvement. To make full use of all the processing power available in a multi-core CPU package, game engines (and their components) need to be designed with concurrency in mind [59].

Concurrency is essential for all modern high performance applications, not just game engines. While all the complexities involved in architecting a concurrent game engine are beyond the scope of this work, they cannot be overlooked when designing a game AI system. A game AI system will reap large benefits from either the simple parallelization of the workload or from running constantly in a secondary processing thread [60] [61] [33].

A concurrent approach to software architecture and algorithms is absolutely essential in this day and age [59] [60], and so needs to be taken into account when designing the AI system architecture.

2.4 The Video Game Artificial Intelligence System

The video game AI system (or simply the game AI system) is part of the game specific layer of a game engine (refer to Figure 2.5). The game AI system is tasked with controlling all NPC present in the game. In doing so the game AI system is responsible for all the decision making of game NPCs as well as executing the decisions made by the NPCs.

The requirements of a game AI system differ greatly across the different game genres, leading to the design of numerous specialized AI architectures to deal with the distinct requirements of each genre [44] [37]. As varied as these architectures are, there is a common high level architectural blueprint present across all the various architectures: AI controlled NPC behaviors are a function of their environmental input.

Since the game AI system is responsible for controlling NPCs, to properly discuss the requirements, constraints and architecture of a game AI system, it is first necessary to discuss how NPC's decisions are made and executed.

This section will provide background on the design and architecture of a game AI system. Subsection 2.4.1 discusses the concept of a game agent as well as game agent architecture. Subsection 2.4.2 discusses the asynchronous task-based updating of game agents. Subsection 2.4.3 discusses the overall game AI system architecture as well as the need for multi-frame distribution of the overall AI processing costs. A discussion on the role of pathfinding within a game AI system concludes the section.

2.4.1 Game Agent Architecture

The AI agent paradigm presented in [2] is critical to the description of the high level AI architecture of these NPCs. In accordance with the agent paradigm, NPCs are renamed "game agents" or simply "agents". An agent is defined as "anything that can be viewed as perceiving its environment through sensors and acting upon the environment through its actuators". A generalized game AI agent architecture has three main components: the agent's inputs (or sensors), the agent-function and the agent's actuators. Figure 2.7(a) illustrates the generalized game agent architecture.

The sensors (commonly referred to as an agent's perception system) act to inform the agent about the state of its environment (the game state). Game agent sensors are not actual sensors but are simple functions that simulate the various agent senses needed to react to the environment [44] [62]. Since perfect information regarding the environment is available, agent sensors are specialized with regards to the information required by the agent-function. For example, if an agent's vision sensor is only used for target selection then that vision sensor will simply return a list of the nearby enemies to which the agent has a line of sight.

The agent-function can then determine the best target from the sensor data. An agent's sensor data represents the current game state and is used by the agent to make decisions as to the agent's future actions. These decisions are made by the agent-function.
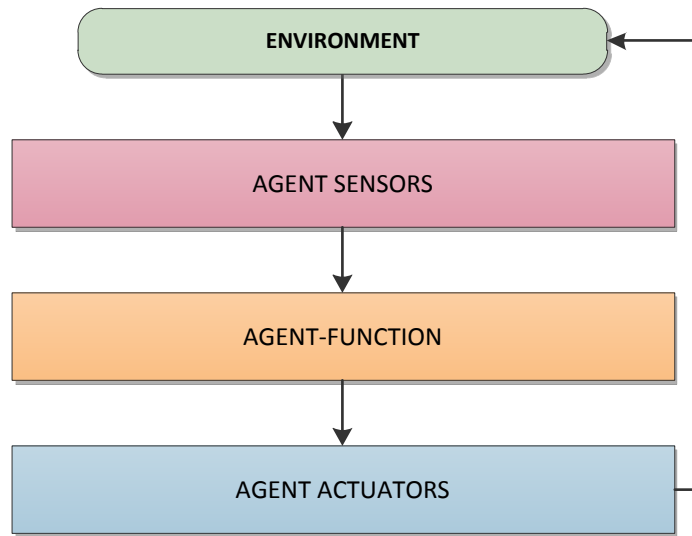
Figure 2.7: The general AI agent architecture.

Just like humans, game agents need to react differently to the same situation based on their behavioral or "emotional" state. All agent decisions and future actions are determined based on the agent's current state as well as the current game state. The decision making process is contained within the agent-function which has two major roles.

The first of which is to determine whether an agent will transition from the agent's current behavioral state to a new state based on the current game state. The second role of the agent-function is to make a decision as to the agent's next actions. This decision is based on the agent's current behavioral state, the agent's goals as well as the game state. Any decisions made will be sent down to the bottom layer of the agent architecture (the agent actuators) for execution.

Agent-functions can be thought of as large state machines which transition between states based on the input data, as such agent-functions are usually implemented as FSMs or fuzzy state machines (FuSM) [44] [63] [37]; although more recently the use of goal oriented and rule-based agent-functions is growing in popularity [3]. A detailed description of the various agent-function architectures is beyond the scope of this work, and interested readers are referred to [2], [64], [65] [61] [66] [3] [37] and [5] for more information on the topic.

The final layer of the agent architecture represents the agent's actuators. These actuators are responsible for all agent actions (such as movement and combat) and are the only means available to the agent for interacting with its environment. Any decision made by the agent-function can be

described by a sequence of commands given to the agent's actuators. Since high level behavior is simply a sequence of actuator actions, the overall quality of the behavior will be dependent on the quality of these actuators.

If, in the above example, the agent encounters an obstacle while moving towards the enemy and makes no effort to circumvent the obstacle resulting in a collision, then this collision not only results in the failure of the actuator but the overall failure of the high level behavior as well. As such it is critical to ensure that the actuators present are of a high quality as they are greatly responsible for the overall quality of a game agent.

2.4.2 The Game Agent Update

Just like humans players, game agents are required to react as the game events and change their behavior accordingly. To do this the game agent will need to continuously loop through all three levels in the agent architecture resulting in a sense-think-act cycle.

Each loop of the agent's architecture is referred to as the agent update and is illustrated in Figure 2.8 [44]. Each stage of the agent update compromises of discrete agent update tasks which need to be completed in order to move onto the next stage [3] [37] [5] [67]. A list of common agent update tasks for each stage is shown in Figure 2.8.

An agent's reaction time is defined as the time taken for the agent to react to a game event, and is measured as the period between agent updates. If the agent update is run each frame then the agent's reaction times will be the length of each frame, which at 30FPS will be 33ms and will only decrease as frame rates increase. Average human reaction times have been measured at around 250ms [68], and having game agents react nearly 10 times faster than a human violates the goal of human level AI.

As such the game agent update period needs to be increased. This is not only essential from a human level AI perspective but also from a computational one as the game agent update can have significant computational costs. In multi-agent games (which account for the vast majority of games) it is simply impossible to update all the game agents in a single frame [67] [3] [62].
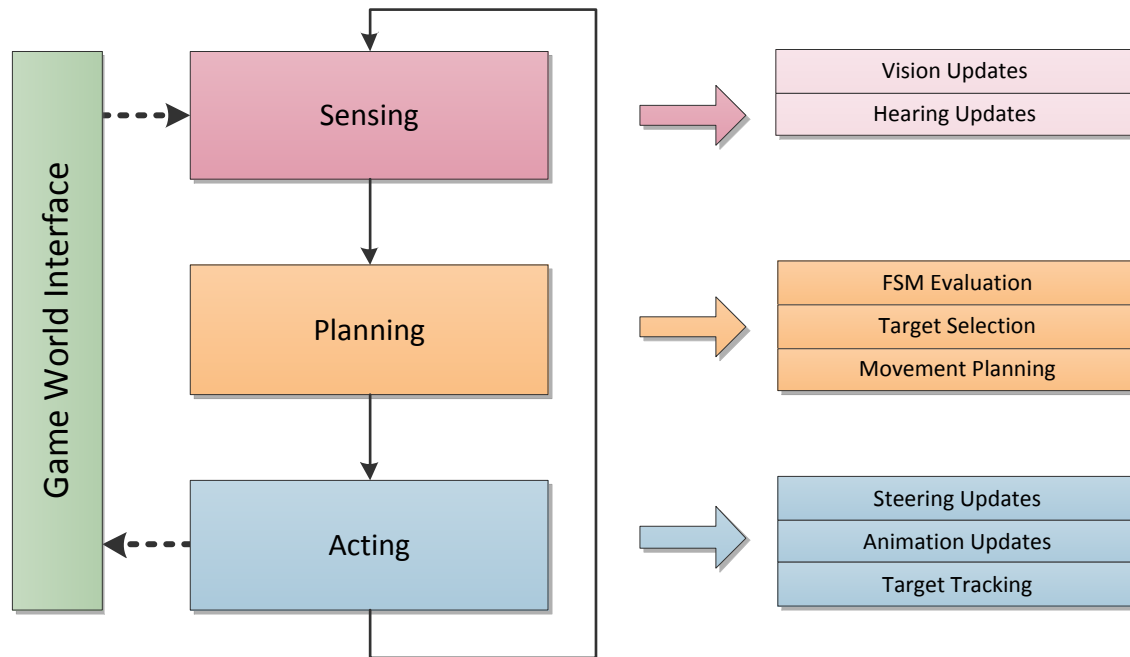
Figure 2.8: The game agent update loop

As mentioned, each stage of the agent update is comprised of discrete agent update tasks and the game agent update should be delayed to simulate human reaction times. Unfortunately, certain agent update tasks, such as movement and animation updates, need to be performed each and every frame. Not doing so will result in jittery movement and animation even though the frame rate is high enough to allow for smooth animation. Agent update tasks are divided into the following categories as defined in [67]:

- **Constant agent update tasks:** These agent update tasks are required to be performed each and every frame for every game agent present. These agent update tasks often include movement and animation updates that are essential for the visual fidelity of the game.

- **Periodic agent update tasks:** These agent update tasks need not be performed every frame, and therefore can be executed periodically with no ill effects. Periodic tasks include the sensing and planning stages' agent update task and account for the majority of the tasks performed during an agent update [67].

Because these two AI task categories have different periodicities, the task categories are required to be executed asynchronously. It is the role of the AI system to manage this asynchronous execution of agent update tasks for multiple agents.

2.4.3 The AI System Architecture and Multi-Frame Distribution

When designing a game AI system, there are numerous system specific considerations to be taken into account, for example behavioral requirements and behavioral modification interfaces [69] [44]. This thesis does not go into too much detail regarding the complete architecture of a game AI, but will focus on the execution of the game agent updates as well as the performance and memory constraints place upon it. For further information on various game AI architectures please refer to [67], [70], [61], [71], [66], [72] and [73].

On each frame (iteration of the game loop), the AI system is called to perform game agent updates (refer to Section 2.3.3). The AI system is allocated a certain percentage of the total processing time available per frame in which to perform these game agent updates. Estimates place this value between 5% to 25% of the total frame time [3] [57] [58].

Consider the scenario illustrated in Figure 2.9. It is impossible to perform all agent updates atomically in the short period of time allocated to the AI system, thereby affecting the frame rate. A naïve approach to the scheduling of agent updates is shown in Figure 2.10 wherein the atomic agent updates are distributed across multiple frames. Unfortunately, this scheduling approach does not take the periodicity of the agent update tasks into account and may result in visual artifacts.

A third approach based on the prioritized task categories scheduler described in [67] as well as the scheduling approach outlined in [3] is shown in Figure 2.11. In this approach the agent updates are now longer atomic and the agent update tasks have been distributed across several frames. In this case both the performance constraints placed upon the AI system as well as the periodicities of the agent update tasks have been taken into account.

It is important to note, that in Figure 2.11, the agent update tasks scheduled to be run on frame 1 exceed the allocated time for the AI system. In production game AI systems, a hard limit for the per-frame computational time is not set, but rather a rough computational time budget on the total computational times for the AI system is set [57].

It is the responsibility of the AI system's scheduling algorithm to schedule agent update tasks per frame such that the total computational time of those agent updates falls within the defined budget. In practice, the computational times of the required agent update tasks varies during runtime resulting in fluctuations in the total time spent on AI updates per frame. In some cases the scheduled AI updates will finish faster than expected, or in the case of Figure 2.11, will exceed the per-frame computational time budget.

A game AI system is nothing more than an agent update scheduler. The scheduling algorithm employed is therefore critical to both the operation and performance of a game AI system. A discussion into various AI scheduling techniques is outside the scope of this thesis. For more information on the topic please refer to [67] [70] [5] [3] [48], [74] and [73].

2.4.4 The Role of Pathfinding in a Game AI System

The course of action decided upon in an agent's planning stage often requires the game agent to move to a new location, be it to pick up a power-up or simply to attack an enemy. Game agent movement, as well as collision avoidance between agents, is usually performed using steering [75] or flocking [76] behaviors [77], [78] [3] [37]. Both steering and flocking behaviors require a goal to move towards. This goal position is determined during the planning stage of an agent's update.

Unfortunately, steering and flocking behaviors are limited in that they simple move agents towards a goal. This uninformed movement can easily result in agents becoming trapped in dead ends, or being moved through undesirable areas. Furthermore, there are no guarantees that an agent simply using a steering behavior will ever reach the goal [3].

Pathfinding (or path planning) is a technique used to plan a route through the game environment from an agent's current position to a required goal position. This planned route can then be followed using a standard steering behavior [3] guaranteeing that an agent will always reach its goal. As such, pathfinding is an essential component of the game agent update's planning stage.

A good pathfinding algorithm will try to find the best route for an agent through the environment. In most cases the best route will be the shortest one, but pathfinding can also be used in a tactical manner to plan routes that are safest for the agent or the most detrimental to the enemy [79].
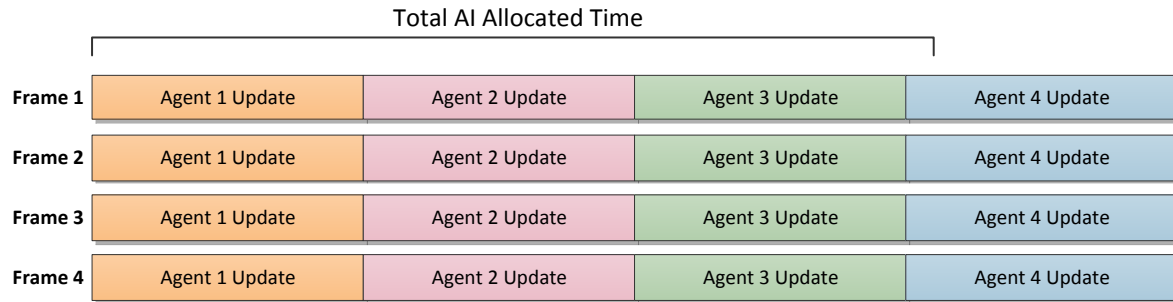
Figure 2.9: Performing agent updates atomically.
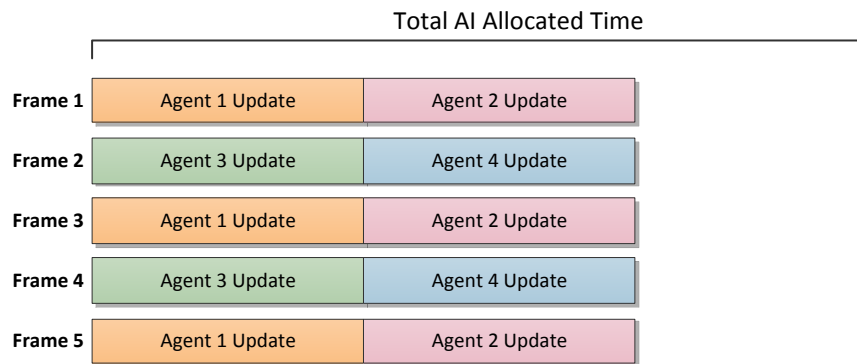
Figure 2.10: A naive approach to agent update scheduling.
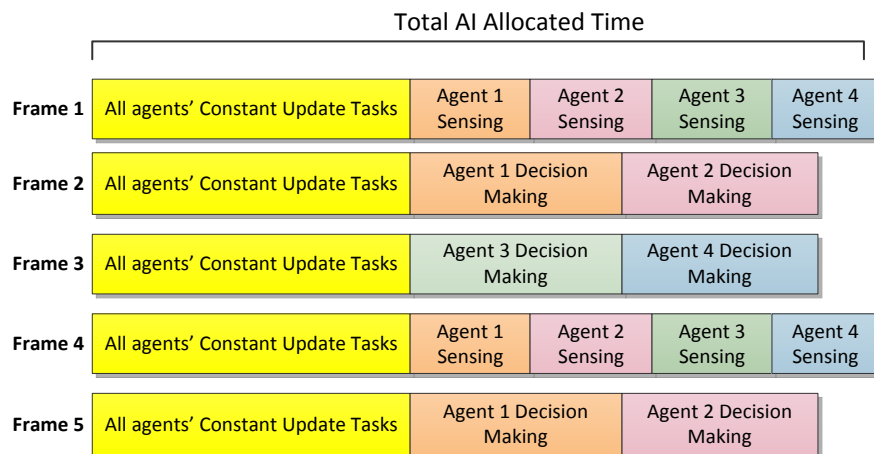
Figure 2.11: A robust scheduling approach for agent updates.

At a high level, a pathfinding algorithm receives a start position, a goal position and a game environment. The pathfinding algorithm performs a search through the game environment and returns the best path possible (if one exists).

This pathfinding search is an atomic action and can be thought of as one of the periodic game update tasks. Pathfinding differs from other periodic agent updates in that pathfinding searches are not performed periodically as are other agent updates such as decision making. Rather, pathfinding actions are only performed as needed.

Pathfinding actions are also significantly expensive from both a computational and a memory viewpoint and can account for a large portion of the total AI computational and memory costs [3]. The high cost of pathfinding means that care needs to be taken with regards to the selection of the pathfinding algorithm employed.

2.5 Summary

This chapter provided a brief background introduction to video games and video game development. Video games have changed significantly over the last few decades. Video games have become more complex and realistic resulting in the creation of complex game development frameworks known as game engines. Game engines perform all the tasks necessary for a game to function but have extremely tight performance constraints placed upon them.

As the levels of complexity and realism increase in modern games, so too does the demand for more intelligent and human-like computer controlled opponents or NPCs. The video game AI system is responsible for the control and behavior of a game's NPCs. Video games feature multiple NPCs, requiring the game AI system to control multiple agents.

Each NPC AI behavior makes use of the game agent architecture discussed in Sections 2.4.1 and 2.4.2. This game agent architecture closely models that of human behavior and has three distinct stages: sensing, planning and acting. These three stages are made up of separate agent update tasks which need to be performed to update an agent's behavior.

As tight as the performance constraints are for a game engine, the constraints placed upon the video game AI system are even tighter. This has resulted in the need for complex game agent architectures

and agent update task scheduling to allow the AI system to function under the tight performance constraints placed upon it.

Video game pathfinding is used to plan a route from one location to another in a game environment. When used in conjunction with an agent steering algorithm, the planned route guarantees that an agent will always reach the agent's intended destination. This is necessary as a steering algorithm alone cannot guarantee that an agent will reach its intended goal. As such, even though pathfinding actions have a high cost, pathfinding is a critical part of the game agent update's planning stage. The pathfinding problem as well as all applicable requirements and constraints are discussed in detail in Chapter 4.

Chapter 3

Graph Representations of Video Game Environments

This chapter discusses the representation of complex 3D video game environments as easy to search navigational graphs. Section 3.1 presents a brief introduction to graph theory and discusses the necessity of using a graph abstraction to represent the game environments. The three most popular navigational graph abstraction techniques, namely waypoint graphs, navigation meshes, and navigational grids are presented in Sections 3.2, 3.3 and 3.4 respectively. Section 3.5 summarizes the chapter.

3.1 The Navigational Graph Abstraction

To be able to search a game environment, the pathfinding search algorithm firstly needs to be able to understand the environment and secondly, the environment needs to be stored in a simple, easy to search format. Unfortunately, as video games have grown in scale and complexity so too have their environments. Game environments contain a high level of visual detail that is necessary for player immersion, but this high level of visual detail is irrelevant for planning a path through the environment.

Consider a human planning a path through a room; a human will only take into account, if and where any environmental obstructions are present and how to avoid them. There is no thought given to the type of obstructions or their properties. It doesn't matter to the person navigating the room whether a coffee table is made of glass or wood, but only that the table is there and that it needs to be avoided.

In most cases, these complex 3D game environments are stored in specialized spatial trees used to facilitate collision detection and renderer optimizations such as view-frustum culling [47] [54]. The complex 3D game environments (and their native storage formats) are highly unsuitable for searching, so game developers are required to generate separate simplified navigational maps from

these complex environments. These navigational maps contain only the essential navigational data required and store the navigational data in a readily searchable format [4].

The simplest data structure able to contain all the required navigational data is a graph [80] [4] [3]. A graph is defined as two finite sets: a set of vertices and a set of edges [81]. A vertex is the actual data being represented by the graph while an edge is simply a link between two vertices. Each vertex may have numerous edges connecting it to other vertices in the graph.

Consider a map of country. Each city on the map is a graph vertex and the roads connecting cities to one another are the graph edges. A graph edge may have a weight associated with it, representing the traversal cost of moving from one vertex to another in the graph across that edge. Using the map analogy, the edge weight would be the length of the road between two cities. Edges may also have a direction associated with them, i.e. only allowing movement between two vertices in a single direction. Graphs that contain edge weights or edge directions are termed weighted graphs and directed graphs respectively. Graphs are a simple method of representing spatial data as well as for mapping state transitions of game problems [81] [2]. In this work, graph vertices will be referred to as nodes to keep in line with the bulk of the pathfinding literature.

A graph containing the navigation data for a game environment is termed a navigational graph (navgraph). These navgraphs serve as search spaces for the pathfinding search algorithm. Each navgraph node represents a possible location in the 3D environment and contains that location's environmental properties, e.g. traversal cost and risk factors.

Each navgraph node also maintains a list of the locations directly accessible from that node (the navgraph edges). Since the navgraph is a simplified version of the game environment, the navgraph can be thought of as an abstraction of the game environment. The method of abstraction used on the 3D game environment will determine the size and complexity of the resulting navgraph. The three most common techniques for the abstraction of 3D game environments follow.

3.2 Waypoint Based Navigation Graphs

The waypoint graph is the traditional method of abstraction for creating a navigational graph from a game environment. Level designers would manually place navigational waypoints (representing the navgraph nodes) throughout a level during the design stages. These waypoints are later linked up either by hand or automatically (by linking nodes that have clear line of sight to one another) to form the final navgraph.

Figure 3.1a shows the placement and linking of waypoints in an example game environment. As these waypoints do not cover the entire range of possible locations, start and end nodes of a search are determined by finding the nearest waypoint that has a clear line of sight to a required location (the start or goal positions).

a) A waypoint based navigational graph created by hand placing waypoints throughout the game environment. **b) A waypoint based navigational graph automatically constructed using a point of visibility approach.**
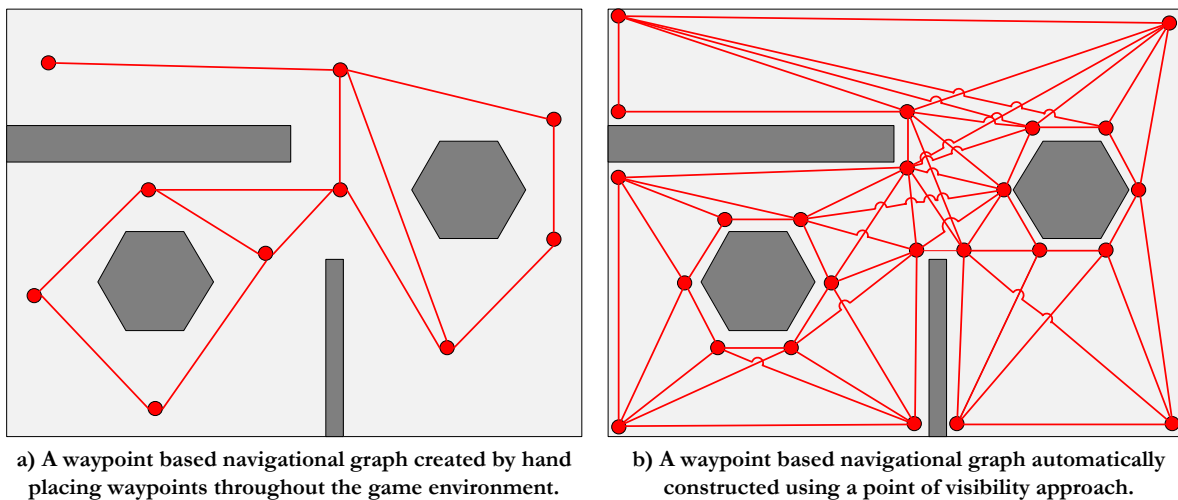
Figure 3.1: Waypoint based navigational graphs

Waypoint placement is usually performed manually. While techniques do exist to automate the generation of waypoints by examining the 3D level, their high computational cost restricts such techniques to pre-processing/offline roles [82]. Waypoint based graphs are also not guaranteed to provide full coverage of the environment (due to human error in waypoint placement) and may also include a large number of redundant nodes, unnecessarily increasing the overall search space.

A variation on the standard waypoint graph is the point of visibility (POV) graph. The POV graph is created by replacing the hand placed waypoint nodes with automatically generated inflection points located at the edges of the convex polygons making up the obstructions in the scene. These inflection points (representing the navgraph nodes) are then automatically connected via line of sight

checks. An example of a waypoint based graph construction using a POV approach is shown in Figure 3.1b. POV generated navgraphs are significantly larger (and have a high branching factor) than standard hand-placed waypoint graphs [82].

Furthermore, POV generated navgraphs exhibit a high degree of redundancy due to the fact that a geometrically complex obstacle such as a circular pillar may result in the generation of dozens of inflection points close to one another. A simple room containing several such circular pillars will generate a navgraph containing several hundred nodes and edges [3] [82]. In such cases, a human developer is usually required to manually merge several nodes together to reduce the size of the navgraph. The POV technique does guarantee full coverage of the traversable area present but the resultant search spaces may be so large and complex, as to be practically unsearchable.

Both methods of waypoint generation, while allowing varying degrees of automation, also require a high degree of human involvement especially in the case of the POV technique. When presented with large scale game environments, the human effort required to create or edit the necessary waypoints can simply be unfeasible.

As automated techniques for the generation of waypoint graphs still require a degree of human interaction, waypoint based navgraphs are unsuitable for use within dynamic environments. In the event of an environmental change, there is no guarantee on the quality of automatically generated waypoints (nor any way to automatically merge redundant nodes) and so waypoint based navgraphs are best used for static environments [83].

3.3 Mesh Based Navigational Graphs

To allow for a high level of automation, the majority of modern games generate navgraphs using polygonal navigation meshes [84] [85] [86] [87]. The navigational mesh (navmesh) technique generates a waypoint graph that successfully minimizes the number of navgraph nodes necessary to represent an environment while guaranteeing near perfect coverage of the traversable environment.

Navmesh based navgraphs are created from a polygonal representation of the 3D game environment's floor (or ground plane). An example game environment is shown Figure 3.2a. The game environment's floor is subdivided into traversable connected polygonal regions. An example of this subdivision using triangles is illustrated in Figure 3.2b. There are numerous subdivision

techniques available, but are quite complicated and beyond the scope of this work. Interested readers are referred to [85], [88], [87], [86] for more information on mesh subdivision techniques.

Once the game environment has been successfully subdivided, the various polygonal regions need to be connected to one another to create the final navgraph. One method of navgraph construction makes use of each polygonal region's centroid as the navgraph node. The connections between the various centroids are determined by the polygonal regions' geometric connection data (the polygons' edges). This method of region centroid connection is illustrated in Figure 3.2c. Triangle based meshes are preferred due to their low level of connection (number of edges <= 3) which results in a low branching factor for the final navgraph, as well as for their flexibility in representing complex environments [85].

a) The base game environment.

b) The game environment subdivided into polygonal traversable regions.

c) The polygonal region centroid calculations and the connection of the centroid to make up the final navigational graph.
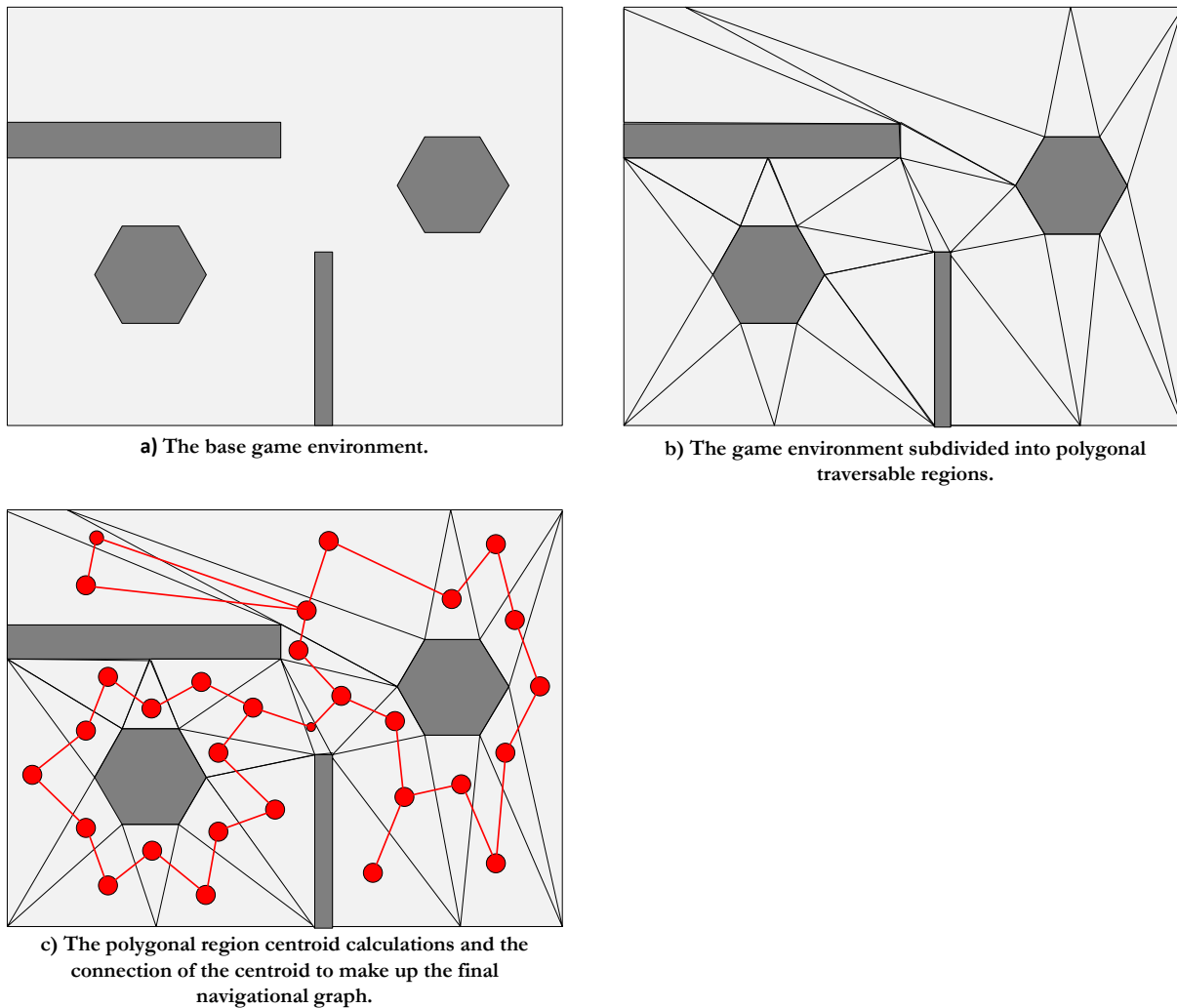
Figure 3.2: Creating a mesh based navigational graph from a game environment

Due to the drastic reduction of waypoint nodes and the reduced branching factor, navigational meshes represent the smallest search space out of all current automatically generated navgraph representations [3] [83]. Navmesh based navgraphs also offer the most accurate representation of the traversable area present in complex game environments [85]. Unfortunately, the initial subdivision of the game floor comes at a very high computational cost and so navmesh based navgraphs are usually created offline [87].

Navigational meshes have a major disadvantage within dynamic environments due to the high computational cost of navmesh generation. Any changes to the game environment will result in numerous mesh subdivisions of any polygonal regions affected (See Figure 3.3) and a rebuilding of the navgraph in that region.

In general, most navmesh generation techniques such as [87], [89] and [86] are not suitable for use in dynamic environments due to their high processing costs and resultant delays in navgraph updates. Several subdivision approaches such as [90] and [85] claim suitability with regards to usage within dynamic game environments. To the author's knowledge, no performance evaluation of said techniques has been performed within dynamic environments. Thus, no evidence exists to support the subdivision techniques' claims of suitability within dynamic environments.
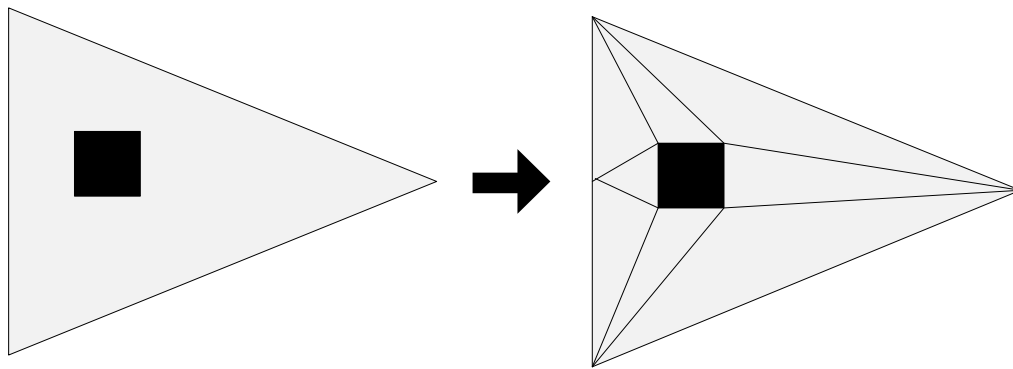
Figure 3.3: The subdivision of a single triangle due to an environmental change

3.4 Grid Based Navigational Graphs

A grid based navgraph is created by superimposing a grid over a game environment, dividing the game environment in grid cells. For each one of these grid cells, a traversability flag is calculated indicating whether a cell is traversable or not. A cell obstruction threshold is used to determine whether a cell is traversable or obstructed. If the obstructed area within a cell is greater than the obstruction threshold, then the cell is considered obstructed; otherwise the cell is considered traversable.

The overlaid grid is converted into a graph by creating an abstract node from each grid cell and then using the grid cells connection geometry (or neighborhood property) to determine the graph edges. The cell connection geometry is determined by the type of cells (or tiles) used to form the grid: a standard tile cell features a 4-neighborhood, hex tiles offer a 6-neighborhood, while the most popular cell type, the octile, features an 8-neighborhood [80]. Figure 3.4 illustrates these three cell connection geometries.

Since the connection geometry is constant for each cell, as well as there being no need for complex examination of the environment (like in navmeshes), the creation of a grid based navgraph from a game environment is simple and efficient [3]. The superimposition of a grid, the calculation of obstructed cells, and the creation of the navgraph is illustrated in Figure 3.5.
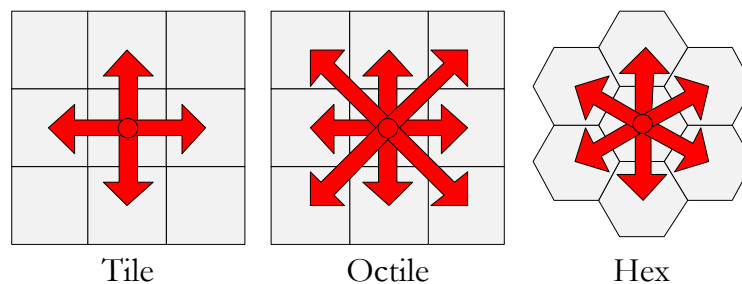
Figure 3.4: The most common grid cell connection geometries

It is important to note that a grid based navgraph will contain information about both the traversable areas as well as the obstructed areas, unlike waypoint and navmesh navgraphs that only contain a representation of the traversable area. This means that a grid-based navgraph will contain a representation of the entire game environment. All possible game locations are represented as

navgraph nodes. The start and goal position will be existing navgraphs nodes, meaning that no additional searches to find the nearest navgraph node from the start or goal positions are necessary.

Reflecting environmental changes on the navgraph is inexpensive; any environmental changes in the game environment result in a recalculation of the traversability flag for the cells overlapping the affected area. If the traversability state of any cell changes, the associated node is updated directly on the navgraph. If the overall size (area) of the environment remains fixed (as is the case in the majority of video games), then a grid based representation will result in a fixed size navgraph. Environmental changes will not require any form of graph modification or rebuilding. By having a simple representation of the entire environment as well as cheap environmental updates, grid maps are primarily used in RTS games [91], especially those featuring dynamic environments.

There are certain drawbacks when using a grid map representation. The resultant navgraph contains the largest search space of all the possible representations. This is both due to the storage of obstructions as well as the complete coverage of the game environment. In the case of hex and octile grid representations, the navgraph will also have a high maximum branching factor.

The large search space presented by these grid-based navgraphs has a significant negative effect on the performance and potentially the memory cost for any searches performed. As such, numerous navgraph abstraction techniques have been developed that reduce the search space presented by grid based navgraphs (and thereby improving search performance) [92]. A detailed discussion of various graph abstraction techniques is presented in Chapter 5.
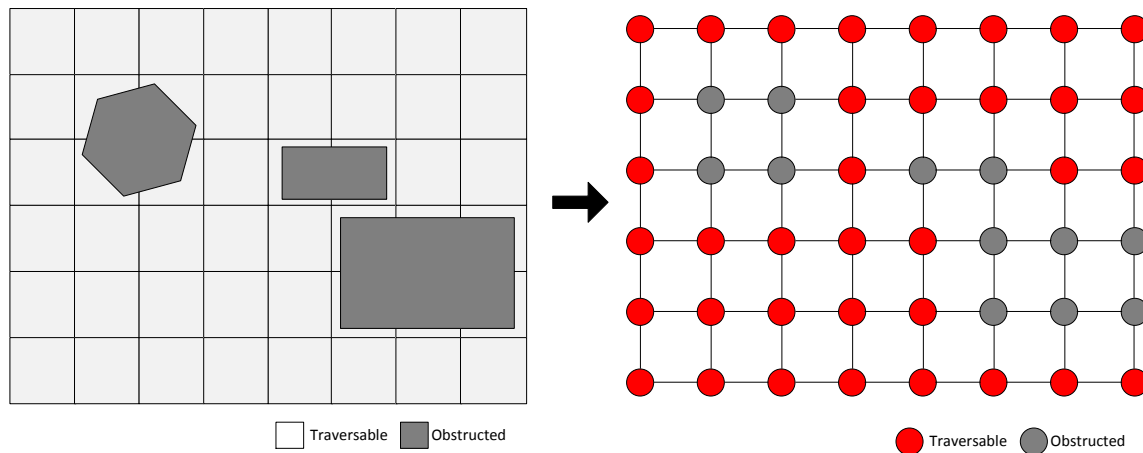
Figure 3.5: A grid-based representation of a Game Environment

3.5 Summary

This chapter discussed the need to convert a complex 3D game environment into a lightweight, easy to search graph representation suitable for pathfinding searches called a navgraph. Three techniques were presented to abstract the underlying game environment into a navgraph. Environment abstraction techniques are compared in regards to the size of the resulting navgraph as well as the suitability of an abstraction technique for use within dynamic environments.

The first technique presented, the waypoint navgraph, creates the navgraph by connecting manually placed waypoints. Waypoint-based navgraphs, while potentially offering the smallest search spaces of the three techniques, require human involvement in their creation and so are unsuitable for dynamic environments.

The second approach uses polygonal meshes to represent the floor of the game environment. These polygonal meshes represent the nodes in the navgraph and are connected to one another based on common edges. The resulting mesh-based navgraphs can be automatically generated, are of high quality, and are of a small size. Unfortunately, using navigational meshes to create the navgraph is an expensive operation, as is updating the navgraph when environmental changes occur. The high costs associated with the creation and updating of mesh-based navgraphs limits their use in dynamic environments.

The final technique presented in this chapter simply overlays a fixed size grid on the game environment and calculates which grid cells are traversable and which are obstructed. These grid cells are connected using a connection geometry to create the final grid-based navgraph. The grid-based technique unfortunately results in the largest navgraphs out of the three techniques presented, but to their benefit, grid-based navgraphs offer extremely cheap navgraph updates and so are highly suitable for dynamic environments.

Chapter 4

The Requirements and Constraints of Video Game Pathfinding

Chapter 3 discussed the need to convert 3D environments into navigational graphs. This chapter discusses the requirements and constraints of searching these navgraphs. Section 4.1 provides background on the need for pathfinding as well as defining the video game pathfinding problem as a graph search problem. Section 4.2 and 4.3 discuss the requirements and constraints placed upon the pathfinding graph search problem respectively. Section 4.4 discusses the additional considerations when pathfinding within a dynamic environment. Guidelines for the selection of a suitable graph search algorithm are provided in Section 4.5. Finally, Section 4.6 summarizes this chapter.

4.1 The Pathfinding Search Problem

The most basic requirement of a game agent AI is a mechanism to allow the game agent to successfully navigate the game environment. Any agent that cannot successfully navigate its environment would appear quite incompetent, irrespective of how many quality high level behaviors are present.

