Laborator12 - Temă - Model 2

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Exerciţiul 1.0.1

Se consideră formula

$$\alpha = (\neg(a \wedge \neg b) \vee (\neg a \to c)) \to (\neg(\neg a \vee b) \to (c \vee a))$$

şi substituţia

$$\sigma = \{(x \vee \neg m)|\alpha, (m \wedge n)|a, (q \vee p)|m, a|q\}$$

Să se determine:

- secvenţa generativă formule (SGF) pentru formula α
- tabelul de adevăr pentru formula α
- arborele de structură pentru formula α
- $\alpha\sigma$ - rezultatul aplicării substituţiei σ pentru formula α şi arborele de structură asociat lui $\alpha\sigma$

Rezolvare

SGF:

$$a, b, c, \neg a, \neg b, a \wedge \neg b, \neg(a \wedge \neg b), \neg a \to c, (\neg(a \wedge \neg b) \vee (\neg a \to c)), \neg a \vee b, \neg(\neg a \vee b), c \vee a, (\neg(\neg a \vee b) \to (c \vee a)),$$
$$(\neg(a \wedge \neg b) \vee (\neg a \to c)) \to (\neg(\neg a \vee b) \to (c \vee a)) = \alpha$$

Tabel de Adevăr:

a	b	c	$a \wedge \neg b$	$\neg a \to c$	$\neg(a \wedge \neg b) \vee (\neg a \to c)$	$\neg a \vee b$	$c \vee a$	$\neg(\neg a \vee b) \to (c \vee a)$	α
T	T	T	F	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T	T
T	F	T	T	T	T	F	T	T	T
T	F	F	T	T	T	F	T	T	T
F	T	T	F	T	T	T	T	T	T
F	T	F	F	F	T	T	F	T	T
F	F	T	F	T	T	T	T	T	T
F	F	F	F	F	T	T	F	T	T

Arbore de Structură:

r

$$\begin{array}{rcl} T(\alpha): & \swarrow \ \searrow & , \varphi(r) = \to, \beta = \neg(a \wedge \neg b) \vee (\neg a \to c), \gamma = \neg(\neg a \vee b) \to (c \vee a) \\ & T(\beta) \quad T(\gamma) & \end{array}$$

$$\begin{array}{rcl} & n_1 & \\ T(\beta): & \swarrow \ \searrow & , \varphi(n_1) = \vee, \beta_1 = \neg(a \wedge \neg b), \beta_2 = \neg a \to c \\ & T(\beta_1) \quad T(\beta_2) & \end{array}$$

$$\begin{array}{rcl} & n_3 & \\ T(\beta_1): & \downarrow & , \varphi(n_3) = \neg, \beta_3 = a \wedge \neg b \\ & T(\beta_3) & \end{array}$$

$$\begin{array}{rcl} & n_5 & \\ T(\beta_3): & \swarrow \ \searrow & , \varphi(n_5) = \wedge, \beta_4 = a, \beta_5 = \neg b \\ & T(\beta_4) \quad T(\beta_5) & \end{array}$$

$$T(\beta_4) = n_6, \varphi(n_6) = a$$

$$\begin{array}{rcl} & n_7 & \\ T(\beta_5): & \downarrow & , \varphi(n_7) = \neg, \beta_6 = b \\ & T(\beta_6) & \end{array}$$

$$T(\beta_6) = n_8, \varphi(n_8) = b$$

$$\begin{array}{rcl} & n_4 & \\ T(\beta_2): & \swarrow \ \searrow & , \varphi(n_4) = \to, \beta_7 = \neg a, \beta_8 = c \\ & T(\beta_7) \quad T(\beta_8) & \end{array}$$

$$T(\beta_8) = n_{10}, \varphi(n_{10}) = c$$

$$\begin{array}{rcl} & n_9 & \\ T(\beta_7): & \downarrow & , \varphi(n_9) = \neg, \beta_9 = a \\ & T(\beta_9) & \end{array}$$

