It's About Time: Implementing a Verified Constant-Time Library

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ABSTRACT

TK when rest of paper is done

Keywords

Keywords

1. INTRODUCTION

Introduction TK When rest of paper is done

2. RELATED WORK

Over the years, we have seen an increase in potential attack vectors for malicious parties. One such attack vector is side channel attacks — utlizing leaked information to undermine the security of applications and systems. Our related work is split into two main areas: a discussion on side channel attacks and mitigations as well as a discussion on constant time implementations.

Starting over 40 years ago [9, 10, 21], there has been a plethora of work on the prevalence of side channel attacks and the problem of confinement. Side channels can be used in power analysis [7, 19] to detect certain computations, or in networking and the TCP/IP protocol [5, 16, 22] and also in memory [6, 11, 12]. Moreover, there has been a extensive amount of work on timing attacks [8, 20]. Our focus is on timing attacks and memory attacks and is thus more closely related to those works.

Moverover, there has been plenty of work completed on avoiding these side-channel attacks, mainly through various programming language institutions. For example, there has been a vast array of work that looks at information flow control in order to avoid or control the covert channels for both timing and memory [4, 13, 15, 17, 18]. We decide to take a slightly different approach and attack this problem at the language level, instead of employing information flow control tactics. The work that is most closely related to us [14] also uses transformations to avoid side channel attacks in C code. The main difference in our work and theirs is that we plan to create a library so that developers do not need to write in our language if they do not desire to do so.

3. FORMALIZATION

We formalize ConstanC by first proving the constant-time properties of a new language (ConstCore), then showing the transformation from ConstanC to ConstCore.

The language ConstCore is a While-like language with highly limited control flow. Notably, there are no conditional branching instructions, and the only loop construct is a for loop with static loop bounds. We adopt the *program-counter security model* from Molnar et al. [14], keeping an instruction count as part of our small-step semantics (see Appendix A). We then show that every function in ConstCore has the following property: For every function f, there exists an instruction count f_κ such that for any input $x_1, \ldots, x_n$ to f and current instruction count κ, the instruction count κ' when f returns is exactly $\kappa + f_\kappa$. That is to say, the number of instructions executed by any function in ConstCore does not depend on the inputs to the function. ***[TODO: program-transcript model instead?]***

We then construct a transformation from ConstanC to ConstCore. Since both ConstCore and ConstanC are side effect-free languages, we simply need to prove that for every function f in ConstanC, the corresponding transformed function f_{core} returns an equivalent value given equivalent arguments.

We ensure this equivalence by keeping a "context", which is a bitmask representing the control flow the of the function at the current statement. This bitmask is either high (all 1s) or low (all 0s). Any variable assignment `x := v` in ConstanC is transformed to the following statement in ConstCore:

```
x := ((ctx & rnset) & v) | ((~(ctx & rnset) & x)
```

where `ctx` is the context bitmask and `&`, `|`, `~` represent bitwise *and*, *or*, and *not* operations respectively. The formal transformation is given in rule Stt-var-assign. The `rnset` variable is an additional bitmask used for tracking early function return and is described below. With this transformation, variables are only updated to new values if the context at the time of execution indicates that the original program control flow would have made it to that statement.

Conditional branches are transformed by executing the statements in both branches. However, before each block is executed, the context bitmask is updated with the branch

condition. Thus the statement `if bexpr then s1 else s2` is transformed to:

```
b := bexpr
oldctx := ctx
ctx := oldctx & b
s1
ctx := oldctx & (~b)
s2
ctx := oldctx
```

where `b` is a fresh temporary variable for each instance of a branching statement. This ensures that nested conditionals still function as expected. The formal transformation is given in rule STT-IF.

Return statements in CONSTANTC are dealt with by constructing two additional variables for every function: *rval* (initialized low) and *rnset* (initialized high). A return statement in CONSTANTC is translated to an assignment to *rval*, gated by the context as above. Additionally, *rnset* ("return value not set") is updated as follows:

```
rnset := rnset & (~ctx)
```

In this way, *rnset* remains high until a "return statement" is executed under an active control flow path, at which point it is set low and remains low for the rest of the function. Since all variable assignments are gated with *rnset* in addition to the context, no further variable assignments will cause updates. The formal transformation is given in rule STT-RET.

