

Aplicatii rezolvate - Grafuri –

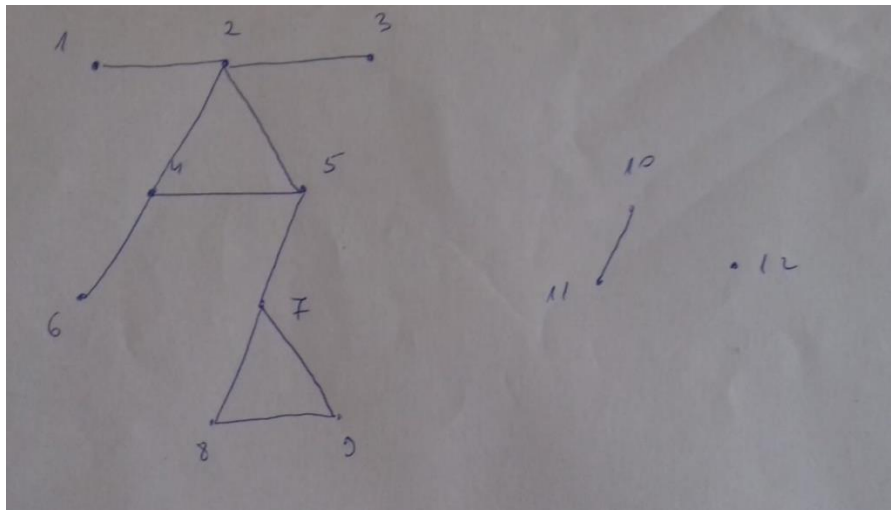
APLICATIA 1: Reprezentati grafic graful neorientat $G=(V,E)$ cu

$V=\{1,2,3,4,5,6,7,8,9,10,11,12\}$ si

$E=\{(1,2),(2,4),(2,3),(2,5),(4,5),(4,6),(5,7),(7,8),(8,9),(7,9),(10,11)\}$. Determinati:

- | | |
|---|---|
| a) Matricea de adiacenta; | e) Parcurgerea BF(1), BF(2) si arborii corespunzatori acestor parcurgeri; |
| b) Gradele nodurilor; | f) Componentele conexe; |
| c) Matricea de incidenta; | g) Matricea drumurilor; |
| d) Parcurgerea DF(1), DF(2) si arborii corespunzatori acestor parcurgeri; | h) Numarul ciclomatic; |

SOLUTIE:

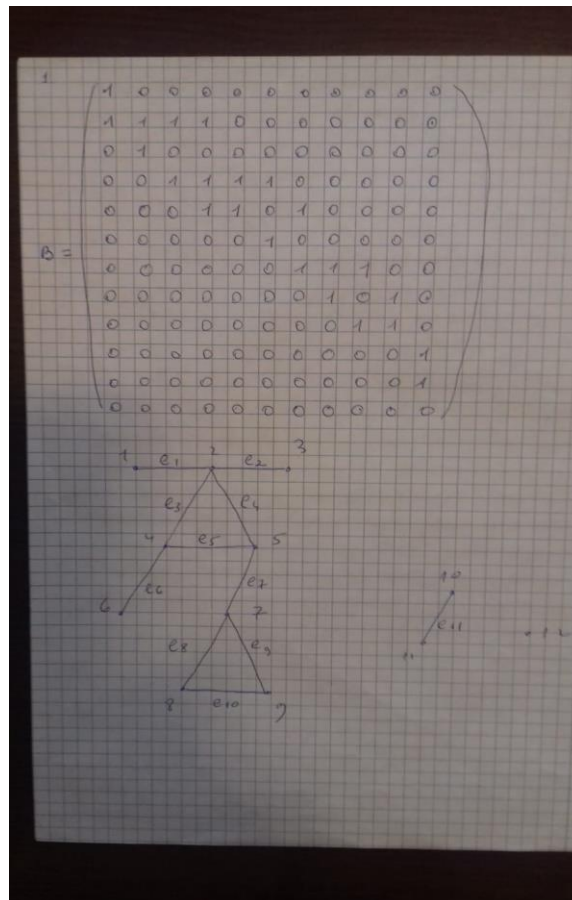


a)

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

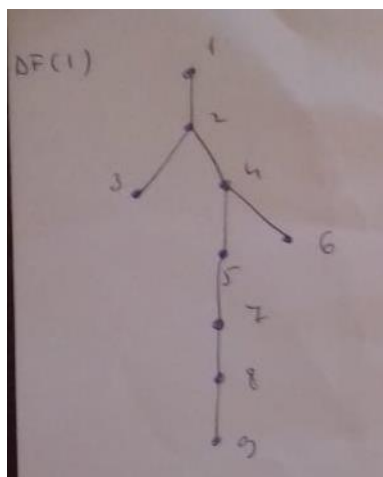
b) $d(1)=1, d(2)=2, d(3)=1, d(4)=3, d(5)=3, d(6)=1,$
 $d(7)=3, d(8)=2, d(9)=2, d(10)=1, d(11)=1, d(12)=0.$

c)

