Laborator09

Ecuații

1.

$$a)\ x^{II} + x^{I} = 3t + 2$$
$$b)\ x^{II} - 4x^{I} + 4x = t^2$$
$$c)\ x^{II} - x^{I} + x = t^3 + 6$$
$$d)\ 3x^{II} - 2x^{I} - x = t^2 - 1$$
$$e)\ x^{IV} - 2x^{III} + x^{II} = t^3$$
$$f)\ x^{III} + 3x^{II} - 4x^{I} = 2t^2 - 3t + 9$$
$$g)\ x^{V} - 4x^{IV} + 5x^{III} = 600t^3 - 240t^2 + 120$$

2.

$$a)\ x^{II} - 4x = te^{3t}$$
$$b)\ x^{II} - 9x = 5t^2 e^{2t}$$
$$c)\ x^{II} + 2x^{I} + x = e^{2t}$$
$$d)\ x^{II} - 4x = t^2 e^{2t}$$
$$e)\ x^{II} - 3x^{I} + 2x = 3t^2 e^{t}$$
$$f)\ x^{II} + 2x^{I} - 3x = 4te^{t}$$

3.

$$a)\ x^{II} + 4x^{I} - 3x = t \cdot sin2t$$
$$b)\ x^{II} - 4x = e^{2t} \cdot cos2t$$
$$c)\ x^{II} - 2x^{I} + 5x = t \cdot e^{t} \cdot sint$$
$$d)\ x^{II} - 2x^{I} + 5x = e^{t} \cdot cos2t$$
$$e)\ x^{II} - x = 2t \cdot sint$$
$$f)\ x^{II} + 4x = e^{2t} \cdot sin2t$$

Rezolvare

Exercițiu 1. d) - [Video](#)

(1) d) $3x'' - 2x' - x = t^2 - 1$

$3x'' - 2x' - x = 0$

$x = e^{nt}, \quad x' = ne^{nt}, \quad x'' = n^2 e^{nt}$

$3n^2 e^{nt} - 2ne^{nt} - e^{nt} = 0 \mid : e^{nt}$

$3n^2 - 2n - 1 = 0$

$\Delta = 4 + 12 = 16$

$n_{1,2} = \frac{2 \pm 4}{6} < \begin{array}{l} n_1 = \boxed{1} \\ n_2 = \boxed{\frac{1}{3}} \end{array}$

$e^{1t}, \quad e^{-\frac{1}{3}t} \quad - S.f.s.$

$\boxed{x_0 = C_1 e^t + C_2 e^{-\frac{1}{3}t}}$

$x_p = \lambda_2 t^2 + \lambda_1 t + \lambda_0$

$x'_p = 2\lambda_2 t + \lambda_1$

$x''_p = 2\lambda_2$

$3 \cdot 2\lambda_2 - 2(2\lambda_2 t + \lambda_1) - (\lambda_2 t^2 + \lambda_1 t + \lambda_0) = t^2 - 1$

$6\lambda_2 - 4\lambda_2 t - 2\lambda_1 - \lambda_2 t^2 - \lambda_1 t - \lambda_0 = t^2 - 1$

$-\lambda_2 = 1$

$-4\lambda_2 - \lambda_1 = 0 \qquad \Rightarrow \lambda_2 = -1$

$6\lambda_2 - 2\lambda_1 - \lambda_0 = -1 \qquad \lambda_1 = 4$

$\lambda_0 = 6\lambda_2 - 2\lambda_1 + 1 = -6 - 8 + 1 = -13$

$\boxed{x_p = -t^2 + 4t - 13}$

$x(t) = x_0 + x_p = C_1 e^t + C_2 e^{-\frac{1}{3}t} - t^2 + 4t - 13$

Exercițiu 1. f)

(1) f) $x''' + 3x'' - 4x' = 2t^2 - 3t + 1$

$x''' + 3x'' - 4x' = 0$

$x = e^{nt}$
$x' = ne^{nt}$
$x'' = n^2 e^{nt}$
$x''' = n^3 e^{nt}$
$\Big\} \Rightarrow n^3 e^{nt} + 3n^2 e^{nt} - 4 \, e^{nt} = 0 \mid : e^{nt}$

$n^3 + 3n^2 - 4 = 0$

$n_1 = 1$

$\begin{array}{r|l} n^3 + 3n^2 \quad -4 & \underline{n-1} \\ -n^3 + n^2 & n^2 + 4n + 4 \\ \hline 4n^2 \quad -4 \\ -4n^2 + 4n \\ \hline 4n - 4 \\ -4n + 4 \\ \hline 0 \end{array}$

