How was the midterm?

- A: Easy
- B: Okay
- C: Hard

Was the midterm fair?

- A: Yes
- B: No

Logistics

- News: HW and PA due Sunday 23:59
- Type Tetris is now extra credit
  - See week4 notes for how to approach problem

Type classes

Problem w/ parametric polymorphism

- Consider the list member function:

```
member x []     = False
member x (y:ys) = if x == y
                   then True
                   else member x ys
```

- Is the type `member :: a -> [a] -> Bool` correct?
  - A: yes, B: no

Problem w/ parametric polymorphism

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Can these work on any type?

- `sort :: [a] -> [a]`
- `(+) :: a -> a -> a`
- `show :: a -> String`
- `serialize :: a -> ByteString`
- `hash :: a -> Int`

No! But we really want to use those same symbols to work on different types

- E.g., `3.4 + 5.5` and `3+5`
- E.g., `show 4` and `show [1,2,3]`
- E.g., `4 == 5` and `Left "w00t" == Right 44`

Motivation for overloading

- Parametric polymorphism doesn’t work…
  - Single algorithm, works on values of any type
  - Type variable may be replaced by any type
- What we want: a form of overloading
  -
  -

Motivation for overloading

- Parametric polymorphism doesn't work…
  - Single algorithm, works on values of any type
  - Type variable may be replaced by any type
- What we want: a form of overloading
  - Single symbol to refer to more than one algorithm
  - Each algorithm may have different type

How should we do overloading?

Non-solution: local choice

- Overload basic operators such as + and $*$

  a * b

- Don't allow for overloading functions defined from them

  - **Problem?**

Non-solution: local choice

- Overload basic operators such as + and *

```
                a `multInt` b
a * b
```

- Don't allow for overloading functions defined from them

  ➤ **Problem?**

Non-solution: local choice

- Overload basic operators such as + and *

```
                a `multInt` b
a * b  <
                a `multFloat` b
```

- Don't allow for overloading functions defined from them
  - **Problem?**

Non-solution: local choice

- Overload basic operators such as + and *

```
a * b   →   a `multInt` b
        →   a `multFloat` b
```

- Don't allow for overloading functions defined from them

  ➤ **Problem?**

first usage tells us that
`square :: Int -> Int`

Non-solution: local choice

- Overload basic operators such as + and *

```
                a `multInt` b
a * b
                a `multFloat` b
```

- Don't allow for overloading functions defined from them
  - **Problem?**

```
square x = x*x
square 3
square 3.14
```

first usage tells us that `square :: Int -> Int`

Non-solution: local choice

- Overload basic operators such as + and *

```
            a `multInt` b
a * b
            a `multFloat` b
```

- Allow for overloading functions defined from them
  - **Problem?**

Non-solution: local choice

- Overload basic operators such as + and *

```
                a `multInt` b
a * b
                a `multFloat` b
```

- Allow for overloading functions defined from them
  - **Problem?**

```
ssquare x y = (square x, square y)
ssquare 3 4
ssquare 3.3 4
...
```

Code blowup!

Non-solution: fully polymorphic

- Make functions like == fully polymorphic
  - `(==) :: a -> a -> Bool`
- At runtime: compare underlying representation
  - `3*3 == 9 => ??`
  - `(\x -> x) == (\x -> x + 1) => ??`
  - `Left 3 == Right “44” => ??`
- **Problems?**