Navigational errors such as grossly sub-optimal routes, failures in obstacle avoidance or even in pathfinding, are more visible from a player perspective than any other AI mistakes, especially higher level errors. Players will tend to be more forgiving of higher level behavioral errors than of lower level errors, mainly due to players not realizing a mistake has been made. For example, a player may not notice an agent's inaccuracy with a ranged weapon but will always notice if an agent attempts to move from one room to another by walking through a wall.

Agent navigation consists of both movement and planning. As discussed in Section 2.4.4, an agent's physical movement (the movement actuator) is performed through the use of steering, flocking, or physics based systems [3]. These movement systems move an agent progressively closer to a goal location with each game frame. Agent movement is largely unintelligent in that an agent is simply

moved directly towards its goal location. As such agent movement systems will fail when an agent's goal is not directly reachable from the agent's current position.

Video game environments have grown both in scale and complexity. For an agent to be able to successfully navigate the game environment, the agent's movement systems need to be augmented with a planning system that will guarantee that an agent will always reach its goal, if that goal is reachable.

The planning component of an agent's navigation system is responsible for finding a list of world positions leading to the agent's final goal; each list position being directly reachable from the previous list position. This list forms a path through the game environment which, if followed by the agent, will lead to the agent's final goal.

Path following is performed by the agent's movement system and is guaranteed to succeed since each position on the path is directly reachable from the previous position on the path. In multi-agent game environments, collisions may occur between agents during the path traversal and so agent movement systems often feature localized collision avoidance mechanisms. A detailed description of agent movement systems is beyond the scope of this thesis and interested readers are referred to [3], [75] [37] [76].

Fundamentally, finding a path through an environment is a search problem and so can be solved through the use of a search algorithm. Chapter 3 discussed the representation of complex 3D game environments in the form of navigation graphs. These navgraphs contain all the traversable areas in the game environment and so represent the search space for the pathfinding search problem. Searching these navgraphs requires the use of a graph search algorithm and in so doing reduces the pathfinding search problem to nothing more than a graph search problem.

The solution to the pathfinding search problem is a path, which is defined as a list of points, cells or nodes that an agent needs to traverse to get from an initial position to a goal position [4]. As with any problem there are various requirements and constraints placed upon it. The act of planning a path for an agent through a game environment is termed a path planning action.

4.2 Pathfinding Search Problem Requirements

At the most basic level a pathfinding search algorithm has two fundamental requirements [3]:

- **Algorithm completeness:** The completeness of a search algorithm refers to the ability of the algorithm to find a solution within the search space. An algorithm is considered complete if the algorithm is guaranteed to find a solution if a solution exists. Conversely an algorithm that is not guaranteed to find a solution is termed incomplete.

- **Algorithm optimality**: The optimality of a search algorithm is determined by the quality of the solutions returned. If an algorithm is guaranteed to return the best (optimal) solution possible, the algorithm is said to be an optimal algorithm. A near optimal search algorithm is one which will return solutions that are close to, but not optimal.

For each pathfinding problem, there may be multiple solutions with some solutions being better than others. To quantify the quality of a given solution and allow the comparison of one solution to another, a path optimality solution metric is used. This path optimality metric is used to quantify the quality of a search algorithm by providing a means of measuring the average quality of solutions returned.

As mentioned, pathfinding search algorithms operate on the search space represented by the navgraph. Depending on the type of representation used for the navgraph, a high level of abstraction of the original game environment may be present [82]. In such cases, the best navgraph path found by the search algorithm can result in an aesthetically unappealing as well as sub-optimal final path through the original game environment [82]. To improve the quality of final paths a path smoothing step can be used.

4.2.1 Path Optimality

The path optimality metric is a measure of how close a particular solution is to the optimal solution available. This means that in order measure the optimality of a proposed solution, the optimal solution will first need to be known. As such, when evaluating the solution optimality of a proposed non-optimal search algorithm, a secondary optimal search algorithm needs to be used to provide the control solutions.

Solutions are then compared to one another using certain properties of the solution. For example, in video game pathfinding, path optimality is usually done on the basis of the path length. The final proposed solution's optimality value is a presented as a percentage of optimality when compared against the optimal solution. The optimality percentage is calculated as follows:

$$Optimality = 1 - \frac{S_{proposed} - S_{optimal}}{S_{optimal}} \qquad (4.1)$$

Where $S_{proposed}$ is the property value of the proposed solution and $S_{optimal}$ is the property value of the optimal solution. An optimal solution is always preferred, but in some cases a compromise has to be made in regards to solution optimality in return for improved algorithmic performance [22] [83] [3].

4.2.2 Path Smoothing

A path returned by the pathfinding search algorithm consists of a list of the navgraph nodes that need to be traversed in order to reach the required destination. Depending on the representation used to create the navgraph (refer to Chapter 3), the navgraph may not represent the environment in great detail. This is especially true in the case of waypoint graphs where each navgraph node represents a large area of the game environment.

Due to the high level of abstraction used in creating these navgraphs, simply following the navgraph path nodes by using a straight line between each node may result in a jagged, sub-optimal final path even if the navgraph path found was optimal [22]. Figure 4.1 illustrates an optimal navgraph path that is clearly a sub-optimal final solution, following the navgraph path results in a sub-optimal final path as well as in aesthetically unappealing agent motion.

To improve the aesthetic quality and the optimality of these paths it is often necessary to apply a post-processing smoothing step to the path [3] [22]. There are several techniques to improve the quality of found paths, for example simply fitting a spline though all the nodes in path will effectively add a level of smoothing to it, although this approach rarely improves optimality [82]. Additional path smoothing techniques can be found in [93], [84], [82].

A path-smoothing step may not always be required depending on the method used for agent movement. For example, if a quality steering behavior is used to move between nodes on the path then the steering technique itself will automatically smooth the path [94].
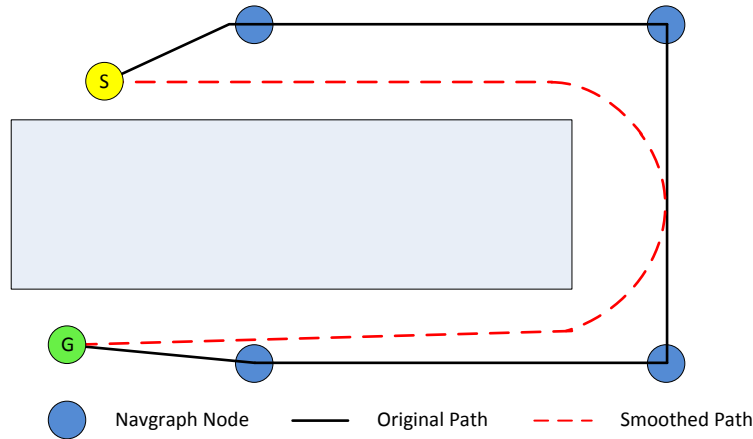
Figure 4.1: The effect of path smoothing on a generated path

4.3 Pathfinding Search Problem Constraints

There are various constraints placed upon the pathfinding search problem and these constraints are divided into two categories:

- **Performance:** The performance constraints of the pathfinding search problem comprise of the processing costs associated with solving the search problem as well the time required to do so. Performance constraints are discussed in detail in Section 4.3.1.

- **Memory:** The memory constraints placed upon the pathfinding search problem comprise of the memory cost incurred when solving a search problem as well as additional memory costs incurred by the search algorithm instance. Memory constraints are discussed in detail in Section 4.3.2.

In all cases, pathfinding constraints are determined by the game type as well as the intended platform. The real-time strategy (RTS) game genre presents a particularly large challenge to the game pathfinding problem [95]. RTS games are top-down war-games featuring hundreds or in some cases, like in Gas Powered Games' "Supreme Commander" series, thousands of semi-autonomous game agents [96]. The majority of higher level agent actions will involve movement and so require path planning actions. Given the large number of in-game agents and the high level of autonomy, the pathfinding system will often be tasked with finding paths for multiple agents at any given moment. Path planning for multiple units simultaneously is a massive task and so path planning actions can, in certain cases, account for more than half of the total computational time of the game [97] [95]. Even

in game types with lower requirements, path planning actions will still account for a significant portion of the game loop's processing time [3] [4] [83].

As mentioned, the intended platform for a game will also place certain constraints on the pathfinding search problem. These constraints are passed on the processing power and memory available on the platform.

4.3.1 Performance Constraints

Path planning actions are atomic actions in that given a start and goal location, the pathfinding search will be run to completion. The scheduling of agent update tasks was discussed in Section 2.4.3. Path planning actions fall into the periodic agent update task category with the distinction that path planning actions are only performed when necessary. As such, a path planning action is required to be able to be performed within the limited time allocated for period agent update tasks per frame.

It is important to note that pathfinding search algorithms can be paused and restarted allowing them to be run over several frames, but the scheduling complexities of doing so outweigh the benefits [3]. As such, multi-frame pathfinding is not commonly used. The atomicity as well as the limit on processing time available for path planning actions can result in a hard time limit being placed upon path planning actions. For example, Bioware Corp. requires that in Dragon Age [25], a single path planning action must be completed in less than 3ms.

A common technique to estimate the performance of an algorithm is the computational complexity measure. The computational complexity refers to an algorithm's computational cost in regards to the number of operations necessary for the algorithm to complete. Computational complexity is represented in Big-O notation. Unfortunately, Big-O is a worst case estimation of the potential performance of an algorithm and modeling algorithmic performance in terms of the number of operations can be misrepresentative of the real world performance profile of an algorithm.

The inaccuracy of the Big-O notation is due in part to the varying costs of memory and numerical operations on different CPU architectures [33] as well as the fact that the cost of a single operation may differ between two algorithms. For example, consider two $O(n)$ algorithms, according to the computational complexity measure the algorithmic performance for both algorithms should be identical. Now, consider that a single operation in the first algorithm takes twice as long to complete

when compared to the second algorithm. In real world terms, the first algorithm will be twice as slow as the second even though their computational complexities are equal. Even though the fringe search algorithm [11], as discussed in Section 5.6, has a much higher worst case computational cost than the A* algorithm [7], the average search time for the fringe search algorithm is significantly lower than that of the A* algorithm.

Due to limited usefulness of the computational complexity, an algorithm's execution time (or search time) is a common method for computational cost comparison between search algorithms [3]. There are two methods of evaluating a search algorithm based on search times:

- **Total search time:** An algorithm is timed over a fixed set of pathfinding problems and the total time needed for the algorithm to solve the problems is used as a comparison metric
- **Average search time:** An algorithm is run over a fixed set of problems and the search time is expressed as the average over the entire set of search problems.

Unfortunately, there is a problem in using execution time as a metric. An algorithm's execution time is dependent on both the implementation of the algorithm as well as the hardware platform the algorithm is run on. As such, algorithm execution times are not a consistent metric and though ubiquitous within the pathfinding literature, should not be used to measure the performance of an algorithm alone.

As will be shown in Chapter 5, there is a direct relationship between a search algorithm's performance and the number of nodes explored (search space exploration) by that algorithm during a given search. In addition to the number of nodes explored, search algorithms may often revisit previously explored nodes. These node revisits will add to the total performance cost of the search algorithm. Node exploration and node revisits have different performance costs, with node revisits often being cheaper to perform [7].

Using the number of nodes explored and the number of nodes visited as algorithm metrics is recommended as both of these metrics are platform and implementation independent. Once again, care needs to be taken when using only the total search space exploration metric since various algorithms have varying node exploration and revisit costs.

Both the execution time and node exploration/revisit metrics have limited usefulness on their own. As such it is recommended that any algorithm performance comparisons be performed using both types of metrics, namely execution time and node exploration/revisits.

4.3.2 Memory Constraints

The memory constraints of a pathfinding search algorithm are based almost entirely on the intended game platform. For example, while allocating 24MB for pathfinding data on a PC platform is acceptable [57], only 1MB to 2MB of memory can be allocated for pathfinding data on a console platform [58]. The memory limit imposed by a game's intended platform will have a significant impact of the choice of search algorithm used to perform path planning actions within the game.

The memory costs of a search algorithm can be divided into two categories:

- **Per-node data**: This is the algorithm specific data created for each node explored by a search algorithm. The per-node data is used in calculating shortest routes as well as constructing the final solution found [7] [10].

- **Per-algorithm data**: For certain types of search algorithms additional memory is needed for the algorithm specific data used to improve the performance or quality of the search algorithm. For example, hierarchical pathfinding algorithms [92] require memory to store the abstraction hierarchies used. Algorithms making use of spatial heuristics [98] also require memory to store the spatial heuristic data lookup tables.

The memory costs of per-algorithm data remain constant for a given static game environment, although environmental change within dynamic environments may affect the memory costs of the per-algorithm data. In most cases, the per-node data memory costs will be significantly higher than the per-algorithm memory costs [25] [98], therefore the reduction of the per-node memory costs is the primary focus of search algorithm memory optimization [10] [8].

As mentioned, per-node data is allocated whenever a node is explored, as such the greater the search space exploration of a search algorithm, the higher the associated memory costs. Node-revisits have no effect on the memory costs of a search algorithm. The memory costs associated with the per-node data are referred to as the space complexity of the algorithm.

Furthermore, the search space exploration performed by a search algorithm is dependent on the search problem being solved. As such there is a high degree of variation of search space exploration across search problems making the estimation of the total memory costs of an algorithm problematic.

A worst case memory costs can be estimated for each algorithm, but as with the computational complexity, the usefulness of such a metric is limited. To effectively compare search algorithms to one another with regards to memory costs, an empirical evaluation needs to be performed on both the average and peak memory costs. Since memory costs are based on search space exploration, space complexity comparisons are made based on the search space exploration of a search algorithm.

There has been a lot of research into creating both reduced-memory and memory-limited search algorithms [10] [25] [11]. Care needs to be taken with memory reduced algorithms as the memory cost reduction often has a negative effect on either the solution optimality or the search times of a search algorithm [10] [11]. The effects of memory reduction on graph search algorithms are further discussed in Sections 5.4 and 5.5.

4.4 Pathfinding in Dynamic Environments

Video games featuring dynamic environments are becoming ever more popular. In these dynamic environments, players have a high degree of interaction with the environment allowing players to drastically change the game environment through their actions. Certain games, like Volition Inc.'s "Red Faction" [99] series as well as Relic Entertainment's "Company of Heroes" series [100] rely heavily on these dynamic environments as a key aspect of their gameplay mechanics.

Figure 4.2 is courtesy of Chris Jurney's GDC2007 presentation [91] detailing the level of destruction present in "Company of Heroes". Figure 4.2a shows an in-game location in its initial state while Figure 4.2b shows the same in-game location after a lengthy battle.

Due to various player actions and the high level of destructibility offered by the game, the game environment has been greatly changed to the point of being near unrecognizable. Areas that were previously traversable are now obstructed and vice versa, as such environmental changes add a high level of complexity to the pathfinding problem.

In dealing with an environmental change, the first priority is to update the navgraph to ensure that any subsequent paths planned will take the environmental change into account. The complexity of updating the navgraph varies depending on the navgraph representation (refer to Chapter 3 for more information) and so care needs to be taken in selecting an appropriate navgraph representation for use within dynamic environments.

a) The initial state of the game environment.	b) The same game environment after a lengthy in-game battle.
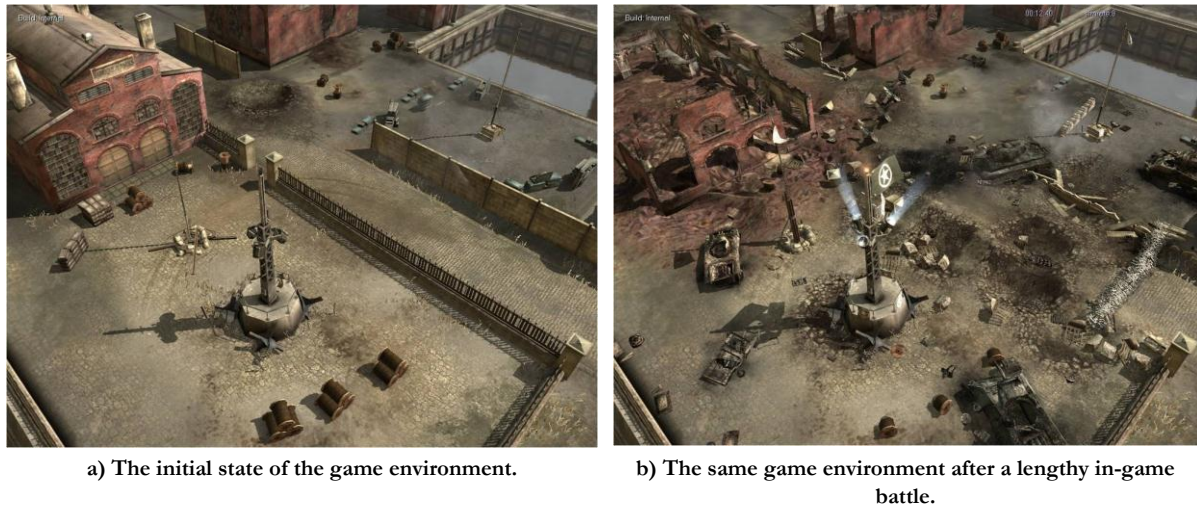
Figure 4.2: An example of the level of Destructibility in 'Company of Heroes'

In navigating within dynamic environments, agent pathfinding is performed as in a static environment with the additional consideration that at any given time an environmental change may affect all currently planned paths. When an environmental change does have an effect on a planned path, the effect will be one of two things: the first being that a portion of the planned path is no longer traversable (refer to Figure 4.3) or that a shortcut (a new optimal path) is now available in the environment (refer to Figure 4.4). In both cases, a planned path that is affected by an environmental change will need to be replanned.

In the case when an environmental change obstructs a path, the detection of such an occurrence is relatively straightforward. Each path in the vicinity of the change is checked to see whether it contains any invalid nodes: either obstructed nodes or non-existent nodes in the case of a navmesh representation. If a path does contain invalid nodes, it needs to be replanned. A path is replanned by performing a new pathfinding search with the same goal node, but originating at the agent's current position in the navgraph. This new pathfinding search is termed a replanning action.
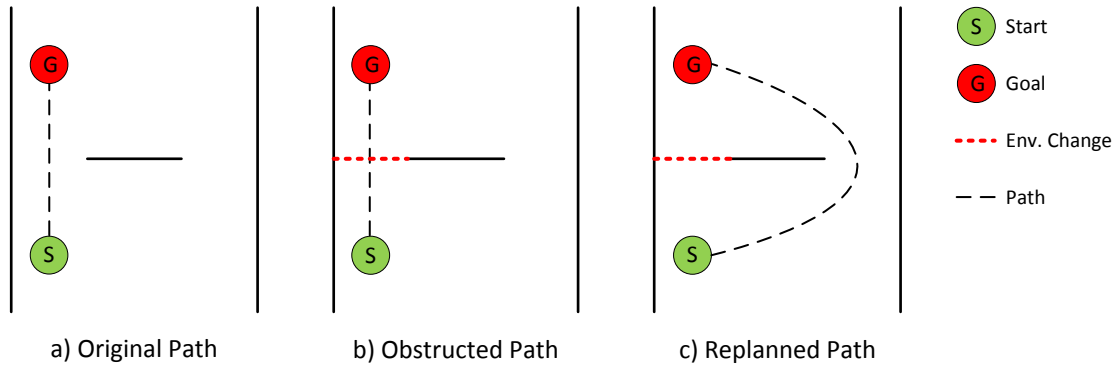
Figure 4.3: The need for path replanning as a result of an environmental change
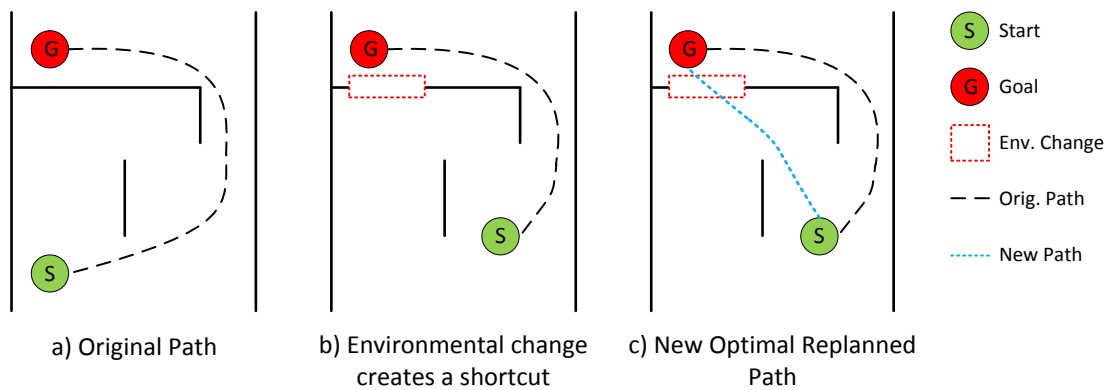
Figure 4.4: An environmental change requiring the replanning of the agent's path

The detection of the existence of a new optimal path for a specific search problem is a more difficult task than simple path validation. Any potential change in the environment irrespective of location may provide a new optimal solution for an already solved pathfinding search problem. Figure 4.4 illustrates such a situation where an environmental change outside of the existing path vicinity results in a new optimal path. The only way to check for the existence of a better path is to perform a replanning action and compare the resulting optimal path found to the existing path. If an optimal (or near-optimal) algorithm is used then the comparison between the new and the existing path is redundant since the new path will always be the same or better than the existing path.

Considering that the shortcut-check will always performs a replanning action for each environmental change, the need for path validity checks will also become redundant as paths will always be replanned. In fact, no checks need to be performed at all and all game agents' paths can simply be replanned once an environment change occurs. Replanning an agent's path each time an

environment change occurs, guarantees that the agent's paths will always be as optimal as the search algorithm allows.

For each environmental change, all previously planned paths will need to be replanned. In games featuring a large number of agents, a single environmental change will result in huge number of path replanning requests. As each path planning action is atomic and since the execution time of the path planning action determines how many path planning actions can be run per frame, a highly efficient search algorithm is required for use in highly dynamic multi-agent games.

Unfortunately, even with a highly efficient search algorithm it may not be possible to execute all the path replanning actions within the agents' required response times and agent response times will be delayed.

4.5 Selecting a Suitable Search Algorithm

The efficiency and suitable of the pathfinding system within certain environments is entirely based upon the efficiency and suitability of the algorithm within those environments. The "perfect" search algorithm will provide optimal paths in next to no time and next to no memory usage. Unfortunately, in practice such a perfect solution does not exist and a real-world system needs to find an appropriate compromise between the three search algorithm metrics and weigh up the importance of path optimality versus time and memory [101].

The selection of a search algorithm for use within a specific game is based on the following factors:

- The game's intended platform,
- the number of game agents required, and
- the dynamicity of the game environment.

The memory constraints placed upon the search algorithm by the platform are the most important factors in the selection of the search algorithm to be used. Depending on the limitation of the environment, drastic compromises with regards to the search speed may need to be made [8].

Using the worst case memory cost for an algorithm as a means of selection will ensure that the algorithm will never exceed the memory budget. Unfortunately, the worst case memory cost is just that, the worst case, and for the majority of search problem solves the memory usage of the search algorithm will be significantly lower than the worst case estimate [3]. Using the worst case memory

cost as a selection factor results in an over reduction of the memory costs of the search algorithm often at the expense of algorithmic performance. The peak and average memory costs of a search algorithm are better selection factors in judging the suitability of a search algorithm for use within memory limited environments.

The peak and average memory costs of an algorithm will need to be empirically calculated for a large and varied set of test problems in a test environment representative of the environments found in the final game. In using the peak memory cost value for algorithm selection care needs to be taken due to the search space exploration variance exhibited across search problems. If a search algorithm's peak memory cost is close to the limit of the platform, the likelihood of a search problem exceeding the platform's hard limit is higher than if the peak memory cost was further from the platform's memory limit.

If a search algorithm exceeds the available memory it will result in a failed search or in the case of a badly implemented algorithm an application crash. An algorithm is a better candidate for a memory limited environment if the algorithm offers a lower peak memory cost when compared to another algorithm even if the algorithm has a higher average memory cost than the other candidate algorithm.

With regards to algorithm performance, the faster the algorithm, the better. As mentioned, the choice of algorithm will be primarily based on the memory constraints. So the selection of the final search algorithm is simply the search algorithm which offers the lowest execution times while obeying the memory constraints of the platform.

4.6 Summary

Due to the representation of the video game environments in the form of a graph, the video game pathfinding problem is reduced to a graph search problem. A graph search problem is solved through the use of a graph search algorithm. Video game pathfinding has specific requirements and constraints with regards to the computational and space complexities of the algorithm.

These requirements and constraints limit the choice of search algorithms which can be successfully used in solving the video game pathfinding problem. Furthermore, when presented with dynamic game environments the workload placed upon the AI system stemming from pathfinding is drastically increased making the choice of search algorithm all that more important.

Chapter 5

Discrete Search Algorithms

As discussed in Chapter 4, the pathfinding search problem is fundamentally a graph search problem, one which is solved through the use of a graph search algorithm. This chapter discusses the most common graph search algorithms employed today. A short introduction to discrete search algorithms is presented in Section 5.1. A discussion of Dijkstra's search algorithm and its memory costs is presented in Sections 5.2 and 5.3. The A* algorithm, the iterative deepening algorithm, the simplified memory-bounded algorithm as well as the fringe search algorithm are discussed in Sections 5.4 through 5.7. This chapter is concluded by a summary presented in Section 5.8.

5.1 Discrete Search Algorithms

This thesis makes novel use of the terms discrete and continuous in the separation of graph search algorithms. The distinction between these two terms is discussed in detail in Section 6.1. It is impossible to define this distinction without first discussing continuous algorithms and so no further mention of the term will be made within this chapter. For all intents and purposes this chapter may simply be titled "graph search algorithms".

A graph search algorithm is an algorithm which, given two nodes in a graph, attempts to find a path between them. If a found path between two nodes in a graph is the shortest possible path, then that path is referred to as the optimal path.

The pathfinding requirements of a graph search algorithm are that the algorithm will always find a path if one exists and that the path found will be optimal or near-optimal. Furthermore, the processing and memory costs of such graph algorithms must be minimized to operate successfully within tight performance and memory constraints[1].

---

[1] A detailed discussion of the requirements and the constraints of the video game pathfinding problem were presented in Chapter 4.

5.2 Dijkstra's Algorithm

Dijkstra's algorithm was presented in Edsger Dijkstra in 1959 as a method to find the shortest routes from a start node to all other nodes in a weighted graph [6]. In such a graph, each edge weight represents the traversal cost incurred moving across that edge.

The basic operation of the algorithm is as follows: The algorithm explores every node in the graph, and while doing so stores shortest route data at each node. Once the algorithm has completed, a path is then be constructed by starting at the goal node and working backwards to the start node using the route data stored at each node. The algorithm operates iteratively and each algorithm iteration will explore (another common term encountered is expand) a single node. The node being explored is referred to as the parent node for that iteration and its neighboring nodes are referred to as the node successors (or child nodes).

5.2.1 The Algorithm

Dijkstra's algorithm makes use of a single metric, the cost-so-far (CSF) value, in determining the shortest paths in the graph. The CSF value is simply the traversal cost incurred in reaching a graph node from the start node. When a node is explored, a CSF value is calculated for each of the successor nodes.

The CSF value for each successor is the sum of the parent's CSF value and the traversal cost of moving from the parent to the successor. Figure 5.1a illustrates the CSF calculation for a node. Multiple routes can exist to a single graph node, implying that when Dijkstra's algorithm explores the graph it may encounter successors that already have CSF values (i.e. a previous route to the node has been found).

In such a case, Dijkstra's algorithm checks whether the current route (from the current parent node) to a successor node is shorter than the previous route found. A new CSF value is calculated from the current parent to the successor node in question. If the new CSF value is smaller than the successor node's CSF value, it means that the current parent offers a shorter route to that successor than the previous route found. The successor node's CSF is then set to the new smaller CSF value (i.e. the new shorter route). This ensures that only the shortest routes found to each node are stored.

In addition to the CSF value stored at each node, a link to the parent node, from which the CSF value originated from, is stored as well. This parent node link is necessary so that routes can be traced back from any node to the start. This parent node link is updated whenever the CSF value of that node is updated. Simply put, whenever a new shorter route to a node is found, the shorter route value is saved as well as the node from which that route originated from.

During the exploration of the search space, Dijkstra's algorithm will encounter nodes that fall into the following categories: unseen, unexplored and explored. When an unseen node is first encountered during the exploration of a parent node to that node, a CSF value originating from the parent node is calculated.

a) The cost-so-far calculation

b) The path generation performed by Dijkstra's algorithm, by means of a back-trace
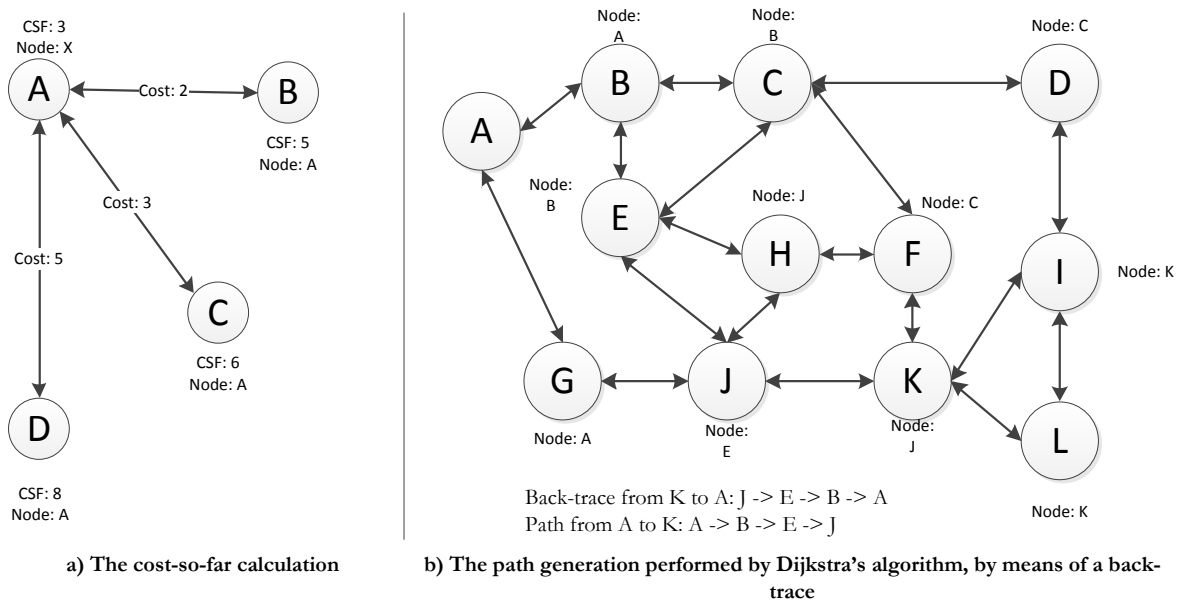
Figure 5.1: The Dijkstra's algorithm's cost-so-far calculation and path generation

This unseen node will now need to be explored, it is placed on a list containing other nodes that the algorithm has encountered but not explored. This list is referred to as the open list; nodes on the open list are all awaiting exploration. Once a node has been explored, it is added to a list containing all other explored nodes. This list is known as the closed list. The open and closed terminology is based on the fact that when a node is encountered, it is opened for exploration and it is only closed once the node has been explored. Nodes awaiting exploration are referred to as open nodes while already explored nodes are referred to as closed nodes.

Dijkstra's algorithm is initialized with only the start node present on the open list and an empty closed list. At each step (iteration) of the algorithm, the node with the lowest CSF value is removed from the open list. This ensures that the closest node to the start node is always explored. When a node N is explored, each of N's successor nodes S is examined. Each S can belong to one of three categories: unseen, open or closed. If S is unseen then a CSF value is calculated originating from N, S's parent link is set to N and finally S is placed on the open list. In the case when S has already been encountered (i.e. either an open or closed node), the new route to S from the start (the CSF value originating from N) is compared to S's existing CSF value, if the new route is shorter than the existing route then S is updated with the new route's CSF value and its parent node link is set to N.

Since a node's successor node's CSF values are based on its CSF value. If the CSF value of a node is updated then all the CSF values of it successor nodes must be updated as well. Instead of explicitly performing this update when an already encountered node is updated, it is simpler to place that node back onto the open list since the successor node's CSF values are automatically checked and updated as necessary during node exploration. In the case of an already encountered node being open, nothing more than the CSF/parent link updates are performed. If an updated node was a closed node, it needs to be removed from the closed list and placed back onto the open list (i.e. the node is reopened).

Dijkstra's algorithm terminates only once the open list is empty, i.e. all the nodes in the graph have been explored. Once the algorithm has terminated, a path can be generated from the start node to any node in the graph (the goal node) by starting at the goal node and tracing the stored parent node links back to the start node (see Figure 5.1b). The complete pseudocode for Dijkstra's Algorithm is shown in Listing 5.1.

From an algorithmic standpoint, Dijkstra's algorithm is both a complete and optimal algorithm. This means that Dijkstra's algorithm will always find a solution if it exists and that the found solution will always be the best solution possible (i.e. the optimal solution).

The memory cost of Dijkstra's algorithm consists of the per-node data values for each node in the graph (CSF and parent link values) and the memory costs of the open and closed lists. Since Dijkstra's algorithm explores each and every node in the graph, per-node data storage is allocated for every graph node as it is guaranteed to be needed.

```
1) Clear the start node's parent and CSF values

2) Add start node to open list

3) While open list is not empty

  3.1) Remove node N with the lowest CSF cost from the open list

  3.2) For each successor node S of N
    3.2.1) Calculate the new CSF value (current CSF + traversal cost)

    3.2.2) If S is unexplored
      a) Set S's CSF to new CSF value
      b) Set S's parent to N
      c) Add S to open list

    3.2.3) Else if S is on the open list and S's CSF is greater than new CSF
      a) Set S's CSF to new CSF value
      b) Set S's parent to N

    3.2.4) Else if S is on the closed list and S's CSF is greater than new CSF
      a) Set S's CSF to new CSF value
      b) Set S's parent to N
      c) Remove S from closed list
      d) Add S to open list

  3.3) Add N to closed list
```

Listing 5.1: Pseudocode for Dijkstra's Algorithm

5.2.2 Optimizations

Dijkstra's algorithm is not without its weaknesses. Since the algorithm was designed to find all routes from a single node in a graph, it explores the entire graph and as such has a very high computational cost, estimated at around $O(n^2)$ where n is the number of nodes in the graph [1].

The bulk of the processing costs incurred by Dijkstra's algorithm are a result of the open/closed list operations. Each time a node is selected from the open list, the entire open list is searched to find the node with the lowest CSF value. With each node explored by the algorithm, the open list grows in size, and so the cost of performing the search for the closest node in the list increases.

The open list search is the most computationally expensive operation in the algorithm, and therefore has been the primary focus of algorithmic optimizations both for Dijkstra's algorithm and all algorithms derived from it such as the A* algorithm [7].

The simplest manner in which to reduce the cost of the open list search is to store the open list in an ordered fashion (usually in order of increasing CSF value). If the open list is ordered, then the first node in the list will always be the closest node, i.e. the next node to be explored. Selecting the closest

node from the list becomes a trivial operation with respect to processing costs. Unfortunately, keeping the opened list ordered introduces additional costs during node exploration.

When an unseen node is added to the open list, the node needs to be inserted at the correct position on the list to maintain the correct ordering. As such, node insertions onto the open list will require an additional search to find the correct insertion position. When an open node is updated, its CSF value will change and so will its position in the open list. The updated open node needs to be removed from the open list and reinserted into the correct position to maintain the correct ordering of the list.

When examining the successors of a node and discovering that a successor node has already been encountered, the open list, the closed list or both lists need to be searched to check whether that successor node is open or closed. These list searches are expensive and their costs quickly add up.

There is a simple optimization which removes the need for searching the open and closed lists during node successor examination. An additional flag value is added to the per-node data of the algorithm. This flag specifies whether the node is unseen, open or closed. A keen reader may have realized that the closed list need not actually be stored as a list. The only role of the closed list is to maintain a record of the explored nodes; if this record is moved into the nodes themselves as detailed above, the closed list becomes redundant and can be removed. The removal of the closed list further counteracts the additional memory cost of the flag value and is a free processing cost optimization.

The only list that needs to be maintained is the open list; moving a node from the closed list to the open list requires an update to the flag value and an open list insertion. A detailed discussion into open list data structures and optimization is presented in Section 5.3.3.

The optimizations discussed above are valid for both Dijkstra's algorithm as well as itsvariants. There is one more optimization that needs to be discussed, one which is specific to Dijkstra's algorithm. Since the closest node to the start is explored each iteration, a closer route to a closed node cannot be found. Successors encountered that are already on the closed list can be safely ignored since it has been shown that Dijkstra's algorithm will never find a better route to an already explored node [3]. The entire pseudo-code contained in section 3.2.4 of Listing 5.1 can be removed.

5.2.3 A Point-to-Point Version of Dijkstra's Algorithm

Video game pathfinding searches are point-to-point searches, i.e. only a path from the start node to the goal node is required. In contrast, Dijkstra's algorithm is an all-points search, finding the shortest path from the start node to every other node in the graph. As such, for any given point-to-point problem, Dijkstra's algorithm will continue exploring the search space even after the shortest path to the goal node has already been found. This continued exploration and its associated computational cost is entirely unnecessary, and the original Dijkstra's algorithm can be modified to create a point-to-point version. The point-to-point version offers significant improvements to the computational cost of the original Dijkstra's algorithm when used for point-to-point searches.

This point-to-point version is based on the fact that once a node is explored (placed on the closed list) no shorter path to that node exists [1], meaning that if the algorithm is terminated once the goal node has been explored, then the current path found to the goal node is guaranteed to be the shortest path possible [1].

This "early" termination of the algorithm prevents any continued (and unnecessary) exploration of the search space while still maintaining the optimality and completeness of the algorithm. Node exploration accounts for the majority of the processing cost associated with Dijkstra's algorithm (and variants) due to the high costs of open list operations. Reducing the number of nodes explored reduces the processing costs and improves search times.

In many cases, the goal node may be encountered early on in the search but may not be immediately explored since there may be other open nodes closer to the start node. In such case, Dijkstra's algorithm will continue to explore the closer open node and ignore the goal node. The goal node will only be explored once it is the closest node to the start node.

This additional searching may be prevented by simply terminating the search as soon as the goal node is encountered. Unfortunately, since the goal node is not fully explored, such an early termination does not guarantee the optimality of the path found and is therefore not recommend. The pseudo-code for the optimal and complete point-to-point version of Dijkstra's algorithm is shown in Listing 5.2.

```
1) Clear the start node parent and CSF value

2) Add start node to open list

3) While open list is not empty and N is not the goal node

  3.1) Remove node N with the lowest CSF cost from the open list

  3.2) For each successor node S of N
    3.2.1) Calculate the new CSF value (current CSF + traversal cost)

    3.2.2) If S is unexplored
      a) Set S's CSF to new CSF value
      b) Set S's parent to N
      b) Add S to open list

    3.2.3) Else if S is on the open list and S's CSF is greater than new CSF
      a) Set S's CSF to new CSF value
      b) Set S's parent to N

  3.3) Add N to closed list

4. Return No Path Exists
```

Listing 5.2: Pseudocode for the point-to-point version of Dijkstra's algorithm

5.3 The A* Algorithm

Even though the point-to-point version of Dijkstra's algorithm is significantly more efficient for point-to-point searches than the original algorithm, it is far from optimal with respect to search space exploration and processing costs. The A* algorithm [7] was presented as an improvement to the point-to-point version of Dijkstra's algorithm. To explain the need for, and the improvements offered by the A* algorithm, it is necessary to first discuss the manner in which the point-to-point Dijkstra's algorithm explores the graph search space. For clarity's sake all references to Dijkstra's algorithm will refer to the point-to-point version unless stated otherwise.

By selecting the closest node to explore, Dijkstra's algorithm explores outwards in all direction from the start node. Figure 5.2a and Figure 5.2b illustrated the search space exploration pattern of Dijkstra's algorithm. The undirected nature of the search pattern means that Dijkstra's algorithm will explore many unnecessary nodes in finding a solution. With only point-to-point searches in mind, if Dijkstra's algorithm's search can be directed to only explore nodes that have a high likelihood of leading to the goal, and to ignore nodes that are unlikely to lead to the goal, then the total search space exploration can be reduced resulting in decreased processing and memory costs.

The search space exploration pattern is directed by the selection of the "best" node from the open list. Dijkstra's algorithm ranks nodes according to the closeness of the nodes to the start node. In a point-to-point search the goal node is just as important as the start node. If the distance to the goal node, as well as the distance to the start node, is taken into account when selecting the next open node to explore, the search space exploration pattern can be directed towards the goal node.

The A* algorithm [7] was presented in 1968 as a directed (best first) point-to-point version of Dijkstra's algorithm. Since then, the A* algorithm has become the benchmark for all point-to-point graph search algorithms, especially in the pathfinding field. The main difference between the A* algorithm and Dijkstra's algorithm is the augmentation of the CSF value with a heuristic value in the selection of the next open node to explore. To stay in line with the literature, the CSF value will henceforth be referred to as the G value.

5.3.1 The Algorithm

As mentioned, the A* algorithm is Dijkstra's algorithm with a simple change to the manner in which nodes are selected from the open list. A heuristic value (H) is created for each node and is stored alongside the other per-node data (the G value and the parent node).

