

c	numeric value
a	array
x	term variable
f	function name
κ	program counter
n	index variable
k	index variable
i	index variable
b_h	boolean value
c_h	numeric value
f_h	function name
$rval$	“return value” variable
$rnset$	“rval-not-set” variable

l	$::=$ $ $ TRUE $ $ FALSE	literals bitmask true (0b1111...) bitmask false (0b0000...)
v	$::=$ $ $ l $ $ c $ $ a	values numeric literal numeric value bytearray
\ominus	$::=$ $ $ \sim	unary operations bitwise not
\oplus	$::=$ $ $ + $ $ - $ $ * $ $ << $ $ >> $ $ & $ $ $ $ == _s $ $!= _s $ $ > _s $ $ < _s $ $ >= _s $ $ <= _s	binary operations bitwise and bitwise or equals (sign extended)
$\{\sigma\}$	$::=$ $ $ $\{x_1/v_1, \dots, x_k/v_k\}$ $ $ $\{\sigma_1\} \cup \{\sigma_2\}$ $ $ $\{\sigma_1\} \cap \{\sigma_2\}$	variable substitution
$fval$	$::=$ $ $ $(x_1, \dots, x_n) : s @ e$	function spec
$fndef$	$::=$ $ $ fdef f $fval$	function definition
$program$	$::=$ $ $ $fndef_1; \dots; fndef_n; \textbf{expose } fndef$	program list of fdefs
Λ	$::=$ $ $ \emptyset_Λ $ $ $\Lambda[f \mapsto fval]$	function store empty function store define function
Γ	$::=$ $ $ \emptyset_Γ	global memory

		$\Gamma[a \mapsto []]$	new array
		$\Gamma[a \mapsto \Gamma(a)[v_1 \mapsto v_2]]$	array update
μ	$::=$		local memory
		\emptyset_μ	empty memory
		$\mu[x \mapsto v]$	add/update variable
		$\mu_1 \triangleright \mu_2$	push stack frame
e	$::=$		expressions
		l	numeric literal
		c	numeric value
		a	bytearray
		x	variable
		$a[e]$	array access
		$\ominus e$	unary operation
		$e_1 \oplus e_2$	binary operation
		$f(e_1, \dots, e_n)$	function application
		$\{\sigma\} e$	variable substitution
		$\{\sigma\} s @ e$	function body
s	$::=$		statements
		skip	skip
		$s_1; s_2$	sequence
		def $x := e$	variable declaration
		ade $x := a$	array declaration
		$x := e$	variable assignment
		$a[e_1] := e_2$	array assignment
		for x from v_1 to $v_2 : s$	for loop
		$(\{\sigma\} s)$	additional variable substitution
v_h	$::=$		values
		b_h	boolean value
		c_h	numeric value
\ominus_h	$::=$		unary operations
		!	logical not
		~	bitwise not
\oplus_h	$::=$		binary operations
		+	
		-	
		*	
		<<	
		>>	
		&	
		 	
		&&	

	$ $ $==$ $!=$ $>$ $<$ $>=$ $<=$	
e_h	$::=$ b_h c_h x $a[e_h]$ $\ominus_h e_h$ $e_{h1} \oplus e_{h2}$ $f_h(e_{h1}, \dots, e_{hn})$	expressions boolean value numeric value variable array access unary operation binary operation function application
s_h	$::=$ hskip $s_{h1}; s_{h2}$ hdef $x := e_h$ hade $x := a$ $x := e_h$ $a[e_{h1}] := e_{h2}$ hfor x from v_{h1} to $v_{h2} : s_h$ hif e_h then s_{h1} else s_{h2} hreturn e_h	statements skip sequence variable declaration array declaration variable assignment array assignment for loop conditional branch return
$hfval$	$::=$ $(x_1, \dots, x_n) : s_h$	function spec
$hfndef$	$::=$ fdef $f_h hfval$	function definition
$hprogram$	$::=$ $hfndef_1; \dots; hfndef_n; \textbf{expose } hfndef$	program list of fdefs
ctx	$::=$ l x $\ominus ctx$ $ctx_1 \oplus ctx_2$	branch context numeric literal variable unary operation binary operation

$$\boxed{\{\Lambda, \Gamma, \mu, \kappa\} e \longrightarrow \{\Lambda', \Gamma', \mu', \kappa'\} e'} \quad e \text{ reduces to } e'$$

