RKF45 Algorithm

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1 Introduction

The RKF45 algorithm is one of the most accurate existing methods for approximating integrals in terms of timesteps. It is a fourth-order method. In our problem, it estimates a given body's position at a given time by integrating the velocity and acceleration of the body at the previous time, and multiplying them by the time-step and various constants. y_{n+1} represents the fourth-order estimate, z_{n+1} the fifth-order estimate.

2 Equations

$$y_{n+1} = y_n + (256/216) * k_1 + (1408/2565) * k_3 + (2197/4104) * k_4 - (1/5) * k_5 \quad (1)$$

$$z_{n+1} = y_n + (16/135) * k_1 + (6656/12825) * k_3 + (28561/56430) * k_4 - (9/50) * k_5 + (22/55) * k_6 \quad (2)$$

$$k_1 = h * f(t_n, y_n) \quad (3)$$

$$k_2 = h * f(t_n + (1/4) * h, y_n + (1/4) * k_1) \quad (4)$$

$$k_3 = h * f(t_n + (3/8) * h, y_n + (3/32) * k_1 + (9/32) * k_2) \quad (5)$$

$$k_4 = h * f(t_n + (12/13) * h, y_n + (1932/2197) * k_1 - (7200/2197) * k_2 + (7296/2197) * k_3) \quad (6)$$

$$k_5 = h * f(t_n + h, y_n + (439/216) * k_1 - 8 * k_2 + (3680/513) * k_3 - (845/4104) * k_4) \quad (7)$$

$$k_6 = h * f(t_n + (1/2) * h, y_n - (8/27) * k_1 + 2 * k_2 - (3544/2565) * k_3 + (18959/4104) * k_4 - (11/40) * k_5) \quad (8)$$

A variable timestep can also be used, with the fifth-order variant of the estimate. The optimal step size is s*h, where:

$$s = ((T * h)/(2 * |z_{n+1} - y_{n+1}|))^{1/4} \quad (9)$$

T represents the tolerance for error.