(11) – Rezolvare.

a) $\Gamma_1 : x^2 - 4xy + 4y^2 - 6x + 2y + 1 = 0$

$$A_\Gamma = \begin{pmatrix} 1 & -2 & -3 \\ -2 & 4 & -2 \\ -3 & -2 & 1 \end{pmatrix}$$

$$\Gamma_1 : \underset{a_{11}}{1} \cdot x^2 \underbrace{-2 \cdot 2}_{a_{12}} xy + \underset{a_{22}}{4} \cdot y^2 \underbrace{-2 \cdot 3}_{a_{10}} x \underbrace{-1 \cdot 2}_{a_{20}} y + \underset{a_{00}}{1} = 0$$

a) b)
$$\Delta = \begin{vmatrix} 1 & -2 & -3 \\ -2 & 4 & -2 \\ -3 & -2 & 1 \end{vmatrix} = \cancel{4} - 12 - 12 - 36 \cancel{+4} - 4 = -64 \neq 0$$
$\Rightarrow$ conică nedegenerată

$$\delta = \begin{vmatrix} 1 & -2 \\ -2 & 4 \end{vmatrix} = 4 - 4 = 0 \Rightarrow \text{tip parabolic}$$
(conică fără centru)

c) $A_\delta = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$

Determinăm vectorii și valorile proprii acestei matrice

$$A - \lambda I_2 = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} 1-\lambda & -2 \\ -2 & 4-\lambda \end{pmatrix}$$

$$\det(A - \lambda I_2) = \begin{vmatrix} 1-\lambda & -2 \\ -2 & 4-\lambda \end{vmatrix} = (1-\lambda)(4-\lambda) - 4$$
$$= 4 - 5\lambda + \lambda^2 - 4$$
$$= \lambda^2 - 5\lambda$$

$\lambda^2 - 5\lambda = 0 \Rightarrow \lambda(\lambda - 5) = 0 \Rightarrow \lambda_1 = 0, \ \lambda_2 = 5$

Pt. $\lambda_1 = 0$

$(A - \lambda_1 I)\vec{u} = \vec{0} \Leftrightarrow A\vec{u} = \vec{0} \Leftrightarrow \begin{pmatrix} 1 & -2 \\ \ldots \end{pmatrix} \begin{pmatrix} u_1 \\ \ldots \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} u_1-2u_2 \\ -2u_1+4u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} u_1-2u_2=0 \\ -2u_1+4u_2=0 \end{matrix} \Rightarrow u_1=2u_2$$

Pt. $u_2=1 \Rightarrow \vec{u}_{\lambda_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Pt. $\lambda_2=5$

$$(A-\lambda_2 I_2)\vec{u} = (A-5I_2)\vec{u} = \begin{pmatrix} -4 & -2 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -4u_1-2u_2 \\ -2u_1-u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\Rightarrow \begin{matrix} -4u_1-2u_2=0 \\ -2u_1-u_2=0 \end{matrix} \Rightarrow u_2=-2u_1$

Pt. $u_1=-1 \Rightarrow \vec{u}_{\lambda_2} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

iii) $\|\vec{u}_{\lambda_1}\| = \sqrt{2^2+1^2} = \sqrt{5}$

$\|\vec{u}_{\lambda_2}\| = \sqrt{(-1)^2+2^2} = \sqrt{5}$

Vectorii proprii ortonormați sunt: $\vec{u}^{\,n}_{\lambda_1} = \begin{pmatrix} \frac{2}{\sqrt5} \\ \frac{1}{\sqrt5} \end{pmatrix}$

$\vec{u}^{\,n}_{\lambda_2} = \begin{pmatrix} \frac{-1}{\sqrt5} \\ \frac{2}{\sqrt5} \end{pmatrix}$

$$\Rightarrow T = \begin{pmatrix} \frac{2}{\sqrt5} & -\frac{1}{\sqrt5} \\ \frac{1}{\sqrt5} & \frac{2}{\sqrt5} \end{pmatrix}$$

