BCQM VII Lab Note
A2: Partition-source sensitivity (core/core vs all/all) (v0.1)

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Purpose

Record the A2 "stake-in-the-ground" sensitivity test: determine how the Stage–2 pivot metrics (Gates 1–3) depend on whether the community partition and super-graph are constructed from (i) **core-only** cloth edges (core/core) or (ii) **all-used** edges (core+halo; all/all). This resolves an ambiguity exposed by Gate–4 localisation at larger N: for trajectories, all-used sources can be needed for coverage, but for geometry it may not be desirable.

Data and scripts used

Dataset. The baseline ensemble used was the Stage–2 pivot dataset:

- hits1, x10 epoch, bins=20, $W_{\text{coh}} = 100$, seeds 56791–56795
- $N \in \{4, 8\}$, $n \in \{0.4, 0.8\}$
- run folder: `outputs_cloth/ensemble_W100_N4N8_hits1_x10_bins20`

CSV inputs (A2.zip). This lab note is based on two output folders (zipped for transfer):

- `csv/A2_core_core/`:
  - `community_partition_stability.csv`
  - `supergraph_edge_stability.csv`
  - `supergraph_ballgrowth_stability.csv`
- `csv/A2_all_all/`:
  - `community_partition_stability.csv`
  - `supergraph_edge_stability.csv`
  - `supergraph_ballgrowth_stability.csv`

Scripts. A2 used the A2-enabled analysis scripts (no simulation reruns). The A2 variants add an `edge_source` switch to select either core-only edges or core+halo (all-used) edges:

- `bcqm_vii_cloth/analysis/community_cloth_stability_A2.py` (adds `-edge_source core` or `-edge_source all`)
- `bcqm_vii_cloth/analysis/supergraph_edge_stability_A2.py` (adds `-edge_source core` or `-edge_source all`)
- `bcqm_vii_cloth/analysis/community_cloth_stability.py` (super-graph ball-growth stability from the same dataset; unchanged script)

Summary of results

Table 1 summarises the key deltas between core/core and all/all, quadrant by quadrant. Quantities are mean values across seed pairs (5 seeds ⇒ 10 pairs).

N	n	K_{core}	K_{all}	NMI_{core}	NMI_{all}	$d_{\text{L2,core}}$	$d_{\text{L2,all}}$
4	0.4	44.2	24.4	0.833	0.810	0.0219	0.0545
4	0.8	22.4	22.4	0.834	0.834	0.0170	0.0170
8	0.4	75.4	23.8	0.838	0.822	0.0085	0.0162
8	0.8	22.4	22.4	0.835	0.835	0.0235	0.0235

Table 1: A2 sensitivity summary: core/core vs all/all. Here K is the mean community count, NMI is partition similarity across seeds, and d_{L2} is the mean L2 distance between normalised super-graph ball-growth curves across seeds. Full CSVs are listed in the text.

Gate 2 invariance. Super-graph edge stability (unweighted Jaccard) and weight correlations are unchanged between core/core and all/all for this dataset. That is, including halo edges does not change the set of community-to-community connections or their relative flow strengths as measured by Gate 2. This indicates that the stable mesoscopic flow skeleton is already captured by the core edges alone.

Low-n sensitivity (n=0.4). At low cross-link pressure, the choice of edge source matters:

- The all/all construction collapses the super-graph to a much smaller connected component (mean component size drops to ~ 24 communities for $N = 8$), and the geometry diagnostic becomes noisier (ball-growth stability degrades).
- The core/core construction yields a larger, richer super-graph (higher K and larger GCC) and more stable ball-growth curves across seeds.

Interpretation: at $n = 0.4$ the halo edges act primarily as "closing" edges that reduce fragmentation and can short-cut the community graph, but this does not improve reproducible geometry; instead it increases variability in the derived super-graph ball-growth curves.

High-n robustness (n=0.8). At high cross-link pressure, core/core and all/all become equivalent: K, NMI/ARI, super-graph edge stability, and super-graph ball-growth stability are the same. This is consistent with the high-n regime being dominated by the coherent core, with relatively little independent halo structure.

Methodological conclusion (recommended defaults)

A2 resolves the "which graph?" ambiguity into a clean practice:

- **Gates 1–3 (geometry object definition):** use **core/core** by default, especially at low n, because it preserves a richer super-graph and yields more stable super-graph geometry diagnostics.
- **Gate 4 (trajectory/localisation tests):** use **all/all** by default when needed for coverage, and explicitly report the mapping coverage fraction. This is particularly important at larger N or weaker regimes where core-only partitions may not cover the traced event IDs.

Next steps

1. Incorporate this A2 conclusion into the Stage–2 pivot record and the "Beyond Stability" plan as a methodological note.
2. For publication clarity, add a short "graph-choice" subsection explaining why core/core is the geometry object and all/all is the localisation substrate (coverage).
3. Optional: repeat A2 at N=16 or N=32 if later needed, but the key qualitative behaviour is already established here.