Laborator3 - Temă

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Exerciţiul 1.0.1. Să se aplice algoritmul Davis-Putnam următoarelor formule (se determină iniţial CNF):

1) $(\neg p \vee q) \wedge (\neg p) \wedge p$

$$\begin{aligned} & S(\alpha) = \{k1 = \neg p \vee q, k2 = \neg p, k3 = p\} \\ Initializare: \ & \gamma \leftarrow \{\neg p \vee q, \neg p, p\};\ sw \leftarrow false;\ T \leftarrow \emptyset \\ Iteratia\ 1: \ & \lambda = p\ clauza\ unitara \\ & \gamma \leftarrow NEG_p(\gamma) = \{q, \square\} \\ Iteratia\ 2: \ & \square \in \gamma\ si\ T = \emptyset \Rightarrow write('invalidalibila'),\ sw \leftarrow true \\ & \Rightarrow STOP \end{aligned}$$

2) $(p \vee q) \wedge (r \vee q) \wedge (\neg r) \wedge (\neg q)$

$$\begin{aligned} & S(\alpha) = \{k1 = p \vee q, k2 = r \vee q, k3 = \neg r, k4 = \neg q\} \\ Initializare: \ & \gamma \leftarrow \{p \vee q, r \vee q, \neg r, \neg q\};\ sw \leftarrow false;\ T \leftarrow \emptyset \\ Iteratia\ 1: \ & \lambda = \neg q\ clauza\ unitara \\ & \gamma \leftarrow NEG_{\neg q}(\gamma) = \{p, r, \neg r\} \\ Iteratia\ 2: \ & \lambda = \neg r\ clauza\ unitara \\ & \gamma \leftarrow NEG_{\neg r}(\gamma) = \{p, \square\} \\ Iteratia\ 3: \ & \square \in \gamma\ si\ T = \emptyset \Rightarrow write('invalidalibila'),\ sw \leftarrow true \\ & \Rightarrow STOP \end{aligned}$$

3) $p \wedge q \wedge r$

$$\begin{aligned} & S(\alpha) = \{k1 = p, k2 = q, k3 = r\} \\ Initializare: \ & \gamma \leftarrow \{p, q, r\};\ sw \leftarrow false;\ T \leftarrow \emptyset \\ Iteratia\ 1: \ & \lambda = p\ clauza\ unitara \\ & \gamma \leftarrow NEG_p(\gamma) = \{q, r\} \\ Iteratia\ 2: \ & \lambda = q\ clauza\ unitara \\ & \gamma \leftarrow NEG_q(\gamma) = \{r\} \\ Iteratia\ 3: \ & \lambda = r\ clauza\ unitara \\ & \gamma \leftarrow NEG_r(\gamma) = \emptyset \\ Iteratia\ 4: \ & \gamma = \emptyset \Rightarrow write('validalibila'),\ sw \leftarrow true \\ & \Rightarrow STOP \end{aligned}$$

4) $(p \vee q) \wedge (\neg p \vee q) \wedge r$

$$
\begin{aligned}
&S(\alpha) = \{k1 = p \vee q, k2 = \neg p \vee q, k3 = r\} \\
Initializare: \ & \gamma \leftarrow \{p \vee q, \neg p \vee q, r\};\ sw \leftarrow false;\ T \leftarrow \emptyset \\
Iteratia\ 1: \ & \lambda = r\ clauza\ unitara \\
& \gamma \leftarrow NEG_r(\gamma) = \{p \vee q, \neg p \vee q\} \\
Iteratia\ 2: \ & \lambda = q\ literar\ pur \\
& \gamma \leftarrow NEG_q(\gamma) = \emptyset \\
Iteratia\ 3: \ & \gamma = \emptyset \Rightarrow write('validalibila'),\ sw \leftarrow true \\
& \Rightarrow STOP
\end{aligned}
$$