Aplicăm rotația $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt5} & -\frac{1}{\sqrt5} \\ \frac{1}{\sqrt5} & \frac{2}{\sqrt5} \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt5}x' - \frac{1}{\sqrt5}y' \\ \frac{1}{\sqrt5}x' + \frac{2}{\sqrt5}y' \end{pmatrix}$

$\Rightarrow x = \frac{2}{\sqrt5}x' - \frac{1}{\sqrt5}y'$

$y = \frac{1}{\sqrt5}x' + \frac{2}{\sqrt5}y'$

Înlocuim în ecuația conicei.

$$\left(\frac{2}{\sqrt5}x'-\frac{1}{\sqrt5}y'\right)^2-4\left(\frac{2}{\sqrt5}x'-\frac{1}{\sqrt5}y'\right)\left(\frac{1}{\sqrt5}x'+\frac{2}{\sqrt5}y'\right)+4\left(\frac{1}{\sqrt5}x'+\frac{2}{\sqrt5}y'\right)^2$$
$$-6\left(\frac{2}{\sqrt5}x'-\frac{1}{\sqrt5}y'\right)+2\left(\frac{1}{\sqrt5}x'+\frac{2}{\sqrt5}y'\right)+1=0$$

$$\frac{4}{5}(x')^2-2\cdot\frac{2}{\sqrt5}\cdot\frac{1}{\sqrt5}x'y'+\frac{1}{5}(y')^2-4\left(\frac{2}{5}(x')^2+\frac{4}{5}x'y'-\frac{1}{5}\cancel{x'}y'\right.$$
$$\left.-\frac{2}{5}(y')^2\right)+4\left(\frac{1}{5}(x')^2+\frac{4}{5}x'y'+\frac{4}{5}(y')^2\right)-\frac{12}{\sqrt5}x'+\frac{6}{\sqrt5}y'$$
$$+\frac{2}{\sqrt5}x'+\frac{4}{\sqrt5}y'+1=0$$

$$\cancel{\frac{4}{5}(x')^2}-\cancel{\frac{4}{5}x'y'}+\frac{1}{5}(y')^2-\cancel{\frac{8}{5}(x')^2}-\cancel{\frac{16}{5}x'y'}+\cancel{\frac{4}{5}x'y'}+\frac{8}{5}(y')^2$$
$$+\cancel{\frac{4}{5}(x')^2}+\cancel{\frac{16}{5}x'y'}+\frac{16}{5}(y')^2-\frac{12}{\sqrt5}x'+\frac{6}{\sqrt5}y'+\frac{2}{\sqrt5}x'+\frac{4}{\sqrt5}y'+1=0$$

$$\frac{25}{5}(y')^2-\frac{10}{\sqrt5}x'+\frac{10}{\sqrt5}y'+1=0.$$

$$\frac{25}{5}(y')^2+\frac{10}{\sqrt5}y'+1-\frac{10}{\sqrt5}x'=0.$$
$$(\sqrt5y'+1)^2=\frac{10}{\sqrt5}x' \iff 5\left(y'+\frac{1}{\sqrt5}\right)^2=\frac{10}{\sqrt5}x'$$

Facem translația: $y=y'+\frac{1}{\sqrt5}$
$x=x'$

$\Rightarrow 5y^2=\frac{10}{\sqrt5}x$ (Ec. parabolei)

$$5y^2=\frac{10\sqrt5}{5}x$$
$$5y^2=2\sqrt5x$$
$$y^2=\frac{2\sqrt5}{5}x$$

h) $\Gamma_2 : 4xy + 3y^2 + 16x + 12y - 36 = 0$

$$\underbrace{4xy}_{2a_{12}} + \underbrace{3y^2}_{a_{22}} + \underbrace{16x}_{a_{10}} + \underbrace{12y}_{a_{20}} \underbrace{-36}_{a_{00}}$$