We have one final problem: even if two functions execute the same number of instructions, they can take different amounts of time ***[TODO: cite]***. Our compiler currently targets Intel x86 assembly, and it is currently unknown ***[TODO: fact check]*** which instructions in this architecture are truly constant time. Our mitigation strategy is to restrict ourselves to assumed "safe" instructions, such as basic arithmetic (except division) and comparison operators, as well as simple loads, stores, and calls. Thus the constant-time nature of compiled CONSTCORE is no less secure than the set of chosen instructions.

4. IMPLEMENTATION

Knowing that we only had eight weeks to implement this project, we made a few key decisions. The first was to go straight from an AST to LLVM to our library. We decided to forego creating a lexer and parser for lack of time and also to show that our preliminary implementations of the language worked and this project was worth pursuing further. Moreover, we figured that as we progressed further into the project, our parser might change rapidly, and decided that, to save time and effort, we would determine our lexer and parser after the language was more stable

Everything is implemented in OCaml [2], because of the existing LLVM bindings between OCaml and LLVM. We have implemented a constantC AST as well as the transformations needed to convert our AST into our constcore AST, which is the representation of the core language. We have created the codegen that converts our constcore code into LLVM IR, as well as a typechecker. A list of our supported operations is listed the Table 1.

Label	Supported Operations
Types	Int/Bool/ByteArr
Statements	VarDec/Assign/ArrAssign/If/For/Return
Expressions	VarExp/ArrExp/Unop/Binop/Primitive/CallExp
Unary Op	B_not
Binary Op	Plus/Minus/GT/B_and/B_or

Figure 1: **Supported Language**—We show the different types, statements, expressions, and operators our language supports.

We have implemented functions from openssl in our language, to show that our system can be practical. We implemented ssl3_cbc_remove_padding.c. Our comparison shows up in Section 5.

4.1 ssl3_cbc_remove_padding

We implemented the ssl3_cbc_remove_padding.c in our language to show that more implementations are possible. Since our language is not fully fleshed out with all the desired features, we ran into a few problems when replicating this function. The original function utilizes structs — instead, we broke up the structs into individual variables, and any mutable objects that needed to be modified were passed in and modified in a byte array. Since we do not support memory referencing or public labels, we had to hardcode the size of the arrays. Moreover, we do not support unsigned types in our language, so wherever an unsigned was used in the original function, we used an int in our code.

5. EVALUATION

We were not able to evaluate our implementation due to a variety of reasons.

[3]

Moreover, we were not able to perform microbenchmarks on this fucntion because it would not have been fair. We only implemented one function that cannot stand on it's own — in order to truly test this, we would like to implement the functinos needed to encrypt and decrypt, so that we can test the function is it's full form.

6. FUTURE WORK

Our goal for this quarter was to create a few implementations in our language to show that this concept is feasible and work further research. As such, we have a plethora of future work that we would like to explore in the months to folow. The first, which is seemingly intuitive, is that we want to implement more functions in our language, so that it becomes a full fledged library, instead of the test library as it stands now. Moreover, we need to create a syntax for our language — in it's current implementation, we have an AST, which can

transform into IR. Having a corresponding syntax is key to stabilize our language.

Currently, our language has only private labels — we would like to expand it to also utilize public labels. Along these lines, we also would like to expand the notion of our side channel safety — in it's current form, our language prevents timing attacks. We would like to expand it to include memory safety and prevention against memory side channels. This includes fixing our language so that it does not index based on a secret value.

Finally, a big future goal of this project is to determine a method of usability testing for programming languages. A huge impediment in this task is that there is no good way to test programming languages — it is not a simple task that you can assign humans, such as with Amazon Mechanical Turk [1]. One needs to be able to find the key group of users, and be able to test it at a large scale in order to get data about it. This is an open question that we are hoping to address in the following months.