The heuristic value is an estimation of how close a given node is to the goal node, or alternatively an estimation of the likelihood of a node leading to the goal. The heuristic value is calculated by a heuristic function that, given two nodes, returns a numeric measure of how close the nodes are together. A simplistic way of describing the heuristic value is to term it the "estimated remaining cost".

At each node, the cost-so-far G value and the estimated-remaining-cost H value are added together to create a new "total estimated cost" value (often referred to as the F value or F cost). Open node selection is then based on this "total estimated cost" value. Open nodes in the direction of the goal node (i.e. closer to the goal node) are explored first, preventing the arbitrary exploration of nodes with small G values that lead away from the goal. Directing the search space exploration towards the goal node reduces the total overall search space exploration of the algorithm significantly.
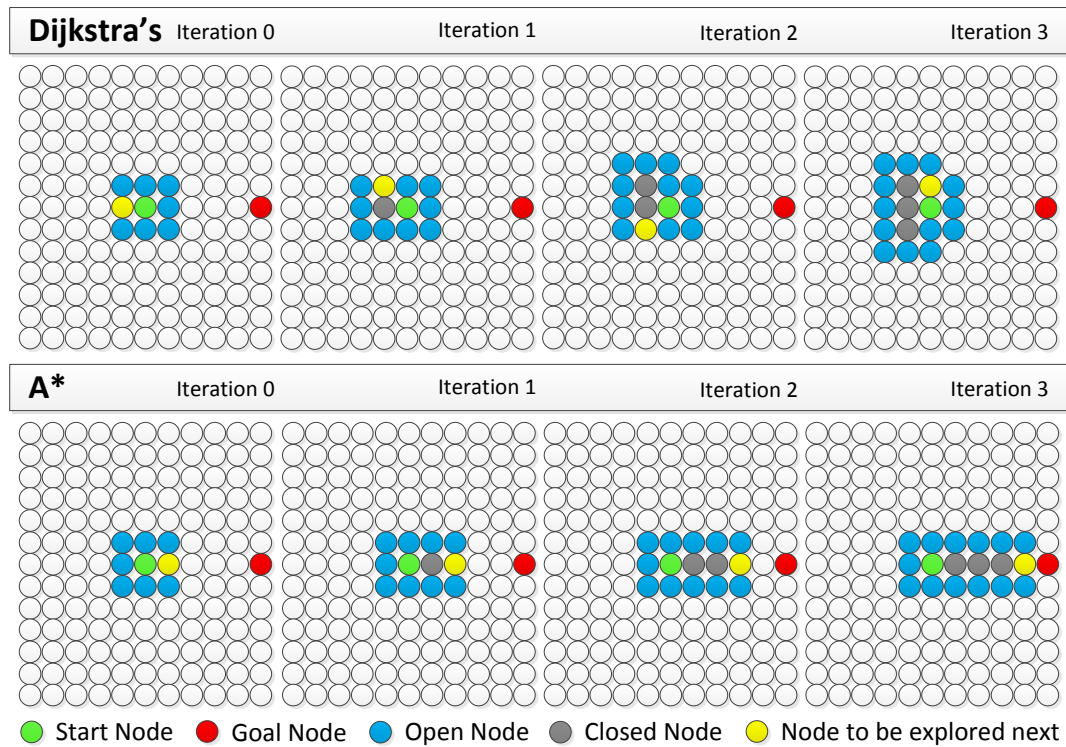
a) Node selection for Dijkstra's algorithm compared to A* for an example search
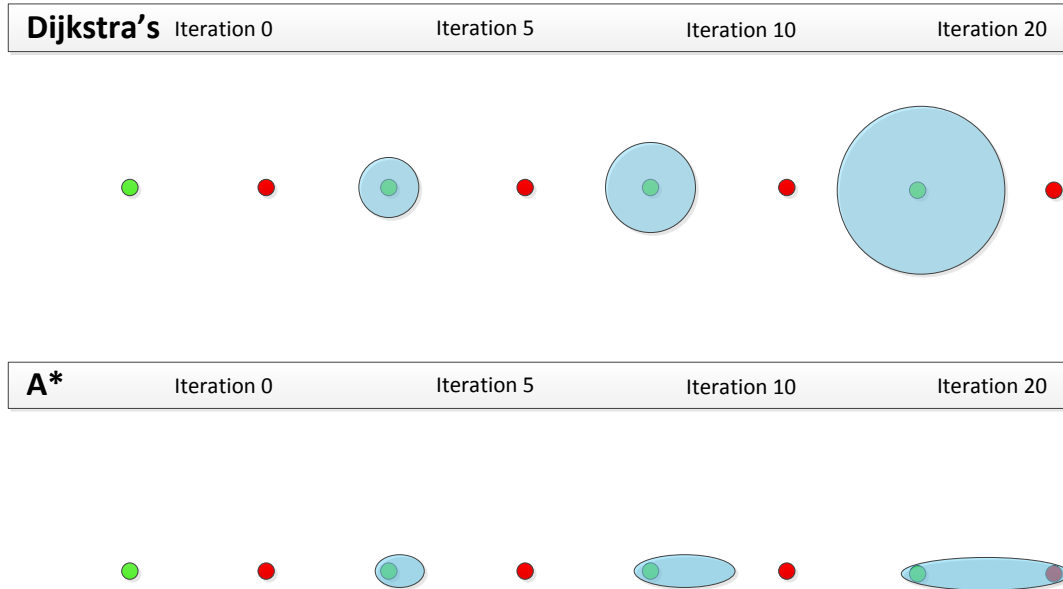
b) Overall search space exploration of Dijkstra's algorithm compared to A*

Figure 5.2: A comparison of search space exploration between Dijkstra's algorithm and A*

During open node selection two open nodes may have the same F cost but different G and H values. In such a case, a tie breaking strategy can be employed to further affect the search behavior. A good tie-breaking strategy to use is to prefer the node with the lowest G value (i.e. nodes that contain shorter paths to the start node). Tie-breaking on the G values can potentially reduce the total search space exploration performed even further [3].

Node exploration in Dijkstra's algorithm should ignore closed nodes. In the case of A*, node exploration occurs based on the heuristic value, which is an estimation of the closeness to the goal. This estimation may often be overly optimistic, and may cause nodes further from the start node (higher G value) to be explored before nodes closer to the start node (lower G value), resulting in the exploration of closed nodes whose G values are overly high and which can still be reduced. This means that closed nodes cannot be ignored as in Dijkstra's algorithm and need to be checked and updated if necessary.

Unfortunately, this introduces the concept of re-exploring closed nodes. Node re-exploration did not occur in Dijkstra's algorithm. Depending on the search problem and search space, the degree of node re-exploration can be quite large. This node re-exploration increases the computational cost of the A* algorithm, but has no effect on the memory costs. Even with this increase in computational cost, A*'s total node exploration (explored nodes + node re-explorations) is still significantly less than that of Dijkstra's algorithm [7] [3].

The A* algorithm terminates as soon as the goal node is selected from the open list (i.e. the goal node has the smallest F cost of all open nodes). This is the same termination condition discussed in the Section 5.2.3, which was stated as being guaranteed to return optimal solutions. This is not true in the case of A*. Since node selection is performed on the F cost and not just the G cost, then A* will need to be run until the open node with the smallest G value has a higher G value than the goal node in order to maintain the guarantee of optimality. This is effectively the same termination condition as used for Dijkstra's algorithm and will exhibit the exact same final search space exploration as Dijkstra's algorithm, thereby nullifying all performance gains offered by A* [3].

It has been shown in [7] that the A* algorithm is guaranteed to be optimal, using the termination condition of Dijkstra's algorithm, if an admissible heuristic function is used. The definition of an admissible heuristic is given in [7] as a one that consistently underestimates the closeness of any node to the goal. If the heuristic value is an underestimate then the F cost will be biased towards the

G value since the H value will always be smaller than the actual distance to the goal. The G-biased F-costs mean that the A* algorithm will prefer to explore nodes closer to the start node, increasing both the search's exploration and the time needed for the search to complete. If the heuristic is always an underestimate of the actual distance to the goal node, A* is guaranteed to return the optimal path (the exact same path returned by Dijkstra's Algorithm).

The A* algorithm is both complete and optimal (when using an admissible heuristic) as well as offering significant improvements over Dijkstra's Algorithm. The pseudo-code for the A* algorithm is given in Listing 5.3.

```
1) Clear Start Node's parent and heuristic values (F, G, H)

2) Add start node to open list

3) While open list is not empty and N is not the goal node

  3.1) Remove node N with the lowest F cost from the open list

  3.2) For each neighbor node S of N
    3.2.1) Calculate the new G value ( N's G value + traversal cost)

    3.2.2) If S is unexplored
      a) Set S's G to new G value
      b) Calculate new H value
      c) Set S's parent to N
      d) Add S to open list

    3.2.3) Else if S is on the open list and S's G is greater than the new G
      a) Set S's G value to new G value
      b) Set S's parent to N

    3.2.4) Else if S is on the closed list and S's G is greater than new G
      a) Set S's G value to new G value
      b) Set S's parent to N
      c) Remove S from closed list
      d) Add S to open list

  3.3) Add N to closed list

4. Return no path exists
```

Listing 5.3: Pseudocode for the A* algorithm

5.3.2 The Heuristic Function

In ensuring A* optimality, an underestimating heuristic function must be used, this is not to say that overestimating (and so inadmissible) heuristics are of no use. If a heuristic function overestimates the difference between a node and the goal, then the F-cost of that node becomes biased towards the H value. Using an overestimating heuristic function, the A* algorithm will prefer exploring

nodes closer to the goal node resulting in a more directed search pattern headed straight towards the goal node. Since the search is more directed, less of the search space, especially surrounding the start node, will be explored resulting in reduced processing and memory costs.

Unfortunately, using an overestimating heuristic removes the optimality property of the A* algorithm meaning that even though solutions will be found quicker, they may now be sub-optimal [83]. Figure 5.3 clearly shows the effect of overestimating and underestimating the search heuristic on the final search space exploration of the same search problem. In the example presented in Figure 5.3, the final solutions are identical, but this is not guaranteed to always be the case. It is the opinion of [3] as well as [83] that a slightly overestimating heuristic's advantages to both performance and memory costs can outweigh the disadvantages and so is a valid method of optimizing the A* algorithm.

Underestimating Heuristic   Overestimating Heuristic

Open List   Closed List
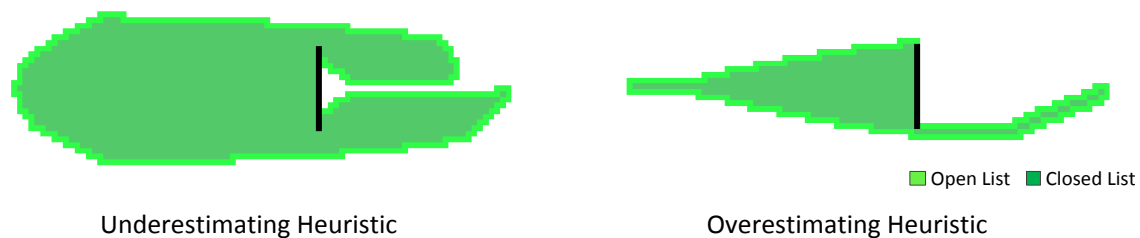
Figure 5.3: The effects of inadmissible A* heuristics

More complex heuristic functions exist, such as the "dead-end" and "gateway" heuristics presented in [98] as well as the cluster heuristic presented in [3]. These complex heuristic functions are not based on the distance between nodes but rather make use of graph spatial information in calculating the heuristic value. Such heuristic functions require a computationally expensive pre-processing step to construct the necessary heuristic information as well as increased memory costs due to the heuristic data storage.

Even though these heuristics can offer benefits in reducing the search space exploration and thereby the overall search time, they are unsuitable for use in dynamic environments as each environmental change will require a complex and expensive update to the heuristic data. Furthermore, to a large degree these complex heuristic functions are rendered redundant through the use of hierarchical approaches to pathfinding which also leverage graph spatial information to reduce search space exploration (refer to Chapter 7 for more details).

5.3.3 Optimizations

In addition to the expensive open list operations performed by the A* algorithm, the computational cost of the heuristic function needs to be discussed. In general, the list operations are the major contributor to the processing costs of solving a search problem, but in certain cases the processing costs of calculating the heuristic function for each node encountered can outweigh that of the list operations [3]. Due to the high degree of variance between search problems and search environment, it is impossible to make a concrete statement regarding the division of processing costs between these two contributors.

The heuristic function is decoupled from the actual algorithm as it is simply a numeric distance estimate between two nodes in a graph. As such, the optimization of the heuristic function can also be performed separately from the A* algorithm. Since the heuristic functions vary greatly, there is no common method of optimization. Heuristic functions need to be chosen so that they minimize the overall explored search space during a search as well as keeping the computational cost of calculating the heuristic value low.

The cost of open list operations has been briefly discussed in Section 5.2.2. These list operations generally account for the majority of the processing costs incurred when searching. As such, open list optimizations have a significant impact on the overall processing costs (and consequently the search time as well) of the A* search algorithm [83] [3].

As in Dijkstra's Algorithm, the only method of removing nodes from the open list is through the selection of the "best" node on the list (i.e. lowest F cost). Since the cost of finding and removing the best open node is always incurred, optimizing the removal of the best node is the first step in optimizing the overall open list operation. Furthermore, during the exploration of the best open node, an additional **M** list operations may be performed (where **M** is the number of successor nodes).

These additional list operations are made up of either list insertions (in the case of unexplored successor nodes or if a better path is found to an already closed node) or list updates (in the case of a better path being found to an already open node). As such, the open list has only four operations that may be performed on it during the execution of the A* algorithm (listed in order of increasing cost): search, removal, insertion and update.

There are various possible implementations of the open list, each with different computational costs per list operation. The first and simplest method of implementing the open list is to make use of an unordered linked list (a dynamic array may also be used). Since the list is unordered, a full search is required each time the best node is required, i.e. at each iteration. The computational cost of such a search is O(N). The benefit of using an unordered structure is that node insertions and updates require no additional care, and are therefore performed in constant time, O(1). Unfortunately, when searching large search spaces, like those present in video games, the open list rapidly grows in size meaning that the O(N) cost incurred at each iteration can quickly become prohibitively expensive.

In an attempt to reduce the overall cost of finding and removing the best open node from the open list, an ordered (best to worst) list may be used. This reduces the cost of finding the best node to O(1) since the best node is always the first node in the list. Due to the list ordering, when a node is inserted or updated the open list needs to be searched to find the correct position of the new or updated node. Consequently node insertions and updates now have a worst case O(N) cost.

The list insertion/updates costs varies depending on the inserted/updated node's F value. Nodes with better F values will have lower costs since they will be inserted closer to the front of the list i.e. their required position will be found faster. In a standard 2D grid in which each node has eight neighbors, the higher update and insertion costs mean that the algorithm may potentially need to perform eight O(N) operations per iteration. This is in stark contrast to the single O(N) operation required in an unordered list. The higher cost of the insertion and update operations makes using an ordered list implementation a potential de-optimization in regards to complexity and is not recommended.

The number of successor operations, per explored node varies, but the removal of the best node always occurs. To holistically optimize the open list, it is necessary to use an implementation that offers lower computational costs for both the removal of the best element as well as the node exploration operations. The recommended implementation of the open list in regards to overall performance is the priority queue data structure [102] [3] [83] [103] [2] [104]. A priority queue is a data structure that contains an unordered collection of elements with the property that the best element is the first element.

Priority queues may be implemented in various ways as shown in [105] and [104]. The skew heap (binary tree) data structure is shown to offer the most efficient implementation [105]. A priority queue implemented using a binary tree is shown to have an amortized cost for each operation of O(log N) [106]. This means that an open list implemented as a binary tree based priority queue will have amortized worst case costs for the removal of the best node, node insertions and updates of O(log N). By using a priority queue with a O(log N) cost for all operations, the effects of an ever growing open list are significantly reduced when compared to the previous two implementations, both featuring O(N) costs for various list operations.

A further A* specific algorithm optimization is presented in [107] offering significant improvements for the A* algorithm and any A* variants. The optimization is simple: when a node is expanded its successors' F values are checked against the parent node's F value. Since the F values of an expanded node are monotonically non-decreasing [108] and the node with the lowest F value is always explored, it is likely that that successor will be expanded next if that successor node has the same F value as the parent node. Since it is known that this successor will likely be expanded next, the expensive insertion and removal open list operations are redundant and that successor is immediately expanded without ever being placed on the open list. Such a successor is said to be expanded fast. This optimization reduces the computational complexity of the A* algorithm at no cost to memory or solution optimality [107].

The primary performance concern with the A* algorithm is not one of computational complexity but rather of memory usage. Numerous later algorithms such as IDA* [8], MA* [109] and SMA* [10] all use the high memory cost of the A* algorithm as the primary motivation for their research. It is shown in [8] that the A* algorithm could not solve any of the given problems due to the algorithm exceeding the hardware limitations on the test platform. Memory sizes have increased significantly since 1985 and the memory cost of the A* algorithm is not as limiting as it once was however this is not to say that A*'s memory cost may not be problematic in certain memory constrained environments.

The primary memory cost of the A* algorithm is in the fact that the algorithm allocates memory for each node encountered in the graph (for the algorithm specific per-node data). The only method available for the reduction of the memory cost for the classical A* algorithm is through the use of a heuristic function that reduces the total number of encountered nodes.

Even though the memory cost is thought to be an issue, the search spaces provided within the video game environment are drastically smaller than those of common academic problems such as the 15 puzzle or a Rubik's cube solver [103]. The most basic per-node data structure size for the A* algorithm is 16 bytes (4 bytes for each of the G, H, parent and list flag values) requiring 16MB of memory to store all the nodes in an 1024x1024 search space.

As mentioned previously, the A* algorithm greatly reduces the number of nodes encountered during the search. When using an upper-bound estimate of search space exploration of around 30% for a search [110], the required memory is reduced to 4.8MB which is trivial on a PC platform and can still be regarded as acceptable on a modern console such as the XBOX 360. Large memories are ubiquitous in modern computing, therefore the A* algorithm should not be discarded based on its memory costs. In fact, A* it is still the primary algorithm in use in the video game pathfinding [4] [111] [102].

There are methods of reducing the total memory cost of the A* algorithm at the expense of processing costs. Additional F and H values do not need to be stored per-node but can simply be calculated each time they are needed. In calculating the H values as needed, the heuristic function to needs to be evaluated for every single encountered node irrespective of whether it has been previously encountered or not. The evaluation of the heuristic function for each encountered node greatly increases the processing costs of the A* algorithm. The calculation of the F cost at each encountered node is cheaper by comparison to the calculation of the H value, but the processing costs associated with the summation of the G and H values can easily add up for complex searches. Due to the increase in processing costs, these memory reduction techniques should only be used when absolutely necessary.

5.4 Iterative Deepening A*

The Iterative Deepening A* (IDA*) algorithm was presented in 1985 [8] as a low memory optimal and admissible alternative to the A* algorithm. It has held some interest from the game development industry as outlined in [103] and [3]. Contrary to the name, IDA* has very little in common with A* and is more akin to a standard depth first search (DFS). The full pseudo-code for the IDA* algorithm is given in Listing 5.4. The IDA* algorithm is very simple: the algorithm consists of a series of depth-limited depth first searches (originating at the start node) with an increasing depth-

limit (referred to as the threshold) for each subsequent iteration of the algorithm. DFS searches are performed recursively.

The initial threshold value is set to that of the estimated cost to the goal (the heuristic value H). This threshold value limits the total search space available for a DFS. A threshold-limited DFS will only explore nodes with a total estimated cost value F, where $F = CSF + H$, that is less than or equal to the specific threshold value for that search. If an admissible heuristic is used, the initial threshold value guarantees that the threshold-limited search space will not contain the goal node since the heuristic will always underestimate the distance to the goal. Whenever a DFS search completes unsuccessfully, it means that the goal node is not contained within the threshold-limited search space, requiring the search space to be enlarged by increasing the threshold value. The threshold value is increased by setting it to the minimum F value encountered during the DFS that exceeds the current threshold value. In doing so, the algorithm is guaranteed to also explore all nodes in order of increasing F costs, thereby ensuring that the algorithm will always find the shortest path (with the lowest F value). If an admissible heuristic is used, then IDA* is guaranteed to be optimal [8].

Path generation is performed once the goal node is found during the unwinding of the DFS recursion. Once a DFS search finds the goal node, a list is created containing only the goal node. As each level of recursion is unwound, the node from which that level of recursion originated is added to the front of the list. Once all the recursion levels are unwound, the list will contain the complete path from the root node to the goal node.

The primary benefit of the IDA* algorithm is its memory cost: the algorithm does not keep records of previously explored nodes, having no per-node storage costs. The memory cost of the algorithm is linear to the number of nodes present in the shortest path [11]. This means that memory is only allocated per recursion of the DFS (i.e. only once per node) and freed once the DFS backtracks. This is a trivial amount of memory when compared to the memory cost of the A* algorithm but this severe memory has some serious performance implications.

In not maintaining any data from previous iterations, each DFS has to be restarted from the start node. This means that the search space explored by the DFS at any given iteration will be re-explored by each subsequent iteration. This results in an extremely high level of node re-exploration and has significant effects on the overall computational efficiency of the algorithm. While A* node re-explorations did not result in heuristic recalculation, IDA* requires that the heuristic function be

evaluated per node encountered by the DFS. The heuristic value can be stored per node to decrease processing costs, at the consequence of increasing the overall memory cost of the algorithm.

Even though the author in [8] claims that IDA* will explore a similar number of nodes as A* and is faster than A* due to removal of the node storage overhead [8], this is shown not to be the case anymore [11]. The authors in [11], compared a memory enhanced IDA* implementation (ME-IDA*) [9] against a standard A* implementation. The ME-IDA* algorithm, when compared against a standard IDA* implementation reduces total node exploration (explored + re-explored nodes) by more than 50% and the overall computational time by more than 30% [9]. This reduction in node exploration is a result of the algorithm storing CSF values for each encountered node.

When a node is encountered that has a better CSF than the current CSF value, that node is ignored preventing the ME-IDA* algorithm from continuing a known sub-optimal search. When benchmarked, the optimized ME-IDA* algorithm was shown to be more than 19 times slower than the standard A* algorithm as well as exhibiting a total node exploration several orders of magnitude larger than standard A*. The reduction in memory cost is simply not worth the massive increase in node re-exploration and heuristic function re-evaluations as well as the resulting decrease in computational performance.

5.5 Simplified Memory-Bounded Algorithm

The simplified memory-bounded algorithm (SMA*) is a simplification of an earlier memory-bounded search algorithm (MA*) [109]. Search algorithms primarily use memory as a means of remembering favorable nodes in the graph thereby reducing the amount of node re-exploration performed [10]. In reducing node re-exploration, algorithm performance is increased. The MA* algorithm is presented as a fixed memory variant of the A* algorithm, in which, the least favorable node from the open list is culled once a predetermined memory limited is reached. In culling the least favorable node, information is lost, resulting in the algorithm has a high degree of node re-exploration to counter this loss of information.

To reduce the degree of re-exploration, a node estimated cost backup system is implemented to try and maintain information regarding culled nodes [109]. The node backup system presented in MA* is extremely complicated and will not be discussed here. SMA* is presented as a simplified version offering a lower computational cost [10].

```
IDA*

1) Set Path to null

2) Set Root Node to Start Node

3) Set Threshold to H value of the Start Node

4) While Goal Not Found

  4.1) Perform Depth First Search at Root Node with Threshold -> DFS(Root, Threshold)

  4.2) If Goal Node not found

    4.2.1) If no F Cost found greater than threshold return no path exits

    4.2.2) Set Threshold to smallest F cost found that is greater than Threshold

5) Return path created by DFS recursion unwinding

Depth First Search (DFS)

1) If Node N is the Goal Node

  1.1) Add Goal Node front of Path
  1.2) Return Goal Found

2) If Node N's F value is greater than the threshold cut off branch by returning Goal Not Found

3) Else for each successor node S of N

  3.1) Perform a depth first search with S and Threshold -> DFS(S, Threshold)
  3.2) If Goal Found returned

    3.2.1) Add node N to front of Path
    3.2.2) Return Goal Found
```

Listing 5.4: Pseudocode for the IDA* Algorithm and depth first search.

The SMA* node backup functions as follows: when a node's successor is created, the estimated cost of the successor is propagated to the successor's parent (this is true for SMA*), then to the successor's parent's parent and so on. While the successor may be pruned in a future step, the node backup allows the algorithm to maintain information about how favorable each node's successors may be, thereby introducing a metric for the favorability of re-exploration of nodes. If a successor's parent has a favorable cost backup value, the parent node may be reinserted into the open list from the closed list, facilitating node re-exploration. The node backup system unfortunately requires that the open list be stored as a binary tree of binary trees to allow the propagation of these successor costs, which further increases the overall computational complexity of open list operations.

The overall MA* and SMA* algorithms are complex and have higher per-node memory costs than A*(one extra F-cost value per node for SMA*, three extra F-cost values per node for MA*).

Furthermore, the node exploration costs are extremely high, estimated at an upper bound of 3.5 times that of A* with the estimated exploration costs for SMA* to be 17.5 times worse [10]. Experimental results show significant improvements in the number of nodes explored by SMA* (at a very small memory cost) when compared against the IDA* algorithm. Increasing the memory limit of SMA* will reduce the number of node re-explorations performed. Unfortunately, the high computational costs of SMA* make the algorithm unsuitable for use in a time constrained environment. This is further augmented by the fact that an ME-IDA* implementation can also provide significant reductions to the degree of node re-exploration at a similar memory cost but offers much cheaper node exploration costs [9]. The reduced node re-exploration rate of ME-IDA* results in reduced computational costs when compared to IDA*.

5.6 Fringe Search

The fringe search algorithm [11] is presented as a means of dealing with the inefficiencies of the IDA* algorithm. For each iteration of the IDA* algorithm, the DFS search is restarted from the root node, requiring that the algorithm re-explores every node until the previous threshold is reached before exploring any unseen nodes [8]. The IDA* algorithm's inability to detect previously explored nodes is its greatest weakness, resulting in an extremely high degree of node re-exploration.

The fringe search algorithm is an IDA* variant that stores the frontier (or fringe) at the end of each threshold-limited search, allowing the subsequent higher threshold search to restart at that fringe. In storing the fringe at the end of each search, the fringe search algorithm removes the need to restart the search, consequently removing the high level of node re-exploration.

Overall algorithmic operation is similar to IDA* in that the algorithm performs a series of threshold-limited searches. As with IDA*, the search threshold limit is increased to the minimum F value encountered during each subsequent search and the threshold limit continues to be increased until the goal is found. The fringe search algorithm makes use of two lists during operation: a "now" and "later" lists. The "now" list is similar to the A* open list in that it contains all the nodes that need to be explored for a specific search iteration. The "later" list serves as the fringe storage containing all the encountered nodes whose F values were greater than or equal to the search threshold.

Each threshold-limited search operates as follows: The head (N_{head}) of the 'now' list is removed (popped) and its F value examined. A threshold-limited search completes once the 'now' list is

empty. The F value of N_{head} is compared against the current search threshold and if the F value is found to be larger or equal to the threshold, N_{head} is added to the back of the 'later' list. In the case of N_{head}'s F value being less than the threshold, N_{head} is expanded and its successors are added to the front of the 'now' list, awaiting immediate examination.

At the end of the threshold-limited search, the 'now' and 'later' lists are swapped, the search threshold increased, and the search restarted on the new 'now' list. The fringe search algorithm also uses the memory enhancement offered by the ME-IDA* algorithm [9] in that CSF values are stored for each node. Any nodes visited that have an existing CSF value that is lower than the current CSF calculated are ignored. In this way the exploration of non-optimal paths are removed from the search. The complete pseudo-code for the fringe search algorithm is given in Listing 5.5.

The order of insertion into the 'now' and 'later' lists determines the node expansion order of the fringe search algorithm. Addition of a node's successor to the front of the 'now' list results in that node's successors being immediately examined, analogous to performing a standard depth first search (only without the need for recursion). Addition of the nodes to the rear of the 'later' list ensures left to right expansion of the nodes. When making use of that list insertion order, the fringe search algorithm has the exact same node expansion order as IDA* [11].

The fringe search algorithm does not require that the 'now' or 'later' lists be sorted, and therefore has the same low node expansion costs as IDA* due to the fact that the 'now' list is unsorted. This is highlighted in the fact that fringe search offered reduced search times (around 25% shorter) compared to A* even though fringe search expanded and visited a much greater number of nodes than A* [11]. The high costs of keeping the A* open list sorted outweigh the reduced computational costs offered by the A* algorithm's reduced node expansions.

Fringe search is both complete and optimal for the same reasons as IDA* (refer to Section 0) while offering improved performance. The resulting computational performance is impressive, providing lower search times than highly optimized versions of A* and ME-IDA* [11]. Unfortunately since the fringe search algorithm doesn't restart the search at the start node, the method of path generation used in IDA* cannot be used. As such, fringe search needs to maintain parent values at each node just like Dijkstra's algorithm and A*.

```
Fringe Search

1) Clear Now and Later List

2) Add Root Node to Now list

3) Set Threshold to H value of the Start Node

4) While Goal Not Found

  4.1) Performed Threshold Limited Fringe Search (TLFS)

  4.2) If Goal Node not found

    4.2.1) If no F Cost found greater than threshold return no path exits
    4.2.2) Set Threshold to smallest F cost found during TLFS that is greater than Threshold
    4.2.3) Swap Now and Later Lists

5) Construct path by backtracking the node parent values from the goal node

Threshold Limited Fringe Search (TLFS)

1) While Now list not empty

  1.1) Pop node N from front of the Now list

  1.2) If N's F value >= threshold

    1.2.1) Add N to the back of the Later list

  1.3) Else if N's F value is < Threshold

    1.3.1) For each successor node S of N

      a) Calculate new G value for S
      b) If new G value is less than S's existing G value or no existing G value present
          i)  Update S's G value to new G value
          ii) Set parent of S to N
          iii) Add S to the front of the Now list
```

Listing 5.5: Pseudocode for the fringe search algorithm and TLFS

Storing the parent value at each node requires that memory to be allocated for every node encountered by the algorithm. Since the number of nodes visited by the fringe search algorithm is significantly larger that the number of nodes visited by A*, the memory costs of the fringe search algorithm will also be greater than the A* algorithm [11].

5.7 Summary

This chapter discussed a variety of graph search algorithms used for finding a path between two nodes in a graph. This chapter highlights the critical fact that search algorithm processing and memory costs are related to the number of nodes explored by said algorithm. Decreasing the search space exploration of an algorithm will result in improvements to both memory and processing costs.

Dijkstra's algorithm was discussed in Section 5.2 as being both a complete and optimal algorithm. Dijkstra's algorithm was not intended for point-to-point searches and so a modification to the algorithm was needed to reduce the processing costs when preforming point-to-point searches. The undirected nature of the search space exploration results in a high degree of exploration resulting in high processing and memory costs. Unfortunately, the high processing and memory costs make it unsuitable for video game pathfinding.

Section 5.3 introduced the A* algorithm as an improvement on Dijkstra's algorithm by directing the search space exploration towards the goal node. This search space direction was facilitated by the use of a heuristic function which acts as an estimation of the likelihood of a node leading to the goal. The A* algorithm is both complete and can be optimal, though optimality can only be guaranteed if an underestimating heuristic is used. Section 5.3.2 discusses various heuristic functions as well as the use of overestimating heuristic functions to speed up graph searches at the expense of solution optimality.

The IDA* algorithm is discussed in Section 5.4 as a complete and optimal low-memory graph search algorithm. Unfortunately, the low memory cost of the algorithm comes with exorbitantly high processing costs. The same can be said of the SMA* algorithm which also offer a significant memory reduction at a high processing cost. The high processing costs of both algorithms (IDA* and SMA*) makes them unusable for use in large video game environments.

The final section presents the fringe search algorithm, which is a form of memory enhanced IDA*. The fringe search algorithm greatly reduces the number of node re-explorations performed, resulting in a greatly reduced overall processing cost. It has been shown to be around 25% faster than the A* algorithm but comes at a greater memory cost. As such the fringe search algorithm is an attractive option for solving the graph search problem when not operating on a limited memory platform.

In conclusion, the A* algorithm still remains the most popular graph search algorithm and its use is ubiquitous in the video game industry. Numerous optimizations can be performed to the algorithm both in terms of reducing memory costs as well as processing costs.

Chapter 6

Continuous Search Algorithms

This chapter reviews continuous graph search algorithms. A short introduction to continuous search algorithms is provided in Section 6.1. The three main continuous search algorithm families are discussed in Sections 6.2, 6.3 and 6.4. The suitability of continuous search algorithms with regards to use for video game pathfinding in dynamic environments is discussed in section 6.5. The final section summarizes the chapter.

6.1 Continuous Search Algorithms

The dynamic pathfinding problem is a fundamental one in the field of robotics [12]. Robot agents are often tasked with pathfinding in unknown or partially unknown environments [12]. Since the agent does not have complete knowledge of its environment, it is impossible for that agent, at any given time, to plan a path that is guaranteed to be valid and simply follow it.

Consider a situation where an agent has only a very basic knowledge of its environment. When given a move order, the agent operates under the free-space assumption in that all unknown nodes are traversable and plans an initial path accordingly. As the agent traverses the environment, the agent will receive more information about the environment through its sensors. Certain nodes that were assumed to be traversable may not be and so the agent's path will need to be corrected to take this new environmental data into account[2].

When a robotic agent encounters an environmental change, the agent's path is replanned and the agent proceeds along the new path. This means that the agent's path is likely to continuously change over the course of the agent's traversal of the path [18] requiring a large number of replanning actions, each replanning action requiring a discrete search.

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[2] Environments which are partially unknown are analogous to highly dynamic video game environments as in both environments, environmental changes may occur over the course of an agent's traversal of the environment. As such, an examination of the applicability of robotic pathfinding search algorithms to dynamic video games environments is necessary.

The costs of running a discrete search for each replanning action can easily overwhelm a pathfinding system resulting in agents' pausing to replan their path each time an environmental change was encountered [13]. A search algorithm is needed that could operate over the course of an agent's path traversal and provide inexpensive localized path corrections as necessary.

This thesis makes novel use of the terms discrete and continuous in the separation of graph search algorithms. Using a discrete search algorithm to solve a problem, will return a solution within an environment at a given time. The algorithm has no means of detecting environmental changes or correcting the solution found if so required. In contrast, a continuous search algorithm will run continuously for the duration of the problem, updating its solution as the environment changes.

Recall, from Chapter 4, that discrete search algorithms are problem-centric in that all environmental knowledge is discarded once a solution to a given problem has been found. In using a discrete search algorithm for replanning, each environmental change will require the search algorithm to re-explore the environment, i.e. restart the search from scratch.

It has been shown that the reuse of environmental knowledge can greatly improve replanning costs [112] [12]. To achieve the re-use of information over the course of the pathfinding problem, a continuous search algorithm is required. This continuous algorithm is further required to interleave path planning with agent movement. An algorithm which interleaves execution (agent movement) with planning is termed an agent-centric algorithm [113].

Continuous algorithms were developed as a means of reducing the re-planning cost incurred in dynamic environments [13]. Although primary developed to deal with dynamic environments, certain continuous algorithms (the real-time family of continuous search algorithms) originated as a means of reducing the memory costs involved in searching large environments [20].

6.2 Incremental Search Algorithms

The dynamic pathfinding problem can be solved by the simple use of a discrete search algorithm such as A*. This means that a new A* search needs to be performed each time an environmental change is discovered by the agent. Since each A* search will be performed atomically, it cannot reuse any of the environmental knowledge of previous A* searches and each environmental change encountered will require that the entire environment be searched again.

This is highly wasteful as a single local change may require a further portion of the environment to be searched again, and so continuous search algorithms have been developed to reuse environmental information across searches. In reusing prior searches' environmental data, the workload of subsequent searches is greatly reduced [21]. Since these continuous search algorithms perform a series of incremental searches, such algorithms are termed incremental search algorithms (also referred to as dynamic search algorithms) [114] [12] [112].

In summary, incremental search algorithms plan an initial complete path and perform (incremental) replanning actions as necessary over the course of the agent's traversal along the path. If an optimal search algorithm is used for the incremental search then each path planned will be optimal, even in fully dynamic environments [15] [112].

Subsection 6.2.1 presents an introduction to the incremental graph search algorithm family. Subsections 6.2.2 and 6.2.3 discuss the two most popular incremental search algorithms: the lifelong planning A* algorithm and the dynamic A* lite algorithm.

6.2.1 Introduction

The most well-known algorithms in the incremental family are the dynamic A* (D*) [12], the focused dynamic A* (focused D*) [13] and the lifelong planning A* (LPA*) [112] algorithms. The D* algorithm was greatly improved after a year by the focused D* algorithm [13]. As such, the term D* is commonly used to refer to the focused D* algorithm and not the actual original D* algorithm.

In 2002, the focused D* and LPA* algorithms were combined with the aim of decreasing the complexity of the D* algorithm. The hybridization resulted in the D* lite algorithm [15]. The D* lite algorithm is significantly simpler and easier to understand than the original focused D* algorithm and is at least as efficient as the focused D* algorithm [15]. The D* lite algorithm is extremely

popular and has been used as a base for several advanced continuous algorithms and are discussed in detail in Section 6.2.3.

There is a recent subfamily of incremental search algorithms originating from the adaptive A* algorithm presented in [16]. The adaptive A* (AA*) algorithm is simpler than other versions of the incremental A* algorithm such as D* and LPA* [16]. Due to its simplicity, the adaptive A* algorithm is easily extensible as shown by the moving target adaptive A* (MTAA*) [115] and the real-time adaptive A* [116].

This family of adaptive search algorithms returns optimal solutions within unknown search spaces as well as within dynamic environments in which the only environmental change allowed is for traversable nodes to become obstructed (or impassable).

In the case of a fully dynamic environment wherein nodes may change their traversability at will, these adaptive algorithms lose their optimality guarantee and have no lower bound on the sub-optimality of the solutions returned. This limits their applicability for use within video game environments. A recent generalized version of the adaptive A* (generalized adaptive A* or GAA*) algorithm was developed to guarantee path optimality within fully dynamic environments [17].

The GAA* algorithm offers worse performance than the D* lite algorithm for static target searches while offering improvements in regards to moving target searches [17]. Since AA* is not suitable for fully dynamic environments and GAA* offers worse performance than D* lite, the adaptive algorithm family will not be discussed further.

6.2.2 Lifelong Planning A* (LPA*)

Almost all continuous search algorithms that can operate within dynamic environments are built upon the D* lite algorithm [19] [21]. D* lite is in turn an LPA* variant that is simpler than and at least as efficient as the standard focused D* algorithm. To discuss the D* lite algorithm, it is necessary to start with a discussion of the LPA* algorithm [112] as the D* lite algorithm is a modified, backwards version of LPA*.

LPA* is the combination of the incremental DynamicSWSF-FP algorithm [117] and the heuristic-based A* algorithm. Fundamentally, LPA* is an incremental version of the A* algorithm that reuses information from previous searches across search iterations.

Per-node data values are roughly the same as for the A* algorithm, namely a CSF G value and a heuristic H value (refer to Section 5.3) with the additional of a new RHS value. The RHS nomenclature originates from the DynamicSWSF-FP algorithm where RHS values were the right hand side (RHS) values of grammar rules.

These RHS values are the minimum next step CSF of a node; except for the special case when the node is the start node, in which case the RHS value for that node is set to 0. The calculation of a node N's RHS value is shown in Equation 6.1.

$$RHS(N) = \begin{cases} 0 & \text{If N is the start node} \\ min_{S \in succ(N)}\big(CSF(S) + traversalCost(S, N)\big) & \text{Otherwise} \end{cases} \quad (6.1)$$

Where node S is a successor of node N and succ(N) is the set of all successors of node N. CSF(N) is the cost-so-far value of node N while traversalCost(S,N) is the cost of moving from node S to node N. The literature for the LPA* algorithm makes use of a node's predecessors for the RHS value calculation but in a standard navgraph (undirected) a node's successors are its predecessors as well, and so for simplicity's sake the mention of node predecessors is omitted.

The next critical concept necessary to understand the operation of the LPA* algorithm is that of local consistency for nodes. A node is locally consistent if and only if its G value is equal to its RHS value. This means that if a node is locally consistent then its G value represents the cost of the shortest path possible to that node from any of its neighboring nodes.

If all the nodes in a graph are locally consistent, the nodes' RHS values equal the exact distances from the start node and so the shortest path to the goal can be traced by starting at the goal node and moving to the neighboring node with the smallest RHS value until the start node is reached. This means that the LPA* algorithm builds a path backwards from the goal node by selecting a node's successor that has the smallest RHS value as the next step in the path.

There are two types of local inconsistencies that can occur: a node is locally over-consistent when its G value is higher than its RHS value and a node is locally under-consistent when its G value is lower than its RHS value. These inconsistencies occur as a result of environmental changes and so need to be corrected at each iteration of the algorithm. A keen reader may have realized that the RHS value

is the minimum of all the 'new G' values calculated for each node successor during A* node exploration.

The LPA* search proceeds in a similar manner to the A* search. The list of potential nodes to explore is contained within an open list. The open list is sorted by the nodes' F values and ties are broken in favor of the lowest G value. When a node is explored, its RHS value is compared to its G value. In the case of an over-consistency occurring, this means that a shorter path to that node has been found.

