- Intro to Quantum Mechanics Homework 2 Question 4: Philip Lucas
- Part A normalization constant for the ground state.
- Defining the wavefunction from Griffiths "Introduction to Quantum Mechanics Second Edition" Page 46

In[26]:=
```
f[x_] = A * E^(-m * ω (x^2) / (2 * ℏ))
```

Out[26]= $A\, e^{-\frac{m x^2 \omega}{2 \hbar}}$

- **Normalizing the wave function requires finding it squared and integrating. That integral must equal one Then solving for the Cofficient A**

In[27]:=
```
Solve[Integrate[f[x]^2, {x, -∞, ∞}, Assumptions → Re[(m * ω) / ℏ] > 0] == 1, A]
```

Out[27]= $\left\{\left\{A \to -\frac{\left(\frac{m\,\omega}{\hbar}\right)^{1/4}}{\pi^{1/4}}\right\}, \left\{A \to \frac{\left(\frac{m\,\omega}{\hbar}\right)^{1/4}}{\pi^{1/4}}\right\}\right\}$

- We know that we just want the positive value because it should be the absolute value of A squared.

In[24]:=
```
Coefficent = ((m ω) / (π * ℏ))^(1 / 4)
```

Out[24]= $\frac{\left(\frac{m\,\omega}{\hbar}\right)^{1/4}}{\pi^{1/4}}$

- This matches the Coefficient found in the Griffiths "Introduction to Quantum Mechanics Second Edition" Textbook pg 46 Eq (2.59) solution.

In[28]:=
```
Φ[x_] = Coefficent * f[x] / A
```

Out[28]= $\frac{e^{-\frac{m x^2 \omega}{2 \hbar}} \left(\frac{m\,\omega}{\hbar}\right)^{1/4}}{\pi^{1/4}}$

- Checking normalization. Re-integrating the wave function squared should give 1

In[∘]:=
```
Integrate[Φ[x]^2, {x, -∞, ∞}, Assumptions → Re[(m * ω) / ℏ] > 0]
```

Out[∘]= 1

- The seems to check out. That is the desired outcome after normalization
- Part 2 Solving for the wave function in the n = 2 state
- Defining operators and wave functions

```
pₓ := -I*ℏ* ∂ₓ# &;
a_raising := (√(1/(2*ℏ*m*ω))) (-I*pₓ@# + m*ω*x #) &;
a_lowering := (√(1/(2*ℏ*m*ω))) (I*pₓ@#+m*ω*x #) &;
Φ₁ = Simplify[C*(1/√(1!))*a_raising@Φ[x]]
```

Out[160]= $\frac{\sqrt{2}\, C\, e^{-\frac{m x^2 \omega}{2\hbar}}\, x \sqrt{\frac{1}{m\,\omega\,\hbar}} \left(\frac{m\,\omega}{\hbar}\right)^{5/4} \hbar}{\pi^{1/4}}$

In[163]:= `Φ₂ = Simplify[C*(1/√(2!))*a_raising@a_raising@Φ[x]]`

Out[163]= $\frac{C\, e^{-\frac{m x^2 \omega}{2\hbar}} \left(2\, m\, x^2\, \omega - \hbar\right) \left(\frac{m\,\omega}{\hbar}\right)^{1/4}}{\sqrt{2}\, \pi^{1/4}\, \hbar}$

- Testing that I can stack operators like I think I can in Mathematica

In[76]:= `Φ₂ₜ = Simplify[C*(1/√(2!))*a_raising@Φ₁]`

Out[76]= $\frac{C^2\, e^{-\frac{m x^2 \omega}{2\hbar}} \left(2\, m\, x^2\, \omega - \hbar\right) \left(\frac{m\,\omega}{\hbar}\right)^{1/4}}{\sqrt{2}\, \pi^{1/4}\, \hbar}$

In[63]:= `Solve[Integrate[Φ₂^2, {x, -∞, ∞}, Assumptions → Re[(m*ω)/ℏ] > 0] == 1, C]`

`{{C → -1}, {C → 1}}`

In[88]:= `Φₙ₌₂ = Simplify[(1/√(2!))*a_raising@a_raising@Φ[x]]`

Out[88]= $\frac{e^{-\frac{m x^2 \omega}{2\hbar}} \left(2\, m\, x^2\, \omega - \hbar\right) \left(\frac{m\,\omega}{\hbar}\right)^{1/4}}{\sqrt{2}\, \pi^{1/4}\, \hbar}$

- making a plot just to see the wave shapes and compare to expected. treating all undefined symbols as equalling 1

In[94]:=
```
ff = (E^(-((x^2 )/(2)))/π^(1/4))
Integrate[ff^2, {x, -∞, ∞}]
```

Out[94]= $\frac{e^{-\frac{x^2}{2}}}{\pi^{1/4}}$

Out[95]= 1

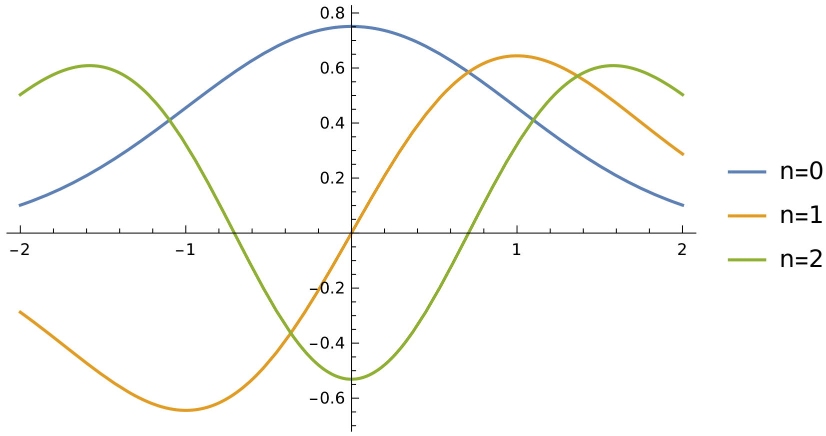
In[92]:= `tt = (E^(-((x^2)/(2))) (2 x^2 - 1)/(Sqrt[2] π^(1/4)))`

$$\frac{e^{-\frac{x^2}{2}}\left(-1+2\,x^2\right)}{\sqrt{2}\;\pi^{1/4}}$$

In[105]:=
```
gg = (Sqrt[2]  E^(-(( x^2 )/ (2 ))) x Sqrt[1/ 1] (1)^(5/ 4) 1)/ π^(1/ 4)
```

Out[105]=
$$\frac{\sqrt{2}\;e^{-\frac{x^2}{2}}\;x}{\pi^{1/4}}$$

In[108]:=
```
Plot[{ff, gg, tt}, {x, -2, 2}, PlotLegends → {"n=0", "n=1", "n=2"}]
```

Out[108]=


- Without plugging in values for m,ω and,$\hbar$. I set them all to 1.These are the shapes I'd expect in relation to a quadratic well. The agree with the wave shapes seen on pg 58 figure 2.7(a) of Griffiths "Introduction to Quantum Mechanics 2nd Edition". As a result I feel pretty confident in my solutions for this problem.