5) $(p \vee q) \wedge (\neg q)$

$$
\begin{aligned}
&S(\alpha) = \{k1 = p \vee q, k2 = \neg q\} \\
Initializare: \ & \gamma \leftarrow \{p \vee q, \neg q\};\ sw \leftarrow false;\ T \leftarrow \emptyset \\
Iteratia\ 1: \ & \lambda = \neg q\ clauza\ unitara \\
& \gamma \leftarrow NEG_{\neg q}(\gamma) = \{p\} \\
Iteratia\ 2: \ & \lambda = p\ clauza\ unitara \\
& \gamma \leftarrow NEG_p(\gamma) = \emptyset \\
Iteratia\ 3: \ & \gamma = \emptyset \Rightarrow write('validalibila'),\ sw \leftarrow true \\
& \Rightarrow STOP
\end{aligned}
$$

6) $(p \vee q) \wedge (\neg p \vee q) \wedge (\neg r \vee \neg q) \wedge (r \vee \neg q)$

$$
\begin{aligned}
&S(\alpha) = \{k1 = p \vee q, k2 = \neg p \vee q, k3 = \neg r \vee \neg q, k4 = r \vee \neg q\} \\
Initializare: \ & \gamma \leftarrow \{p \vee q, \neg p \vee q, \neg r \vee \neg q, r \vee \neg q\};\ sw \leftarrow false;\ T \leftarrow \emptyset \\
Iteratia\ 1: \ & Nu\ exista\ literar\ pur\ sau\ clauza\ unitara \\
& alegem \lambda = q\ literar \\
& \gamma \leftarrow NEG_q(\gamma) = \{\neg r, r\} \\
& T \leftarrow POS_q(\gamma) = \{p, \neg p\} \\
Iteratia\ 2: \ & \lambda = r\ clauza\ unitara \\
& \gamma \leftarrow NEG_q(\gamma) = \{\square\} \\
Iteratia\ 3: \ & \square \in \gamma, T \leftarrow POS_q(\gamma) = \{p, \neg p\} \\
& \gamma \leftarrow T = \{p, \neg p\} \\
& T = \emptyset \\
Iteratia\ 4: \ & \lambda = p\ clauza\ unitara \\
& \gamma \leftarrow NEG_q(\gamma) = \{\square\} \\
Iteratia\ 5: \ & \square \in \gamma, T = \emptyset \Rightarrow write\ 'invalidabila', sw \leftarrow rue \\
& \Rightarrow STOP
\end{aligned}
$$

Exerciţiul 1.0.2. Aplicând algoritmul lui Davis-Putnam, demonstraţi că următoarea formulă este validabilă:

$$(((q \to p) \wedge (p \to q)) \to (\neg q \wedge \neg r)) \vee (((r \to p) \wedge (q \to s)) \to ((p \to r) \to (r \wedge s)))$$

Determinare CNF:

$$
\begin{aligned}
T2: \ & (\neg((\neg q \vee p) \wedge (\neg p \vee q)) \vee (\neg q \wedge \neg r)) \vee (\neg((\neg r \vee p) \wedge (\neg q \vee s)) \vee (\neg(\neg p \vee r) \vee (r \wedge s))) \\
T3: \ & (q \wedge \neg p) \vee (p \wedge \neg q) \vee (\neg q \wedge \neg r) \vee (r \wedge \neg p) \vee (\neg q \wedge s) \vee (p \wedge \neg r) \vee (r \wedge s)
\end{aligned}
$$

Mapă Karnaugh:

rs\qp	00	01	10	11
00	1	1	1	1
01	1	1	1	1
10	1	1	1	0
11	1	1	1	1

CNF: $\neg q \vee p \vee \neg r \vee \neg s$

Aplicăm algoritmul *Davis-Putnam*:

$$\begin{aligned} & S(\alpha) = \{k1 = \neg q \vee p \vee \neg r \vee \neg s\} \\ Initializare: \ & \gamma \leftarrow \{\neg q \vee p \vee \neg r \vee \neg s\};\ sw \leftarrow false;\ T \leftarrow \emptyset \\ Iteratia\ 1: \ & \lambda = \neg q\ literal\ pur \\ & \gamma \leftarrow NEG_{\neg q}(\gamma) = \emptyset \\ Iteratia\ 2: \ & \gamma = \emptyset \Rightarrow write('Validabila'),\ sw \leftarrow true \\ & \Rightarrow STOP \end{aligned}$$