Laborator11 - Temă

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Exerciţiul 1.0.1
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Exerciţiul 1.0.1

Se consideră formula

$$\alpha = ((\neg a \lor (b \land \neg c)) \leftrightarrow (a \lor (\neg b \to \neg(c \land a))))$$

şi substituţia

$$\sigma = \{(x \lor \neg m)|\alpha, (d \land \neg t)|a, (q \lor p)|m, a|q\}$$

Să se determine:

- secvenţa generativă formule (SGF) pentru formula α
- tabelul de adevăr pentru formula α
- arborele de structură pentru formula α
- $\alpha\sigma$ - rezultatul aplicării substituţiei σ şi arborele de structură asociat lui $\alpha\sigma$

SGF:

$$a, b, c, \neg a, \neg b, \neg c, b \land \neg c, \neg a \lor (b \land \neg c), c \land a, \neg(c \land a), \neg b \to \neg(c \land a), a \lor (\neg b \to \neg(c \land a))$$
$$(\neg a \lor (b \land \neg c)) \leftrightarrow (a \lor (\neg b \to \neg(c \land a))) = \alpha$$

Tabel de Adevăr:

a	b	c	$b \land \neg c$	$\neg a \lor (b \land \neg c)$	$\neg(c \land a)$	$\neg b \to \neg(c \land a)$	$a \lor (\neg b \to \neg(c \land a))$	α
T	T	T	F	F	F	T	T	F
T	T	F	T	T	T	T	T	T
T	F	T	F	F	F	F	T	F
T	F	F	F	F	T	T	T	F
F	T	T	F	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Arbore de Structură:

$$T(\alpha):\ \begin{array}{c} r \\ \swarrow\ \searrow \\ T(\beta)\quad T(\gamma) \end{array}, \varphi(r) = \leftrightarrow, \beta = \neg a \vee (b \wedge \neg c), \gamma = a \vee (\neg b \to \neg(c \wedge a))$$

$$T(\beta):\ \begin{array}{c} n_1 \\ \swarrow\ \searrow \\ T(\beta_1)\quad T(\beta_2) \end{array}, \varphi(n_1) = \vee, \beta_1 = \neg a, \beta_2 = b \wedge \neg c$$

$$T(\beta_1):\ \begin{array}{c} n_2 \\ \downarrow \\ T(\beta_3) \end{array}, \varphi(n_2) = \neg, \beta_3 = a$$

$$T(\beta_3) = n_3, \varphi(n_3) = a$$

$$T(\beta):\ \begin{array}{c} n_4 \\ \swarrow\ \searrow \\ T(\beta_4)\quad T(\beta_5) \end{array}, \varphi(n_4) = \wedge, \beta_4 = b, \beta_5 = \neg c$$

$$T(\beta_4) = n_5, \varphi(n_5) = b$$

$$T(\beta_5):\ \begin{array}{c} n_6 \\ \downarrow \\ T(\beta_6) \end{array}, \varphi(n_6) = \neg, \beta_6 = c$$

$$T(\beta_6) = n_7, \varphi(n_7) = c$$

$$T(\gamma):\ \begin{array}{c} n_8 \\ \swarrow\ \searrow \\ T(\gamma_1)\quad T(\gamma_2) \end{array}, \varphi(n_8) = \wedge, \gamma_1 = a, \gamma_2 = \neg b \to \neg(c \wedge a)$$

$$T(\gamma_1) = n_9, \varphi(n_9) = a$$

$$T(\gamma_2):\ \begin{array}{c} n_{10} \\ \swarrow\ \searrow \\ T(\gamma_3)\quad T(\gamma_4) \end{array}, \varphi(n_{10}) = \to, \gamma_3 = \neg b, \gamma_4 = \neg(c \wedge a)$$

$$T(\gamma_3):\ \begin{array}{c} n_{11} \\ \downarrow \\ T(\gamma_5) \end{array}, \varphi(n_{11}) = \neg, \gamma_5 = b$$