7. CONCLUSION

Conclusion **TK** when rest of paper is done

8. REFERENCES

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APPENDIX

A. SEMANTICS OF CONSTANC

A.1 CONSTCORE Grammar

$\ominus$	::=		unary operations
	|	`~`	bitwise not
$\oplus$	::=		binary operations
	|	`+`	
	|	`-`	
	|	`*`	
	|	`«`	
	|	`»`	
	|	`&`	bitwise and
	|	`|`	bitwise or
	|	$==_s$	equals (sign extended)
	|	$!=_s$	
	|	$>_s$	
	|	$<_s$	
	|	$>=_s$	
	|	$<=_s$	
e	::=		expressions
	|	TRUE	bitmask true (`0b1111...`)
	|	FALSE	bitmask false (`0b0000...`)
	|	c	numeric value
	|	a	bytearray
	|	x	variable
	|	$a[e]$	array access
	|	$\ominus e$	unary operation
	|	$e_1 \oplus e_2$	binary operation
	|	$f(e_1,..,e_n)$	function application
s	::=		statements
	|	**skip**	skip
	|	$s_1;s_2$	sequence
	|	**def** $x := e$	variable declaration
	|	**adef** $x := a$	array declaration
	|	$x := e$	variable assignment
	|	$a[e_1] := e_2$	array assignment
	|	**for** x **from** v_1 **to** $v_2 : s$	for loop
fval	::=		function spec
	|	$(x_1,..,x_n) : s$ `@` e	
fndef	::=		function definition
	|	**fdef** f *fval*	
program	::=		program
	|	$fndef_1;..;fndef_n;$ **expose** *fndef*	list of fdefs

A.2 CONSTCORE Small-Step Semantics

Λ	::=		function store
	|	$\emptyset_\Lambda$	empty function store
	|	$\Lambda[f \mapsto fval]$	define function
Γ	::=		global memory
	|	$\emptyset_\Gamma$	
	|	$\Gamma[a \mapsto [\,]]$	new array
	|	$\Gamma[a \mapsto \Gamma(a)[v_1 \mapsto v_2]]$	array update
μ	::=		local memory
	|	$\emptyset_\mu$	empty memory
	|	$\mu[x \mapsto v]$	add/update variable
	|	$\mu_1 \triangleright \mu_2$	push stack frame
$\{\sigma\}$	::=		variable substitution
	|	$\{x_1/v_1, .., x_k/v_k\}$	

$\boxed{\{\Lambda,\Gamma,\mu,\kappa\}\, e \longrightarrow \{\Lambda',\Gamma',\mu',\kappa'\}\, e'}$ *(e reduces to e′)*

EXR-VAR
$$\frac{\mu = \mu'[x \mapsto v] \qquad \kappa' = \kappa + 1}{\{\Lambda,\Gamma,\mu,\kappa\}\, x \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, v}$$

EXR-ARR-GET-EXPR
$$\frac{\{\Lambda,\Gamma,\mu,\kappa\}\, e \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, e'}{\{\Lambda,\Gamma,\mu,\kappa\}\, a[e] \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, a[e']}$$

EXR-ARR-GET-VAL
$$\frac{v' = \Gamma(a)[v] \qquad \kappa' = \kappa + 1}{\{\Lambda,\Gamma,\mu,\kappa\}\, a[v] \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, v'}$$

EXR-UNOP-EXPR
$$\frac{\{\Lambda,\Gamma,\mu,\kappa\}\, e \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, e'}{\{\Lambda,\Gamma,\mu,\kappa\}\, \ominus e \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, \ominus e'}$$

EXR-UNOP-VAL
$$\frac{v' \equiv [\![\ominus v]\!] \qquad \kappa' = \kappa + 1}{\{\Lambda,\Gamma,\mu,\kappa\}\, \ominus v \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, v'}$$

EXR-BINOP-L
$$\frac{\{\Lambda,\Gamma,\mu,\kappa\}\, e_1 \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, e_1'}{\{\Lambda,\Gamma,\mu,\kappa\}\, e_1 \oplus e_2 \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, e_1' \oplus e_2}$$

