- c numeric value
- a array
- x term variable
- f function name
- n index variable
- k index variable
- i index variable
- b_h boolean value
- c_h numeric value
- f_h function name rval "return value" variable
- rnset "rval-not-set" variable

```
values
v
                       True
                                                                        bitmask true (0b1111...)
                       False
                                                                        bitmask false (0b0000...)
                                                                        numeric value
                        c
                                                                        bytearray
                                                                     unary operations
\ominus
                                                                        bitwise not
                                                                     binary operations
\oplus
                       >>
                                                                        bitwise and
                                                                        bitwise or
                                                                        equals (sign extended)
\{\sigma\}
                                                                     variable substitution
                ::=
                       \{x_1/v_1, \ldots, x_k/v_k\}
fval
                ::=
                                                                     function spec
                        (x_1, \ldots, x_n) : s @ e
                                                                     function definition
fndef
                ::=
                       \mathbf{fdef}\,f\,fval
program
                                                                     program
                       fndef_1; ...; fndef_n; expose fndef
                                                                        list of fdefs
Λ
                                                                     function store
                                                                        empty function store
                       \emptyset_{\Lambda}
                       \Lambda[f \mapsto fval]
                                                                        define function
Γ
                ::=
                                                                     global memory
                       \emptyset_{\Gamma}
                       \Gamma[a \mapsto \text{\tt []]}
                                                                        new array
                       \Gamma[a \mapsto \Gamma(a)[v_1 \mapsto v_2]]
                                                                        array update
                                                                     local memory
\mu
                       \emptyset_{\mu}
                                                                        empty memory
```

```
\mu[x \mapsto v]
                                                        add/update variable
                                                        push stack frame
                 \mu_1 \triangleright \mu_2
                                                     program transcript
\kappa
          ::=
                 \emptyset_{\kappa}
                                                        empty transcript
                 \kappa \triangleright \ominus
                                                        add \ominus to \ transcript
                                                        add \oplus to transcript
                  \kappa \triangleright \oplus
                 \kappa \triangleright \mathbf{load}
                                                        add memory load to transcript
                 \kappa \triangleright \mathbf{store}
                                                        add memory store to transcript
                                                        add call to f to transcript
                 \kappa \triangleright f
                                                        add function return to transcript
                 \kappa \triangleright \mathbf{ret}
                                                     expressions
                 TRUE
                                                        bitmask true (0b1111...)
                                                        bitmask false (0b0000...)
                 False
                  c
                                                        numeric value
                  a
                                                        bytearray
                                                        variable
                 \boldsymbol{x}
                  a[e]
                                                        array access
                                                        unary operation
                 \ominus e
                                                        binary operation
                  e_1 \oplus e_2
                 f(e_1,\ldots,e_n)
                                                        function application
                                                     statements
s
                 skip
                                                        skip
                                                        sequence
                 s_1; s_2
                 \operatorname{\mathbf{def}} x := e
                                                        variable declaration
                 \mathbf{adef}\ x := a
                                                        array declaration
                 x := e
                                                        variable assignment
                  a[e_1] := e_2
                                                        array assignment
                 for x from v_1 to v_2: s
                                                        for loop
                                                     values
v_h
                                                        boolean value
                 b_h
                                                        numeric value
                 c_h
                  a
                                                        bytearray
                                                     unary operations
\ominus_h
          ::=
                  ļ
                                                        logical not
                                                        bitwise not
                                                     binary operations
\oplus_h
```

```
&
                             &&
                             | |
                             !=
                             <
                             >=
                             <=
                                                                                     expressions
e_h
                                                                                        boolean value
                             b_h
                                                                                        numeric value
                             c_h
                                                                                        bytearray
                             a
                             \boldsymbol{x}
                                                                                        variable
                             a[e_h]
                                                                                        array access
                                                                                        unary operation
                             \ominus_h e_h
                                                                                        binary operation
                             e_{h1} \oplus e_{h2}
                             f_h(e_{h1},\ldots,e_{hn})
                                                                                        function application
                                                                                    statements
s_h
                            \mathbf{skip}_h
                                                                                        skip
                             s_{h1}; s_{h2}
                                                                                        sequence
                             \mathbf{def}_h \ x := e_h
                                                                                        variable declaration
                             \mathbf{adef}_h \ x := a
                                                                                        array declaration
                             x := e_h
                                                                                        variable assignment
                             a[e_{h1}] := e_{h2}
                                                                                        array assignment
                            for x from v_{h1} to v_{h2}
                                                                                        for loop
                                                                                        conditional branch
                            if<sub>h</sub> e_h then s_{h1} else s_{h2}
                                                                                        return
                             \mathbf{return}_h \ e_h
hfval
                    ::=
                                                                                     function spec
                            (x_1,\ldots,x_n):s_h
hfndef
                    ::=
                                                                                     function definition
                            \mathbf{fdef}_h \ f_h \ hfval
hprogram
                                                                                     program
                             hfndef_1; ...; hfndef_n; expose hfndef
                                                                                        list of fdefs
ctx
                                                                                     branch context
                                                                                        variable
                            \ominus ctx
                                                                                        unary operation
                             ctx_1 \oplus ctx_2
                                                                                        binary operation
\{\Lambda, \Gamma, \mu, \kappa\} \ e \longrightarrow \{\Lambda', \Gamma', \mu', \kappa'\} \ e'
                                                  e reduces to e'
                                                  \mu = \mu'[x \mapsto v]
                                                 \kappa' = \kappa \triangleright \mathbf{load}
                                                                                      Exr_var
                                      \overline{\{\Lambda,\Gamma,\mu,\kappa\}\ x\longrightarrow \{\Lambda,\Gamma,\mu,\kappa'\}\ v}
```

