Laborator 08 - TEMĂ                Petculescu Mihai-Liviu

I. Să se rezolve urm. ec. dif. liniare de ordin n omogene cu coeficienți constanți.

b) $2x''' - 3x'' + x' = 0, \quad x(0) = -1, \; x'(0) = 2, \; x''(0) = 1$

$$\left.\begin{array}{l} x = e^{rt} \\ x' = re^{rt} \\ x'' = r^2 e^{rt} \\ x''' = r^3 e^{rt} \end{array}\right\} \Rightarrow 2r^3 e^{rt} - 3r^2 e^{rt} + re^{rt} = 0 \,\Big|\, : e^{rt}$$

$$2r^3 - 3r^2 + r = 0$$

$$r(2r^2 - 3r + 1) = 0 \longrightarrow \boxed{r_1 = 0}$$

$$2r^2 - 3r + 1 = 0$$

$$\Delta = 9 - 8 = 1 \Rightarrow r_{2,3} = \frac{3 \pm 1}{2} \quad \boxed{r_2 = 2} \quad \boxed{r_3 = 1}$$

- $e^{0\cdot t}, e^{2\cdot t}, e^{1\cdot t}$ - sist. fundamental de sol.
- $x(t) = C_1 + C_2 e^{2t} + C_3 e^t \Leftrightarrow x(0) = \boxed{C_1 + C_2 + C_3 = -1}$
- $x'(t) = C_2 2 e^{2t} + C_3 e^t \Leftrightarrow x'(0) = \boxed{2C_2 + C_3 = 2}$
- $x''(t) = C_2 4 e^{2t} + C_3 e^t \Leftrightarrow x''(0) = \boxed{4C_2 + C_3 = 1}$

$$\begin{cases} C_1 + C_2 + C_3 = -1 \Leftrightarrow C_1 = -1 - 3 + \frac{1}{2} = \boxed{\frac{-7}{2} = C_1} \\ \left.\begin{array}{l} 2C_2 + C_3 = 2 \\ 4C_2 + C_3 = 1 \end{array}\right\} \Rightarrow 2C_2 = -1 \Leftrightarrow \boxed{C_2 = \frac{-1}{2}} \Leftrightarrow \boxed{C_3 = 3} \end{cases}$$

- $x_{PC} = \frac{-7}{2} - \frac{1}{2} e^{2t} + 3e^t$

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c) $x''' - 7x'' + 14x' - 8x = 0, \quad x(0) = 1, \; x'(0) = 0, \; x''(0) = 1$

$$\left.\begin{array}{l} x = e^{rt} \\ x' = re^{rt} \\ x'' = r^2 e^{rt} \\ x''' = r^3 e^{rt} \end{array}\right\} \Rightarrow r^3 e^{rt} - 7r^2 e^{rt} + 14re^{rt} - 8e^{rt} = 0 \,\Big|\, : e^{rt}$$

$$r^3 - 7r^2 + 14r - 8 = 0$$

$$\begin{array}{l|l} r^3 - 7r^2 + 14r - 8 & r - 1 \\ \cline{2-2} \underline{-r^3 + r^2} & r^2 - 6r + 8 \\ \quad -6r^2 + 14r - 8 & \\ \quad \underline{+6r^2 - 6r} & \\ \quad / \quad -8r - 8 & \end{array}$$

$$= r^2 - 4r - 2r + 8$$

$$= (r - 4)(r - 2)$$

- $(r-1)(r-2)(r-4)=0 \Leftrightarrow \begin{cases} r_1=1 \\ r_2=2 \\ r_3=4 \end{cases}$

- $e^{1t}, e^{2t}, e^{4t}$ – sist. fundam. de sol.