$$\begin{array}{c}
\mu = \mu'[x \mapsto v] \\
\kappa' = \kappa + 1 \\
\hline
\{\Lambda, \Gamma, \mu, \kappa\} x \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} v \quad \text{EXR_VAR} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'}{\{\Lambda, \Gamma, \mu, \kappa\} a[e] \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} a[e']} \quad \text{EXR_ARR_GET_EXPR}
\end{array}$$

$$\begin{array}{c}
\frac{v' = \Gamma(a)[v] \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} a[v] \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} v'} \text{EXR_ARR_GET_VAL} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'}{\{\Lambda, \Gamma, \mu, \kappa\} \ominus e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \ominus e'} \text{EXR_UNOP_EXPR} \\
\\
\frac{v' \equiv \llbracket \ominus v \rrbracket \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} \ominus v \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} v'} \text{EXR_UNOP_VAL} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} e_1 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'_1}{\{\Lambda, \Gamma, \mu, \kappa\} e_1 \oplus e_2 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'_1 \oplus e_2} \text{EXR_BINOP_L} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} e_2 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'_2}{\{\Lambda, \Gamma, \mu, \kappa\} v \oplus e_2 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} v \oplus e'_2} \text{EXR_BINOP_R} \\
\\
\frac{v_3 \equiv \llbracket v_1 \oplus v_2 \rrbracket \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} v_1 \oplus v_2 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} v_3} \text{EXR_BINOP_VAL} \\
\\
\frac{}{\{\Lambda, \Gamma, \mu, \kappa\} \{\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa\} e} \text{EXR_SUBST_EMPTY} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} e'} \text{EXR_SUBST_EXPR} \\
\\
\frac{\kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} \{x_1/v_1, \dots, x_k/v_k\} x_i \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} v_i} \text{EXR_SUBST_VAR} \\
\\
\frac{}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} x \longrightarrow \{\Lambda, \Gamma, \mu, \kappa\} x} \text{EXR_SUBST_NO_VAR} \\
\\
\frac{}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} v \longrightarrow \{\Lambda, \Gamma, \mu, \kappa\} v} \text{EXR_SUBST_VAL} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} e_1 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'_1}{\{\Lambda, \Gamma, \mu, \kappa\} f(v_1, \dots, v_k, e_1, e_2, \dots, e_n) \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} f(v_1, \dots, v_k, e'_1, e_2, \dots, e_n)} \text{EXR_FN_EXPR} \\
\\
\frac{\Lambda = \Lambda'[f \mapsto (x_1, \dots, x_k) : s @ e] \quad \mu' = \mu \triangleright \emptyset_\mu \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} f(v_1, \dots, v_k) \longrightarrow \{\Lambda, \Gamma, \mu', \kappa'\} \{x_1/v_1, \dots, x_k/v_k\} s @ e} \text{EXR_FN_CALL} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{skip} @ e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{skip} @ e'} \text{EXR_SKIP_EXPR} \\
\\
\frac{\mu = \mu_1 \triangleright \mu_2}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{skip} @ v \longrightarrow \{\Lambda, \Gamma, \mu_1, \kappa\} v} \text{EXR_SKIP_VAL} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} s_1 @ e_0 \longrightarrow \{\Lambda, \Gamma, \mu', \kappa'\} \{\sigma'\} s'_1 @ e_0}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} s_1; s_2 @ e_0 \longrightarrow \{\Lambda, \Gamma, \mu', \kappa'\} \{\sigma'\} s'_1; s_2 @ e_0} \text{EXR_SEQ} \\
\\