EXR-BINOP-R
$$\frac{\{\Lambda,\Gamma,\mu,\kappa\}\, e_2 \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, e_2'}{\{\Lambda,\Gamma,\mu,\kappa\}\, v \oplus e_2 \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, v \oplus e_2'}$$

EXR-BINOP-VAL
$$\frac{v_3 \equiv [\![v_1 \oplus v_2]\!] \qquad \kappa' = \kappa + 1}{\{\Lambda,\Gamma,\mu,\kappa\}\, v_1 \oplus v_2 \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, v_3}$$

EXR-SUBST-EMPTY
$$\frac{}{\{\Lambda,\Gamma,\mu,\kappa\}\, \{\,\}\, e \longrightarrow \{\Lambda,\Gamma,\mu,\kappa\}\, e}$$

EXR-SUBST-EXPR
$$\frac{\{\Lambda,\Gamma,\mu,\kappa\}\, e \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, e'}{\{\Lambda,\Gamma,\mu,\kappa\}\, \{\sigma\}\, e \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, \{\sigma\}\, e'}$$

EXR-SUBST-VAR
$$\frac{\kappa' = \kappa + 1}{\{\Lambda,\Gamma,\mu,\kappa\}\, \{x_1/v_1, \ldots, x_k/v_k\}\, x_i \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, v_i}$$

EXR-SUBST-NO-VAR
$$\frac{}{\{\Lambda,\Gamma,\mu,\kappa\}\, \{\sigma\}\, x \longrightarrow \{\Lambda,\Gamma,\mu,\kappa\}\, x}$$

EXR-SUBST-VAL
$$\frac{}{\{\Lambda,\Gamma,\mu,\kappa\}\, \{\sigma\}\, v \longrightarrow \{\Lambda,\Gamma,\mu,\kappa\}\, v}$$

EXR-FN-EXPR
$$\frac{\{\Lambda,\Gamma,\mu,\kappa\}\, e_1 \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, e_1'}{\{\Lambda,\Gamma,\mu,\kappa\} f(v_1, .., v_k, e_1, e_2, .., e_n) \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\} f(v_1, .., v_k, e_1', e_2, .., e_n)}$$

EXR-FN-CALL
$$\frac{\begin{array}{c}\Lambda = \Lambda'[f \mapsto (x_1, .., x_k) : s \;@e] \\ \mu' = \mu \triangleright \emptyset_\mu \qquad \kappa' = \kappa + 1\end{array}}{\{\Lambda,\Gamma,\mu,\kappa\} f(v_1, .., v_k) \longrightarrow \{\Lambda,\Gamma,\mu',\kappa'\}\, \{x_1/v_1, .., x_k/v_k\}\, s \;@e}$$

EXR-SKIP-EXPR
$$\frac{\{\Lambda,\Gamma,\mu,\kappa\}\, \{\sigma\}\, e \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\, e'}{\{\Lambda,\Gamma,\mu,\kappa\}\, \{\sigma\}\, \textbf{skip} \;@e \longrightarrow \{\Lambda,\Gamma,\mu,\kappa\}\, \{\sigma\}\, \textbf{skip} \;@e'}$$

EXR-SKIP-VAL
$$\frac{\mu = \mu_1 \triangleright \mu_2}{\{\Lambda,\Gamma,\mu,\kappa\}\, \{\sigma\}\, \textbf{skip} \;@v \longrightarrow \{\Lambda,\Gamma,\mu_1,\kappa\}\, v}$$

EXR-SEQ
$$\frac{\{\Lambda,\Gamma,\mu,\kappa\}\, \{\sigma\}\, s_1 \;@e_0 \longrightarrow \{\Lambda,\Gamma,\mu',\kappa'\}\, \{\sigma'\}\, s_1' \;@e_0}{\{\Lambda,\Gamma,\mu,\kappa\}\, \{\sigma\}\, s_1; s_2 \;@e_0 \longrightarrow \{\Lambda,\Gamma,\mu',\kappa'\}\, \{\sigma'\}\, s_1'; s_2 \;@e_0}$$