- $x(t)=c_1e^{t}+c_2e^{2t}+c_3e^{4t} \Leftrightarrow x(0)=\boxed{c_1+c_2+c_3=1}$

- $x'(t)=c_1e^{t}+c_2\cdot 2e^{2t}+c_3\cdot 4e^{4t} \Leftrightarrow x'(0)=\boxed{c_1+2c_2+4c_3=0}$

- $x''(t)=c_1e^{t}+c_2\cdot 4e^{2t}+c_3\cdot 16e^{4t} \Leftrightarrow x''(0)=\boxed{c_1+4c_2+16c_3=1}$

$$\begin{cases} c_1+c_2+c_3=1 \\ c_1+2c_2+4c_3=0 \\ c_1+4c_2+16c_3=1 \end{cases} \Leftrightarrow \begin{matrix} c_2+3c_3=-1 \\ 2c_2+12c_3=1 \end{matrix} \Leftrightarrow 6c_3=3 \Leftrightarrow \boxed{c_3=\frac{1}{2}}$$

$$c_2=-1-\frac{3}{2}=\frac{-2-3}{2}=\boxed{\frac{-5}{2}=c_2}$$

$$c_1-\frac{5}{2}+\frac{1}{2}=1 \Leftrightarrow c_1=1-\frac{1}{2}+\frac{5}{2}=\frac{7-1}{2}=\boxed{3=c_1}$$

- $\underline{x_{PC}=3e^{t}-\frac{5}{2}e^{2t}+\frac{1}{2}e^{4t}}$

e) $x'''-x'=0$

$$\left.\begin{matrix} x=e^{rt} \\ x'=re^{rt} \\ x''=r^2e^{rt} \\ x'''=r^3e^{rt} \end{matrix}\right\} \Leftrightarrow r^3e^{rt}-re^{rt}=0 \;\Big|\, : e^{rt}$$

$r^3-r=0$

$r(r^2-1)=0 \Leftrightarrow r_1=0$

$r^2-1=0 \Leftrightarrow (r+1)(r-1)=0 \Leftrightarrow r_2=-1, r_3=1$

- $e^{0t}, e^{-1t}, e^{1t}$ – sist. fundamental de sol.

$x(t)=c_1+c_2e^{-t}+c_3e^{t}$

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II b) $x^{IV} - 5x'' + 4x = 0$

$$\left.\begin{array}{l} x = e^{rt} \\ x' = re^{rt} \\ x'' = r^2e^{rt} \\ x''' = r^3e^{rt} \\ x^{IV} = r^4e^{rt} \end{array}\right\} \Rightarrow \begin{array}{l} r^4e^{rt} - 5r^2e^{rt} + 4re^{rt} = 0 \;\Big|: e^{rt} \\ r^4 - 5r^2 + 4r = 0 \\ r(r^3 - 5r + 4) = 0 \quad \Leftrightarrow r_1 = 0 \\ r^3 - r \cdot 5 + 4 = 0 \end{array}$$

- $r^3 - 5r + 4 = 0 \Leftrightarrow r(r^2-1) - 4(r-1) = 0 \Leftrightarrow r(r+1)(r-1) - 4(r-1) = 0$

$(r-1)(r(r+1)-4) = 0 \Leftrightarrow \underline{(r-1)}(r^2+r-4) = 0 \Leftrightarrow r_2 = 1$

- $r^2 + r - 4 = 0$

$\Delta = 1 - 4(-4) = 17$

$r_{3,4} = \frac{-1 \pm \sqrt{17}}{2}$ — $r_3 = \frac{-1-\sqrt{17}}{2}$

$r_4 = \frac{-1+\sqrt{17}}{2}$

- $e^{0t}, e^{1t}, e^{\frac{-1-\sqrt{17}}{2} \cdot t}, e^{\frac{-1+\sqrt{17}}{2} \cdot t}$ – sist. fund. de sol.
- $x(t) = C_1 + C_2e^{t} + C_3e^{\frac{(-1-\sqrt{17})t}{2}} + C_4e^{\frac{(-1+\sqrt{17})t}{2}}$

c) $x''' - 6x'' + 12x' - 8x = 0$

$$\left.\begin{array}{l} x = e^{rt} \\ x' = re^{rt} \\ x'' = r^2e^{rt} \\ x''' = r^3e^{rt} \end{array}\right\} \Leftrightarrow \begin{array}{l} r^3e^{rt} - 6r^2e^{rt} + 12re^{rt} - 8e^{rt} = 0 \;\Big|: e^{rt} \\ r^3 - 6r^2 + 12r - 8 = 0 \end{array}$$

