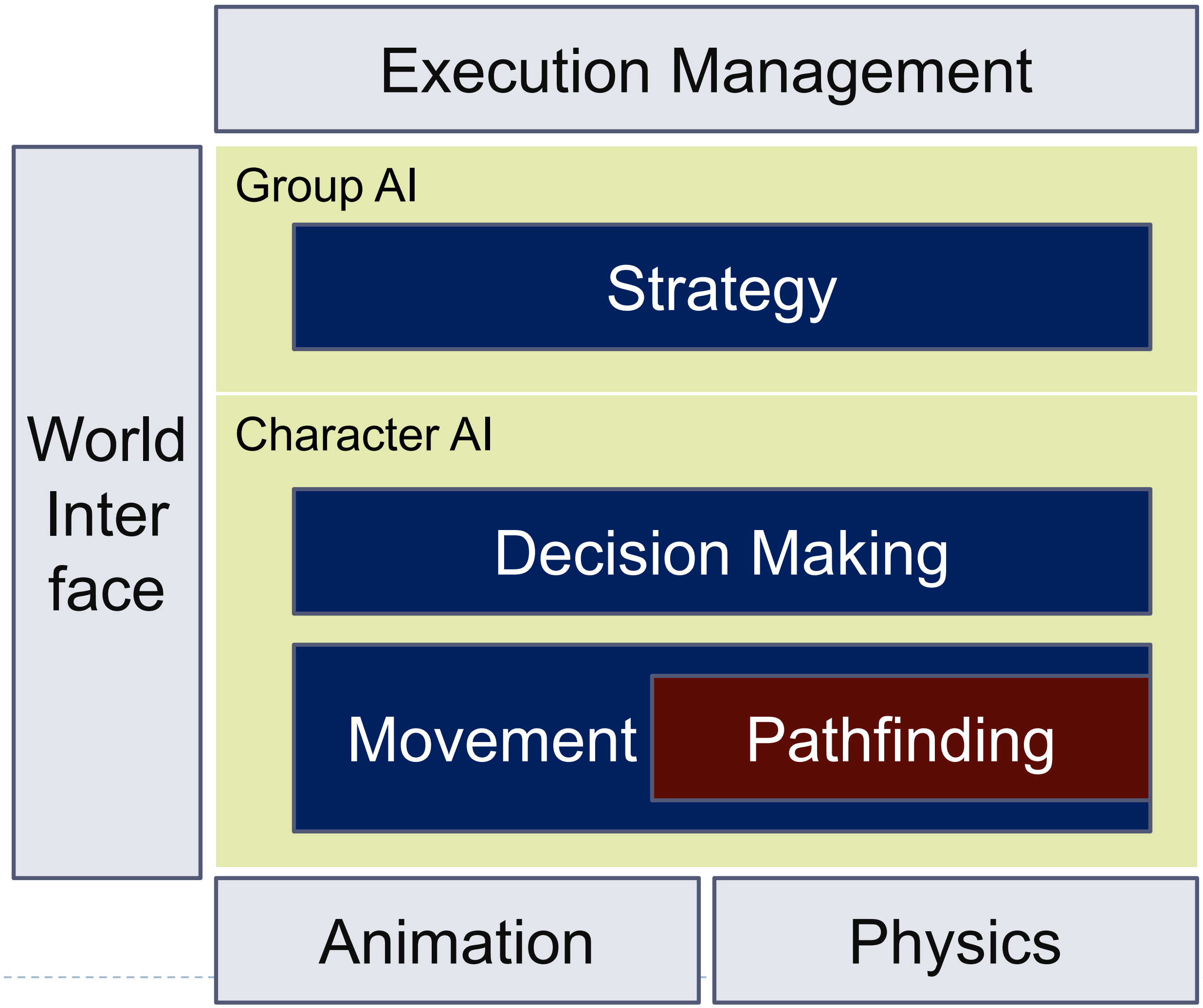
Pathfinding

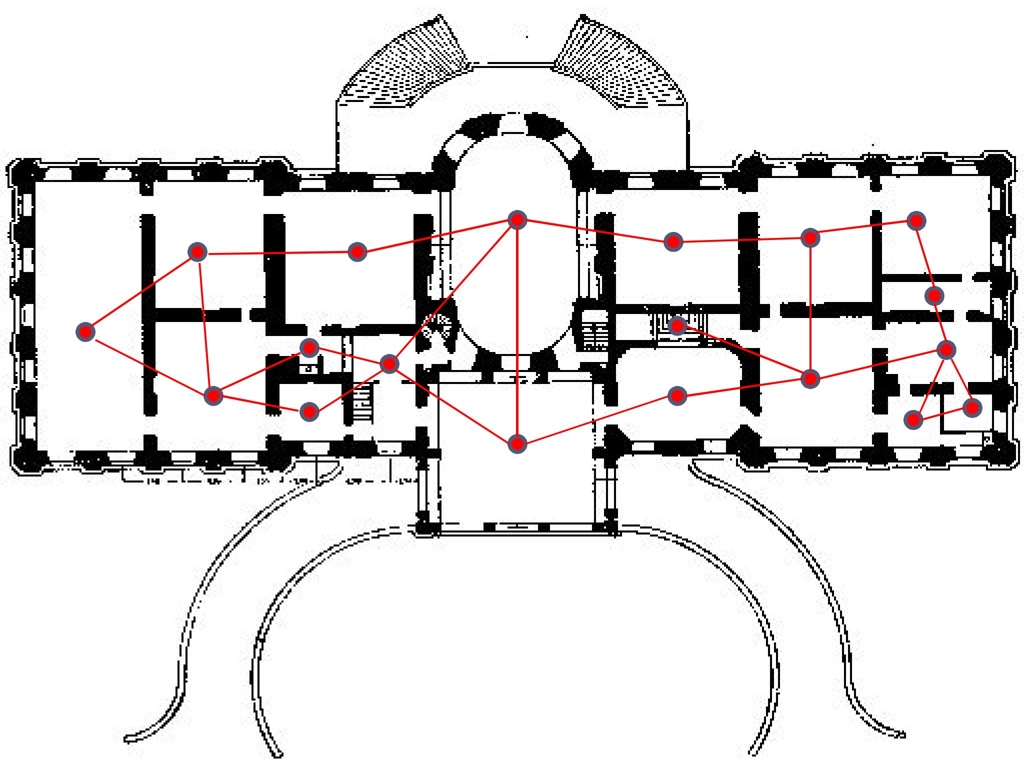
Artificial Intelligence for gaming

Pathfinding



Pathfinding Graphs

- Pathfinding does not work directly on Geometry
- Simplification:
  - Abstraction of movement possibilities in a graph

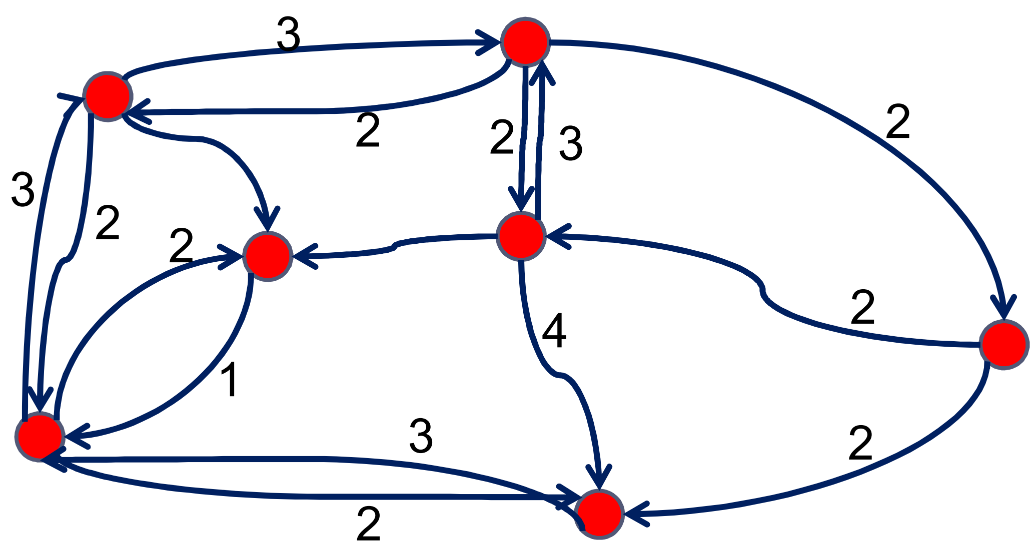


Pathfinding Graphs

- Nodes: Important places
  - Sometimes just rooms
  - Sometimes different places in a single room
  - Preview:
    - For tactical planning, need to
- Edges: Connections through which we can travel
- Weights: Costs of traveling through a certain connection
  - General assumption for developing algorithms:
    - Weights are positive numbers

Pathfinding Graphs

- If travelling costs depend on the direction:
  - Use a directed graph
  - All algorithms support directed graphs

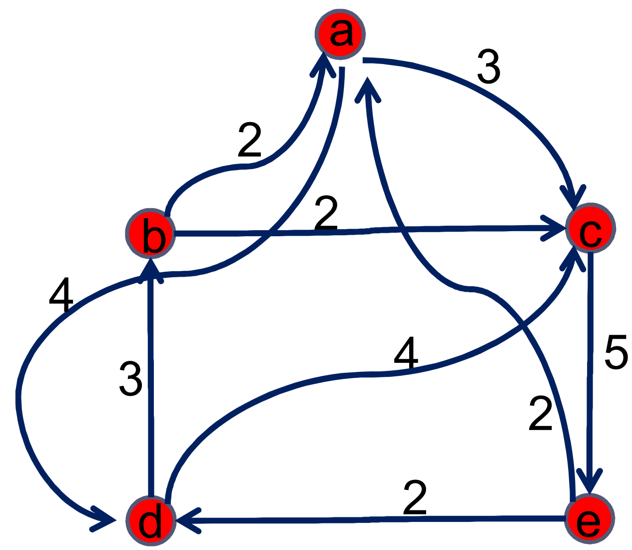


Pathfinding Graphs

- Representation of (directed, weighted) graphs
  - Adjacency List
  - Adjacency Matrix
  - List of edges

```
a:  [(c,3),  (d,4)]
b:  [(a,2),  (c,2)]
c:  [(e,5)]
d:  [(b,3),  (c,4)]
e:  [(a,2),(d,2)]
```

$$\begin{pmatrix} 0 & 0 & 3 & 4 & 0 \\ 2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 \\ 0 & 3 & 4 & 0 & 0 \\ 2 & 0 & 0 & 2 & 0 \end{pmatrix}$$



Pathfinding Graphs

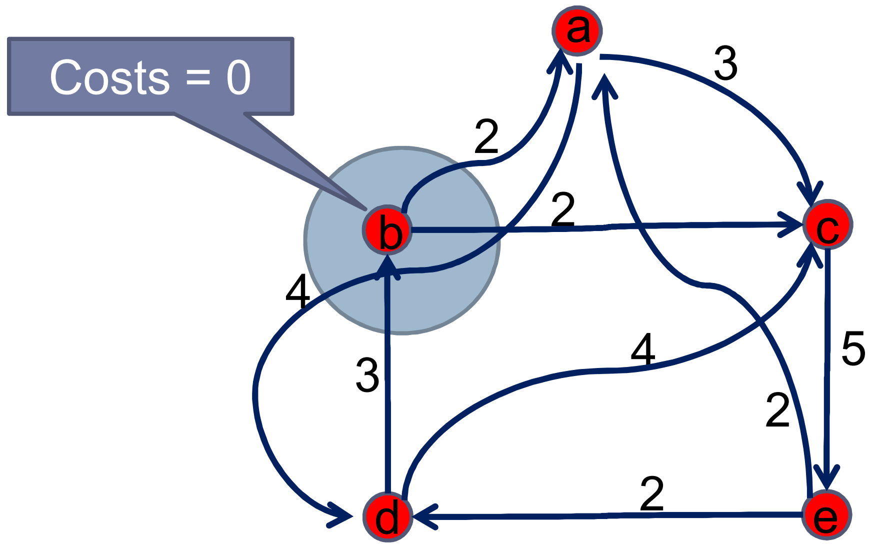
- Dijkstra Algorithm
  - Input: Weighted, directed graph, starting and ending vertex
  - Output: A minimum path between starting and ending vertex
    - Definition: Costs of a path is the sum of the weights

Pathfinding Graphs

- Disjkstra Algorithm
  - Informal Description:
    - Processes nodes one-by-one, starting with the starting node
    - Whenever a node is visited, puts neighboring node not yet visited into a list of “seen” nodes
    - Maintains a list of costs to reach each nodes.
    - Whenever a node is visited:
      - Update all costs when visiting through the node

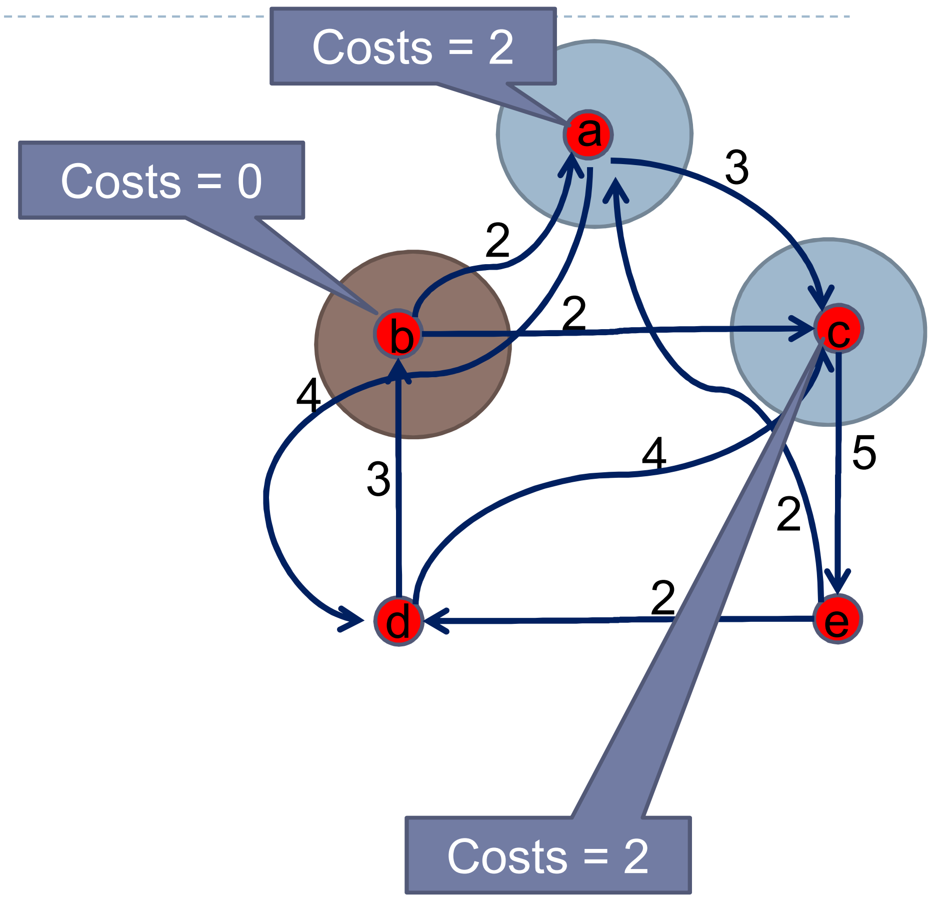
Pathfinding Graphs

- Example:
  - Seen = [b]
  - Costs in b = 0



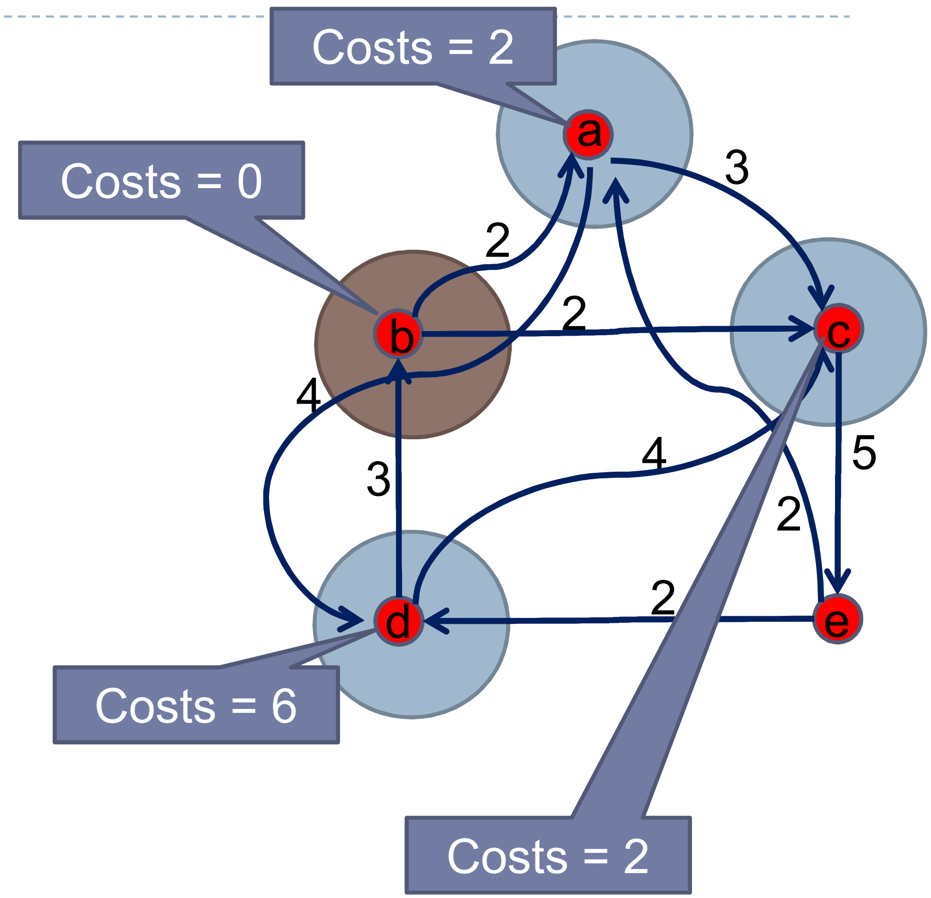
Pathfinding Graphs

- Example:
  - Go to all the edges leaving b
  - Mark the target nodes as seen
  - Give the target nodes costs equal to the weight of the edge
  - Mark node b as done



