Temă - Seminar 6 Ietculescu Mihai-Liviu

1. Să se aducă la forma canonică următoarele forme pătratice.

e) $q(x) = x_1^2 - 4x_1x_2 + 2x_1x_3 + 2x_2^2 + 5x_3^2$

Metoda 1:

$$
\begin{aligned}
q(x) &= x_1^2 - 4x_1x_2 + 2x_1x_3 + 2x_2^2 + 5x_3^2 \\
&= (x_1^2 - 4x_1x_2 + 2x_1x_3) + 2x_2^2 + 5x_3^2 \\
&= (x_1^2 - 4x_1x_2 + 2x_1x_3 - 4x_2x_3 + 4x_2^2 + x_3^2) + 4x_2x_3 - 4x_2^2 - x_3^2 + 2x_2^2 + 5x_3^2 \\
&= (x_1 - 2x_2 + x_3)^2 + 4x_2x_3 - 2x_2^2 + 4x_3^2 \\
&= (x_1 - 2x_2 + x_3)^2 + (-2)(x_2^2 - 2x_2x_3) + 4x_3^2 \\
&= (x_1 - 2x_2 + x_3)^2 - 2(x_2^2 - 2x_2x_3 + x_3^2) + 2x_3^2 + 4x_3^2 \\
&= (\underbrace{x_1 - 2x_2 + x_3}_{\xi_1})^2 - 2(\underbrace{x_2 - x_3}_{\xi_3})^2 + 6\underbrace{x_3}_{\xi_3}{}^2
\end{aligned}
$$

$\Rightarrow q(\xi) = \xi_1^2 - 2\xi_2^2 + 6\xi_3^2$

Metoda 2

$A = \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$; $\begin{pmatrix} \boxed{1} & 2 & 1 \\ -2 & 2 & 0 \\ 1 & 0 & 5 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 \\ 0 & \boxed{-2} & 2 \\ 0 & 2 & 4 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & \boxed{6} \end{pmatrix}$

$\Rightarrow q(x) = 1\cdot(x_1 - 2x_2 + x_3)^2 - 2(x_2 - x_3)^2 + 6x_3^2$

$\Rightarrow q(\xi) = \xi_1^2 - 2\xi_2^2 + 6\xi_3^2$

Metoda 3

$A = \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$; $D_0 = 1$; $D_1 = 1$; $D_2 = \begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix} = -2$

$D_3 = 10 - 2 - 20 = -12$

$\Rightarrow q(\xi) = \frac{D_1}{D_0}\xi_1^2 + \frac{D_2}{D_1}\xi_2^2 + \frac{D_3}{D_2}\xi_3^2 = \xi_1^2 - 2\xi_2^2 + 6\xi_3^2$

g) $q(x) = -2x_1x_2 + x_2x_3 - x_1x_3$

Se face schimbarea de variabilă:

$$
\begin{cases} x_1 = \eta_1 - \eta_2 \\ x_2 = \eta_1 + \eta_2 \\ x_3 = \eta_3 \end{cases}
$$

Metoda 1

$$q(x) = -2(y_1-y_2)(y_1+y_2) + (y_1+y_2)\cdot y_3 - y_3(y_1-y_2)$$
$$= -2y_1^2 + 2y_2^2 + y_1y_3 + y_2y_3 - y_1y_3 + y_2y_3$$
$$= -2y_1^2 + 2y_2^2 + 2y_2y_3$$
$$= -2y_1^2 + 2\left(y_2^2 + y_2y_3 + \frac{1}{4}y_3^2\right) - \frac{1}{2}y_3^2$$
$$= \underbrace{-2y_1^2}_{\xi_1} + 2\underbrace{\left(y_2+\frac{1}{2}y_3\right)^2}_{\xi_2} - \frac{1}{2}\underbrace{y_3^2}_{\xi_3} \Rightarrow q(\xi) = -2\xi_1^2 + 2\xi_2^2 - \frac{1}{2}\xi_3^2.$$