$$\frac{\langle \Lambda.\Gamma,\mu,\kappa\rangle}{\{\Lambda.\Gamma,\mu,\kappa\}}\frac{\sigma}{\sigma}|e| \to \langle \Lambda.\Gamma,\mu,\kappa'\rangle\frac{\sigma'}{\sigma}|e|} = \text{Exr_arr.get_expr}$$

$$\frac{\langle \Gamma(\alpha)|v|}{\kappa' = \kappa \circ \text{load}}$$

$$\frac{\langle \Lambda.\Gamma,\mu,\kappa\rangle}{\{\Lambda.\Gamma,\mu,\kappa\}}\frac{\sigma}{\sigma}| \to \langle \Lambda.\Gamma,\mu,\kappa'\rangle\frac{\sigma'}{\sigma'} = \text{Exr_arr.get_expr}$$

$$\frac{\langle \Lambda.\Gamma,\mu,\kappa\rangle}{\{\Lambda.\Gamma,\mu,\kappa\}}\frac{\sigma}{\sigma} \to \langle \Lambda.\Gamma,\mu,\kappa'\rangle\frac{\sigma'}{\sigma'} = \text{Exr_unop_expr}$$

$$\frac{\langle \Lambda.\Gamma,\mu,\kappa\rangle}{\{\Lambda.\Gamma,\mu,\kappa\}}\frac{\sigma}{\sigma} \to \langle \Lambda.\Gamma,\mu,\kappa'\rangle\frac{\sigma'}{\sigma'} = \text{Exr_unop_expr}$$

$$\frac{\kappa' = |\Phi|}{\{\Lambda.\Gamma,\mu,\kappa\}}\frac{\sigma}{\sigma} \to \langle \Lambda.\Gamma,\mu,\kappa'\rangle\frac{\sigma'}{\sigma'} = \text{Exr_unop_expr}$$

$$\frac{\langle \Lambda.\Gamma,\mu,\kappa\rangle}{\{\Lambda.\Gamma,\mu,\kappa\}}\frac{\sigma}{\sigma} \to \langle \Lambda.\Gamma,\mu,\kappa'\rangle\frac{\sigma'}{\sigma'} \to \sigma_{\sigma} = \text{Exr_unop_expr}$$

$$\frac{\langle \Lambda.\Gamma,\mu,\kappa\rangle}{\{\Lambda.\Gamma,\mu,\kappa\}}\frac{\sigma}{\sigma} \to \langle \Lambda.\Gamma,\mu,\kappa'\rangle\frac{\sigma'}{\sigma'} \to \sigma_{\sigma} = \text{Exr_unop_expr}$$

$$\frac{\langle \Lambda.\Gamma,\mu,\kappa\rangle}{\{\Lambda.\Gamma,\mu,\kappa\}}\frac{\sigma}{\sigma} \to \langle \Lambda.\Gamma,\mu,\kappa'\rangle\frac{\sigma'}{\sigma} \to \sigma_{\sigma} = \text{Exr_unop_expr}$$