$$T(\beta_9) = n_{11}, \varphi(n_{11}) = a$$

$$\begin{array}{rcl} & n_2 & \\ T(\gamma): & \swarrow \ \searrow & , \varphi(n_2) = \to, \gamma_1 = \neg(\neg a \vee b), \gamma_2 = c \vee a \\ & T(\gamma_1) \quad T(\gamma_2) & \end{array}$$

$$\begin{array}{rcl} & n_{12} & \\ T(\gamma_1): & \downarrow & , \varphi(n_{12}) = \neg, \gamma_3 = \neg a \vee b \\ & T(\gamma_3) & \end{array}$$

$$\begin{array}{rcl} & n_{14} & \\ T(\gamma_3): & \swarrow \ \searrow & , \varphi(n_{14}) = \vee, \gamma_4 = \neg a, \gamma_5 = b \\ & T(\gamma_4) \quad T(\gamma_5) & \end{array}$$

$$T(\gamma_5) = n_{16}, \varphi(n_{16}) = b$$

$$\begin{array}{rcl} & n_{15} & \\ T(\gamma_1): & \downarrow & , \varphi(n_{15}) = \neg, \gamma_6 = a \\ & T(\gamma_6) & \end{array}$$

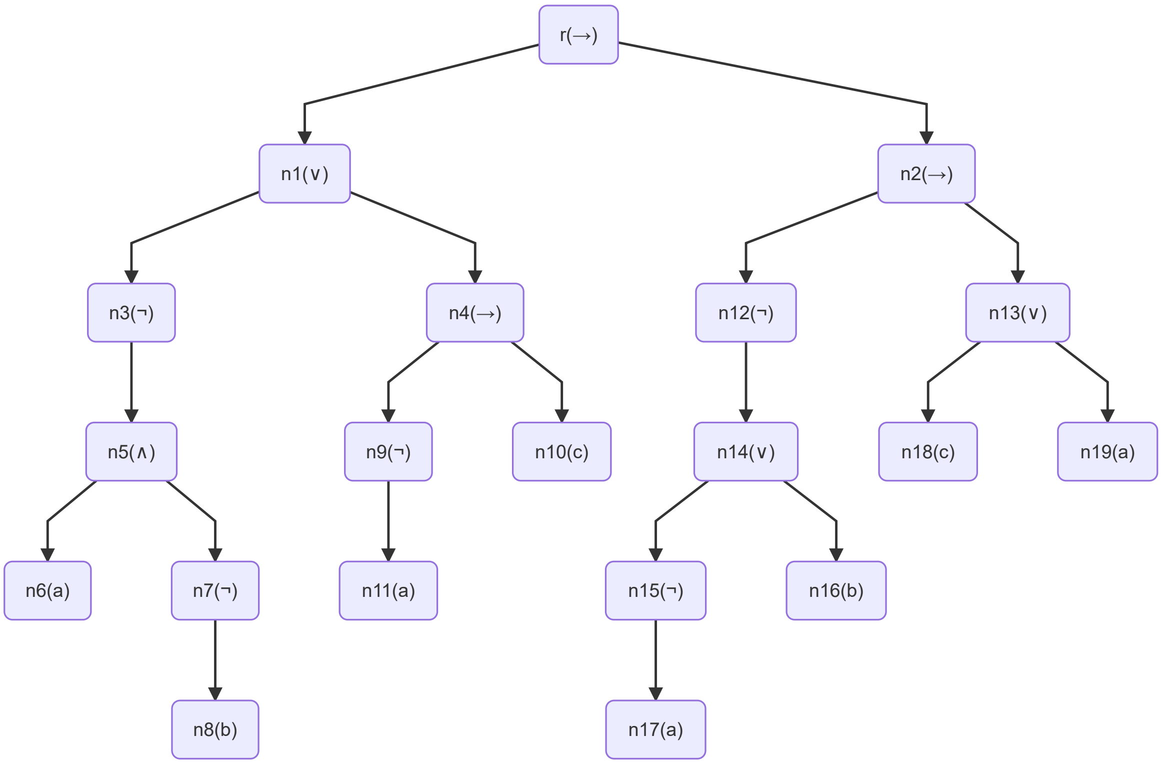
$$T(\gamma_6) = n_{17}, \varphi(n_{17}) = a$$