\frac{}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{skip}; s @ e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} s @ e_0} \text{EXR_SEQ_SKIP} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{def} x := e @ e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} \mathbf{def} x := e' @ e_0} \text{EXR_DEF_EXPR} \\
\\
\frac{\mu' = \mu[x \mapsto v] \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{def} x := v @ e_0 \longrightarrow \{\Lambda, \Gamma, \mu', \kappa'\} \{\sigma\} \mathbf{skip} @ e_0} \text{EXR_DEF_VAL}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma' = \Gamma[a \mapsto []] \quad \mu' = \mu[x \mapsto a] \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{a\!def} \ x := a \ @e_0 \longrightarrow \{\Lambda, \Gamma', \mu', \kappa'\} \{\sigma\} \mathbf{skip} \ @e_0} \text{EXR_DEF_ARR} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} x := e \ @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} x := e' \ @e_0} \text{EXR_ASSIGN_EXPR} \\
\\
\frac{\mu' = \mu[x \mapsto v] \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} x := v \ @e_0 \longrightarrow \{\Lambda, \Gamma, \mu', \kappa'\} \{\sigma\} \mathbf{skip} \ @e_0} \text{EXR_ASSIGN_VAL} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} e_1 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'_1}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} a[e_1] := e_2 \ @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} a[e'_1] := e_2 \ @e_0} \text{EXR_ARR_ASSIGN_EXPR_L} \\
\\
\frac{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} e_2 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'_2}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} a[v_1] := e_2 \ @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} a[v_1] := e'_2 \ @e_0} \text{EXR_ARR_ASSIGN_EXPR_R} \\
\\
\frac{\Gamma' = \Gamma[a \mapsto \Gamma(a)[v_1 \mapsto v_2]] \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} a[v_1] := v_2 \ @e_0 \longrightarrow \{\Lambda, \Gamma', \mu, \kappa'\} \{\sigma\} \mathbf{skip} \ @e_0} \text{EXR_ARR_ASSIGN_VAL} \\
\\
\frac{v_1 < v_2 \quad v'_1 = v_1 + 1 \quad \kappa' = \kappa + 1}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{for} \ x \mathbf{from} \ v_1 \mathbf{to} \ v_2 : s \ @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} (\{x/v_1\} s); \mathbf{for} \ x \mathbf{from} \ v'_1 \mathbf{to} \ v_2 : s \ @e_0} \text{EXR_FOR} \\
\\
\frac{\{\sigma_1\} \cap \{\sigma_2\} = \{\} \quad \{\sigma_3\} = \{\sigma_1\} \cup \{\sigma_2\}}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma_1\} (\{\sigma_2\} s) \ @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma_3\} s \ @e_0} \text{EXR_ADD_SUBST} \\
\\
\frac{v_1 = v_2}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \mathbf{for} \ x \mathbf{from} \ v_2 \mathbf{to} \ v_2 : s \ @e_0 \longrightarrow \{\Lambda, \Gamma, \mu', \kappa\} \{\sigma\} \mathbf{skip} \ @e_0} \text{EXR_FOR_BASE} \\
\\
\boxed{\llbracket e_h \rrbracket_t = e} \quad e_h \text{ is transformed to } e \\
\\
\frac{v \equiv \llbracket v_h \rrbracket_{int}}{\llbracket v_h \rrbracket_t = v} \text{EXT_VAL} \\
\\
\frac{}{\llbracket x \rrbracket_t = x} \text{EXT_VAR} \\
\\
\frac{\llbracket e_h \rrbracket_t = e}{\llbracket a[e_h] \rrbracket_t = a[e]} \text{EXT_ARR_GET} \\
\\
\frac{\llbracket \mathbf{fdef} \ f_h \ hfval \rrbracket_t = \mathbf{fdef} \ f \ fval \quad \llbracket e_{h1} \rrbracket_t = e_1 \quad \dots \quad \llbracket e_{hk} \rrbracket_t = e_k}{\llbracket f_h(e_{h1}, \dots, e_{hk}) \rrbracket_t = f(e_1, \dots, e_k)} \text{EXT_FN_CALL} \\
\\
\boxed{\llbracket s_h \rrbracket_{ctx} = s} \quad s_h \text{ is transformed to } s \\
\\
\frac{}{\llbracket \mathbf{hskip} \rrbracket_{ctx} = \mathbf{skip}} \text{STT_SKIP} \\
\\
\frac{\llbracket s_{h1} \rrbracket_{ctx} = s_1 \quad \llbracket s_{h2} \rrbracket_{ctx} = s_2}{\llbracket s_{h1}; s_{h2} \rrbracket_{ctx} = s_1; s_2} \text{STT_SEQ}
\end{array}$$