$(n-1)(n^2 + 4n + 4) = 0 \qquad n_1 = \boxed{1}$

$(n-1)(n+2)^2 = 0 < \begin{array}{l} n_2 = n_3 = \boxed{2} \end{array}$

$e^{1t}, \quad e^{2t}, \quad te^{2t} \quad - \text{sist. fundam. de sol.}$

$\boxed{x_0 = C_1 e^t + C_2 e^{2t} + C_3 t e^{2t}}$

$x_p = \lambda_2 t^2 + \lambda_1 t + \lambda_0$

$x'_p = 2\lambda_2 t + \lambda_1$

$x''_p = 2\lambda_2$

$x'''_p = 0.$

$3 \cdot 2\lambda_2 - 4(\lambda_2 t^2 + \lambda_1 t + \lambda_0) = 2t^2 - 3t + 1$

$-4\lambda_2 t^2 - 4\lambda_1 t + 6\lambda_2 - 4\lambda_0 = 2t^2 - 3t + 1$

$-4\lambda_2 = 2 \qquad \rightarrow \boxed{\lambda_2 = -\frac{1}{2}}$

$-4\lambda_1 = -3 \qquad \rightarrow \boxed{\lambda_1 = \frac{3}{4}}$

$6\lambda_2 - 4\lambda_0 = 1 \qquad \Rightarrow 4\lambda_0 = 6\lambda_2 - 1 = 6 \cdot -\frac{1}{2} - 1 = -4$

$\boxed{\lambda_0 = -1}$

$\boxed{x_p = -\frac{1}{2} t^2 + \frac{3}{4} t - 1}$

$x(t) = x_0 + x_p = C_1 e^t + C_2 e^{2t} + C_3 t e^{2t} - \frac{1}{2} t^2 + \frac{3}{4} t - 1$

Exercițiu 1. g)

① g) $x^V - 4x^{IV} + 5x''' = 600t^3 - 240t^2 + 120$

$x^V - 4x^{IV} + 5x''' = 0$

$x = e^{nt}, \ x' = ne^{nt}, \ x'' = n^2 e^{nt}, \ x''' = n^3 e^{nt}, \ x^{IV} = n^4 e^{nt}, \ x^V = n^5 e^{nt}$

$n^5 e^{nt} - 4n^4 e^{nt} + 5n^3 e^{nt} = 0 \quad |: e^{nt}$

$n^5 - 4n^4 + 5n^3 = 0$

$n^3 (n^2 - 4n + 5) = 0$

$n_1 = n_2 = n_3 = \boxed{0}$

$n^2 - 4n + 5 = 0$

$\Delta = 16 - 20 = -4$

$n_{4,5} = \dfrac{4 \pm 2i}{2} = \boxed{2 \pm i} \quad \left(\alpha \pm i\beta, \ \begin{array}{l} \alpha = 2 \\ \beta = 1 \end{array} \right)$

$e^{0t}, \ te^{0t}, \ t^2 e^{0t}, \ e^{2t}\cos 1 \cdot t, \ e^{2t}\sin 1 \cdot t - S.f.A$

$\boxed{x_0 = C_1 + C_2 t + C_3 t^2 + C_4 e^{2t}\cos t + C_5 e^{2t}\sin t}$

$\boxed{x_p = t^3 (t^3 + 4t^2 + 10t + 20)}$

$x(t) = x_0 + x_p = C_1 + C_2 t + C_3 t^2 + C_4 e^{2t}\cos t + C_5 e^{2t}\sin t$
$\qquad\qquad + t^3 (t^3 + 4t^2 + 10t + 20)$

$x_p = t^3 (\lambda_3 t^3 + \lambda_2 t^2 + \lambda_1 t + \lambda_0) = \lambda_3 t^6 + \lambda_2 t^5 + \lambda_1 t^4 + \lambda_0 t^3$