Metoda 2

$$A = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix}; \begin{pmatrix} \boxed{-2} & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & \boxed{2} & 1 \\ 0 & 1 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/2 \\ 0 & 0 & \boxed{-1/2} \end{pmatrix}$$

$$\Rightarrow q(y) = -2y_1^2 + 2\left(y_2^2 + \frac{1}{2}\cdot y_3\right)^2 - \frac{1}{2}y_3^2$$
$$\Rightarrow q(\xi) = -2\xi_1^2 + 2\xi_2^2 - \frac{1}{2}\xi_3^2$$

Metoda 3

$$A = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix}; \quad D_0 = 1;\ D_1 = -2;\ D_2 = -4;\ D_3 = 2.$$
$$q(\xi) = -2\xi_1^2 + 2\xi_2^2 - \frac{1}{2}\xi_3^2$$

b) $q(x) = 2x_1^2 - 5x_2^2 + 4x_1x_3 - 4x_1x_4 + 6x_2x_3 - 8x_3x_4.$

Metoda 1

$$q(x) = (2x_1^2 + 4x_1x_3 - 4x_1x_4) - 5x_2^2 + 6x_2x_3 - 8x_3x_4$$
$$= 2(x_1^2 + 2x_1x_3 - 2x_1x_4 - 2x_3x_4 + x_3^2 + x_4^2) + 4x_3x_4 - 2x_3^2 - 2x_4^2 - 5x_2^2 + 6x_2x_3 - 8x_3x_4$$
$$= 2(x_1+x_3-x_4)^2 - 5\left(x_2^2 - \frac{6}{5}x_2x_3 + \frac{9}{25}x_3^2\right) + \frac{9}{5}x_3^2 - 2x_3^2 - 2x_4^2 - 4x_3x_4.$$
$$= 2(x_1+x_3-x_4)^2 - 5\left(x_2 - \frac{3}{5}x_3\right)^2 - \frac{1}{5}x_3^2 - 2x_4^2 - 4x_3x_4$$
$$= 2(x_1+x_3-x_4)^2 - 5\left(x_2 - \frac{3}{5}x_3\right)^2 - \frac{1}{5}(x_3^2 + 20x_3x_4 + 100x_4^2) + 20x_4^2 - 2x_4^2$$
$$= 2\underbrace{(x_1+x_3-x_4)^2}_{\xi_1} - 5\underbrace{\left(x_2 - \frac{3}{5}x_3\right)^2}_{\xi_2} - \frac{1}{5}\underbrace{(x_3+10x_4)^2}_{\xi_3} + 18\underbrace{x_4^2}_{\xi_4}$$
$$\Rightarrow q(\xi) = 2\xi_1^2 - 5\xi_2^2 - \frac{1}{5}\xi_3^2 + 18\xi_4^2$$

Metoda 2

$$A=\begin{pmatrix} \boxed{2} & 0 & 2 & -2 \\ 0 & -5 & 3 & 0 \\ 2 & 3 & 0 & -4 \\ \boxed{-2} & 0 & -4 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & \boxed{-5} & 3 & 0 \\ 0 & 3 & -2 & -2 \\ 0 & 0 & -2 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -3/5 & 0 \\ 0 & 0 & \boxed{-1/5} & -2 \\ 0 & 0 & -2 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -3/5 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & \boxed{18} \end{pmatrix}$$

$$\Rightarrow q(x) = 2(x_1+x_3-x_4)^2 - 5\left(x_2 - \frac{3}{5}x_3\right)^2 - \frac{1}{5}\left(x_3+10x_4\right)^2 + 18x_4^2$$

$$\Rightarrow q(\xi) = 2\xi_1^2 - 5\xi_2^2 - \frac{1}{5}\xi_3^2 + 18\xi_4^2$$

Metoda 4

$$A=\begin{pmatrix} 2 & 0 & 2 & -2 \\ 0 & -5 & 3 & 0 \\ 2 & 3 & 0 & -4 \\ -2 & 0 & -4 & 0 \end{pmatrix}$$

$D_0 = 1;$

$D_1 = 2;$

$D_2 = -10;$

$D_3 = 20 - 18 = 2;$

$D_4 = 36;$

$$\Rightarrow q(\xi) = 2\xi_1^2 - 5\xi_2^2 - \frac{1}{5}\xi_3^2 + 18\xi_4^2$$