$$T(\gamma_2): \begin{array}{c} n_{13} \\ \swarrow \ \searrow \\ T(\gamma_7) \quad T(\gamma_8) \end{array}, \varphi(n_{13}) = \vee, \gamma_7 = c, \gamma_8 = a$$

$$T(\gamma_7) = n_{18}, \varphi(n_{18}) = a$$

$$T(\gamma_8) = n_{19}, \varphi(n_{19}) = a$$

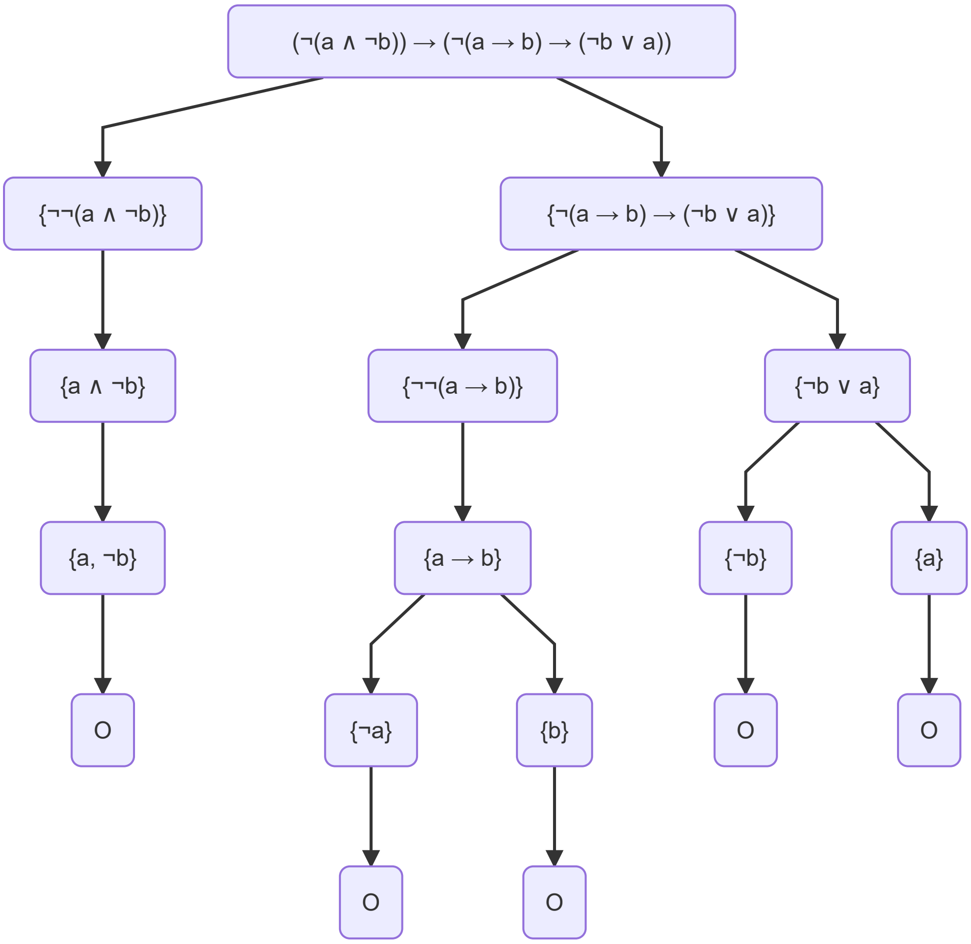
Final:



Exerciţiul 1.0.2

Se consideră formula $\alpha = (\neg(a \wedge \neg b)) \rightarrow (\neg(a \rightarrow b) \rightarrow (\neg b \vee a))$

a) Să se verifice validabilitatea formulei α prin aplicarea metodei arborilor semantici.



b) Să se determine rezultatul aplicării funcţiei de interpretare $I(\alpha)$ asupra formulei α.