$x_p' = 6\lambda_3 t^5 + 5\lambda_2 t^4 + 4\lambda_1 t^3 + 3\lambda_0 t^2$

$x_p'' = 30\lambda_3 t^4 + 20\lambda_2 t^3 + 12\lambda_1 t^2 + 6\lambda_0 t$

$x_p''' = 120\lambda_3 t^3 + 60\lambda_2 t^2 + 24\lambda_1 t + 6\lambda_0$

$x_p^{IV} = 360\lambda_3 t^2 + 120\lambda_2 t + 24\lambda_1$

$x_p^V = 720\lambda_3 t + 120\lambda_2$

$720\lambda_3 t + 120\lambda_2 - 4 \cdot 360\lambda_3 t^2 - 4 \cdot 120\lambda_2 t - 4 \cdot 24\lambda_1 + 5 \cdot 120\lambda_3 t^3$
$+ 300\lambda_2 t^2 + 5 \cdot 24\lambda_1 t + 30\lambda_0 = 600t^3 - 240t^2 + 120$

$600\lambda_3 = 600 \qquad \Rightarrow \boxed{\lambda_3 = 1}$

$\qquad\qquad\qquad\qquad 300\lambda_2 = -240 + 1440$
$\qquad\qquad\qquad\qquad\qquad = 1200$
$-4 \cdot 360\lambda_3 + 300\lambda_2 = -240 \qquad \boxed{\lambda_2 = 4}$

$720\lambda_3 - 480\lambda_2 + 120\lambda_1 = 0$

$120\lambda_2 - 96\lambda_1 + 30\lambda_0 = 120 \qquad 120\lambda_1 = 480 \cdot 4 - 720$
$\qquad\qquad\qquad\qquad\qquad\qquad = 1200$
$\qquad\qquad\qquad\qquad\qquad\qquad \boxed{\lambda_1 = 10}$

$30\lambda_0 = 120 - 120 \cdot 4 + 96 \cdot 10 = 120 - 480 + 960 = 600$
$\boxed{\lambda_0 = 20}$

Exercițiu 2. a) - [Video](#)

② a) $x'' - 4x = te^{3t} \qquad\qquad \alpha = 3$

$x'' - 4x = 0$

$x = e^{nt}, \ x' = ne^{nt}, \ x'' = n^2 e^{nt}$

$n^2 e^{nt} - 4e^{nt} = 0 \quad |: e^{nt}$

$n^2 - 4 = 0 \Rightarrow n^2 = 4$

$n_{1,2} = \boxed{\pm 2}$

$e^{2t}, \ e^{-2t} - S.f.D.$

$\boxed{x_0 = C_1 e^{2t} + C_2 e^{-2t}}$

$x_p = e^{3t}(\lambda_1 t + \lambda_0)$

$x_p' = 3e^{3t}(\lambda_1 t + \lambda_0) + e^{3t} \cdot \lambda_1 = e^{3t}(3\lambda_1 t + 3\lambda_0 + \lambda_1)$

$x_p'' = 3e^{3t}(3\lambda_1 t + 3\lambda_0 + \lambda_1) + e^{3t} \cdot 3\lambda_1 = e^{3t}(9\lambda_1 t + 9\lambda_0 + 3\lambda_1 + 3\lambda_1)$

$e^{3t}(9\lambda_1 t + 9\lambda_0 + 6\lambda_1) - 4e^{3t}(\lambda_1 t + \lambda_0) = te^{3t} \quad |: e^{3t}$

$9\lambda_1 t + 9\lambda_0 + 6\lambda_1 - 4\lambda_1 t - 4\lambda_0 = t$

$5\lambda_1 = 1 \qquad\qquad \Rightarrow \lambda_1 = \dfrac{1}{5}$

$5\lambda_0 + 6\lambda_1 = 0$

$\qquad\qquad\qquad \lambda_0 = -\dfrac{6}{5} \cdot \dfrac{1}{5} = -\dfrac{6}{25}$

$\boxed{x_p = e^{3t}\left(\dfrac{1}{5}t - \dfrac{6}{25} \right)}$

$x = x_0 + x_p = C_1 e^{2t} + C_2 e^{-2t} + e^{3t}\left(\dfrac{1}{5}t - \dfrac{6}{25} \right)$

Exercițiu 2. d)

② d) $x'' - 4x = t^2 e^{2t} \qquad\qquad \alpha = 2$

$x'' - 4x = 0$

$n_{1,2} = \boxed{\pm 2}$

$\boxed{x_0 = C_1 e^{2t} + C_2 e^{-2t}}$

$x_p = t \cdot e^{2t}(\lambda_2 t^2 + \lambda_1 t + \lambda_0) = e^{2t}(\lambda_2 t^3 + \lambda_1 t^2 + \lambda_0 t)$