Non-solution: fully polymorphic

- Make functions like == fully polymorphic
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- At runtime: compare underlying representation
  - `3*3 == 9` => ??
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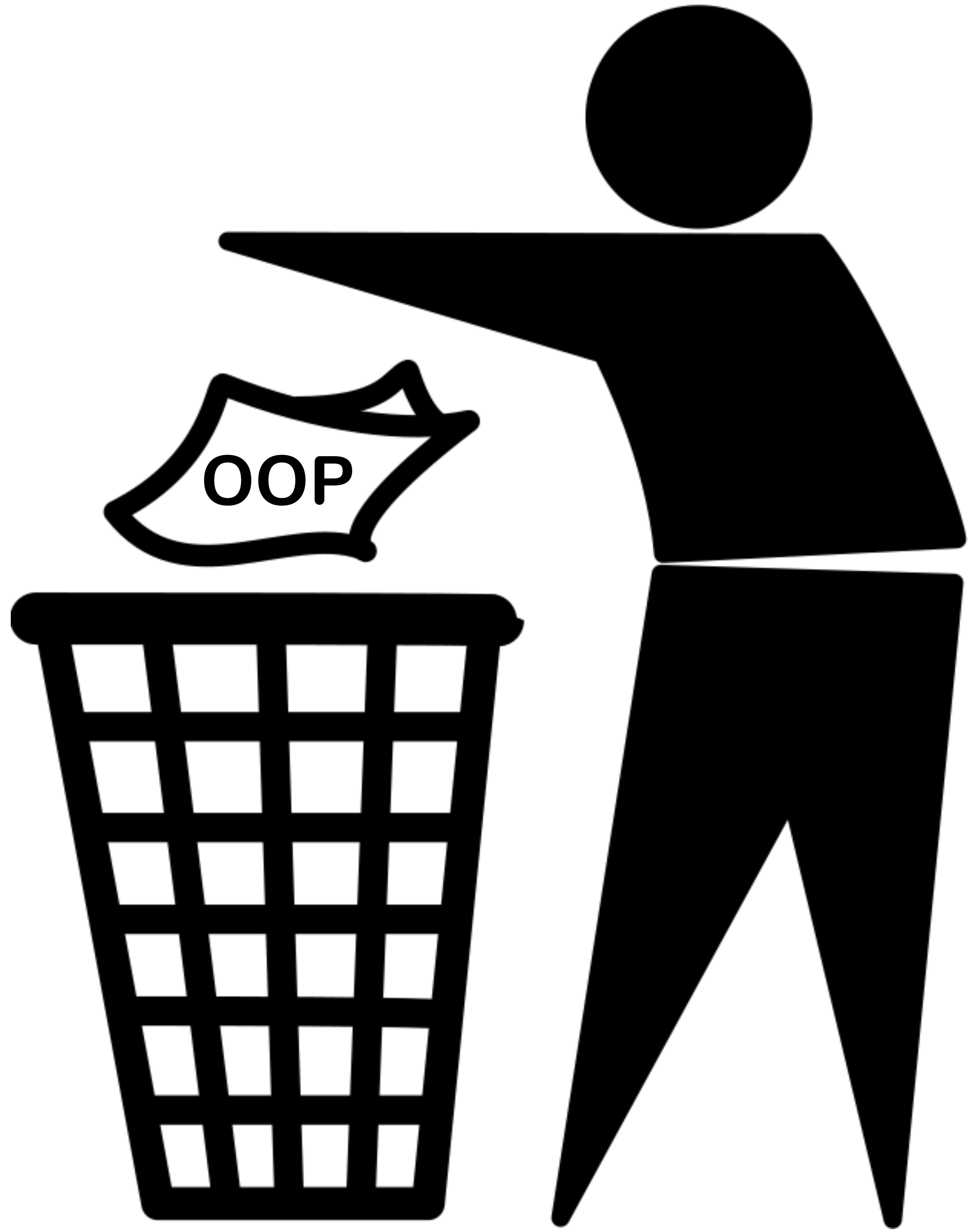
  - `Left 3 == Right “44”` => ??
- **Problems?** Breaks abstraction!

Non-solution: “eqtype” polymorphism [SML]

- Make equality polymorphic in a limited way
  - `(==) :: a== -> a== -> Bool`
  - `member :: a== -> [a==] -> Bool`
- $a_{==}$ are special type variables restricted to types with equality
- **Problem?**

Solution: type classes


OOP

Solution: type classes

- Idea: generalize eqtypes to arbitrary types
- Provide concise types to describe overloaded functions
  - Solves:
- Allow users to define functions using overloaded ones
  - Solves:
- Allow users to declare new collections of overloaded functions

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Solution: type classes

- Idea: generalize eqtypes to arbitrary types
- Provide concise types to describe overloaded functions
  - Solves: exponential blow up
- Allow users to define functions using overloaded ones
  - Solves: monomorphism
- Allow users to declare new collections of overloaded functions

Back to our old examples

- `square ::          a -> a`
- `sort ::          [a] -> [a]`
- `serialize ::          a -> ByteString`
- `member ::          a -> [a] -> Bool`

Back to our old examples

- `square :: Num a => a -> a`
- `sort ::                [a] -> [a]`
- `serialize ::                  a -> ByteString`
- `member ::              a -> [a] -> Bool`

Back to our old examples

- `square :: Num a => a -> a`
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- `square :: Num a => a -> a`
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Back to our old examples

- `square :: Num a => a -> a`
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- `member :: Eq a => a -> [a] -> Bool`

Type classes

- Class declaration: what are the Num operations?

- Instance declaration: how are the ops implemented?

Type classes

- Class declaration: what are the Num operations?

```
class Num a where
  (+) :: a -> a -> a
  (*) :: a -> a -> a
  ...
```

- Instance declaration: how are the ops implemented?