$$T(\gamma_5) = n_{12}, \varphi(n_{12}) = b$$

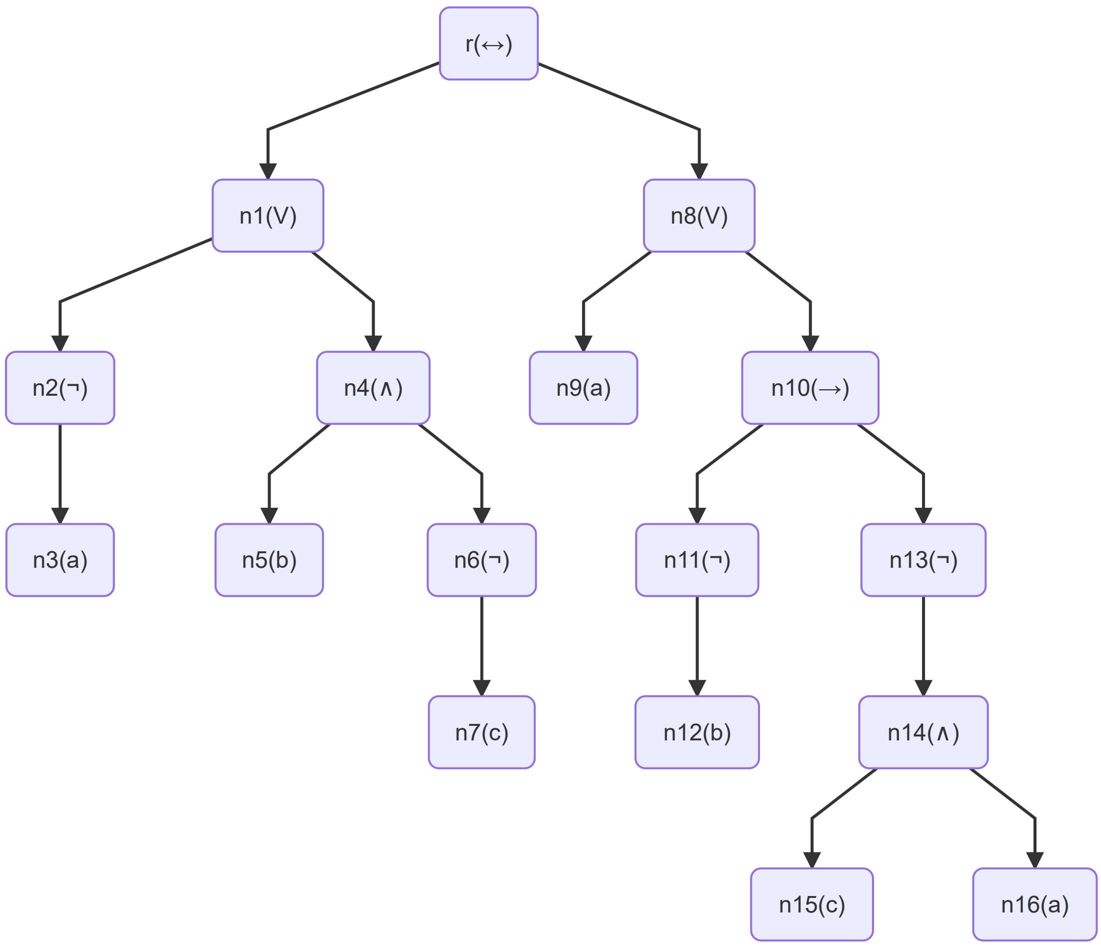
$$T(\gamma_4):\ \begin{array}{c} n_{13} \\ \downarrow \\ T(\gamma_6) \end{array}, \varphi(n_{13}) = \neg, \gamma_6 = c \wedge a$$