$$\frac{\sigma}{\langle \Lambda.\Gamma,\mu,\kappa\rangle}\frac{\sigma}{\sigma} \to \langle \Lambda.\Gamma,\mu,\kappa'\rangle\frac{\sigma'}{\sigma} \to \sigma_{\sigma} = \text{Exr_unop_expr}$$

$$\frac{\sigma}{\langle \Lambda.\Gamma,\mu,\kappa\rangle}\frac{\sigma}{\sigma} \to \langle \Lambda.\Gamma,\mu,\kappa'\rangle\frac{\sigma'}{\sigma} \to \sigma_{\sigma} = \text{Exr_unop_exp}$$

$$\frac{\sigma}{\langle \Lambda.\Gamma,\mu,\kappa\rangle}\frac{\sigma}{\sigma} \to \langle \Lambda.\Gamma,\mu,\kappa'\rangle\frac{\sigma'}{\sigma} \to \text{Exr_unop_exp}$$

$$\frac{\sigma}{\langle \Lambda.\Gamma,\mu,\kappa\rangle}\frac{\sigma}{\sigma} \to \langle \Lambda.\Gamma,\mu,\kappa\rangle^{\sigma} \to \text{Exr_unop_exp}$$

$$\frac{\sigma}{\langle \Lambda.\Gamma,\mu,\kappa\rangle}\frac{\sigma}{\sigma} \to \text{Exr_unop_exp}$$

$$\frac{\sigma}{\langle \Lambda.\Gamma,\mu,\kappa\rangle}\frac{\sigma}{\sigma} \to \langle \Lambda.\Gamma,\mu,\kappa\rangle^{\sigma} \to \text{Exr_unop_exp}$$

$$\frac{\sigma}{\langle \Lambda.\Gamma,\mu,\kappa\rangle}\frac{\sigma}{\sigma} \to \text{Exr_unop_exp}$$

$$\frac{\sigma}{\langle \Lambda.\Gamma,\mu,\kappa\rangle}\frac{\sigma}{\sigma} \to \langle \Lambda.\Gamma,\mu,\kappa\rangle^{\sigma} \to \text{Exr_unop_exp}$$

$$\frac{\sigma}{\langle \Lambda.\Gamma,\mu,\kappa\rangle}\frac{\sigma}{\sigma} \to \text{Exr_unop_exp}$$

$$\frac{\sigma}{\langle \Lambda.\Gamma,\mu,\kappa\rangle}\frac{\sigma}{\sigma} \to \langle \Lambda.\Gamma,\mu,\kappa\rangle^{\sigma} \to \text{Exr_unop_exp}$$