$$A_\Gamma = \begin{pmatrix} 0 & 2 & 8 \\ 2 & 3 & 6 \\ 8 & 6 & -36 \end{pmatrix}$$

a) $\Delta = \begin{vmatrix} 0 & 2 & 8 \\ 2 & 3 & 6 \\ 8 & 6 & -36 \end{vmatrix} = 2 \cdot 2 \begin{vmatrix} 0 & 1 & 4 \\ 2 & 3 & 6 \\ 4 & 3 & -18 \end{vmatrix} = 8 \begin{vmatrix} 0 & 1 & 4 \\ 1 & 3 & 6 \\ 2 & 3 & -18 \end{vmatrix}$

$= 8\,(12 + 12 - 24 - 0 + 18) = 8 \cdot 18 \neq 0 \Rightarrow$ conică nedegenerată

$\delta = \begin{vmatrix} 0 & 2 \\ 2 & 3 \end{vmatrix} = -4 < 0 \Rightarrow$ tip hiperbolic

$\delta \neq 0 \Rightarrow$ conică cu centru

b) $g(x,y) = 4xy + 3y^2 + 16x + 12y - 36$

$\frac{1}{2} g'_x(x,y) = \frac{1}{2}(4y + 16) = 2y + 8$

$\frac{1}{2} g'_y(x,y) = \frac{1}{2}(4x + 6y + 12) = 2x + 3y + 6$

$\begin{cases} 2y + 8 = 0 \\ 2x + 3y + 6 = 0 \end{cases} \Rightarrow \begin{array}{l} y = -4 \\ 2x - 12 + 6 = 0 \Rightarrow 2x = 6 \Rightarrow x = 3 \end{array}$

$\Rightarrow C(3, -4) \rightarrow$ centrul conicei

c) $A_\delta = \begin{pmatrix} 0 & 2 \\ 2 & 3 \end{pmatrix}$

Determinăm vectorii și valorile proprii acestei matrice.

$A - \lambda I_2 = \begin{pmatrix} 0 & 2 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} -\lambda & 2 \\ 2 & 3-\lambda \end{pmatrix}$

$$\det(A-\lambda I_2)=\begin{vmatrix}-\lambda & 2\\ 2 & 3-\lambda\end{vmatrix}=-\lambda(3-\lambda)-4=-3\lambda+\lambda^2-4$$

$$\lambda^2-3\lambda-4=0$$

$$\Delta_\lambda=9+16=25.$$

$$\lambda_{1,2}=\frac{3\pm\sqrt{25}}{2}=\frac{3\pm5}{2}\begin{cases}4\\-1\end{cases}$$

Pt. $\lambda_1=4$.

$$(A-\lambda_1 I_2)\vec{u}=\begin{pmatrix}-4 & 2\\ 2 & -1\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix}=\begin{pmatrix}-4u_1+2u_2\\ 2u_1-u_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$\begin{cases}-4u_1+2u_2=0\\ 2u_1-u_2=0\end{cases}\Rightarrow u_2=2u_1$$

Fie $u_1=1 \Rightarrow u_2=2 \Rightarrow \vec{u}_{\lambda_1}=\begin{pmatrix}1\\2\end{pmatrix}$

Pt. $\lambda_2=-1$

$$(A-\lambda_2 I_2)\vec{u}=\begin{pmatrix}1 & 2\\ 2 & 4\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix}=\begin{pmatrix}u_1+2u_2\\ 2u_1+4u_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$\begin{cases}u_1+2u_2=0\\ 2u_1+4u_2=0\end{cases}\quad u_1=-2u_2$$

