

$c$	numeric value
$a$	array
$x$	term variable
$f$	function name
$\kappa$	program counter
$n$	index variable
$k$	index variable
$i$	index variable
$b_h$	boolean value
$c_h$	numeric value
$f_h$	function name
$rval$	“return value” variable
$rnset$	“rval-not-set” variable

$v$	$::=$ $ $ TRUE $ $ FALSE $ $ $c$ $ $ $a$	values bitmask true (0b1111...) bitmask false (0b0000...) numeric value bytearray
$\ominus$	$::=$ $ $ ~	unary operations bitwise not
$\oplus$	$::=$ $ $ + $ $ - $ $ * $ $ << $ $ >> $ $ & $ $   $ $ == <sub>s</sub> $ $ != <sub>s</sub> $ $ > <sub>s</sub> $ $ < <sub>s</sub> $ $ >= <sub>s</sub> $ $ <= <sub>s</sub>	binary operations      bitwise and bitwise or equals (sign extended)
$\{\sigma\}$	$::=$ $ $ $\{x_1/v_1, \dots, x_k/v_k\}$ $ $ $\{\sigma_1\} \cup \{\sigma_2\}$ $ $ $\{\sigma_1\} \cap \{\sigma_2\}$	variable substitution
$fval$	$::=$ $ $ $(x_1, \dots, x_n) : s @ e$	function spec
$fndef$	$::=$ $ $ <b>fdef</b> $f$ $fval$	function definition
$program$	$::=$ $ $ $fndef_1; \dots; fndef_n; \textbf{expose } fndef$	program list of fdefs
$\Lambda$	$::=$ $ $ $\emptyset_\Lambda$ $ $ $\Lambda[f \mapsto fval]$	function store empty function store define function
$\Gamma$	$::=$ $ $ $\emptyset_\Gamma$ $ $ $\Gamma[a \mapsto []]$ $ $ $\Gamma[a \mapsto \Gamma(a)[v_1 \mapsto v_2]]$	global memory   new array array update

$\mu$	$::=$ $\emptyset_\mu$ $\mu[x \mapsto v]$ $\mu_1 \triangleright \mu_2$	local memory empty memory add/update variable push stack frame
$e$	$::=$ $\text{TRUE}$ $\text{FALSE}$ $c$ $a$ $x$ $a[e]$ $\ominus e$ $e_1 \oplus e_2$ $f(e_1, \dots, e_n)$ $\{\sigma\} e$ $\{\sigma\} s @ e$	expressions bitmask true (0b1111...) bitmask false (0b0000...) numeric value bytearray variable array access unary operation binary operation function application variable substitution function body
$s$	$::=$ $\text{skip}$ $s_1; s_2$ $\text{def } x := e$ $\text{adeft } x := a$ $x := e$ $a[e_1] := e_2$ $\text{for } x \text{ from } v_1 \text{ to } v_2 : s$ $(\{\sigma\} s)$	statements skip sequence variable declaration array declaration variable assignment array assignment for loop additional variable substitution
$v_h$	$::=$ $b_h$ $c_h$ $a$	values boolean value numeric value bytearray
$\ominus_h$	$::=$ $!$ $\sim$	unary operations logical not bitwise not
$\oplus_h$	$::=$ $+$ $-$ $*$ $<<$ $>>$ $\&$ $ $ $\&\&$ $  $	binary operations

	$\mid$ $\mid$ $\mid$ $\mid$ $\mid$ $\mid$	$==$ $!=$ $>$ $<$ $>=$ $<=$	
$e_h$	$::=$ $\mid$ $\mid$ $\mid$ $\mid$ $\mid$ $\mid$ $\mid$ $\mid$ $\mid$	$b_h$ $c_h$ $a$ $x$ $a[e_h]$ $\ominus_h e_h$ $e_{h1} \oplus e_{h2}$ $f_h(e_{h1}, \dots, e_{hn})$	expressions boolean value numeric value bytearray variable array access unary operation binary operation function application
$s_h$	$::=$ $\mid$ $\mid$ $\mid$ $\mid$ $\mid$ $\mid$ $\mid$ $\mid$ $\mid$	<b>hskip</b> $s_{h1}; s_{h2}$ <b>hdef</b> $x := e_h$ <b>hade</b> $x := a$ $x := e_h$ $a[e_{h1}] := e_{h2}$ <b>hfor</b> $x$ <b>from</b> $v_{h1}$ <b>to</b> $v_{h2} : s_h$ <b>hif</b> $e_h$ <b>then</b> $s_{h1}$ <b>else</b> $s_{h2}$ <b>hreturn</b> $e_h$	statements skip sequence variable declaration array declaration variable assignment array assignment for loop conditional branch return
$hfval$	$::=$ $\mid$	$(x_1, \dots, x_n) : s_h$	function spec
$hfndef$	$::=$ $\mid$	<b>fdef</b> $f_h$ $hfval$	function definition
$hprogram$	$::=$ $\mid$	$hfndef_1; \dots; hfndef_n; \textbf{expose } hfndef$	program list of fdefs
$ctx$	$::=$ $\mid$ $\mid$ $\mid$	$x$ $\ominus ctx$ $ctx_1 \oplus ctx_2$	branch context variable unary operation binary operation

