

Curs08 - Rezolvare

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Exercițiu 1 și 2.

Exercițiu 4

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Ex 1 (PC) $x(0) = 1, x'(0) = 2$

$$x(t) = C_1 \cos t + C_2 \sin t + t^2 + t - 2$$

$$x(0) = C_1 \cos 0 + C_2 \sin 0 + (-2) = C_1 - 2 = 1 \Rightarrow \boxed{C_1 = 3}$$

$$x'(t) = C_1 \cdot (-\sin t) + C_2 \cdot \cos t + 2t + 1$$

$$x'(0) = C_1 \cdot (-\sin 0) + C_2 \cdot \cos 0 + 1 = C_2 + 1 = 2 \Rightarrow \boxed{C_2 = 1}$$

$$x_{PC} = 3 \cos t + \sin t + t^2 + t - 2$$

Ex 2 (PC) $x(0) = 2, x'(0) = -1$ $\rightarrow \frac{1}{2}t^2 - 3t$

$$x(t) = C_1 + C_2 e^{-t} + t\left(\frac{1}{2}t - 3\right)$$

$$x(0) = C_1 + C_2 \cdot e^0 = \boxed{C_1 + C_2 = 2}$$

$$x'(t) = C_2 \cdot e^{-t} \cdot (-1) + t - 3$$

$$x'(0) = -C_2 - 3 = -1 \Rightarrow -C_2 = 2 \Rightarrow \boxed{C_2 = -2} \Rightarrow \boxed{C_1 = 4}$$

$$x_{PC} = 4 - 2e^{-t} + t\left(\frac{1}{2}t - 3\right)$$

Exercițiu 4

$$x'' + 4x = t \sin 2t$$

$$x_p = (\lambda_1 t^2 + \lambda_0 t) \cos 2t + (\beta_1 t^2 + \beta_0 t) \sin 2t$$

$$x_p' = (2\lambda_1 t + \lambda_0) \cos 2t + (\lambda_1 t^2 + \lambda_0 t) \cdot (-2 \sin 2t) + (2\beta_1 t + \beta_0) \cdot \sin 2t + (\beta_1 t^2 + \beta_0 t) \cdot 2 \cos 2t$$

$$= (2\lambda_1 t + \lambda_0 + 2\beta_1 t^2 + 2\beta_0 t) \cos 2t + (-2\lambda_1 t^2 - 2\lambda_0 t + 2\beta_1 t + \beta_0) \sin 2t$$

$$x'' = (2\lambda_1 + 4\beta_1 t + 2\beta_0) \cos 2t + (2\lambda_1 t + \lambda_0 + 2\beta_1 t^2 + 2\beta_0 t) \cdot (-2 \sin 2t) + (-4\lambda_1 t - 2\lambda_0 + 2\beta_1) \sin 2t + (-2\lambda_1 t^2 - 2\lambda_0 t + 2\beta_1 t + \beta_0) \cdot 2 \cos 2t$$

$$= (2\lambda_1 + 4\beta_1 t + 2\beta_0 - 4\lambda_1 t^2 - 4\lambda_0 t + 4\beta_1 t + 2\beta_0) \cos 2t + (-4\lambda_1 t - 2\lambda_0 - 4\beta_1 t^2 - 4\beta_0 t - 4\lambda_1 t - 2\lambda_0 + 2\beta_1) \sin 2t$$

$$= (-4\lambda_1 t^2 - 4\lambda_0 t + 8\beta_1 t + 2\lambda_1 + 4\beta_0) \cos 2t + (-4\beta_1 t^2 - 8\lambda_1 t - 4\beta_0 t - 4\lambda_0 + 2\beta_1) \sin 2t$$

$$(-4\lambda_1 t^2 - 4\lambda_0 t + 8\beta_1 t + 2\lambda_1 + 4\beta_0) \cos 2t + (-4\beta_1 t^2 - 8\lambda_1 t - 4\beta_0 t - 4\lambda_0 + 2\beta_1) \sin 2t = t \sin 2t$$

$$(8\beta_1 t + 2\lambda_1 + 4\beta_0) \cos 2t + (-8\lambda_1 t + 4\lambda_0 + 2\beta_1) \sin 2t = t \sin 2t$$

$$8\lambda_1 t + 2\lambda_1 + 4\beta_0 = 0$$

$$\Rightarrow 8\lambda_1 = 0 \Rightarrow \lambda_1 = 0$$

$$4\lambda_0 + 2\beta_1 = 0$$

$$-8\lambda_1 t + 4\lambda_0 + 2\beta_1 = t$$

$$2\lambda_1 + 4\beta_0 = 0$$

$$-8\lambda_1 = 1 \Rightarrow \lambda_1 = -\frac{1}{8}$$

