My research area is the ***Representation Theory*** and ***Number Theory***, in particular the Automorphic Forms. I am also interested in Algebraic Geometry, including Complex Geometry, Arithmetic Geometry, and Harmonic Analysis. These 'hobbies' serve me some estimation techniques from analysis (eg. *Langlands classification, Local Trace Formula*), and the geometric realization of abstract objects (eg. *Borel-Weil theorem, Elliptic Curves with Complex Multiplication and Class Field Theory*).

Modular Forms and Automorphic Forms

It's the 'Undergraduate Research Opportunities' Project with a series of reading groups. Our presentation note can be found here. The main theorem of this note is the following:

Theorem 1 (Multiplicity One)**.** *The unitary representation* $(R, L^2_{\text{cusp}}([\text{GL}_n]))$*, can be decomposed as the completion direct sum of irreducible representations, and the multiplicity of each irreducible class* ≤ 1.

Then we use the stronger version (isomorphism for almost all places v implies isomorphism of two irreducible components in $(R, L^2_{\text{cusp}}([\text{GL}_n]))$) to study Whittaker models and the L-functions. Besides, we studied the representation theory with an emphasis on GL_2, including Langland's Classification of Real Reductive Groups, Automorphic Forms and Representations, Tate Thesis, Harmonic Analysis on Reductive p-adic Groups and Lie Algebras: The Local Trace Formula, Jacquet Langlands Correspondence for $\text{GL}_2(\mathbb{Q}_p)$. The respective references are by Wallach, Bump, Tate, Kottwitz, Taïbi. In 2023 fall, we studied Galois and Cartan Cohomology of Real Groups following Taïbi's paper.

Theta Lifts

My bachelor thesis aims to study the non-vanishing problem of global *theta lifts*. Its idea follows from Gan, Qiu and Takeda, who proved the second term identity of the regularized Siegel-Weil formula, thereby obtaining the Rallis inner product formula in complete generality. First, we need to study the irreducible points of the degenerate principal series representation. Its structure is connected with the theory of local theta correspondence. Using Rallis Inner Product Formula, the non-vanishing of theta lift is equivalent to the non-vanishing of the local doubling zeta integral associated with the representation on a certain submodule of the degenerate principal series. This nonvanishing property of the integral can be used to construct an explicit realization of theta correspondence and study its matrix coefficients. The real case is studied by Zhe Li and Shanwen Wang: here. The quaternion algebra case is unsettled.

Local Gan-Gross-Prasad Conjecture (in archimedean places)

We aim to replace the trace formula step of Beuzart-Plessis (who proved the generalized GGP conjecture (extends the multiplicity one result to a whole L-packet of tempered representations) for unitary group in both archimedean and non-archimedean case uniformly) by theta correspondence, to solve the symplectic-metapletic restriction problems. First we apply the correspondence between the irreducible genuine unitary representations of $\text{Mp}(W)$ and the irreducible unitary representation of the Jacobi group $\text{J}(W)$, see Binyong Sun's work. The local GGP conjecture for the real symplectic-Jacobi case can be reformulated via period integral: as in Beuzart-Plessis's work, $m(\pi, \psi) \neq 0$ if and only if the specific functional (a period integral of trace along the subgroup $H(F)$) nonvanishes. In [BP], this result is crucial in proving the spectral expansion of $J(f)$. Then he used its geometric expansion (present as an integral along conjugacy class $\Gamma(G, H)$) to prove the multiplicity formula (Theorem 2 in [BP]) of the desired disjoint union of L-packets. Then we need to calculate the period integral via metaplectic theta correspondence (by J. Adams and D. Barbasch):

Theorem 2. *Let* W *be a* $2n$*-dimensional real symplectic vector space and let* S_{2n+1} *be the set of isomorphism classes of real orthogonal spaces with* $\dim V' = 2n+1$ *and* $\text{disc}(V') = 1$*. Then the dual pair* $(\text{Sp}_{2n}(\mathbb{R}), \text{O}(p, q))$ *gives rise to a bijection between the genuine representations of metaplectic group* $\text{Mp}(W)$ *and the union of representations of odd special group of the same rank.*

Interest: Perfectoid Spaces

In the spring 2023 course "Topics in Number Theory", Prof. Yiwen Ding introduced perfectoid spaces to me (my note). The main philosophy of this story is to reduce certain problems about mixed characteristic rings to problems about rings in characteristic p. Tilt functor plays a crucial role here. The study on the structure sheaf allows one to define general perfectoid spaces by gluing affinoid perfectoid spaces. Further, we studied an improvement on Faltings's almost purity theorem. Afterthen, Prof. Ding asked me to give him a talk on an application of perfectoid spaces theory, as the further study of this course. I gave a detailed construction of the "fundamental curve of p-adic Hodge theory" together with sketches of proofs of the main properties (e.g. fundamental exact sequence). Details can be found: Chapter 6 in note. Even though sometimes these topics fly way above my head, I find the progressive discovery of new theories (for example,) by Fargues and Scholze (Geometrization of the local Langlands correspondence), and by Ben-Zvi, Sakellaridis and Venkatesh (Relative Langlands Duality) fascinating to study. I am always ready for the beautiful and thrilling journey during mathematical research.