$$\boxed{\{\Lambda, \Gamma, \mu, \kappa\} e \longrightarrow \{\Lambda', \Gamma', \mu', \kappa'\} e'} \quad e \text{ reduces to } e'$$

$$\begin{array}{c}
\mu = \mu'[x \mapsto v] \\
\kappa' = \kappa + 1 \\
\hline
\{\Lambda, \Gamma, \mu, \kappa\} x \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} v \quad \text{EXR\_VAR} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'}{\{\Lambda, \Gamma, \mu, \kappa\} a[e] \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} a[e']} \quad \text{EXR\_ARR\_GET\_EXPR}
\end{array}$$

$$\begin{array}{c}
\frac{v' = \Gamma(a)[v] \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} a[v] \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} v'} \text{EXR\_ARR\_GET\_VAL} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'}{\{\Lambda, \Gamma, \mu, \kappa\} \ominus e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \ominus e'} \text{EXR\_UNOP\_EXPR} \\
\\
\frac{v' \equiv \llbracket \ominus v \rrbracket \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} \ominus v \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} v'} \text{EXR\_UNOP\_VAL} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} e_1 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'_1}{\{\Lambda, \Gamma, \mu, \kappa\} e_1 \oplus e_2 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'_1 \oplus e_2} \text{EXR\_BINOP\_L} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} e_2 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'_2}{\{\Lambda, \Gamma, \mu, \kappa\} v \oplus e_2 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} v \oplus e'_2} \text{EXR\_BINOP\_R} \\
\\
\frac{v_3 \equiv \llbracket v_1 \oplus v_2 \rrbracket \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} v_1 \oplus v_2 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} v_3} \text{EXR\_BINOP\_VAL} \\
\\
\frac{}{\{\Lambda, \Gamma, \mu, \kappa\} \{\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa\} e} \text{EXR\_SUBST\_EMPTY} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} e'} \text{EXR\_SUBST\_EXPR} \\
\\
\frac{\kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} \{x_1/v_1, \dots, x_k/v_k\} x_i \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} v_i} \text{EXR\_SUBST\_VAR} \\
\\
\frac{}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} x \longrightarrow \{\Lambda, \Gamma, \mu, \kappa\} x} \text{EXR\_SUBST\_NO\_VAR} \\
\\
\frac{}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} v \longrightarrow \{\Lambda, \Gamma, \mu, \kappa\} v} \text{EXR\_SUBST\_VAL} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} e_1 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'_1}{\{\Lambda, \Gamma, \mu, \kappa\} f(v_1, \dots, v_k, e_1, e_2, \dots, e_n) \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} f(v_1, \dots, v_k, e'_1, e_2, \dots, e_n)} \text{EXR\_FN\_EXPR} \\
\\
\frac{\Lambda = \Lambda'[f \mapsto (x_1, \dots, x_k) : s @e] \quad \mu' = \mu \triangleright \emptyset_\mu \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} f(v_1, \dots, v_k) \longrightarrow \{\Lambda, \Gamma, \mu', \kappa'\} \{x_1/v_1, \dots, x_k/v_k\} s @e} \text{EXR\_FN\_CALL} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{skip} @e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{skip} @e'} \text{EXR\_SKIP\_EXPR} \\
\\
\frac{\mu = \mu_1 \triangleright \mu_2}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{skip} @v \longrightarrow \{\Lambda, \Gamma, \mu_1, \kappa\} v} \text{EXR\_SKIP\_VAL} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} s_1 @e_0 \longrightarrow \{\Lambda, \Gamma, \mu', \kappa'\} \{\sigma'\} s'_1 @e_0}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} s_1; s_2 @e_0 \longrightarrow \{\Lambda, \Gamma, \mu', \kappa'\} \{\sigma'\} s'_1; s_2 @e_0} \text{EXR\_SEQ} \\
\\