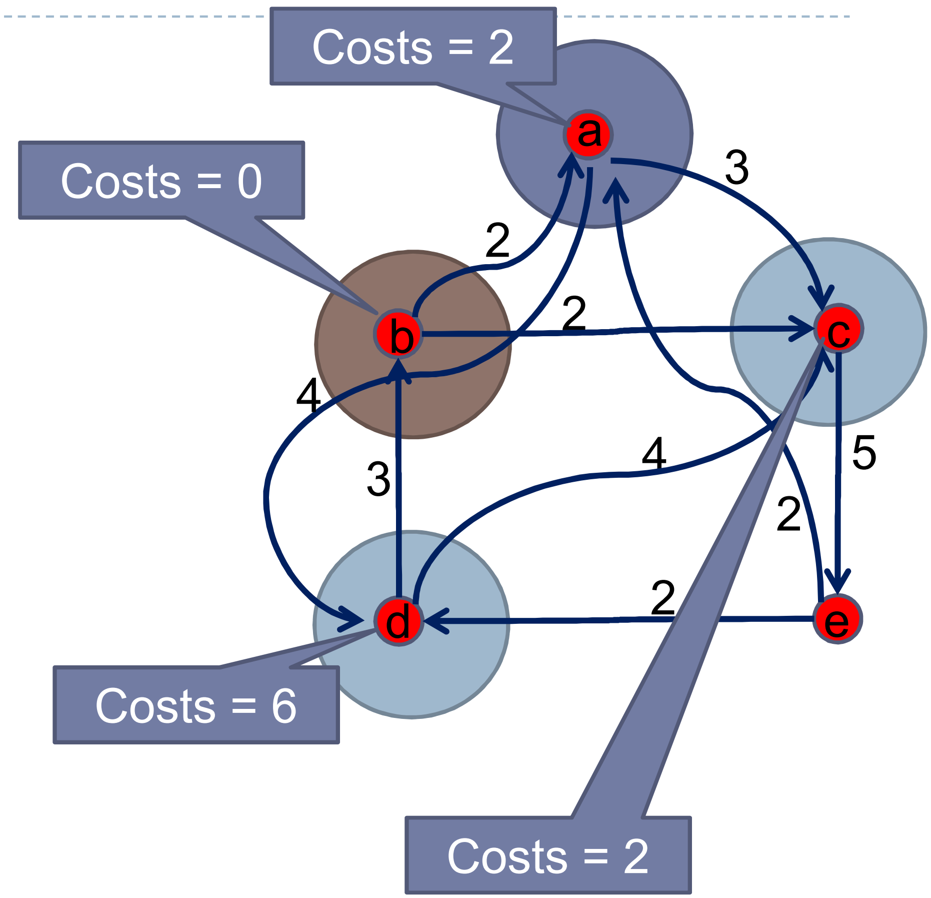
Pathfinding Graphs

- Example:
  - Pick the node with lowest costs in the seen set
    - We break tie by picking a
  - Repeat what we did for b
    - Go to all edges leaving a
    - The edge to d put d into the seen list and gives it costs 2+4



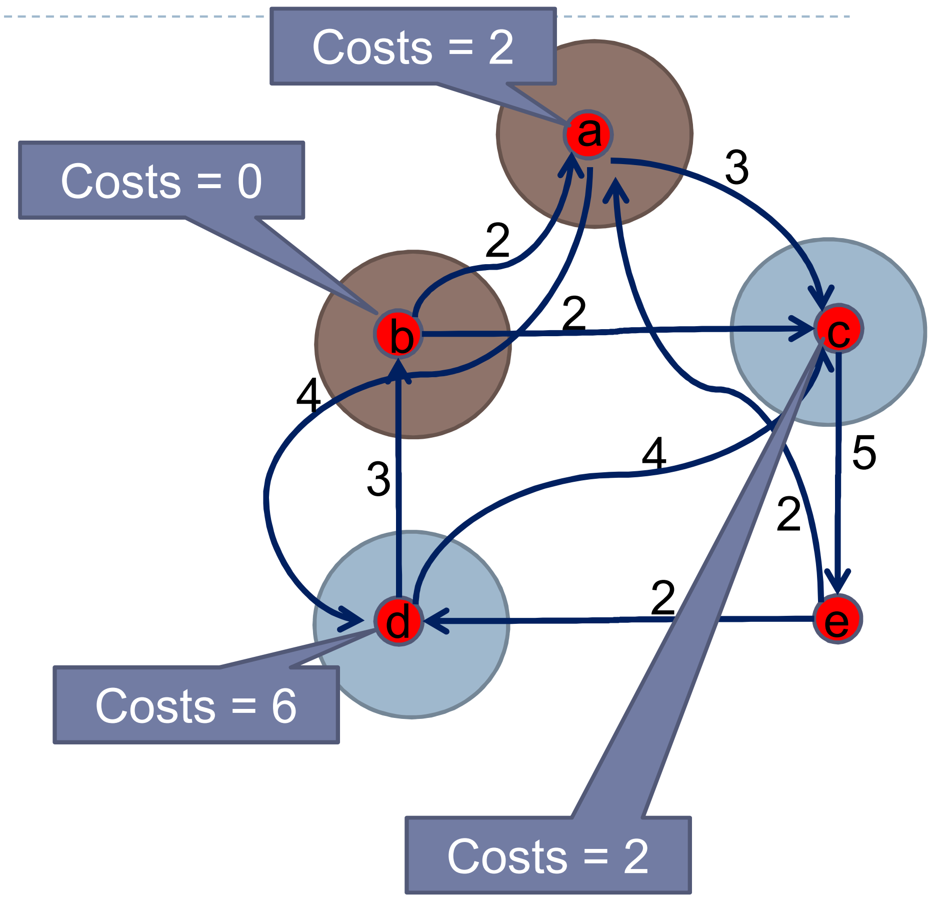
Pathfinding Graphs

- Example:
  - Processing a
  - The edge to c goes to a node already seen
    - In this case, we need to compare the costs through a (costs of a + weight of edge) to the costs previously obtained (in this case 2).
    - We give it the minimum costs 2 = min(5,2)



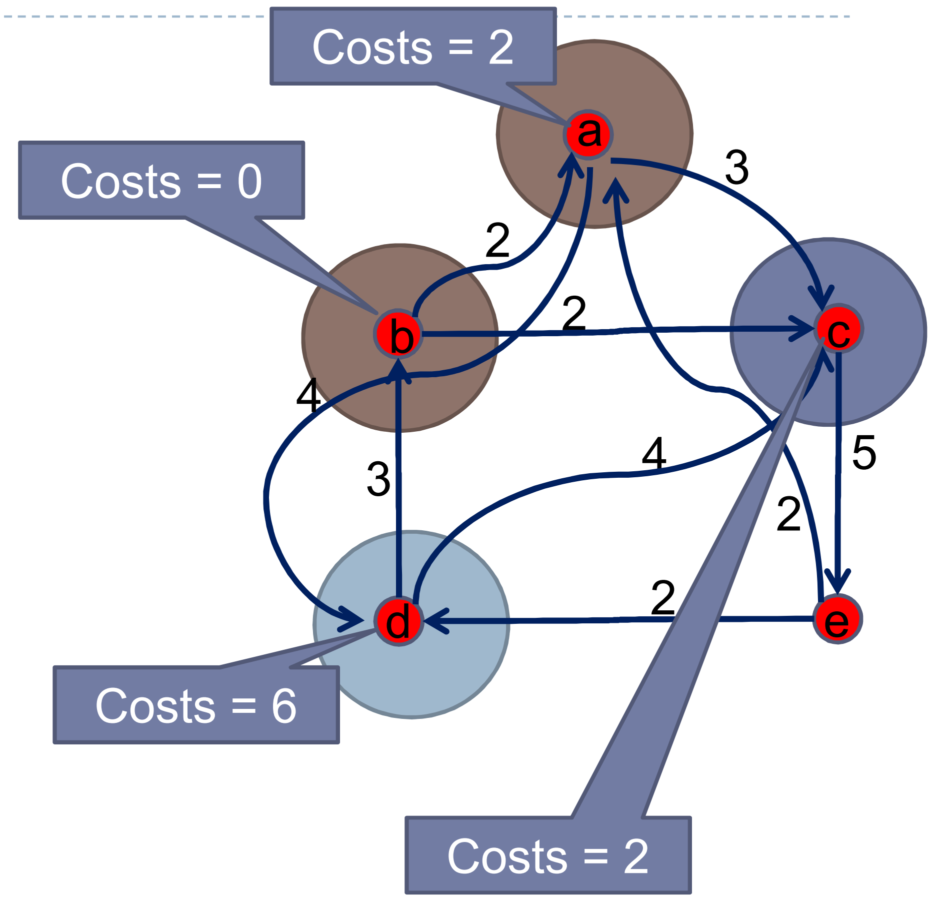
Pathfinding Graphs

- Example:
  - Now we can mark a as done



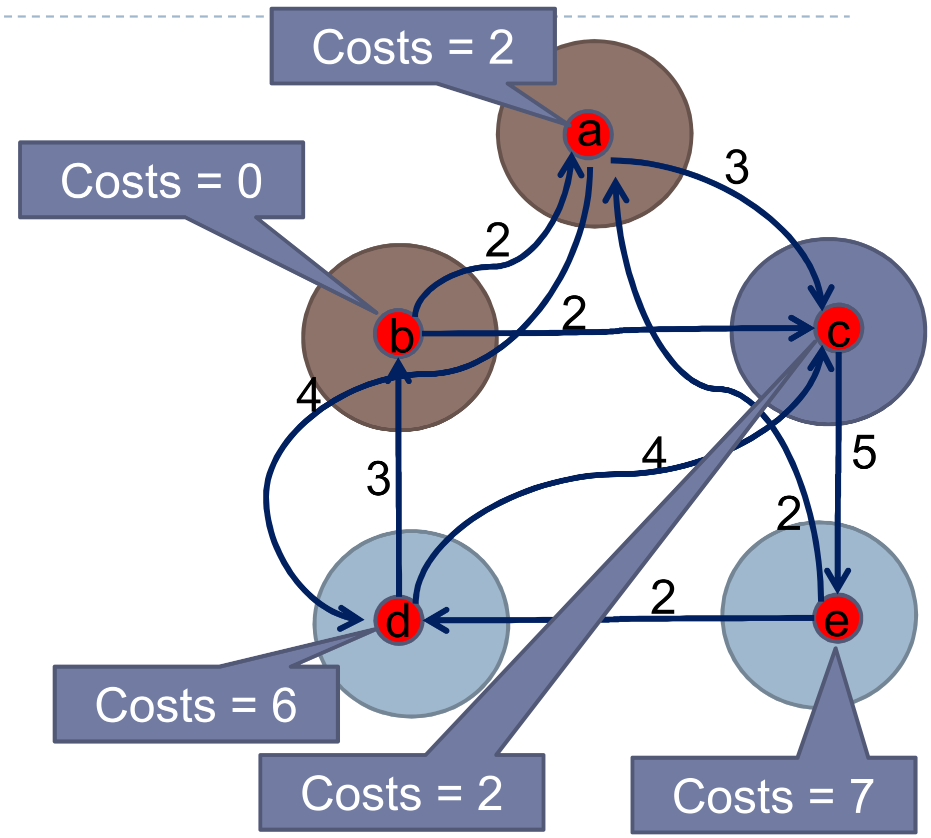
Pathfinding Graphs

- Example:
  - The seen list has two elements, c and d
  - c has minimum costs, so we use it



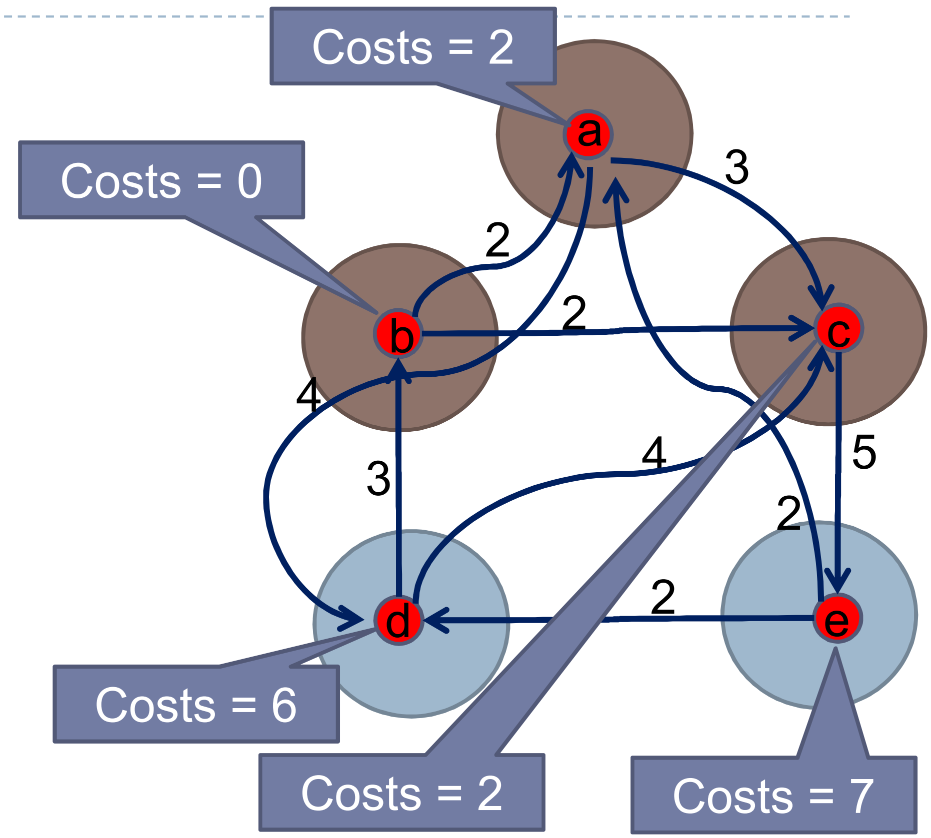
Pathfinding Graphs

- Example:
  - Processing c
  - There is an edge to e, which places e in the seen list with costs 2+5