EXR-SEQ-SKIP

$$\frac{}{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,\mathbf{skip};s\ @e_0 \longrightarrow \{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,s\ @e_0}$$

EXR-DEF-EXPR

$$\frac{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,e \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\,e'}{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,\mathbf{def}\,x := e\ @e_0 \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\,\{\sigma\}\,\mathbf{def}\,x := e'\ @e_0}$$

EXR-DEF-VAL

$$\frac{\mu' = \mu[x \mapsto v] \qquad \kappa' = \kappa + 1}{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,\mathbf{def}\,x := v\ @e_0 \longrightarrow \{\Lambda,\Gamma,\mu',\kappa'\}\,\{\sigma\}\,\mathbf{skip}\ @e_0}$$

EXR-DEF-ARR

$$\frac{\begin{array}{c}\Gamma' = \Gamma[a \mapsto [\,]] \\ \mu' = \mu[x \mapsto a] \qquad \kappa' = \kappa + 1\end{array}}{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,\mathbf{adef}\,x := a\ @e_0 \longrightarrow \{\Lambda,\Gamma',\mu',\kappa'\}\,\{\sigma\}\,\mathbf{skip}\ @e_0}$$

EXR-ASSIGN-EXPR

$$\frac{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,e \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\,e'}{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,x := e\ @e_0 \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\,\{\sigma\}\,x := e'\ @e_0}$$

EXR-ASSIGN-VAL

$$\frac{\mu' = \mu[x \mapsto v] \qquad \kappa' = \kappa + 1}{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,x := v\ @e_0 \longrightarrow \{\Lambda,\Gamma,\mu',\kappa'\}\,\{\sigma\}\,\mathbf{skip}\ @e_0}$$

EXR-ARR-ASSIGN-EXPR-L

$$\frac{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,e_1 \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\,e_1'}{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,a[e_1] := e_2\ @e_0 \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\,\{\sigma\}\,a[e_1'] := e_2\ @e_0}$$

EXR-ARR-ASSIGN-EXPR-R

$$\frac{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,e_2 \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\,e_2'}{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,a[v_1] := e_2\ @e_0 \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\,\{\sigma\}\,a[v_1] := e_2'\ @e_0}$$

EXR-ARR-ASSIGN-VAL

$$\frac{\begin{array}{c}\Gamma' = \Gamma[a \mapsto \Gamma(a)[v_1 \mapsto v_2]] \\ \kappa' = \kappa + 1\end{array}}{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,a[v_1] := v_2\ @e_0 \longrightarrow \{\Lambda,\Gamma',\mu,\kappa'\}\,\{\sigma\}\,\mathbf{skip}\ @e_0}$$

EXR-FOR

$$\frac{\begin{array}{c}v_1 < v_2 \\ v_1' = v_1 + 1 \qquad \kappa' = \kappa + 1\end{array}}{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,\mathbf{for}\,x\,\mathbf{from}\,v_1\,\mathbf{to}\,v_2 : s\ @e_0 \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\,\{\sigma\}\,(\{x/v_1\}\,s);\mathbf{for}\,x\,\mathbf{from}\,v_1'\,\mathbf{to}\,v_2 : s\ @e_0}$$

EXR-ADD-SUBST

$$\frac{\begin{array}{c}\{\sigma_1\} \cap \{\sigma_2\} = \{\,\} \\ \{\sigma_3\} = \{\sigma_1\} \cup \{\sigma_2\}\end{array}}{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma_1\}\,(\{\sigma_2\}\,s)\ @e_0 \longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\,\{\sigma_3\}\,s\ @e_0}$$

EXR-FOR-BASE

$$\frac{v_1 = v_2}{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,\mathbf{for}\,x\,\mathbf{from}\,v_2\,\mathbf{to}\,v_2 : s\ @e_0 \longrightarrow \{\Lambda,\Gamma,\mu',\kappa\}\,\{\sigma\}\,\mathbf{skip}\ @e_0}$$