\frac{}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{skip}; s @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} s @e_0} \text{EXR\_SEQ\_SKIP} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{def} x := e @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} \mathbf{def} x := e' @e_0} \text{EXR\_DEF\_EXPR} \\
\\
\frac{\mu' = \mu[x \mapsto v] \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{def} x := v @e_0 \longrightarrow \{\Lambda, \Gamma, \mu', \kappa'\} \{\sigma\} \mathbf{skip} @e_0} \text{EXR\_DEF\_VAL}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma' = \Gamma[a \mapsto []] \quad \mu' = \mu[x \mapsto a] \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{adef} \ x := a \ @e_0 \longrightarrow \{\Lambda, \Gamma', \mu', \kappa'\} \{\sigma\} \mathbf{skip} \ @e_0} \text{EXR\_DEF\_ARR} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} x := e \ @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} x := e' \ @e_0} \text{EXR\_ASSIGN\_EXPR} \\
\\
\frac{\mu' = \mu[x \mapsto v] \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} x := v \ @e_0 \longrightarrow \{\Lambda, \Gamma, \mu', \kappa'\} \{\sigma\} \mathbf{skip} \ @e_0} \text{EXR\_ASSIGN\_VAL} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} e_1 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'_1}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} a[e_1] := e_2 \ @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} a[e'_1] := e_2 \ @e_0} \text{EXR\_ARR\_ASSIGN\_EXPR\_L} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} e_2 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'_2}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} a[v_1] := e_2 \ @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} a[v_1] := e'_2 \ @e_0} \text{EXR\_ARR\_ASSIGN\_EXPR\_R} \\
\\
\frac{\Gamma' = \Gamma[a \mapsto \Gamma(a)[v_1 \mapsto v_2]] \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} a[v_1] := v_2 \ @e_0 \longrightarrow \{\Lambda, \Gamma', \mu, \kappa'\} \{\sigma\} \mathbf{skip} \ @e_0} \text{EXR\_ARR\_ASSIGN\_VAL} \\
\\
\frac{v_1 < v_2 \quad v'_1 = v_1 + 1 \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{for} \ x \mathbf{from} \ v_1 \mathbf{to} \ v_2 : s \ @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} (\{x/v_1\} s); \mathbf{for} \ x \mathbf{from} \ v'_1 \mathbf{to} \ v_2 : s \ @e_0} \text{EXR\_FOR} \\
\\
\frac{\{\sigma_1\} \cap \{\sigma_2\} = \{\} \quad \{\sigma_3\} = \{\sigma_1\} \cup \{\sigma_2\}}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma_1\} (\{\sigma_2\} s) \ @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma_3\} s \ @e_0} \text{EXR\_ADD\_SUBST} \\
\\
\frac{v_1 = v_2}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{for} \ x \mathbf{from} \ v_2 \mathbf{to} \ v_2 : s \ @e_0 \longrightarrow \{\Lambda, \Gamma, \mu', \kappa\} \{\sigma\} \mathbf{skip} \ @e_0} \text{EXR\_FOR\_BASE} \\
\\
\boxed{\llbracket e_h \rrbracket_t = e} \quad e_h \text{ is transformed to } e \\
\\
\frac{v \equiv \llbracket v_h \rrbracket_{int}}{\llbracket v_h \rrbracket_t = v} \text{EXT\_VAL} \\
\\
\frac{}{\llbracket x \rrbracket_t = x} \text{EXT\_VAR} \\
\\
\frac{}{\llbracket a \rrbracket_t = a} \text{EXT\_ARR} \\
\\
\frac{\llbracket e_h \rrbracket_t = e}{\llbracket a[e_h] \rrbracket_t = a[e]} \text{EXT\_ARR\_GET} \\
\\
\frac{\llbracket \mathbf{fdef} \ f_h \ hfval \rrbracket_t = \mathbf{fdef} \ f \ fval \quad \llbracket e_{h1} \rrbracket_t = e_1 \quad \dots \quad \llbracket e_{hk} \rrbracket_t = e_k}{\llbracket f_h(e_{h1}, \dots, e_{hk}) \rrbracket_t = f(e_1, \dots, e_k)} \text{EXT\_FN\_CALL} \\
\\
\boxed{\llbracket s_h \rrbracket_{ctx} = s} \quad s_h \text{ is transformed to } s \\
\\
\frac{}{\llbracket \mathbf{hskip} \rrbracket_{ctx} = \mathbf{skip}} \text{STT\_SKIP} \\
\\
\frac{\llbracket s_{h1} \rrbracket_{ctx} = s_1 \quad \llbracket s_{h2} \rrbracket_{ctx} = s_2}{\llbracket s_{h1}; s_{h2} \rrbracket_{ctx} = s_1; s_2} \text{STT\_SEQ}
\end{array}$$

