b) $x'' + 9x = t\sin t + \cos 3t$

- $x'' + 9x = 0$

$\left.\begin{array}{l} x = e^{nt} \\ x' = n e^{nt} \\ x'' = n^2 e^{nt} \end{array}\right\}$ $\Rightarrow$ $n^2 e^{nt} + 9 e^{nt} = 0 \mid : e^{nt}$

$n^2 + 9 = 0 \Leftrightarrow n^2 = -9 \Leftrightarrow n_{1,2} = \pm 3i \; (\alpha \pm \beta i, \alpha = 0, \beta = 3)$

• $\sin 3t, \cos 3t$ - sist. fund. de sol.

$$\boxed{x_0 = c_1 \sin 3t + c_2 \cos 3t}$$

• $\alpha = 0, \beta = 3$

$x_p = t \cdot e^{0t}\left((\lambda_1 t + \lambda_0)\cos 3t + (\beta_1 t + \beta_0)\sin 3t\right)$

$\quad = (\lambda_1 t^2 + \lambda_0 t)\cos 3t + (\beta_1 t^2 + \beta_0 t)\sin 3t$

$x_p' = (2\lambda_1 t + \lambda_0)\cos 3t + (\lambda_1 t^2 + \lambda_0 t)\cdot(-3)\sin 3t + (2\beta_1 t + \beta_0)\sin 3t +$

$\quad + (\beta_1 t^2 + \beta_0 t) 3\cos 3t$

$\quad = \cos 3t(2\lambda_1 t + \lambda_0 + 3\beta_1 t^2 + 3\beta_0 t) + \sin 3t(-3\lambda_1 t^2 - 3\lambda_0 t + 2\beta_1 t + \beta_0)$

$x_p'' = (2\lambda_1 + 6\beta_1 t + 3\beta_0)\cos 3t + (2\lambda_1 t + \lambda_0 + 3\beta_1 t^2 + 3\beta_0 t)\cdot(-3)\sin 3t +$

$\quad + (-6\lambda_1 t - 3\lambda_0 + 2\beta_1)\sin 3t + (-3\lambda_1 t^2 - 3\lambda_0 t + 2\beta_1 t + \beta_0)\cos 3t \cdot 3$

$\quad = (2\lambda_1 + 12\beta_1 t - 9\lambda_0 t - 9\lambda_1 t^2 + 6\beta_0)\cos 3t + (-12\lambda_1 t - 6\lambda_0 - 9\beta_1 t^2 - 9\beta_0 t + 2\beta_1)\cdot$

$\quad \cdot \sin 3t$

• $(-9\lambda_1 t^2 + 12\beta_1 t - 9\lambda_0 t + 2\lambda_1 + 6\beta_0)\cos 3t + (-9\beta_1 t^2 - 12\lambda_1 t - 9\beta_0 t - 6\lambda_0 + 2\beta_1)\cdot$

$\cdot \sin 3t + (9\lambda_1 t^2 + 9\lambda_0 t)\cos 3t + (9\beta_1 t^2 + 9\beta_0 t)\sin 3t = t\sin 3t + \cos 3t$

$\Leftrightarrow \begin{cases} -9\lambda_1 t^2 + 12\beta_1 t - 9\lambda_0 t + 2\lambda_1 + 6\beta_0 + 9\lambda_1 t^2 + 9\lambda_0 t = 1 \\ -9\beta_1 t^2 - 12\lambda_1 t - 9\beta_0 t - 6\lambda_0 + 2\beta_1 + 9\beta_1 t^2 + 9\beta_0 t = t \end{cases}$

$\Rightarrow \begin{cases} 12\beta_1 = 0 \\ 2\lambda_1 + 6\beta_0 = 1 \\ -12\lambda_1 = 1 \\ -6\lambda_0 + 2\beta_1 = 0 \end{cases}$ $\Leftrightarrow \begin{cases} \beta_1 = 0 \\ \lambda_0 = 0 \\ \lambda_1 = \dfrac{-1}{12} \\ \beta_0 = \dfrac{7}{36} \end{cases}$

• $x_p = t\left(\dfrac{-1}{12}t\cos 3t + \dfrac{7}{36}\sin 3t\right)$

• $x(t) = x_0 + x_p = c_1\cos 3t + c_1\sin 3t + t\left(\dfrac{-1}{12}t\cos 3t + \dfrac{7}{36}\sin 3t\right)$