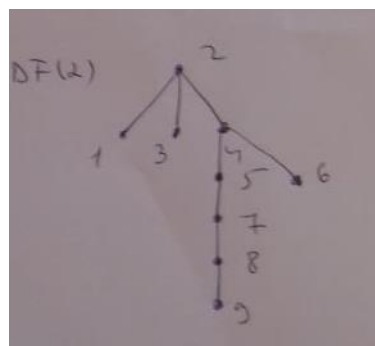


d) Intre vecinii unui nod, prin conventie se considera ordinea data de ordinea crescatoare a numerelor acestora. Astfel:

DF(1): 1,2,3,4,5,7,8,9,6

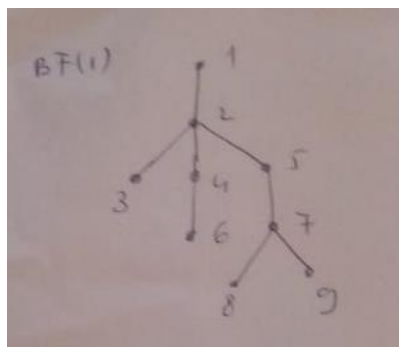


DF(2): 2,1,3,4,5,7,8,9,6

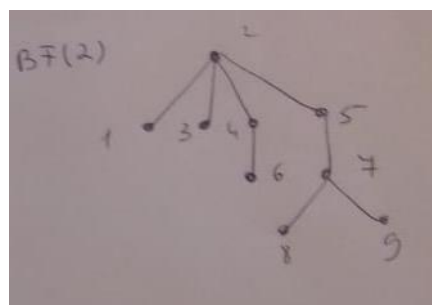


e)

BF(1): 1,2,3,4,5,6,7,8,9



BF(2): 2,1,3,4,5,6,7,8,9



f) $V_1 = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$; $V_2 = \{10, 11\}$; $V_3 = \{12\}$;

g)

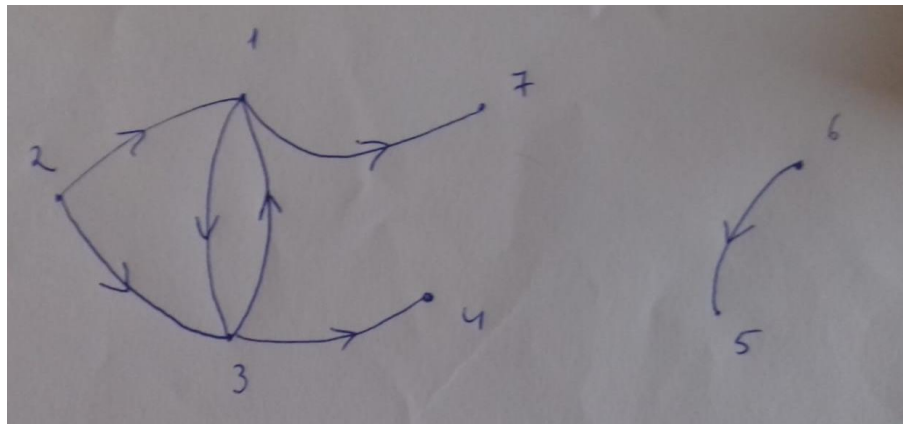
$$D = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

h) $\gamma(G) = m - n + k = 11 - 12 + 3 = 2$

APLICATIA 2: Reprezentati grafic graful orientat $G=(V,E)$ cu $V=\{1,2,3,4,5,6,7\}$ si $E=\{(1,7),(1,3),(2,1),(2,3),(3,1),(3,4),(6,5)\}$. Determinati:

- | | |
|---|-------------------------|
| a) Matricea de adiacenta; | e) Parcurgerea DF(1); |
| b) Gradele de intrare, de iesire si totale ale nodurilor; | f) Parcurgerea BF(1); |
| c) Matricea de incidenta; | g) Componentele conexe; |
| d) Listele de adiacenta ale nodurilor; | h) Numarul ciclomatic; |

SOLUTIE:

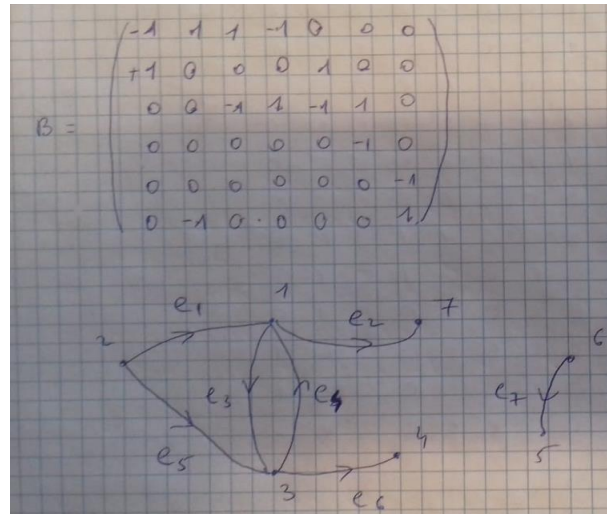


a)

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- b) $d^+(1)=2, d^-(1)=2, d(1)=4;$
 $d^+(2)=2, d^-(2)=0, d(2)=2;$
 $d^+(3)=2, d^-(3)=2, d(3)=4;$
 $d^+(4)=0, d^-(4)=1, d(4)=1;$
 $d^+(5)=0, d^-(5)=1, d(5)=1;$
 $d^+(6)=1, d^-(6)=0, d(6)=1;$
 $d^+(7)=1, d^-(7)=0, d(7)=1;$

c)



d) $L(1)=\{7\}$, $L(2)=\{1,3\}$, $L(3)=\{1,4\}$, $L(4)=\emptyset$, $L(5)=\emptyset$; $L(6)=\{5\}$, $L(7)=\emptyset$;

e) $DF(1)$: 1,3,4,7;

f) $BF(1)$: 1,3,7,4;

g) $V_1=\{1,2,3,4,7\}$; $V_2=\{5,6\}$;

h) $\gamma(G)=m-n+k=7-7+2=2$

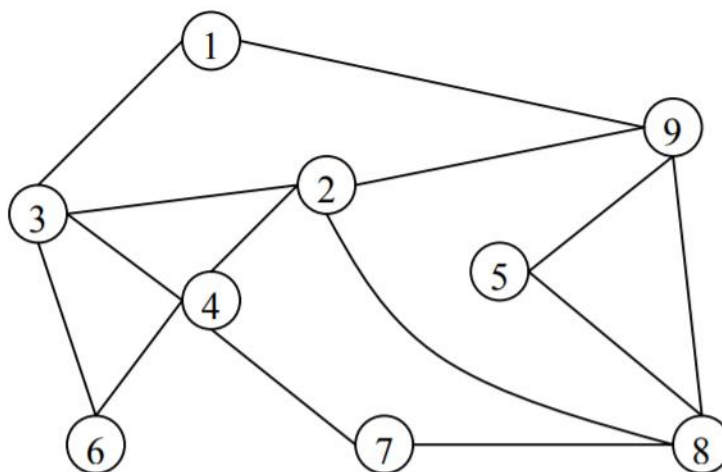
APLICATIA 3: Consideram graful neorientat dat prin matricea de adiacenta:

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Reprezentati grafic graful neorientat si determinati

daca graful are cicluri euleriene sau lanturi euleriene; in caz afirmativ determinati un ciclu eulerian, respectiv un lant eulerian.

SOLUTIE:



În acest graf putem considera ciclul $C=[1,3,2,4,6,3,4,7,8,2,9,8,5,9,1]$ în care nu se repetă nicio muchie și astfel ciclul este unul simplu. Ciclul C conține toate muchiile din graful G și astfel ciclul C este un ciclu eulerian și astfel graful este un graf eulerian.

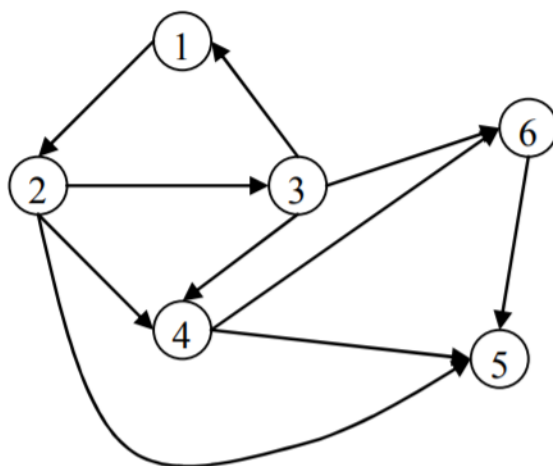
APLICATIA 4: Consideram graful orientat dat prin matricea de adiacenta:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Reprezentati grafic graful orientat si determinati

daca graful orientat are circuite hamiltoniene sau drumuri hamiltoniene.

SOLUTIE:



Acest graf contine ciclul $C=[1,2,4,5,6,3,1]$ care trece prin toate cele 6 vârfuri și astfel este un ciclu hamiltonian. Ca urmare și graful considerat este unul hamiltonian. Facem observatia că graful G nu contine și circuite hamiltoniene.