

# BCQM VII Lab Note

## Test 2.4: Effective dimensionality from ball growth ( $d_{\text{eff}}$ ) (v0.1)

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### Purpose

Document the Stage-2 “Beyond Stability” Test 2.4: estimate an effective dimensionality  $d_{\text{eff}}$  from ball growth, and record the outcome for cloth-level and super-graph-level objects.

### Inputs

This test uses existing run outputs; no new simulations were required. The analysis reads `RUN_METRICS_*.json` and uses:

- **Cloth object:** `RUN_METRICS["cloth"]["ball_growth"]` ...
- **Super-graph object:** community detection on the undirected projection of the hits1 cloth core edge set (`core_edges_used`), then build the corresponding community super-graph and compute its ball growth.

The analysis script used was:

- `bcqm\_vii\_cloth/analysis/d\_eff\_ball\_growth.py`

### Method

Ball growth provides a curve  $|B(r)|$  (mean ball volume at radius  $r$ ). We estimate an effective exponent  $d_{\text{eff}}$  by fitting

$$\log |B(r)| \approx d_{\text{eff}} \log r + c$$

over an automatically chosen window that (i) excludes very small  $r$ , (ii) avoids near-saturation where  $|B(r)| \approx |C|$ , and (iii) has at least a minimum number of contiguous radii.

We report the chosen window  $[r_{\text{lo}}, r_{\text{hi}}]$  and fit quality via  $R^2$ . If no admissible window exists,  $d_{\text{eff}}$  is reported as NaN (undefined).

### Runs analysed

- Cloth: A3 scale-ups at  $n = 0.8$ , hits1, x10 epoch, bins=20:
  - $N = 16$ : `gateA3\_N16\_hits1\_x10\_bins20\_n0p8`
  - $N = 32$ : `gateA3\_N32\_hits1\_x10\_bins20\_n0p8`
- Super-graph: Pivot baseline ensemble (hits1, x10 epoch, bins=20,  $N \in \{4, 8\}$ ,  $n \in \{0.4, 0.8\}$ ):
  - `ensemble\_W100\_N4N8\_hits1\_x10\_bins20`

## Results

### Cloth: no usable scaling window at high connectivity

For the cloth object at  $n = 0.8$  (both  $N = 16$  and  $N = 32$ ), the window selection finds no admissible intermediate scaling region, and  $d_{\text{eff}}$  is undefined (NaN) for all seeds. This remains true even under progressively loosened selection parameters (“loose” and “ultra-loose” attempts).

**Interpretation.** In the high- $n$  hits1 cloth regime, the cloth core saturates extremely rapidly:  $|B(r)|/|C|$  reaches unity at very small radius (often  $r \leq 2$ ). This indicates a small-world/shortcut-rich cloth at the primitive edge level. Under such rapid saturation there is no intermediate window where  $|B(r)| \sim r^d$  holds over multiple consecutive radii, so a power-law  $d_{\text{eff}}$  estimate is not a meaningful diagnostic at the cloth level in this regime.

### Super-graph: stable effective exponents with excellent fit quality

At the community super-graph level, the same fitting procedure yields stable effective exponents with excellent fit quality (very high  $R^2$ ). The fitted window is short but consistent (typically  $r \approx 3$ – $6$ ), reflecting finite-size constraints at  $K \sim 20$ – $30$  for the super-graph.

**Interpretation.** The coarse-grained community super-graph is sparse enough to admit an intermediate growth window, making  $d_{\text{eff}}$  a meaningful diagnostic at this scale. This supports the Stage-2 pivot: geometry-like scaling behaviour can be clearer on coarse objects than on molecular edge sets.

## Outcome and decision

- Cloth-level  $d_{\text{eff}}$  (power-law fit) is **undefined** in the high- $n$  hits1 cloth regime due to immediate saturation. This should be reported as a diagnostic signature (small-world cloth) rather than treated as a failure.
- Super-graph  $d_{\text{eff}}$  is **defined** and stable with excellent fit quality, and is therefore the preferred effective-dimensionality diagnostic at this stage.
- If a cloth-level effective dimension is required later, it should be sought either (i) in weaker-connectivity regimes where saturation is delayed, or (ii) via alternative diagnostics (e.g. local-slope curves) designed for rapid saturation.

## Generated artefacts

The following CSV outputs were produced by the analysis:

- `gateA3\_N16\_cloth\_d\_eff\_runs.csv` and `gateA3\_N16\_cloth\_d\_eff\_summary.csv`
- `gateA3\_N32\_cloth\_d\_eff\_runs.csv` and `gateA3\_N32\_cloth\_d\_eff\_summary.csv`
- `gateA3\_N32\_cloth\_loose\_d\_eff\_runs.csv` and `gateA3\_N32\_cloth\_loose\_d\_eff\_summary.csv`
- `gateA3\_N32\_cloth\_ultra\_loose\_d\_eff\_runs.csv` and `gateA3\_N32\_cloth\_ultra\_loose\_d\_eff\_summary.csv`
- `pivot\_supergraph\_deff\_d\_eff\_runs.csv` and `pivot\_supergraph\_deff\_d\_eff\_summary.csv`