In that case the G value is set to the RHS value (the new shortest path) and all that node's successors are updated to take the change into account. If the node is consistent or locally under-consistent then its G value is set to infinity (making it locally over-consistent) and the node, as well its successors, is updated. A node update is simply a recalculation of that node's RHS, F and G values. The updated node is then inserted into the open list (or its position in the open list is updated if the node is already present on the open list).

For the LPA* algorithm described above, the initial search performed by the algorithm is exactly the same as an A* search. The exact nodes are explored in the exact same order as would have occurred during an A* search [112]. Subsequent searches simply act to repair any detected local inconsistencies within the graph that may have resulted from environmental changes. When an environmental change is detected, the changed node is updated (added to the open list) and the search is continued. The LPA*'s open list serves to prioritize the repair of locally inconsistent nodes that are part of the solution (just like A* explores nodes that are more likely to be part of the solution first).

The termination conditions of each LPA* search, are as follows: the search terminates once the goal node is locally consistent and the goal's F value is smaller than or equal to lowest F value on the open list (i.e. no node with a potential better route to the goal node exists). A more detailed discussion of the termination conditions and proof of correctness can be found in section 5.1 of [112]. The pseudocode for the LPA* algorithm is shown in Listing 6.1.

```
Calculate Key for Node N
1. G = the minimum of N's G value and N's RHS value
2. F = G + heuristic estimate from N to goal node
3. Return ( F , G )


Update Node N
1. If N not equal to start node calculate RHS value
2. If N is on the open list, remove N from open list
3. If N's G value not equal to N's RHS value (i.e. locally inconsistent)
  3.1 calculate key for N
  3.2 insert N into open List


Compute Shortest Path from Start Node to Goal Node
1. While ( goal node is inconsistent and the goal node's key is greater than the best node's key )
  1.1 Remove the best node N from the open list

  1.2 If N is locally over-consistent (N's G value > N's RHS value)
    1.2.1 Set N's G value to N's RHS value
    1.2.2 Update N's successor nodes

  1.3 Else
    1.3.1 Set N's g value to infinity
    1.3.2 Update N
    1.3.3 Update N's successor nodes

LPA*
1. Set all nodes' RHS and G values to infinity
2. Set RHS of start node to 0
3. Calculate start node's key and insert start node into open list

4. Do forever
  4.1 Compute Shortest Path
  4.2 Wait for environmental changes
  4.3 For all changed nodes C
    4.3.1 Update Node C
```

Listing 6.1: Pseudocode for the LPA* Algorithm

6.2.3 Dynamic A* Lite (D* Lite)

To solve a pathfinding problem as the agent moves through the environment requires that the start node of the search problem changes over time. Unfortunately, the LPA* algorithm estimates all G values as being from the start to the goal. As the agent moves from one node to another, this move invalidates all previous search data as the G values of each node need to be recalculated from the new start node. To do this would be the same as running a new search for each movement of the agent, akin to simply using a discrete search algorithm for each replanning action.

The D* lite algorithm is presented as a means to solve the real-time pathfinding problem (moving agent) in both partially unknown and dynamic environments [15]. To maintain search information

validity even across searches with varied start nodes, the D* lite algorithm simply reverses the search direction of LPA* (searches are now performed from the goal node to the start node) resulting in the per-node G values now representing the cost from goal (CSG) values instead of CSF values. Paths are constructed forwards from the start node to the goal node by selecting the node with the lowest RHS as the next step.

Simply reversing the algorithm's search direction is not enough to ensure the optimality of the algorithm [15], as the heuristic values for each node in the graph will also be inaccurate as the agent's start node changes over the course of the agent's traversal. If the heuristic values change then the ordering of the open list of the D* lite algorithm will be invalidated and so the open list will need to be reordered taking into account the heuristic value changes. D* lite uses a method derived from the focused D* algorithm to prevent open list reordering, by using node priorities that are lower bounds to those used in LPA* [13]. To remove the need for open list reordering, the heuristic function used must satisfy the following properties:

$$H(S, S') \leq traversalCost(S, S') \ and \ H(S, S'') \leq \ H(S, S') + H(S', S'') \tag{6.2}$$

Where H(S,S') is the heuristic value between nodes S and S' and traversalCost(S,S') is the cost of moving from node S to S'. This means that the heuristic function must be an underestimation or exact estimation of the costs between two nodes.

Due to the properties shown in Equation 6.2, when an agent moves from a node S to a node S', the heuristic values will decrease by at most H(S, S'). This means that the F values in the open list will have also have decreased by at most H(S, S').

To correct the F values of the nodes in the open list, each node's F and H values need to have H(S, S') subtracted from them. Since H(S, S') is constant, this means that the ordering of the priority queue will remain unchanged and so the open list will require no reordering upon agent movement. As with LPA*, repairs need to be performed on any node inconsistencies that may have resulted from environmental changes.

These repairs may need to insert new nodes into the open list. Even though the open list ordering is correct, all the nodes' H values (and as a result the F values) are now H(S, S') too high. Updating each F and H value of all the nodes in the open list is an expensive operation and can be avoided. When an environmental change occurs and the agent has changed location (from S to S'), the H(S,

S') value is added to a K_m variable which tracks the changes in heuristic values over the course of the agent's movement. An agent may have moved several steps between environmental changes and so S and S' are not necessarily directly connected.

To prevent the updating of every node in the open list to reflect the decrease in the heuristic values, the K_m variable is simply added to any new nodes' F values when they are inserted into the open list. Adding K_m has the effect of maintaining the consistency of the heuristic values across agent moves. The addition of the K_m value is performed automatically by the node update function (refer to the "calculate key" function shown in Listing 6.2) for new nodes being inserted into the open list.

During the search stage, the node with the lowest F value is selected from the open list and expanded, to maintain node F value correctness, an extra check is performed prior to a node being selected from the open list. The node with the lowest F value is selected and a new F value is calculated (taking into account a possibly modified K_m value). If the old F value is lower than the new F value, then the old F value is updated to the new F value and the node's position is reinserted in the open list. Node exploration occurs in the same fashion as in the LPA* algorithm. The pseudocode for D* lite is presented in Listing 6.2.

The node exploration rate of D* lite is extremely similar to that to that of A* [15] and the per node memory cost is also similar (the exact same per-node data values, differing only in that the parent link of A* is replaced with the RHS value in the D* lite algorithm). These similarities result in the D* lite algorithm's overall memory cost being very similar to that of A*.

In summary, the D* lite algorithm has more in common with LPA* than with focused D*. It is significantly simpler and easier to understand than focused D* and so easier to extend [15]. Furthermore, D* lite is shown to be at least as efficient as focused D*, in many cases being significantly more efficient [15]. The D* Lite algorithm has been successfully used to map out unknown environments as well as for robot navigation [15]. The D* lite algorithm has also served as the base for several state-of-the-art continuous search algorithms (AD* and RTD*) [21] [19].

```
Calculate Key for Node N
1. G = the minimum of N's G value and N's RHS value
2. F = G + heuristic estimate from N to goal node + Km
3. Return ( F , G )


Update Node N
1. If N not Sgoal then calculate RHS value
2. If N is on the open list, remove N from open list
3. If N's G value not equal to N's RHS value (i.e. locally inconsistent)
  3.1 calculate key for N
  3.2 insert N into open List


Compute Shortest Path from Start Node to Goal Node
1. While ( Sstart is inconsistent and Sstart's key is greater than the best open node's key )
  1.1. Set Kold to the best open node's key
  1.2. Remove the best node N from the open list

  1.3. if Kold < calculateKey(N)
    1.3.1. Insert N into the open list

  1.4 Else if N is locally over-consistent (N's G value > N's RHS value)
    1.4.1 Set N's G value to N's RHS value
    1.4.2 Update N's successor nodes

  1.5 Else
    1.5.1 Set N's G value to infinity
    1.5.2 Update N
    1.5.3 Update N's successor nodes


D* Lite

1. Set the last position Slast to the start node Sstart
2. Set all nodes' RHS and G values to infinity
3. Set RHS of Sgoal to 0
4. Calculate Sgoal's key and insert start node into open list
5. Set Km to 0

6. Compute Shortest Path

7. While (Sstart != Sgoal )
  7.1. Sstart = minimum( G value + traversal cost ) of all of Sstart's successor
  7.2. Move to Sstart
  7.3. Check for any environmental changes

  7.4. If ( environmental changes have occurred )

    7.4.1. Set Km to Km + H(Slast, Sstart)
    7.4.2. Set Slast to Sstart
    7.4.3. For all changed nodes N
      a) update node N
    7.4.4. Compute Shortest Path
```

Listing 6.2: Pseudocode for the D* Lite algorithm

6.3 Anytime Search Algorithms

An anytime search algorithm is a variant of the incremental search algorithm with a key difference. Whereas an incremental search algorithm will generate an optimal solution and correct the solution by replanning a new optimal route when environmental changes occur, an anytime algorithm will begin by generating a grossly sub-optimal solution and proceeds to improve the solution over time [19] [18]. The justification for the incremental improvement of a grossly sub-optimal route is to allow the algorithm to always return a solution irrespective of the total time allowed for planning.

For a given time constraint, it may not be possible to perform an optimal search for each planning action and so a sub-optimal solution is preferred over no solution at all [18]. Anytime algorithms attempt to provide a sub-optimal path quickly and improve it as much as the allocated planning time allows. The anytime repair A* (ARA*) algorithm presented in [18] is an improvement on the anytime A* algorithm presented in [118] by presenting sub-optimality bounds for the solutions.

The ARA* algorithm operates by performing a series of fast incremental searches using an over-estimated heuristic function (refer to Section 5.3.2 for a discussion on the effects of the over-estimation of the heuristic function). Information from past searches is reused by subsequent searches in a similar manner to that used in the GAA* algorithm.

The method of over-estimation used in [18] and [19] is a simple multiplication of the heuristic value by an inflation factor termed ε, where $\varepsilon > 1$. The level of over-estimation of the heuristic function (the ε value) is reduced for each subsequent search until the heuristic function is admissible (i.e. exact or under estimate; achieved by setting $\varepsilon = 1$).

In this manner, the ARA* algorithm will eventually return an optimal solution if it is given sufficient time to complete. In the case when there is insufficient time to perform a search with $\varepsilon=1$, a sub-optimal path is returned whose length is no more than ε times the optimal path length [18]. As such, final optimality of the solution depends on the allocated planning time.

Anytime algorithms are intended for use in complex static search environments, and as such cannot operate within dynamic environments as there is no mechanism present to deal with environmental changes occurring during the planning time [19]. To allow for operation within dynamic environments, the ARA* algorithm was combined with the D* lite algorithm resulting in the anytime dynamic A* (AD*) algorithm [19].

The AD* algorithm allows for anytime solution improvement within dynamic environments. At the current time, the author of this thesis is not aware of any other anytime pathfinding algorithms capable of operating within dynamic environments. The AD* algorithm is essentially the same as the ARA* algorithm in that it performs a series of heuristic searches with decreasing inflation factors. The difference between ARA* and AD* is subtle, in that when environmental changes are detected, any locally affected nodes are placed back onto the open list and the search for the current inflation value is repeated (the path is repaired) exactly as in the D* lite algorithm. In the case of substantial environmental changes, it may be more efficient to replan a more sub-optimal path (higher inflation value) than repair the path at the current level of sub-optimality [19]; in such a case, the AD* algorithm increases the heuristic inflation factor and re-plans at a higher level of sub-optimality and then proceeds to improve the resultant sub-optimal path in the normal fashion.

The AD* algorithm offer no performance or memory cost improvements over the D* lite algorithm. What it does offer is an anytime capability at the cost of sub-optimality. Furthermore, there is an expensive open list reordering between searches that may be removed by the inclusion of a bias term (K_m) as in the D* lite algorithm. In a video game environment, there is no anytime requirement in place as the pathfinding workload may be distributed over a number of frames, making the added complexity of the AD* redundant when compared to the simplicity of the D* lite algorithm. The added complexity as well the fact that AD* offers no performance or memory benefits over the D* lite algorithm within the required application results in there being no benefit in using the AD* algorithm for a video game application.

6.4 Real-Time Search Algorithms

In an attempt to solve larger search problems by using a heuristic search algorithm (previously unsolvable due to the memory limitations of the time), the real-time A* algorithm, a real-time heuristic algorithm, was developed by Richard Korf in 1990 [20].

Real-time heuristic search algorithms make use of depth limited look-ahead search around the agent's current node and determine the next best move using a heuristic function. The depth limit placed upon the look-ahead search is referred to as the search horizon (or look-ahead depth). As such the agent will always choose the best move visible to it, where the range of visibility is defined by the search horizon.

In only selecting the best move from within a limited visibility, agents face the risk of heading towards a dead end or selecting sub-optimal moves. To overcome these problems a real-time heuristic search algorithm will update (or learn) their heuristic function with experience. This led to the development of the learning real-time A* algorithm (LRTA*) [20], which has become the basis for the real-time heuristic search algorithm family. Real-time search algorithms places limits on the memory and the computational cost of each agent planning action.

Real-time planning actions are atomic, fast and use little memory. Since an agent decides its next move only once the previous move completes, no path is generated. An agent will wander the environment selecting the best move it can see until it reaches its goal. Furthermore, agents will often backtrack upon discovering that the route followed has led to a sub-optimal solution [20]. This results in final paths, constructed by tracing the agent's movements from the start to the goal, that are highly sub-optimal [21] [119] [120].

Real-time algorithms are also intended for use in known, static graphs and are unsuitable for use in dynamic environments [20] [21]. Real-time algorithms can typically handle finding out that a previously traversable cell is not blocked, but have no means of dealing with the converse, that is, when a previously blocked cell becomes traversable [21].

Current research has attempted to hybridize a real-time search algorithm with an incremental search algorithm for use within dynamic environments resulting in the real-time dynamic A* (RTD*) algorithm [21]. RTD* is another hybridization of the D* lite algorithm, except that in this case, the incremental D* lite algorithm is combined with a state-of-the-art real-time algorithm: the local search space learning real-time A* (LSS-LRTA) [121].

RTD* will not be discussed in detail as it only offers a performance improvement over the D* lite algorithm in unknown static worlds and worse performance than D* lite in known dynamic worlds (such as those commonly found within video games) [21].

Due the inherent unsuitability of this family of algorithms for use in dynamic environments, real-time algorithms will not be discussed in any more detail and interested readers are referred to [20], [120], [119], [122], [123] for more information on the topic.

6.5 The Suitability of Continuous Search Algorithms for use in Dynamic Video Game Environments

Numerous authors casually mention the suitability of their continuous search algorithms for use within video games [120] [116] [21] without any substantiation of their claims. The basis of these statements is that algorithms can successfully operate within video game navgraphs as well as the fact that video game maps are usually used as test environments for these algorithms. Unfortunately, while the algorithms may be able to successfully operate on video game environments, making a broad statement on the suitability of the algorithms without taking into account the performance and memory restrictions of the game platform is short-sighted and potentially misleading.

To explain why continuous search algorithms may be potentially unsuitable in video game environments, it is necessary to discuss the reasons for the development of continuous search algorithms. As continuous pathfinding search algorithms originate primarily from the robotics field, it is necessary to briefly discuss the differences between the two pathfinding problems. A detailed discussion regarding the robotics field is beyond the scope of this thesis and interested readers are referred to [2] for more information.

The primary difference between robotics and video game pathfinding is the availability of environmental information to the agent. In robotics, robotic agents (robots) are often required to navigate unknown or partially known environments. These robots are required to plan an initial path based only on what little environmental information the agent has at that time as well the free space assumption which states that any unknown locations in the environment are traversable [19].

This means that a robot's primary path has no guarantee of correctness and the robot is required to continuously update (or correct) its path as the robot receives more information regarding the environment during its traversal. This continuous update requirement has led to the development of continuous pathfinding search algorithms [12]. In contrast, game agents operate in fully known environments meaning that the agents are not required to make any assumptions when planning paths. Furthermore game agents are immediately notified of environmental changes irrespective of the location of the environmental change.

The second difference between the two pathfinding problems is at a system level. Robots each have their own processing hardware and memories dedicated to their AI systems. Each robot's AI system is only responsible for that single robot and so most robotic pathfinding search algorithms are agent-centric [113]. An agent-centric algorithm maintains data relevant to that specific agent and cannot be used for any other agent [123].

On the other hand, video game AI systems operate in multi-agent environments and are responsible for control of numerous game agents. Since video game AI systems are multi-agent systems, the processing time and memory available need to be split across all the game agents. In splitting the processing power and memory across multiple agents, much tighter performance and memory constraints are placed upon the game AI system. As will be shown, the use of agent centric algorithms (especially pathfinding algorithms) in multi-agent environments is extremely problematic from a memory cost viewpoint.

In the case of pathfinding, the continuous search algorithm is executed over the entire course of the agent's movement from the start node to the goal node in a graph. The memory costs of continuous algorithms have been briefly discussed and are estimated to be similar to those of discrete algorithms [21] [15]. Since continuous algorithms operate over the period of time taken for the agent to move from its start position to its goal position, the associated memory costs of these algorithms are also present for that period of time.

Game agents will move through the game environments at various speeds and so it is not uncommon for path traversals to take extended periods of time to complete. This means that, for the entire time period of the agent's path traversal, the continuous algorithms memory costs are present for that agent. Each agent will require its own memory storage for the necessary data need for the continuous search algorithm employed.

To illustrate this better, consider a standard RTS game in which a game agent is required to move from one side of a moderately size game world to the other. The game world consists of 1024x1024 navgraph nodes and the game agent uses a continuous search algorithm for path planning. Assume that the continuous search algorithm used by the agent has an average search space exploration rate of 20% (similar to A*) and that the agent requires a minute to complete the movement across the game world. Taking an optimistic estimate of 8 bytes of memory for the each of algorithm's per-

node data, the algorithm will use around 1.6MB of memory for the duration of the agent's path traversal.

This is a moderate amount of memory and will easy fit within the memory budgets of most games, even those run on memory limited platforms such as game consoles, but keep in mind this is the memory cost for a single game agent.

Unfortunately, games do not usually feature a single game agent and as mentioned in chapter 4, it is not uncommon for RTS games to contain dozens (if not hundreds) of game agents. When using an agent-centric continuous search algorithm in a multi-agent game environment, each agent will have its own instance of the search algorithm and each search algorithm instance will have its own memory costs.

In a multi-agent environment the memory costs will increase as the number of agents increases. Using the above example and increasing the number of agent to 50, the total memory cost for pathfinding will be around 80MB (50 agents x 1.6MB). A memory cost that will definitely exceed the AI memory budgets of any video game. As such, continuous search algorithms are highly unsuitable for use in multi-agent memory constrained game environments [124].

6.6 Summary

Discrete search algorithms are more problem-centric in that they simply return a solution for a specific problem (a start-goal pair) and nothing more. The processing costs and memory costs are paid once per problem, meaning that a pathfinding system using a discrete algorithm will have a constant memory cost irrespective of the number of game agents present. Unfortunately, once an environmental change is detected, a new discrete search is required in response to the environmental change and so using a discrete search algorithm in a dynamic environment means that each replanning action will require a new discrete search.

Performing numerous discrete searches is analogous to a simplistic incremental search algorithm that offers no data reuse across searches. This results in very expensive replanning actions [13]. When used in a highly dynamic multi-agent environment, the processing load presented by the replanning actions of numerous agents can easily overwhelm the pathfinding system resulting in delayed agent response times.

Continuous search algorithms were developed to reduce the cost of these replanning actions by reusing environmental data across the agent's traversal. The reuse of environmental data works quite well and it has been shown that the processing costs of using a continuous search algorithm for a dynamic environment are several orders of magnitude lower than those of discrete algorithms in the same environment [13].

Unfortunately as discussed in Section 6.5, continuous search algorithms are unsuitable for use within dynamic video game environments due to memory restrictions. Furthermore, the high processing costs of replanning actions when using discrete algorithms make discrete algorithms unsuitable for use within dynamic environments.

Chapter 7

Hierarchical Search Algorithms

This chapter discusses the most common hierarchical approaches to video game pathfinding, namely the hierarchical pathfinding algorithm (HPA*), dynamic HPA* (DHPA*) algorithms, the partial-refinement A* algorithm (PRA*), and the minimal memory (MM) abstraction. This chapter is structured as follows: Section 7.1 provides an introduction to hierarchical approaches and discusses the pros and cons of such approaches. Sections 7.2, 7.3 and 7.4 discuss the HPA* algorithm, PRA* algorithm and the MM abstraction respectively. Section 7.5 summarizes this chapter.

7.1 Hierarchical Approaches

A hierarchical approach to graph search was first presented with the hierarchical A* algorithm [125]. However, it wasn't until the hierarchical pathfinding A* (HPA*) algorithm [22] that the suitability of such approaches for use within video games was shown. Hierarchical pathfinding approaches provide significant reductions both in terms of the processing and the memory costs associated with solving a search problem at the cost of path optimality and an increased algorithm memory cost [92].

The following subsection discusses the technique of problem sub-division as a means of improving performance. The remaining subsections cover the various stages present in a hierarchical search algorithm as well as additional consideration to keep in mind when using a hierarchical search algorithm.

7.1.1 Problem Sub-division

One method to decrease the processing costs of performing a discrete search within an environment is to split the original problem into series of smaller, connected sub-problems [22]. Each sub-problem is defined by a start and goal node. These start and goal nodes are both referred to as sub-goals, in part due to the fact that the goal node of each sub-problem will be the start node for the next sub-problem in the series.

Furthermore since these sub-problems are smaller and simpler than the initial overall problem, the processing and memory costs required in solving each sub-problem are trivial when compared to the costs of solving the overall problem [22].

Figure 7.1 illustrates the splitting of a search problem into smaller sub-problems as well as illustrating why splitting the problem into sub-problems offers significant cost reductions. An example pathfinding problem is shown in Figure 7.1a. The optimal solution for that problem as found by using an A* discrete search is also shown. In solving the example search problem the A* algorithm has explored around 50% of the total search space. Consider the four smaller problems as illustrated in Figure 7.1c. Solving these smaller problems requires very little search space exploration.

The reduced search space exploration results in lower processing and memory costs. If the four sub-problems' solutions are connected to one another, a solution to the original problem is created, (refer to Figure 7.1b). The total search space examined in solving these sub-problems is a small fraction of the original search space explored in solving the original problem.

Since each sub-problem is self-contained, the memory costs associated with solving the search problem are freed once a solution for that sub-problem has been found when using a discrete search algorithm. This means that the peak memory usage in solving the overall problem by subdivision into sub-problems is the maximum of the peak memory usages encountered when solving the sub-problems. Looking at Figure 7.1c, the largest search space explored in solving a single sub-problem is significantly smaller in comparison to the total search space explored in Figure 7.1a.

It is important to note that the solution presented in Figure 7.1b differs to the solution shown in Figure 7.1a. In fact, the solution in Figure 7.1b is slightly sub-optimal. The sub-optimality of the solution in Figure 7.1b occurs as a result of the selection of the sub-goals defining the sub-problems. If the sub-goals lie on the optimal path then the returned solution will be optimal. This means that the sub-optimality of the returned solution is dependent on the selection of the sub-goals that define the sub-problems.

To keep the solution sub-optimality low, sub-goals must be well chosen, but unfortunately it is impossible to find the optimal sub-goals ahead of time without performing an expensive optimal graph search. As such all techniques that split the overall problem when solving it are inherently sub-optimal, i.e. all solutions returned will exhibit a degree of sub-optimality.

Hierarchical approaches create a hierarchy of abstract graphs from the base navgraph. These abstract graphs are then used for the selection of sub-goals allowing a hierarchical approach to quickly and cheaper split a problem into sub-problems. Each abstract graph is created in a manner similar to that in which the navgraph was created from the 3D environment (refer to Chapter 3 for more details).

Hierarchical approaches make use of environmental knowledge to create abstract nodes (representing the sub-goals). These abstract nodes are then interconnected and form the final abstract graph. Searching the abstract graph will result in an abstract path or simply put a list of sub-goals defining the sub-problems needed to be solved to create a solution to the original problem.

As such, hierarchical approaches all have three key stages:

- **The Build Stage:** The build stage is responsible for creating an abstract graph hierarchy from the navgraph.
- **The Abstract Planning Stage:** The abstract planning stage searches the abstract graph to find an abstract path. This abstract path determines the list of sub-problems that need to be solved.
- **The Path Refinement Stage:** The path refinement stage refines the abstract path found during the abstract planning stage into a low-level path.

To put it plainly, the build stage creates and connects the sub-goals. The abstract planning stage then identifies the sub-goals to be used in solving a specific problem. The path refinement stage solves the sub-problems originating from the sub-goal identified in the abstract planning stage and combines their solutions into a single low-level path. This low-level path is the final solution to the original problem.

a) The search space explored to solve the overall pathfinding problem

b) The combined search space required to solve the same problem using subdivision

c) The search space required to solve each of the smaller sub-problems
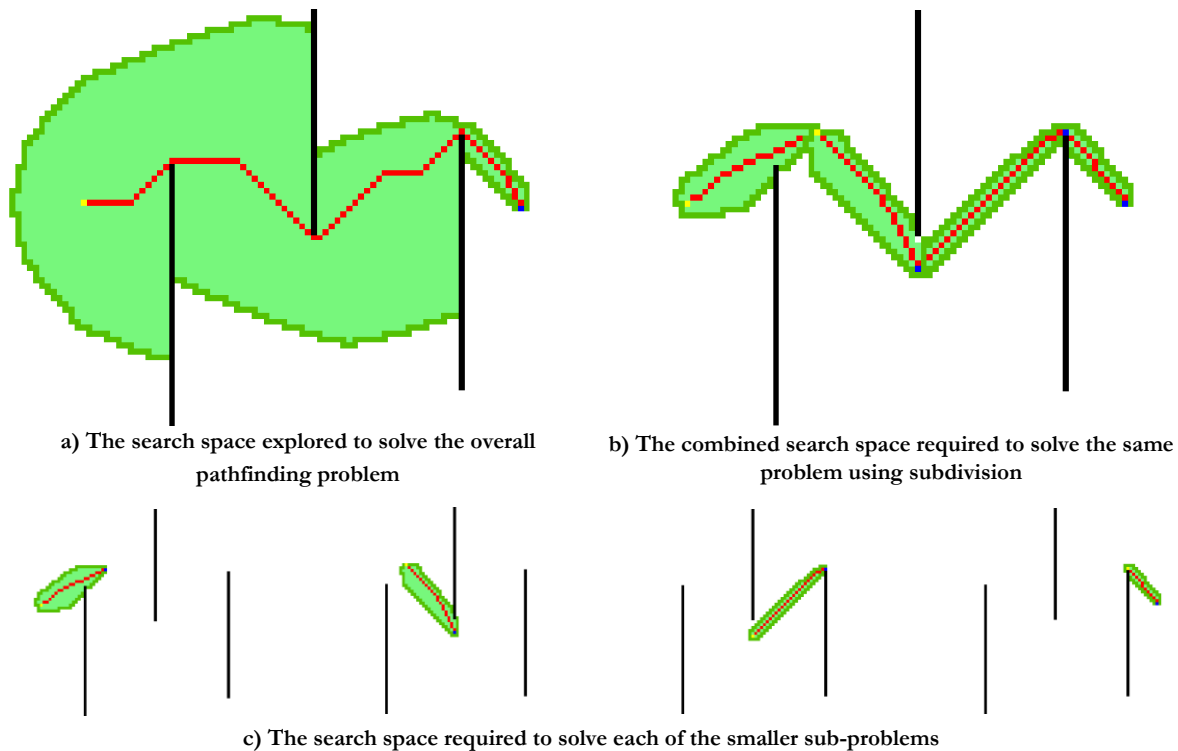
Figure 7.1: The subdivision of a large pathfinding problem into smaller sub-problems

7.1.2 The Abstraction Build Stage

When dealing with the build stage, there are various abstraction techniques that can be used to build the abstract graphs. These abstraction techniques primarily differentiate the various hierarchical pathfinding approaches [22] [25] [24]. The abstract graphs created by any of the various abstraction techniques will be significantly smaller than the underlying navgraph from which the abstract graph was created, often containing several orders of magnitude less nodes [22] [24].

Typical abstraction techniques make use of environmental features and represent entire regions with a single abstract node [22] [25]. Other techniques such as the connection hierarchies (CH) presented in [126] make use of the actual node topology in their abstractions.

The processing costs for building the abstract graph vary depending on the abstraction technique employed. In certain cases like HPA* and C, the processing costs are expensive and so the abstract graph construction is performed offline.

When operating within dynamic environments, any environmental changes will result in a change to the abstract graph as well. As such, abstract graph modifications need to be performed during runtime, requiring a short simplified build stage to perform any necessary corrections to the abstract graph. This is an important consideration when choosing an abstraction technique for use in dynamic environments as complex abstraction techniques will result in higher abstract graph update costs.

In extremely large environments, it may be beneficial to create several layers of abstraction (i.e. create an abstract graph from an already abstract graph) to reduce the processing costs even further. Unfortunately, the added complexity of implementation for multi-level abstraction hierarchies as well as the additional memory costs for storing the abstraction can outweigh the minimal processing cost improvements introduced [23] [22].

This is not always the case, as was discussed in [126] where a second level of abstraction was required to ensure that pathfinding actions completed within the time constraints placed upon a commercial video game. When dealing with a dynamic environment, having additional levels of abstraction further increases the costs of environmental changes as all levels of abstraction will need to be checked and potentially corrected. The benefits of higher levels of abstraction are highly situational and need to be evaluated per application.

A term often encountered when dealing with abstraction techniques is granularity. Granularity refers to the severity of the search space reduction performed during abstraction. A coarse granularity technique would reduce the search space drastically, for example replacing an entire 16x16 portion of the navgraph with a single abstract node. Such coarse techniques lose a lot of the low-level detail and are not accurate representations of the underlying graph.

The minimal memory abstraction (refer to section 7.4) is a good example of a coarse abstraction technique. A fine granularity technique on the other hand would result in a much smaller degree of search space reduction. Finer techniques such as the "clique and orphans" abstraction technique used by PRA* (refer to Section 7.3) keep many of the underlying graph features, though at a much higher build cost. In general, finer abstractions represent the underlying graph in more detail at the expense of low search space reduction and increased build costs.

7.1.3 The Abstract Search Stage

Once the abstract graph has been created from the underlying navgraph, the abstract graph becomes the starting point for all path planning actions. When presented with a pathfinding problem, an abstract path needs to be created which will define the number of sub-problems needed as well as the sub-goals for each sub-problem. In the following explanation of the operation of a hierarchical algorithm, only a single layer of abstraction is assumed.

The first step to finding an abstract path is to find the two nearest and reachable abstract nodes (start$_{abstract}$ & goal$_{abstract}$) to the low-level start (start$_{navgraph}$) and goal (goal$_{navgraph}$) nodes of the overall problem. The nearest abstract nodes to the original start and goal nodes are usually found by performing a series of discrete searches originating from the original start and goal nodes.

A discrete search is then performed on the abstract graph to find the abstract path between the abstract start$_{abstract}$ and goal$_{abstract}$ nodes. Since the abstract graph is of small size, performing this abstract search is trivial from both a processing and memory cost perspective when compared to searching the underlying navgraph [22]. The abstract path returned from the abstract planning stages contains a list of sub-goals between the start$_{navgraph}$ and goal$_{navgraph}$ nodes as well as the start$_{navgraph}$ and the goal$_{navgraph}$ nodes which are positioned at the start and end of the abstract path respectively.

7.1.4 Path Refinement

To create the final solution, the abstract path needs to be refined (or translated) into a low-level path. Since each node in the abstract path represents a sub-goal, the transition (or edge) between two abstract nodes defines a sub-problem.

The two abstract nodes are the start and goal nodes of the sub-problem. As the abstract path is traversed, the edges between the abstract nodes are refined into low-level paths by solving the sub-problem (refinement action). This procedure is termed path refinement.

To solve a sub-problem, a discrete search is performed on the underlying navgraph. The discrete search action results in a low-level path. This low-level path is only a portion of the final low-level path and so is termed a partial path. Both the sub-problems and the abstract search problem can be solved using any discrete graph search algorithms.

Figure 7.2 illustrates a simplified example of solving a search problem, similar to the problem presented in Figure 7.1, through the use of a hierarchical approach. The navgraph is abstracted using a very simplistic abstraction technique which divides the navgraph into 10x10 regions and places a single sub-goal within each region.

Three such neighboring regions are shown in Figure 7.2, in which a path is required from the yellow start node to the purple goal node. The green and blue nodes are identified as the closest reachable abstract nodes from the start (yellow) and goal (purple) nodes respectively.

An abstract search is performed on the abstract graph (the abstract graph is not shown for simplicity's sake) and returns the abstract path shown in the figure. This abstract path is then refined into the final solution shown by the beige nodes. The labels within the beige nodes signify the sub-problem from which each of the beige nodes originate from.

Once again the final solution presented is grossly sub-optimal, but this is a result of the simplistic abstraction technique used. The sub-optimality of the final path can be improved through the use of a better abstraction technique.
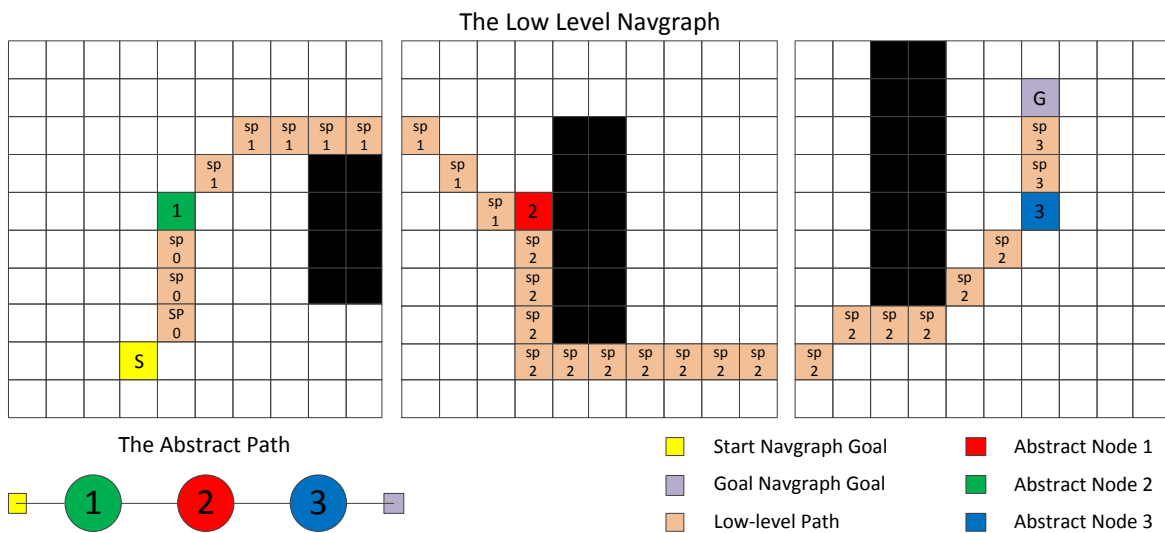
Figure 7.2: A basic overview of a hierarchical search.

7.1.5 Considerations for Hierarchical Pathfinding

Decreased memory and processing costs are not the only benefits offered by a hierarchical approach. An additional benefit is that a complete path is not needed before an agent can begin moving. Since the complete path is made up of a series of partial paths (sub-problem solutions), an agent can proceed along the path as soon as the first partial path is found (i.e. not completing the entire path refinement stage). In allowing the game agent to move before the complete path is planned an agent's response time is decreased.

Furthermore, the next partial path need only be solved once an agent approaches the end of its current partial path. Unlike continuous search algorithms, no information is reused by a hierarchical approach, meaning that all planning actions, be they abstract or refinement, are self-contained and atomic. The atomicity of the planning actions as well as the lack of information re-use allows for the pseudo-continuous operation of a hierarchical pathfinding algorithm over an extended period of time without any memory implications like those encountered with continuous search algorithms.

The low cost of hierarchical planning actions allows for more planning actions to be performed in the same time needed for a single planning action in a non-hierarchical approach. The time taken for the complete path planning (abstract search and complete refinement) using the HPA* algorithm has been shown to be up to nine times lower than a non-hierarchical approach [22]. The low costs of planning actions allow multiple agents to receive complete paths (and so begin moving) in the same time that was previously needed for a single agent to plan a complete path.

As a result, the increased agent responsiveness in multi-agent environments (without any increase in memory costs) makes a hierarchical approach attractive for use in production multi-agent video games. Hierarchical approaches are currently used (with great success) in "Relic Entertainment's" "Dawn of War" and "Company of Heroes" series as well as in "Bioware's" critically acclaimed "Dragon Age" role playing game [126] [91].

7.2 Hierarchical Pathfinding A*

The HPA* algorithm is a discrete hierarchical search algorithm and was the first work to highlight the suitability of a hierarchical approach for video game pathfinding [22]. The HPA* uses a fixed-size clustering abstraction to create the abstract graph. This abstraction is created by first dividing the navgraph into fixed size regions (or clusters). Figure 7.3a shows an example 30x30 game environment, which is then divided into nine 10x10 clusters. These clusters are shown in Figure 7.3b. Each cluster contains within it a fixed size region of the game environment.

a) A test game environment navgraph

b) The game environment divided into fixed size clusters by overlaying a grid onto the base navgraph
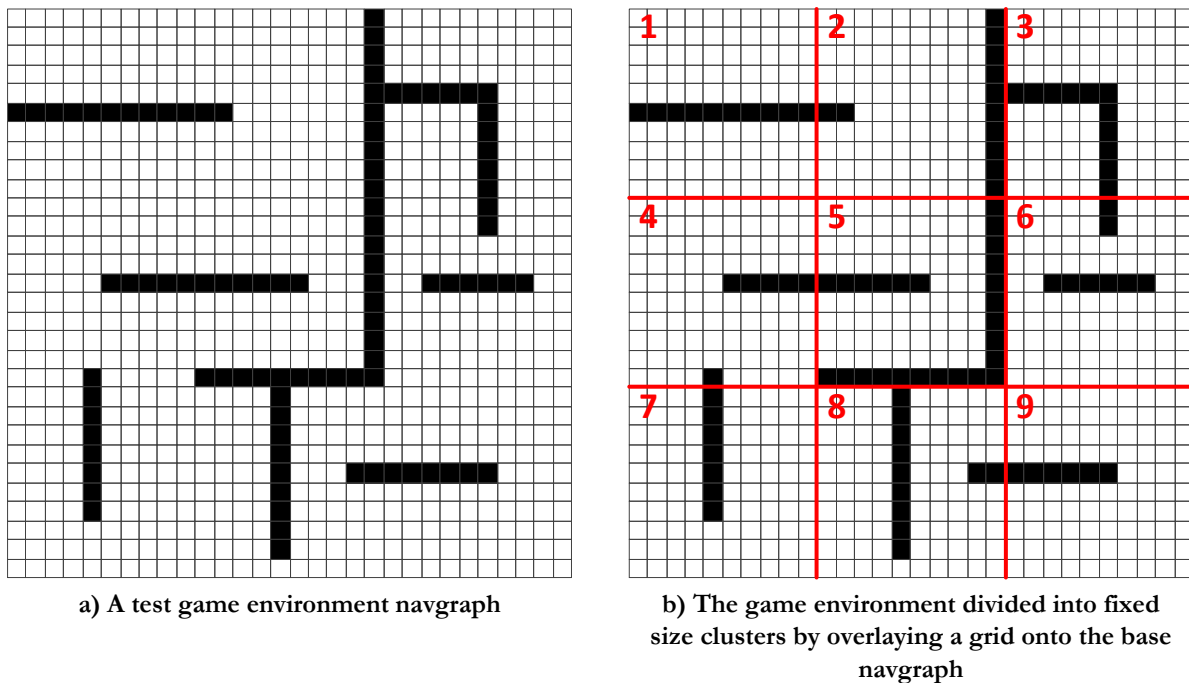
Figure 7.3: The HPA* graph abstraction cluster creation

HPA* uses the connections between clusters to create the abstract nodes. An "entrance" is defined as a "maximal obstacle-free segment along a common border of two adjacent clusters" [22]. For each node on a cluster's border, there is a symmetrical node with respect to the cluster's border. Entrances are created for each segment of traversable nodes on a cluster's border whose symmetrical counterparts are also traversable (i.e. both adjacent nodes are traversable). Figure 7.4a shows the upper right of the example map shown in Figure 7.3, as well as the identification of the entrance segments present between clusters 3, 6 and 9.

The transitions between entrances are defined as follows: if the entrance segment has a length less than a predefined constant (in this case 6), a single transition is created in the center of the entrance segment (refer to the entrances between clusters 3 and 6 in Figure 7.4b). If the entrance segment length is greater than the predefined constant then two transitions are created for the entrance segment, one at each end (refer to the entrance between clusters 6 and 9 in Figure 7.4b). Transitions across clusters are connected by intra-edges and transitions contained within a single cluster are connected using intra-edges. These transitions, intra-edges and inter-edges make up the final abstract graph (shown in Figure 7.4c). The creation of cluster entrances (and their inter-edges) as well as searching for intra-edges has expensive processing costs and so the abstract graph is constructed offline when a game environment is loaded.

