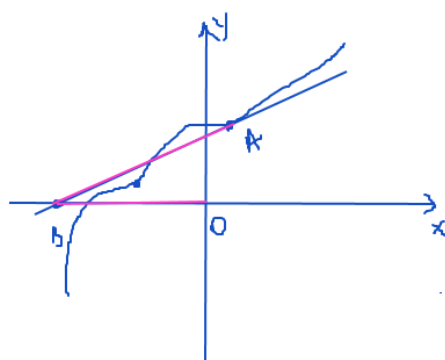


Seminar04 - Rezolvare

Exercițiu02

2



$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2} = \frac{2 \frac{y}{x}}{1 - \left(\frac{y}{x}\right)^2} = f\left(\frac{y}{x}\right)$$

$$u = \frac{y}{x} \Rightarrow y = ux \Rightarrow y' = u'x + u$$

$$u'x + u = \frac{2u}{1-u^2} \Rightarrow u'x = \frac{2u}{1-u^2} - u$$

$$u'x = \frac{2u - u + u^3}{1-u^2}$$

$$\frac{1-u^2}{u^3+u} = \frac{1-u^2}{u(u^2+1)} = \frac{A}{u} + \frac{Bu+C}{u^2+1} = \frac{Au^2+1+Bu^2+Cu}{u(u^2+1)} = \frac{(A+B)u^2+Cu+1}{u(u^2+1)}$$

$$\frac{du}{dx} \cdot x = \frac{u^3+u}{1-u^2} \Rightarrow \frac{1-u^2}{u^3+u} du = \frac{1}{x} dx$$

$$\int \frac{1-u^2}{u^3+u} du = \int \frac{1}{x} dx$$

$$= \frac{(A+B)u^2 + Cu + 1}{u(u^2+1)}$$

2

$$A+B=-1 \Rightarrow B=-2$$

$$C=0$$

$$A=1$$

$$\frac{1-u^2}{u^3+u} = \frac{1}{u} - \frac{2u}{u^2+1}$$

$$\int \left(\frac{1}{u} - \frac{2u}{u^2+1} \right) du = \int \frac{1}{x} dx$$

$$\ln|u| - \ln|u^2+1| = \ln|x| + C$$

$$\ln \left| \frac{y}{x^2+1} \right| = \ln |x| + C$$

$$\frac{y}{x^2+1} = Cx$$

$$\frac{y}{x} = Cx$$

$$\frac{y^2}{x^2} = C^2 x^2$$

$$\frac{y}{x} \cdot \frac{x^2}{x^2+y^2} = Cx \quad | : x$$

$$yx = C(x^2+y^2) \Leftrightarrow C(x^2+y^2) - xy = 0$$