$$T(\gamma_6):\ \begin{array}{c} n_{14} \\ \swarrow\ \searrow \\ T(\gamma_7)\quad T(\gamma_8) \end{array}, \varphi(n_{14}) = \wedge, \gamma_7 = c, \gamma_8 = a$$

$$T(\gamma_7) = n_{15}, \varphi(n_{15}) = c$$

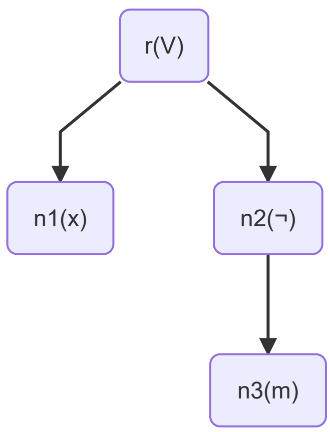
$$T(\gamma_8) = n_{16}, \varphi(n_{16}) = a$$

Final:



Aplicare substituţie $\alpha\sigma$:

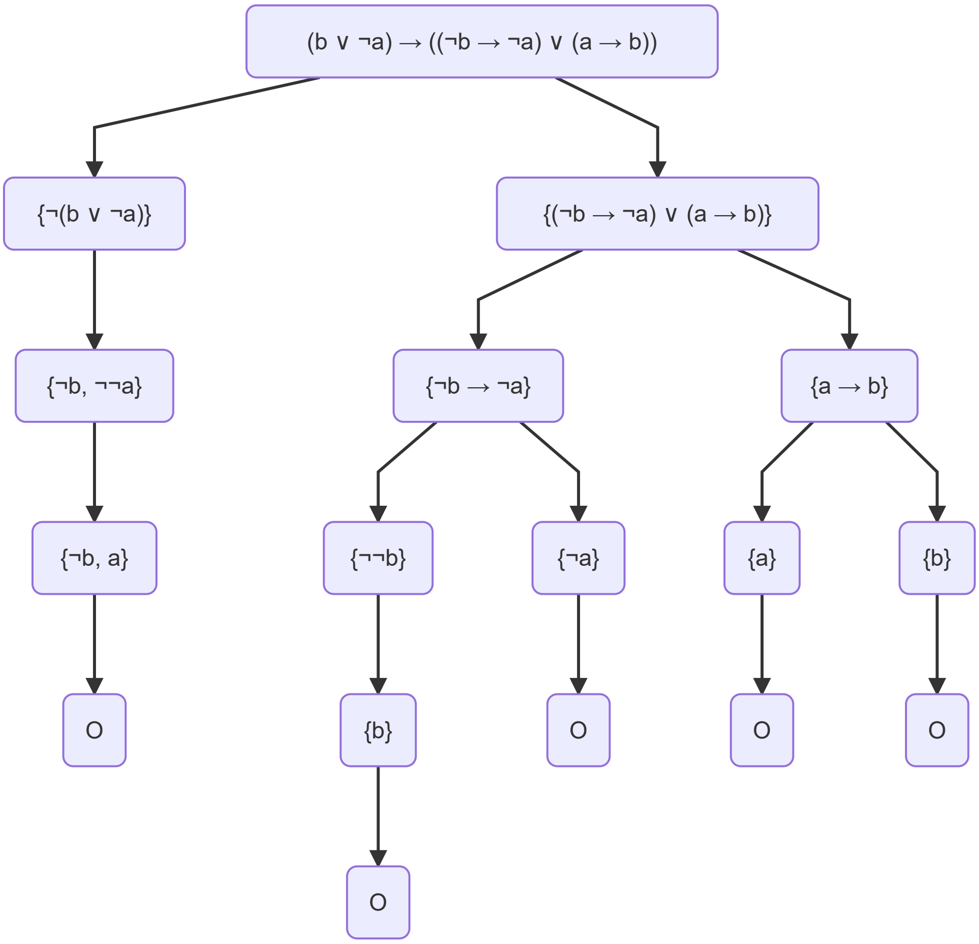
$$\alpha\sigma = x \vee \neg m$$



Exerciţiul 1.0.2

Se consideră formula $\alpha = (b \vee \neg a) \rightarrow ((\neg b \rightarrow \neg a) \vee (a \rightarrow b))$

a) Să se verifice validabilitatea formulei α prin aplicarea metodei arborilor semantici.