- $r^3 - 6r^2 + 12r - 8 \;\Big|\; \underline{r-2}$

$$\begin{array}{l|l} r^3 - 6r^2 + 12r - 8 & r-2 \\ \hline \underline{-r^3 + 2r^2} & r^2 - 4r + 4 = (r-2)^2 \\ / \quad -4r^2 + 12r - 8 & \\ \quad\;\; \underline{4r^2 - 8r} & \\ \quad\;\; / \quad +4r - 8 & \\ \quad\quad\quad \underline{-4r + 8} & \\ \quad\quad\quad / \quad\; / & \end{array}$$

- $(r-2)^3 = 0 \iff r_1 = r_2 = r_3 = 2$
- $e^{2t}, \; te^{2t}, \; t^2 e^{2t}$ – sist. fund. de sol.

$x(t) = c_1 e^{2t} + c_2 t e^{2t} + c_3 e^{2t}$

e) $x^{IV} + 2x''' + x'' = 0$

$$\left.\begin{array}{l} x = e^{rt} \\ x' = r e^{rt} \\ x'' = r^2 e^{rt} \\ x''' = r^3 e^{rt} \\ x^{IV} = r^4 e^{rt} \end{array}\right\} \Rightarrow \begin{array}{l} r^4 e^{rt} + 2r^3 e^{rt} + r^2 e^{rt} = 0 \mid : e^{rt} \\ r^4 + 2r^3 + r^2 = 0 \\ r^2 (r^2 + 2r + 1) = 0 \Rightarrow r_1 = r_2 = 0 \\ r^2 + 2r + 1 = (r+1)^2 = 0 \iff r_3 = r_4 = -1 \end{array}$$

- $e^{0t}, \; te^{0t}, \; e^{-t}, \; te^{-t}$ – sist. fund. de sol.

$x(t) = c_1 + c_2 t + c_3 e^{-t} + c_4 t e^{-t}$

d) $x^{IV} - 2x'' + x = 0$

$$\left.\begin{array}{l} x = e^{rt} \\ x' = r e^{rt} \\ x'' = r^2 e^{rt} \\ x''' = r^3 e^{rt} \\ x^{IV} = r^4 e^{rt} \end{array}\right\} \Rightarrow \begin{array}{l} r^4 e^{rt} - 2r^2 e^{rt} + r e^{rt} = 0 \mid : e^{rt} \\ r^4 - 2r^2 + r = 0 \\ r (r^3 - 2r + 1) = 0 \Rightarrow r_1 = 0 \\ r^3 - 2r + 1 = 0 \iff (r-1)(r^2 + r - 1) = 0 \Rightarrow r_2 = 1 \end{array}$$

$$\begin{array}{l|l} \bullet\; r^3 - 2r + 1 & r - 1 \\ \cline{2-2} \underline{-r^3 + r^2} & r^2 + r - 1 \\ /\; r^2 - 2r + 1 & \\ \underline{-r^2 + r} & \\ /\quad -r + 1 & \\ \quad\;\; \underline{+r - 1} & \\ \quad\;\; /\quad / & \end{array}$$

$r^2 + r - 1 = 0$

$\Delta = 1 - 4(-1) = 5$

$r_{3,4} = \dfrac{-1 \pm \sqrt{5}}{2}$ — $r_3 = \dfrac{-1-\sqrt{5}}{2}$, $r_4 = \dfrac{-1+\sqrt{5}}{2}$

- $e^{0t}, \; e^{1t}, \; e^{\frac{-1-\sqrt{5}}{2}t}, \; e^{\frac{-1+\sqrt{5}}{2}t}$ – sist. fund. de sol.

$x(t) = c_1 + c_2 e^{t} + c_3 e^{\frac{-1-\sqrt{5}}{2}t} + c_4 e^{\frac{-1+\sqrt{5}}{2}t}$

g) $x^{(7)}+3x^{(6)}+3x^{(5)}+x^{(4)}=0$

$$\left.\begin{aligned} x&=e^{rt}\\ x'&=re^{rt}\\ x''&=r^2e^{rt}\\ x'''&=r^3e^{rt}\\ x^{(4)}&=r^4e^{rt}\\ x^{(5)}&=r^5e^{rt}\\ x^{(6)}&=r^6e^{rt}\\ x^{(7)}&=r^7e^{rt}\end{aligned}\right\} \Rightarrow r^7e^{rt}+3\cdot r^6e^{rt}+3r^5e^{rt}+r^4e^{rt}=0 \quad \Big|:e^{rt}$$

