Variant 2

(1)

1. Nonlinear optimization problem is given in the following form

$$\min\{f(x) = 10\,{x_1}^2{x_2}^2 + 2\,{x_1}^2{x_3}^2 + {x_2}^4 - 10\,{x_2}^2{x_3}^2 - 3\,{x_3}^4\},$$

subject to

$$h_1(x) = 13\,{x_1}^2 + 8\,x_1x_2 - 2\,x_1x_3 + 1063\,x_1 + 3\,{x_2}^2 + 2\,x_2x_3 + 951\,x_2 + 10\,{x_3}^2 - 232\,x_3 - 120 = 0,$$

$$g_1(x) = 538\,x_1 + 479.5\,x_2 - 118\,x_3 - 58.5 \leqslant 0,$$

$$g_2(x) = 930.75\,x_1 + 755.25\,x_2 - 22.5\,x_3 - 657 \leqslant 0.$$

Check whether $\bar{x} = (1, -1, 0)^\top$, $\tilde{x} = (116.514008, -424.475095, 52.491911)^\top$, $\hat{x} = (4, -4, 2)^\top$ are stationary points. (1.0)

$$(h_1(x))'_{x_1} = 26x_1 + 8x_2 - 2x_3 + 1063$$

$$(h_1(x))'_{x_2} = 8x_1 + 6x_2 + 2x_3 + 951$$

$$(h_1(x))'_{x_3} = -2x_1 + 2x_2 + 20x_3 - 232$$

$$(f(x))'_{x_1} = 20x_1x_2^2 + 4x_1x_3^2$$

$$(f(x))'_{x_2} = 20x_1^2x_2 + 4x_2^3 - 20x_2x_3^2$$

$$(f(x))'_{x_3} = 4x_1^2x_3 - 20x_2^2x_3 - 12x_3^3$$

1) Substitute $\bar{x}$ in ↑ : 1081, 953, -236,

20, -24, 0

Checking stationarity conditions:

Values of the points in constraints:

* h_1, g_1, g_2 respectively

```
True 0 0.0 -481.5
False 9.67132109508384e-06 -147103.8172465 -213977.47055025003
True 0 -60.5 0.0
```

stationarity

$$\begin{cases} 20 + \lambda \cdot 1081 + 538 \cdot \mu_1 + 1930{,}75 \cdot \underset{=0}{\mu_2} = 0 \\ -24 + \lambda \cdot 953 + 479{,}5\mu_1 + 755{,}25 \cdot \underset{=0}{\mu_2} = 0 \\ 0 + \lambda \cdot (-236) - 118\mu_1 - 22{,}5\underset{=0}{\mu_2} = 0 \end{cases}$$

$$\begin{cases} 1081\lambda + 538\mu_1 + 20 = 0 \\ 953\lambda + 479{,}5\mu_1 - 24 = 0 \\ -236\lambda - 118\mu_1 = 0 \end{cases}$$

$\lambda = -4; \ \mu_1 = 8$ so $\begin{cases} \mu_1 \geq 0 \\ \mu_2 \geq 0 \end{cases}$

$\mu_2 = 0$ $\Rightarrow$ $\bar{x}$ is stationary point

2) Substitute $\tilde{x}$ in ↑:

```
591.5796259999998 -558.7546840000002 -264.1399859999999 421151967.72610205 -397783545.7692034 -188044128.50252962
```

$\tilde{x}_1 \quad \tilde{x}_2 \quad \tilde{x}_3 \quad \tilde{x}_4 \quad \tilde{x}_5 \quad \tilde{x}_6$

$$\begin{cases} \tilde{x}_4 + \lambda \tilde{x}_1 = 0 \\ \tilde{x}_5 + \lambda \tilde{x}_2 = 0 \\ \tilde{x}_6 + \lambda \tilde{x}_3 = 0 \end{cases}$$

as $\mu_1 = \mu_2 = 0$ for $\tilde{x}$ — solution doesn't exist

$\Rightarrow \tilde{x}$ is not stationary point

3) Substitute $\hat{x}$ in $\uparrow$:

$1131 \quad 963 \quad -208 \quad 1344 \quad -1216 \quad -608$

Stationarity conditions:

$$\begin{cases} 1344 + 1131\lambda + 930{,}75 \cdot \mu_2 = 0 \\ -1216 + 963\lambda + 755{,}25 \cdot \mu_2 = 0 \\ -608 - 208\lambda - 22{,}5\mu_2 = 0 \end{cases}$$

as $\mu_1 = 0$ for $\hat{x}$

No solution exists

$\Rightarrow \hat{x}$ is not stationary point

③

3. Find a stationary point of function

$$f(x) = \sum_{i=1}^{n-1} \left(x_i^2 - 2\right)^2 + \left(\sum_{i=1}^{n} x_i^2 - 0.5\right)^2$$

for $n = 10$ using Broyden-Fletcher-Goldfarb-Shanno (BFGS) quasi-Newton method with the Wolfe inexact line search. Starting point $x_0 = (1, 1, \ldots, 1)^\top$, convergence tolerance $\varepsilon = 0.001$. Give as the answer the obtained point, the corresponding value of the objective function f, and the number of iterations. (2.0)

Code in GitHub,

Output:

```
Final Result (obtained point): [-5.00001514e-01 -5.00001514e-01 -5.00001514e-01 -5.00001514e-01
 -5.00001514e-01 -5.00001514e-01 -5.00001514e-01 -5.00001514e-01
 -5.00001514e-01 -1.43849011e-06]
Iteration Count: 7
Corresponding value of the objective function f: 30.625000000213618
```

2

2. Find a stationary point of function

$$f(x) = \sum_{j=1}^{\frac{n}{2}} (x_{2j-1}^2 + x_{2j} - 11)^2 + (x_{2j-1}^2 + x_{2j}^2 - 7)^2$$

for $n = 10$ using Polak-Ribière conjugate gradient method with the Goldstein inexact line search. Starting point $x_0 = (1, 1, \ldots, 1)^\top$, convergence tolerance $\varepsilon = 0.001$. Give as the answer the obtained point, the corresponding value of the objective function f, and the number of iterations. (2.0)

Code in Git Hub,

Output:

```
Obtained point: [0.49997437 2.93683935 0.49997437 2.93683935 0.49997437 2.93683935
 0.49997437 2.93683935 0.49997437 2.93683935]
Value of the function: 35.15625001231371
Number of interation: 30
```