$x_p' = 2e^{2t}(\lambda_2 t^3 + \lambda_1 t^2 + \lambda_0 t) + e^{2t}(3\lambda_2 t^2 + 2\lambda_1 t + \lambda_0) = e^{2t}(2\lambda_2 t^3 + 2\lambda_1 t^2 + 2\lambda_0 t + 3\lambda_2 t^2 + 2\lambda_1 t + \lambda_0)$

$x_p'' = 2e^{2t}(2\lambda_2 t^3 + 2\lambda_1 t^2 + 2\lambda_0 t + 3\lambda_2 t^2 + 2\lambda_1 t + \lambda_0) + e^{2t}(6\lambda_2 t^2 + 4\lambda_1 t + 2\lambda_0 + 6\lambda_2 t + 2\lambda_1)$

$= e^{2t}(4\lambda_2 t^3 + 4\lambda_1 t^2 + 4\lambda_0 t + 6\lambda_2 t^2 + 4\lambda_1 t + 2\lambda_0 + 6\lambda_2 t^2 + 4\lambda_1 t + 2\lambda_0 + 6\lambda_2 t + 2\lambda_1)$

$4\lambda_2 t^3 + 4\lambda_1 t^2 + 12\lambda_2 t^2 + 4\lambda_0 t + 8\lambda_1 t + 6\lambda_2 t + 4\lambda_0 + 2\lambda_1 - 4\lambda_2 t^3 - 4\lambda_1 t^2 - 4\lambda_0 t = t^2$

$\begin{cases} 12\lambda_2 = 1 & \Rightarrow \lambda_2 = \dfrac{1}{12} \\ 8\lambda_1 + 6\lambda_2 = 0 & \lambda_1 = -\dfrac{6}{8} \cdot \dfrac{1}{12} = -\dfrac{1}{16} \\ 4\lambda_0 + 2\lambda_1 = 0 & \lambda_0 = -\dfrac{2}{4} \cdot -\dfrac{1}{16} = \dfrac{1}{32} \end{cases}$

$\Rightarrow \boxed{x_p = t \cdot e^{2t}\left(\dfrac{1}{12}t^2 - \dfrac{1}{16}t + \dfrac{1}{32} \right)}$

$x(t) = x_0 + x_p = C_1 e^{2t} + C_2 e^{-2t} + te^{2t}\left(\dfrac{1}{12}t^2 - \dfrac{1}{16}t + \dfrac{1}{32} \right)$

Exercițiu 3. a) - <u>Video</u>

③ a) $x'' + 4x' - 3x = t \sin 2t$

$$\alpha = 0 \qquad \beta = 2 \qquad \alpha \pm i\beta = \pm 2i$$

$x'' + 4x' - 3x = 0$

$x = e^{rt}, \quad x' = r\,e^{rt}, \quad x'' = r^2 e^{rt}$

$r^2 e^{rt} + 4r\,e^{rt} - 3e^{rt} = 0 \quad |: e^{rt}$

$r^2 + 4r - 3 = 0$

$\Delta = 16 + 12 = 28$

$r_{1,2} = \dfrac{-4 \pm 2\sqrt{7}}{2} = \boxed{-2 \pm \sqrt{7}}$

$e^{(-2+\sqrt{7})t}, \quad e^{(-2-\sqrt{7})t}$ — S.f.S.

$$\boxed{x_o = c_1 e^{(-2+\sqrt{7})t} + c_2 e^{(-2-\sqrt{7})t}}$$

$x_p = (\lambda_1 t + \lambda_0)\sin 2t + (\beta_1 t + \beta_0)\cos 2t$

$x_p' = \lambda_1 \cdot \sin 2t + (\lambda_1 t + \lambda_0)\cdot 2\cos 2t + \beta_1 \cos 2t + (\beta_1 t + \beta_0)\cdot(-2)\sin 2t$

$\quad = (\lambda_1 - 2\beta_1 t - 2\beta_0)\sin 2t + (2\lambda_1 t + 2\lambda_0 + \beta_1)\cos 2t$

$x_p'' = -2\beta_1 \cdot \sin 2t + (\lambda_1 - 2\beta_1 t - 2\beta_0)\cdot 2\cos 2t + 2\lambda_1 \cos 2t + (2\lambda_1 t + 2\lambda_0 + \beta_0)\cdot(-2)\sin 2t$