$$\begin{aligned}
I(\alpha) &= \neg I(a \wedge \neg b) \to I(\neg(a \to b) \to (\neg b \vee a)) \\
&= \neg\neg(I(a) \wedge \neg I(b)) \vee (\neg\neg I(\neg a \vee b) \vee I(\neg b \vee a)) \\
&= (I(a) \wedge \neg I(b)) \vee (\neg I(a) \vee I(b) \vee \neg I(b) \vee I(a)) \\
&= (I(a) \wedge \neg I(b)) \vee (T \vee T) \\
&= (I(a) \wedge \neg I(b)) \vee T = T
\end{aligned}$$

Exerciţiul 1.0.3

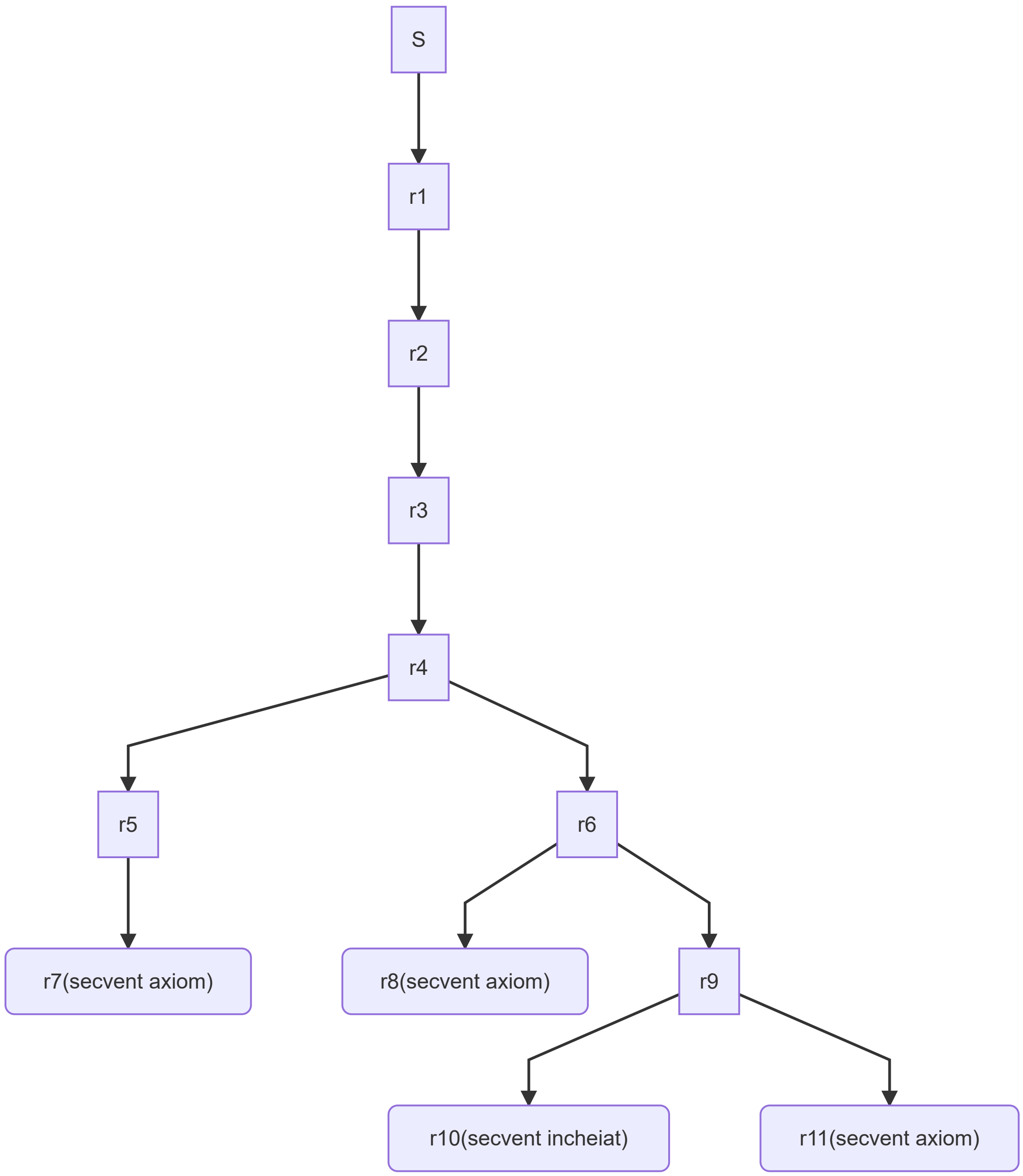
a) Să se verifice dacă următorul secvent este demonstrabil:

$$S = \{(a \vee (b \to c)), (a \to \neg c)\} \Rightarrow \{\neg(d \vee \neg b) \to \neg c\}$$