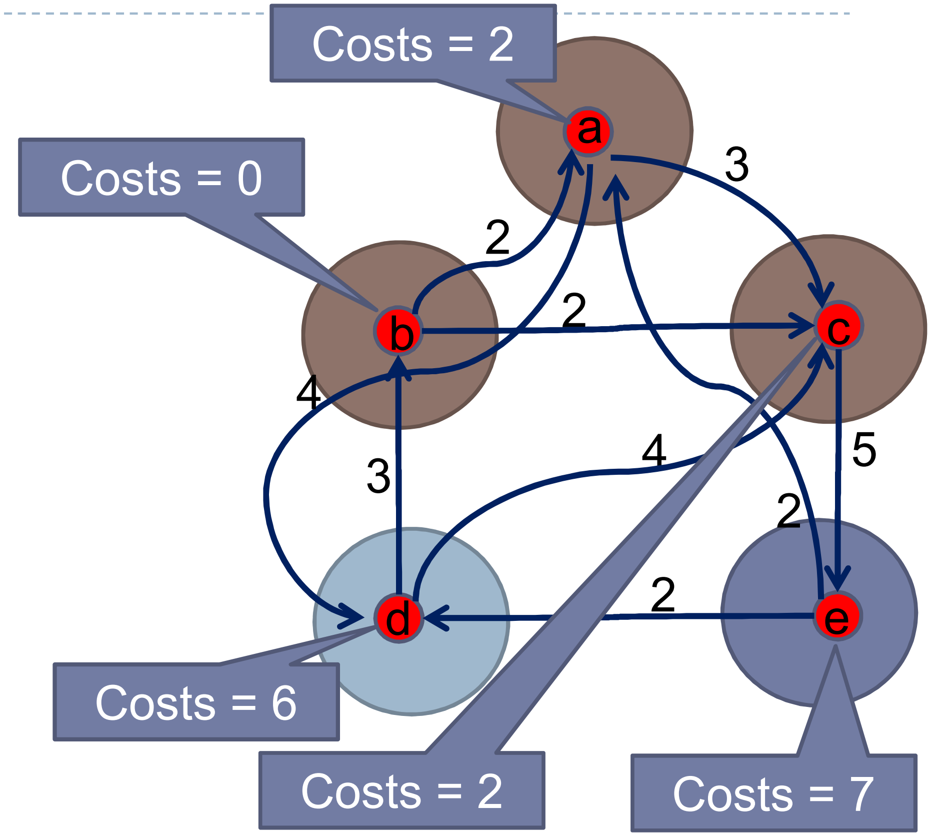
Pathfinding Graphs

- Example:
  - We select d in the seen list



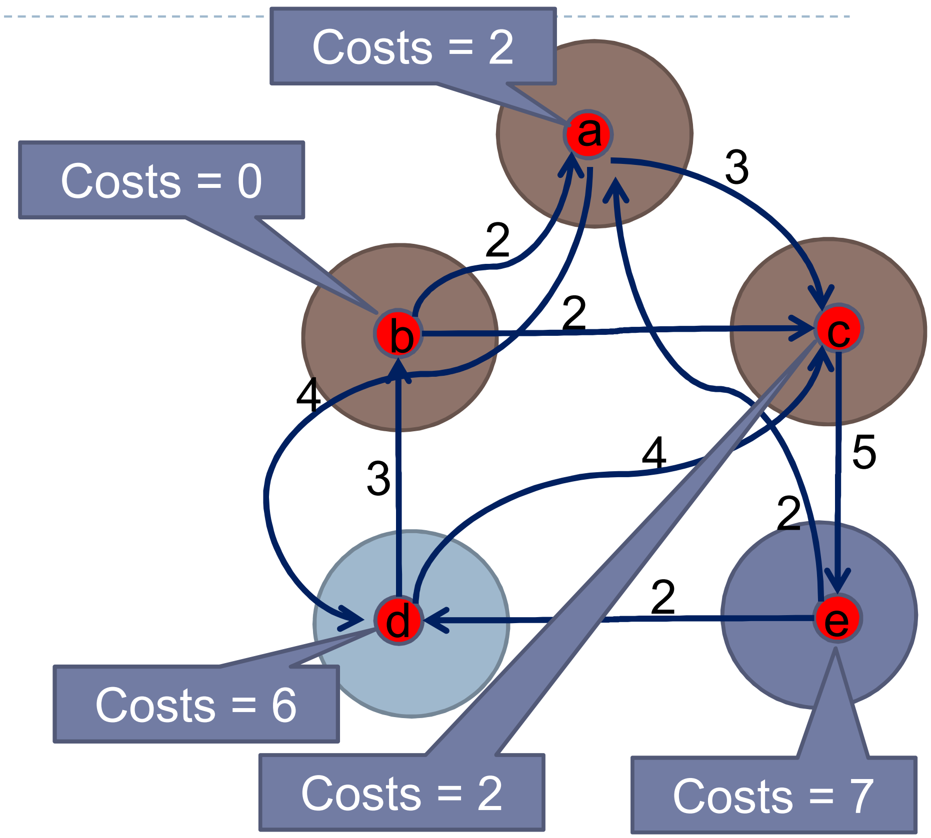
Pathfinding Graphs

- Example:
  - We select d in the seen list



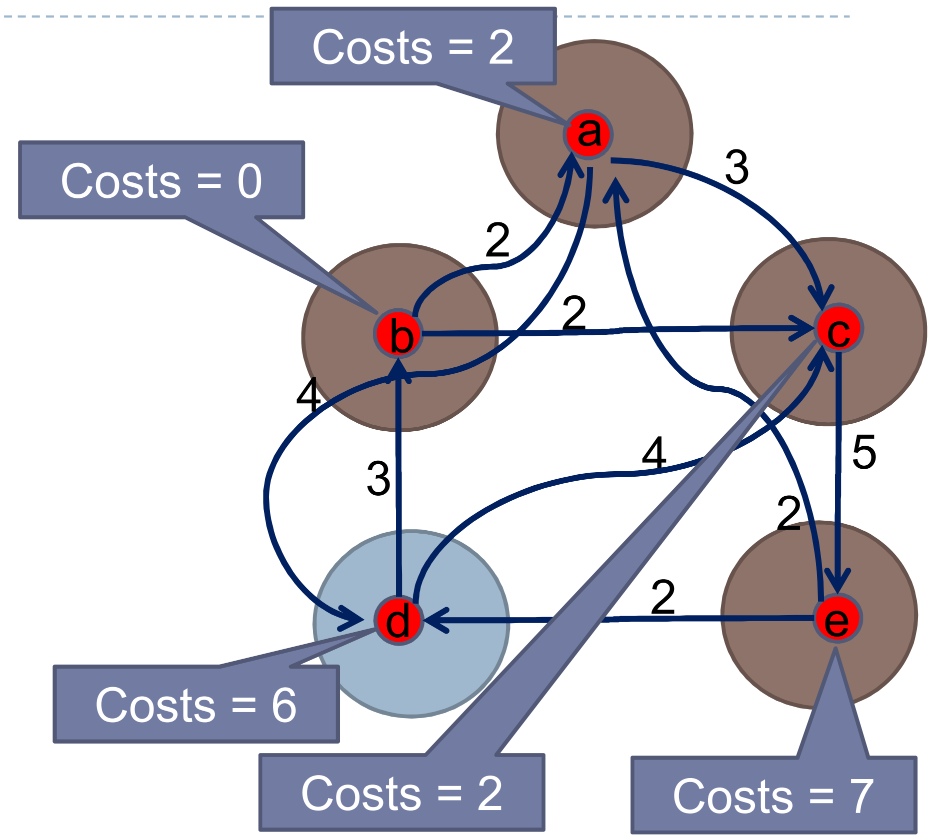
Pathfinding Graphs

- Example:
  - We have an edge to *a*
  - However, a is already done
  - Because of the way we select from the seen list, we ***know*** that we cannot beat the costs to get to a by going through e
    - Because we know that the costs to go to a is lower than the costs to go to e



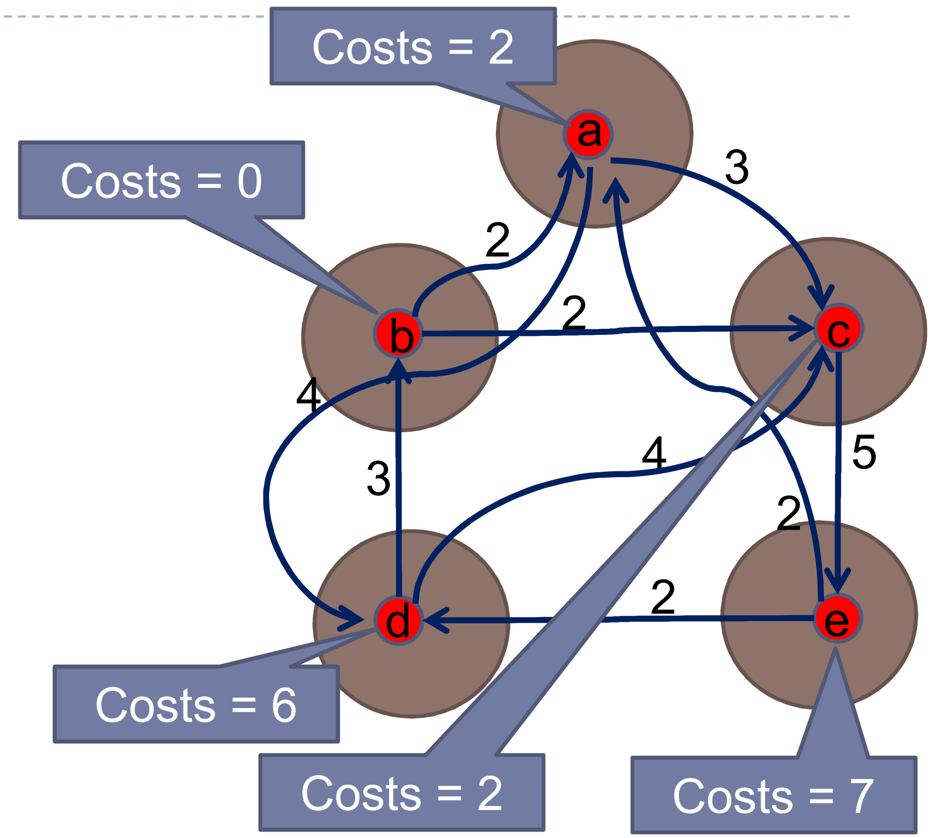
Pathfinding Graphs

- Example:
  - We process the edge to *d*
  - The alternative costs to *d* is 9, so we do not change the costs in d
  - We now can mark *e* as done



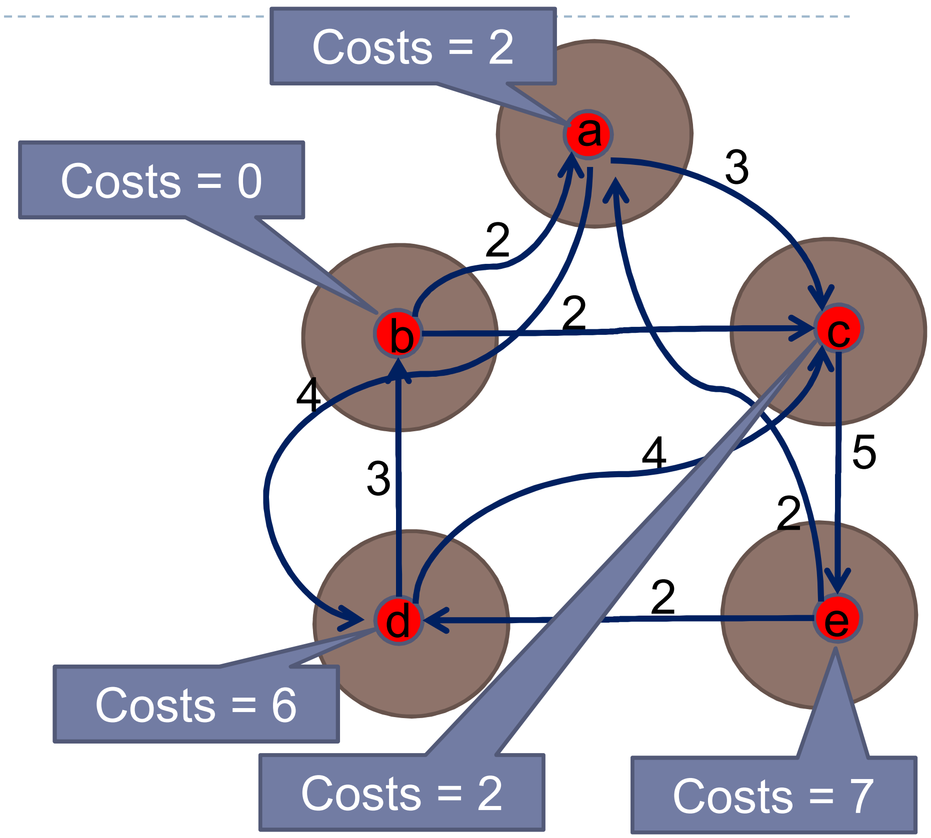
Pathfinding Graphs

- Example:
  - There is only *d* left in the seen list
  - All other nodes are done
  - Can stop



Pathfinding Graphs

- Example:
  - We know the costs of going to *d*
  - But we do not know how to get there.
  - Solution:
    - Decorate each node with the predecessor when updating the costs.



Pathfinding Graphs

- Algorithm for Dijkstra

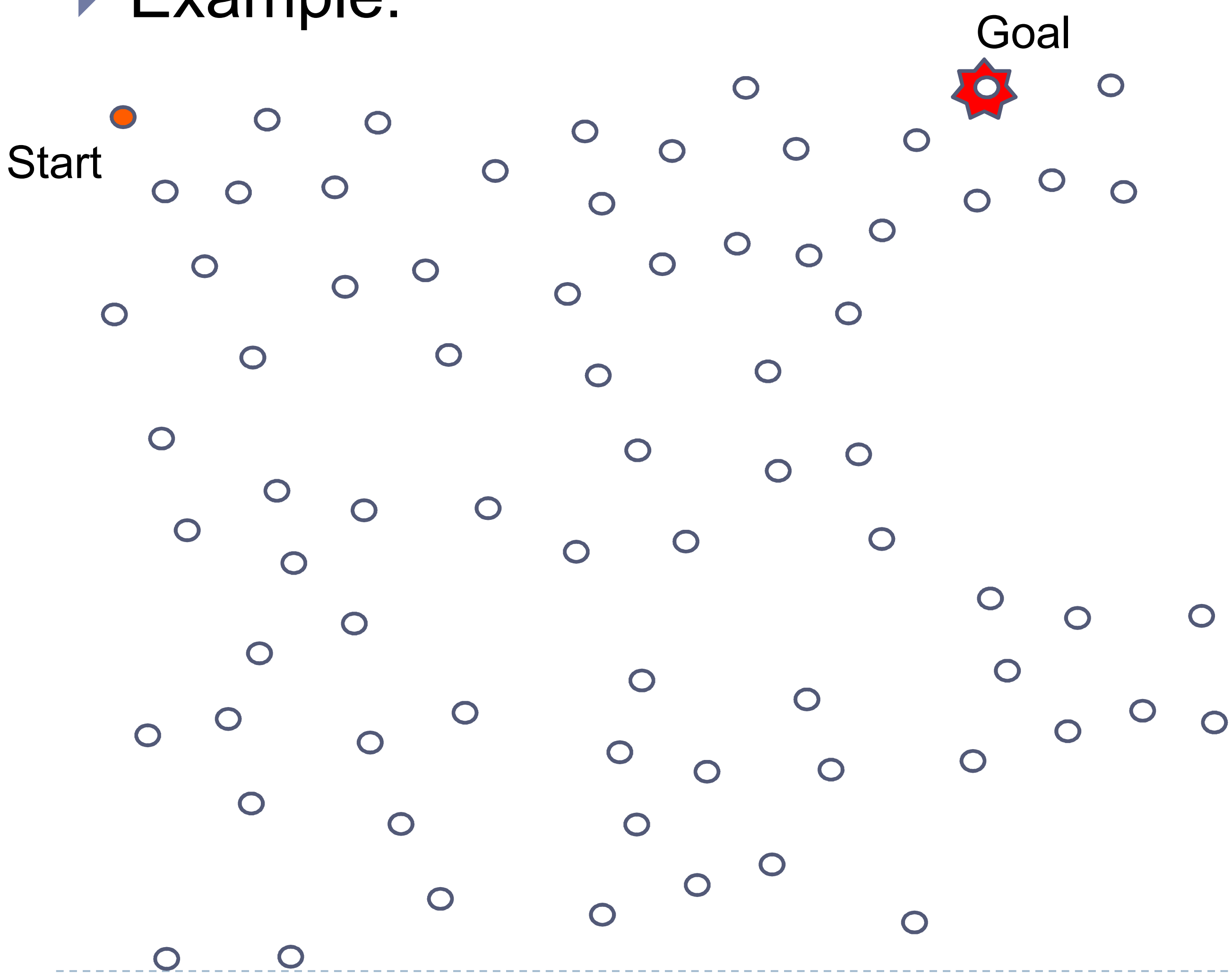
```
Seen = [Start]
While Seen:
  current = minimum([seen])
   seen.remove(current); done.add(current)
  for edge in current.edgesOut:
  newVertex = edge.getFinish()
  if newVertex not in seen and not in processed:
      seen.append(newVertex)
      newVertex.costs = current.costs + edge.weight
       newVertex.predecessor = current
  elif newVertex in seen:
       altCosts = current.costs + edge.weight
       if altCosts < newVertex.costs:
       newVertex.costs = alt.costs
       newVertex.predecessor = current
```

Pathfinding Graphs

- Problems with Dijkstra
  - Dijskstra looks at every edge
  - Can stop Dijkstra when the goal edge has a costs smaller than any of the nodes in the seen category, i.e. when we process the goal
  - But this does not mean that we do not have to look at lots of nodes

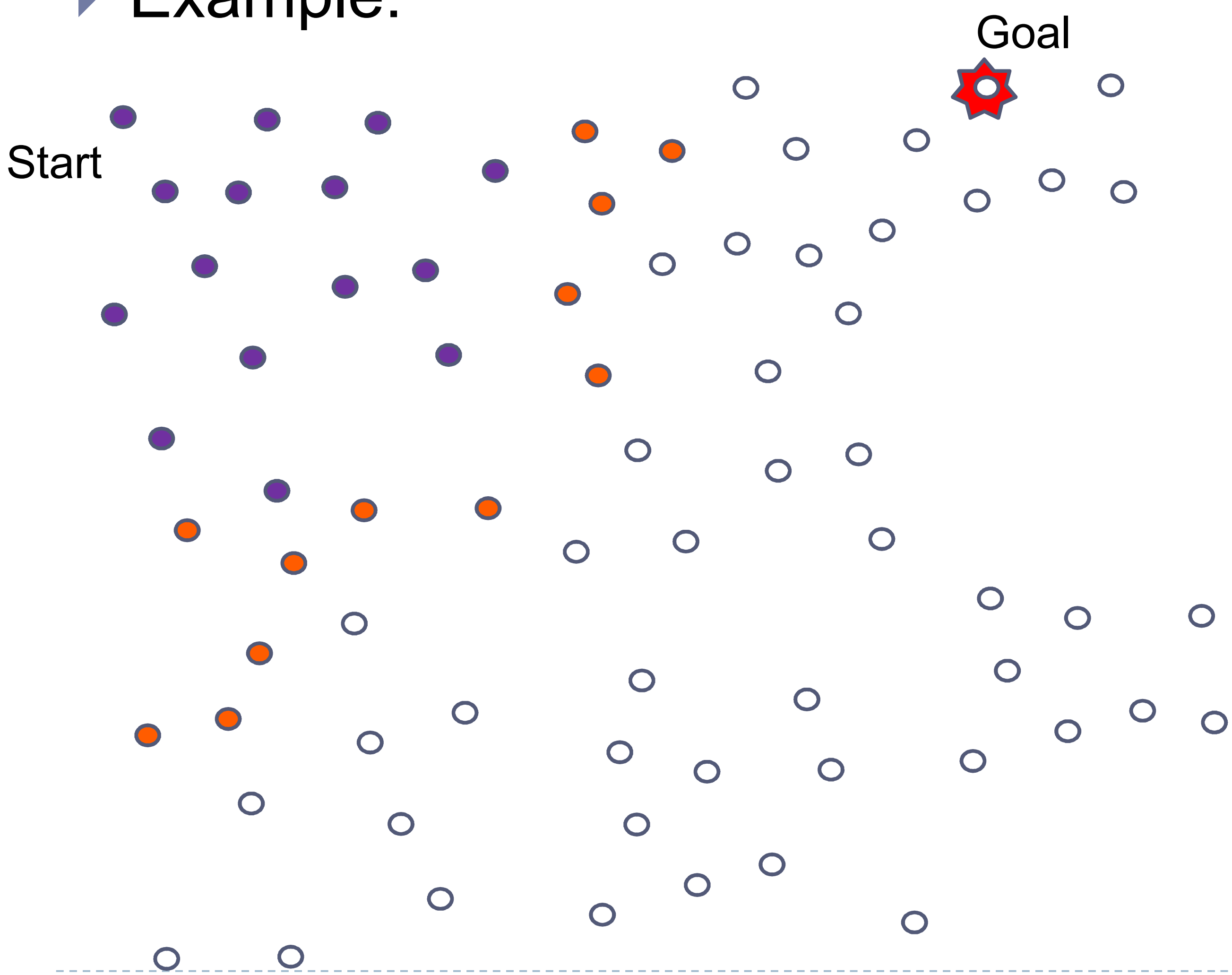
Pathfinding Graphs

- Example:



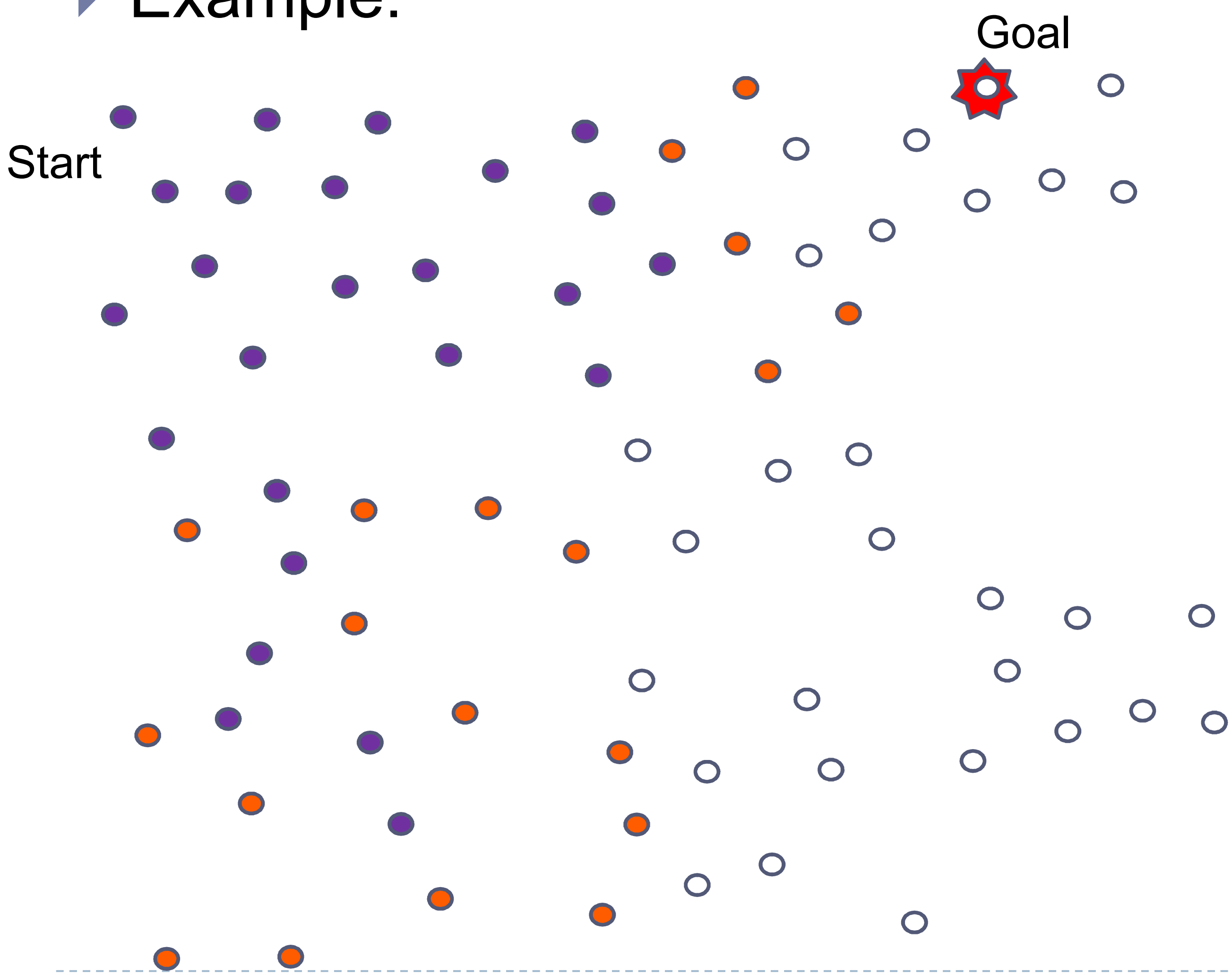
Pathfinding Graphs

- Example:



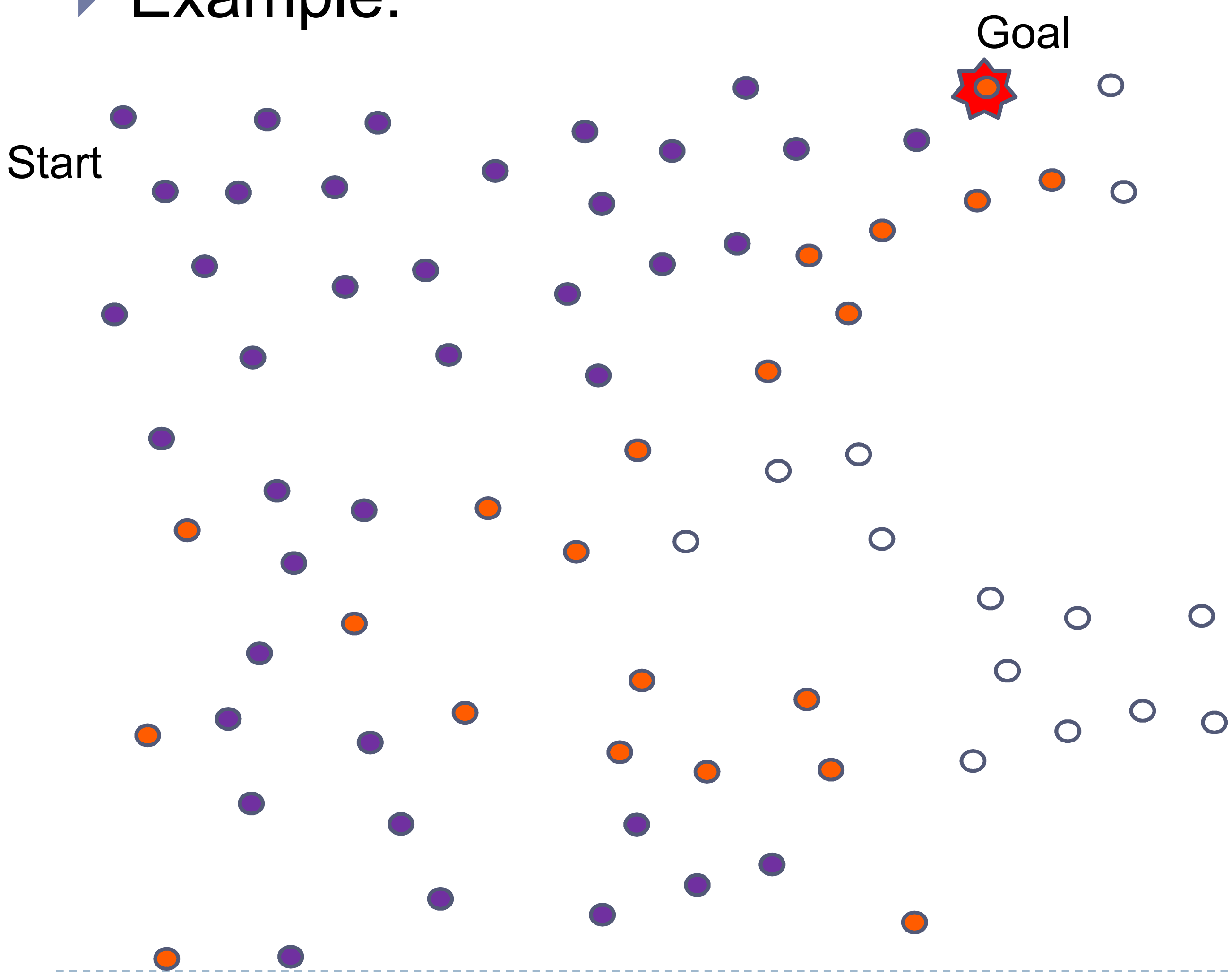
Pathfinding Graphs

- Example:



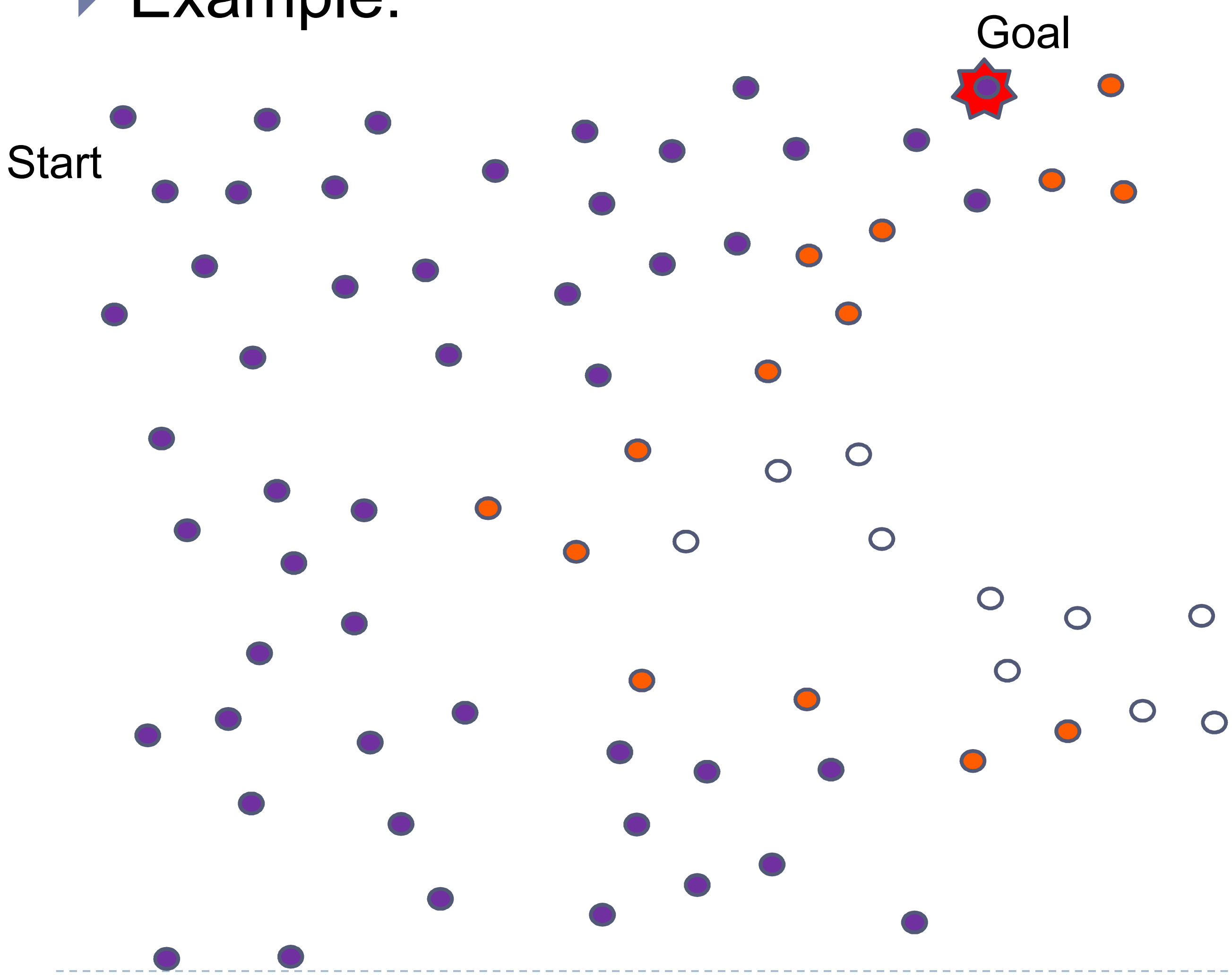
Pathfinding Graphs

- Example:



Pathfinding Graphs

- Example:



Pathfinding Graphs

- It might take processing lots of “obviously” uninteresting nodes before we can process the goal node
- Dijkstra works well to find minimum paths to all nodes, but not so well for finding a minimum path to a specific node

Implementing Dijkstra

- Graph data structure:
  - List of vertices
  - For each vertex, a list of weighted edges
    - Each edge is defined by a weight and a destination

- Lists data structure:
  - Need to find the minimum cost node in the seen list
    - Use priority queue
      - In Python: module queue has PriorityQueue

- Defining edges:
  - To use PriorityQueue, need to define a class Vertex, with sorting methods __le__, __ge__

Implementing Dijkstra

- Algorithm
  - Initialize lists:
    - Seen contains starting node
    - Done is empty

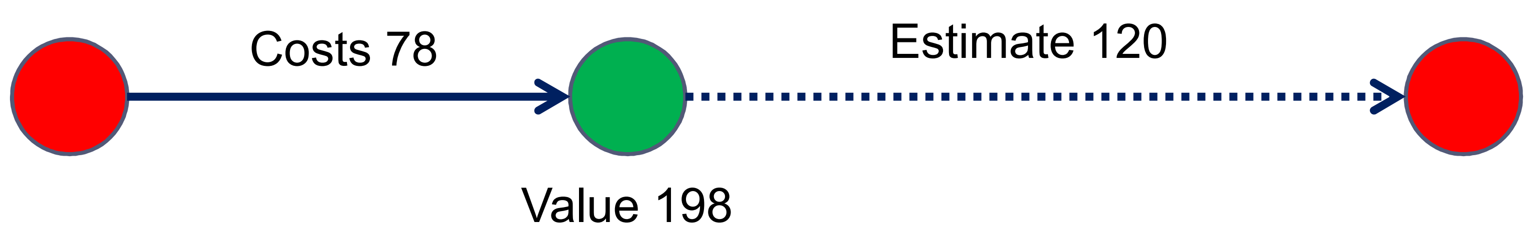
```
while not seen.empty()
  current = seen.get()
  if current = goal:
    #create path from information
  for edge in current.edges():
    vertex = edge.getDestination()
    if vertex not in done:
      # update vertex,
```

Pathfinding Graphs

- A*
  - Uses bounds in order to eliminate uninteresting branches
  - Is very interesting for any search problem that can be described with a graph
  - Is a very generic algorithm used in gaming also for strategic planning by avatars.

Pathfinding Graphs

- A* Idea
  - Use a heuristic to *estimate* the costs of a node to the goal
  - Similar to Dijkstra
    - Use value of node to select from seen list



Pathfinding Graphs

- In path-finding, the heuristics can be related to bird-flight distance

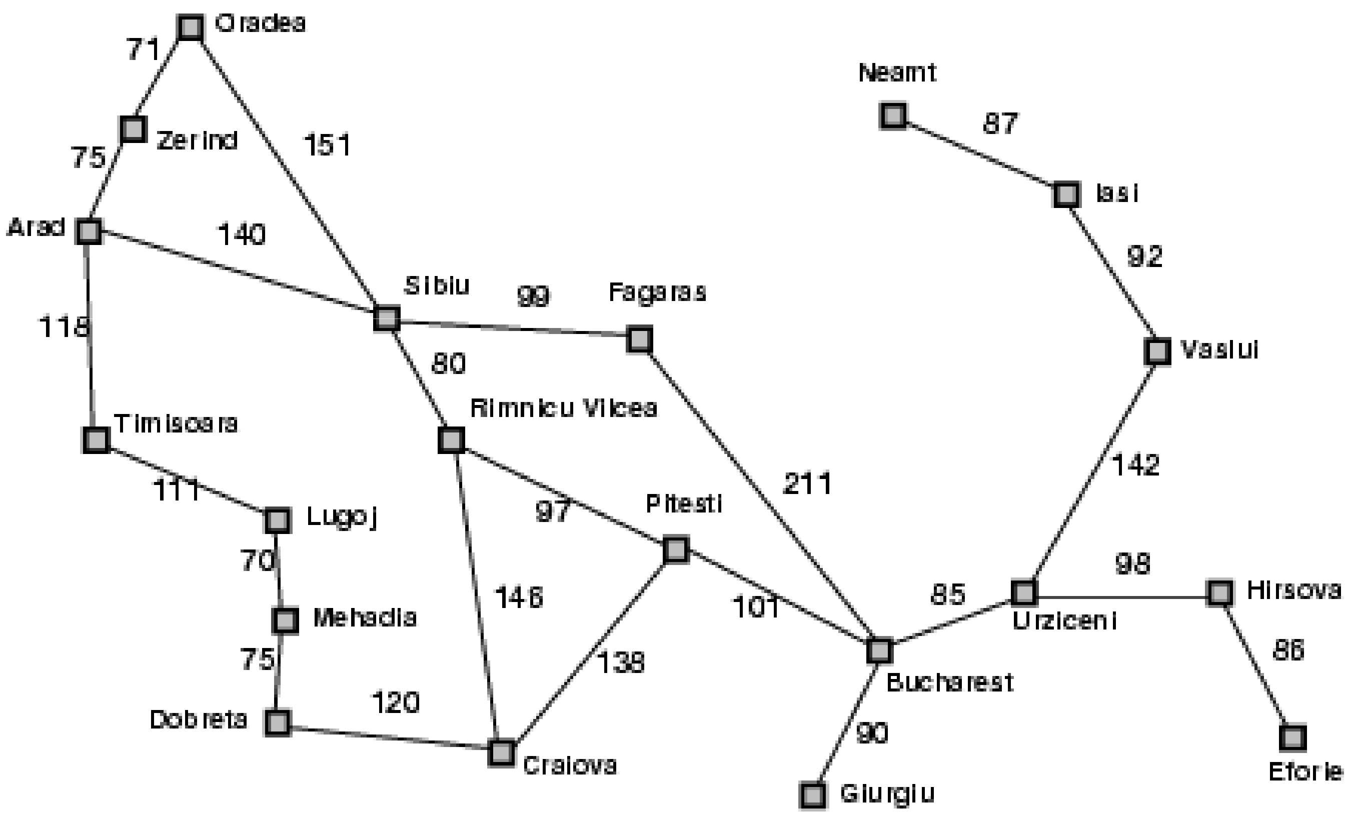
Pathfinding Graphs

- A*: Stopping Rule
  - In Dijkstra:
    - Stop when the minimum cost in the Seen-list is larger than the current costs to the goal
  - In A*:
    - Goal node has smallest estimated costs in the Seen-list
    - To be sure, need established smallest costs

Pathfinding Graphs

- Algorithm:
  - Tree algorithm:
    - Nodes characterized by location (original graph node) and costs to get there.
    - Additionally evaluated by costs estimate

