Temă - Seminar 3 Petculescu Mihai-Silviu

1. Să se verifice cu ajutorul lemei substituției dacă următoarele sisteme de vectori formează baze și în caz afirmativ să se det. coord vect. $\vec{x}$ în aceste baze.

b) $B = \{(1,2,-1)^T, (1,5,0)^T, (6,3,-3)^T\}$, $\vec{x} = (0,3,2)^T$.

| | $\vec{v_1}$ | $\vec{v_2}$ | $\vec{v_3}$ | $\vec{x}$ |
|---|---|---|---|---|
| $\vec{e_1}$ | $\boxed{1}$ | 1 | 6 | 0 |
| $\vec{e_2}$ | 2 | 5 | 3 | 3 |
| $\vec{e_3}$ | -1 | 0 | -3 | 2 |
| $\vec{v_1}$ | 1 | 1 | 6 | 0 |
| $\vec{e_2}$ | 0 | $\boxed{3}$ | -9 | 3 |
| $\vec{e_3}$ | 0 | 1 | 3 | 2 |
| $\vec{v_1}$ | 1 | 0 | 9 | -1 |
| $\vec{v_2}$ | 0 | 1 | -3 | 1 |
| $\vec{e_3}$ | 0 | 0 | $\boxed{6}$ | 1 |
| $\vec{v_1}$ | 1 | 0 | 0 | -15/6 |
| $\vec{v_2}$ | 0 | 1 | 0 | 9/6 |
| $\vec{v_3}$ | 0 | 0 | 1 | 1/6 |

Deoarece obț. $I_3 \Rightarrow B$ formează o bază, iar $\vec{x}_B = \left(\frac{-15}{6}, \frac{9}{6}, \frac{1}{6}\right)$

c) $B = \{(1,-2,3)^T, (-2,5,0)^T, (2,-3,2)^T\}$, $\vec{x} = (0,-3,8)$.

| | $\vec{v_1}$ | $\vec{v_2}$ | $\vec{v_3}$ | $\vec{x}$ |
|---|---|---|---|---|
| $\vec{e_1}$ | $\boxed{1}$ | -2 | 2 | 0 |
| $\vec{e_2}$ | -2 | 5 | -3 | -3 |
| $\vec{e_3}$ | 3 | 0 | 2 | 8 |
| $\vec{v_1}$ | 1 | -2 | 2 | 0 |
| $\vec{e_2}$ | 0 | $\boxed{9}$ | 1 | -3 |
| $\vec{e_3}$ | 0 | 6 | -4 | 8 |
| $\vec{v_1}$ | 1 | 0 | 20/9 | -2/3 |
| $\vec{v_2}$ | 0 | 1 | 1/9 | -1/3 |
| $\vec{e_3}$ | 0 | 0 | $\boxed{-14/3}$ | 10 |
| $\vec{v_1}$ | 1 | 0 | 0 | 86/21 |
| $\vec{v_2}$ | 0 | 1 | 0 | -2/21 |
| $\vec{v_3}$ | 0 | 0 | 1 | -15/7 |

Deoarece am obținut $I_3 \Rightarrow$ B formează o bază, iar $\vec{x}_B = (86/21, -2/21, -15/7)$.

d) $B = \{(1,-2,1)^T, (2,5,4)^T, (3,-1,2)^T\}$, $\vec{x} = (0,3,1)^T$.

| | $\vec{v_1}$ | $\vec{v_2}$ | $\vec{v_3}$ | $\vec{x}$ |
|---|---|---|---|---|
| $\vec{l_1}$ | $\boxed{1}$ | 2 | 3 | 0 |
| $\vec{l_2}$ | -2 | 5 | -1 | 3 |
| $\vec{l_3}$ | 1 | 4 | 2 | 1 |
| $\vec{v_1}$ | 1 | 2 | 2 | 0 |
| $\vec{l_2}$ | 0 | $\boxed{9}$ | 5 | 3 |
| $\vec{l_3}$ | 0 | 2 | -1 | 1 |
| $\vec{v_1}$ | 1 | 0 | 17/9 | -2/3 |
| $\vec{v_2}$ | 0 | 1 | 5/9 | 1/3 |
| $\vec{l_3}$ | 0 | 0 | $\boxed{-19/9}$ | 1/3 |
| $\vec{v_1}$ | 1 | 0 | 0 | -7/19 |
| $\vec{v_2}$ | 0 | 1 | 0 | 8/19 |
| $\vec{v_3}$ | 0 | 0 | 1 | -3/19 |