Sistem:

$$\begin{aligned}
& S = \{a \vee (b \to c), a \to \neg c\} \Rightarrow \{\neg(d \vee \neg b) \to \neg c\} \\
G8:\ & r1 = \{a \vee (b \to c), a \to \neg c, \neg(d \vee \neg b)\} \Rightarrow \{\neg c\} \\
G1:\ & r2 = \{a \vee (b \to c), a \to \neg c\} \Rightarrow \{d \vee \neg b, \neg c\} \\
G7:\ & r3 = \{a \vee (b \to c), a \to \neg c\} \Rightarrow \{d, \neg b, \neg c\} \\
G5:\ & r4 = \{a \vee (b \to c), a \to \neg c, c, b\} \Rightarrow \{d\} \\
G4:\ & r5 = \{a \vee (b \to c), \neg c, c, b\} \Rightarrow \{d\} \\
& r6 = \{a \vee (b \to c), c, b\} \Rightarrow \{a, d\} \\
G1:\ & r7 = \{a \vee (b \to c), c, b\} \Rightarrow \{d, c\}\ secvent\ axiom \\
G3:\ & r8 = \{a, c, b\} \Rightarrow \{a, d\}\ secvent\ axiom \\
& r9 = \{b \to c, c, b\} \Rightarrow \{a, d\} \\
G4:\ & r10 = \{c, b\} \Rightarrow \{a, d\}\ secvent\ incheiat \\
& r11 = \{c, b\} \Rightarrow \{b, a, d\}\ secvent\ axiom \\
& S\ nu\ e\ tautologie
\end{aligned}$$

Schema:



b) Să se calculeze mulţimile α_λ^+, α_λ^-, α_λ^0, $POS_\lambda(\alpha)$, $NEG_\lambda(\alpha)$, $REZ_\lambda(\alpha)$ unde $\lambda = \eta$, respectiv $\lambda = \neg\theta$, iar

$$S(\alpha) = \{\neg\gamma \vee \beta \vee \neg\delta, \neg\beta \vee \eta \vee \neg\gamma, \neg\theta, \beta, \theta \vee \beta \vee \neg\eta, \delta \vee \beta \vee \neg\theta, \gamma \vee \eta \vee \neg\delta\}$$

Pentru $\lambda = \eta$

$$\begin{aligned} \alpha_\lambda^+ &= \{\neg\beta \vee \eta \vee \neg\gamma, \gamma \vee \eta \vee \neg\delta\} \\ \alpha_\lambda^- &= \{\theta \vee \beta \vee \neg\eta\} \\ \alpha_\lambda^0 &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\theta, \beta, \delta \vee \beta \vee \neg\theta\} \\ POS_\lambda(\alpha) &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\theta, \beta, \delta \vee \beta \vee \neg\theta, \neg\beta \vee \neg\gamma, \gamma \vee \neg\delta\} \\ NEG_\lambda(\alpha) &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\theta, \beta, \delta \vee \beta \vee \neg\theta, \theta \vee \beta\} \\ REZ_\lambda(\alpha) &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\theta, \beta, \delta \vee \beta \vee \neg\theta, \theta \vee \neg\gamma, \gamma \vee \neg\delta \vee \theta \vee \beta\} \end{aligned}$$