Romania with step costs in km

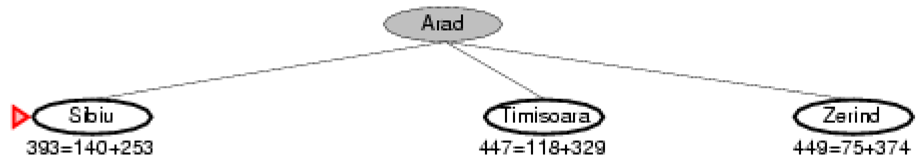


Straight–line distance to Bucharest

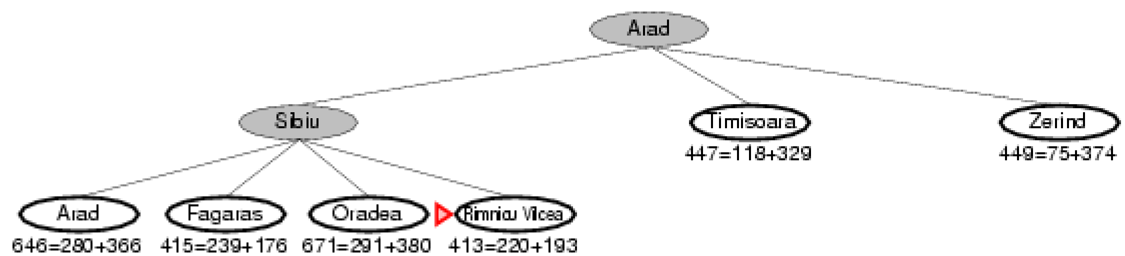
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

A* search example

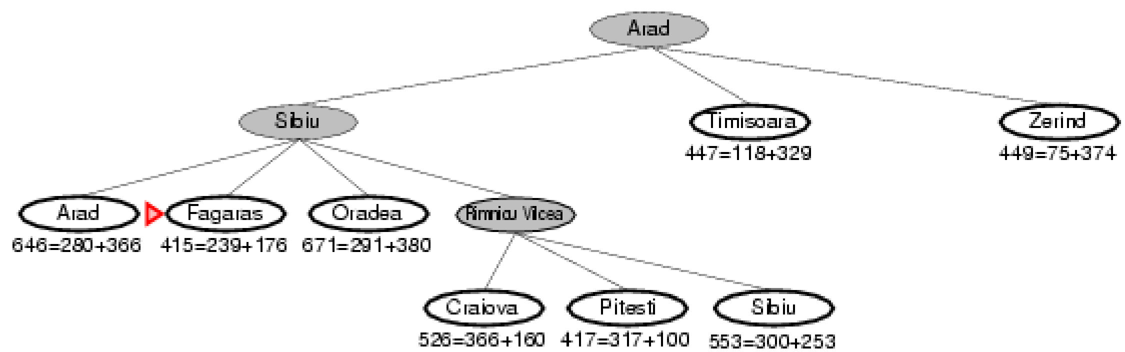
A* search example



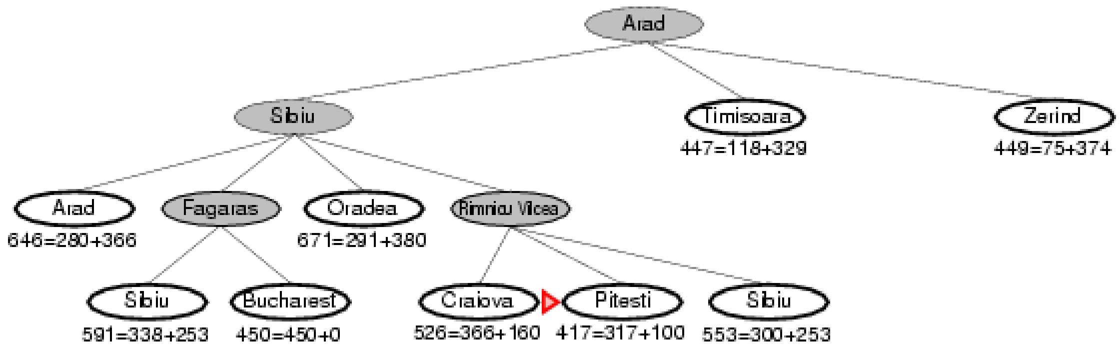
A* search example



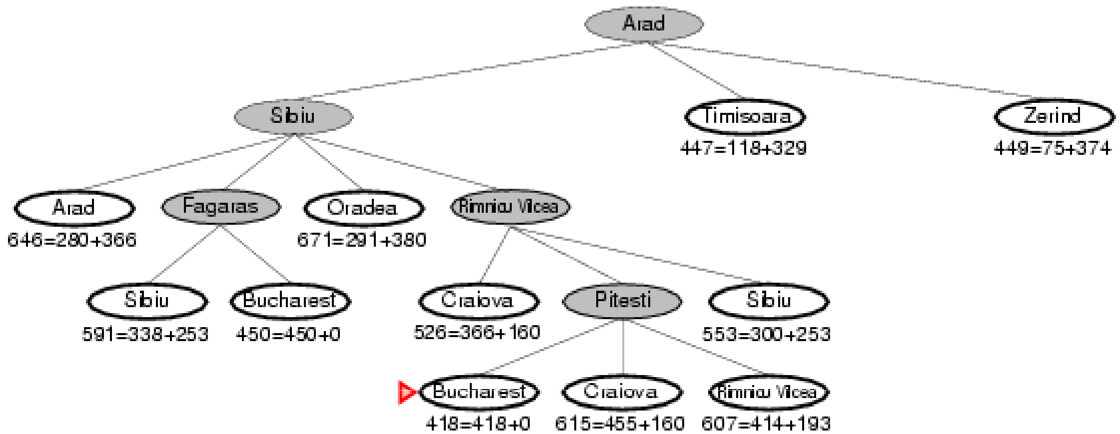
A* search example



A^* search example



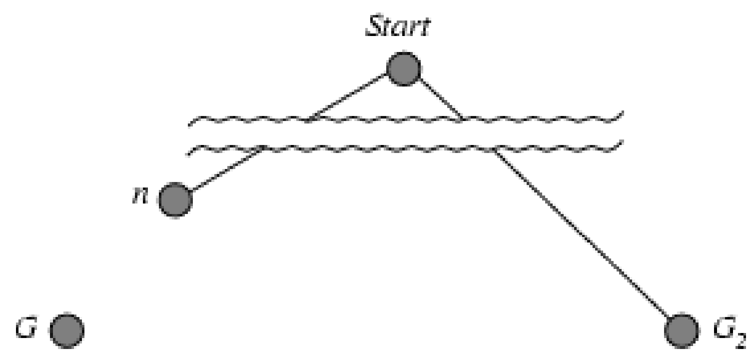
A* search example



Admissible heuristics

- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- Theorem: If $h(n)$ is admissible, A^* using `TREE-SEARCH` is optimal

Optimality of A^* (proof)

- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let *n* be an unexpanded node in the fringe such that *n* is on a shortest path to an optimal goal *G*.
- 

- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $g(G_2) > g(G)$ since G_2 is suboptimal
- $f(G) = g(G)$ since $h(G) = 0$
- $f(G_2) > f(G)$ from above

Optimality of A^* (proof)

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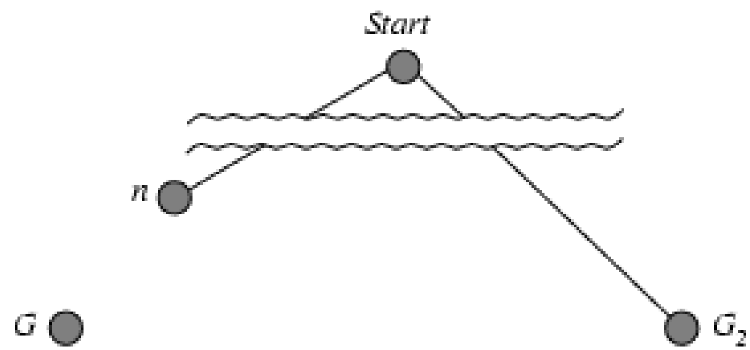
- 

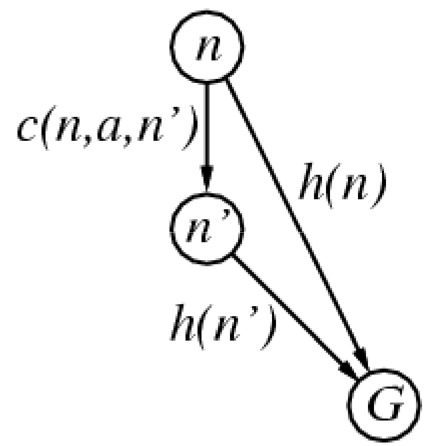
$f(G_2)$	$> f(G)$	from above
$h(n)$	$\leq h^*(n)$	since h is admissible
$g(n) + h(n)$	$\leq g(n) + h^*(n)$	
$f(n)$	$\leq f(G)$	

Hence $f(G_2) > f(n)$, and A^* will never select G_2 for expansion

Consistent heuristics

- A heuristic is consistent if for every node n, every successor n' of n generated by any action a,
-

  $h(n) \leq c(n,a,n') + h(n')$



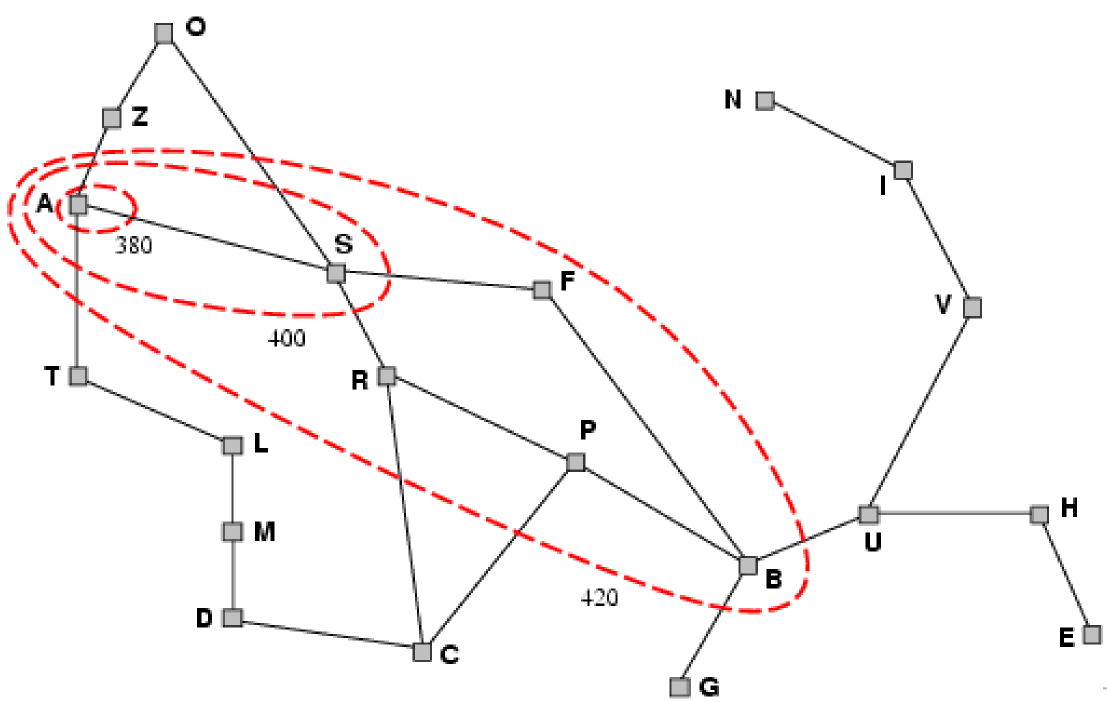
- If h is consistent, we have
-

$$
\begin{aligned}
f(n') &= g(n') + h(n') \\
&= g(n) + c(n,a,n') + h(n') \\
&\geq g(n) + h(n) \\
&= f(n)
\end{aligned}
$$

- i.e., $f(n)$ is non-decreasing along any path.
-
- Theorem: If $h(n)$ is consistent, A* using `GRAPH-SEARCH` is optimal
-

Optimality of A*

- A* expands nodes in order of increasing *f* value
  - Gradually adds "*f*-contours" of nodes
  - Contour *i* has all nodes with $f=f_i$, where $f_i < f_{i+1}$



Pathfinding Graphs

- Heuristic selection
  - Heuristic always too low
    - A* takes longer to run
  - Heuristic sometimes overestimates
    - A* can produce wrong result
    - Might still be OK for gaming

Pathfinding Graphs

- Heuristics selection
  - Euclidean distance
    - Admissible heuristic (always underestimates)
      - In an indoor environment, can lead to long run-times
  - Cluster heuristic
    - Groups nodes in clusters
    - Can be automatic, but is often provided by level design
    - Look-up table gives smallest distance between members of two different clusters
    - Heuristic:
      - If start and end node are in the same cluster: Euclidean distance
      - Otherwise, use minimum distance between points in both clusters
    - Trade-off for choosing cluster size
      - Clusters small → Large lookup table
      - Clusters big → Inaccurate

World Representation

- Needed to translate from level design to graph representation
  - Generation:
    - Manual
    - Automatic
  - Validity:
    - If graph tells avatar to move from region A to region B then it should be possible to reach any point in B from any point in A
    - Validity is not always enforced

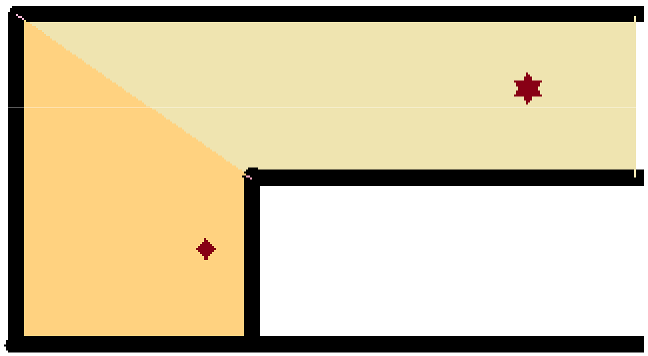
World Representation

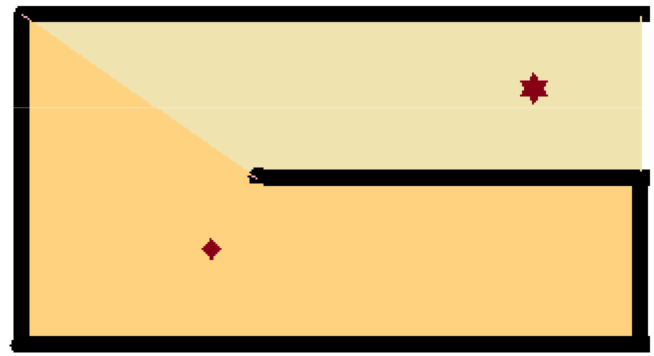
Poor quantization

But wall avoidance results in useful path

Bad quantization

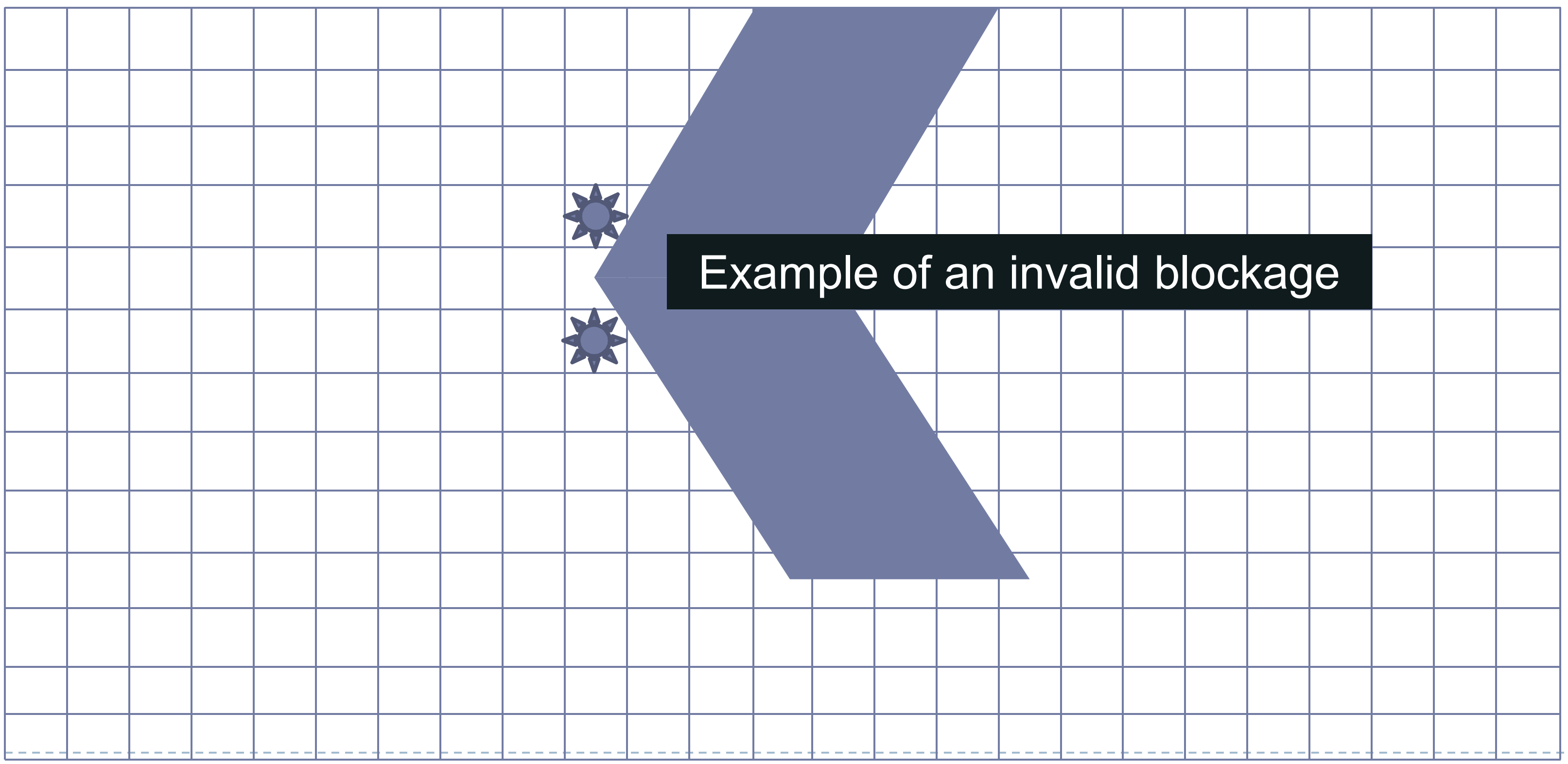
Path ends up in dead end





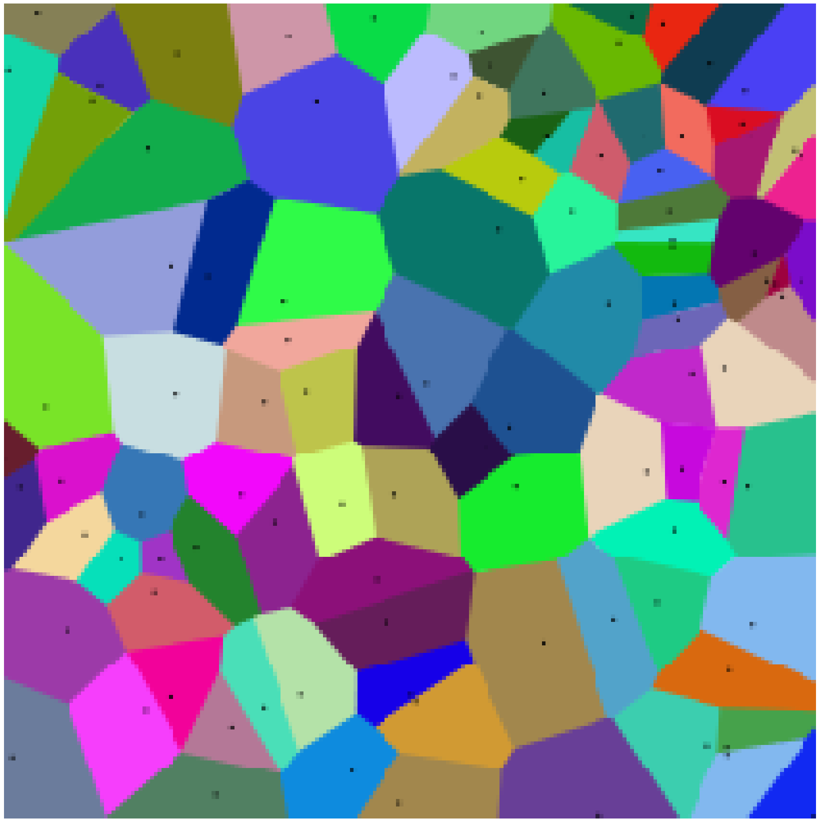
World Representation

- Tiling
  - Tile-based graphs are generated automatically
  - Validity problems can arise if tiles are only partially blocked