In a dynamic environment, the abstract graph is created offline just as in a static environment but needs to be modified as the environmental changes occur. When an environmental change occurs, the HPA* needs to identify the clusters affected by the change and recalculate all the cluster entrances as well as find all intra-edges. The abstract graph is then updated with this new cluster data.

a) The top right hand portion
of the subdivided navgraph

b) The creation of cluster
entrances as well as inter and
intra-edges

c) The final abstract graph
from clusters 3 and 6
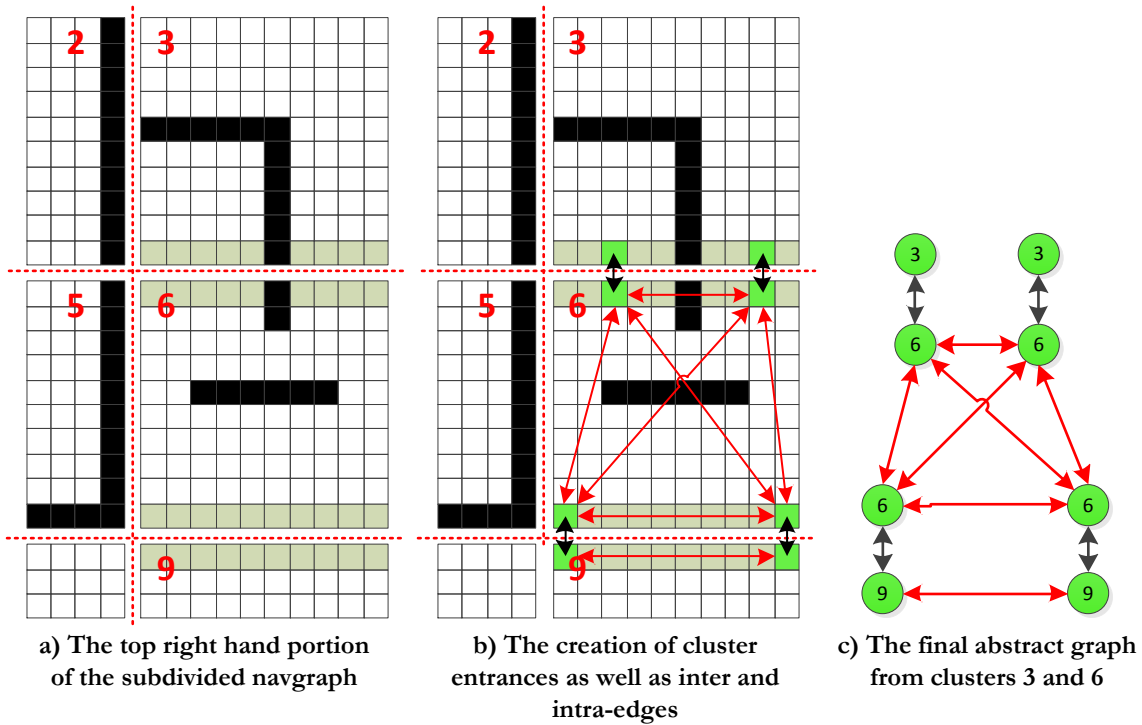
Figure 7.4: The HPA* graph inter-cluster and intra-cluster links

Since the searches performed to find the intra-edges have high processing costs, a lazy approach to their generation is presented in [127]. Since there is a likelihood the changes may not be immediately required, the expensive intra-edge searches are postponed until that abstract node is encountered; at which point the required searches are performed to identify all intra-edges. Simply put, intra-edges are found as needed. Using a lazy approach does not decrease the workload required when environmental changes occur, but simply postpones the work in the hope that intra-edges may not be needed or that the clusters will change again prior to the intra-edges being needed.

HPA* is capable of building multiple levels of abstraction, but it has been shown that increasing the cluster size decreases processing costs more so than adding extra layers of abstraction (which increase the memory cost of HPA*) [127]. The choice of an additional abstraction layer instead of simply increasing the cluster size should therefore be made carefully.

To perform an abstract search using HPA*, [22] propose the insertion of the start and goal nodes into the abstract graph. The insertion is performed by first creating abstract nodes from the start node, then performing a series of A* searches to find the nearest abstract node contained within the same cluster as the start node. Once the nearest reachable abstract node has been found, an edge is created between the abstract node and the start node [22]. The goal node is similarly inserted into the abstract graph. Once both the start and goal nodes are inserted into the abstract graph, the abstract path is found and refined (as described in the previous section). The abstract planning as well as the path refinement stages are both completed before a path is returned. As such, the HPA* algorithm as is cannot return partial solutions and so is a discrete search algorithm.

The abstract graph node insertion performed at the start of each search is problematic due to the cost of the numerous A* searches performed to find the nearest reachable abstract node. The cost of inserting a node into the abstract graph can be reduced by using Dijkstra's algorithm instead of A* to find the shortest path to all neighboring abstract nodes as outlined in [127], but this optimization only works within clusters featuring numerous abstract nodes with long or complex paths between them. In the case of clusters featuring large open spaces or few entrances, using A* searches to find the nearest reachable nodes is faster [127].

A recent paper [23] presents the dynamic HPA* (DHPA*) algorithm which entirely removes the cost of inserting the start and goal nodes into the abstract graph. DHPA* achieves this by maintaining a node cache within a cluster. This cluster node cache stores information regarding the

optimal next node to move to when trying to reach any abstract node within that cluster (i.e. the shortest path to each abstract node) [23]. DHPA* also removes the need for using A* search for path refinement as each node within a cluster already knows the shortest path to any abstract node.

While DHPA* is on average 1.9 times faster than HPA*, it uses B times more memory than HPA* where B is the number of abstract nodes. Unfortunately DHPA* produces less optimal solutions than HPA* [23]. The high memory costs of DHPA* severely restrict the algorithm's applicability for use in memory-limited environments.

The HPA* algorithm has been shown to return paths that are less than 10% sub-optimal [22]. The path sub-optimality results from the placement of the abstract nodes within the cluster entrances, as illustrated in Figure 7.5a. To improve the optimality of the return HPA* solutions, a simple post-processing path smoothing phase is introduced. This smoothing phase simply starts at one end of the path and tries to replace sub-optimal sections in the path with straight lines.

For each node in the path, the smoothing phase sends out rays in all directions. These rays are then traced until an obstructed node is encountered. During this ray tracing, if a node on the returned HPA* path is encountered, the initial sub-optimal path segment between the two nodes is replaced with the straight line between them (see Figure 7.5b). The smoothing phase then continues from two nodes prior to the encountered path node. When this path smoothing phase is employed, the sub-optimality of HPA* is reduced to less than 1% [22].
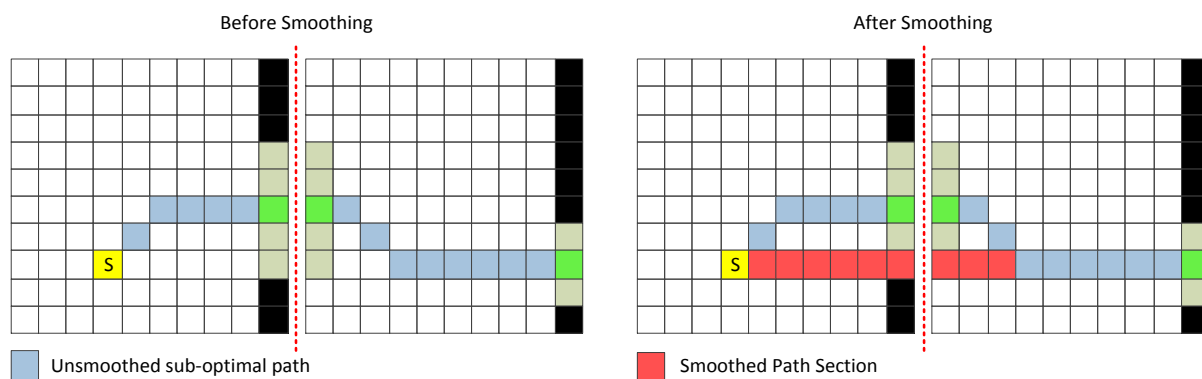
Figure 7.5: The effects of the post-processing smoothing phase on path optimality

Path smoothing can be an expensive operation and so an optimization is introduced in [127] to reduce the processing costs of the path smoothing phase. This is achieved by placing a bounding box around the node from which the rays originate. The bounding box places limits on how far out the rays extend, thereby reducing the processing costs of the ray tracing operation. In decreasing the processing costs, this smoothing optimization trades path optimality for performance. No processing cost comparison was made between the original and the "optimized" smoothing algorithms and only the increase in sub-optimality was shown [127].

7.3 Partial Refinement A*

As mentioned in section 7.1.5, being able to find quick partial solutions is beneficial in regards to agent response times. The HPA* algorithm returns complete paths for each search problem and cannot return partial paths. The HPA* algorithm can be modified such that the abstract path refinement actions are distributed over the course of an agent's movement, thereby allowing HPA* to return partial solutions (termed incremental hierarchical pathfinding).

Unfortunately, this introduces a large degree of sub-optimality that cannot be reduced through post-processing techniques such as path smoothing. The low degree of sub-optimality in the HPA* algorithm is a result of the post-processing smoothing stage. Without this additional stage, the HPA* algorithm returns paths that are on average approximately 6% sub-optimal (smoothing reduces this to under 1%) [22]. The PRA* algorithm was presented as the first partial hierarchical search algorithm, and can find partial solutions that are more optimal than those returned by HPA* [24].

PRA* is based on a different abstraction technique to HPA* and a multi-level abstraction hierarchy is necessary for the operation of the algorithm. To abstract the base navgraph, PRA* uses a "clique and orphans" based abstraction [24]. A clique is defined as a set of nodes wherein each node is both reachable and at most a single step away from every other node in the set (i.e. each node has an edge to every other node in the set). Cliques are shown in Figure 7.6 and are outlined in red. An orphan is a node that is reachable from only a single other node (the blue outlined node in Figure 7.6c). PRA* uses a maximum clique size of 4 in abstracting the navgraph.

PRA* also requires that the navgraph be fully abstracted until only a single node remains at the highest abstraction level. Each clique represents a node on the abstract graph one level above it. Orphan nodes are attached to the cliques leading to the orphan node. Since orphan nodes can only

be reached by a single edge, they can only be connected to a single clique. A complete example of a navgraph abstraction using cliques and orphans is presented in Figure 7.6. This abstraction scheme offers very fine granularity and so at best can only offer a four times reduction in search space over a lower level. In contrast, the abstraction technique used in the HPA* algorithm creates an abstract graph several orders of magnitude smaller than the original. The fine granularity of the abstraction technique also greatly increases the processing and memory costs of creating and storing the abstraction layers.

PRA* performs the path refinement slightly differently to HPA*, the first step being to identify the abstraction level SL in which both the start and goal nodes are abstracted into the same abstract node (refer to Figure 7.6d). Path refinement then commences from level SL/2. This level is chosen by the authors in [24] to provide low processing costs while still maintaining high path optimality.

An A* search is performed at the SL/2 abstraction level. The resulting abstract path is then refined as in HPA*. In completing the path-refinement in one step, a complete solution is returned. This variant is termed PRA*(∞) or infinite refinement PRA*.

a) The first level of abstraction is created by dividing a navgraph into 4-cliques.

b) The first abstraction level

c) The second abstraction level (including an orphan node which is outlined in blue)

d) The final single node level of abstraction. The start and goal nodes are now both contained within the abstract node
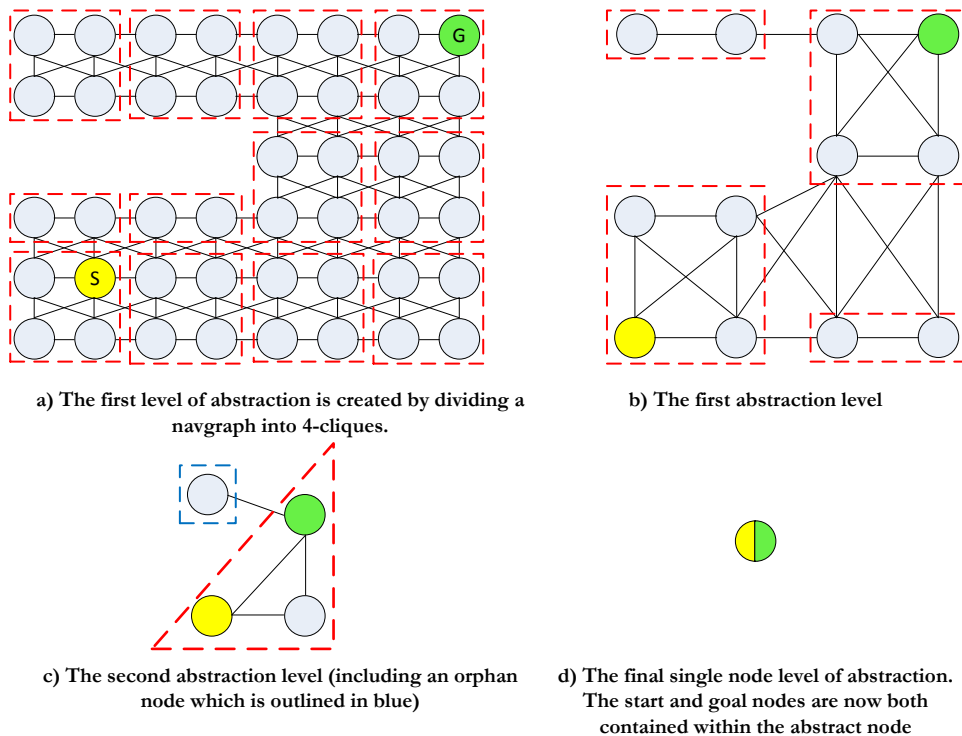
Figure 7.6: The PRA* Abstraction

PRA* uses an extremely fine granularity for its navgraph abstractions compared to that used in HPA*. This means that the search space reductions between abstract levels is significantly lower for the PRA* abstraction than the HPA* abstraction. However the fine granularity also means that abstract paths will contain more-optimal solutions that those within the abstract paths found using HPA* [24] [128]. Since searching a large search space is expensive, PRA* leverages the cost reductions offered by the higher levels of abstraction.

Partial path refinement is possible by simply truncating the abstract path found at each abstract level to a fixed length K and using the tail node of the truncated path as the new goal for lower level refinement. Applying path truncation during refinement is referred to as the PRA*(K) algorithm. The PRA*(K) algorithm returns a partial path in a single atomic planning action while still guaranteeing that an agent following the resulting partial path, as well as subsequently found partial paths, will eventually reach the goal node [24]. By truncating the path at each abstract level, PRA*(K) reduces the scope of the abstract search problem for the lower levels, further decreasing search costs.

The fine granularity allows PRA*(∞) to return paths with a higher optimality (without the need for an expensive smoothing phase) in similar time to HPA*. Paths returned by PRA*(∞) are within 1% of optimal 98% of the time. Furthermore, partial paths can be returned, using PRA*(K), offering only minimal reduction in optimality when compared to the paths returned by PRA*(∞). The path optimality of paths returned by PRA*(K) is dependent on the K value. A K value of 16 was shown to produce complete paths within 5% of optimal 98% of the time.

The fine granularity of the clique abstraction as well as the need for a full abstraction hierarchy makes PRA* unsuitable for dynamic environments. When an environmental change occurs, depending on the severity of the change, all of the abstract layers will need to be recreated. The fine granularity also increases the memory storage of an abstract level when compared to HPA* making the PRA* abstraction unsuitable for memory limited platforms.

As such, the PRA* algorithm is not recommended for use within dynamic environments or on low memory platforms. The idea of returning partial paths instead of a complete solution must not be discarded since it can potentially decrease the amount of wasted effort spent on replanning a complete path each time an environmental change occurs. Partial hierarchical pathfinding will be further examined in Chapter 8.

7.4 Minimal Memory Abstractions

The minimal memory (MM) abstraction approach presented in [25] was developed for use within memory limited environments. The abstraction technique is similar to the HPA* abstraction method in that the first step is to overlay a grid onto the navgraph, dividing the navgraph into fixed sized clusters (referred to as sectors). These sectors are then divided into a set of fully traversable regions, meaning that each node in the region is reachable from any other node in the region. Figure 7.7b shows three sectors divided into full traversable regions. A region is defined as the search space explored by a breadth-first search originating at a traversable node within a sector. Repeating this search for each traversable node not already in a region returns all the regions within a sector.

The centroid node of a region's node set is termed the region center (the red nodes in Figure 7.7b). Abstract nodes are created from these region centers, representing the regions in the abstract graph. The regions within a sector cannot be connected to one another. This means that there are no intra-edges present within sectors. Regions are connected to other regions in adjacent sectors via inter-edges between their region centers (refer to Figure 7.7b). These region centers and inter-edges make up the final abstract graph.

This abstraction method strives to reduce the number of abstract nodes by representing an entire region as a single node unlike the HPA* abstraction technique which uses the sector transitions to create abstract nodes, thereby possibly creating multiple abstract nodes per region. A comparison of the abstraction used by HPA* and the MM abstraction is shown in Figure 7.8. There is a significant reduction in the size and complexity of the abstract graph between the two abstraction techniques, especially evident in sector 2. This reduction in size and complexity decreases both the memory costs of storing the abstraction as well as the processing costs when searching the abstraction.

Pathfinding using the MM abstraction is performed similarly to that of HPA*, with the region centers now being the sub-problem sub-goals. The placement of these region centers has been shown to affect the processing costs of using the MM abstraction [25]. By optimizing region center positions to minimize the search space exploration of all refinement actions possible from that node, the overall search space exploration for the overall search was decreased by at least a factor of two [126]. Further research into optimal region center placement techniques is still needed.

There is a drawback to this simplified abstraction with regards to the optimality of the paths returned. Since region centers are central to the region, using them to plan abstract paths can result in grossly sub-optimal low-level paths (refer to Figure 7.9a). To improve on the sub-optimality of paths returned, a technique for trimming the ends of sub-problem solutions is presented in [25]. Trimming the ends of the sub-problem solutions returned during path refinement results in slightly more direct low level paths (refer to Figure 7.9b).

While this trimming does decrease the path sub-optimality, it can potentially increase the processing costs in refining the sub-problems [25] as well as introducing a degree of wasted effort. This wasted effort results from the fact that when complete sub-problem solutions are planned, then a portion of those solutions is subsequently deleted. Trimming 10% of the sub-problem paths results in final paths that are as little as 5% sub-optimal [25]. Further improvements to path sub-optimality can be achieved by increasing the trimming to 15% and solving two sub-problems per step instead of one. This improvement will not be discussed in detail here and interested readers are referred to [25] for more information.

This trimming technique is supplemented by a post-processing smoothing phase like that present in HPA*. A comparison between the optimality for paths that have been trimmed compared to paths without trimming after this smoothing step has not been performed. As such no conclusions can be made on the necessity of this trimming stage when used in conjunction with a smoothing phase.

In a follow up paper, it was shown that the implementation of this memory efficient abstraction in the critically acclaimed game "Dragon Age" had certain problems [126]. In large game environments, the generated abstract graphs were too large and complex to be searched efficiently. Pathfinding searches would run out of time on these large environments.

In smaller environments the abstraction was too coarse and did not accurately represent the smaller environment. Increasing the granularity would improve path quality (especially for smaller environments), but would also further increase the abstract search times which were already problematic for larger environments [126].

Underlying Navgraph

Top Right Section of Navgraph

a) An example game environment navgraph

b) The top left hand corner of the game environment with the separate regions identified. The green nodes represent the abstract nodes in the final abstract graph

Region Centers

Inter-edges
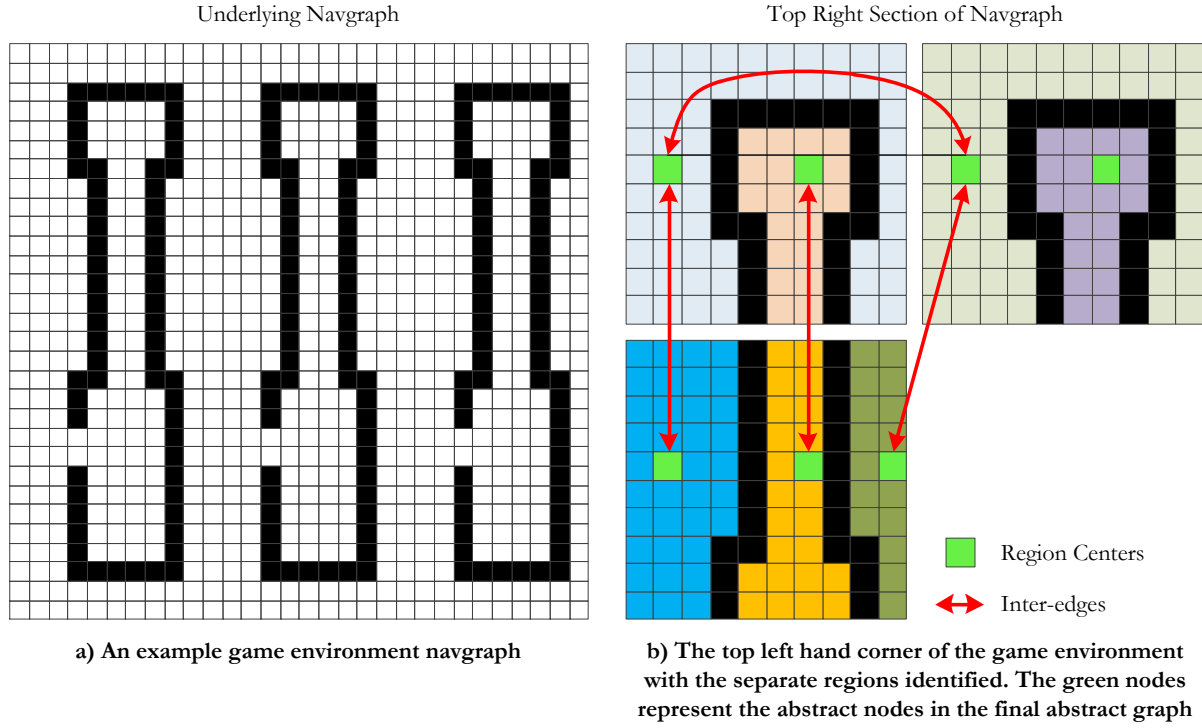
Figure 7.7: The Minimal Memory Abstraction

HPA* Abstraction

MM Abstraction

4 Abstract nodes,
6 Intra-edges,
4 Inter-edges for Sector 1

Entrances

Abstract Nodes

Intra-edges

Inter-edges

a) The HPA* abstraction

2 Abstract nodes,
3 Inter-edges for Sector 1

Region Centers

Inter-edges

b) The MM abstraction
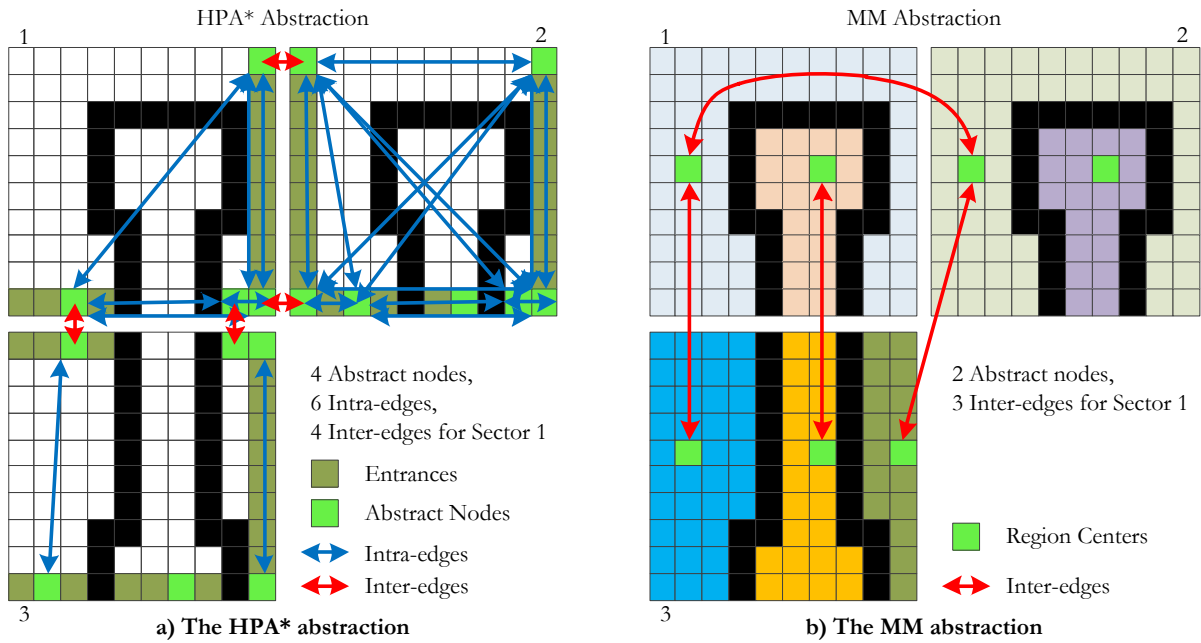
Figure 7.8: A comparison between the HPA* and MM* abstractions. (a). (b).

a) Original sub-problem paths   b) The trimmed sub-problem paths
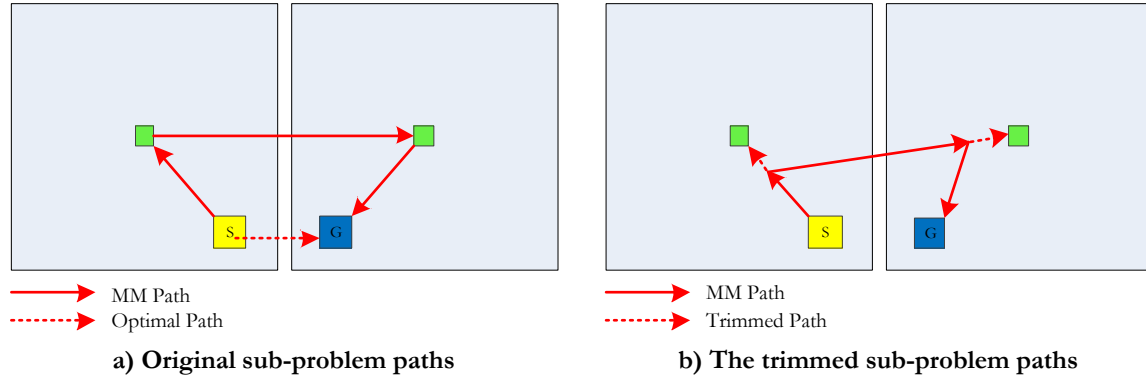
Figure 7.9: Improving path sub-optimality through the trimming of sub-problem solutions

To solve this granularity problem, a second level of abstraction was introduced that allowed for a finer granularity at the first abstraction level while also offering improvements to the abstract search times. Furthermore, the sector sizes were dynamically modified according to map size. The second level of abstraction allowed for the abstract searches on both abstraction levels to complete faster allowing paths to be generated quicker and so decreasing agent response times [126]. As mentioned in Section 7.1.2, the choice for implementing multiple layers of abstraction is situational and needs to be evaluated per application.

The MM abstraction technique is suitable for use within dynamic environments, though the addition of a second level of abstraction may prove problematic with regards to the cost of abstract graph updates as a result of environmental changes. The simplicity and low memory cost of this abstraction technique (even with two levels of abstraction) makes it highly suitable for memory-limited environments.

7.5 Summary

Hierarchical search algorithms are discrete graph search algorithms which greatly improve search efficiency by splitting a problem into multiple sub-problems. Hierarchical approaches using a discrete search algorithm have been shown to offer a search time reduction of up to nine times compared to using the discrete search algorithm alone. In addition to the improved search efficiency, hierarchical approaches also greatly reduce the peak memory usage of the search algorithm.

Hierarchical search algorithms make use of an abstract graph to split the original problem into sub-problems. The abstract graph is created from the base navgraph through the use of an abstraction technique. The techniques of abstraction vary between the hierarchical algorithms.

The first hierarchical pathfinding search algorithm developed was the HPA* algorithm, which uses the connection between fixed size clusters to create the abstract graph. The paths returned by the HPA* algorithm are on average only 6% sub-optimal. The sub-optimality of these paths can be further reduced to be, on average, 1% sub-optimal through the use of a post-processing path smoothing stage. The HPA* algorithm is suitable for use within dynamic environments and depending on the size of the game environment may also be applicable to memory limited environments.

The PRA* algorithm uses a fine granularity in its abstraction technique as well as requiring a complete abstraction hierarchy to be created. There are two PRA* variants: the PRA(∞) variant which return complete paths and the PRA*(K) variant which returns partial paths of abstract length K. The paths returned by PRA* exhibit a very low level of sub-optimality. The PRA* is unsuitable for highly dynamic environments due to the high costs associated with updating the abstraction hierarchy each time an environmental change occurs. The PRA* algorithm is also unsuitable for memory limited platforms due to the need for a complete abstract hierarchy as well as the low level of search space reduction between abstract levels.

The final hierarchical approach is the MM abstraction. This is basically an HPA* variant that makes use of a coarse (and therefore more memory efficient) abstraction technique. Unfortunately, the coarseness of the abstraction technique increases the sub-optimality of the algorithm. To correct the high level of sub-optimality introduced, a sub-problem trimming system was implemented resulting in final paths only being 5% sub-optimal. Due to the simplicity and low memory costs of the MM abstraction technique, it is suitable for use within dynamic environments and on memory limited platforms.

Chapter 8

The Spatial Grid A* Algorithm

This chapter discusses the novel approach for performing discrete pathfinding searches within grid-based navgraphs presented by this thesis: the spatial A* algorithm. Section 8.1 discusses the domain specific nature of video game pathfinding. Section 8.2 presents the spatial grid A* algorithm and variants. Section 8.3 presents an empirical performance evaluation of the spatial grid A* algorithm on a variety of game maps representing various domains. Section 8.4 summarizes this chapter.

8.1 The Domain Specificity of Video Game Pathfinding

The requirements and constraints of the video game pathfinding problem were discussed in detail in Chapter 4. An important fact that was alluded to but not explicitly mentioned was the domain specificity of video game pathfinding. Video game pathfinding problems are domain specific in that the domain, the game and its intended platforms, will set the requirements and constraints for the pathfinding problem [3]. The type of game environments present are determined by the game type and the choice of navgraph representation of the environment is made based on the performance and memory constraints of the game (platform restrictions, number of game agents, etc.).

Even in the case of two games making use of the same navgraph representation, the navgraph topologies and scale can vary between the two games and will affect navgraph representation specific parameters such as the polygon type for navmeshes or the grid cell scale in grid-based navgraphs. Figures 8.1 and 8.2 illustrate four grid-based navgraphs from four popular games. It is clear to see that map topologies change drastically across the games. In fact, map topologies can change drastically within a single genre as "Company of Heroes" and "Starcraft" are both RTS games and yet have radically different map topologies. As such, the choice of navgraph representation, representation parameters and the search algorithm employed is made based on the requirements of a specific game [129]. There is no "silver bullet" pathfinding approach that is suitable for all games.

a) Sample RTS map from the Company of Heroes game

b) The resulting grid-based navgraph

Figure 8.1: Sample map and resulting gridmap from the Company of Heroes RTS game
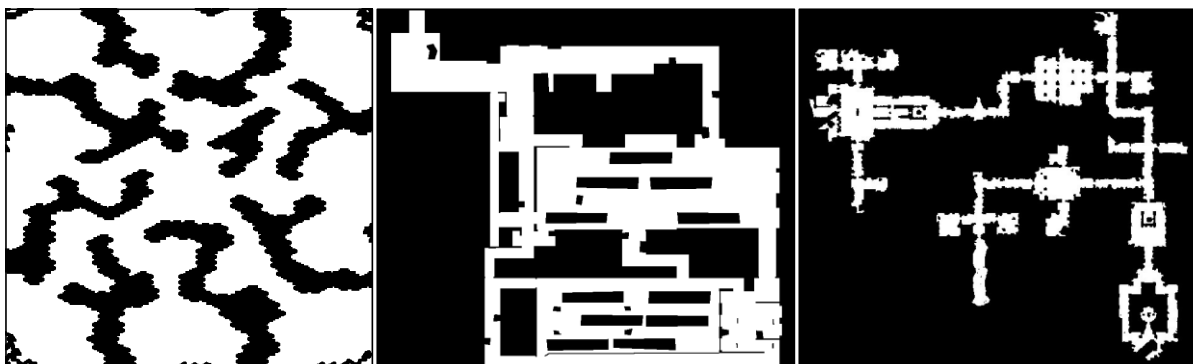
a) 'BigGameHunters' from Starcraft   b) 'de_train' from Counter-Strike   c) 'brc202d' from Dragon Age: Origins

Figure 8.2: Sample maps from various game genres

Another critical factor defining the domain of the pathfinding problem is the dynamicity of the game environments. When dealing with dynamic game environments all previously planned paths need to be replanned upon any environmental changes. The higher the number of game agents present, the higher the number of potential replanning actions that need to be performed per environmental change.

The greater the dynamicity of the environment, the greater the number of path replanning actions that need to be performed. Path replanning is especially problematic in the RTS genre due to the extremely high numbers of game agents present [95]. The only method to mitigate the impact of path replanning on a game's performance is to ensure that path planning actions are performed as quickly and efficiently as possible. Pathfinding search algorithms that minimize the path planning time are absolutely essential for use within dynamic environments.

In addition to the speed of path planning actions, the processing costs of applying environmental changes to the navgraph need to be taken into account. The high costs of performing updates to navmesh based navgraphs limits the use of navmesh in games featuring dynamic environments. This is not to say that dynamic navmeshes do not exist and are not in use.

In fact, Ubisoft Toronto's "Splinter Cell: Conviction" makes use of exactly such an approach since both the scale and dynamicity of the environments present in the game is quite low [130]. Waypoint base navgraphs are highly unsuitable for dynamic environments (as discussed in Chapter 3) while grid-based navgraphs are highly suitable for dynamic environments but contain massive search spaces resulting in increased processing and memory costs when searching grid-based navgraphs.

Hierarchical search algorithms were discussed in Chapter 7 as a means of improving both the performance and the memory costs of discrete search path planning actions especially for large search spaces. The performance and memory cost improvements of hierarchical approaches result from the subdivision of the search problem into several smaller sub-problems, which are then solved individually. In solving each smaller sub-problem individually, hierarchical approaches reduce the total search space explored in solving the overall problem.

This search space exploration reduction makes hierarchical approaches perfect for dealing with the massive search spaces contained within grid-based navgraphs. Furthermore, since hierarchical approaches make use of discrete search algorithms to solve the sub-problems, memory costs are

now paid per sub-problem rather than for the overall problem. This means that the overall memory cost of solving a single search problem is simply the highest (peak) memory cost encountered in individually solving each of the sub-problems.

Hierarchical search algorithms are not without their drawbacks. Hierarchical search algorithms maintain a navgraph abstraction hierarchy which can significantly increase the overall memory costs of the hierarchical search algorithm. Furthermore, the abstraction hierarchy requires an expensive pre-processing step to create, and in the case of a dynamic environment, any environmental change will require an expensive abstraction hierarchy update or rebuild before any path replanning can commence [22].

When using a simple discrete search algorithm such as A* within a dynamic environment, the only difference between a planning and a replanning action is the state of the game environment. Environmental changes require only a modification to the navgraph and in the case of using grid-based navgraphs, navgraph updates are essentially free [83]. Requiring an expensive abstraction update or rebuild upon an environmental change will, to a certain degree, negate the benefits of using grid-based underlying navgraph representation for a dynamic environment.

The spatial grid A* algorithm is presented as a memory reduced version of the A* algorithm for use on grid-based navgraphs. The spatial grid A* will be shown to offer significant improvements (at the cost of path optimality) over the A* algorithm in both performance and memory costs within certain pathfinding problem domains.

8.2 The Spatial Grid A* Algorithm

The spatial grid A* (SGA*) algorithm is presented as a means of performing search problem subdivision on grid-based navgraphs without incurring the processing and memory costs of maintaining a navgraph abstraction hierarchy, thereby keeping the cheap environmental updates afforded by grid-based navgraphs. SGA*'s sub-goal selection is performed by making use of the spatial data contained within a grid-based navgraph rather than a navgraph abstraction. The three SGA* variants are presented below while extensive performance evaluations of the three SGA* variants are presented in Section 8.3.

8.2.1 Spatial Grid A* Variant 1

Grid-based navgraphs differ from other navgraph representations in that both the traversable and blocked areas are stored within the navgraph. As such, grid-based navgraphs are the only navgraph representations which contain the entire game environment and therefore contain more spatial information about the game environment than any other navgraph representation.

This section discusses the basic SGA* algorithm referred to as variant 1. Sub-section 8.2.1.1 discusses the straight line sub-goal selection technique employed by SGA* variant 1. Sub-section 8.2.1.2 discusses the completeness and optimality of the SGA* variant 1 while Sub-section 8.2.1.3 discusses the partial pathfinding ability of SGA*. A discussion into the path optimality and necessity of path smoothing concludes this section.

8.2.1.1 Straight Line Sub-goal Selection

Since the division of the game environment into grid cells is uniform, grid-based navgraphs can simply be represented as two-dimensional traversability maps. Grid-based navgraphs are therefore often represented as 2D bitmap images (refer to Figure 8.1b). The SGA* algorithm is based on this idea of representing a grid-based navgraph as a bitmap image, making use of the spatial data and uniformity of grid cell connections to make decisions regarding sub-problem sub-goal selection.

If a grid-based navgraph (for simplicity's sake henceforth referred to as a gridmap) can be represented as a two dimensional bitmap, then any two nodes (pixels) in the gridmap can be connected by drawing a straight line between them. The line drawing can be achieved using any standard 2D line drawing algorithm such as Bresenham's line algorithm [131]. If no obstacles exist in the path of the line between two gridmap nodes then that line is the optimal path between the two nodes.

In the case when obstacles are present in the path of a straight line between two gridmap nodes, the obstacles will serve to split the line into line segments. These line segments can then be stored in an ordered list. Figure 8.3a illustrates the subdivision of a line into line segments due to obstacles present between two gridmap nodes.

a) Initial line segment creation stage   a) Connection of line segments using A*
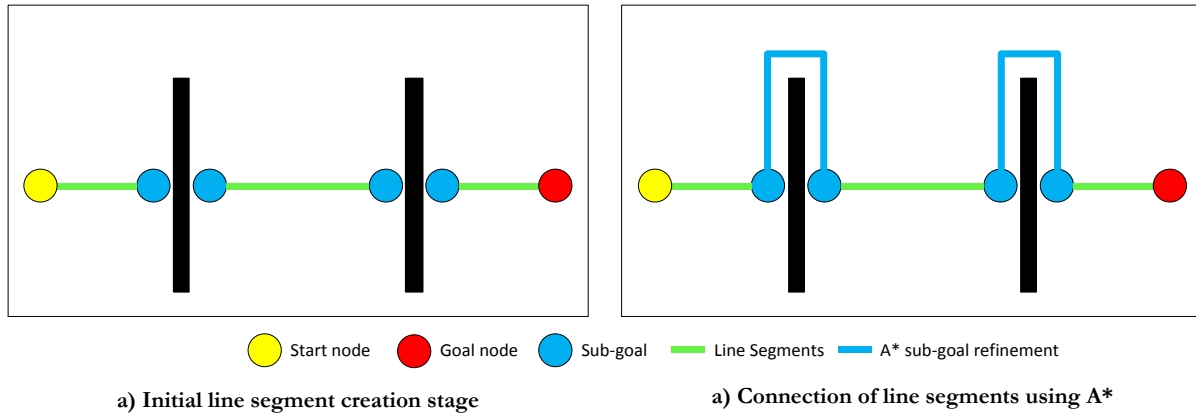
Figure 8.3: SGA* sub-goal creation and refinement

If the line segments can be joined together by finding a path from the end of one line segment to the start of the subsequent line segment then once all the line segments are connected, a path will exist between the two original gridmap nodes (illustrated in Figure 8.3b). As mentioned, if only a single line segment is created then that line segment is the optimal path between the two gridmap nodes and so no searching is required.

This basic line drawing and line segment connection is the basis of the basic SGA* algorithm, referred to as variant 1 (SGA* V1). The motivation behind the development of the SGA* is based on the observed topology of certain RTS maps, specifically the type of RTS maps found in the "Company of Heroes" games. These "Company of Heroes" RTS maps feature large traversable regions containing small scattered obstacles.

An example "Company of Heroes" map is illustrated in Figure 8.1b. Furthermore, each of these large traversable regions is highly connected to the rest of the map meaning that there exist two or more entrances to each region. These types of RTS maps do not feature any winding tunnels/corridors or dead ends (single entrance regions), map features which are ubiquitous in FPS and RPG styles maps (refer to Figure 8.2b and Figure 8.2c).

When performing a standard A* search within maps featuring large traversable regions, the search space exploration may be unnecessarily high when obstacles are encountered towards the end of a path. This unnecessary search space exploration is due to the admissible heuristic which, when presented with an obstacle, will start to explore open nodes closer to the start node rather than open nodes closer to the goal node.

The admissible heuristic used under-estimates the H value of an open node meaning that in most cases the under-estimation will be greater further away from the goal node. Since the H values of open nodes further from the goal are underestimated, the combined F cost (G+H) will often be lower than the F costs closer to the goal resulting in the further open nodes being explored. Figure 8.4a illustrates the unnecessary search space exploration performed away from the goal node.