$$\begin{array}{c}
\frac{\llbracket e_h \rrbracket_t = e}{\llbracket \mathbf{hdef} \ x := e_h \rrbracket_{ctx} = \mathbf{def} \ x := e} \quad \text{STT\_VAR\_DEC} \\
\\
\frac{}{\llbracket \mathbf{hade} \ x := a \rrbracket_{ctx} = \mathbf{ade} \ x := a} \quad \text{STT\_ARR\_DEC} \\
\\
\frac{\begin{array}{l} \llbracket e_h \rrbracket_t = e \\ e' = ctx \ \& \ rnset \\ e'' = e \ \& \ e' \\ e''' = x \ \& \ (\sim e') \end{array}}{\llbracket x := e_h \rrbracket_{ctx} = x := (e'' \mid e''')} \quad \text{STT\_VAR\_ASSIGN} \\
\\
\frac{\begin{array}{l} \llbracket e_{h1} \rrbracket_t = e_1 \\ \llbracket e_{h2} \rrbracket_t = e_2 \\ e' = ctx \ \& \ rnset \\ e'' = e_2 \ \& \ e' \\ e''' = a[e_1] \ \& \ (\sim e') \end{array}}{\llbracket a[e_{h1}] := e_{h2} \rrbracket_{ctx} = a[e_1] := (e'' \mid e''')} \quad \text{STT\_ARR\_ASSIGN} \\
\\
\frac{\begin{array}{l} \llbracket v_{h1} \rrbracket_t = v_1 \\ \llbracket v_{h2} \rrbracket_t = v_2 \\ \llbracket s_h \rrbracket_{ctx} = s \end{array}}{\llbracket \mathbf{hfor} \ x \ \mathbf{from} \ v_{h1} \ \mathbf{to} \ v_{h2} : s_h \rrbracket_{ctx} = \mathbf{for} \ x \ \mathbf{from} \ v_1 \ \mathbf{to} \ v_2 : s} \quad \text{STT\_FOR} \\
\\
\frac{\begin{array}{l} \llbracket e_h \rrbracket_t = e \\ \llbracket s_{h1} \rrbracket_{(ctx' \ \& \ ctx)} = s_1 \\ \llbracket s_{h2} \rrbracket_{(ctx' \ \& \ ctx)} = s_2 \end{array}}{\llbracket \mathbf{hif} \ e_h \ \mathbf{then} \ s_{h1} \ \mathbf{else} \ s_{h2} \rrbracket_{ctx} = \mathbf{def} \ ctx' := e; s_1; ctx' := (\sim ctx'); s_2} \quad \text{STT\_IF} \\
\\
\frac{\begin{array}{l} \llbracket e_h \rrbracket_t = e \\ e' = ctx \ \& \ rnset \\ e'' = e \ \& \ e' \end{array}}{\llbracket \mathbf{hreturn} \ e_h \rrbracket_{ctx} = rval := (e'' \mid rval); rnset := (rnset \ \& \ (\sim ctx))} \quad \text{STT\_RET} \\
\\
\boxed{\llbracket hfndef \rrbracket_t = fndef} \quad hfndef \text{ is transformed to } fndef \\
\\
\frac{\llbracket s_h \rrbracket_{\text{TRUE}} = s}{\llbracket \mathbf{fdef} \ f_h(x_1, \dots, x_k) : s_h \rrbracket_t = \mathbf{fdef} \ f(x_1, \dots, x_k) : \mathbf{def} \ rval := \text{FALSE}; \mathbf{def} \ rnset := \text{TRUE}; s \ @rval} \quad \text{FDEFT\_FDEF} \\
\\
\boxed{\llbracket \ominus_h \rrbracket_t = \ominus} \quad \ominus_h \text{ is transformed to } \ominus \\
\\
\frac{}{\llbracket ! \rrbracket_t = \sim} \quad \text{UNOPT\_LNOT} \\
\\
\frac{}{\llbracket \ominus_h \rrbracket_t = \ominus} \quad \text{UNOPT\_UNOP} \\
\\
\boxed{\llbracket \oplus_h \rrbracket_t = \oplus} \quad \oplus_h \text{ is transformed to } \oplus \\
\\
\frac{}{\llbracket \&\& \rrbracket_t = \&} \quad \text{BINOPT\_LAND} \\
\\
\frac{}{\llbracket \mid \mid \rrbracket_t = \mid} \quad \text{BINOPT\_LOR} \\
\\
\frac{}{\llbracket == \rrbracket_t = ==_s} \quad \text{BINOPT\_EQ} \\
\\
\frac{}{\llbracket != \rrbracket_t = !=_s} \quad \text{BINOPT\_NEQ}
\end{array}$$

$$\begin{array}{l}
\overline{\llbracket > \rrbracket_t = >_s} \quad \text{BINOPT\_GT} \\
\overline{\llbracket < \rrbracket_t = <_s} \quad \text{BINOPT\_LT} \\
\overline{\llbracket >= \rrbracket_t = >=_s} \quad \text{BINOPT\_GTE} \\
\overline{\llbracket <= \rrbracket_t = <=_s} \quad \text{BINOPT\_LTE} \\
\overline{\llbracket \oplus_h \rrbracket_t = \oplus} \quad \text{BINOPT\_BINOP}
\end{array}$$

Definition rules: 56 good 0 bad  
Definition rule clauses: 122 good 0 bad