A.3 CONSTANC Grammar

$\ominus_h$	::=		unary operations
	|	`!`	logical not
	|	`~`	bitwise not
$\oplus_h$	::=		binary operations
	|	`+`	
	|	`-`	
	|	`*`	
	|	`«`	
	|	`»`	
	|	`&`	
	|	`|`	
	|	`&&`	
	|	`||`	
	|	`==`	
	|	`!=`	
	|	`>`	
	|	`<`	
	|	`>=`	
	|	`<=`	
e_h	::=		expressions
	|	b_h	boolean value
	|	c_h	numeric value
	|	a	bytearray
	|	x	variable
	|	$a[e_h]$	array access
	|	$\ominus_h e_h$	unary operation
	|	$e_{h1} \oplus e_{h2}$	binary operation
	|	$f_h(e_{h1}, .., e_{hn})$	function application
s_h	::=		statements
	|	$\textbf{skip}_h$	skip
	|	$s_{h1}; s_{h2}$	sequence
	|	$\textbf{def}_h\ x := e_h$	variable declaration
	|	$\textbf{adef}_h\ x := a$	array declaration
	|	$x := e_h$	variable assignment
	|	$a[e_{h1}] := e_{h2}$	array assignment
	|	$\textbf{for}_h\ x\ \textbf{from}\ v_{h1}\ \textbf{to}\ v_{h2}$	for loop
	|	$\textbf{if}_h\ e_h\ \textbf{then}\ s_{h1}\ \textbf{else}\ s_{h2}$	conditional branch
	|	$\textbf{return}_h\ e_h$	return
hfval	::=		function spec
	|	$(x_1, .., x_n) : s_h$	
hfndef	::=		function definition
	|	$\textbf{fdef}_h\ f_h\ hfval$	
hprogram	::=		program
	|	$hfndef_1; ..; hfndef_n; \textbf{expose}\, hfndef$	list of fdefs

A.4 Transformations from CONSTANC to CONSTCORE

$\boxed{[\![\ominus_h]\!]_t = \ominus}$ *($\ominus_h$ is transformed to $\ominus$)*

UNOPT-LNOT
$$\overline{[\![\texttt{!}]\!]_t = \texttt{~}}$$

UNOPT-UNOP
$$\overline{[\![\ominus_h]\!]_t = \ominus}$$

$\boxed{[\![\oplus_h]\!]_t = \oplus}$ *($\oplus_h$ is transformed to $\oplus$)*

BINOPT-LAND
$$\overline{[\![\texttt{\&\&}]\!]_t = \texttt{\&}}$$

BINOPT-LOR
$$\overline{[\![\texttt{||}]\!]_t = \texttt{|}}$$

BINOPT-EQ
$$\overline{[\![\texttt{==}]\!]_t = \texttt{==}_\texttt{s}}$$

BINOPT-NEQ
$$\overline{[\![\texttt{!=}]\!]_t = \texttt{!=}_\texttt{s}}$$

BINOPT-GT
$$\overline{[\![\texttt{>}]\!]_t = \texttt{>}_\texttt{s}}$$

BINOPT-LT
$$\overline{[\![\texttt{<}]\!]_t = \texttt{<}_\texttt{s}}$$

BINOPT-GTE
$$\overline{[\![\texttt{>=}]\!]_t = \texttt{>=}_\texttt{s}}$$

BINOPT-LTE
$$\overline{[\![\texttt{<=}]\!]_t = \texttt{<=}_\texttt{s}}$$

BINOPT-BINOP
$$\overline{[\![\oplus_h]\!]_t = \oplus}$$

$\boxed{[\![e_h]\!]_t = e}$ *(e_h is transformed to e)*

EXT-VAL
$$\frac{v \equiv [\![v_h]\!]_{int}}{[\![v_h]\!]_t = v}$$

EXT-VAR
$$\overline{[\![x]\!]_t = x}$$

EXT-ARR
$$\overline{[\![a]\!]_t = a}$$

EXT-ARR-GET
$$\frac{[\![e_h]\!]_t = e}{[\![a[e_h]]\!]_t = a[e]}$$

EXT-FN-CALL
$$\frac{[\![\mathbf{fdef}_h\ f_h\ hfval]\!]_t = \mathbf{fdef} f\,fval \quad [\![e_{h1}]\!]_t = e_1 \quad .. \quad [\![e_{hk}]\!]_t = e_k}{[\![f_h(e_{h1}, .., e_{hk})]\!]_t = f(e_1, .., e_k)}$$