World Representation

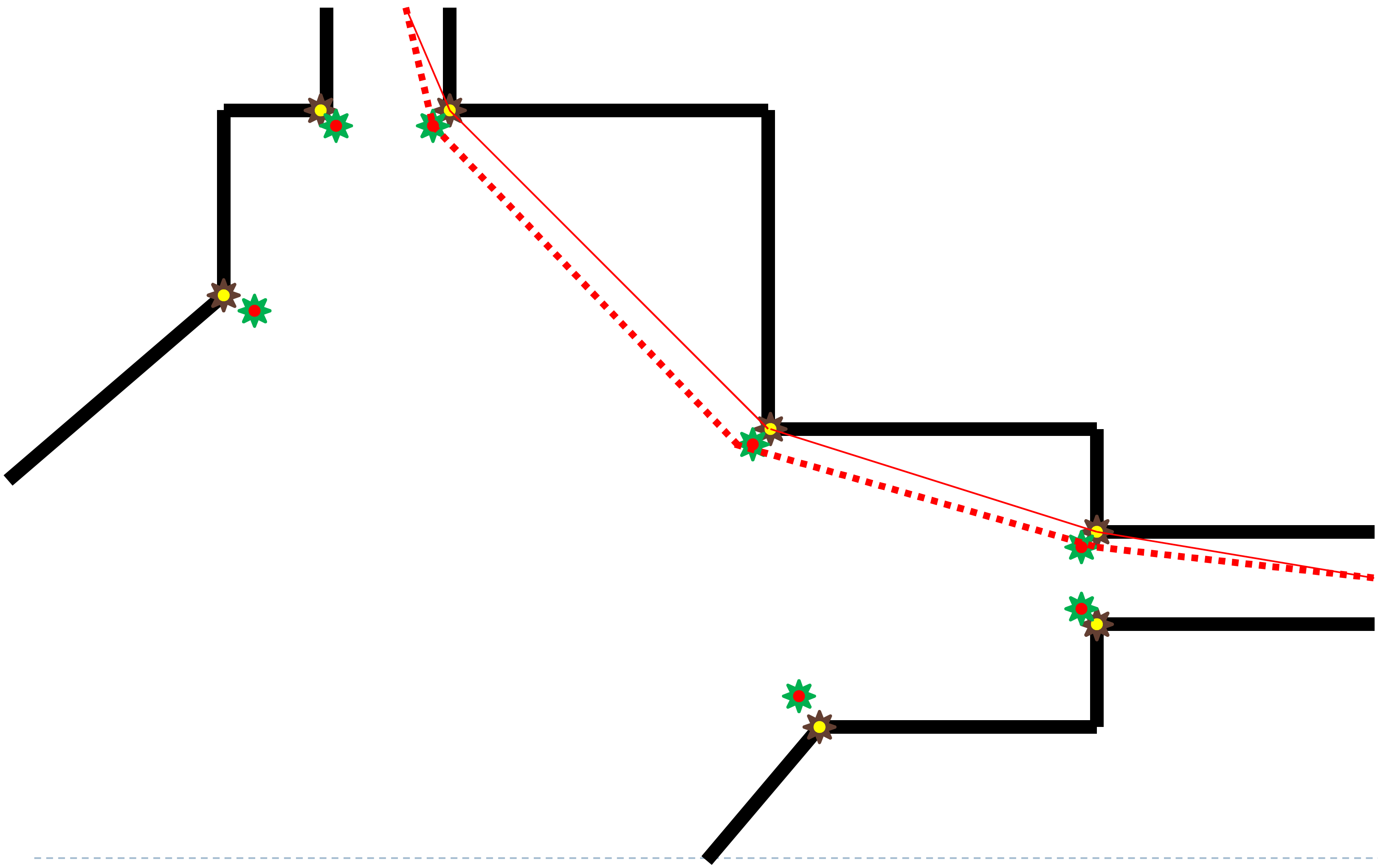
- Dirichlet Domains
  - aka Voronoi poygons
- Tiles the plane into regions defined by “characteristic points”
- Regions made up of the points nearest to a characteristic point
- In general: do not give valid tiling
- Are popular because they can be automatically generated



World Representation

- Points of visibility
  - Observation: Optimal path has inflection points at convex vertices in the environment
  - Generate points at convex vertices
    - If avatars have girth, move away from vertex

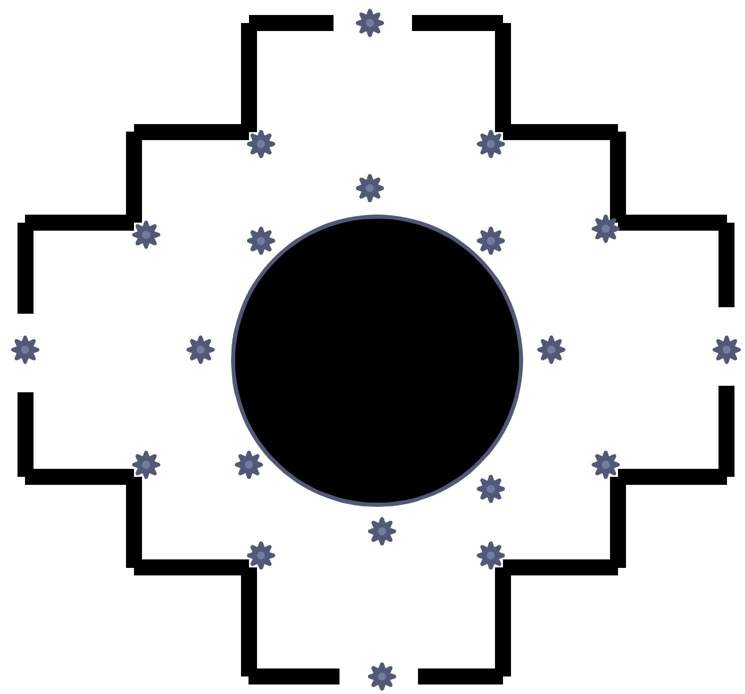
World Representation



World Representation

- Points of Visibility Graph
  - Edges between points
    - If one can be seen from the other
    - Cast rays
  - Can be taken to represent the centers of Dirichlet domain
  - Can generate too many points

World Representation



Vertices in a bloated visibility graph

World Representation

- Division Schemes
  - Games can have floor polygons (designed by level artist) as regions
  - Each polygon becomes a graph
  - Difficult for artists to maintain validity
  - Very popular
    - Pathengine middleware

World Representation

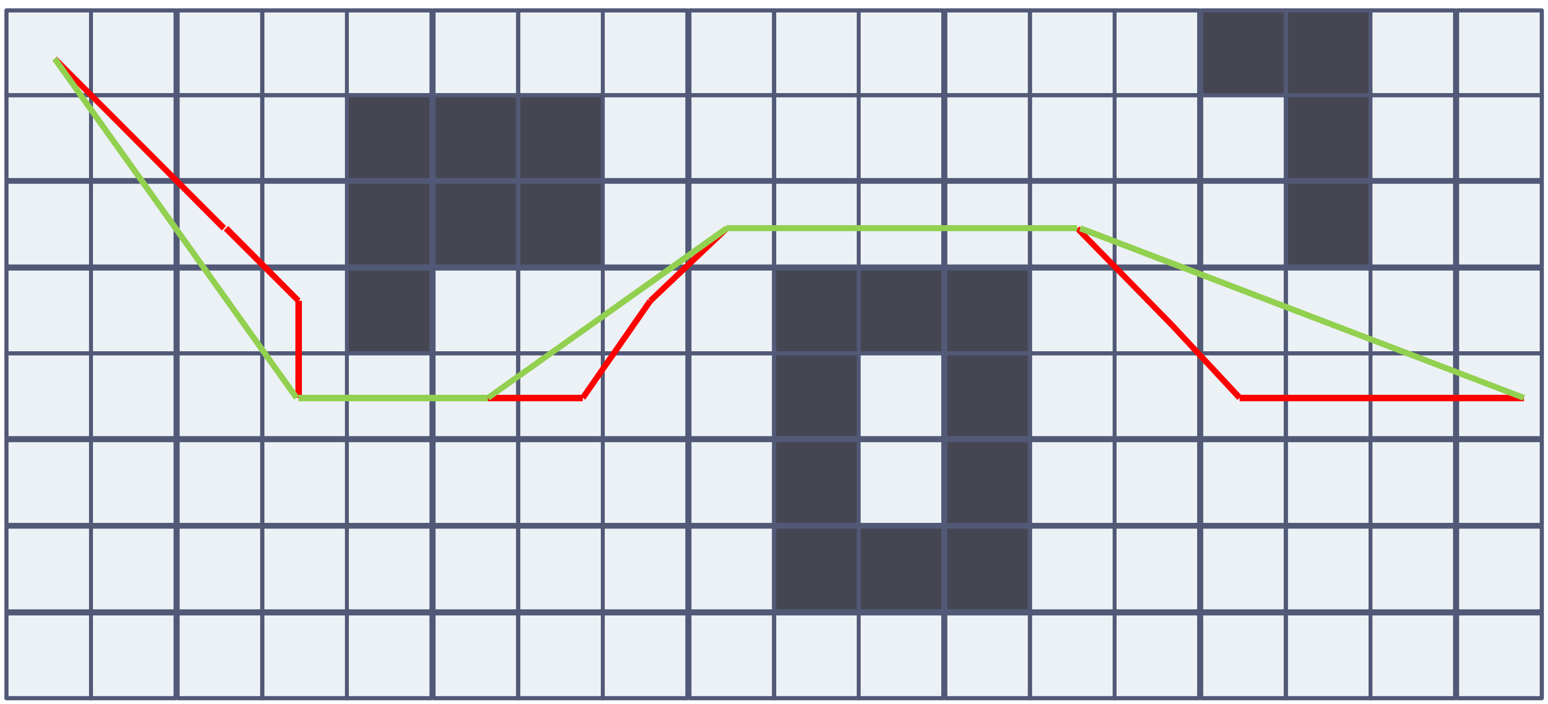
- Additional information
  - Graph vertex can represent more than just a position
  - Example: Ship
    - Cannot turn sharply
    - Vertex represents position and orientation

World Representation

- Cost functions
  - Usually, cost function (edge weight) represents distance
  - Can represent costs of moving
    - Moving through swamp takes more time, …

Path Smoothing

- Paths generated by path-finding can be erratic
- Path smoothing gives more believable paths



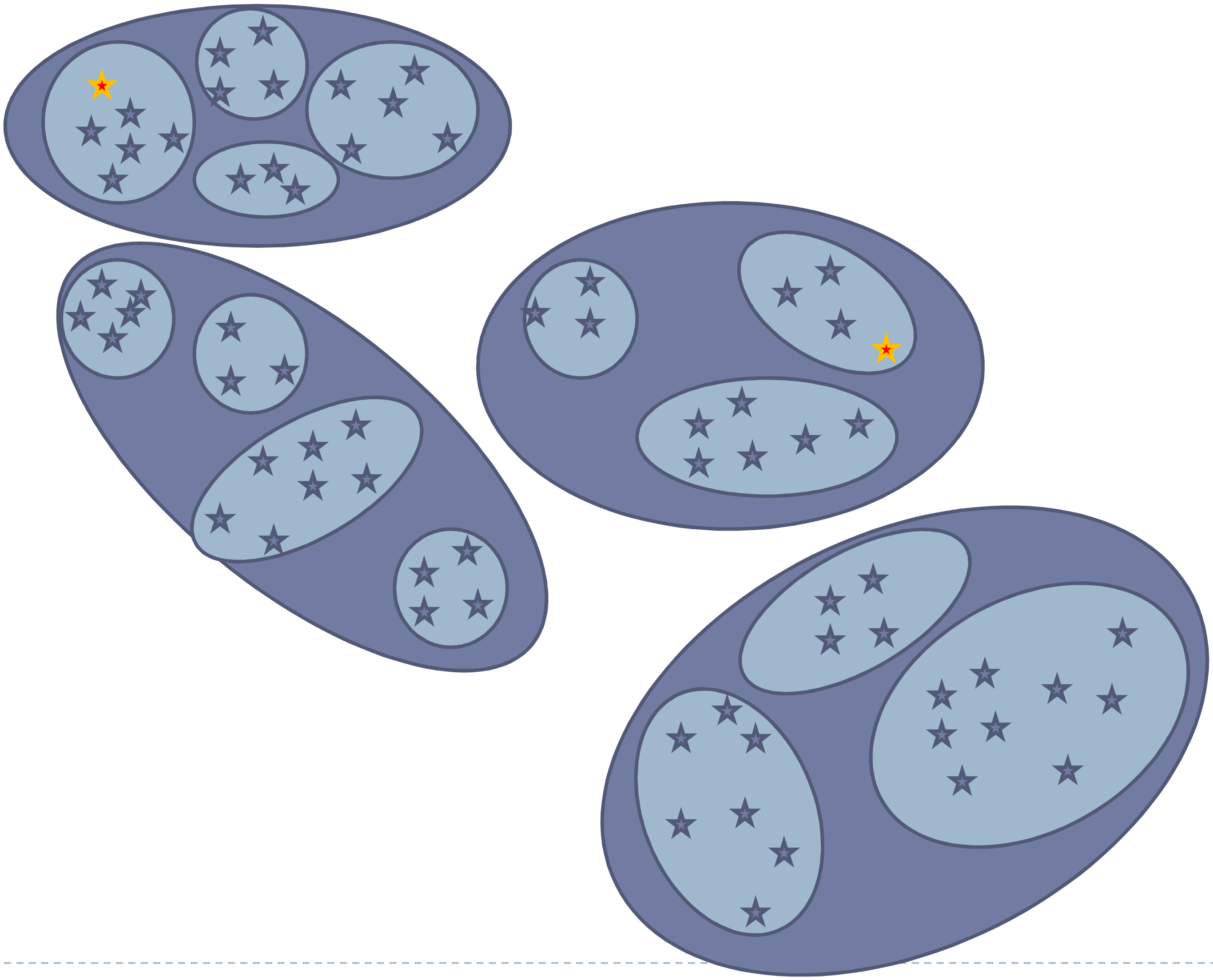
Path Smoothing

- Algorithm
  - Starting with the third node, cast a ray towards the beginning node, the second node, …
  - If ray goes not through, add the node to the smoothed path node list
  - Generates a reasonable smooth path, but not all possible ones