$$\begin{array}{c}
\frac{\llbracket e_h \rrbracket_t = e}{\llbracket \mathbf{hdef} \ x := e_h \rrbracket_{ctx} = \mathbf{def} \ x := e} \quad \text{STT_VAR_DEC} \\
\\
\frac{}{\llbracket \mathbf{hade} \ x := a \rrbracket_{ctx} = \mathbf{ade} \ x := a} \quad \text{STT_ARR_DEC} \\
\\
\frac{\begin{array}{l} \llbracket e_h \rrbracket_t = e \\ e' = ctx \ \& \ rnset \\ e'' = e \ \& \ e' \\ e''' = x \ \& \ (\sim e') \end{array}}{\llbracket x := e_h \rrbracket_{ctx} = x := (e'' \mid e''')} \quad \text{STT_VAR_ASSIGN} \\
\\
\frac{\begin{array}{l} \llbracket e_{h1} \rrbracket_t = e_1 \\ \llbracket e_{h2} \rrbracket_t = e_2 \\ e' = ctx \ \& \ rnset \\ e'' = e_2 \ \& \ e' \\ e''' = a[e_1] \ \& \ (\sim e') \end{array}}{\llbracket a[e_{h1}] := e_{h2} \rrbracket_{ctx} = a[e_1] := (e'' \mid e''')} \quad \text{STT_ARR_ASSIGN} \\
\\
\frac{\begin{array}{l} \llbracket v_{h1} \rrbracket_t = v_1 \\ \llbracket v_{h2} \rrbracket_t = v_2 \\ \llbracket s_h \rrbracket_{ctx} = s \end{array}}{\llbracket \mathbf{hfor} \ x \mathbf{from} \ v_{h1} \mathbf{to} \ v_{h2} : s_h \rrbracket_{ctx} = \mathbf{for} \ x \mathbf{from} \ v_1 \mathbf{to} \ v_2 : s} \quad \text{STT_FOR} \\
\\
\frac{\begin{array}{l} \llbracket e_h \rrbracket_t = e \\ \llbracket s_{h1} \rrbracket_{(ctx' \ \& \ ctx)} = s_1 \\ \llbracket s_{h2} \rrbracket_{(ctx' \ \& \ ctx)} = s_2 \end{array}}{\llbracket \mathbf{hif} \ e_h \mathbf{then} \ s_{h1} \mathbf{else} \ s_{h2} \rrbracket_{ctx} = \mathbf{def} \ ctx' := e; s_1; ctx' := (\sim ctx'); s_2} \quad \text{STT_IF} \\
\\
\frac{\begin{array}{l} \llbracket e_h \rrbracket_t = e \\ e' = ctx \ \& \ rnset \\ e'' = e \ \& \ e' \end{array}}{\llbracket \mathbf{hreturn} \ e_h \rrbracket_{ctx} = rval := (e'' \mid rval); rnset := (rnset \ \& \ (\sim ctx))} \quad \text{STT_RET} \\
\\
\boxed{\llbracket hfndef \rrbracket_t = fndef} \quad hfndef \text{ is transformed to } fndef \\
\\
\frac{\llbracket s_h \rrbracket_{\text{TRUE}} = s}{\llbracket \mathbf{fdef} \ f_h(x_1, \dots, x_k) : s_h \rrbracket_t = \mathbf{fdef} \ f(x_1, \dots, x_k) : \mathbf{def} \ rval := \text{FALSE}; \mathbf{def} \ rnset := \text{TRUE}; s \ @rval} \quad \text{FDEFT_FDEF} \\
\\
\boxed{\llbracket \ominus_h \rrbracket_t = \ominus} \quad \ominus_h \text{ is transformed to } \ominus \\
\\
\frac{}{\llbracket ! \rrbracket_t = \sim} \quad \text{UNOPT_LNOT} \\
\\
\frac{}{\llbracket \ominus_h \rrbracket_t = \ominus} \quad \text{UNOPT_UNOP} \\
\\
\boxed{\llbracket \oplus_h \rrbracket_t = \oplus} \quad \oplus_h \text{ is transformed to } \oplus \\
\\
\frac{}{\llbracket \&\& \rrbracket_t = \&} \quad \text{BINOPT_LAND} \\
\\
\frac{}{\llbracket \mid \mid \rrbracket_t = \mid} \quad \text{BINOPT_LOR} \\
\\
\frac{}{\llbracket == \rrbracket_t = ==_s} \quad \text{BINOPT_EQ} \\
\\
\frac{}{\llbracket != \rrbracket_t = !=_s} \quad \text{BINOPT_NEQ}
\end{array}$$

$$\begin{array}{rcl}
\overline{\llbracket > \rrbracket_t = >_s} & \text{BINOPT_GT} \\
\overline{\llbracket < \rrbracket_t = <_s} & \text{BINOPT_LT} \\
\overline{\llbracket >= \rrbracket_t = >= _s} & \text{BINOPT_GTE} \\
\overline{\llbracket <= \rrbracket_t = <= _s} & \text{BINOPT_LTE} \\
\overline{\llbracket \oplus_h \rrbracket_t = \oplus} & \text{BINOPT_BINOP}
\end{array}$$

Definition rules: 55 good 0 bad
Definition rule clauses: 121 good 0 bad