

$$\begin{aligned} \text{a)} \quad & x'' - 2x' + 5x = e^t (\cos 2t - 2e^{it} \sin 2t) \quad (f(t) = e^{at} (\underbrace{p_1(t) \cos \beta t + p_2(t) \sin \beta t}_{\text{particular solution}})) \quad \begin{matrix} \alpha = 1 \\ \beta = 2 \end{matrix} \rightarrow \alpha \pm i\beta = 1 \pm 2i \\ & x'' - 2x' + 5x = 0 \quad \begin{matrix} x = e^{nt} \\ x' = n e^{nt} \\ x'' = n^2 e^{nt} \end{matrix} \quad \begin{matrix} n^2 e^{nt} - 2n e^{nt} + 5e^{nt} = 0 \quad | : e^{nt} \\ n^2 - 2n + 5 = 0 \\ \Delta = 4 - 20 = -16 \\ n_{1,2} = \frac{2 \pm 4i}{2} = 1 \pm 2i \quad (\alpha \pm i\beta, \beta = 2) \\ e^{1 \pm 2i} \cos 2t, e^{1 \pm 2i} \sin 2t \rightarrow \text{S. f. d.} \end{matrix} \\ & x_0 = c_1 e^t \cos 2t + c_2 e^t \sin 2t \end{aligned}$$