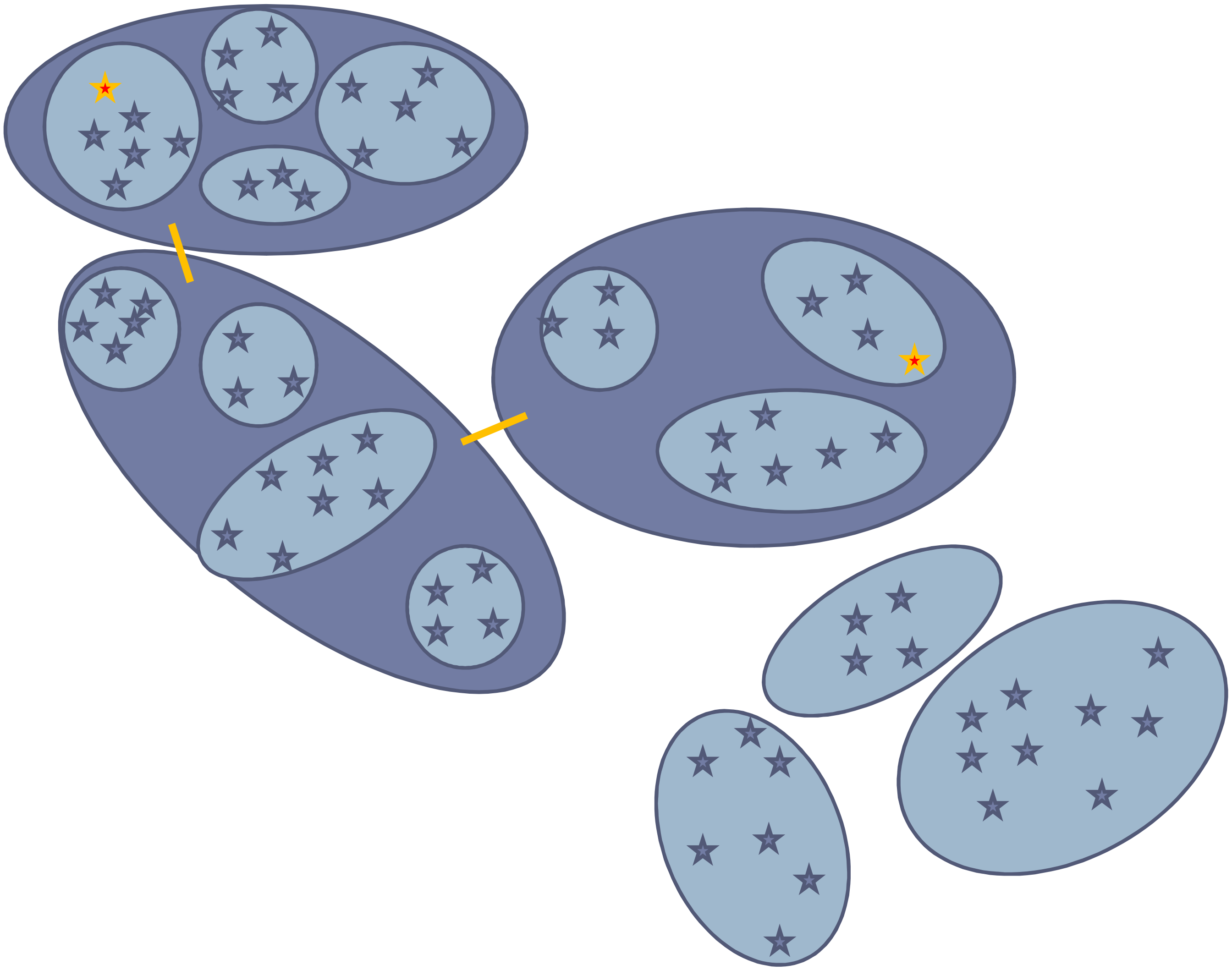
Hierarchical Pathfinding

- Idea:
  - Clustering:
    - Group nodes into clusters
    - Clusters for nodes of a higher level graph
    - Continue
  - Pathfinding:
    - Find path on highest level
    - For each super-node, find path inside super-node
    - continue to lowest level

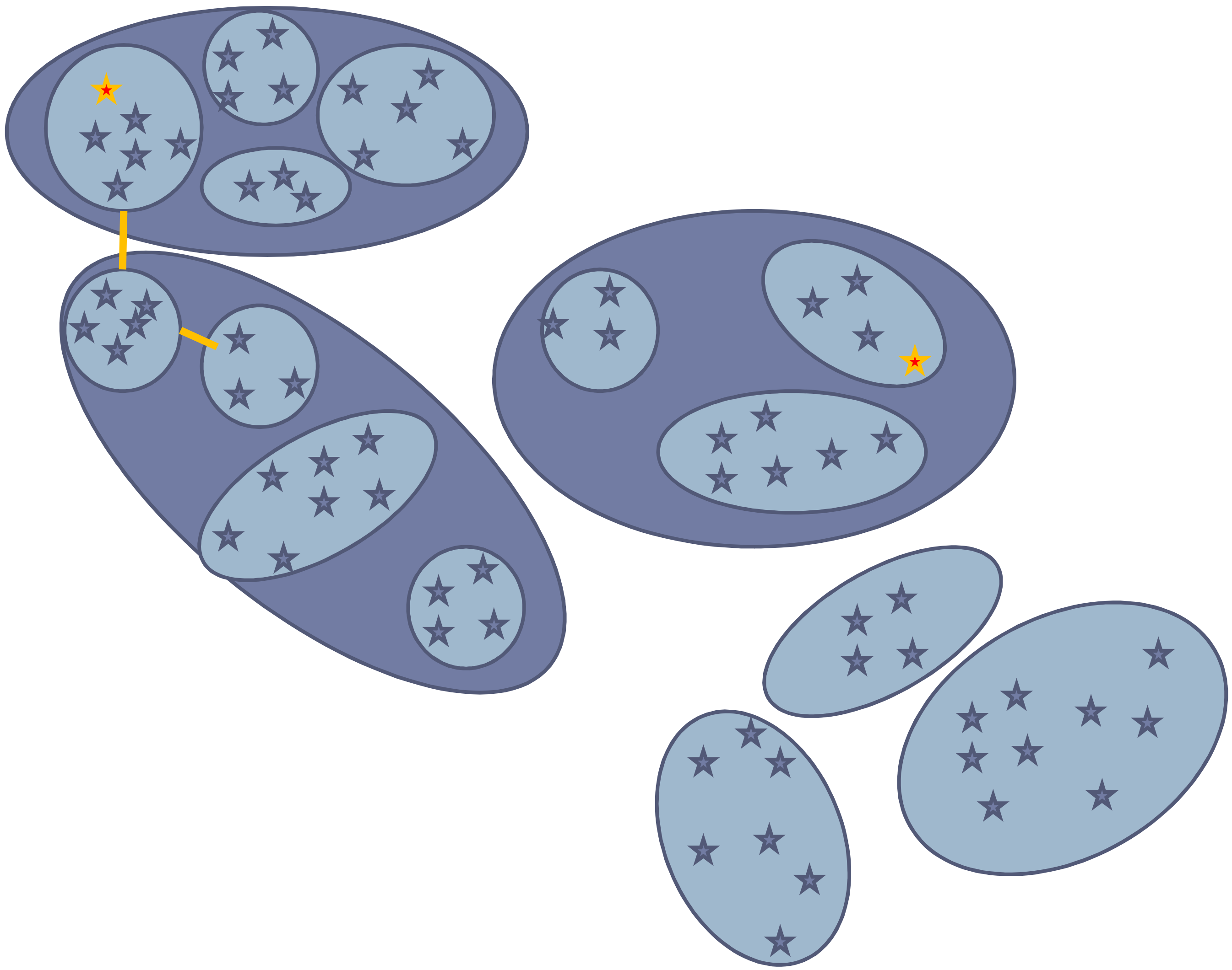
Hierarchical Pathfinding



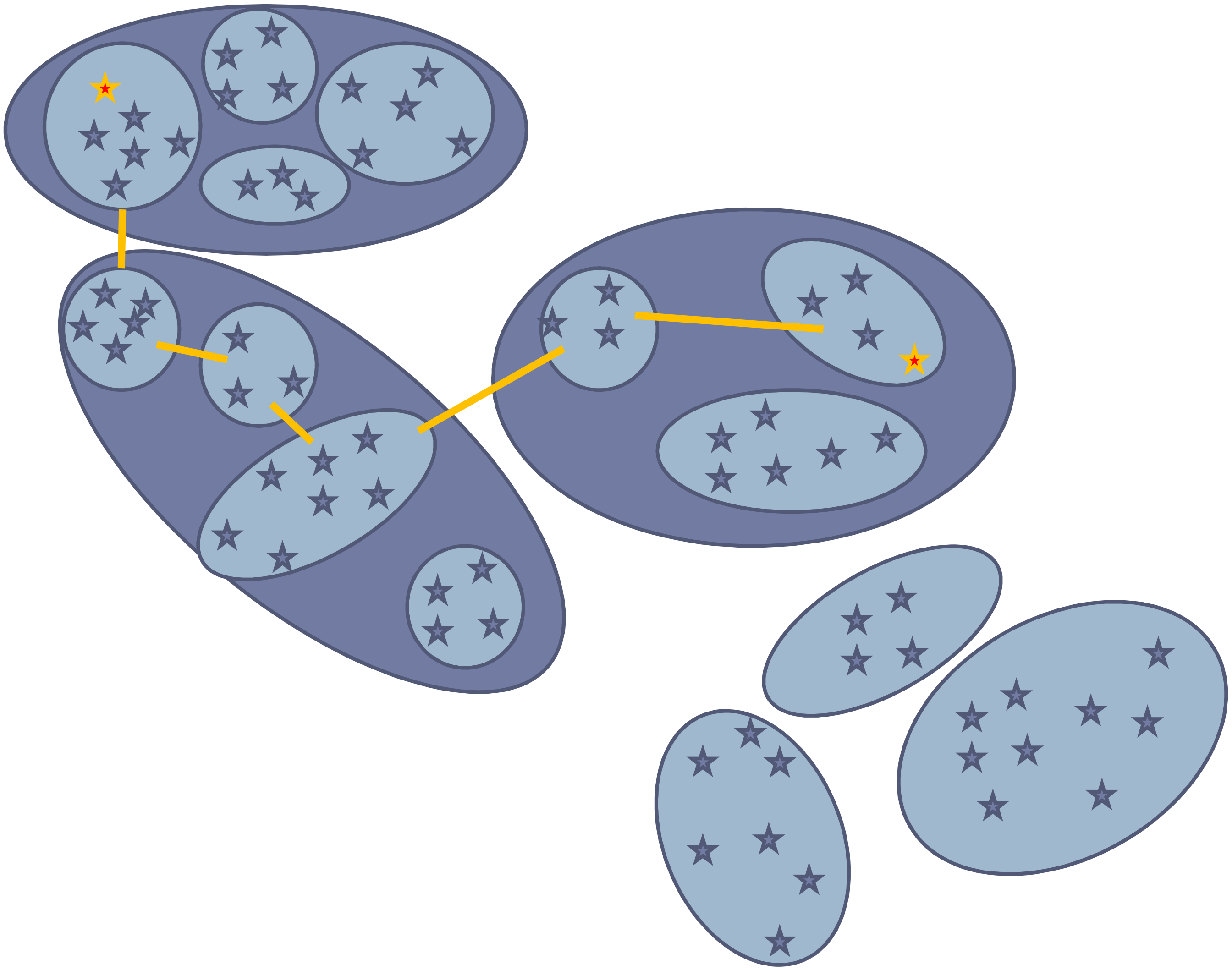
Hierarchical Pathfinding



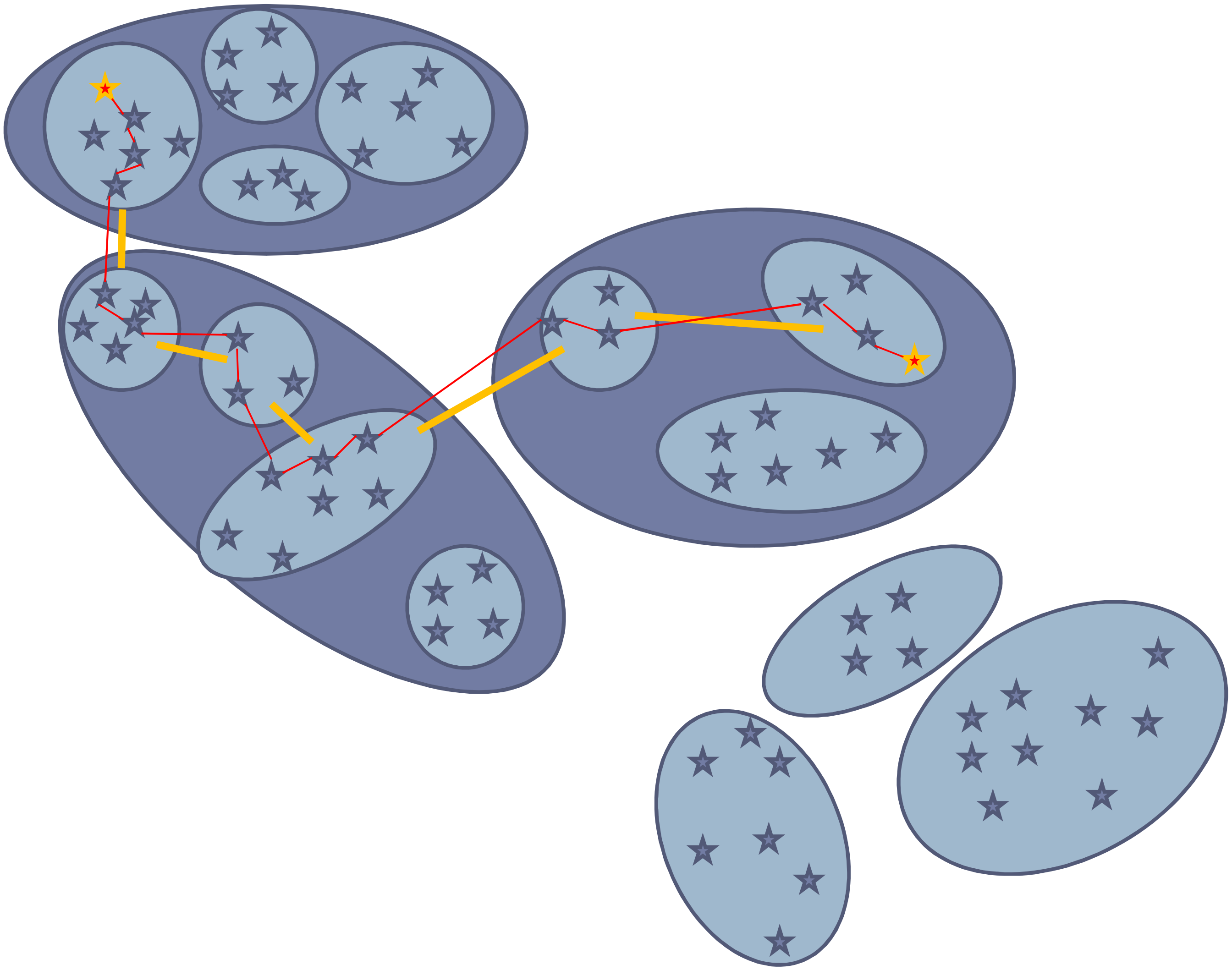
Hierarchical Pathfinding



Hierarchical Pathfinding



Hierarchical Pathfinding



Hierarchical Pathfinding

- Advantages
  - Pathfinding in small graphs: Quick
- Disadvantages
  - Distance between clusters are hard to measure

Hierarchical Pathfinding

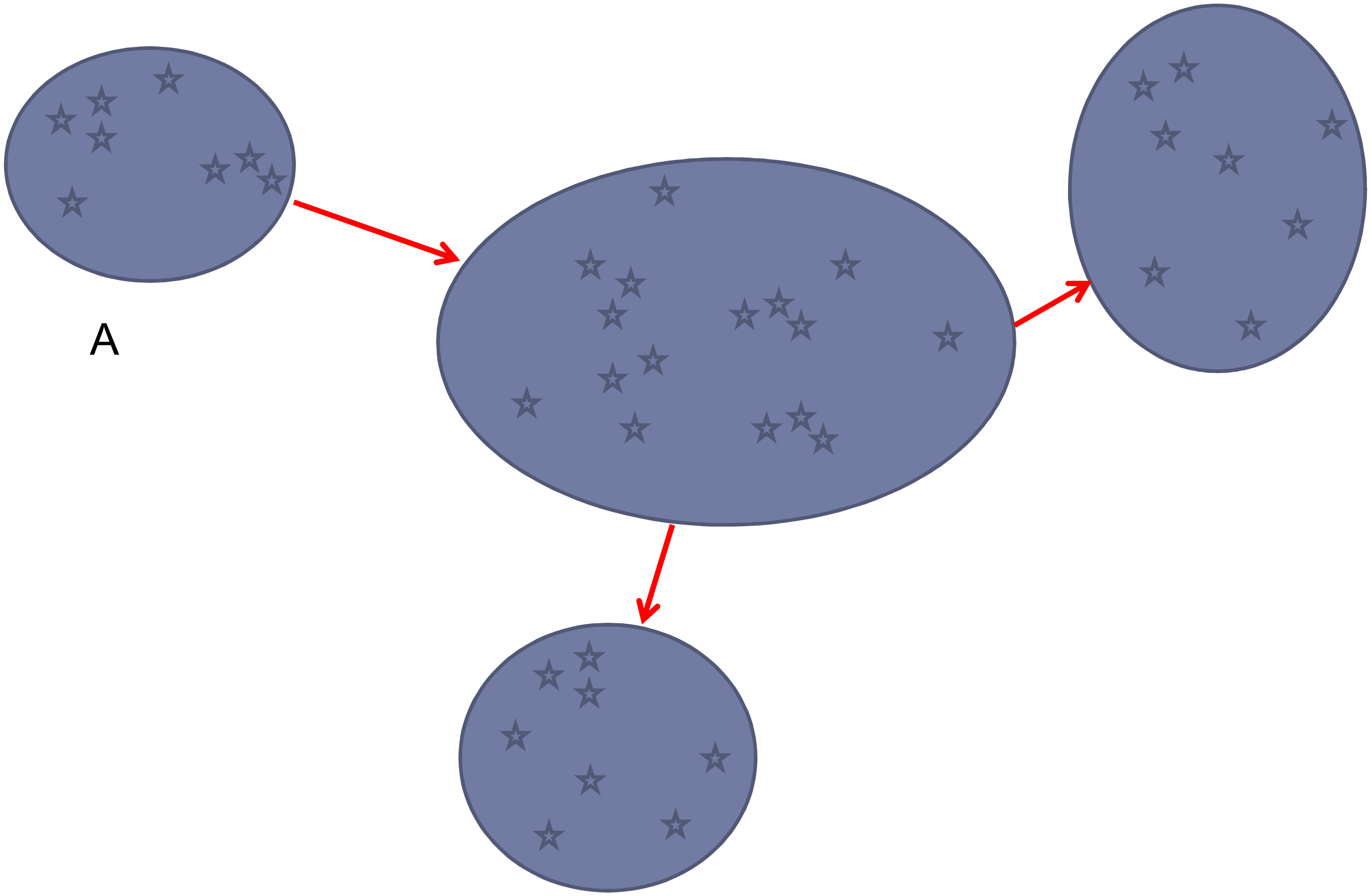
- Distance between clusters:
  - Depends on from where you enter the cluster
  - Using this information destroys advantages of hierarchical pathfinding
- Heuristics
  - Minimum distance
  - Maximin distance
  - Average minimum distance

Hierarchical Pathfinding

- Minimum distance
  - `Distance(A,B) = min{|a-b|, a∈A, b∈B}`
- Maximin distance
  - For each incoming link into *B* and for each outgoing link from *B*:
    - Calculate distance within *B* from incoming to outgoing link
    - Add the maximum of these distance to the cost of the incoming link

