Laborator5 - Temă - Model2

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Exerciţiu 1.0.4.

Se consideră formula

$$\alpha = (\neg(a \wedge (\neg b)) \vee (\neg a \rightarrow c)) \rightarrow (\neg(\neg a \vee b) \rightarrow (c \vee a))$$

şi substituţia

$$\sigma = \{(x \vee \neg m)|\alpha, (m \wedge n)|a, (q \wedge p)|m, a|q\}$$

Să se determine:

- secvenţa generativă formule (SGF) pentru formula α
- tabelul de adevăr pentru formula α
- arborele de structură pentru formula α
- $\alpha\sigma$ - rezultatul aplicării substituţiei σ pentru formula α şi arborele de structură asociat lui $\alpha\sigma$

SGF:

$$a, b, c, \neg a, \neg b, a \wedge \neg b, \neg(a \wedge \neg b), \neg a \rightarrow c, (\neg(a \wedge \neg b)) \vee (\neg a \rightarrow c), \neg a \vee b, \neg(\neg a \vee b), c \vee a, \neg(\neg a \vee b) \rightarrow (c \vee a)$$
$$(\neg(a \wedge (\neg b)) \vee (\neg a \rightarrow c)) \rightarrow (\neg(\neg a \vee b) \rightarrow (c \vee a)) = \alpha$$

Tabel de Adevăr:

a	b	c	$\neg(a \wedge \neg b)$	$\neg a \rightarrow c$	$\neg(a \wedge (\neg b)) \vee (\neg a \rightarrow c)$	$\neg(\neg a \vee b)$	$c \vee a$	$\neg(\neg a \vee b) \rightarrow (c \vee a)$	α
T	T	T	T	T	T	F	T	T	T
T	T	F	T	T	T	F	T	T	T
T	F	T	F	T	T	T	T	T	T
T	F	F	F	T	T	T	T	T	T
F	T	T	T	T	T	F	T	T	T
F	T	F	T	F	T	F	F	T	T
F	F	T	T	T	T	F	T	T	T
F	F	F	T	F	T	F	F	T	T

Arbore de structură:

$$T(\alpha): \begin{array}{c} r \\ \swarrow \quad \searrow \\ T(\beta) \quad T(\gamma) \end{array}, \varphi(r) = \rightarrow, \beta = \neg(a \wedge \neg b) \vee (\neg a \rightarrow c), \gamma = \neg(\neg a \vee b) \rightarrow (c \vee a)$$

$$T(\beta):\ \begin{array}{c} n_1 \\ \swarrow\ \searrow \\ T(\beta_1)\quad T(\beta_2) \end{array},\varphi(n_1)=\vee,\beta_1=\neg(a\wedge\neg b),\beta_2=\neg a\to c$$

$$T(\beta_1):\ \begin{array}{c} n_2 \\ \downarrow \\ T(\beta_3) \end{array},\varphi(n_2)=\neg,\beta_3=a\wedge\neg b$$

$$T(\beta_3):\ \begin{array}{c} n_3 \\ \swarrow\ \searrow \\ T(\beta_4)\quad T(\beta_5) \end{array},\varphi(n_3)=\wedge,\beta_4=a,\beta_5=\neg b$$

$$T(\beta_4)=n_4,\varphi(n_4)=a$$

$$T(\beta_5):\ \begin{array}{c} n_5 \\ \downarrow \\ T(\beta_6) \end{array},\varphi(n_5)=\neg,\beta_6=b$$

$$T(\beta_6)=n_6,\varphi(n_6)=b$$

$$T(\beta_2):\ \begin{array}{c} n_7 \\ \swarrow\ \searrow \\ T(\beta_7)\quad T(\beta_8) \end{array},\varphi(n_7)=\to,\beta_7=\neg a,\beta_8=c$$

$$T(\beta_7):\ \begin{array}{c} n_8 \\ \downarrow \\ T(\beta_9) \end{array},\varphi(n_8)=\neg,\beta_9=a$$

$$T(\beta_9)=n_9,\varphi(n_9)=a$$

$$T(\beta_8)=n_{10},\varphi(n_{10})=c$$

$$T(\gamma):\ \begin{array}{c} n_{11} \\ \swarrow\ \searrow \\ T(\gamma_1)\quad T(\gamma_2) \end{array},\varphi(n_{11})=\to,\gamma_1=\neg(\neg a\vee b),\gamma_2=c\vee a$$