Fie $u_2=1 \Rightarrow u_1=-2 \Rightarrow \vec{u}_{\lambda_2}=\begin{pmatrix}-2\\1\end{pmatrix}$

$$\|\vec{u}_{\lambda_1}\|=\sqrt{1+4}=\sqrt{5}$$

$$\|\vec{u}_{\lambda_2}\|=\sqrt{4+1}=\sqrt{5}$$

Vectorii ortonormați: $\frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt{5}}\\ \frac{2}{\sqrt{5}}\end{pmatrix}$

$$\frac{1}{\sqrt{5}}\begin{pmatrix}-2\\1\end{pmatrix}=\begin{pmatrix}-\frac{2}{\sqrt{5}}\\ \frac{1}{\sqrt{5}}\end{pmatrix}$$

$$\Rightarrow T=\begin{pmatrix}\frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}}\\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}}\end{pmatrix}$$

Aplicăm rotația $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt5} & \frac{-2}{\sqrt5}\\ \frac{2}{\sqrt5} & \frac{1}{\sqrt5}\end{pmatrix}\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt5}x' - \frac{2}{\sqrt5}y'\\ \frac{2}{\sqrt5}x' + \frac{1}{\sqrt5}y'\end{pmatrix}$

$$x = \frac{1}{\sqrt5}x' - \frac{2}{\sqrt5}y'$$

$$y = \frac{2}{\sqrt5}x' + \frac{1}{\sqrt5}y'$$

Înlocuim în ecuația conicei:

$$\Rightarrow 4\left(\frac{1}{\sqrt5}x' - \frac{2}{\sqrt5}y'\right)\left(\frac{2}{\sqrt5}x' + \frac{1}{\sqrt5}y'\right) + 3\left(\frac{2}{\sqrt5}x' + \frac{1}{\sqrt5}y'\right)^2$$

$$+ 16\left(\frac{1}{\sqrt5}x' - \frac{2}{\sqrt5}y'\right) + 12\left(\frac{2}{\sqrt5}x' + \frac{1}{\sqrt5}y'\right) - 36 = 0$$

$$4\left(\frac{2}{5}(x')^2 + \frac{1}{5}x'y' - \frac{4}{5}x'y' - \frac{2}{5}(y')^2\right) + 3\left(\frac{4}{5}(x')^2 + \frac{4}{5}x'y' + \frac{1}{5}(y')^2\right)$$

$$+ \frac{16}{\sqrt5}x' - \frac{32}{\sqrt5}y' + \frac{24}{\sqrt5}x' + \frac{12}{\sqrt5}y' - 36 = 0$$

$$\frac{8}{5}(x')^2 + \cancel{\frac{4}{5}x'y'} - \cancel{\frac{16}{5}x'y'} - \frac{8}{5}(y')^2 + \frac{12}{5}(x')^2 + \cancel{\frac{12}{5}x'y'} + \frac{3}{5}(y')^2$$

$$+ \frac{16}{\sqrt5}x' - \frac{32}{\sqrt5}y' + \frac{24}{\sqrt5}x' + \frac{12}{\sqrt5}y' - 36 = 0$$

$$4(x')^2 - (y')^2 + \frac{40}{\sqrt5}x' - \frac{20}{\sqrt5}y' - 36 = 0$$

$$4(x')^2 - (y')^2 + \frac{40\sqrt5}{5}x' - \frac{20\sqrt5}{5}y' - 36 = 0$$

$$4(x')^2 - (y')^2 + 8\sqrt5 x' - 4\sqrt5 y' - 36 = 0$$

$$\left(4(x')^2 + 2\cdot\underbrace{2\sqrt5}\cdot 2x' + 20\right) - \left((y')^2 + 2\cdot y'\cdot 2\sqrt5 + 20\right) = 36$$

$$(2x' + 2\sqrt5)^2 - (y' + 2\sqrt5)^2 = 36 \Leftrightarrow 4(x' + \sqrt5)^2 - (y' + 2\sqrt5)^2 = 36$$