$$\frac{\sigma}{\langle \Lambda.\Gamma,\mu,\kappa\rangle}\frac{\sigma}{\sigma} \to \text{$$

```
\overline{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,\mathbf{skip};s\,\,@e_0\longrightarrow\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,s\,\,@e_0}\quad \text{Exr\_seq\_skip}
                   \frac{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \operatorname{\mathbf{def}} x := e @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} \operatorname{\mathbf{def}} x := e' @e_0}
                                                                                                                                                                                                           EXR_DEF_EXPR
                                                                                         \mu' = \mu[x \mapsto v]
                                                                                         \kappa' = \kappa \triangleright \mathbf{store}
                            \overline{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,\mathbf{def}\,x:=v\,\,@e_0\longrightarrow\{\Lambda,\Gamma,\mu',\kappa'\}\,\{\sigma\}\,\mathbf{skip}\,\,@e_0}
                                                                                                                                                                                                     EXR_DEF_VAL
                                                                                         \Gamma' = \Gamma[a \mapsto []]
                                                                                         \mu' = \mu[x \mapsto a]
                                                                                        \kappa' = \kappa \triangleright \mathbf{store}
                                                                                                                                                                                                        EXR_DEF_ARR
                         \overline{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \text{ adef } x := a @ e_0 \longrightarrow \{\Lambda, \Gamma', \mu', \kappa'\} \{\sigma\} \text{ skip } @ e_0}
                         \frac{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} e \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e'}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} x := e @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} x := e' @e_0}
                                                                                                                                                                                            EXR_ASSIGN_EXPR
                                                                                    \mu' = \mu[x \mapsto v]
                             \frac{\kappa' = \kappa \triangleright \mathbf{store}}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \, x := v \, @e_0 \longrightarrow \{\Lambda, \Gamma, \mu', \kappa'\} \{\sigma\} \, \mathbf{skip} \, @e_0}
                                                                                                                                                                                             EXR_ASSIGN_VAL
     \frac{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,e_1\longrightarrow\{\Lambda,\Gamma,\mu,\kappa'\}\,e_1'}{\{\Lambda,\Gamma,\mu,\kappa\}\,\{\sigma\}\,a[e_1]:=e_2\,\,@e_0\longrightarrow\{\Lambda,\Gamma,\mu,\kappa'\}\,\{\sigma\}\,a[e_1']:=e_2\,\,@e_0}
                                                                                                                                                                                             EXR_ARR_ASSIGN_EXPR_L
     \frac{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} e_2 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} e_2'}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} a[v_1] := e_2 @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} a[v_1] := e_2' @e_0}
                                                                                                                                                                                             EXR_ARR_ASSIGN_EXPR_R
                                                              \Gamma' = \Gamma[a \mapsto \Gamma(a)[v_1 \mapsto v_2]]
                                                              \kappa' = \kappa \triangleright \mathbf{store}
                 \frac{\kappa' = \kappa \triangleright \mathbf{store}}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \ a[v_1] := v_2 \ @e_0 \longrightarrow \{\Lambda, \Gamma', \mu, \kappa'\} \{\sigma\} \mathbf{skip} \ @e_0}
                                                                                                                                                                                          EXR_ARR_ASSIGN_VAL
                                                                                                               v_1 < v_2

v_1' = v_1 + 1

\kappa' = \kappa \triangleright \mathbf{store}
                                                                                                                                                                                                                                                                                EXR_FOR
\overline{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \text{ for } x \text{ from } v_1 \text{ to } v_2 : s @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma\} (\{x/v_1\} s); \text{ for } x \text{ from } v'_1 \text{ to } v_2 : s @e_0\}}
                                                                               \{\sigma_1\} \cap \{\sigma_2\} = \{\}
                              \frac{\{\sigma_3\} = \{\sigma_1\} \cup \{\sigma_2\}}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma_1\} (\{\sigma_2\} s) @e_0 \longrightarrow \{\Lambda, \Gamma, \mu, \kappa'\} \{\sigma_3\} s @e_0} \quad \text{EXR\_ADD\_SUBST}
            \frac{v_1 = v_2}{\{\Lambda, \Gamma, \mu, \kappa\} \{\sigma\} \text{ for } x \text{ from } v_2 \text{ to } v_2 : s @e_0 \longrightarrow \{\Lambda, \Gamma, \mu', \kappa\} \{\sigma\} \text{ skip } @e_0}
                                                                                                                                                                                                               EXR_FOR_BASE
     [e_h]_t = e | e_h is transformed to e
                                                                                                  \frac{v \equiv \llbracket v_h \rrbracket_{int}}{\llbracket v_h \rrbracket_t = v} \quad \text{EXT_VAL}
                                                                                                        \overline{[\![x]\!]_t = x} \quad \text{Ext_var}
                                                                                                        \overline{[\![a]\!]_t = a} \quad \text{Ext\_ARR}
                                                                                         \frac{\llbracket e_h \rrbracket_t = e}{\llbracket a[e_h] \rrbracket_t = a[e]} \quad \text{Ext\_arr\_get}
```

 $[\![hfndef]\!]_t = fndef$ hfndef is transformed to fndef

 $\llbracket \ominus_h \rrbracket_t = \ominus \middle| \quad \ominus_h \text{ is transformed to } \ominus$

$$\frac{[\![\!]\!]_t = \sim}{[\![\![\!]\!]\!]_t = \sim} \quad \text{Unopt_unop}$$

$$\frac{[\![\![\!]\!]\!]_t = \ominus}{[\![\![\!]\!]\!]_t = \ominus} \quad \text{Unopt_unop}$$

 $\llbracket \oplus_h \rrbracket_t = \oplus$ \oplus_h is transformed to \oplus

Definition rules: 57 good 0 bad Definition rule clauses: 124 good 0 bad