$$T(\gamma_1):\ \begin{array}{c} n_{12} \\ \downarrow \\ T(\gamma_3) \end{array},\varphi(n_{12})=\neg,\gamma_3=\neg a\vee b$$

$$T(\gamma):\ \begin{array}{c} n_{13} \\ \swarrow\ \searrow \\ T(\gamma_4)\quad T(\gamma_5) \end{array},\varphi(n_{13})=\vee,\gamma_4=\neg a,\gamma_5=b$$

$$T(\gamma_4):\ \begin{array}{c} n_{14} \\ \downarrow \\ T(\gamma_6) \end{array},\varphi(n_{14})=\neg,\gamma_6=a$$

$$T(\gamma_6)=n_{15},\varphi(n_{15})=a$$

$$T(\gamma_5)=n_{16},\varphi(n_{16})=b$$

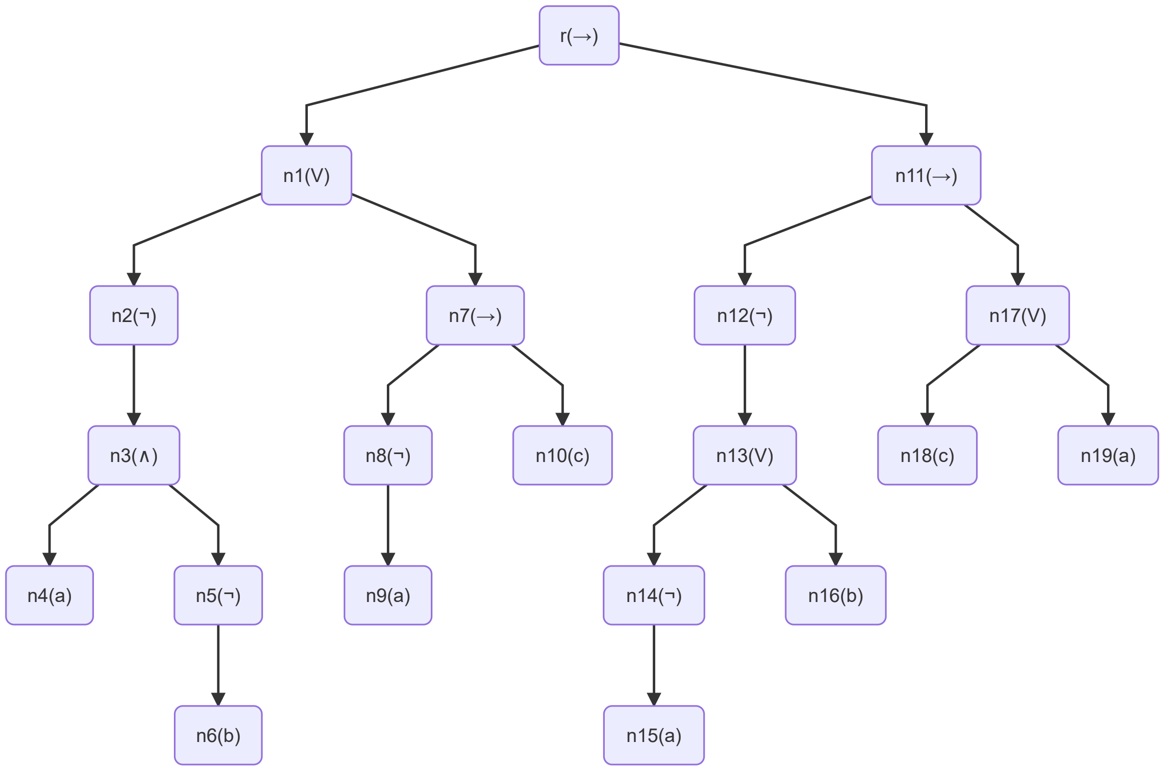
$$n_{17}$$

$$\begin{array}{l} T(\gamma): \ \swarrow \searrow \ , \varphi(n_{17}) = \vee, \gamma_7 = c, \gamma_8 = a \\ \quad T(\gamma_7) \ \ T(\gamma_8) \end{array}$$