$\quad = (-2\beta_1 - 4\lambda_1 t - 4\lambda_0 - 2\beta_0)\sin 2t + (2\lambda_1 - 4\beta_1 t - 4\beta_0 + 2\lambda_1)\cos 2t$

$(-2\beta_1 - 4\lambda_1 t - 4\lambda_0 - 2\beta_0)\sin 2t + (2\lambda_1 - 4\beta_1 t - 4\beta_0 + 2\lambda_1)\cos 2t$
$+ 4(\lambda_1 - 2\beta_1 t - 2\beta_0)\sin 2t + 4(2\lambda_1 t + 2\lambda_0 + \beta_1)\cos 2t - 3(\lambda_1 t + \lambda_0)\sin 2t$
$- 3(\beta_1 t + \beta_0)\cos 2t = t \sin 2t$

$-2\beta_1 - 4\lambda_1 t - 4\lambda_0 - 2\beta_0 + 4\lambda_1 - 8\beta_1 t - 8\beta_0 - 3\lambda_1 t - 3\lambda_0 = t$

$2\lambda_1 - 4\beta_1 t - 4\beta_0 + 2\lambda_1 + 8\lambda_1 t + 8\lambda_0 + 4\beta_1 - 3\beta_1 t - 3\beta_0 = 0$

$$\begin{cases} -7\lambda_1 - 8\beta_1 = 1 \\ -2\beta_1 - 7\lambda_0 - 8\beta_0 + 4\lambda_1 = 0 \\ -7\beta_1 + 8\lambda_1 = 0 \\ 4\lambda_1 - 7\beta_0 + 8\lambda_0 + 4\beta_1 = 0 \end{cases}$$

$$\begin{cases} -7\lambda_1 - 8\beta_1 = 1 & |\,8 \\ 8\lambda_1 - 7\beta_1 = 0 & |\,7 \end{cases} \Longleftrightarrow \begin{cases} -56\lambda_1 - 64\beta_1 = 8 \\ 56\lambda_1 - 49\beta_1 = 0 \end{cases}$$

$-113\beta_1 = 8 \Rightarrow \boxed{\beta_1 = \dfrac{-8}{113}}$

$8\lambda_1 = 7 \cdot \dfrac{-8}{113} \Rightarrow \boxed{\lambda_1 = \dfrac{-7}{113}}$

$$\begin{cases} -7\lambda_0 - 8\beta_0 = 2\beta_1 - 4\lambda_1 \\ 8\lambda_0 - 7\beta_0 = -4\beta_1 - 4\lambda_1 \end{cases} \Longleftrightarrow$$

$$\begin{cases} -7\lambda_0 - 8\beta_0 = 2\cdot\dfrac{-8}{113} - 4\cdot\dfrac{-7}{113} = \dfrac{-16+28}{113} = \dfrac{12}{113} \Rightarrow \\ 8\lambda_0 - 7\beta_0 = -4\cdot\dfrac{-8}{113} - 4\cdot\dfrac{-7}{113} = \dfrac{32+28}{113} = \dfrac{60}{113} \Rightarrow \end{cases}$$

$$\begin{cases} -7\lambda_0 - 8\beta_0 = \dfrac{12}{113} \;|\,r \\ 8\lambda_0 - 7\beta_0 = \dfrac{60}{113} \Big/ 7 \end{cases}$$

$$\begin{cases} 56\lambda_0 - 64\beta_0 = \dfrac{96}{113} \\ 56\lambda_0 - 49\beta_0 = \dfrac{420}{113} \end{cases}$$

$-113\beta_0 = \dfrac{516}{113} \Rightarrow \beta_0 = \dfrac{-516}{113^2}$

$8\lambda_0 = \dfrac{60}{113} + 7\cdot\dfrac{-516}{113^2} = \dfrac{3168}{113^2} \Rightarrow \lambda_0 = \dfrac{396}{113^2}$

$$\boxed{x_p = \left(\dfrac{-7}{113}t + \dfrac{396}{113^2}\right)\sin 2t + \left(\dfrac{-8}{113}t - \dfrac{516}{113^2}\right)\cos 2t}$$

$X(t) = x_o + x_p =$
$= c_1 e^{(-2+\sqrt{7})t} + c_2 e^{(-2-\sqrt{7})t} + \left(\dfrac{-7}{113}t + \dfrac{396}{113^2}\right)\sin 2t$
$+ \left(\dfrac{-8}{113}t - \dfrac{516}{113^2}\right)\cos 2t$