1 Să se dezvolte funcțiile în jurul pct indicate:

a) $f(x)=\frac{1}{7x+3}$, $a=0$; b) $f(x)=\frac{1}{7x+3}$, $a=1$; c) $f(x)=\frac{1}{x^2-5}$, $a=0$.

$$f(x)=\frac{1}{3x+2}=\frac{1}{3(x-1)+5}=\frac{1}{5\left(\frac{3}{5}(x-1)+1\right)}=\frac{1}{5}\cdot\frac{1}{1+\frac{3}{5}(x-1)}=\frac{1}{5}\cdot\frac{1}{1-\underbrace{\left(-\frac{3}{5}(x-1)\right)}_{q}}=$$

$$\sum_{n=0}^{\infty} q^n=\frac{1}{1-q},\ |q|<1.$$

$$=\frac{1}{5}\sum_{n=0}^{\infty}\left(\frac{-3}{5}\right)^n\cdot(x-1)^n$$

$$=\cancel{\tfrac{1}{5}}\sum_{n=0}^{\infty}\frac{(-3)^n\cdot(x-1)^n}{5^{n+1}}$$

$$\left|\frac{-3}{5}(x-1)\right|<1 \Leftrightarrow (x-1)>\frac{5}{-3} \Leftrightarrow x>\frac{-5}{3}+1=\frac{-2}{3}.$$

$$-1<\frac{-3}{5}(x-1) \Leftrightarrow \frac{5}{3}>(x-1) \Leftrightarrow x<\frac{5}{3}+1=\frac{8}{3}.$$

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a) $$f(x)=\frac{1}{7x+3}=\frac{1}{1+\frac{7}{3}x}\cdot\frac{1}{3}=\frac{1}{3}\cdot\frac{1}{1-\left(\frac{-7}{3}x\right)}=\frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{-7}{3}\right)^n\cdot x^n=\sum_{n=0}^{\infty}\frac{(-7)^n\cdot x^n}{3^{n+1}}$$ ✓

b) $$f(x)=\frac{1}{7(x-1)+10}=\frac{1}{10}\cdot\frac{1}{1+\frac{7}{10}(x-1)}=\frac{1}{10}\cdot\frac{1}{1-\left(-\frac{7}{10}(x-1)\right)}=\frac{1}{10}\sum_{n=0}^{\infty}\left(\frac{-7}{10}\right)^n\cdot(x-1)^n=$$

$$=\cancel{\tfrac{1}{10}}\sum_{n=0}^{\infty}(-1)^n\cdot\frac{(-7)^n\cdot(x-1)^n}{10^{n+1}}.$$

c) $$f(x)=\frac{1}{x^2-5}=\frac{1}{5}\cdot\frac{1}{\frac{1}{5}x^2-1}=\frac{-1}{5}\cdot\frac{1}{1-\frac{1}{5}x^2}=\frac{-1}{5}\cdot\sum_{n=0}^{\infty}\left(\frac{1}{5}\right)^n\cdot(x^2)^n=$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}\cdot x^{2n}}{5^{n+1}}$$

$$(2x+1)(x-3)=2x^2-6x+x-3=$$
$$=2x^2-5x-3.$$

$$f(x)=\frac{3x-2}{2x^2-5x-2}\overset{?}{=}\frac{1}{2x+1}+\frac{1}{x-3};$$

$2x^2-5x-2=$ $\quad (2x+1)(x-3)=2x^2-6x+x-3.=2x^2-5x-3.$

$\cancel{2x^2-6x+x-2}=$

$$2x^2-5x-3=2x^2-2x-3x-3=2x(x+1)-3(x-1)=(2x-3)(x-1).$$

$=$

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2. Scrieți J_f și aplic. dif df:

a) $f:\mathbb{R}^2\to\mathbb{R}^2$, $f(x,y)=(xy^3-4xy,\ 5x^3y-y^2)$

$$J_f=\begin{pmatrix} y^3-4y, & 3y^2x-4x \\ 15x^2y & ,\ 5x^3-2y \end{pmatrix};\ \forall (x,y)\in\mathbb{R}^2 \qquad df=\begin{pmatrix} (y^3-4y)dx+(3y^2x-4x)dy \\ (15x^2y)dx+(5x^3-2y)dy \end{pmatrix}$$

b) $f:\mathbb{R}^2\to\mathbb{R}^3$, $f(x,y)=(\sin(11x+3y),\ x^3y^4,\ e^{x-2y^2})$;

$$J_f=\begin{pmatrix} \cos(11x+3y)\cdot 11, & \cos(11x+3y)\cdot 3 \\ 3x^2y^4 & ,\ 4y^3x^3 \\ e^{x-2y^2} & ,\ e^{x-2y^2}\cdot(-4y) \end{pmatrix}$$

c) $f:\mathbb{R}\times\mathbb{R}^*\times\mathbb{R}\to\mathbb{R}^2$, $f(x,y,z)=\left(y^2z-z^2e^{x^2+y^2},\ \frac{-x^2z}{y}\right)$.

$$J_f=\begin{pmatrix} -z^2e^{x^2+y^2}\cdot 2x, & 2yz-z^2e^{x^2+y^2}\cdot 2y, & y^2-2ze^{x^2+y^2} \\ \frac{z\cdot -2x}{y}, & \frac{x^2z}{y^2}, & \frac{-x^2}{y} \end{pmatrix} \checkmark$$