Pentru $\lambda = \neg\theta$

$$
\begin{aligned}
\alpha_\lambda^+ &= \{\neg\theta, \delta \vee \beta \vee \neg\theta\} \\
\alpha_\lambda^- &= \{\theta \vee \beta \vee \neg\eta\} \\
\alpha_\lambda^0 &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\beta \vee \eta \vee \neg\gamma, \beta, \gamma \vee \eta \vee \neg\delta\} \\
POS_\lambda(\alpha) &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\beta \vee \eta \vee \neg\gamma, \beta, \gamma \vee \eta \vee \neg\delta, \square, \delta \vee \beta\} \\
NEG_\lambda(\alpha) &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\beta \vee \eta \vee \neg\gamma, \beta, \gamma \vee \eta \vee \neg\delta, \beta \vee \neg\eta\} \\
REZ_\lambda(\alpha) &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\beta \vee \eta \vee \neg\gamma, \beta, \gamma \vee \eta \vee \neg\delta, \square \vee \beta \vee \neg\eta, \delta \vee \beta \vee \neg\eta\}
\end{aligned}
$$

Exerciţiul 1.0.4

Să se determine forma normală conjunctivă (CNF) şi să se aplice algoritmul Davis-Putnam pentru formula $\alpha = ((\neg a \vee b) \leftrightarrow (d \to c))$

CNF:

$$
\begin{gathered}
((\neg a \vee b) \to (d \to c)) \wedge ((d \to c) \to (\neg a \vee b)) \\
(\neg(\neg a \vee b) \vee (\neg d \vee c)) \wedge (\neg(\neg d \vee c) \vee (\neg a \vee b)) \\
((a \wedge \neg b) \vee (\neg d \vee c)) \wedge ((d \wedge \neg c) \vee (\neg a \vee b)) \\
(a \vee \neg d \vee c) \wedge (\neg b \wedge \neg d \vee c) \wedge (d \vee \neg a \vee b) \wedge (\neg c \vee \neg a \vee b)
\end{gathered}
$$

Davis-Putnam:

$$
\begin{aligned}
Initializare : &\ \gamma \leftarrow \{a \vee \neg d \vee c, \neg b \wedge \neg d \vee c, d \vee \neg a \vee b, \neg c \vee \neg a \vee b\} \\
&\ sw \leftarrow false, T \leftarrow \emptyset \\
Iteratia\ 1 : &\ Nu\ exista\ literar\ pur\ sau\ clauza\ unitara \\
&\ alegem\ \lambda = b\ literar \\
&\ \gamma \leftarrow NEG_b(\gamma) = \{a \vee \neg d \vee c, \neg d \vee c\} \\
&\ T \leftarrow POS_b(\gamma) = \{a \vee \neg d \vee c, d \vee \neg a, \neg c \vee \neg a\} \\
Iteratia\ 2 : &\ \lambda = c\ literar\ pur \\
&\ \gamma \leftarrow NEG_c(\gamma) = \emptyset \\
Iteratia\ 3 : &\ \gamma = \emptyset, \gamma \leftarrow T = \{a \vee \neg d \vee c, d \vee \neg a, \neg c \vee \neg a\} \\
&\ T = \emptyset \\
Iteratia\ 4 : &\ Nu\ exista\ literar\ pur\ sau\ clauza\ unitara \\
&\ alegem\ \lambda = a\ literar \\
&\ \gamma \leftarrow NEG_a(\gamma) = \{d, \neg c\} \\
&\ T \leftarrow POS_a(\gamma) = \{\neg d \vee c\} \\
Iteratia\ 5 : &\ \lambda = d\ clauza\ unitara \\
&\ \gamma \leftarrow NEG_d(\gamma) = \{\neg c\} \\
Iteratia\ 6 : &\ \lambda = \neg c\ clauza\ unitara \\
&\ \gamma \leftarrow NEG_{\neg c}(\gamma) = \emptyset \\
Iteratia\ 7 : &\ \gamma = \emptyset, \gamma \leftarrow T = \{\neg d \vee c\} \\
&\ T = \emptyset \\
Iteratia\ 8 : &\ \lambda = c\ literar\ pur \\
&\ \gamma \leftarrow NEG_c(\gamma) = \emptyset \\
Iteratia\ 9 : &\ \gamma = \emptyset \Rightarrow write('validalibila'),\ sw \leftarrow true
\end{aligned}
$$