$$r^7+3r^6+3r^5+r^4=0$$

$$r^4(r^3+3r^2+3r+1)=0 \Rightarrow r_1=r_2=r_3=r_4=0$$

$$r^3+3r^2+3r+1=0 \Leftrightarrow (r+1)^3=0$$

$$\Leftrightarrow r_5=r_6=r_7=-1.$$

- $e^{0t}, te^{0t}, t^2e^{0t}, t^3e^{0t}, e^{-t}, te^{-t}, t^2e^{-t}$

  – sist. fundam. de sol.

- $x(t)=c_1+c_2t+c_3t^2+c_4t^3+c_5e^{-t}+c_6te^{-t}+c_7t^2e^{-t}$

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III b) $x^{IV}+4x=0$

$$\left.\begin{array}{l} x=e^{rt} \\ x'=re^{rt} \\ x''=r^2e^{rt} \\ x'''=r^3e^{rt} \\ x^{IV}=r^4e^{rt} \end{array}\right\} \Rightarrow r^4e^{rt}+4e^{rt}=0 \;\Big|:e^{rt}$$

$$r^4+4=0$$

$$r^4+4=(r^2+2r+2)(r^2-2r+2)=0$$

$$D=4-4\cdot 2=-4$$

$$r_{1,2}=\frac{-2\pm 2i}{2}=-1\pm i \quad (\alpha\pm\beta,\ \alpha=-1,\ \beta=1)$$

$$r_{3,4}=\frac{2\pm 2i}{2}=1\pm i \quad (\alpha\pm\beta,\ \alpha=1,\ \beta=1)$$

- $e^{-1t}\cos t,\ e^{-1t}\sin t,\ e^{1t}\cos t,\ e^{1t}\sin t$ - sist. fund de soluții
- $x(t)=C_1e^{-t}\cos t+C_2e^{-t}\sin t+C_3e^{t}\cos t+C_4e^{t}\sin t$

d) $x''+4x'+13x=0$

$$\left.\begin{array}{l} x=e^{rt} \\ x'=re^{rt} \\ x''=r^2e^{rt} \end{array}\right\} \Rightarrow r^2e^{rt}+4re^{rt}+13e^{rt}=0 \;\Big|:e^{rt}$$

$$r^2+4r+13=0$$

$$D=16-4\cdot 13=-36$$

$$r_{1,2}=\frac{-4\pm 6i}{2}=-2\pm 3i \quad (\alpha\pm\beta,\ \alpha=-2,\ \beta=3)$$

- $e^{-2t}\cos 3t,\ e^{-2t}\sin 3t$ - sist. fund. de soluții
- $x(t)=C_1e^{-2t}\cos 3t+C_2e^{-2t}\sin 3t$

e) $x''+4x'+5x=0$

$$\left.\begin{array}{l} x=e^{rt} \\ x'=re^{rt} \\ x''=r^2e^{rt} \end{array}\right\} \Rightarrow r^2e^{rt}+4re^{rt}+5e^{rt}=0 \;\Big|:e^{rt}$$

$$r^2+4r+5=0$$

$$D=16-4\cdot 5=-4$$

$$r_{1,2}=\frac{-4\pm 2i}{2}=-2\pm i \quad (\alpha\pm\beta,\ \alpha=-2,\ \beta=1)$$

- $e^{-2t}\cos t$, $e^{-2t}\sin t$ – sist. fund de sol.

$x(t) = C_1 e^{-2t}\cos t + C_2 e^{-2t}\sin t$

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IV. b) $x''' - 3x'' + 9x' + 13x = 0$

$$\left.\begin{aligned} x &= e^{rt} \\ x' &= re^{rt} \\ x'' &= r^2 e^{rt} \\ x''' &= r^3 e^{rt} \end{aligned}\right\} \Rightarrow r^3 e^{rt} - 3r^2 e^{rt} + 9re^{rt} + 13e^{rt} = 0 \quad \Big| : e^{rt}$$

$$r^3 - 3r^2 + 9r + 13 = 0$$

$$(r+1)(r^2 - 4r + 13) = 0 \Rightarrow r_1 = -1.$$

- $r^3 - 3r^2 + 9r + 13 \;\big|\; r+1$

$$\begin{array}{l|l} r^3 - 3r^2 + 9r + 13 & r+1 \\ \hline -r^3 - r^2 & r^2 - 4r + 13 \\ \quad / \;\; -4r^2 + 9r + 13 & \\ \qquad 4r^2 + 4r & \\ \qquad / \quad +13r + 13 & \\ \qquad\quad -13r - 13 & \\ \qquad\qquad / \quad / & \end{array}$$