b) Să se determine rezultatul aplicării funcţiei de interpretare $I(\alpha)$ asupra formulei α

$$\begin{aligned} I(\alpha) &= I(b \vee \neg a) \rightarrow I((\neg b \rightarrow \neg a) \vee (a \rightarrow b)) \\ &= \neg(I(b) \vee \neg I(a)) \vee ((I(b) \vee \neg I(a)) \vee (\neg I(a) \vee I(b))) \\ &= (\neg I(b) \wedge I(a)) \vee (I(b) \vee \neg I(a)) \\ &= (\neg I(b) \vee I(b) \vee \neg I(a)) \wedge (I(a) \vee I(b) \vee \neg I(a)) \\ &= T \vee T = T \end{aligned}$$

Exerciţiul 1.0.3

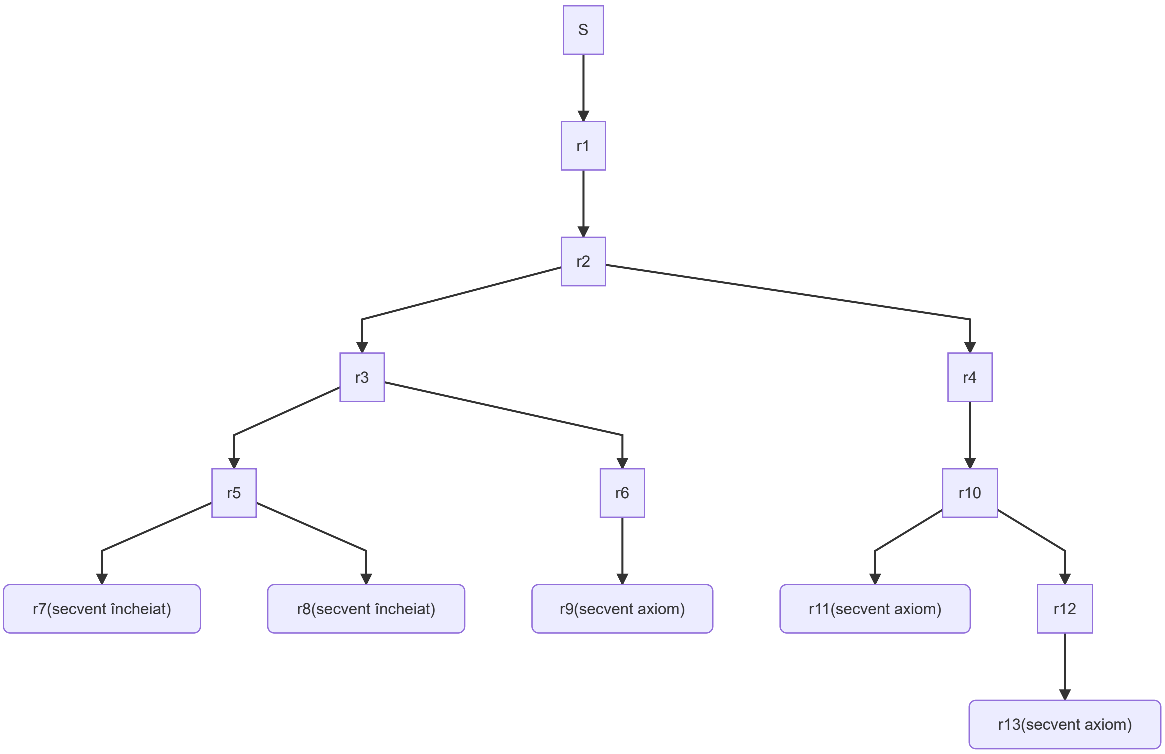
a) Să se verifice dacă următorul secvent este demonstrabil:

$$S = \{(\alpha \vee \neg\beta), (\beta \vee (\gamma \wedge \theta))\} \Rightarrow \{\neg\alpha \rightarrow (\theta \wedge \gamma)\}$$