Facem translația $X = x' + \sqrt5 \Rightarrow 4X^2 - Y^2 = 6^2$

c) $\Gamma_3: \; \underset{a_{11}}{\underset{\downarrow}{5x^2}} + \underset{2a_{12}}{\underset{\downarrow}{8xy}} + \underset{a_{22}}{\underset{\downarrow}{6y^2}} - \underset{2a_{10}}{\underset{\downarrow}{14x}} - \underset{2a_{20}}{\underset{\downarrow}{18y}} + \underset{a_{00}}{\underset{\downarrow}{9}} = 0$

$$A_\Gamma = \begin{pmatrix} 5 & 4 & -7 \\ 4 & 6 & -9 \\ -7 & -9 & 9 \end{pmatrix}$$

a) $\Delta = \begin{vmatrix} 5 & 4 & -7 \\ 4 & 6 & -9 \\ -7 & -9 & 9 \end{vmatrix} = -69 \neq 0 \Rightarrow$ conică nedegenerată

$\delta = \begin{vmatrix} 5 & 4 \\ 4 & 6 \end{vmatrix} = 6 > 0$ − tip eliptic

$\delta \neq 0 \Rightarrow$ conică cu centru

a) $g(x,y) = 5x^2 + 8xy + 6y^2 - 14x - 18y + 9$

$\frac{1}{2} g'_x(x,y) = \frac{1}{2}(10x + 8y - 14) = 5x + 4y - 7$

$\frac{1}{2} g'_y(x,y) = \frac{1}{2}(8x + 12y - 18) = 4x + 6y - 9$

$\begin{cases} 5x + 4y = 7 & \mid -4 \\ 4x + 6y = 9 & \mid 5 \end{cases} \Leftrightarrow \begin{cases} -20x - 16y = -28 \\ 20x + 30y = 45 \end{cases}$

$14y = 17 \Rightarrow y = \frac{17}{14} \Rightarrow x = \frac{3}{7}$

$C\left(\frac{3}{7}, \frac{17}{14}\right)$

c) Calculăm valorile și vectorii proprii matricei $A = \begin{pmatrix} 5 & 4 \\ 4 & 6 \end{pmatrix}$

$A - \lambda I = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} 5-\lambda & 4 \\ 4 & 5-\lambda \end{pmatrix}$

$\det(A - \lambda I) = (5-\lambda)(5-\lambda) - 16 = 25 - 10\lambda + \lambda^2 - 16 = \lambda^2 - 10\lambda + 9$

$\lambda^2 - 10\lambda + 9 = 0$

$\Delta = 100 - 36 = 64$

$\ldots 10 \pm 8 \ldots = 9$

Pt. $\lambda_1 = 9$

$$(A-\lambda_1 I_2)\begin{pmatrix}u_1\\u_2\end{pmatrix} = \begin{pmatrix}-4 & 4\\ 4 & -4\end{pmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix} = \begin{pmatrix}-4u_1+4u_2\\ 4u_1-4u_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$-4u_1+4u_2=0 \Rightarrow u_1=u_2$

Fie $u_1=1 \Rightarrow u_2=1 \quad \Rightarrow \vec{u}_{\lambda_1} = \begin{pmatrix}1\\1\end{pmatrix}$

$$\|\vec{u}_{\lambda_1}\| = \sqrt{1+1} = \sqrt{2}$$

$$\vec{u}^{\,n}_{\lambda_1} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}$$

Pt. $\lambda_2 = 1$

$$(A-\lambda_2 I_2)\begin{pmatrix}u_1\\u_2\end{pmatrix} = \begin{pmatrix}4 & 4\\ 4 & 4\end{pmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix} = \begin{pmatrix}4u_1+4u_2\\ 4u_1+4u_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$4u_1+4u_2=0 \Rightarrow u_2=-u_1$

