(12) - Rezolvare

a) $\Gamma_1 : \underset{\underset{a_{11}}{\downarrow}}{5}x^2 + \underset{\underset{2a_{12}}{\downarrow}}{8}xy + \underset{\underset{a_{22}}{\downarrow}}{5}y^2 - 14x - 18y + 9 = 0 \quad (= g(x,y))$

Stabilim mai întâi natura conicei

$$A = \begin{pmatrix} 5 & 4 & -7 \\ 4 & 5 & -9 \\ -7 & -9 & 9 \end{pmatrix}, \quad \delta = \begin{vmatrix} 5 & 4 \\ 4 & 5 \end{vmatrix} = 9 > 0 \Rightarrow \text{tip eliptic}$$

(are centru)

Determinăm coordonatele centrului

$$\begin{cases} \frac{1}{2}(10x + 8y - 14) = 0 \\ \frac{1}{2}(8x + 10y - 18) = 0 \end{cases} \Rightarrow \begin{cases} 5x + 4y = 7 \\ 4x + 5y = 9 \end{cases} \Rightarrow \begin{aligned} x &= -\frac{1}{9} \\ y &= \frac{17}{9} \end{aligned}$$

$$\Rightarrow C\left(-\frac{1}{9}, \frac{17}{9}\right).$$

Determinăm pantele axelor de simetrie:

$$a_{12}k^2 + (a_{11} - a_{22})k - a_{12} = 0$$

$$4k^2 + (5-5)k - 4 = 0 \Leftrightarrow 4k^2 - 4 = 0 \Leftrightarrow k^2 = 1 \Rightarrow k = \pm 1$$

Determinăm ecuațiile axelor de simetrie:

Pt. $k = 1$: $y - \frac{17}{9} = 1\left(x + \frac{1}{9}\right)$

$$y - \frac{17}{9} = x + \frac{1}{9} \Leftrightarrow x - y + 2 = 0.$$

Pt. $k = -1$: $y - \frac{17}{9} = -1\left(x + \frac{1}{9}\right)$

$$y - \frac{17}{9} = -x - \frac{1}{9} \Leftrightarrow x + y - \frac{16}{9} = 0$$

Determinăm punctele de intersecție ale axelor de simetrie cu elipsa

$$\begin{cases} x - y + 2 = 0 \\ 5x^2 + 8xy + 5y^2 - 14x - 18y + 9 = 0 \end{cases} \Rightarrow x = y - 2$$

$$5(y^2-4y+4)+8(y^2-2y)+5y^2-32y+37=0$$

$$18y^2-68y+57=0$$

$$y_{1,2}=\frac{17}{9}\pm\frac{\sqrt{130}}{18}$$

$$y_1=\frac{17}{9}+\frac{\sqrt{130}}{18} \Rightarrow x_1=\frac{17}{9}+\frac{\sqrt{130}}{18}-2=-\frac{1}{9}+\frac{\sqrt{130}}{18}$$

$$y_2=\frac{17}{9}-\frac{\sqrt{130}}{18} \Rightarrow x_2=-\frac{1}{9}-\frac{\sqrt{130}}{18}$$

$$\Rightarrow A\left(-\frac{1}{9}+\frac{\sqrt{130}}{18}, \frac{17}{9}+\frac{\sqrt{130}}{18}\right), \quad A'\left(-\frac{1}{9}-\frac{\sqrt{130}}{18}, \frac{17}{9}-\frac{\sqrt{130}}{18}\right)$$

$$\begin{cases} x+y-\frac{16}{9}=0 & \Rightarrow x=\frac{16}{9}-y \\ 5x^2+8xy+5y^2-14x-18y+9=0 \end{cases}$$