Sistem:

$$
\begin{aligned}
& S = \{(\alpha \vee \neg\beta), (\beta \vee (\gamma \wedge \theta))\} \Rightarrow \{\neg\alpha \to (\theta \wedge \gamma)\} \\
G8: \ & r1 = \{\alpha \vee \neg\beta, \beta \vee (\gamma \wedge \theta), \neg\theta\} \Rightarrow \{\theta \wedge \gamma\} \\
G1: \ & r2 = \{\alpha \vee \neg\beta, \beta \vee (\gamma \wedge \theta)\} \Rightarrow \{\theta, \theta \wedge \gamma\} \\
G3: \ & r3 = \{\alpha, \beta \vee (\gamma \wedge \theta)\} \Rightarrow \{\theta, \theta \wedge \gamma\} \\
& r4 = \{\neg\beta, \beta \vee (\gamma \wedge \theta)\} \Rightarrow \{\theta, \theta \wedge \gamma\} \\
G3: \ & r5 = \{\alpha, \beta\} \Rightarrow \{\theta, \theta \wedge \gamma\} \\
& r6 = \{\alpha, \gamma \wedge \theta)\} \Rightarrow \{\theta, \theta \wedge \gamma\} \\
G6: \ & r7 = \{\alpha, \beta\} \Rightarrow \{\theta\} \ secvent \ incheiat \\
& r8 = \{\alpha, \beta\} \Rightarrow \{\theta, \gamma\} \ secvent \ incheiat \\
G2: \ & r9 = \{\alpha, \gamma, \theta)\} \Rightarrow \{\theta, \theta \wedge \gamma\} \ secvent \ axiom \\
G1: \ & r10 = \{\beta \vee (\gamma \wedge \theta)\} \Rightarrow \{\beta, \theta, \theta \wedge \gamma\} \\
G3: \ & r11 = \{\beta\} \Rightarrow \{\beta, \theta, \theta \wedge \gamma\} \ secvent \ axiom \\
& r12 = \{\gamma \wedge \theta\} \Rightarrow \{\beta, \theta, \theta \wedge \gamma\} \\
G2: \ & r13 = \{\gamma, \theta\} \Rightarrow \{\beta, \theta, \theta \wedge \gamma\} \ secvent \ axiom \\
& S \ nu \ e \ tautologie
\end{aligned}
$$

Schema:



b) Să se calculeze mulţimile α_λ^+, α_λ^-, α_λ^0, $POS_\lambda(\alpha)$, $NEG_\lambda(\alpha)$, $REZ_\lambda(\alpha)$ unde $\lambda = \beta$, respectiv $\lambda = \neg\delta$, iar

$$S(\alpha) = \{\neg\gamma \vee \beta \vee \neg\eta, \neg\beta \vee \delta \vee \neg\gamma, \neg\delta, \beta, \theta \vee \beta, \delta \vee \beta \vee \neg\theta, \gamma \vee \eta \vee \neg\delta\}$$

Pentru $\lambda = \beta$:

$$
\begin{aligned}
\alpha_\lambda^+ &= \{\neg\gamma \vee \beta \vee \neg\eta, \beta, \theta \vee \beta, \delta \vee \beta \vee \neg\theta\} \\
\alpha_\lambda^- &= \{\neg\beta \vee \delta \vee \neg\gamma\} \\
\alpha_\lambda^0 &= \{\neg\delta, \gamma \vee \eta \vee \neg\delta\} \\
POS_\lambda^\alpha &= \{\neg\delta, \gamma \vee \eta \vee \neg\delta, \neg\gamma \vee \neg\eta, \Box, \theta, \delta \vee \neg\theta\} \\
NEG_\lambda^\alpha &= \{\neg\delta, \gamma \vee \eta \vee \neg\delta, \delta \vee \neg\gamma\} \\
REZ_\lambda^\alpha &= \{\neg\delta, \gamma \vee \eta \vee \neg\delta, \delta \vee \neg\gamma \vee \neg\eta, \delta \vee \neg\gamma \vee \Box, \delta \vee \neg\gamma \vee \theta, \delta \vee \neg\gamma \vee \neg\theta\}
\end{aligned}
$$