3. a) $f:\mathbb{R}^3\to\mathbb{R}$

3. $\begin{cases} \frac{\partial f}{\partial x}(x,y,z) = 18x - 9y + 9 = 0 \\ \frac{\partial f}{\partial y}(x,y,z) = 18y - 9x = 0 \\ \frac{\partial f}{\partial z}(x,y,z) = 18z - 18 = 0 \end{cases} \Leftrightarrow \begin{cases} 2x - y + 1 = 0 \\ 2y - x = 0 \\ z - 1 = 0 \end{cases} \Leftrightarrow \begin{cases} 2x - \frac{x}{2} + 1 = 0 \\ y = \frac{x}{2} \\ z = 1 \end{cases} \Leftrightarrow \begin{cases} \frac{3x}{2} = -1 \Rightarrow x = \frac{-2}{3} \\ y = \frac{x}{2} = \frac{-1}{3} \\ z = 1. \end{cases}$

$\Leftrightarrow S = \left\{ \left(\frac{-2}{3}, \frac{-1}{3}, 1 \right) \right\}.$

$Hf(x,y,z) = \begin{pmatrix} 18 & -9 & 0 \\ -9 & 18 & 0 \\ 0 & 0 & 18 \end{pmatrix} = Hf\left(\frac{-2}{3}, \frac{-1}{3}, 1\right) = \begin{cases} D_1 = 18 > 0 \\ D_2 = \begin{vmatrix} 18 & -9 \\ -9 & 18 \end{vmatrix} > 0 \\ D_3 = \begin{vmatrix} \cdots \end{vmatrix} > 0 \end{cases}$ | punct de minim. loc strict.

b) $f: \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^3 + y^3 - \frac{1}{3}xy$;

$\begin{cases} \frac{\partial f}{\partial x}(x,y,z) = 3x^2 - \frac{1}{3}y \\ \frac{\partial f}{\partial y}(x,y,z) = 3y^2 - \frac{1}{3}x \\ \cancel{\frac{\partial f}{\partial z}(x,y,z) = 0} \end{cases} \Leftrightarrow \begin{cases} 9x^2 - y = 0 \\ 9y^2 - x = 0 \\ \cancel{z = 0} \end{cases} \Leftrightarrow \begin{cases} x = \sqrt{\frac{y}{9}} = \frac{\sqrt{y}}{3} \\ y = \sqrt{\frac{x}{9}} = \frac{\sqrt{x}}{3} \Leftrightarrow \frac{\sqrt{y}}{3} = \frac{\sqrt{x}}{3} \Leftrightarrow \\ \cancel{z = 0.} \end{cases}$

$3x^2 - \frac{1}{3}y = 0 \mid \cdot 3 \Leftrightarrow (3x)^2 - y = 0 \Leftrightarrow y = (3x)^2$
$x = (3y)^2 \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \Leftrightarrow \begin{cases} x = \frac{1}{9} \\ y = \frac{1}{9} \end{cases}$? ✓

$\cancel{(3x)^2 = (3y)^2}$
$\cancel{-3x = 3y \Leftrightarrow -9x = y \Leftrightarrow}$

$x = (3x)^2 \Leftrightarrow x = 9 \cdot x^2. \Leftrightarrow 9 \cdot x = 1$

$Hf = \begin{pmatrix} 6x, & -\frac{1}{3} \\ \frac{-1}{3}, & 6y \end{pmatrix} \Rightarrow Hf(0,0) = \begin{pmatrix} 0 & \frac{-1}{3} \\ \frac{-1}{3} & 0 \end{pmatrix} \begin{cases} D_1 = 0. \\ D_2 < 0 \text{ punct. şa.} \end{cases}$

$Hf = \frac{1}{3}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} = \begin{matrix} D_1 = \frac{2}{3} > 0 \\ D_2 = \frac{3}{3} > 0 \end{matrix}$ | punct de minim.

$= \begin{pmatrix} \frac{2}{3}, & \frac{-1}{3} \\ \frac{-1}{3}, & \frac{2}{3} \end{pmatrix}$ $\quad \frac{4}{9} - \frac{1}{9} = \frac{3}{9} = \frac{1}{3} > 0$ ✓.

4.b) Fie $c: R^3 \to R$, $c(x,y,z) = u(x^2yz, yz - 3zx - xy)$.

$c \quad \begin{matrix} u \\ v \end{matrix} \quad x \; y$

$$\frac{\partial c}{\partial x} = \frac{\partial c}{\partial u}(x,y,z) \cdot \frac{\partial u}{\partial x}(x,y,z) + \frac{\partial c}{\partial v}(x,y,z) \cdot \frac{\partial v}{\partial x}(x,y,z)$$

$$= \frac{\partial c}{\partial u}\overset{u,v}{(\cancel{x,y,z})} \cdot 2xyz + \frac{\partial c}{\partial v}(u,v) \cdot (-3z - y).$$

$$\frac{\partial c}{\partial y} = \frac{\partial c}{\partial u}(u,v) \cdot x^2 z + \frac{\partial c}{\partial v}(u,v) \cdot (z - x)$$