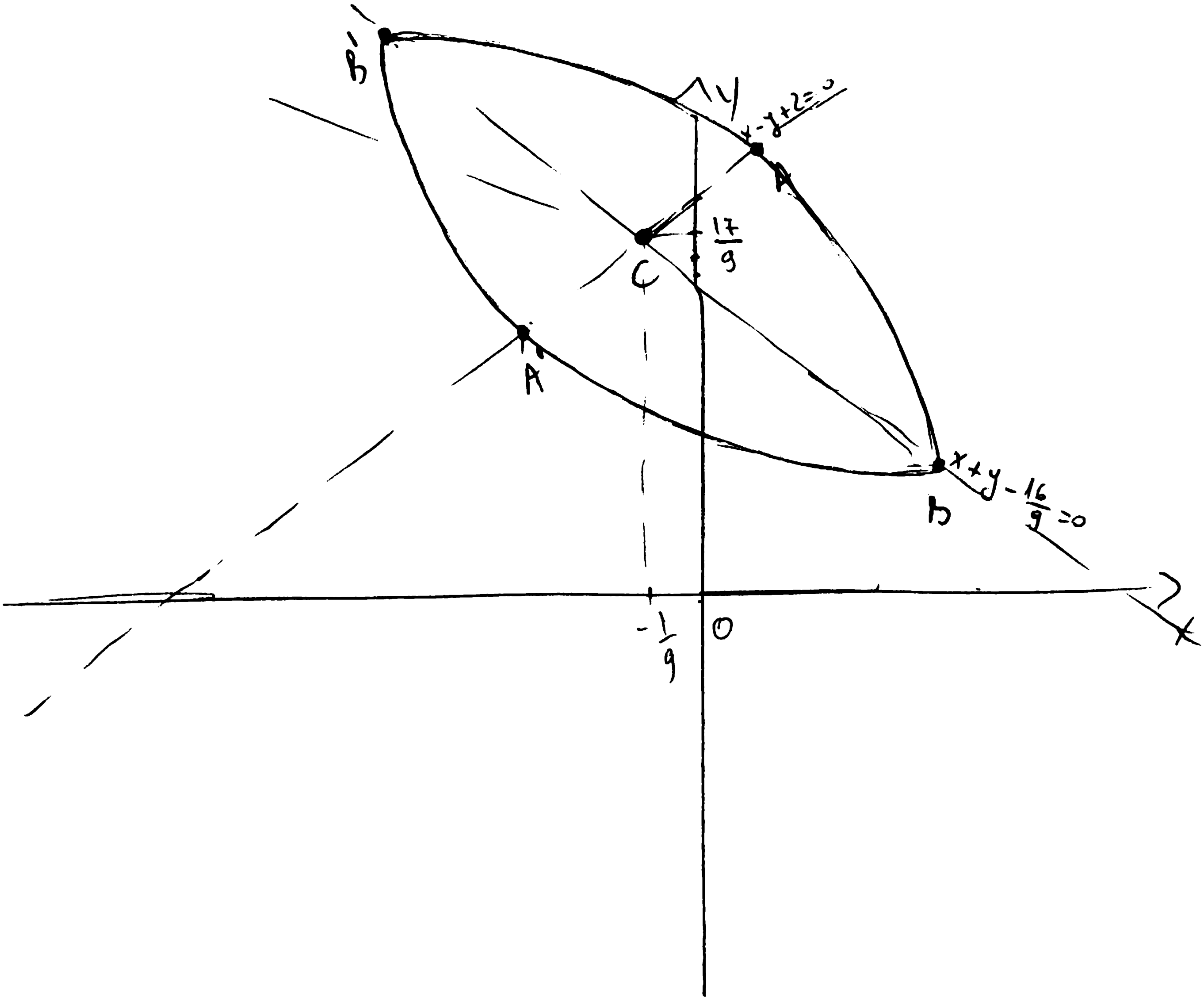
$$5\left(\frac{16}{9}-y\right)^2+8\left(\frac{16}{9}-y\right)y+5y^2-14\left(\frac{16}{9}-y\right)-18y+9=0$$

$$= \dots = 2y^2-\frac{68}{9}y-\frac{7}{81}=0.$$

$$y_{1,2}=\frac{17}{9}\pm\frac{\sqrt{130}}{6} \Rightarrow x_{1,2}=\frac{16}{9}-\frac{17}{9}\mp\frac{\sqrt{130}}{6}=-\frac{1}{9}\mp\frac{\sqrt{130}}{6}$$

$$\Rightarrow B\left(-\frac{1}{9}+\frac{\sqrt{130}}{6}, \frac{17}{9}-\frac{\sqrt{130}}{6}\right), \quad B'\left(-\frac{1}{9}-\frac{\sqrt{130}}{6}, \frac{17}{9}+\frac{\sqrt{130}}{6}\right)$$

Pt. a desena elipsa fixam centrul, ducem cele 2 axe de simetrie și fixam punctele



y
x - y + 2 = 0
A
$\frac{17}{9}$
C
A
x + y - $\frac{16}{9}$ = 0
B
B'
$-\frac{1}{9}$
O
x