Fie $u_1=1 \Rightarrow \vec{u}_{\lambda_2} = \begin{pmatrix}1\\-1\end{pmatrix}$

$$\|\vec{u}_{\lambda_2}\| = \sqrt{1+1} = \sqrt{2}$$

$$\vec{u}^{\,n}_{\lambda_2} = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix}$$

$$\Rightarrow T = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}$$

Aplicăm Rotația $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}x'+\frac{1}{\sqrt{2}}y'\\ \frac{1}{\sqrt{2}}x'-\frac{1}{\sqrt{2}}y'\end{pmatrix}$

$$x = \frac{1}{\sqrt{2}}x' + \frac{1}{\sqrt{2}}y'$$

$$y = \frac{1}{\sqrt{2}}x' - \frac{1}{\sqrt{2}}y'$$

Înlocuim în ec. conicei

$$5\left(\frac{1}{\sqrt{2}}x'+\frac{1}{\sqrt{2}}y'\right)^2+8\left(\frac{1}{\sqrt{2}}x'+\frac{1}{\sqrt{2}}y'\right)\left(\frac{1}{\sqrt{2}}x'-\frac{1}{\sqrt{2}}y'\right)+$$

$$+5\left(\frac{1}{\sqrt{2}}x'-\frac{1}{\sqrt{2}}y'\right)^2-14\left(\frac{1}{\sqrt{2}}x'+\frac{1}{\sqrt{2}}y'\right)-18\left(\frac{1}{\sqrt{2}}x'-\frac{1}{\sqrt{2}}y'\right)+9=0$$

$$5\left(\frac{1}{2}(x')^2+2\cdot\frac{1}{2}x'y'+\frac{1}{2}(y')^2\right)+8\left(\frac{1}{2}(x')^2-\frac{1}{2}(y')^2\right)$$

$$+5\left(\frac{1}{2}(x')^2-x'y'+\frac{1}{2}(y')^2\right)-\frac{14}{\sqrt{2}}x'-\frac{14}{\sqrt{2}}y'-\frac{18}{\sqrt{2}}x'$$

$$+\frac{18}{\sqrt{2}}y'+9=0$$

$$\frac{5}{2}(x')^2\cancel{+5x'y'}+\frac{5}{2}(y')^2+4(x')^2-4(y')^2+\frac{5}{2}(x')^2\cancel{-5x'y'}$$

$$+\frac{5}{2}(y')^2-\frac{14}{\sqrt{2}}x'-\frac{14}{\sqrt{2}}y'-\frac{18}{\sqrt{2}}x'+\frac{18}{\sqrt{2}}y'+9=0$$

$$9(x')^2+(y')^2-\frac{32}{\sqrt{2}}x'+\frac{4}{\sqrt{2}}y'+9=0$$

$$9(x')^2+(y')^2-\frac{32\sqrt{2}}{2}x'+\frac{4\sqrt{2}}{2}y'+9=0$$

$$9(x')^2-16\sqrt{2}x'+(y')^2+2\sqrt{2}y'+9=0$$

$$\left(9(x')^2-2\cdot 3x'\cdot\frac{8}{3}\sqrt{2}+\frac{128}{9}\right)+\left((y')^2+2\cdot y'\cdot\sqrt{2}+2\right)=-9+\frac{128}{9}+2$$

$$\left(3x'+\frac{8\sqrt{2}}{3}\right)^2+\left(y'+\sqrt{2}\right)^2=\frac{-81+128+18}{9}=\frac{65}{9}=\left(\frac{\sqrt{65}}{3}\right)^2$$

$$9\left(x'+\frac{8\sqrt{2}}{9}\right)^2+\left(y'+\sqrt{2}\right)^2=\left(\frac{\sqrt{65}}{3}\right)^2$$

Aplicăm translația $x=x'+\frac{8\sqrt{2}}{9}$

$y=y'+\sqrt{2}$

$$\Rightarrow x^2+y^2=\left(\frac{\sqrt{65}}{3}\right)^2\Rightarrow\frac{9x^2}{\left(\frac{\sqrt{65}}{3}\right)^2}+\frac{y^2}{\left(\frac{\sqrt{65}}{3}\right)^2}=1$$