As mentioned in Chapter 5, this unnecessary search space exploration can be improved by making use of a non-admissible over-estimating heuristic but will result in sub-optimal final paths [3]. SGA* tries to reduce the unnecessary search space exploration by replacing large straight sections within the A* search with line segments and only performing A* searches around obstacles, thereby limiting the overall search areas explored to the immediate area around the obstacle. In reducing the overall search space explored in solving a search problem, the SGA* algorithm offers improvements to both the processing and memory costs of the A* algorithm.

In fact, the SGA* algorithm can simply be thought of as a technique to improve the costs of a standard A* algorithm. Figure 8.5 illustrates the difference in search space exploration between the A* and SGA* algorithms for a search problem within the "Company of Heroes" illustrated in Figure 8.1b. It is important to note that the A* sub-problem searches illustrated in Figure 8.4 and Figure 8.5 are performed backwards. The reasons for this are discussed in Section 8.2.1.2.

Unfortunately, only finding paths between line segment end points result in highly sub-optimal final paths as illustrated in Figure 8.4b. These highly sub-optimal paths are not as problematic and as will be discussed in Section 8.2.1.4 can be converted into near-optimal paths through the use of a post-processing path smoothing step.

Unfortunately, the naïve line drawing technique discussed above performs really poorly within maps featuring dead ends and disjoint regions, requiring modifications to the algorithm to improve performance. The problematic topologies and algorithm modifications are discussed in detail in Section 8.2.2.

a) A* search space
exploration

b) SGA* search space exploration
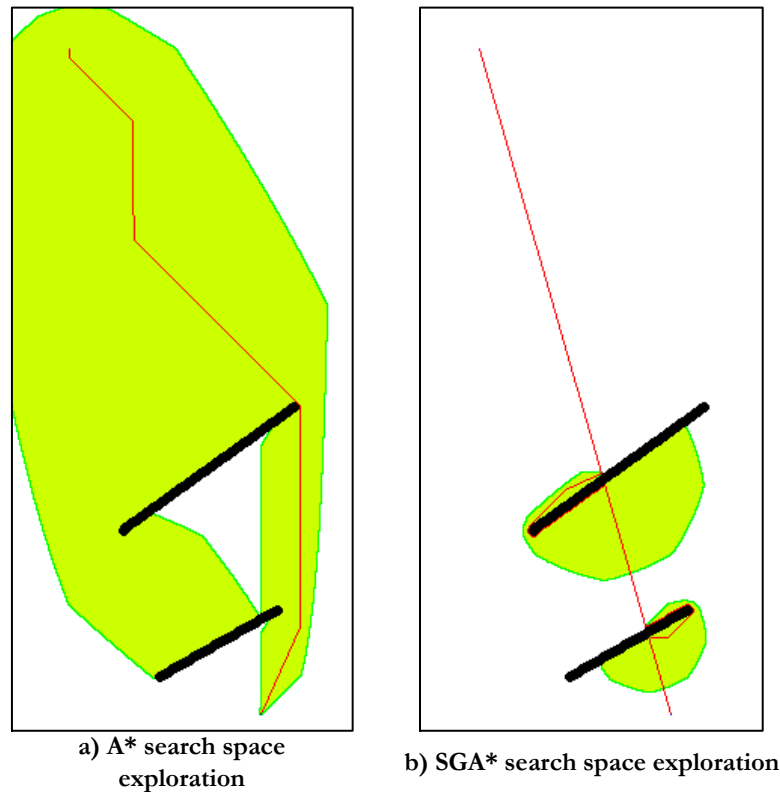
Figure 8.4: Search space comparison between A* and SGA* for a simple search problem

8.2.1.2 Guarantee of Completeness

A guarantee of completeness is essential for any pathfinding search algorithm. If a path does exists between two nodes in the environment it must be found by the algorithm. The SGA* algorithm guarantee of completeness is based on the guarantee of completeness of the A* algorithm.

A line segment is composed of a set of connected gridmap nodes, meaning that any node is reachable from any other node in that line segment. If two line segments are connected to one another via a connecting path, then the resulting set will also have the property that all nodes within that set are reachable from one another. If all the line segments created from the start node to the goal node are connected to one another then the resulting final set will contain within it a path from the goal node to the start node. Since the connections are made using a complete algorithm if a connection between two line segments cannot be made then the SGA* algorithm will fail since a complete connection of the line segments cannot be completed and so a path to the goal node cannot be returned.

a) A* optimal path

b) SGA* variant 1 unsmoothed path

c) SGA* variant 1 smoothed path
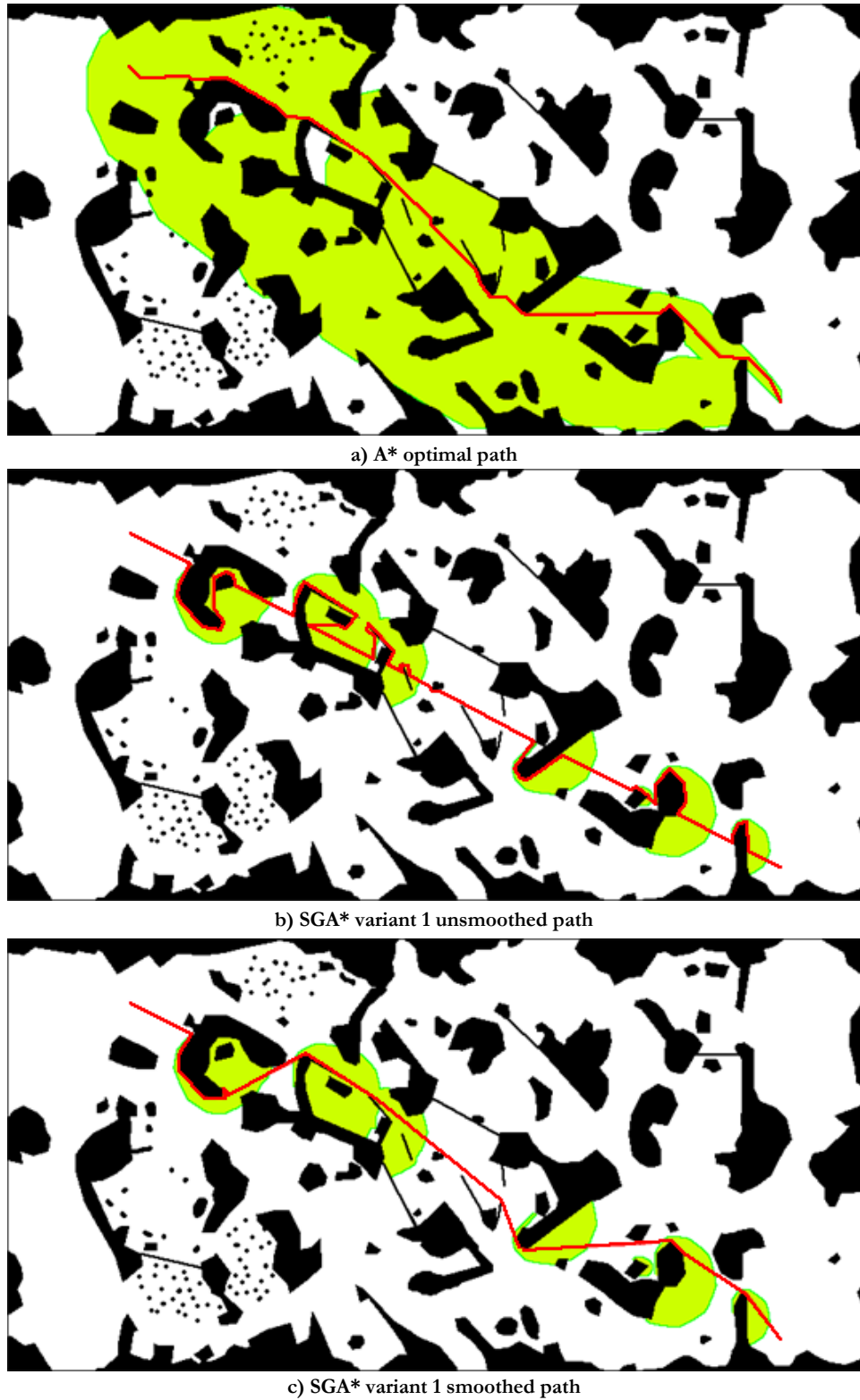
Figure 8.5: The search space reduction of SGA* and effect of path smoothing on a SGA* path
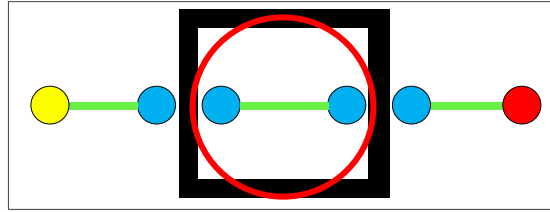
Figure 8.6: Invalid sub-goal creation

Unfortunately, a failed connection between line segments is not a guarantee of completeness as there exists a pathological special case which will cause the algorithm to fail even though a valid path does in fact exist. The special case occurs when, during the line segment creation stage, a line segment is created within a disjoint region of the gridmap. A work-around needs to be included into the SGA* algorithm to guarantee completeness.

Since this line segment is in a disjoint region of the map, the line segment is unreachable from any of the other line segments created and any attempts to connect the line segment to the previous or subsequent line segments will fail. Figure 8.6 illustrates such a case in which the unreachable line segment is highlighted.

If the SGA* algorithm fails to make a connection between two line segments, then the algorithm will fail prematurely even though a solution does exist. The workaround for this special case is fairly straightforward: if a connection between a line segment i and the subsequent line segment $i+1$ fails, then a connection is attempted with the subsequent line segment $i+2$. If that too fails, then a connection with line segment $i+2$ is attempted, and so on. In this manner any unreachable line segments which lie within disjoints regions are ignored. If all attempts to complete a connection between the path segment containing the start node and a subsequent line segment fail, then, due to the A* algorithm's completeness property, no path exists between the start node and the goal node.

Unfortunately, for an A* search to fail, the entire traversable environment will need to be searched to ensure that no path exists to the A* search's goal node. Meaning that, for a connection to a disjoint region to fail, the entire traversable environment in the same traversable region as the start node will need to be explored. This full environment search is extremely expensive, more so than simply performing an A* search from the start node to the goal node, and should be avoided if at all possible. A simple optimization is presented below as a means of reducing the unnecessary search space exploration.

The assumption is made that, in general, straight line paths from the start node will pass through disjoint areas and not originate within them. If the connection between two line segments is performed in a backwards fashion, i.e. from the subsequent *i+1-th* line segment's first node to the current *i-th* line segment's last node, then only the disjoint area would need be explored for the A* search to fail (refer to Figure 8.7b).

Unfortunately, this optimization only works in cases when line segments pass through enclosed areas or terminate within enclosed areas. In the case when a search originates in an enclosed area, the optimization will result in a full search of the environment outside that disjoint region before failing, meaning that SGA* will require significantly more processing time than A* to fail (refer to Figure 8.7a). Furthermore, since all the line segments are created prior to performing the line segment connections, a connection will be attempted with each and every other line segment present before the SGA* search will fail, if the line segment containing the start node originates within a disjoint region. As such, the processing costs associated with a failed SGA* search will be significantly higher than those for A*.

There is another solution available which will completely remove this problem. A preprocessing step can be introduced which will label each node in a fully traversable region with a region ID. After line segment creation all line segments containing nodes in a different region to the start node can be discarded. Furthermore, labeling connected regions allows for an instant search failure check since if the start and goal nodes are in different regions, then no path is possible between them. The downside of this approach is that it increases the memory cost of the grid storing albeit minimally as well as requiring potentially expensive region recalculations upon environmental changes within dynamic environments. If a static environment is used, then this is a valid optimization and should be implemented.

It is important to reiterate that SGA* is a domain specific search algorithm designed around a specific map topology, namely map topologies not containing disjoint regions. The SGA* is therefore not expected to be an efficient search algorithm for all game map topologies. In fact, since disjoint regions are not expected in the intended domain, only the backwards connection optimization is implemented in the experimental versions of the SGA* variants. A performance evaluation of the region labeling optimization within dynamic environments featuring disjoint regions is left for future work.

a) SGA* search starting within a disjoint region
(the shaded area represents the search space
explored prior to SGA* termination)

b) SGA* search ending within a disjoint region
(the shaded area represents the search space
explored prior to SGA* termination)
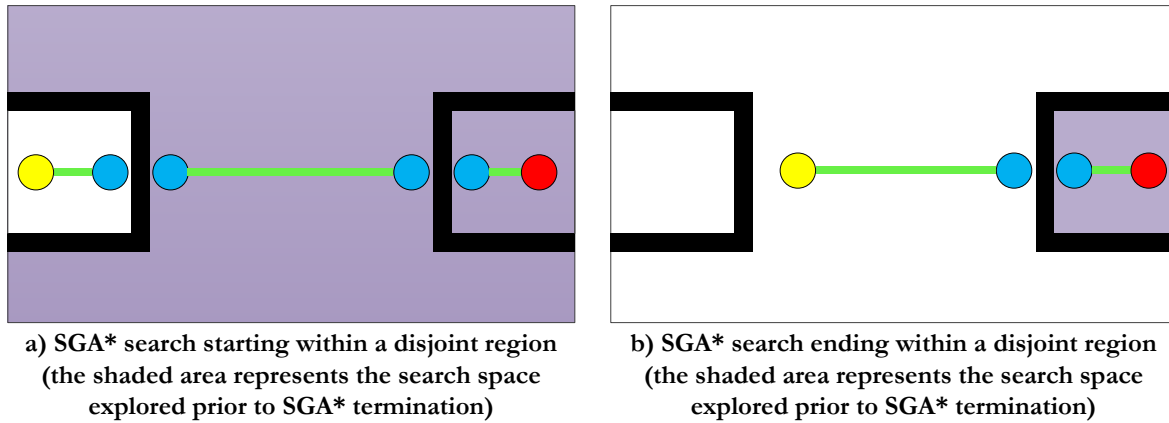
Figure 8.7: Disjoint region SGA* special case

The pseudocode for the SGA* variant 1 (SGA* V1) algorithm is presented in Listing 8.1. Details regarding the 2D line drawing algorithms are beyond the scope of this thesis and readers are referred to [131], [132] and [133] for more details on the topic.

8.2.1.3 Partial Pathfinding

There is a beneficial side-effect of the SGA* algorithm which may help to mitigate the severity of the disjoint region worst case scenario. Unlike standard discrete and hierarchical search algorithms, the SGA* algorithm is able return a partial path for any failed searches at no additional cost. The partial pathfinding ability of SGA* is illustrated in Figure 8.7b when, even though the SGA* search fails to find a complete path to the goal node, a partial path (the initial line segment) to the border of the enclosed area was found and returned.

Partial pathfinding can be beneficial in simulating human level AI behavior. Any game agent attempting to move to an unreachable goal node will be given a partial path towards that goal node. In having the agent follow that partial path, a player is given the impression that the agent only fails to find a path once the agent has neared its goal. Without the ability for partial path planning, the agent will simply remain stationary whenever a path planning request fails highlighting the fact that a game agent has perfect knowledge of the environment. To avoid this unwanted behavior, secondary path planning actions are often performed in order to find a path to a reachable node that is close to the goal. The ability of the SGA* algorithm to perform partial pathfinding can reduce any additional processing costs that may be incurred in finding a reachable node close to the goal.

```
1) Create line segments between START and GOAL

2) If ( number of line segments > 1 )

  2.1) Current line segment = 0

  2.2) For ( i = 1; i < number of line segments; i++ )

      2.2.1) If ( A* search from the beginning of line segment i to the end of current line segment failed )

                a) If ( i = last line segment ) return FAILED

      2.2.2) Else
                a) Add A* path between line segments to PATH
                b) Add line segment i to PATH
                c) Set current line segment to i

  2.3) Return PATH

3) Else if ( number of line segments == 1 )

  3.1) Return line segment as PATH

4) Else return FAILED
```

Listing 8.1: Pseudocode for SGA* variant 1

8.2.1.4 SGA* Path Optimality and the Necessity of Path Smoothing

A keen reader may have noticed the similarity of the SGA* algorithm to a hierarchical search algorithm. The line segment creation is analogous to the abstract planning stage of a hierarchical approach in that it creates a list of sub-goals defining a set of sub-problems to be solved. The line segment connection stage is analogous to the refinement stage in that it solves the sub-problems defined by the abstract planning stage. Even though the SGA* algorithm can be thought of as a hierarchical search algorithm, the lack of an abstraction hierarchy distinctly classifies SGA* as a discrete search algorithm.

Even though SGA* does not use an abstraction hierarchy for sub-goal selection, most of the advantages and disadvantages associated with hierarchical approaches such as reduced search space exploration and path sub-optimality also apply to the SGA* algorithm. A major concern for both SGA* and hierarchical approaches is the optimality and the visual quality of the final paths.

A hierarchical search algorithm uses an abstraction hierarchy to ensure that sub-goal selection is optimal with regards to the abstraction. In refining an optimal abstract path, hierarchical search algorithms ensure that agents only select sub-goals that are guaranteed to lead to the goal node. Since the SGA* algorithm does not make use of any type of abstraction hierarchy and only uses the

straight line between the start and goal nodes, the SGA* algorithm can be said to make naïve or uninformed decisions regarding the selection of sub-goals.

The straight line sub-goal selection technique can result in zigzag style final paths (refer to Figure 8.9b) of poor quality from both visual and optimality perspectives. Furthermore, the naïve sub-goal selection of SGA* can result in the algorithm exploring areas which do not lead to the goal whereas hierarchical approaches will only explore areas guaranteed to lead to the goal. SGA*'s poor sub-goal selection is especially prevalent in interior-style game maps such as those found in RPG and FPS games resulting in highly sub-optimal paths. Since the SGA* algorithm is not guaranteed to return optimal paths, it is a sub-optimal search algorithm.

Even though the sub-goal selection of hierarchical search algorithms is optimal with regards to the abstraction layer, the final refined low level paths will also often exhibit a high degree of sub-optimality [82] [22]. In fact, the minimal level of sub-optimality exhibited by hierarchical search algorithms is only achieved through the use of a post-processing path smoothing step [22].

It is also important to note that path smoothing is not only used to improve the optimality of paths but also to improve the visual quality of optimal paths. Note that there may exist multiple optimal paths (solutions) for any given search problem, with these paths differing only in visual quality.

Figure 8.8a illustrates an optimal path found through the use of the A* algorithm, whose visual quality is quite poor. Figure 8.8b illustrates the same path after path smoothing has been applied. While the path optimality of both paths is identical, the visual quality of the smoothed path in Figure 8.8b is superior. The effects of path smoothing on an example SGA* path is illustrated in Figure 8.5.

Due to there not existing any guarantee on the visual quality of found paths, most games will make use of a path smoothing stage even if the pathfinding search algorithm employed is guaranteed to be optimal from a length perspective [129] [82]. Even though path smoothing can be a potentially expensive operation, the fact remains that some sort of path smoothing must be employed, irrespective of the optimality of the search algorithm used, in order to have visually appealing final paths.

b) Optimal A* path before smoothing   **b) Optimal A* path after smoothing**
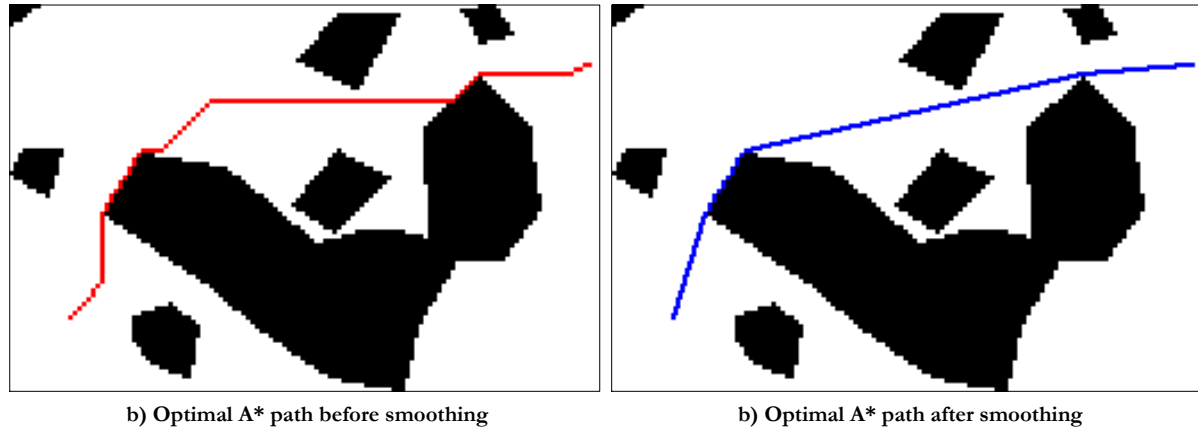
Figure 8.8: The need for path-smoothing for optimal paths

The application of a path smoothing step offers dramatic improvements to both the visual quality and the path optimality of the final SGA* path (refer to Figure 8.9). In some cases, such as the case illustrated in Figure 8.9b, path smoothing will convert a grossly sub-optimal SGA* path into an optimal final path. The fact that a path smoothing technique will be employed irrespective of the choice of search algorithm means that the path optimality improvements of the SGA* come at no additional cost. Actual statistics on the optimality of SGA* paths and on the improvements to those path due to path smoothing are presented in Section 8.3.

8.2.2 Spatial Grid A* Variant 2

Due to the naïve straight line sub-goal selection methods, certain map topologies may prove problematic for the SGA* as discussed in Section 8.2.1.2. For example, in the situation illustrated in Figure 8.9, the straight line sub-goal selection will result in an extremely poor path both in optimality and visual quality. As discussed in Section 8.2.1.3, a path smoothing step can be used to improve both optimality and visual quality of the path. Unfortunately, selection of the sub-goal in such a naïve fashion causes the SGA* algorithm to unnecessarily explore a large portion of the search space. A comparison of the search space exploration of A* and SGA* V1 is shown in Figure 8.9.

When navigating around obstacles (sub-goal refinement using A*), the SGA* algorithm will find a path to a node directly opposite the start node resulting in the saw-tooth path pattern illustrated in Figure 8.9. This saw-tooth path pattern is one of the main reasons behind the poor path optimality exhibited by the SGA* V1 algorithm.

a) A* search space exploration for an example problem

b) SGA* Variant 1 search space exploration
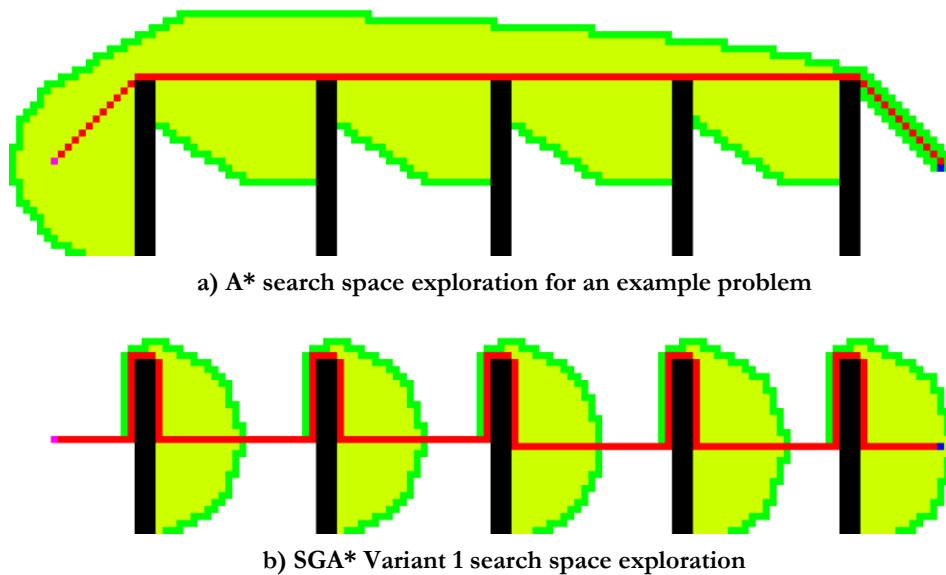
Figure 8.9: Unnecessary SGA* variant 1 search space exploration due to naïve sub-goal selection

Even though the saw tooth pattern can be removed through path smoothing, greatly improving path optimality, many unnecessary path refinements were performed to find that initial saw-tooth pattern.

The SGA* variant 2 (SGA* V2) algorithm is presented as a modified SGA* algorithm which uses dynamic sub-goaling as well as a double ended search approach to both reduce search space exploration and improve path optimality over the SGA* V1 algorithm. This section discusses the modifications (dynamic sub-goaling and refinement path trimming) made to the SGA* algorithm resulting in the SGA* variant 2 as well as a discussion into the effects of these modifications on the partial pathfinding ability of the SGA* variant 1.

This section discusses the modifications made to the SGA* variant 1 algorithm resulting in the SGA* variant 2 algorithm. Sub-section 8.2.2.1 discusses the addition of dynamic sub-goaling to the SGA* variant 1 algorithm. Sub-section 8.2.2.2 discusses the need for and implementation of refinement path trimming. A discussion on the effects that the SGA* variant 2 modifications have had on the partial pathfinding ability of the SGA* variant 1 algorithm concludes this section.

8.2.2.1 Dynamic Sub-Goaling

Consider a modified version of the SGA* V1 algorithm, where each subsequent line segment is only drawn as needed, i.e. at the end of each refinement step, and not in advance as presented in Section 8.2.1. Line segments are drawn from the end of the refinement path towards the goal node, terminating once an obstacle has been encountered. The subsequent sub-goal is simply the first traversable node found, along the same line, after the encountered obstacle. Since the refinement path is simply appended to the currently found SGA* path, the end of the refinement path is also the end of the final path. This simple modification introduces the concept of dynamic sub-goal selection without modifying the operation or resultant paths of the SGA* V1 algorithm.

On its own, dynamic sub-goaling has no effect on the SGA* V1 algorithm. An observation was made that refinement paths do not need to reach the sub-goal but can in fact be trimmed once the refinement path goes around an obstacle. In fact, allowing refinement paths to reach each sub-goal is what results in the saw tooth pattern exhibited in Figure 8.9. Refinement path trimming when combined with the dynamic sub-goaling modification results in a selection of sub-goals that differ to those of the SGA* V1 algorithm, thereby altering the operation of the algorithm.

Dynamic sub-goaling and refinement path trimming result in more direct final paths, thereby improving path optimality as well as reducing overall search space exploration. The path and search space exploration reduction resulting from these modifications are illustrated in Figure 8.10a. Two refinement path trimming techniques are presented in Section 8.2.2.2.

Dynamic sub-goaling and refinement path trimming have improved the search space exploration significantly but as illustrated Figure 8.10a, the search space exploration reduction offered over the original SGA* algorithm decreases as the modified SGA* search approaches the goal node. This observation led to one further modification being made to the SGA* V1 algorithm to create the final version of the SGA* V2 algorithm: a double ended search approach.

In addition to the refinement path trimming and the dynamic sub-goaling, the SGA* search is now performed in a double ended manner. In addition to the original forward path, now termed $path_{start}$, there now exists a second path, $path_{tail}$, which originates at the goal node and heads towards the start node. For each step of the SGA* V2 algorithm, line segment creation (and sub-goal determination) is performed from the end of $path_{start}$ to the end of $path_{tail}$, as well as from the end of $path_{tail}$ to the end of $path_{start}$.

a) SGA* search space exploration with dynamic sub-goaling and refinement path trimming

b) SGA* Variant 2 search space exploration
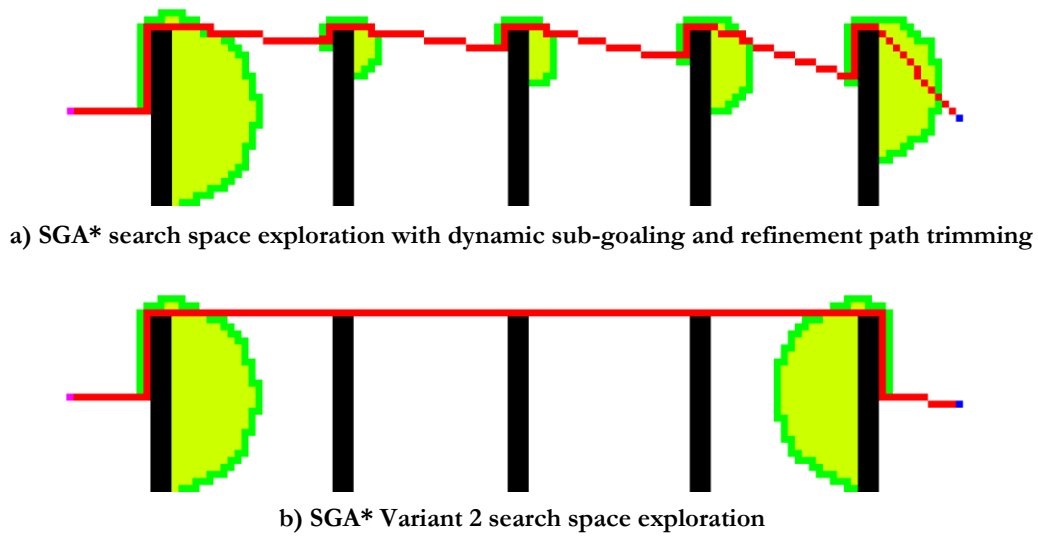
Figure 8.10: SGA* Variant 2 Search Space Exploration

As such for each step of the SGA* V2 algorithm two line segment generation and refinement actions are performed. The SGA* V2 algorithm terminates immediately once path$_{start}$ and path$_{tail}$, are connected either through line segment creation or through a refinement path. The search space exploration for the SGA* V2 algorithm for the example problem is illustrated in Figure 8.10b.

The double-ended nature of the SGA* V2 algorithm serves to decrease search space exploration resulting in decreased processing and memory costs, as well as improving path optimality due to the improved sub-goals selection.

Once again, just like the SGA* V1 algorithm, SGA* V2 is both a complete and a sub-optimal gridmap search algorithm. The completeness is based on the completeness property of the A* algorithm. The final connection made between path$_{start}$ and path$_{tail}$ (if not directly connectable via a straight line) is made via A*. Since the start and the goal nodes are contained within and are reachable from any node in path$_{start}$ and path$_{tail}$ respectively, then A* is guaranteed to find a path between path$_{start}$ and path$_{tail}$ if one exists.

8.2.2.2 Refinement Path Trimming

As mentioned above, refinement paths, once found, are trimmed. Two refinement path trimming techniques are presented below:

- **Maximum Distance Trimming**: The node within the refinement path which has the furthest distance from the start of the path is found and is termed the maximum distance node. All nodes from this maximum distance node until the end of the refinement path are trimmed.

- **Maximum Deviation Trimming**: The node within the refinement path which exhibits the maximum deviation (in both dimensions) from the start of the path is found and is termed the maximum deviation node. All nodes from this maximum deviation node until the end of the refinement path are trimmed.

Figure 8.11 illustrates the difference between the trimming techniques. Figure 8.11a illustrates a path trimmed to the furthest point on the path from the start while Figure 8.11b illustrates a path trimmed to the point that exhibits the maximum deviation from the start of the path.

In addition to the trimming techniques presented above, an additional condition needs to be placed upon the refinement path trimming techniques to ensure that the SGA* V2 algorithm does not become trapped within U shaped map topologies. In Figure 8.12a, the refinement path trimming trims the refinement path to the node at the bottom right of the U shaped region. The subsequent line segment generation directs the path back into the region after which the refinement path trimming results in the exact same sub-goal being selected as before and the search is once again directed into the U shaped region.

a) Max distance trimming   **b) Max deviation trimming**
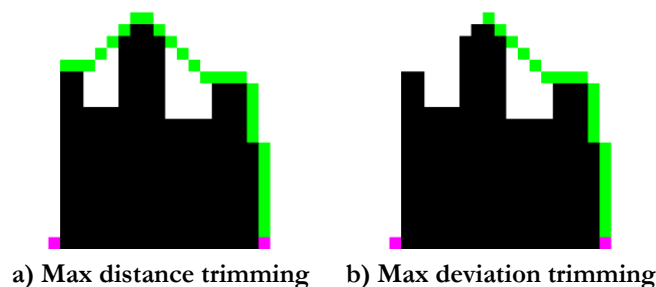
Figure 8.11: The difference between the two refinement path trimming techniques

The refinement path trimming will always result in the exact same sub-goal being selected causing the SGA* V2 algorithm to remain trapped within the U shaped map region. An additional condition is placed on the refinement path trimming in that the refinement paths may only be trimmed from a node that is in front of the start of the refinement path in the direction of the final goal node (the end node of path$_{tail}$ or path$_{start}$).

In Figure 8.12b, trimming paths to points in front of the start node results in the path trimming always moving the subsequent sub-goal forward ensuring that, even though the path will enter the U shaped region multiple times, the algorithm will eventually leave the U shaped region.

Unfortunately, having the path entering and leaving the U shaped region numerous times will dramatically increase the overall path length as well as negatively affect the visual quality of the path. A saving grace to this is that the SGA* V2 search is double ended. Meaning that the final sub-goal (the end node of path$_{tail}$ or path$_{start}$) for the SGA* V2 algorithm will keep changing and may serve to direct the search out of the U shaped region as well as the fact that the search may connect with the opposite path, further reducing the number of entry and exits from the U shaped region.

a) Standard refinement path trimming

b) Refinement path trimming ensuring trimming past start point in the direction of the sub-goal
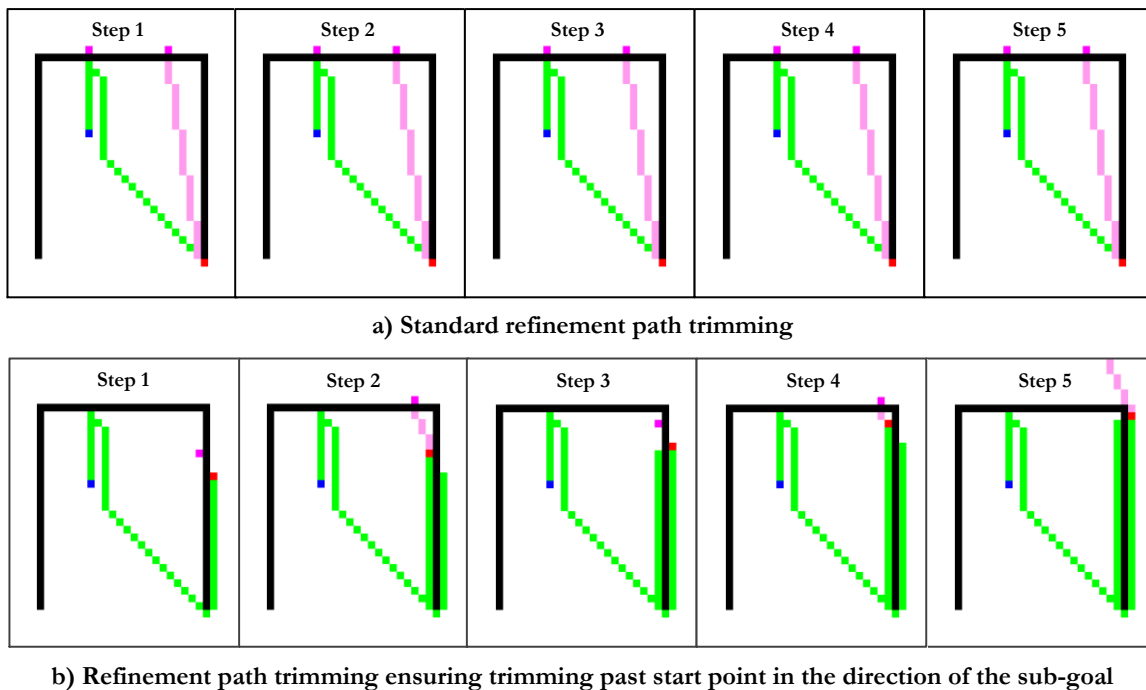
Figure 8.12: The effect of the additional condition on refinement path trimming

8.2.2.3 Partial Pathfinding

Like the SGA* V1 algorithm, the SGA* V2 algorithm also provides the ability to return a partial path upon a search failure. Since the SGA* V2 algorithm searches in a double ended manner, search failures due to map features close to the goal node will be detected earlier than by the forward-only search performed by the SGA* V1 algorithm.

This early failure detection can help to decrease the processing costs involved in detecting failed searches, but has a negative effect on the partial paths returned. If a failure is detected close to the goal, the SGA* V2 algorithm will terminate resulting in a partial path that is not guaranteed to terminate close to the goal node. Figure 8.13a illustrates the poor partial path resulting from an early termination of the SGA* V2 algorithm.

The quality of the partial paths returned by the SGA* V2 algorithm can be improved by setting the search problem's start or goal node to the most recent sub-goal found for the failed end of the SGA* V2 algorithm double ended search.

For example, in Figure 8.13b the backwards (from the goal node to the start node) search failed. The SGA* V2 search is then continued using the most recently found sub-goal for the backwards search as the goal node for the overall search problem. The result from this modified search is the partial path illustrated in Figure 8.13b. Modifying the overall search problem in such a manner allows for the generation of partial paths of a similar quality to those returned by the SGA* V1 algorithm.

This dynamic end goal selection was not implemented in the experimental version of the SGA* V2 algorithm as partial paths were not required. The discussion regarding the improvement of SGA* V2 partial paths was included for sake of completeness. The final pseudocode for the SGA* V2 algorithm is presented in Listing 8.2.

**a) Early termination of the SGA* V2 algorithm
resulting in a poor partial path.**

**b) Improved partial pathfinding through a new goal
node selection**
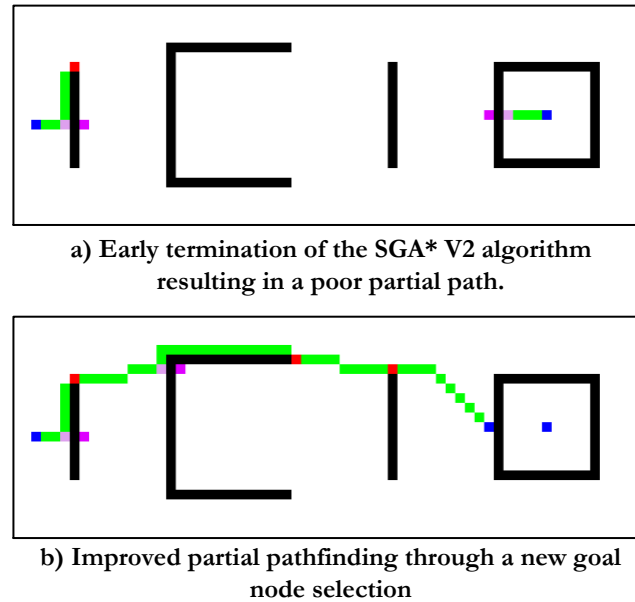
Figure 8.13: SGA* V2 Partial Pathfinding

8.2.3 Spatial Grid A* Variant 3

The SGA* V3 was developed as a means to research the overall cost savings resulting from the use of the straight line interconnects in between selected sub-goals. The SGA* Variant 3 (V3) is identical to the SGA* V2 algorithm with only a single modification: the created line segments are not appended to the path after creation and are only used for sub-goal selection. In removing the straight line segments, optimal paths are generated between sub-goals and so the overall path optimality of the final path will be improved.

Path refinement is performed from the end of the current found path to the next sub-goal. In removing the addition of the line segments between sub-goals, SGA* V3 is simply reduced to a series of connected A* searches as performed in hierarchical approaches such as HPA* [22].