$$\begin{aligned}
& e^t \left[(\lambda_2 t^3 + \lambda_1 t^2 + \lambda_0 t + 3\lambda_2 t^2 + 2\lambda_1 t + \lambda_0 + 2\beta_2 t^3 + 2\beta_1 t^2 + 2\beta_0 t + 3\lambda_2 t^2 + 2\lambda_1 t + \lambda_0 + 6\lambda_2 t + 2\lambda_1 + 6\beta_2 t^2 + 4\beta_1 t + 2\beta_0) \cos 2t \right. \\
& \quad + (\beta_2 t^3 + \beta_1 t^2 + \beta_0 t - 2\lambda_2 t^3 - 2\lambda_1 t^2 - 2\lambda_0 t + 3\beta_2 t^2 + 2\beta_1 t + \beta_0 - 2\lambda_2 t^2 - 2\lambda_1 t^2 - 2\lambda_0 t - 6\lambda_2 t^2 - 4\lambda_1 t \\
& \quad \left. - 2\lambda_0 - 4\beta_2 t^3 - 4\beta_1 t^2 - 4\beta_0 t + 3\beta_2 t^2 + 2\beta_1 t + \beta_0 - 6\lambda_2 t^2 - 4\lambda_1 t - 2\lambda_0 + 6\beta_2 t^2 + 2\beta_1 t) \sin 2t \right] \\
& - 2e^t \left[(\lambda_2 t^3 + \lambda_1 t^2 + \lambda_0 t + 3\lambda_2 t^2 + 2\lambda_1 t + \lambda_0 + 2\beta_2 t^3 + 2\beta_1 t^2 + 2\beta_0 t) \cos 2t \right. \\
& \quad + (\beta_2 t^3 + \beta_1 t^2 + \beta_0 t - 2\lambda_2 t^3 - 2\lambda_1 t^2 - 2\lambda_0 t + 3\beta_2 t^2 + 2\beta_1 t + \beta_0) \sin 2t \\
& \left. + 5e^t \left[(\lambda_2 t^3 + \lambda_1 t^2 + \lambda_0) \cos 2t + (\beta_2 t^3 + \beta_1 t^2 + \beta_0) \sin 2t \right] = e^t (t \cos 2t - t^2 \sin 2t) \right] \quad | : e^t \\
& \left[-3\lambda_2 t^3 + 4\beta_2 t^3 - 3\lambda_1 t^2 + 6\lambda_2 t^2 + 4\beta_1 t^2 + 12\beta_2 t^2 + 4\lambda_1 t + 4\beta_0 t + 6\lambda_2 t + 8\beta_1 t - 3\lambda_0 t + 2\lambda_0 + 2\lambda_1 + 4\beta_0 - 2\lambda_2 t^3 \right. \\
& \quad - 2\lambda_1 t^2 - 2\lambda_0 t - 6\lambda_2 t^2 - 4\lambda_1 t - 2\lambda_0 - 4\beta_2 t^3 - 4\beta_1 t^2 - 4\beta_0 t + 5\lambda_2 t^3 + 5\lambda_1 t^2 + 5\lambda_0 t \left. \right] \cos 2t \\
& + \left[-3\beta_2 t^3 - 4\lambda_2 t^3 - 4\lambda_1 t^2 + 6\beta_2 t^2 - 12\lambda_1 t^2 - 3\beta_1 t^2 - 3\beta_0 t - 4\lambda_0 t + 4\beta_1 t - 8\lambda_1 t + 6\beta_2 t + 2\beta_0 - 4\lambda_0 + 2\beta_1 \right. \\
& \quad \left. - 2\beta_2 t^3 - 2\beta_1 t^2 - 2\beta_0 t + 4\lambda_2 t^3 + 4\lambda_1 t^2 + 4\lambda_0 t - 6\beta_2 t^2 - 4\beta_1 t - 2\beta_0 + 5\beta_2 t^3 + 5\beta_1 t^2 + 5\beta_0 t \right] \sin 2t = \\
& 12\beta_2 t^2 + 6\lambda_2 t + 8\beta_1 t + 2\lambda_1 + 4\beta_0 = t \\
& -12\lambda_2 t^2 \quad -4\lambda_1 t - 4\lambda_0 + 2\beta_1 = -t^2 \\
& 12\beta_2 = 0 \quad \rightarrow \beta_2 = 0 \\
& 6\lambda_2 + 8\beta_1 = 1 \quad \rightarrow 8\beta_1 = 1 - 6 \cdot \frac{1}{12} = 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow \beta_1 = \frac{1}{16} \\
& 2\lambda_1 + 4\beta_0 = 0 \quad \rightarrow \beta_0 = 0 \\
& -12\lambda_2 = -1 \quad \rightarrow \lambda_2 = \frac{1}{12} \\
& -8\lambda_1 = 0 \quad \rightarrow \lambda_1 = 0 \\
& -4\lambda_0 + 2\beta_1 = 0 \quad \rightarrow 4\lambda_0 = 2\beta_1 = 2 \cdot \frac{1}{16} = \frac{1}{8} \Rightarrow \lambda_0 = \frac{1}{32} \\
& x_p = t e^t \left[\left(\frac{1}{12} t^2 + \frac{1}{32} \right) \cos 2t + \frac{1}{16} t \sin 2t \right] \\
& x = x_0 + x_p = c_1 e^t \cos 2t + c_2 e^t \sin 2t + t e^t \left[\left(\frac{1}{12} t^2 + \frac{1}{32} \right) \cos 2t + \frac{1}{16} t \sin 2t \right]
\end{aligned}$$

Exercițiu 1. c) - Video

c) $x'' - 9x = e^t + \frac{t}{2} e^t - \frac{t^2 - 2}{8} e^{2t}$

$x'' - 9x = 0$ $\lambda^2 - 9 = 0 \Rightarrow \lambda_1 = 3, \lambda_2 = -3$

$x_0 = c_1 e^{3t} + c_2 e^{-3t}$

$x_p = \frac{1}{5} e^{2t}$

$x_{p2} = \frac{1}{8} t^2 + \frac{20}{81}$

$x = x_0 + x_{p1} + x_{p2} + x_{p3}$

$= c_1 e^{3t} + c_2 e^{-3t} - \frac{1}{5} e^{2t} + e^t \left(\frac{1}{8} t - \frac{1}{32} \right) + \frac{1}{9} t^2 + \frac{20}{81}$

Maple