$$T(\gamma_7) = n_{18}, \varphi(n_{18}) = c$$

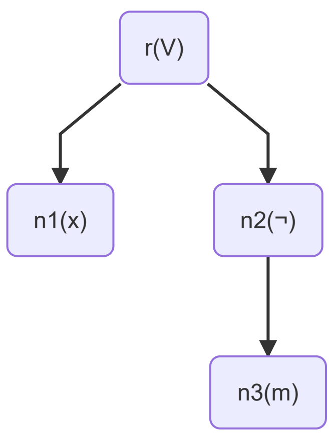
$$T(\gamma_8) = n_{19}, \varphi(n_{19}) = a$$

Final:



Aplicare substituţie $\alpha\sigma$:

$$\alpha\sigma = x \vee \neg m$$



Exerciţiul 1.0.5.

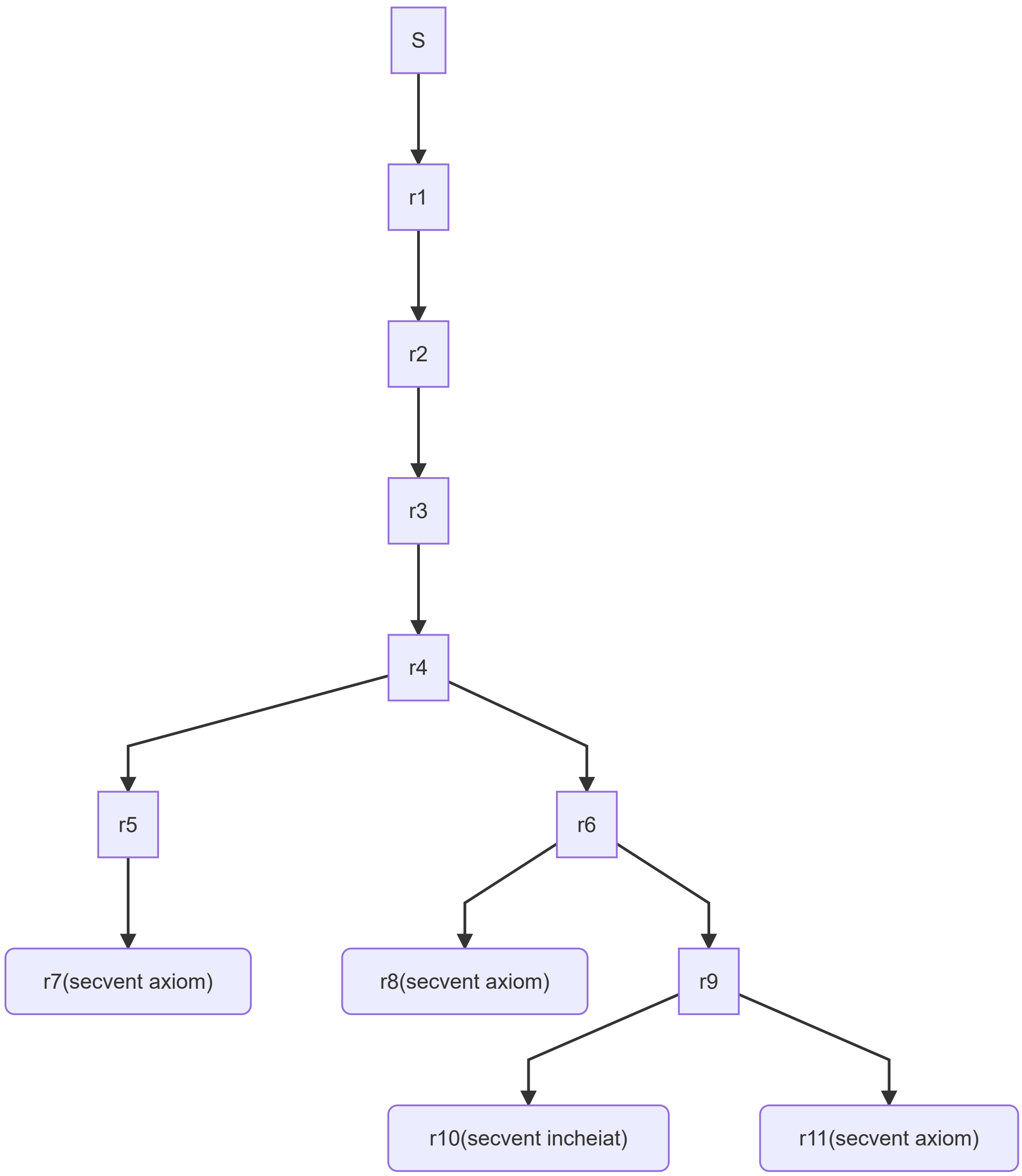
a) Să se verifice dacă următorul secvent este demonstrabil:

$$S = \{(a \vee (b \to c)), (a \to (\neg c))\} \Rightarrow \{\neg(d \vee (\neg(b)) \to (\neg c)\}$$

Sistem:

$$
\begin{aligned}
& S = \{a \vee (b \to c), a \to \neg c\} \Rightarrow \{\neg(d \vee \neg b) \to \neg c\} \\
G8: \ & r1 = \{a \vee (b \to c), a \to \neg c, \neg(d \vee \neg b)\} \Rightarrow \{\neg c\} \\
G1: \ & r2 = \{a \vee (b \to c), a \to \neg c\} \Rightarrow \{d \vee \neg b, \neg c\} \\
G7: \ & r3 = \{a \vee (b \to c), a \to \neg c\} \Rightarrow \{d, \neg b, \neg c\} \\
G5: \ & r4 = \{a \vee (b \to c), a \to \neg c, c, b\} \Rightarrow \{d\} \\
G4: \ & r5 = \{a \vee (b \to c), \neg c, c, b\} \Rightarrow \{d\} \\
& r6 = \{a \vee (b \to c), c, b\} \Rightarrow \{a, d\} \\
G1: \ & r7 = \{a \vee (b \to c), c, b\} \Rightarrow \{d, c\} \ secvent \ axiom \\
G3: \ & r8 = \{a, c, b\} \Rightarrow \{a, d\} \ secvent \ axiom \\
& r9 = \{b \to c, c, b\} \Rightarrow \{a, d\} \\
G4: \ & r10 = \{c, b\} \Rightarrow \{a, d\} \ secvent \ incheiat \\
& r11 = \{c, b\} \Rightarrow \{b, a, d\} \ secvent \ axiom \\
& S \ nu \ e \ tautologie
\end{aligned}
$$

Schema:



b) Să se calculeze mulţimile α_λ^+,α_λ^-, α_λ^0,POS_λ^α, NEG_λ^α, REZ_λ^α, unde $\lambda = \eta$, respectiv $\lambda = \neg\theta$, iar:

$$S(\alpha) = \{\neg\gamma \vee \beta \vee \neg\delta, \neg\beta \vee \eta \vee \neg\gamma, \neg\theta, \beta, \theta \vee \beta \vee \neg\eta, \delta \vee \beta \vee \neg\theta, \gamma \vee \eta \vee \neg\delta\}$$

Pentru $\lambda = \eta$:

$$\begin{aligned}
\alpha_\lambda^+ &= \{\neg\beta \vee \eta \vee \neg\gamma, \gamma \vee \eta \vee \neg\delta\} \\
\alpha_\lambda^- &= \{\theta \vee \beta \vee \neg\eta\} \\
\alpha_\lambda^0 &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\theta, \beta, \delta \vee \beta \vee \neg\theta\} \\
POS_\lambda^\alpha &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\beta \vee \neg\gamma, \neg\theta, \beta, \delta \vee \beta \vee \neg\theta, \gamma \vee \neg\delta\} \\
NEG_\lambda^\alpha &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\theta, \beta, \theta \vee \beta, \delta \vee \beta \vee \neg\theta\} \\
REZ_\lambda^\alpha &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\theta, \beta, \delta \vee \beta \vee \neg\theta, \theta \vee \neg\gamma, \theta \vee \beta \vee \gamma \vee \neg\delta\}
\end{aligned}$$

