BCQM VII Lab Note
Test 2.3 (Curvature proxy): Forman–Ricci curvature on the community super-graph (v0.1)

Peter M. Ferguson
Independent Researcher

3 February 2026

Purpose

Implement and evaluate a first curvature-like diagnostic for Stage–2 by computing (i) Forman–Ricci curvature on edges of the community super-graph, (ii) augmented Forman curvature including triangle contributions, and (iii) clustering/transitivity sanity checks. The intent is not to claim a continuum curvature, but to test whether the coarse-grained geometry object exhibits loop/triangle structure in a regime-dependent, reproducible way.

Dataset and scripts

Dataset. The pivot baseline ensemble (hits1, x10 epoch, bins=20, 5 seeds per quadrant) in:

- `outputs_cloth/ensemble_W100_N4N8_hits1_x10_bins20/`

Script.

- `bcqm_vii_cloth/analysis/curvature_supergraph_forman.py`

CSV artefacts. Two runs of the script were performed, differing only in edge-source choice:

- **core/core geometry object:** `curvature_pivot_core_supergraph_curvature_summary.csv` and `curvature_pivot_core_supergraph_curvature_runs.csv`
- **all/all geometry object:** `curvature_pivot_all_supergraph_curvature_summary.csv` and `curvature_pivot_all_supergraph_curvature_runs.csv`

Method (summary)

For each seed and quadrant (N, n):

1. Build an undirected cloth graph from the chosen edge set:
   - core: `core_edges_used`
   - all: `core_edges_used` $\cup$ `halo_edges_used`
2. Apply Louvain community detection (resolution $\gamma = 1.0$) on this undirected graph to obtain a partition π : event $\mapsto$ community.
3. Build the *undirected* community super-graph: add an edge between communities if any cloth edge crosses between them.
4. Compute:
   - transitivity and mean clustering coefficient on the super-graph,
   - per-edge triangle counts,

- Forman edge curvature $F(e) = 4 - \deg(u) - \deg(v)$,
- augmented Forman curvature $F_{\text{aug}}(e) = F(e) + 3T(e)$, where $T(e)$ is triangles containing edge e.

Results

Table 1 compares the core/core and all/all outcomes (mean over seeds) for each quadrant.

N	n	source	K	trans	clust	F_{mean}	$F_{\text{aug,mean}}$
4	0.4	core	44.6	0.0258	0.0150	−0.0080	+0.0730
4	0.4	all	24.6	0.0000	0.0000	+0.0852	+0.0852
4	0.8	core	21.8	0.0000	0.0000	+0.0962	+0.0962
4	0.8	all	21.8	0.0000	0.0000	+0.0962	+0.0962
8	0.4	core	76.6	0.0881	0.0552	−0.1857	+0.1086
8	0.4	all	23.6	0.0000	0.0000	+0.0888	+0.0888
8	0.8	core	22.2	0.0000	0.0000	+0.0945	+0.0945
8	0.8	all	22.2	0.0000	0.0000	+0.0945	+0.0945

Table 1: Curvature-proxy summary on the community super-graph. "source" indicates whether the partition and super-graph were constructed from core-only edges or core+halo (all-used) edges. trans = transitivity; clust = mean clustering coefficient. F is unweighted Forman edge curvature; F_{aug} includes triangle contributions.

Key empirical pattern. At $n = 0.8$, core/core and all/all are identical: the super-graph is sparse and triangle-free (transitivity = 0, clustering = 0), and $F_{\text{aug}} = F$ because no triangles exist.

At $n = 0.4$, the choice of edge source becomes decisive. Under core/core, the super-graph exhibits nonzero triangle closure (especially for $N = 8$: transitivity ≈ 0.088, clustering ≈ 0.055), and augmented Forman curvature differs substantially from degree-only Forman curvature. Under all/all, the super-graph collapses to a much smaller community graph ($K \approx 24$) and becomes triangle-free (transitivity = 0, clustering = 0), eliminating the triangle contribution entirely.

Interpretation

What curvature is detecting here. Forman curvature $F(e) = 4 - \deg(u) - \deg(v)$ is primarily a degree/branching proxy; negative mean values indicate that many edges connect higher-degree nodes (expansive branching). Augmented Forman curvature F_{aug} adds a positive correction proportional to triangle closure, separating "branching" from "loop closure".

Low-n sensitivity and the role of halo edges. The all/all construction at $n = 0.4$ removes triangle closure by introducing shortcut edges that collapse the community graph. This is consistent with earlier A2 findings: all/all at low n shrinks the community super-graph and worsens super-graph ball-growth stability. In curvature terms, halo edges act as closing shortcuts that suppress mesoscopic loop structure detectable under core/core.

Methodological consequence. Curvature-like proxies are meaningful only when the super-graph retains mesoscopic structure. For geometry tests (Gate 2), core/core is the preferred default; for localisation/trajectory tests (Gate 4), all/all may still be necessary for coverage and should be used with explicit coverage reporting.

Conclusion

Test 2.3 (curvature proxy) has been implemented at the super-graph level and reveals a clear regime dependence:

- High n: sparse, triangle-free super-graph backbone (core and all coincide).
- Low n: core/core retains a richer, triangle-bearing super-graph; all/all collapses the super-graph and eliminates triangle closure.

This supports the Stage–2 pivot: geometry-relevant structure is clearer on the coarse object, and the choice of "which graph" (core-only versus all-used) is itself a principled part of the validation pipeline.