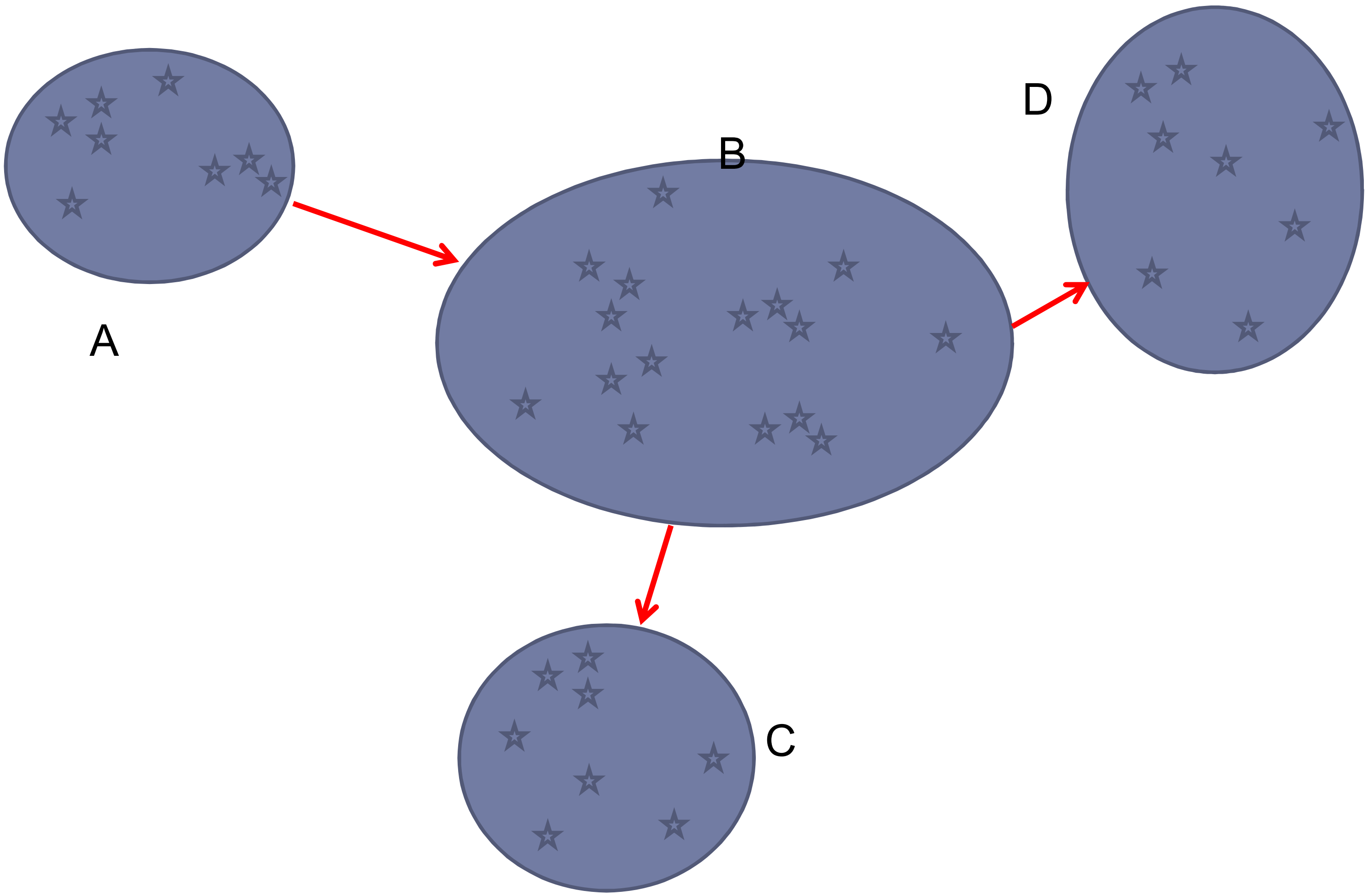
Hierarchical Pathfinding

- Maximin distance



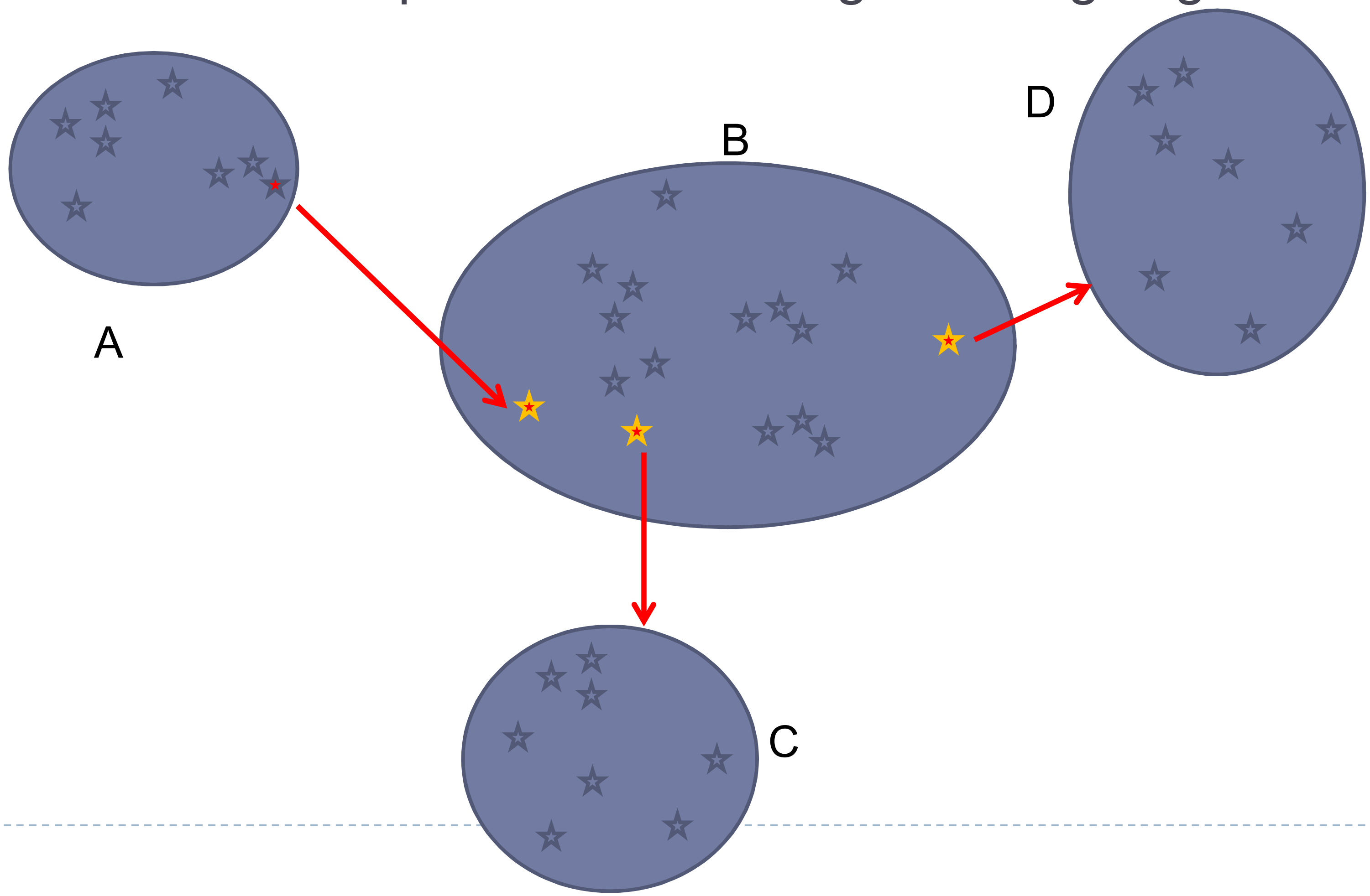
Hierarchical Pathfinding

- Maximin distance: Inlink A → B



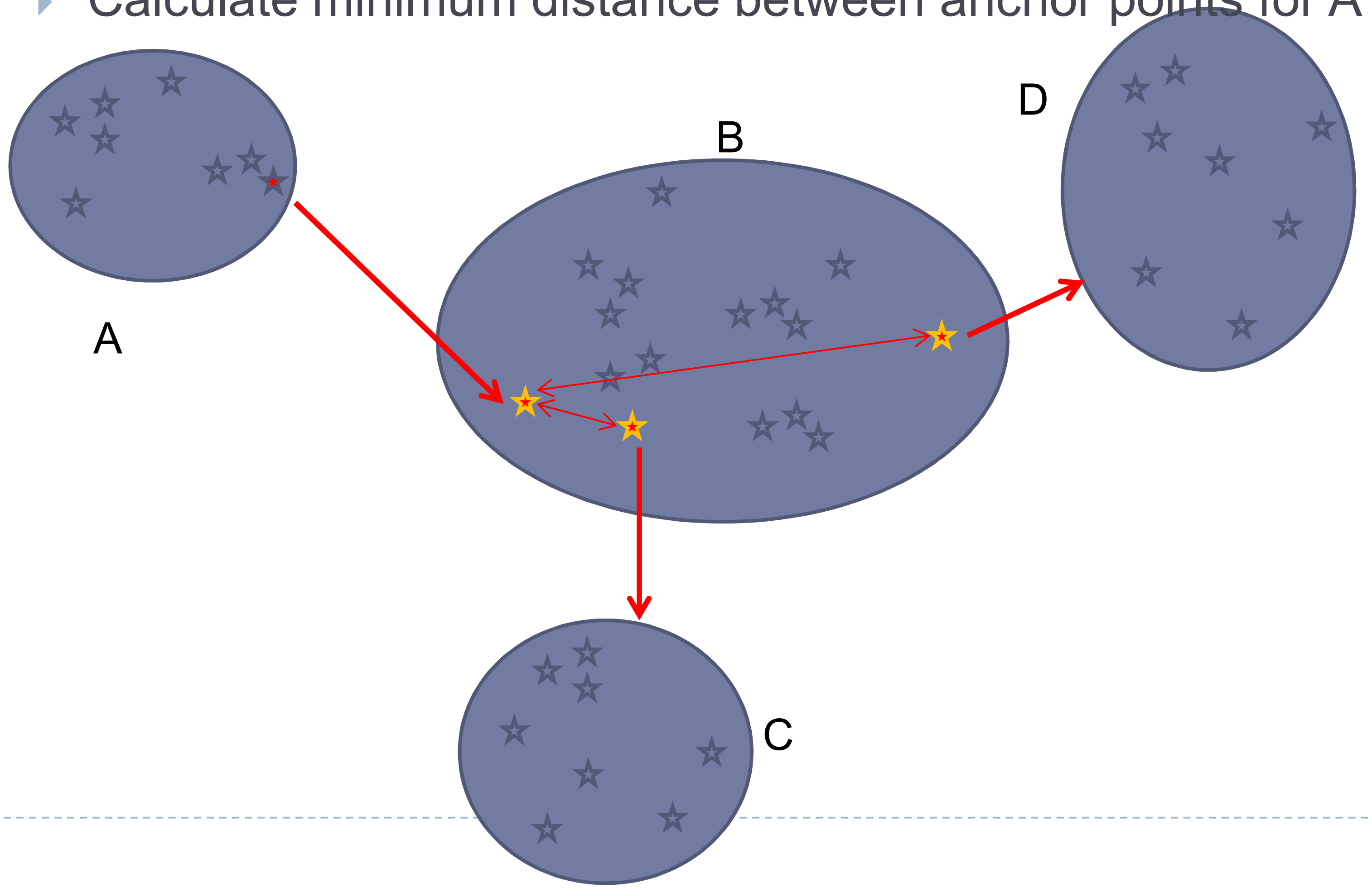
Hierarchical Pathfinding

- Maximin distance: Inlink A → B
  - Find anchor points for incoming and outgoing links



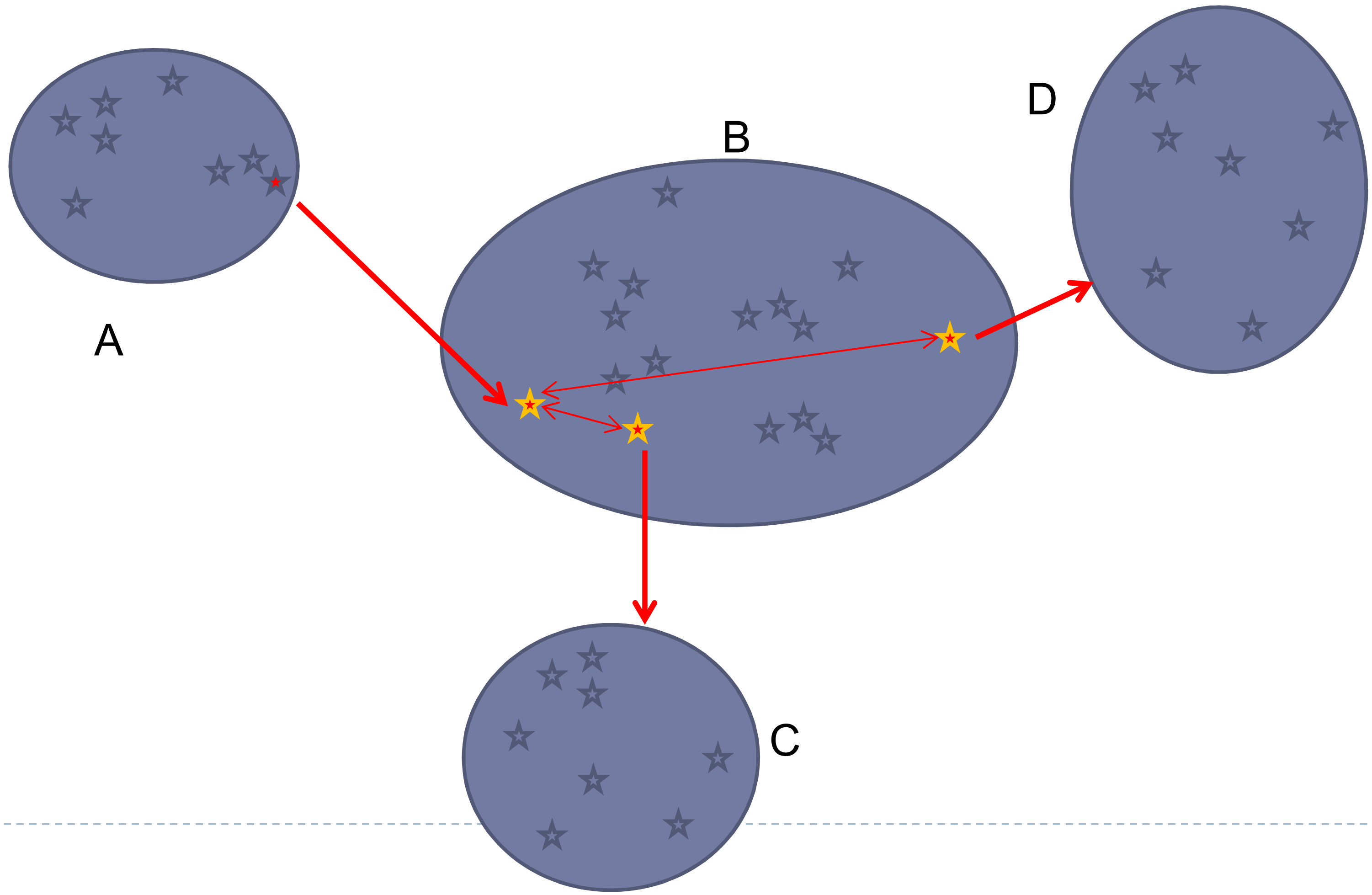
Hierarchical Pathfinding

- Maximin distance: Inlink A → B
  - Find anchor points for incoming and outgoing links
  - Calculate minimum distance between anchor points for A



Hierarchical Pathfinding

- Maximin distance:
  - Add maximum of these distances to the costs of the inlink



Hierarchical Pathfinding

- Average Minimum Distance
  - Add the average distance between anchor points to the costs of the inlink
- Minimum distance:
  - connections within the cluster are free
- Maximin distance
  - connections within the cluster are taken to be the maximum possible
- Average Minimum Distance
  - Is a compromise

Open Goal Pathfinding

- Goal nodes are not necessarily unique
  - e.g.: Avatar needs to find ammunition
    - Ammunition dumps at various locations
    - Pathfinding needs to give path to the ***nearest*** point with ammunition
  - A* heuristic is problematic
    - Assume nearest goal point is blocked
    - A* uses the distance to nearest goal point in its heuristic
    - A* will not investigate nodes in direction of alternative goal post until late

Hierarchical Pathfinding

- Dynamic pathfinding
  - Situation can change
    - Pathfinding needs to run again
  - D* algorithm
    - Similar to A*, but updates costs in the open node when the environment changes
    - Stentz, Anthony (1995), "The Focussed D* Algorithm for Real-Time Replanning", *In Proceedings of the International Joint Conference on Artificial Intelligence*: 1652–1659, http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.41.8257

Adaptive A*

- A* can use lots of memory
  - IDA*
    - Starts with a cut-off value
    - Explores path only if they are below the cut-off value
    - Uses A* heuristics to determine nodes that should be considered
  - SMA*
    - Uses a fixed limit on the number of “open” nodes

Continuous Time Pathfinding

- Pathfinding task can change quickly, but predictably
  - Example: Police car in a freeway chase
    - Other cars move at constant speed in same lane, police car changes lanes and speeds
  - Solution:
    - Limit problem heuristically
      - Change lanes as soon as possible
      - Move in current lane to potential lane change as quickly as possible
    - Create a graph where each node corresponds to a possible situation
      - Connections:
        - Lane change nodes: Time required to change lanes at current speed
        - Boundary node: Travel in same lane as fast as possible, but brake before slamming into preceding car
        - Safe opportunity nodes: Car travels in same lane as fast as possible until a point where a lane change can be made
        - Unsafe opportunity nodes: Same, but do not brake in order not to slam in the preceding car.

Movement Planning

- Extensions of nodes taking into account different types of motion
  - Animation defines different types of movement
  - Only certain speeds / rotations can be represented
- Create a *movement graph*
  - Each node represents
    - position
    - velocity
    - possible animations
  - Connections only if there is an animation

Footfall

- Movement graph where a node corresponds to a certain foot setting