Pentru $\lambda = \neg\theta$:

$$\begin{aligned}
\alpha_\lambda^+ &= \{\neg\theta, \delta \vee \beta \vee \neg\theta\} \\
\alpha_\lambda^- &= \{\theta \vee \beta \vee \neg\eta\} \\
\alpha_\lambda^0 &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\beta \vee \eta \vee \neg\gamma, \beta, \gamma \vee \eta \vee \neg\delta\} \\
POS_\lambda^\alpha &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\beta \vee \eta \vee \neg\gamma, \square, \beta, \delta \vee \beta, \gamma \vee \eta \vee \neg\delta\} \\
NEG_\lambda^\alpha &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\beta \vee \eta \vee \neg\gamma, \beta, \beta \vee \neg\eta, \gamma \vee \eta \vee \neg\delta\} \\
REZ_\lambda^\alpha &= \{\neg\gamma \vee \beta \vee \neg\delta, \neg\beta \vee \eta \vee \neg\gamma, \beta, \gamma \vee \eta \vee \neg\delta, \square \vee \beta \vee \neg\eta, \delta \vee \beta \vee \neg\eta\}
\end{aligned}$$

Exerciţiul 1.0.6.

Să se determine forma normală conjunctivă (CNF) şi să se aplice algoritmul *Davis-Putnam* pentru formula

$$\alpha = ((\neg a \vee b) \leftrightarrow (d \to c))$$

CNF:

$$\begin{gathered}
((\neg a \vee b) \to (d \to c)) \wedge ((d \to c) \to (\neg a \vee b)) \\
(\neg(\neg a \vee b) \vee (\neg d \vee c)) \wedge (\neg(\neg d \vee c) \vee (\neg a \vee b)) \\
((a \wedge \neg b) \vee (\neg d \vee c)) \wedge ((d \wedge \neg c) \vee (\neg a \vee b)) \\
(a \vee \neg d \vee c) \wedge (\neg b \wedge \neg d \vee c) \wedge (d \vee \neg a \vee b) \wedge (\neg c \vee \neg a \vee b)
\end{gathered}$$

Davis-Putnam:

$$
\begin{aligned}
Initializare : & \gamma \leftarrow \{a \vee \neg d \vee c, \neg b \wedge \neg d \vee c, d \vee \neg a \vee b, \neg c \vee \neg a \vee b\} \\
& sw \leftarrow false, T \leftarrow \emptyset \\
Iteratia\ 1 : & Nu\ exista\ literar\ pur\ sau\ clauza\ unitara \\
& alegem\ \lambda = b\ literar \\
& \gamma \leftarrow NEG_b(\gamma) = \{a \vee \neg d \vee c, \neg d \vee c\} \\
& T \leftarrow POS_b(\gamma) = \{a \vee \neg d \vee c, d \vee \neg a, \neg c \vee \neg a\} \\
Iteratia\ 2 : & \lambda = c\ literar\ pur \\
& \gamma \leftarrow NEG_c(\gamma) = \emptyset \\
Iteratia\ 3 : & \gamma = \emptyset, \gamma \leftarrow T = \{a \vee \neg d \vee c, d \vee \neg a, \neg c \vee \neg a\} \\
& T = \emptyset \\
Iteratia\ 4 : & Nu\ exista\ literar\ pur\ sau\ clauza\ unitara \\
& alegem\ \lambda = a\ literar \\
& \gamma \leftarrow NEG_a(\gamma) = \{d, \neg c\} \\
& T \leftarrow POS_a(\gamma) = \{\neg d \vee c\} \\
Iteratia\ 5 : & \lambda = d\ clauza\ unitara \\
& \gamma \leftarrow NEG_d(\gamma) = \{\neg c\} \\
Iteratia\ 6 : & \lambda = \neg c\ clauza\ unitara \\
& \gamma \leftarrow NEG_{\neg c}(\gamma) = \emptyset \\
Iteratia\ 7 : & \gamma = \emptyset, \gamma \leftarrow T = \{\neg d \vee c\} \\
& T = \emptyset \\
Iteratia\ 8 : & \lambda = c\ literar\ pur \\
& \gamma \leftarrow NEG_c(\gamma) = \emptyset \\
Iteratia\ 9 : & \gamma = \emptyset \Rightarrow write('validalibila'),\ sw \leftarrow true
\end{aligned}
$$