

BCQM VII Lab Note

Test 2.4: Effective dimensionality from ball growth (d_{eff}) (v0.1)

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Purpose

Document the Stage-2 “Beyond Stability” Test 2.4: estimate an effective dimensionality d_{eff} from ball growth, and record the outcome for cloth-level and super-graph-level objects.

Inputs

This test uses existing run outputs; no new simulations were required. The analysis reads `RUN_METRICS_*.json` and uses:

- **Cloth object:** `RUN_METRICS["cloth"]["ball_growth"] ...`
- **Super-graph object:** community detection on the undirected projection of the `hits1` cloth core edge set (`core_edges_used`), then build the corresponding community super-graph and compute its ball growth.

The analysis script used was:

- `bcqm__vii__cloth\analysis\d__eff__ball__growth.py`

Method

Ball growth provides a curve $|B(r)|$ (mean ball volume at radius r). We estimate an effective exponent d_{eff} by fitting

$$\log |B(r)| \approx d_{\text{eff}} \log r + c$$

over an automatically chosen window that (i) excludes very small r , (ii) avoids near-saturation where $|B(r)| \approx |C|$, and (iii) has at least a minimum number of contiguous radii.

We report the chosen window $[r_{\text{lo}}, r_{\text{hi}}]$ and fit quality via R^2 . If no admissible window exists, d_{eff} is reported as NaN (undefined).

Runs analysed

- Cloth: A3 scale-ups at $n = 0.8$, `hits1`, x10 epoch, bins=20:
 - $N = 16$: `gateA3__N16__hits1__x10__bins20__n0p8`
 - $N = 32$: `gateA3__N32__hits1__x10__bins20__n0p8`
- Super-graph: Pivot baseline ensemble (`hits1`, x10 epoch, bins=20, $N \in \{4, 8\}$, $n \in \{0.4, 0.8\}$):
 - `ensemble__W100__N4N8__hits1__x10__bins20`

Results

Cloth: no usable scaling window at high connectivity

For the cloth object at $n = 0.8$ (both $N = 16$ and $N = 32$), the window selection finds no admissible intermediate scaling region, and d_{eff} is undefined (`NaN`) for all seeds. This remains true even under progressively loosened selection parameters (“loose” and “ultra-loose” attempts).

Interpretation. In the high- n hits1 cloth regime, the cloth core saturates extremely rapidly: $|B(r)|/|C|$ reaches unity at very small radius (often $r \leq 2$). This indicates a small-world/shortcut-rich cloth at the primitive edge level. Under such rapid saturation there is no intermediate window where $|B(r)| \sim r^d$ holds over multiple consecutive radii, so a power-law d_{eff} estimate is not a meaningful diagnostic at the cloth level in this regime.

Super-graph: stable effective exponents with excellent fit quality

At the community super-graph level, the same fitting procedure yields stable effective exponents with excellent fit quality (very high R^2). The fitted window is short but consistent (typically $r \approx 3\text{--}6$), reflecting finite-size constraints at $K \sim 20\text{--}30$ for the super-graph.

Interpretation. The coarse-grained community super-graph is sparse enough to admit an intermediate growth window, making d_{eff} a meaningful diagnostic at this scale. This supports the Stage-2 pivot: geometry-like scaling behaviour can be clearer on coarse objects than on molecular edge sets.

Outcome and decision

- Cloth-level d_{eff} (power-law fit) is **undefined** in the high- n hits1 cloth regime due to immediate saturation. This should be reported as a diagnostic signature (small-world cloth) rather than treated as a failure.
- Super-graph d_{eff} is **defined** and stable with excellent fit quality, and is therefore the preferred effective-dimensionality diagnostic at this stage.
- If a cloth-level effective dimension is required later, it should be sought either (i) in weaker-connectivity regimes where saturation is delayed, or (ii) via alternative diagnostics (e.g. local-slope curves) designed for rapid saturation.

Generated artefacts

The following CSV outputs were produced by the analysis:

- `gateA3_N16_cloth_d_eff_runs.csv` and `gateA3_N16_cloth_d_eff_summary.csv`
- `gateA3_N32_cloth_d_eff_runs.csv` and `gateA3_N32_cloth_d_eff_summary.csv`
- `gateA3_N32_cloth_loose_d_eff_runs.csv` and `gateA3_N32_cloth_loose_d_eff_summary.csv`
- `gateA3_N32_cloth_ultra_loose_d_eff_runs.csv` and `gateA3_N32_cloth_ultra_loose_d_eff_summary.csv`
- `pivot_supergraph_deff_d_eff_runs.csv` and `pivot_supergraph_deff_d_eff_summary.csv`