$\boxed{[\![s_h]\!]_{ctx} = s}$ *(s_h is transformed to s)*

STT-SKIP
$$\overline{[\![\mathbf{skip}_h]\!]_{ctx} = \mathbf{skip}}$$

STT-SEQ
$$\frac{[\![s_{h1}]\!]_{ctx} = s_1 \qquad [\![s_{h2}]\!]_{ctx} = s_2}{[\![s_{h1}; s_{h2}]\!]_{ctx} = s_1; s_2}$$

STT-VAR-DEC
$$\frac{[\![e_h]\!]_t = e}{[\![\mathbf{def}_h\ x := e_h]\!]_{ctx} = \mathbf{def}\, x := e}$$

STT-ARR-DEC
$$\overline{[\![\mathbf{adef}_h\ x := a]\!]_{ctx} = \mathbf{adef}\, x := a}$$

STT-VAR-ASSIGN
$$\frac{[\![e_h]\!]_t = e \qquad e' = ctx\,\&\,rnset \qquad e'' = e\,\&\,e' \qquad e''' = x\,\&\,(\texttt{~}e')}{[\![x := e_h]\!]_{ctx} = x := (e'' \mid e''')}$$

STT-ARR-ASSIGN
$$\frac{[\![e_{h1}]\!]_t = e_1 \qquad [\![e_{h2}]\!]_t = e_2 \qquad e' = ctx\,\&\,rnset \qquad e'' = e_2\,\&\,e' \qquad e''' = a[e_1]\,\&\,(\texttt{~}e')}{[\![a[e_{h1}] := e_{h2}]\!]_{ctx} = a[e_1] := (e'' \mid e''')}$$

STT-FOR
$$\frac{[\![v_{h1}]\!]_t = v_1 \qquad [\![v_{h2}]\!]_t = v_2 \qquad [\![s_h]\!]_{ctx} = s}{[\![\mathbf{for}_h\ x\ \mathbf{from}\ v_{h1}\ \mathbf{to}\ v_{h2}]\!]_{ctx} = \mathbf{for}\, x\, \mathbf{from}\, v_1\, \mathbf{to}\, v_2 : s}$$

STT-IF
$$\frac{[\![e_h]\!]_t = e \qquad [\![s_{h1}]\!]_{(ctx'\,\&\,ctx)} = s_1 \qquad [\![s_{h2}]\!]_{(ctx'\,\&\,ctx)} = s_2}{[\![\mathbf{if}_h\ e_h\ \mathbf{then}\ s_{h1}\ \mathbf{else}\ s_{h2}]\!]_{ctx} = \mathbf{def}\, ctx' := e; s_1; ctx' := (\texttt{~}ctx'); s_2}$$

STT-RET
$$\frac{[\![e_h]\!]_t = e \qquad e' = ctx\,\&\,rnset \qquad e'' = e\,\&\,e'}{[\![\mathbf{return}_h\ e_h]\!]_{ctx} = rval := (e'' \mid rval); rnset := (rnset\,\&\,(\texttt{~}ctx))}$$

$\boxed{[\![hfndef]\!]_t = fndef}$ *(hfndef is transformed to fndef)*

FDEFT-FDEF
$$\frac{[\![s_h]\!]_{\text{TRUE}} = s}{[\![\mathbf{fdef}_h\ f_h\ (x_1, .., x_k) : s_h]\!]_t = \mathbf{fdef} f\,(x_1, .., x_k) : \mathbf{def}\, rval := \text{FALSE}; \mathbf{def}\, rnset := \text{TRUE}; s\ @rval}$$