SGA* V3 attempts to improve path optimality at the expense of both the processing and memory costs. The increase to processing and memory costs is due to the fact that the search space reduction offered by the line segment addition has been removed and that the sub-problems created are now more complex. As with SGA* V1 and V2, V3 is also a complete and sub-optimal search algorithm. The reasons for completeness are identical to those discussed for SGA* V2. The pseudocode for SGA* V3 is presented in Listing 8.3.

```
1) Set SUBGOAL_start = START, SUBGOAL_goal = GOAL, SUBGOAL_temp = 0

2) While ( true )

   2.1) Create line segment between SUBGOAL_start and SUBGOAL_goal, and find SUBGOAL_temp
   2.2) Add line segment to PATH
   2.3) If ( line segment doesn't reach SUBGOAL_goal )


        2.3.1) While ( A* search between SUBGOAL_temp and end of line segment = FAILED )

               a) If (SUBGOAL_temp = SUBGOAL_goal) RETURN FAILED
               b) Else create line segment between SUBGOAL_temp and SUBGOAL_goal, and find new SUBGOAL_temp

        2.3.2) If ( A* path doesn't connect PATH to PATH_TAIL)
               a) If ( PATH_TAIL is directly reachable from A* path)
                          a)create straight line from A* path to PATH_TAIL
                          b)add straight line to PATH
                          c)GOTO 3
               b) ELSE
                          a) Trim A* path and add trimmed A* path to PATH
                          b) SUBGOAL_start = end of trimmed A* path
        2.3.3) Else
               a) Add A* path to PATH
               b) GOTO 3

        2.3.4) Create line segment between SUBGOAL_goal and SUBGOAL_start, and find SUBGOAL_temp
        2.3.5) Add line segment to PATH_TAIL
        2.3.6) If ( line segment doesn't reach SUBGOAL_start )

               a) While ( A* search between SUBGOAL_temp and end of line segment = FAILED )

                          a) If (SUBGOAL_temp = SUBGOAL_start) RETURN FAILED
                          b) Else create line segment between SUBGOAL_temp and SUBGOAL_start, and find new SUBGOAL_temp

               c) If ( A* path doesn't connect PATH to PATH_TAIL)
                          a)If ( PATH is directly reachable from A* path)
                                   a)create straight line from A* path to PATH
                                   b)add straight line to PATH_TAIL
                                   c)GOTO 3
                          b) ELSE
                                   a) Trim A* path and add trimmed A* path to PATH_TAIL
                                   b) SUBGOAL_goal = end of trimmed A* path

               d) Else
                          a) Add A* path to PATH_TAIL
                          b) GOTO 3

        2.3.7) Else GOTO 3

   2.4) Else GOTO 3

3) Add PATH_TAIL to PATH
4) RETURN PATH
```

Listing 8.2: Pseudocode for SGA* variant 2

```
1) Set SUBGOAL_start = START, SUBGOAL_goal = GOAL, SUBGOAL_temp = 0

2) While ( true )

  2.1) Create line segment between SUBGOAL_start and SUBGOAL_goal, and find SUBGOAL_temp
  2.2) If ( line segment doesn't reach SUBGOAL_goal )

     2.2.1) While ( A* search between SUBGOAL_temp and SUBGOAL_start = FAILED )

          a) If (SUBGOAL_temp = SUBGOAL_goal) RETURN FAILED
          b) Else create line segment between SUBGOAL_temp and SUBGOAL_goal, and find new SUBGOAL_temp

     2.2.2) If ( A* path doesn't connect PATH to PATH_TAIL)
          a) If ( PATH_TAIL is directly reachable from A* path)
                    a)create straight line from A* path to PATH_TAIL
                    b)add straight line to PATH
                    c)GOTO 3
          b) ELSE
                    a) Trim A* path and add trimmed A* path to PATH
                    b) SUBGOAL_start = end of trimmed A* path
     2.2.3) Else
          a) Add A* path to PATH
          b) GOTO 3

     2.2.4) Create line segment between SUBGOAL_goal and SUBGOAL_start, and find SUBGOAL_temp
     2.2.6) If ( line segment doesn't reach SUBGOAL_start )

          a) While ( A* search between SUBGOAL_temp and SUBGOAL_goal = FAILED )

                    a) If (SUBGOAL_temp = SUBGOAL_start) RETURN FAILED
                    b) Else create line segment between SUBGOAL_temp and SUBGOAL_start, and find new SUBGOAL_temp

          c) If ( A* path doesn't connect PATH to PATH_TAIL)
                    a) If ( PATH directly reachable from A* path)
                              a)create straight line from A* path to PATH
                              b)add straight line to PATH_TAIL
                              c)GOTO 3
                    b) ELSE
                              a) Trim A* path and add trimmed A* path to PATH_TAIL
                              b) SUBGOAL_goal = end of trimmed A* path
          d) Else
                    a) Add A* path to PATH_TAIL
                    b) GOTO 3

     2.2.7) Else

          a) Add line segment to PATH
          b) GOTO 3

  2.3) Else GOTO 3

     2.3.1) Add line segment to PATH
     2.3.2) GOTO 3

3) Add PATH_TAIL to PATH
4) RETURN PATH
```

Listing 8.3: Pseudocode for SGA* variant 3

8.3 Experimental Results

This section provides detailed performance evaluations of the SGA* algorithm variants on various game map test sets representing various pathfinding domains. A discussion of the results is presented per test set. The overall goals for the performance evaluations are to show that the SGA* variants offer significant improvements to the processing and memory costs of a standard discrete search algorithm at a minimal cost to path optimality within the intended domains. Evaluations are further performed on both the best and the worst case problem domains.

This section is broken up into several sub-sections. The first subsection discusses the testing procedures and testing hardware used for the evaluation of the SGA* variants. The remaining five sub-sections are identical in structure, each sub-section describing a test set, presenting the experimental results and concluding with a discussion of the results for that test set.

8.3.1 Testing Procedures

The SGA* variants should be thought of as a set of techniques aimed at improving the overall performance of a discrete search algorithm through problem sub-division rather than new standalone algorithms. In fact, SGA* is search algorithm agnostic and any complete discrete graph search algorithm may be used in the place of the A* algorithm. The A* algorithm is used as it is the current industry standard algorithm [3] [83] as well as being the control algorithm for all discrete search literature [11]. The SGA* algorithm variants make use of an A* search algorithm to perform path refinement and are therefore compared against the exact same optimized A* implementation used.

All the problems contained within each test set were first solved using A* and then solved using the three SGA* variants. The effects of the refinement path trimming techniques presented in Section 8.2.2.2 were also investigated. The SGA* problem solutions are then compared to the control A* solutions across the performance metrics discussed in Chapter 4. The results of these comparisons are presented as a percentage-of-A* (POA) value, representing an SGA* solution metric's value as a percentage of the control A* solution metric value. For example, a POA of 100% means that the SGA* solution metric value is identical to the A* solution metric value while a POA of 50% means that the SGA* solution metric value is 50% smaller than the A* solution metric value. The POA values are then averaged over the set of problems for each map within a test set as well as over the entire test set.

For simplicity's sake, the SGA* designation will be omitted and the variants will henceforth be referred to as V1, V2 and V3. Furthermore the SGA* V2 with maximum distance trimming will be referred to as V2 MaxDist and SGA* V2 with maximum deviation trimming will be referred to as V2 MaxDev.

All experiments were run on an overclocked Intel i7 920 processor running at 4.2GHz. Algorithm implementations were done in C++ and compiled using the Microsoft VC++ 2010 compiler. It is important to note that the SGA* experimental versions were not heavily optimized and so further improvements to the processing costs (search times) may still be possible. Code optimization will not affect the SGA* variants' search space exploration or memory costs.

8.3.2 Counter-Strike Maps

"Counter-Strike" is a team based first person shooter game released in 2000 by Valve Software [134]. "Counter-Strike" Maps feature both large open areas and narrow corridors. The Counter-Strike maps were converted, by the author, into gridmaps by using the D level overviews. Example maps from the Counter-Strike test set are illustrated in Figure 8.14.
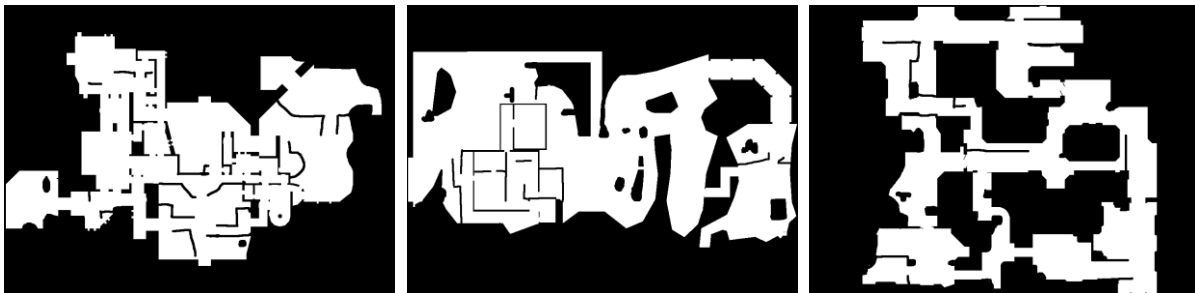
Figure 8.14: Example maps from the "Counter-Strike" test set

The "Counter-Strike" test set consists of 19 1024x768 game maps. There are 2000 test problems per-map for a total of 38000 test problems across all maps. Per-map results are presented as the average result over the 2000 test problems for that map.

The search time and memory usages results of the SGA* variants are illustrated in Figure 8.16. In general, there are significant improvements (of over 50%) to both search times and memory usage over A* search, except in the case of map 4 (cs_militia).

The cs_militia map, illustrated in Figure 8.15, while featuring large open regions has a low level of connectivity between the map regions. In fact, a large portion of the map (the highlighted left hand corner) is only accessible through a single entrance. Any searches originating on the right hand sight of that region and ending either within or to the left of it, will require that the entire region be explored needlessly each time a path refinement action is taken. This is a worst case scenario for the SGA* variants and results in the extremely poor performance exhibited. The V1 algorithm resulted in search times of over 130% slower than those of A*. The V2 and V3 algorithms performed better than the V1 but still resulted in nearly 50% slower search times.

As expected, the V2 variant performs better and uses less memory than both V1 and V3. The presence of dynamic sub-goaling and a double-ended search in the V2 and V3 algorithm are shown to offer improvements over V1 especially in the case of the problematic map cs_militia.
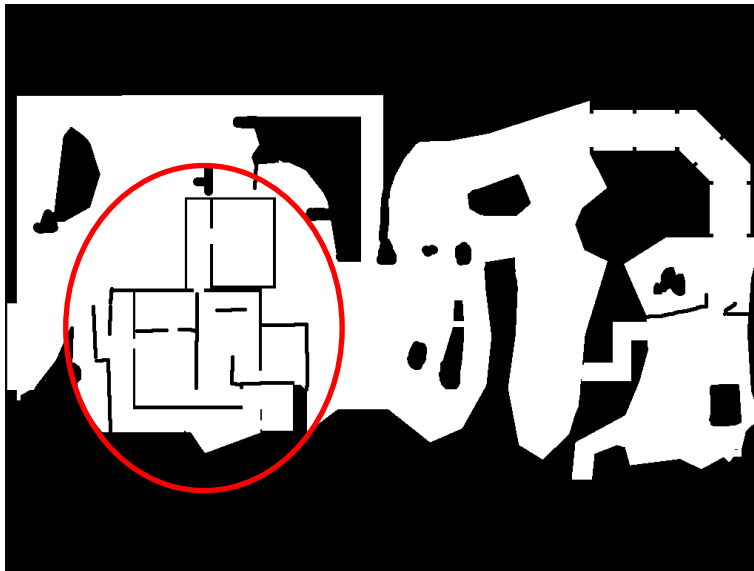
Figure 8.15: The cs_militia "Counter-Strike" Map"

The performance and memory improvements of the SGA* variants over A* result from the search space reduction stemming from the search problem subdivision. The SGA* search space reduction results are shown in Figure 8.17. The SGA* memory usage values shown in Figure 8.16 are based on the peak search space exploration performed by any single refinement action. It is important to note the correlation between the graphs in Figure 8.16 and those in Figure 8.17.

The search space exploration data is contained within the search time and peak memory graphs. In fact the total number of nodes encountered during the operation of the SGA* is irrelevant considering that each path refinement action is atomic and the only valuable measure is the total memory reduction afforded by the SGA* variants. Providing results of the total number of nodes encountered and explored by SGA* is redundant as it is a duplication of the data already presented through the search time and peak memory results and are therefore omitted from the remainder of the test set performance discussions.

Path optimality measurements are shown in Figure 8.18. Unsmoothed path optimality is quite poor for the V1 algorithm with an average sub-optimality of over 35%. The dynamic sub-goaling   and double-ended search in V2 have improved the unsmoothed path optimality by, on average, around 15% over V1. The V3 algorithm provides the highest unsmoothed path optimality but has resulted in increased search times.

Search Time Per Map - "Counter-Strike"

Map	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
V1	48.97	56.72	84.20	237.9	24.54	27.83	57.43	50.03	40.01	49.85	54.60	39.10	50.29	42.29	70.14	34.47	40.26	36.61	33.50
V2 MaxDist	48.32	53.20	76.50	143.5	24.13	25.75	57.28	44.57	37.07	48.73	49.76	38.59	49.65	35.56	67.05	33.08	38.02	33.21	33.04
V2 MaxDev	48.96	54.33	80.77	151.2	24.18	25.85	56.67	45.28	36.96	48.98	51.75	38.97	50.11	35.81	69.48	34.40	38.72	33.48	33.18
V3	61.56	58.44	83.16	136.3	30.19	31.76	56.76	42.29	40.93	49.75	52.34	41.32	50.59	36.28	71.24	35.68	45.07	37.61	39.94

Peak Memory Per Map - "Counter-Strike"

Map	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
V1	53.00	57.81	69.86	113.5	31.42	33.65	57.05	51.42	45.45	50.26	54.89	43.87	48.84	43.98	70.69	38.86	41.93	42.05	37.12
V2 MaxDist	53.65	55.42	66.62	99.57	30.43	32.15	55.76	47.95	43.65	48.89	51.28	43.53	47.86	40.84	69.33	38.39	40.20	39.62	37.31
V2 MaxDev	53.62	55.51	67.21	101.2	30.58	32.08	55.70	47.89	43.30	48.71	52.14	43.46	47.78	40.81	69.19	38.90	40.34	39.67	37.27
V3	58.76	56.07	67.93	92.13	33.90	36.08	53.92	44.77	43.73	46.84	49.88	42.98	48.15	39.72	67.72	37.96	44.05	40.33	40.62

Figure 8.16: Per-map SGA* performance results for the "Counter-Strike" test set

The application of a post-processing path smoothing stage has a significant impact on the path optimality for all three variants. Once again the cs_militia map is problematic and path optimality values for that map are significantly lower than for the other maps. Path smoothing resulted in near-optimal final paths (around 4% sub-optimal).

The overall results for the entire "Counter-Strike" test set are presented in Figure 8.19. The SGA* algorithm on average provides a 50% improvement in memory usage and search times over A* at a cost of around 37% and 21% in path optimality for the V1 and V2 algorithms respectively. The V3 algorithm has the highest path optimality values of the three variants but has a slightly higher search time cost than the V2 algorithm from which it is derived.

Once path smoothing has been applied, the path optimality between V2 and V3 is insignificant. There is also no significant difference to performance and path optimality between the refinement path trimming methods used for theV2 algorithm.

Total Nodes Explored - "Counter-Strike"

Map	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
V1	58.83	66.23	96.1	160.9	37.34	40.7	72.62	62.8	55.33	65.6	68.7	51.78	60.56	55.62	83.99	50.62	52.22	49.12	46.18
V2 MaxDist	59.76	62.89	97.05	147.3	36.74	38.31	70.94	59.34	53.62	64.43	65.23	52.35	56.9	49.22	81.84	50.47	52.3	46.88	46
V2 MaxDev	61.27	64.52	103.5	150.9	37.31	38.73	71.74	61.22	53.77	65.49	68.81	53.44	58.36	50.11	86.17	52.92	53.08	47.62	46.32
V3	66.26	65.8	100.6	139.8	43.74	45.28	69.33	58.74	56.31	64.32	65.65	53.52	59.4	51.25	81.44	51.36	58.33	50.17	52.53

Total Nodes Encountered - "Counter-Strike"

Map	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
V1	58.64	66.61	95.49	155.3	37.94	41.29	71.95	62.84	55.8	65.63	69.01	52.44	60.67	55.55	83.51	50.99	52.39	49.74	46.81
V2 MaxDist	59.66	63.35	96.86	144.2	37.38	38.91	70.3	59.53	54.2	64.64	65.69	53.07	57.11	49.42	81.81	50.91	52.63	47.59	46.67
V2 MaxDev	61.18	65.03	103.4	147.7	37.99	39.39	71.2	61.43	54.4	65.75	69.33	54.23	58.63	50.36	86.26	53.41	53.44	48.39	47.06
V3	66.58	67.08	101.1	137.4	45.26	46.64	69.78	60.15	57.81	65.38	66.75	55.17	60.85	52.71	82.11	52.5	59.4	51.74	54

Figure 8.17: Search space exploration statistics for SGA* over A*

Unsmoothed Path Optimality Per Map - "Counter-Strike"

Map	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
V1	67.97	62.54	50.28	23.97	76.77	73.62	60.14	56.70	70.16	59.54	63.19	73.12	61.01	58.52	58.55	64.01	68.44	72.13	72.70
V2 MaxDist	83.53	82.33	63.57	45.48	86.97	87.41	77.49	77.35	82.62	81.14	79.36	84.06	81.53	78.80	76.91	77.35	79.77	84.58	85.93
V2 MaxDev	83.20	82.48	62.20	49.45	87.34	87.57	78.37	77.53	83.51	81.75	78.29	84.34	81.74	78.77	75.63	76.19	80.23	84.70	86.67
V3	93.63	90.77	75.70	65.61	94.10	94.15	89.78	90.19	91.45	92.31	89.35	93.00	91.97	91.11	87.31	90.89	89.76	93.09	92.94

Smoothed Path Optimality Per Map - "Counter-Strike"

Map	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
V1	94.49	94.42	90.69	78.75	95.69	96.21	93.14	94.06	93.73	92.27	94.09	96.27	95.33	95.50	89.44	94.47	91.36	95.43	93.32
V2 MaxDist	97.43	96.86	93.16	91.07	97.09	97.36	96.38	96.35	95.65	96.60	96.84	97.66	97.38	97.06	93.75	97.09	94.19	96.86	96.49
V2 MaxDev	97.35	96.91	92.79	89.26	97.11	97.45	96.20	96.48	96.18	96.69	96.48	97.80	97.42	97.04	93.83	97.15	94.35	96.87	96.83
V3	97.50	97.12	94.06	95.14	97.74	97.74	97.39	96.87	95.97	97.36	97.28	98.06	97.52	97.03	94.49	97.45	95.75	97.29	96.57

Figure 8.18: Per-map SGA* path optimality measurements for the "Counter-Strike" test set

Overall Results - "Counter-Strike"

Algorithm	V1	V2 MaxDist	V2 MaxDev	V3
Search Times	56.78395836	49.32045374	50.48528389	52.70422005
Unsmoothed Path Optimality	62.81204954	78.7506566	78.9496674	89.32502966
Smoothed Path Optimality	93.09224628	96.07095111	96.01572631	96.75842455
Memory	51.88268951	49.60709391	49.76189794	49.76931529

Figure 8.19: Overall SGA* results for the "Counter-Strike" test set

8.3.3 Company of Heroes Maps

"Company of Heroes" is a real time strategy game released in 2006 by Relic Entertainment [100]. "Company of Heroes" maps feature large open maps with no disjoint regions and a high degree of connectivity between map regions. These maps are the intended domain for the SGA* algorithm variants. Example maps from the "Company of Heroes" test set are illustrated in Figure 8.20.
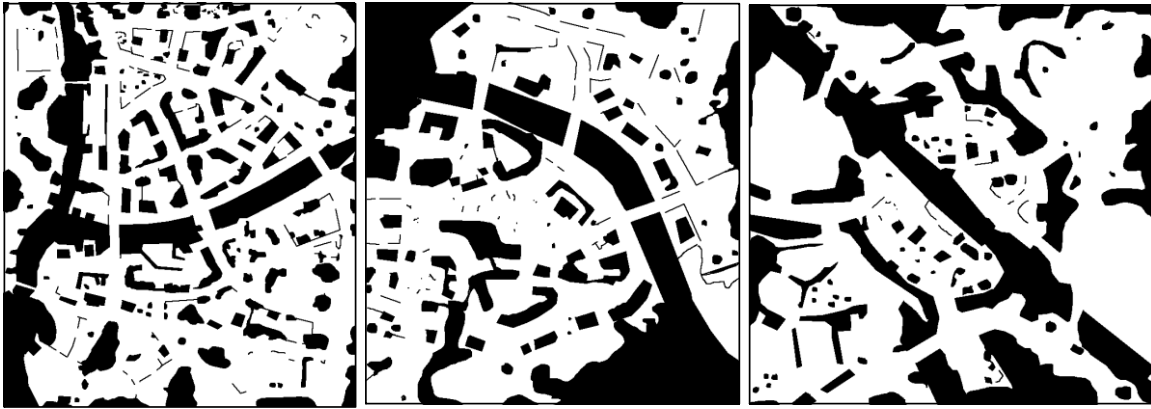
Figure 8.20: Example maps from the "Company of Heroes" test set

The "Company of Heroes" maps were converted, by the author, into gridmaps by using the 2D level overviews. The "Company of Heroes" test set includes 17 maps of varying dimensions. The most common map dimensions within the test set are 658x768 and 768x768. There were 2500 test problems per-map for a total of 42500 test problems across all maps. Per-map results are presented as the average result over the 2500 test problems for that map.

The results for the "Company of Heroes" test set are quite positive. Once again the search time and memory usage statistics per-map are illustrated in Figure 8.21. There is some variance in the overall search results across the test set due to the varying complexity of the test maps. Overall performance for the V2 algorithm was excellent offering at worst a 39% improvement to search times and at best a 96% improvement.

Peak memory usages are also significantly improved across the board, offering at worse a 35% reduction in memory usage (offered by V1 in map 4) and at best a 94% reduction in memory usage (V2 on maps 4 and 6).

SGA* Search Time Per Map - "Company of Heroes"

Map	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
V1	33.72	34.53	32.48	86.27	17.93	4.00	15.54	3.52	58.16	28.00	55.33	6.18	19.32	54.95	37.84	12.08	54.64
V2 MaxDist	32.36	29.61	26.59	61.95	13.04	3.86	14.39	3.26	48.90	27.49	46.86	5.75	17.51	53.80	36.70	10.66	54.40
V2 MaxDev	32.90	30.25	27.51	61.44	13.09	3.87	14.60	3.29	50.64	28.19	47.26	5.76	17.79	55.71	36.62	11.01	55.83
V3	37.85	33.09	27.59	63.42	20.16	16.36	20.42	16.57	48.98	33.02	46.26	15.19	22.10	60.48	39.29	20.57	56.52

SGA* Peak Memory Per Map - "Company of Heroes"

Map	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
V1	39.60	39.50	34.51	64.82	23.56	7.56	23.53	6.75	56.04	35.05	54.93	10.53	26.67	58.95	45.39	17.28	59.07
V2 MaxDist	39.13	35.94	32.61	57.43	20.76	7.66	22.97	6.62	51.81	34.82	50.21	10.45	25.59	57.72	44.58	16.55	58.83
V2 MaxDev	39.25	35.41	32.62	56.79	20.42	7.66	23.02	6.65	51.53	35.10	50.17	10.43	25.76	57.84	44.46	16.58	58.53
V3	42.38	38.99	32.38	56.69	27.60	23.48	27.10	22.79	52.61	37.97	49.12	21.69	27.67	60.93	44.59	26.91	57.81

Figure 8.21: Per-map SGA* performance results for the "Company of Heroes" test set

The unsmoothed path sub-optimality is at around 16% for the V2 variant, reaching a sub-optimality of only 4% in some cases. There is no significant difference between the trimming techniques used for the V2 variant with regards to path optimality. Path optimality is quite low for the V1 variant in all cases. As expected, the path-optimality for V3 is higher than that of V2 but once path smoothing has been applied, the difference in optimality between V2 and V3 is insignificant. Per-map path optimality statistics are illustrated in Figure 8.22.

The overall performance profile of the SGA* variants is similar to that of the "Counter-Strike" test set and a trend is starting to emerge, with the V2 variant consistently providing lower search times and lower memory usage than the other variants. Furthermore, the V2 and V3 variants are shown to handle harder maps (featuring low levels of connectivity) more efficiently than the V1 variant.

Unsmoothed Path Optimality Per Map - "Company of Heroes"

Map	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
V1	64.30	61.57	60.17	40.27	71.10	89.83	68.81	89.55	59.72	77.72	60.96	84.21	64.55	66.70	65.37	74.47	65.20
V2 MaxDist	83.87	82.40	78.43	71.71	85.79	94.00	83.24	94.25	78.76	87.28	77.59	90.76	81.33	82.37	78.77	86.79	79.10
V2 MaxDev	85.00	83.87	79.55	72.99	86.39	94.33	83.96	94.54	79.02	87.79	78.64	91.17	82.06	82.78	79.64	86.79	79.42
V3	91.19	90.16	88.73	84.11	93.22	97.59	91.33	97.45	88.12	95.01	87.45	94.88	90.13	91.12	89.68	94.03	89.58

Smoothed Path Optimality Per Map - "Company of Heroes"

Map	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
V1	90.18	93.36	90.93	84.35	94.38	98.83	93.51	98.77	90.12	96.22	92.67	96.89	93.07	90.30	91.68	95.48	91.92
V2 MaxDist	95.81	96.46	94.62	92.05	96.82	99.30	96.19	99.20	95.30	97.87	95.40	97.98	95.72	94.95	94.69	97.37	94.99
V2 MaxDev	95.91	96.64	94.79	92.40	96.84	99.34	96.33	99.23	95.19	97.93	95.59	98.05	95.84	94.93	94.72	97.35	94.89
V3	96.50	96.60	95.56	93.05	97.22	99.18	96.54	99.07	95.60	98.00	96.03	98.36	95.92	96.07	95.78	97.88	95.93

Figure 8.22: Per-map SGA* path optimality measurements for the "Company of Heroes" test set

SGA* Overall Results- "Company of Heroes"

Algorithm	V1	V2 MaxDist	V2 MaxDev	V3
Search Times	32.61751463	28.65440104	29.16196071	33.99286403
Unsmoothed Path Optimality	68.49961415	83.32066802	83.99765038	91.39920713
Smoothed Path Optimality	93.09854505	96.16007261	96.2328381	96.66470809
Memory	35.51446893	33.74522993	33.65935131	38.27751355
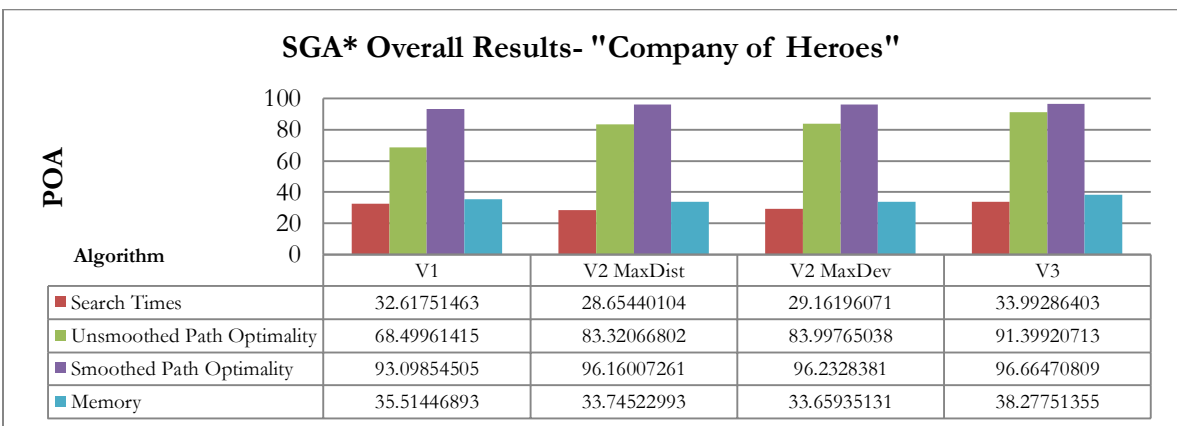
Figure 8.23: Overall SGA* results for the "Company of Heroes" test set

8.3.4 Baldur's Gate Maps

"Baldur's Gate" is an RPG game released in 1998 by Bioware Corp [135]. "Baldur's Gate" maps are primarily indoor maps featuring long winding corridors and single entrance rooms. These maps do contain disjoint regions as well as winding dead end corridors, falling outside the intended domain for the SGA* algorithm variants and therefore present a significant challenge. Example maps from the "Baldur's Gate" test set are illustrated in Figure 8.24.
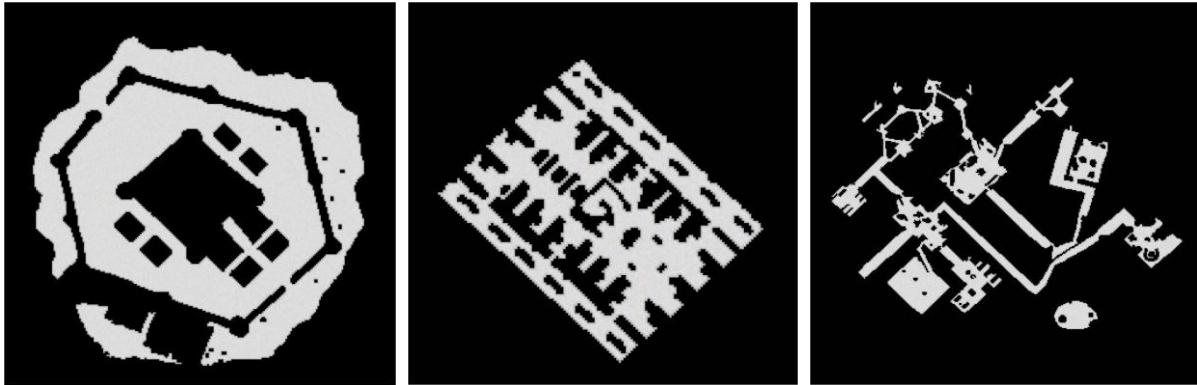
Figure 8.24: Example maps from the "Baldur's Gate" test set

The Baldur's Gate test maps are courtesy of Nathan Sturtevant's Moving AI Lab at the University of Denver [136]. These maps are used as a standard test set within the field of academic video game pathfinding research and have been used as the test set in the following articles: [22], [24], [25], [98], [11].

The Baldur's Gate maps test set consists of 75 maps scaled to 512x512 nodes. There are approximately 1280 test problems per-map for a total of 93162 test problems across all maps. Per-map results are presented as the average result over the approximately 1280 test problems for that map.

Due to the number of maps in the "Baldur's Gate" test set, detailed per-map results are difficult to represent concisely. Therefore, line graphs of the search times and memory usage are presented in Figure 8.25 which span the entire test set. A detailed graph of the performance for a subsection of the test set results is presented in Figure 8.26. This subsection was selected as it includes a section of maps upon which the SGA* variants' performance varied greatly.

The map topologies across the test set vary greatly. Figure 8.24 illustrates a large difference in map topology across the three example maps. Furthermore, it is already established that maps with low connectivity and disjoint regions are especially problematic for the SGA* variants.

The "Baldur's Gate" test sets features maps with both of those features and it is these maps that account for the large variation in SGA* performance across the test set.

On average, the SGA* variants offer significant improvements over A* for the "Baldur's Gate" test set except in the case of three test maps which feature high numbers of disjoint regions. On average there is around a 55% reduction in both search times and memory usage over A*.

Figure 8.25: Per-map SGA* performance results for the "Baldur's Gate" test set

It is also important to note that this is the first test set in which significant differences between V2 MaxDist and V2 MaxDev have been evident. V2 MaxDev results in extremely poor performance over V2 MaxDist on map 32 in Figure 8.26 while on map 33 the opposite is true. Also note that some cases provided an over 90% improvement to both memory usage and search times.

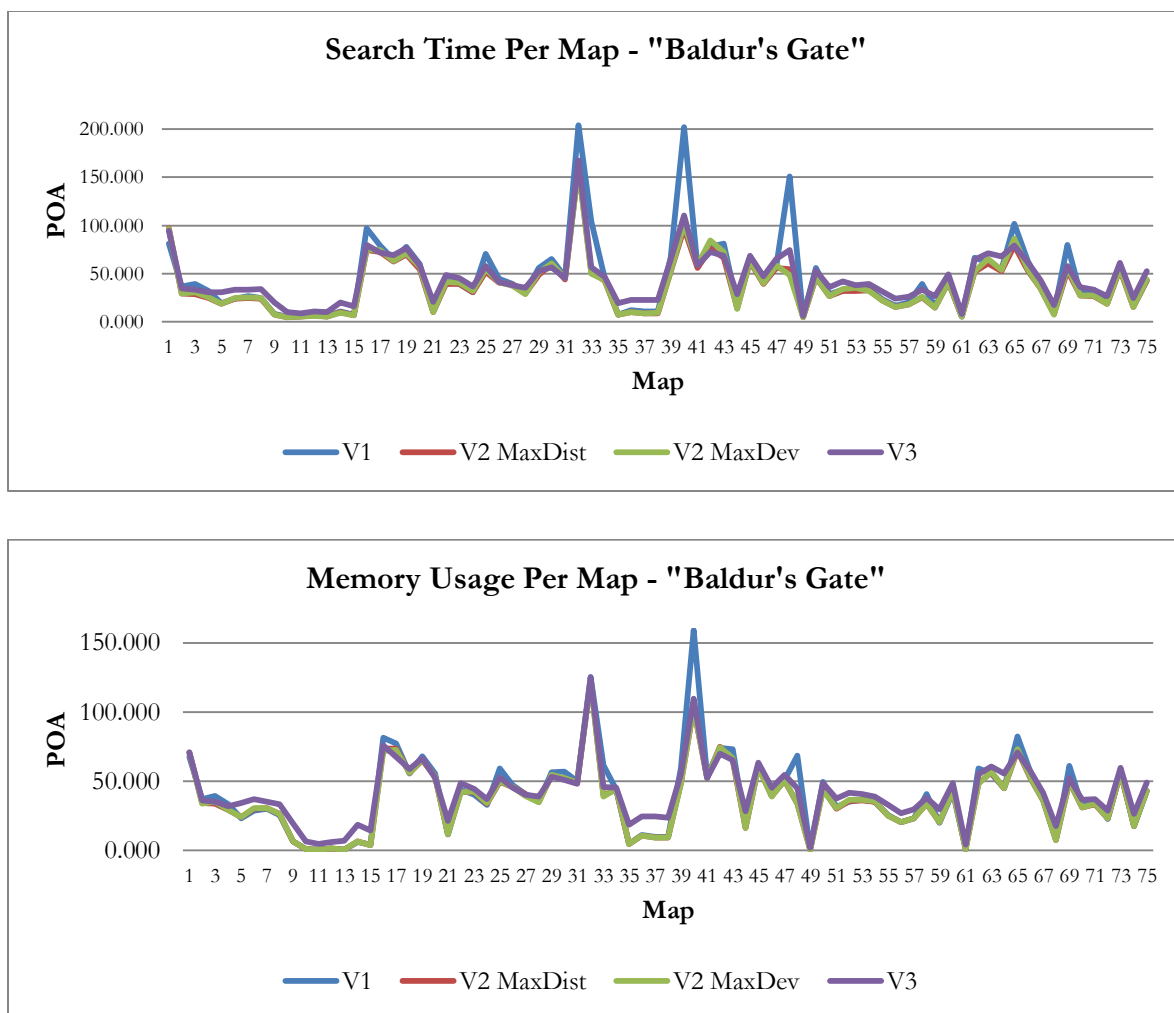Memory reductions correlate to the performance improvements over the test set. Once again a similar pattern emerges with V1 offering worse performance than V2 and V3, with V2 resulting in the best overall performance.

The extremely poor performance exhibited by the SGA* variants on several maps is also extended to the path optimality for those maps. The path optimality results are illustrated in Figure 8.27 and Figure 8.28.

Search Times for Maps 29 to 43 - "Baldur's Gate"

Map	29	30	31	32	33	24	35	36	37	38	39	40	41	42	43
V1	30.370	55.973	65.180	46.512	204.34	104.07	45.167	7.274	11.626	10.880	10.537	66.488	202.32	57.110	77.674
V2 MaxDist	29.113	48.524	59.591	44.057	161.00	51.607	42.439	7.021	10.017	8.791	8.964	52.084	96.924	55.743	75.872
V2 MaxDev	51.194	60.267	45.256	159.86	49.702	43.416	7.056	10.082	8.877	9.019	52.570	101.13	59.065	84.185	72.514
V3	35.496	52.220	56.340	45.308	167.78	56.429	46.418	19.305	22.840	22.687	22.485	64.789	110.19	58.454	72.272

Memory Usage for Maps 29 to 43 - "Baldur's Gate"

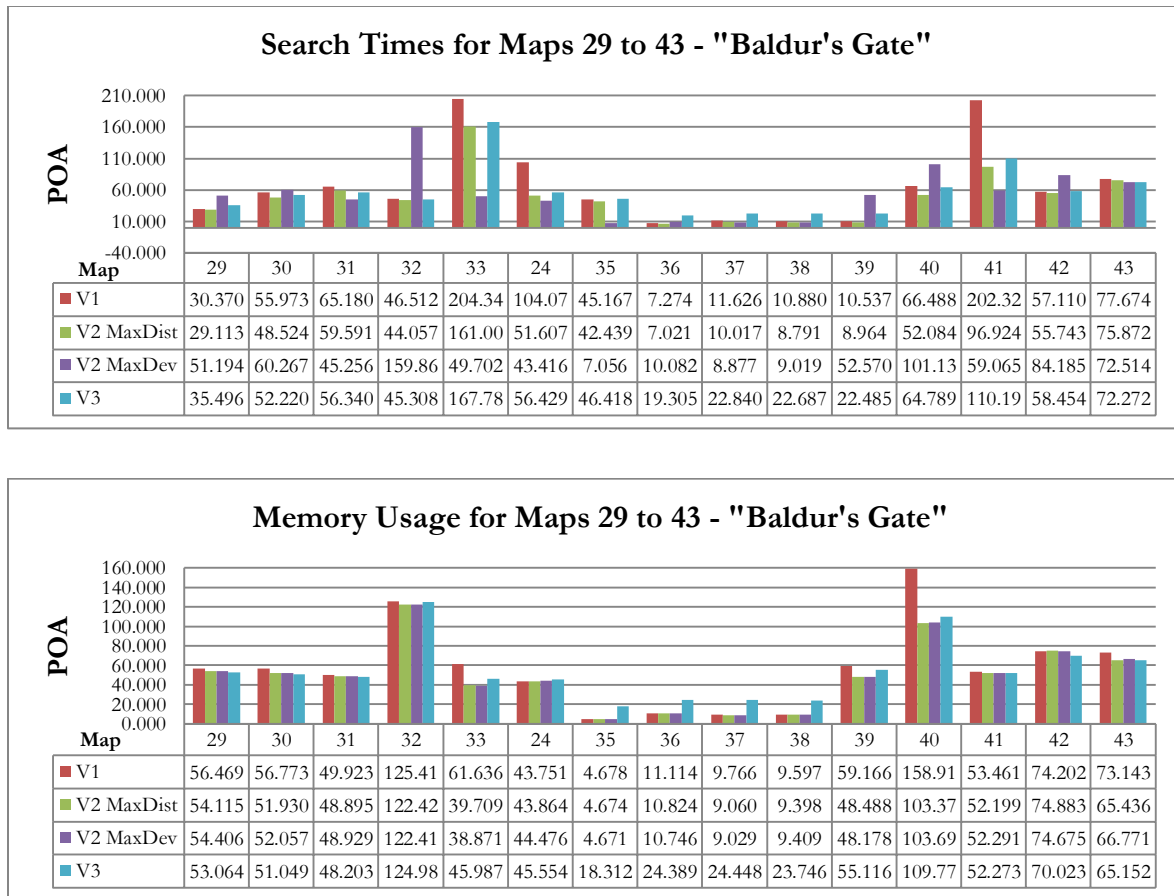| Map	29	30	31	32	33	24	35	36	37	38	39	40	41	42	43
V1	56.469	56.773	49.923	125.41	61.636	43.751	4.678	11.114	9.766	9.597	59.166	158.91	53.461	74.202	73.143
V2 MaxDist	54.115	51.930	48.895	122.42	39.709	43.864	4.674	10.824	9.060	9.398	48.488	103.37	52.199	74.883	65.436
V2 MaxDev	54.406	52.057	48.929	122.41	38.871	44.476	4.671	10.746	9.029	9.409	48.178	103.69	52.291	74.675	66.771
V3	53.064	51.049	48.203	124.98	45.987	45.554	18.312	24.389	24.448	23.746	55.116	109.77	52.273	70.023	65.152

Figure 8.26: Per-map SGA* performance results for maps 29-43 of the "Baldur's Gate" test set

For example, on map 40 (AR0400SR) of the test set, the average path-optimality is over 100% sub-optimal. This map exhibits the exact same problem as the "cs_militia" map in the "Counter-Strike" test set, namely a large central region with a single point of entry. Furthermore, there exists several more single entry regions within the map. The problematic test map is illustrated in Figure 8.29.

