RK45 Notes

Fall 2020

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1 About RK45

RK represents the Runge-Kutta (RK) class of integrators for estimating the value of an equation. For the purposes of this article, we will be assuming that we are attempting to find $y(x_1)$ from $y(x_0)$. Additionally, although this equation, $y(x)$, is only a function of one variable, these methods can be applied to multi-dimensional equations.

1.1 First Order RK

The first order of the RK class is simply the Euler approximation to the equation.

$$y(x_1) \approx y(x_0) + \Delta_x * y'(x_0), \text{ where } \Delta_x = x_1 - x_0$$

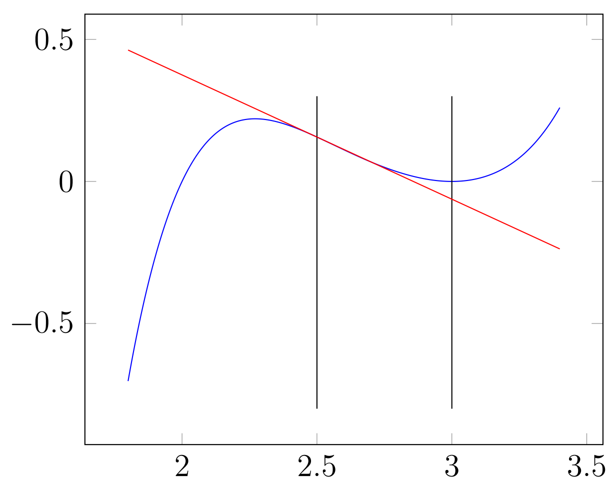

Figure 1: $(x-3)^5 + (x-3)^4 + (x-3)^3 + (x-3)^2$ with first derivative shown, and the approximation at $x_1 = 3$, given $x_0 = 2.5$.

Although the inaccuracy in this can be fairly large unless a sufficiently small timestep (Δ_x) is used, there is a large benefit in terms of computational power required to compute this result. To calculate this normally, one would have to compute first, $x - 3$ and store that in a register. Next, compute $(x-3)^5$, during which the other powers will be computed. 1

computation for each of the following operations produces 8 operations to evaluate the given equation at a value.

1. $a = x - 3$
2. $b = a * a$
3. $c = a * b$
4. $d = a * c$
5. $e = a * d$
6. $f = b + c$
7. $f = f + d$
8. $y(x_1) = f + e$

Meanwhile, using the first order RK, and a first order taylor series, we are able to approximate the value of $y(x_1)$ based on $y(x_0)$ by doing the following.

1. Given $y(x_0)$...
2. How do we calculate the derivative?
3. Is this only more efficient not for higher order polynomials, but rather more complex equations, such as when sin/cos or fractions with polynomials everywhere exist?
4. Somehow calculate the derivative
5. $\Delta_y = \Delta_x * \frac{dy}{dx}$
6. $y(x_1) = y(x_0) + \Delta_y$