Type classes

- Class declaration: what are the Num operations?

```
class Num a where
  (+) :: a -> a -> a
  (*) :: a -> a -> a
  ...
```

- Instance declaration: how are the ops implemented?

```
instance Num Int where
  (+) a b = plusInt a b
  (*) a b = mulInt a b
  ...
```

```
instance Num Float where
  (+) a b = plus Float a b
  (*) a b = mul Float a b
  ...
```

Type classes

- Basic usage: how do you use the overloaded ops?
  - `3 + 4`
  - `3.3 + 4.4`
  - `“4” + “5”`
- Functions using these ops can be polymorphic too
  - E.g.,

```
square ::
square x = x * x
```

Type classes

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  - E.g.,
    ```
    square :: Num x => x -> x
    square x = x * x
    ```

Type classes can have subclasses

- Example: consider `Eq` and `Ord` classes
  - Eq:
  - Ord:
- Subclass declaration can express relationship:
  - E.g., `class Eq a => Ord a where …`
- When you declare functions you just need to specify `Ord`, we know that it must also be `Eq`

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How do type classes work?

- Basic idea:

```
square :: Num x => x -> x
square x = x * x
```

- Intuition from C's qsort:

```
void qsort(void *base, size_t nel, size_t width,
           int (*compar)(const void *, const void *));
```

  - Pass operator as argument!

How do type classes work?

- Basic idea:

```
square :: Num x => x -> x
square x = x * x
```

```
square :: Num x -> x -> x
square dic x = (*) dic x x
```

- Intuition from C's qsort:

```
void qsort(void *base, size_t nel, size_t width,
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```

  - Pass operator as argument!

type-classes-use.hs

How do type classes work?

- Class declaration: desugar to dictionary type decl

```
class Num a where
  (+) :: a -> a -> a
  (*) :: a -> a -> a
  ...
```

- Instance declaration: desugar to dictionary values

```
instance Num Int where
  (+) a b = plusInt a b
  (*) a b = mulInt a b
  ...
```

How do type classes work?

- Class declaration: desugar to dictionary type decl

```
class Num a where                data Num a = MkNumDict
  (+) :: a -> a -> a                          (a -> a -> a)
  (*) :: a -> a -> a                          (a -> a -> a)
  ...                                         ...
```

- Instance declaration: desugar to dictionary values

```
instance Num Int where
  (+) a b = plusInt a b
  (*) a b = mulInt a b
  ...
```

How do type classes work?

- Class declaration: desugar to dictionary type decl

```
class Num a where                data Num a = MkNumDict
  (+) :: a -> a -> a                           (a -> a -> a)
  (*) :: a -> a -> a                           (a -> a -> a)
  ...                                          ...
```

- Instance declaration: desugar to dictionary values

```
instance Num Int where          dictNumInt = MkNumDict
  (+) a b = plusInt a b                          plusInt
  (*) a b = mulInt a b                           mulInt
  ...
```

How do type classes work?

- Basic usage: whenever you use operator you must pass it a dictionary value:
  - E.g., (*)
  - E.g., (==)
- Defining polymorphic functions: always take dictionary values, so type and definition must reflect this
  - E.g.,
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How do type classes work?

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  - E.g.,
    ```
    square :: Num x -> x -> x
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    ```
  - E.g., `square dictNumFloat 4.4`

type-classes-desugar.hs

How does this affect type inference?

- Type inference infers a qualified type: Q => τ
- τ is ordinary Hindley-Miner type, inferred as usual
- Q is a constraint set/set of type class predicates
- Consider:

```
f ::                    => a -> Bool
f x = x + 2 == 3
```

How does this affect type inference?

- Type inference infers a qualified type: $Q \Rightarrow \tau$
- τ is ordinary Hindley-Miner type, inferred as usual
- Q is a constraint set/set of type class predicates
- Consider:

```
f :: (Eq a, Num a) => a -> Bool
f x = x + 2 == 3
```

Modification to our TI algorithm

- Modify the “Generate constraints” step to include type class constraints
- Simplify constraint set in final step

Generate constraints

- Example: `f x y = x == y`
  - Assign τ_0 to x
  - Assign τ_1 to y
  - Contraints:
    -
    -

Simplify constraints

- Eliminate duplicates:
  - `{Num a, Num a}` =
- Use more general instance declaration
  - `{Eq [a], Eq a}` =
- Use sub-class declaration declaration
  - `{Ord a, Eq a}` =
- Example: `{Eq a, Eq [a], Ord a}` =

Simplify constraints

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  - `{Eq [a], Eq a}` = `{Eq a}` if `instance Eq a => Eq [a]`
- Use sub-class declaration declaration
  - `{Ord a, Eq a}` = `{Ord a}` if `class Eq a => Ord a`
- Example: `{Eq a, Eq [a], Ord a}` = `{Ord a}`

Are these the same as in OO?

```
class String a where
  show :: a -> String
```

$\stackrel{?}{=}$

```
interface Show {
   String show();
}
```

Are these the same as in OO?

```haskell
class String a where
  show :: a -> String
```

$\stackrel{?}{=}$

```java
interface Show {
   String show();
}
```

type-based dispatch vs value-based dispatch

Summary

- Type classes are a good approach to the overloading
- They provide a form of polymorphism: ad-hoc
- More flexible than designers first realized
  - The type-driven, dictionary approach
- Not the same as OO classes/interfaces