Overall path optimality follows the same trend as for the previous test sets. V1 exhibits the worst path optimality both smoothed and unsmoothed. V3 exhibits better unsmoothed path optimality than V2 but once smoothing has been applied, there is no significant difference between the final path optimality of V2 and V3. In many cases the smoothed path optimality is actually optimal which until now has not occurred. The reason for the optimality is that, due to the map topologies, there is often only a single route between map regions and so A* and SGA* will both return the exact same final paths.
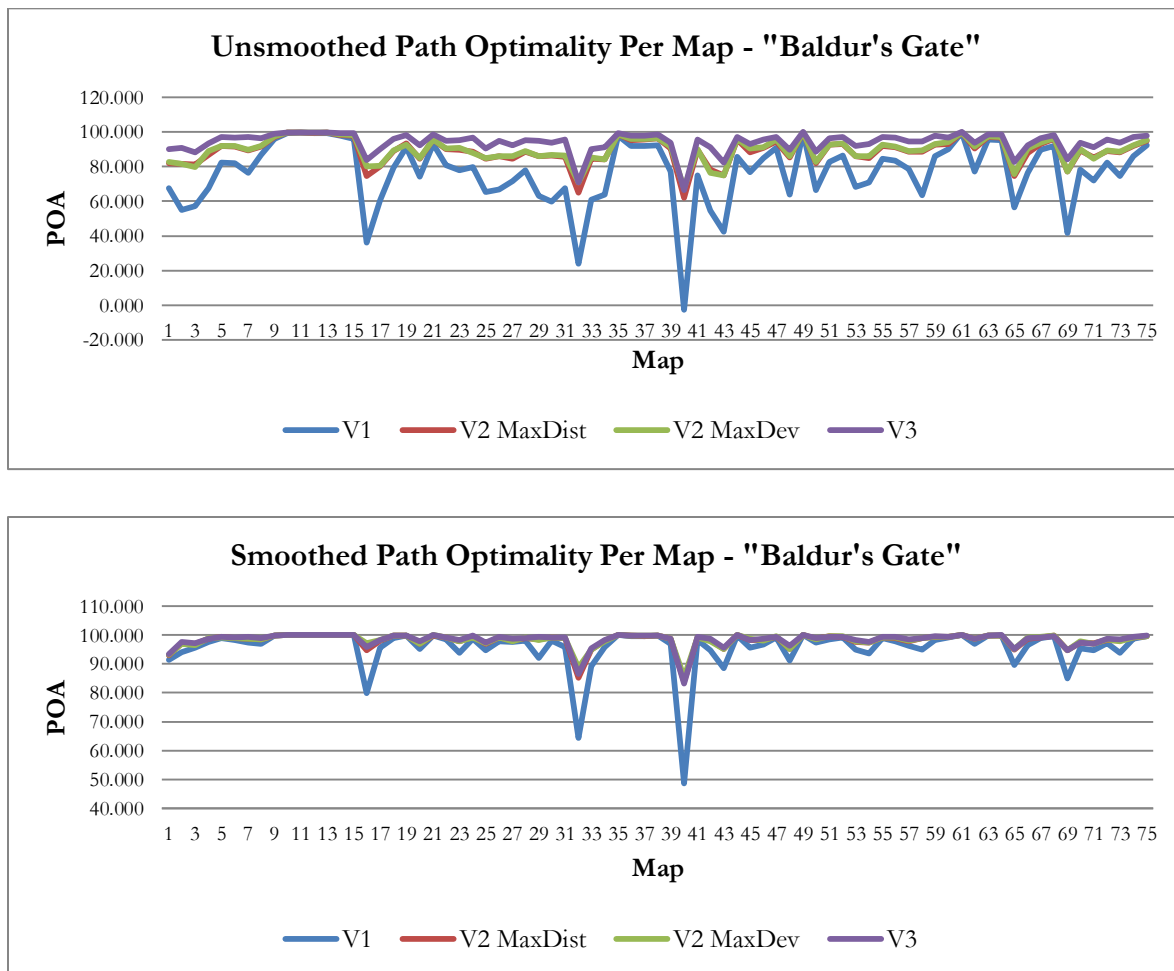

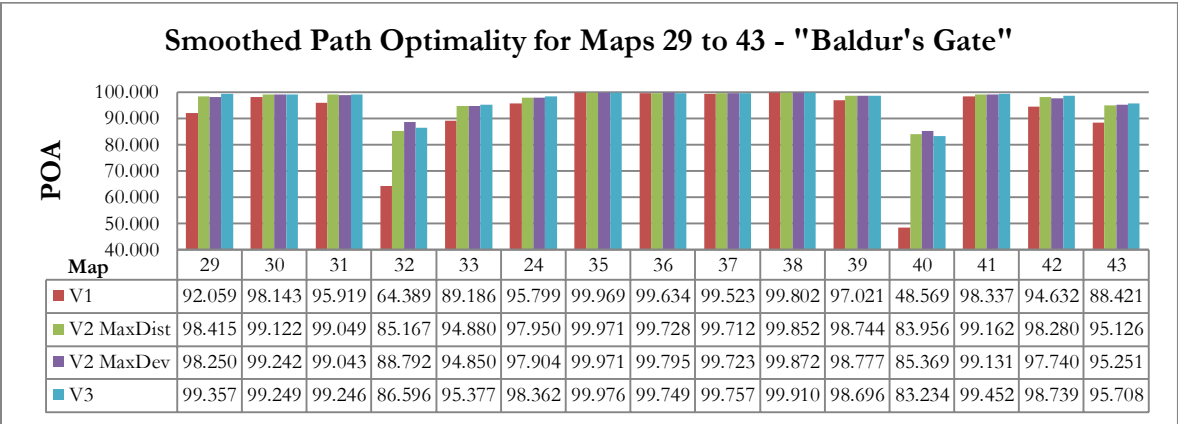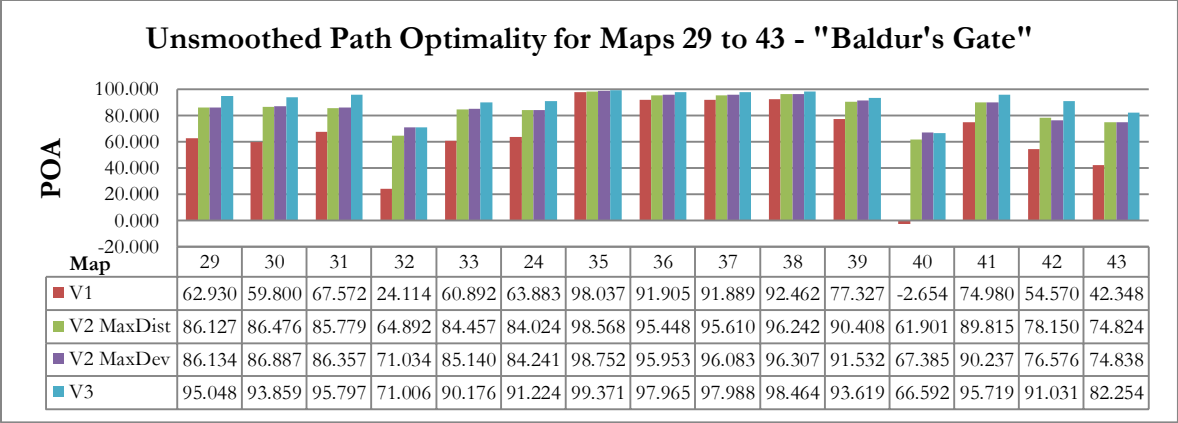Figure 8.27: Per-map SGA* path optimality measurements for the "Baldur's Gate" test set

Unsmoothed Path Optimality for Maps 29 to 43 - "Baldur's Gate"

Map	29	30	31	32	33	24	35	36	37	38	39	40	41	42	43
V1	62.930	59.800	67.572	24.114	60.892	63.883	98.037	91.905	91.889	92.462	77.327	-2.654	74.980	54.570	42.348
V2 MaxDist	86.127	86.476	85.779	64.892	84.457	84.024	98.568	95.448	95.610	96.242	90.408	61.901	89.815	78.150	74.824
V2 MaxDev	86.134	86.887	86.357	71.034	85.140	84.241	98.752	95.953	96.083	96.307	91.532	67.385	90.237	76.576	74.838
V3	95.048	93.859	95.797	71.006	90.176	91.224	99.371	97.965	97.988	98.464	93.619	66.592	95.719	91.031	82.254

Smoothed Path Optimality for Maps 29 to 43 - "Baldur's Gate"

Map	29	30	31	32	33	24	35	36	37	38	39	40	41	42	43
V1	92.059	98.143	95.919	64.389	89.186	95.799	99.969	99.634	99.523	99.802	97.021	48.569	98.337	94.632	88.421
V2 MaxDist	98.415	99.122	99.049	85.167	94.880	97.950	99.971	99.728	99.712	99.852	98.744	83.956	99.162	98.280	95.126
V2 MaxDev	98.250	99.242	99.043	88.792	94.850	97.904	99.971	99.795	99.723	99.872	98.777	85.369	99.131	97.740	95.251
V3	99.357	99.249	99.246	86.596	95.377	98.362	99.976	99.749	99.757	99.910	98.696	83.234	99.452	98.739	95.708

Figure 8.28: Per-map SGA* path optimality measurements for maps 29 to 43 of the "Baldur's Gate" test set

Figure 8.29: Map AR0400SR from the "Baldur's Gate" test set

The overall results for the "Baldur's Gate" test set are quite promising but are misleading without the per-map evaluations presented above. SGA* does not offer improvement across the entire test set as the overall averages would suggest. The poor performance of the SGA* on several of the test maps is welcome as it further strengthens the ability to accurately state the domain restrictions for the SGA* variants. Even though the SGA* variants are not suitable for the entire test set, there are quite significant overall improvements across the test set.
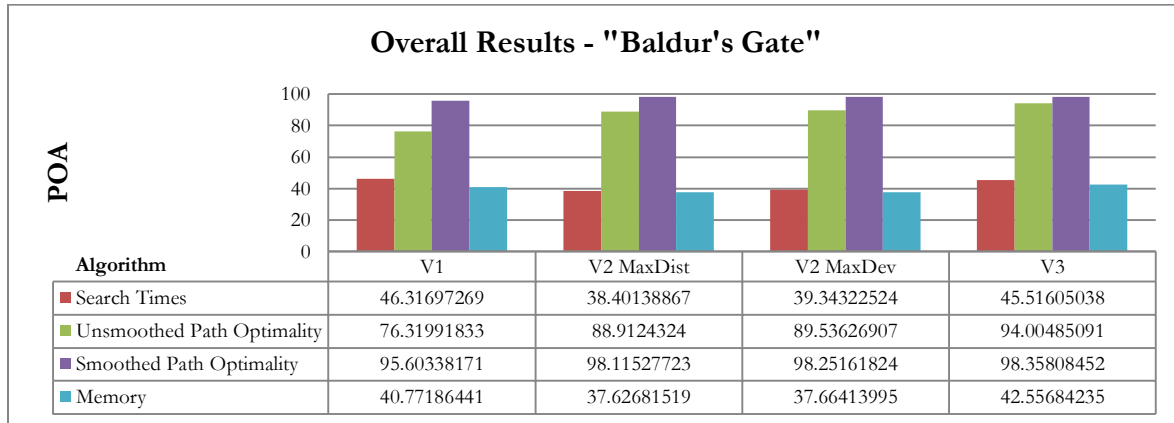

Overall Results - "Baldur's Gate"

Algorithm	V1	V2 MaxDist	V2 MaxDev	V3
Search Times	46.31697269	38.40138867	39.34322524	45.51605038
Unsmoothed Path Optimality	76.31991833	88.9124324	89.53626907	94.00485091
Smoothed Path Optimality	95.60338171	98.11527723	98.25161824	98.35808452
Memory	40.77186441	37.62681519	37.66413995	42.55684235

Figure 8.30: Overall SGA* results for the "Baldur's Gate" test set

8.3.5 Starcraft Maps

"Starcraft" is currently the most popular real time strategy game in the world and was released in 1998 by Blizzard Entertainment [137]. "Starcraft" maps are highly symmetrical, with most maps featuring a large open central area surrounded by single entrance "base" areas. Example maps from the Starcraft test set are illustrated in Figure 8.31.

The "Starcraft" test maps also include a massive number of disjoint regions within the maps. As such, the "Starcraft" maps are an excellent test set for highlighting the additional processing costs of sub-goal selection within these disjoint regions (refer to 8.2.1.1). The "Starcraft" test set represents the absolute worst case scenario for the SGA* variants. The performance of the SGA* variants is therefore expected to be extremely poor across this test set.

The "Starcraft" maps are once again courtesy of Nathan Sturtevant and are included as part of the hierarchical open graph 2 (HOG2) project [138].

The "Starcraft" test set contains 125 maps of varying dimensions. The most common map dimensions are 512x512 and 768x768. There are 1280 test problems per-map for a total of 160000 test problems across all maps. Per-map results are presented as the average result over the approximately 1280 test problems for that map.

Figure 8.14

Figure 8.31: Example maps from the "Starcraft" test set

The per-map performance and path optimality results are illustrated in Figure 8.32 and Figure 8.33 respectively. As expected, SGA* performance on the "Starcraft" test set is extremely poor with over 2000% increases to A* search times in certain cases. Memory usage is also negatively affected although not to the same degree as the search times. What is interesting is that in some of the maps, SGA* does in fact provide improvements to both search times and memory usage, these improvements occur on maps that do not feature disjoint regions and have a high degree of connectivity between map regions.

"Starcraft" does not feature dynamic environments and preprocessing the grid map is possible to allow for the SGA* region labeling optimization from Section 8.2.1.2 to be employed. This optimization is primarily useful for static environments as any environmental changes may require region relabeling and so incur additional costs. The region labeling optimization has not been employed and has been left for future work.

The overall performance results are illustrated in Figure 8.34 and serve to illustrate the unsuitability of the SGA* variants within such environments.
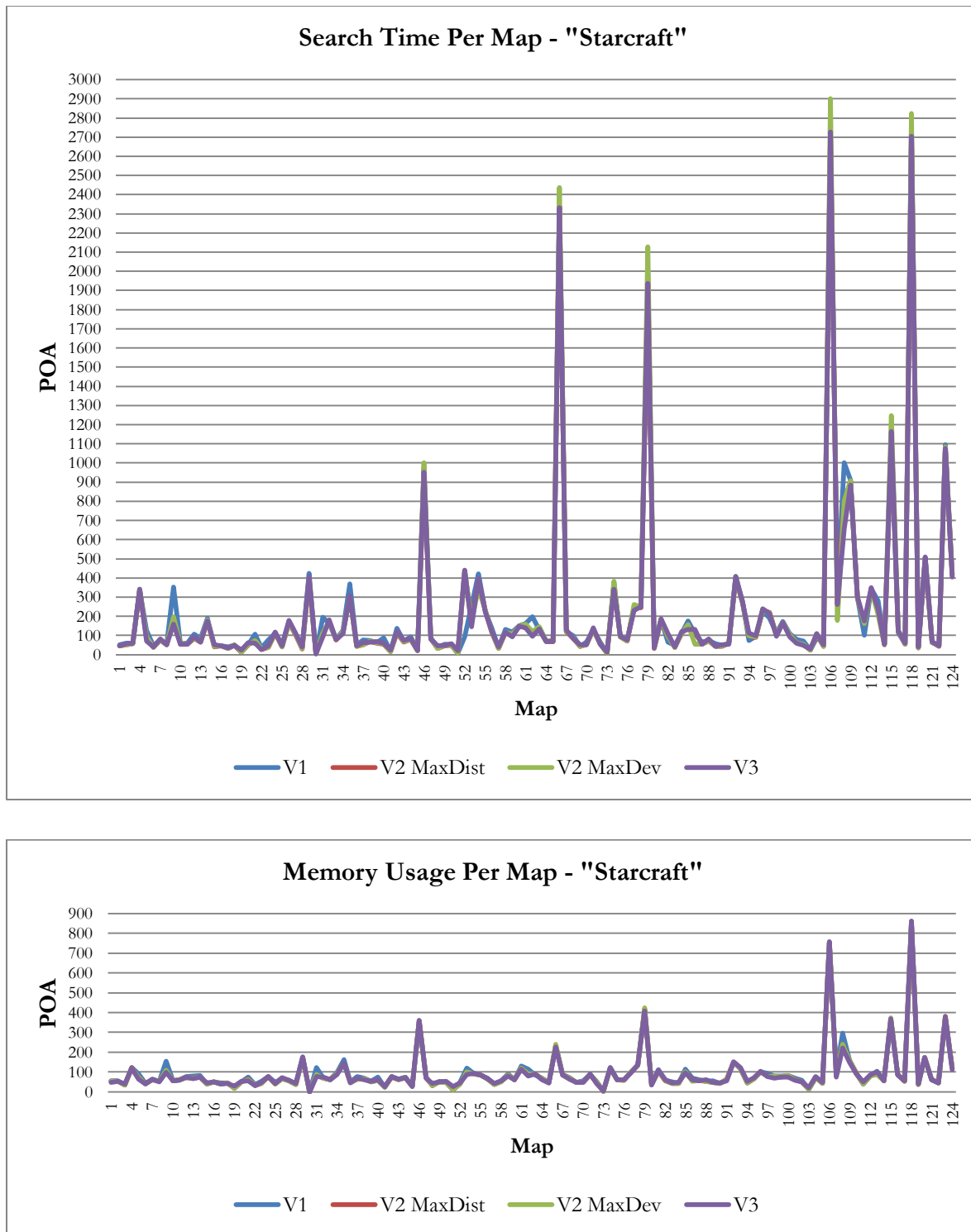
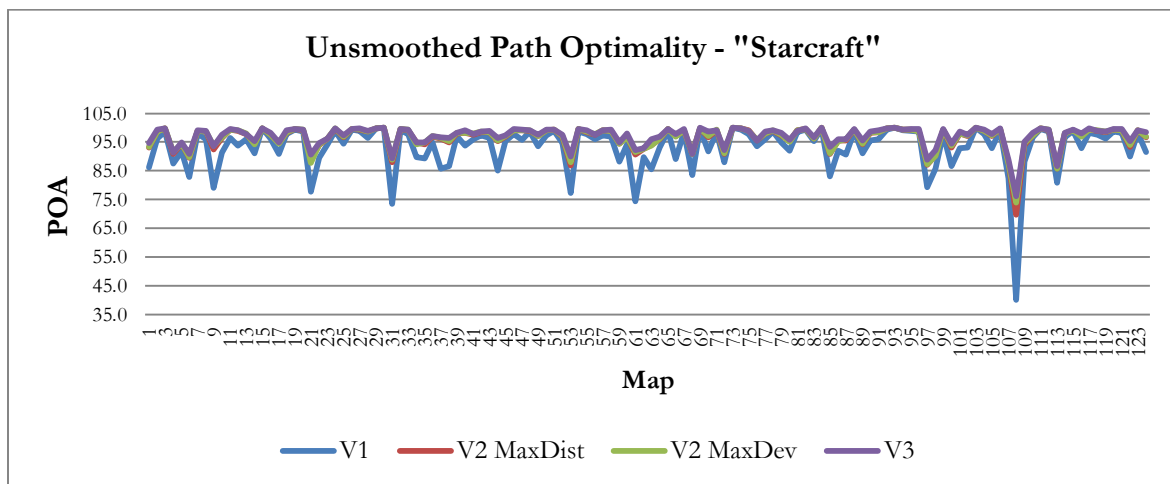**Figure 8.32: Per-map SGA* performance measurements for the "Starcraft" test set**
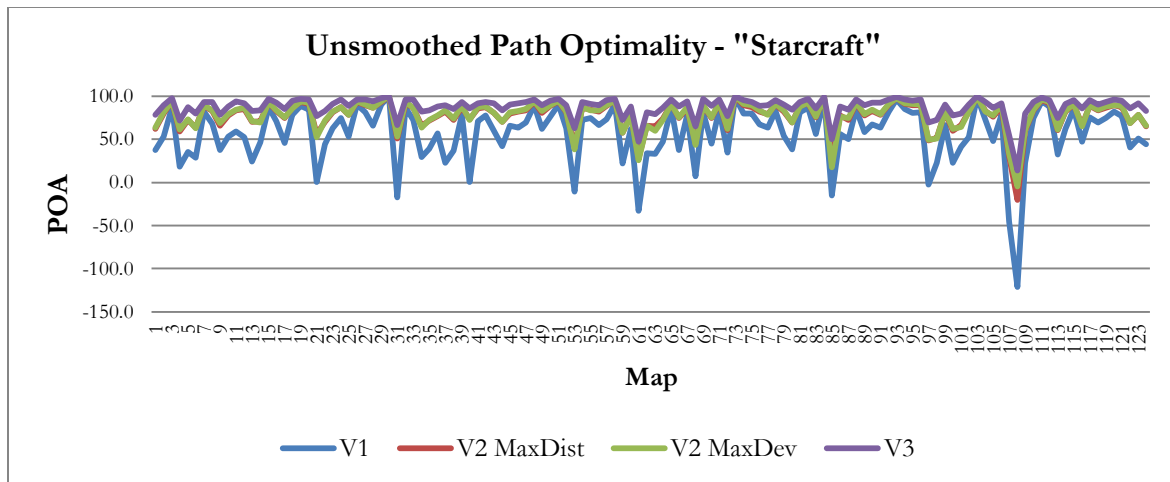
Figure 8.33: Per-map SGA* path optimality measurements for the "Starcraft" test set

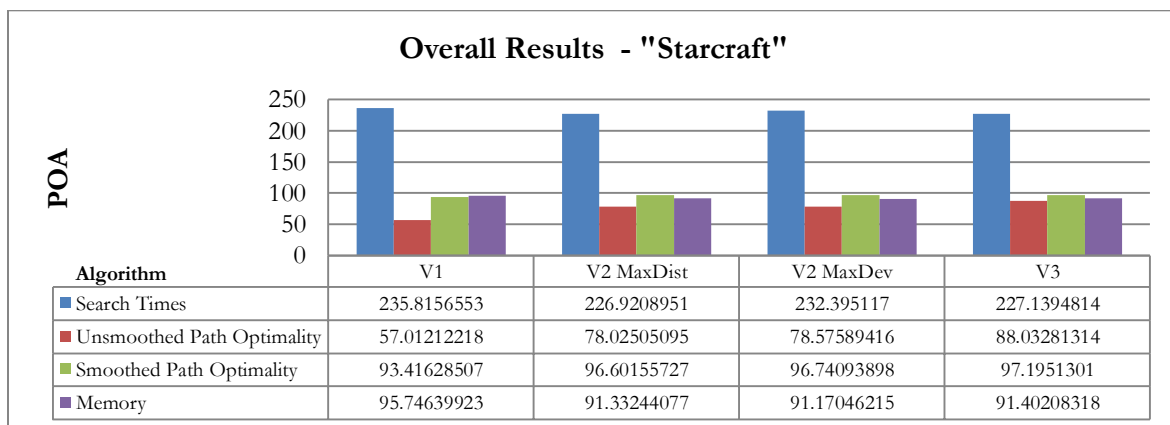| Algorithm	V1	V2 MaxDist	V2 MaxDev	V3
■ Search Times	235.8156553	226.9208951	232.395117	227.1394814
■ Unsmoothed Path Optimality	57.01212218	78.02505095	78.57589416	88.03281314
■ Smoothed Path Optimality	93.41628507	96.60155727	96.74093898	97.1951301
■ Memory	95.74639923	91.33244077	91.17046215	91.40208318

Figure 8.34: Overall SGA* results for the "Starcraft" test set

8.3.6 Pseudo Real Time Strategy Maps

In addition to the standard game map test sets, a further test set was used featuring highly connected maps with an extremely high obstacle density. These maps represent a test set similar to the one presented in the "Company of Heroes" maps except with a much lower degree of difficulty due to the high level of connectivity and small obstacle size. These maps represent the absolute best case scenario for the SGA* algorithm variants and the results were included to show off the potential benefits of the SGA* variants within the intended domains. Example maps from the "Pseudo RTS" test set are illustrated in Figure 8.35.
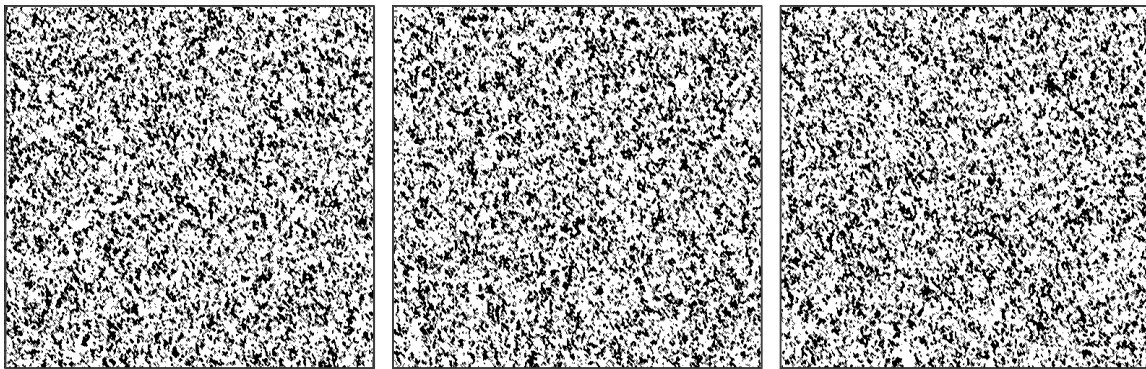

Figure 8.35: Example maps from the pseudo RTS test set

The pseudo RTS maps were generated by the author using a pseudo-random map generator. The test set includes 50 maps with dimension 512x512. There are 1280 test problems per-map for a total of 64 000 test problems across all maps. Per-map results are presented as the average result over the 1280 test problems for that map.

The SGA* variants provide huge improvements (over 90%) to both the search time as well the memory usage of A* within the intended problem domain. This test set further highlights the improvements to both search time and path optimality of the V2 variant over the V1 variant. The per-map performance and path optimality results are illustrated in Figure 8.36 and Figure 8.37 respectively. Furthermore, within these environments there is a clear effect on the unsmoothed path optimality of the V2 variant resulting from the choice of path refinement trimming technique. The maximum deviation trimming technique results in higher optimality paths with no detrimental effect on the search times.

The final smoothed path optimality of the V2 and V3 variants is on average around 7% sub-optimal with no significant difference between the path optimality of the variants. The V3 variant once again results in longer search times than either of the V2 variants. The overall performance results are illustrated in Figure 8.38.
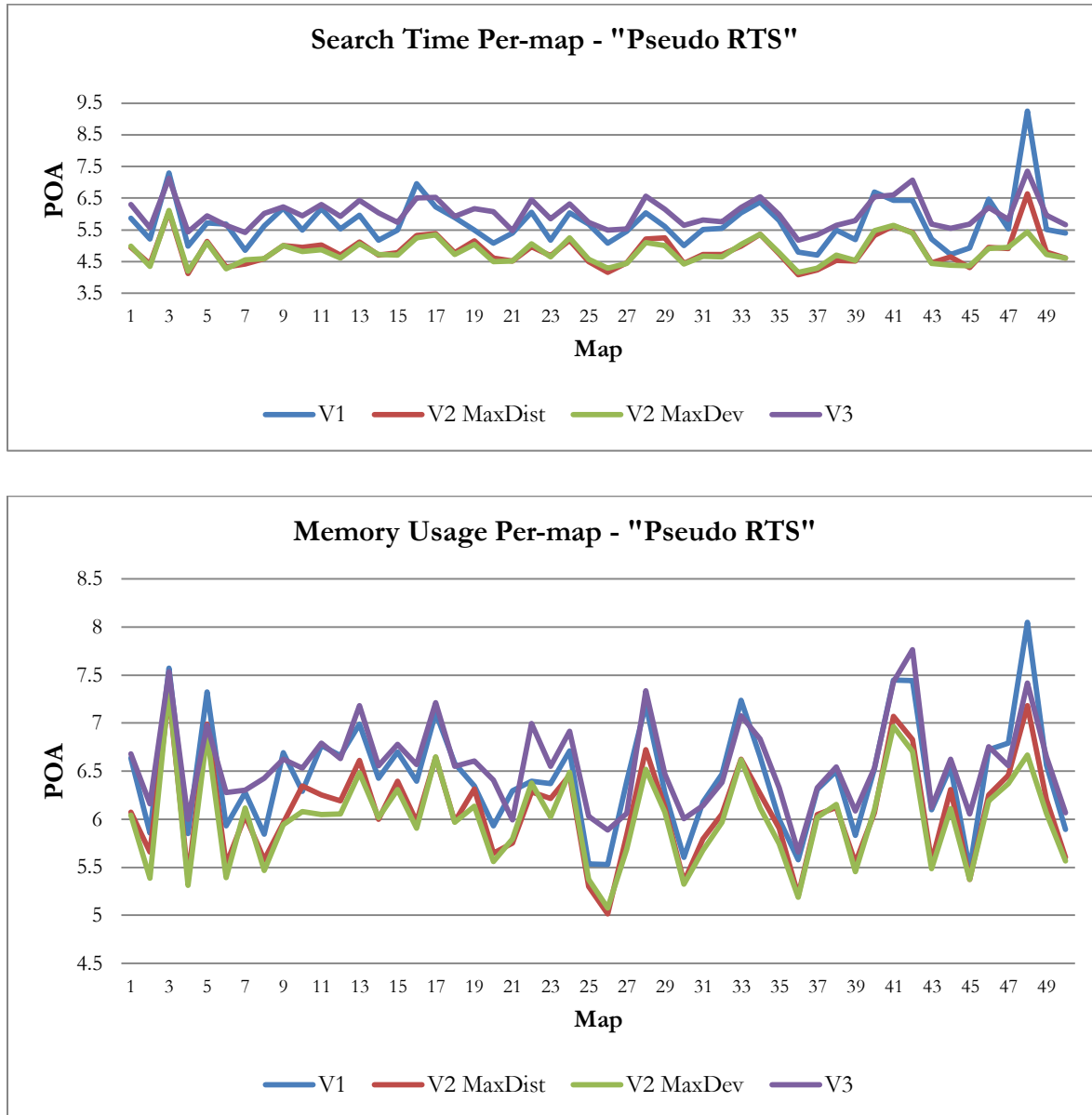

Figure 8.36: Per-map SGA* performance measurements for the "Pseudo RTS " test set
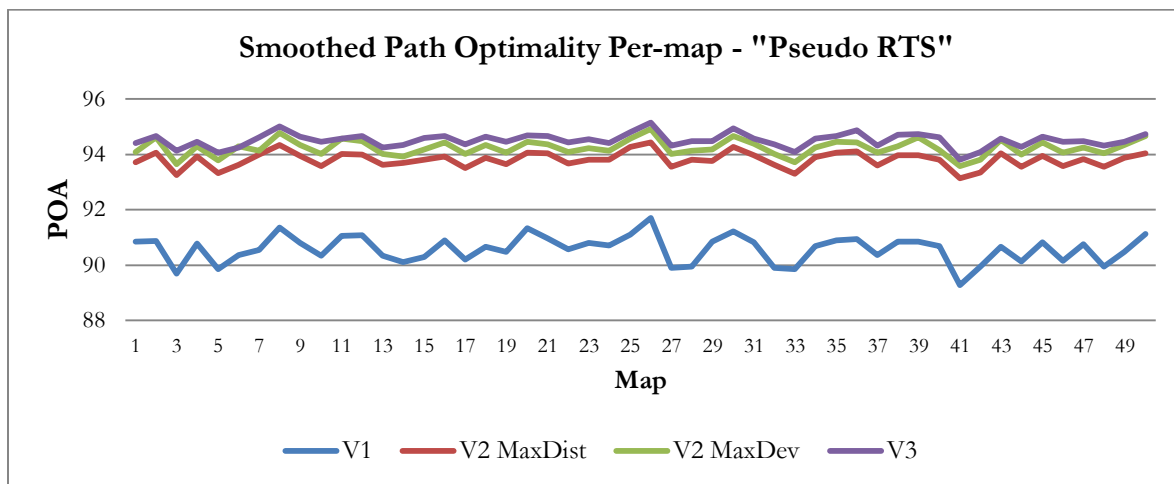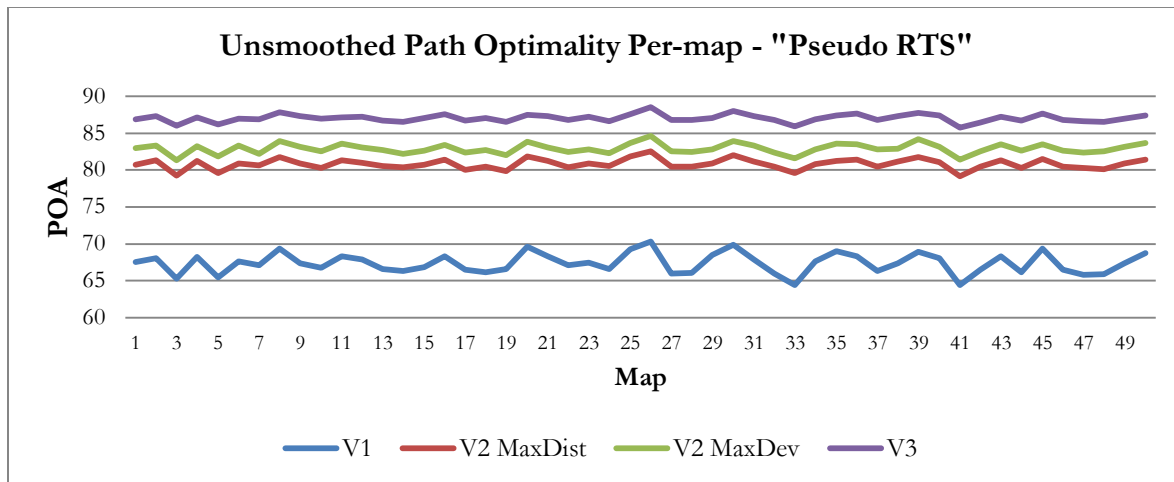
Figure 8.37: Per-map SGA* path optimality measurements for the "Pseudo RTS" test set

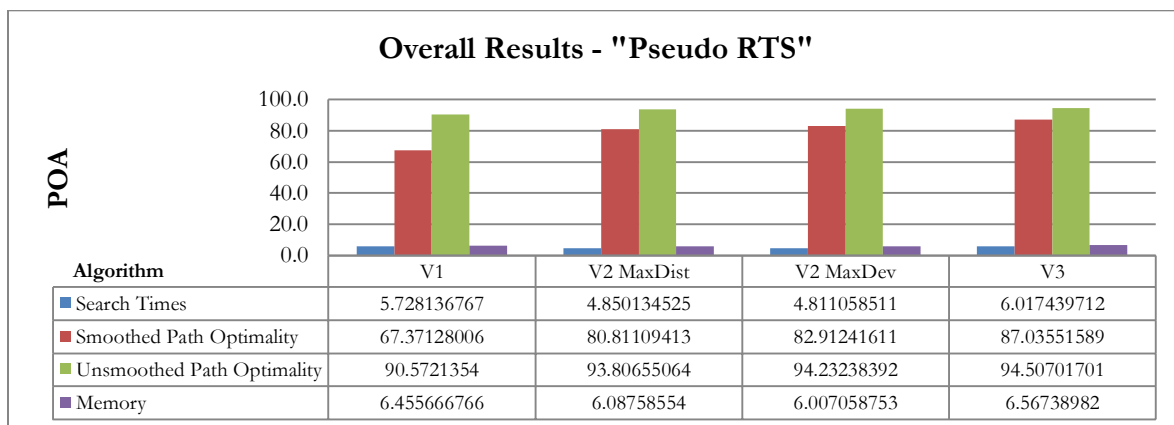| Algorithm	V1	V2 MaxDist	V2 MaxDev	V3
■ Search Times	5.728136767	4.850134525	4.811058511	6.017439712
■ Smoothed Path Optimality	67.37128006	80.81109413	82.91241611	87.03551589
■ Unsmoothed Path Optimality	90.5721354	93.80655064	94.23238392	94.50701701
■ Memory	6.455666766	6.08758554	6.007058753	6.56738982

Figure 8.38: Overall SGA* results for the "Pseudo RTS" test set

8.4 Summary and Conclusion

This chapter presented the SGA* algorithm as a complete, domain specific, sub-optimal gridmap search technique for improving the performance of an A* search algorithm. The SGA* is based on a problem sub-division approach commonly found in hierarchical search algorithms. The SGA* algorithm makes use of the game map's spatial data to perform sub-goal selection rather than using a navgraph abstraction hierarchy. The SGA* algorithm was developed as a means of decreasing the processing and memory costs of the A* algorithm at the expense of path optimality.

SGA* is intended for use in extremely memory limited environments in which the memory cost of maintaining an abstraction hierarchy may be too expensive. Abstraction graphs can be significantly more expensive to maintain than gridmaps since the lack of uniformity in the graph require edge data to be stored for each node.

Unlike hierarchical approaches when used within a dynamic environment, there is no need for any special considerations with regards to environmental changes apart from directly updating the gridmap. Furthermore SGA* offers the ability to return partial paths from failed searches, something which the A* algorithm cannot achieve.

As mentioned, the SGA* variants are domain specific and the intended domain requires that the environment contain the following features:

- Large open regions with high levels of connectivity between the various regions.
- No disjoint regions within the map environment.

Three SGA* variants were presented:

- **Variant 1:** Variant 1 is the basic forward-only version of the SGA* algorithm. Sub-goals are selected in advance after which path refinement occurs.
- **Variant 2:** Variant 2 is a double ended version of the SGA* which makes use of dynamic sub-goal selection. Sub-goal refinement paths are now trimmed to create the required sub-goals. Two path refinement trimming techniques were presented in Section 8.2.2.2: maximum distance trimming and maximum deviation trimming.

- **Variant 3:** Variant 3 is a modification of variant 2 in that the created line segments are not added to the final path. A* path refinement occurs from sub-goal to sub-goal and not from the end of a line-segment to a sub-goal.

The SGA* variants were evaluated on four distinct real world game map test sets as well as on a best case test set. On the test sets which meet the domain specific criteria of the SGA* algorithm significant improvements to the standard A* algorithm were achieved.

Certain trends were observed across all the test sets. The V2 variant performed significantly better with respect to search times, memory costs and path optimality than the V1 variant. The V3 variant, while improving upon the unsmoothed path optimality, did not have any significant effect on the path optimality of the final smoothed paths and results in longer search times than the V2 variants.

The choice of the path refinement trimming technique used by the V2 variant seems to be largely inconsequential with only one test set showing any significant difference between trimming techniques. Even in that single test set, the difference between trimming techniques was only to the unsmoothed path optimality which, once path smoothing was applied, was nullified.

In light of the results of the simulations, there are no benefits to the V1 or V3 variants over that of the V2 variant. It is therefore the recommendation of the author that the term SGA* algorithm be taken to imply the V2 variant.

In conclusion, the SGA* algorithm has been shown to improve the search times and memory usage by up to 90% within domain specific environments at the cost of around 6% path optimality.

Chapter 9

Conclusions and Future Work

This short chapter summarizes the findings of this work. In addition, future work resulting from these findings is suggested.

9.1 Conclusions

The aim of this thesis was to provide a complete and detailed review of the video game pathfinding field, with a focus on the pathfinding search algorithms employed in said field. Video game AI was overviewed, highlighting the difference between video game AI and academic AI. A brief introduction to video game engines was given to investigate the requirements and constraints placed upon the video game AI system.

During the discussing of the game agent model, it was shown that the majority of higher level agent behaviors rely on the ability of an agent to successfully navigate the game environment making pathfinding an integral component of any video game AI system.

Due to the high complexity of modern video game environments, to be able to find a route through a game environment, that game environment must be represented in a lightweight simple graph format. Since game environments are reduced to graphs, the pathfinding search problem is subsequently reduced to a graph search problem.

There are three methods of graph abstraction used to represent a complex 3D environment into a graph format: waypoint graphs, navmesh graphs and grid-based graph. The choice of graph abstraction is dependent on the size and complexity of the environments present within a game.

The requirements and constraints of the pathfinding search problem were investigated and a set of metrics for the comparison of search algorithms was presented. There are three primary metrics used for comparing graph search algorithms: performance, memory cost and solution optimality. It was revealed that improving algorithm performance, a reduction in solution optimality was acceptable. Furthermore, it was shown that the pathfinding search problem is highly domain

specific. The domain boundaries are defined by the game's performance and memory constraints as well as the type and complexity of the in-game environments. The problem domain is responsible for both the choice of navgraph abstraction technique and choice of graph search algorithm employed.

The three main families of graph search algorithms (discrete, continuous, and hierarchical) employed today were reviewed next. There is a significant difference between discrete and continuous search algorithms while hierarchical search algorithms can be seen as an extension of the discrete search family. Discrete search algorithms return a path between two nodes in a graph through a single atomic action. Discrete search algorithms are problem centric, meaning that performance and memory costs are paid per problem and are released upon solving that problem.

On the other hand, continuous search algorithms are agent centric in that a path is found and updated over the course of an agent's movement from the start location to the goal location. The agent centric nature of the continuous algorithm means that the memory allocated to solve a problem is now allocated per agent for the duration of the agent's move. In multi-agent environments, the total memory cost of a continuous search algorithm's per-agent memory allocation model can easily exceed the total memory available to the game.

Hierarchical search algorithms were investigated and shown to be a technique allowing for the subdivision of a complex pathfinding problem into a series of simpler problems. The problem subdivision was achieved through the use of a navgraph abstraction hierarchy with each higher abstraction level offering a significant reduction in the total search space over the lower level. Hierarchical search algorithms are applicable for use within dynamic game environments, but require that the abstraction hierarchy be updated once an environmental change occurs. This abstraction hierarchy update can be a potentially expensive operation and may restrict the use of hierarchical algorithms within dynamic environments.

During the investigation of hierarchical search algorithms the observation was made that pathfinding problem subdivision may be achieved on grid maps by using the spatial data contained within them instead of using an abstraction hierarchy. This observation led to the development of the SGA* algorithm, a technique for the improvement of A* search times through problem subdivision using a straight line spatial rule.

The SGA* algorithm is highly domain specific, being tailored to game environments which contain highly interconnected regions and do not contain disjoint regions. The SGA* algorithm was evaluated on five distinct sets of game environments and was shown to improve upon the search times and memory costs of the industry standard A* search algorithm by over 94% at only a 6% cost to path optimality within the intended search domain.

9.2 Future Research

Research that may follow from this work includes:

SGA* Region Labeling for the SGA* Anti-domain

Although the SGA* algorithm performed admirably within the intended domains, the SGA* algorithm's performance was extremely poor within the "Starcraft" test set. The "Starcraft" test set features a high number of disjoint regions as well as an extremely low connectivity between regions. In fact, the "Starcraft" test set can be seen as representing the anti-domain for the SGA* algorithm. A potential optimization (the region labeling optimization – refer to Section 8.2.1.1) was presented as a means of improving SGA* performance within maps that feature disjoint regions. This region labeling optimization should have no effect on the SGA* performance within maps that feature low connectivity between regions and so was not implemented. It is hoped that the addition of this region labeling will significantly improve the performance of the SGA* algorithm on anti-domain environments.

Alternate Spatial Sub-division Criteria

The SGA* algorithm makes use of a straight line drawing technique to allow for sub-goal selection during problem sub-division. The straight line drawing technique is naïve and is responsible for various anomalies in the resulting unsmoothed SGA* paths (refer to Chapter 8). Future work may investigate the use of other subdivision techniques and the effects thereof on the SGA* algorithm.

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