$r^2 - 4r + 13 = 0$

$D = 16 - 4 \cdot 13 = -36$

$r_{2,3} = \frac{4 \pm 6i}{2} = 2 \pm 3i$

$(\alpha \pm \beta, \alpha = 2, \beta = 3)$

- $e^{-t}, e^{2t} \cos 3t, e^{2t} \sin 3t$

- $x(t) = C_1 e^{-t} + C_2 e^{2t} \cos 3t + C_3 e^{2t} \sin 3t$

d) $x''' - 5x'' + 17x' - 13x = 0$

$$\left.\begin{aligned} x &= e^{rt} \\ x' &= re^{rt} \\ x'' &= r^2 e^{rt} \\ x''' &= r^3 e^{rt} \end{aligned}\right\} \Rightarrow r^3 e^{rt} - 5r^2 e^{rt} + 17re^{rt} - 13e^{rt} = 0 \quad \Big| : e^{rt}$$

$$r^3 - 5r^2 + 17r - 13 = 0$$

$$(r-1)(r^2 - 4r + 13) = 0 \Rightarrow r_1 = 1$$

$$\begin{array}{l|l} \bullet\; r^3 - 5r^2 + 17r - 13 & r-1 \\ \hline -r^3 + r^2 & r^2 - 4r + 13 \\ \quad / \;\; -4r^2 + 17r - 13 & \\ \qquad +4r^2 - 4r & \\ \qquad / \quad +13r - 13 & \\ \qquad\quad -13r + 13 & \\ \qquad\qquad / \quad / & \end{array}$$

- $r^2 - 4r + 13 = 0$

$D = 16 - 4 \cdot 13 = -36$

$r_{2,3} = \frac{+4 \pm 6i}{2} = 2 \pm 3i$

$(\alpha \pm \beta, \alpha = 2, \beta = 3)$

- $e^{1t}, e^{2t}\cos 3t, e^{2t}\sin 3t$ - sist. fund. de sol.

$X(t) = c_1 e^{t} + c_2 e^{2t}\cos 3t + c_3 e^{2t}\sin 3t$

e) $x^{V} + 4x^{IV} + 3x''' - 6x' - 2x = 0$

$$\left.\begin{array}{l} x = e^{rt} \\ x' = r e^{rt} \\ x'' = r^2 e^{rt} \\ x''' = r^3 e^{rt} \\ x^{IV} = r^4 e^{rt} \\ x^{V} = r^5 e^{rt} \end{array}\right\} \Rightarrow r^5 e^{rt} + 4r^4 e^{rt} + 3r^3 e^{rt} - 6r e^{rt} - 2e^{rt} = 0 \ \Big| : e^{rt}$$

$$r^5 + 4r^4 + 3r^3 - 6r - 2 = 0$$

$$(r-1)(r^4 + 5r^3 + 8r^2 + 8r + 2) = 0 \Rightarrow r_1 = 1$$

- $$\begin{array}{l|l} r^5 + 4r^4 + 3r^3 - 6r - 2 & r-1 \\ \cline{2-2} -r^5 + r^4 & r^4 + 5r^3 + 8r^2 + 8r + 2 \\ \hline / \ +5r^4 + 3r^3 - 6r - 2 & \\ \quad -5r^4 + 5r^3 & \\ \hline \quad / \quad +8r^3 - 6r - 2 & \\ \qquad -8r^3 + 8r^2 & \\ \hline \qquad / \ +8r^2 - 6r - 2 & \\ \qquad -8r^2 + 8r & \\ \hline \qquad / \quad 2r - 2 & \\ \qquad\quad -2r + 2 & \\ \hline \qquad\quad / \quad / & \end{array}$$