Pentru $\lambda = \neg\delta$:

$$
\begin{aligned}
\alpha_\lambda^+ &= \{\neg\delta, \gamma \vee \eta \vee \neg\delta\} \\
\alpha_\lambda^- &= \{\neg\beta \vee \delta \vee \neg\gamma, \delta \vee \beta \vee \neg\theta\} \\
\alpha_\lambda^0 &= \{\neg\gamma \vee \beta \vee \neg\eta, \beta, \theta \vee \beta\} \\
POS_\lambda^\alpha &= \{\neg\gamma \vee \beta \vee \neg\eta, \beta, \theta \vee \beta, \Box, \gamma \vee \eta\} \\
NEG_\lambda^\alpha &= \{\neg\gamma \vee \beta \vee \neg\eta, \beta, \theta \vee \beta, \neg\beta \vee \neg\gamma, \beta \vee \neg\theta\} \\
REZ_\lambda^\alpha &= \{\neg\gamma \vee \beta \vee \neg\eta, \beta, \theta \vee \beta, \neg\beta \vee \neg\gamma \vee \Box, \neg\beta \vee \eta, \beta \vee \neg\theta \vee \Box, \beta \vee \neg\theta \vee \gamma \vee \eta\}
\end{aligned}
$$

Exerciţiul 1.0.4

Să se determine forma normală conjunctivă (CNF) şi să se aplice algoritmul bazat pe rezoluţie pentru formula

$$\alpha = ((b \to \neg a) \leftrightarrow (\neg c \to d))$$

CNF:

$$
\begin{gathered}
((b \to \neg a) \to (\neg c \to d)) \wedge ((\neg c \to d) \to (b \to \neg a)) \\
(\neg(\neg b \vee \neg a) \vee (c \vee d)) \wedge (\neg(c \vee d) \vee (\neg b \vee \neg a)) \\
((b \wedge a) \vee (c \vee d)) \wedge ((\neg c \wedge \neg d) \vee (\neg b \vee \neg a)) \\
(a \vee c \vee d) \wedge (b \vee c \vee d) \wedge (\neg c \vee \neg b \vee \neg a) \wedge (\neg d \vee \neg b \vee \neg a)
\end{gathered}
$$

Soluţie:

$$
\begin{aligned}
Initializare: &\ \gamma \leftarrow \{a \vee c \vee d, b \vee c \vee d, \neg c \vee \neg b \vee \neg a, \neg d \vee \neg b \vee \neg a\} \\
Iteratia\ 1: &\ Nu\ exista\ clauze\ unitare\ si\ nici\ literali\ puri \\
&\ alegem\ \lambda = a\ literal \\
&\ \gamma \leftarrow REZ_a(\gamma) = \{b \vee c \vee d, \neg b \vee d, \neg b \vee c\} \\
Iteratia\ 2: &\ \lambda = c\ literal\ pur \\
&\ \gamma \leftarrow NEG_c(\gamma) = \{\neg b \vee d\} \\
Iteratia\ 3: &\ \lambda = d\ literal\ pur \\
&\ \gamma \leftarrow NEG_d(\gamma) = \emptyset \\
Iteratia\ 4: &\ \gamma = \emptyset \Rightarrow write\ "\ Validabila\ ", sw \leftarrow true \\
&\ \Rightarrow STOP
\end{aligned}
$$