Deoarece am obținut $I_3 \Rightarrow B$ formează o bază, iar $\vec{x}_B = \left(\frac{-7}{19}, \frac{8}{19}, \frac{-3}{19}\right)$

e) $B = \{(1,5,1)^T, (0,1,3)^T, (1,6,5)^T\}$, $\vec{x} = (1,2,3)^T$.

| | $\vec{v_1}$ | $\vec{v_2}$ | $\vec{v_3}$ | $\vec{x}$ |
|---|---|---|---|---|
| $\vec{l_1}$ | $\boxed{1}$ | 0 | 1 | 1 |
| $\vec{l_2}$ | 5 | 1 | 6 | 2 |
| $\vec{l_3}$ | 1 | 3 | 5 | 3 |
| $\vec{v_1}$ | 1 | 0 | 1 | 1 |
| $\vec{l_2}$ | 0 | $\boxed{1}$ | 1 | -3 |
| $\vec{l_3}$ | 0 | 3 | 4 | 2 |
| $\vec{v_1}$ | 1 | 0 | 1 | 1 |
| $\vec{v_2}$ | 0 | 1 | 1 | -3 |
| $\vec{l_3}$ | 0 | 0 | $\boxed{1}$ | 11 |
| $\vec{v_1}$ | 1 | 0 | 0 | -10 |
| $\vec{v_2}$ | 0 | 1 | 0 | -14 |
| $\vec{v_3}$ | 0 | 0 | 1 | 11 |

Deoarece am obținut $I_3 \Rightarrow B$ form. o bază iar $\vec{x}_B = (-10, -14, 11)$.

2. Să se determine matricea de la $B_1$ la $B_2$:

b) $B_1 = \{(2,-1,0)^T, (1,1,1)^T, (0,2,3)^T\}$

$B_2 = \{(-1,0,1)^T, (0,1,1)^T, (3,1,-1)^T\}$

| | $\vec{v_1}$ | $\vec{v_2}$ | $\vec{v_3}$ | $\vec{w_1}$ | $\vec{w_2}$ | $\vec{w_3}$ |
|---|---|---|---|---|---|---|
| $\vec{e_1}$ | $\boxed{2}$ | 1 | 0 | -1 | 0 | 3 |
| $\vec{e_2}$ | -1 | 1 | 2 | 0 | 1 | 1 |
| $\vec{e_3}$ | 0 | 1 | 3 | 1 | 1 | 1 |
| $\vec{v_1}$ | 1 | 1/2 | 0 | -1/2 | 0 | 3/2 |
| $\vec{e_2}$ | 0 | $\boxed{3/2}$ | 2 | -1/2 | 1 | 5/2 |
| $\vec{e_3}$ | 0 | 1 | 3 | 1 | 1 | -1 |
| $\vec{v_1}$ | 1 | 0 | 2/3 | -1/3 | -1/3 | 2/3 |
| $\vec{v_2}$ | 0 | 1 | 4/3 | -1/3 | 2/3 | 5/3 |
| $\vec{e_3}$ | 0 | 0 | $\boxed{5/3}$ | 4/3 | 1/3 | -8/3 |
| $\vec{v_1}$ | 1 | 0 | 0 | -1/5 | -1/5 | 26/15 |
| $\vec{v_2}$ | 0 | 1 | 0 | -7/5 | 2/5 | 19/5 |
| $\vec{v_3}$ | 0 | 0 | 1 | 4/5 | 1/5 | -8/5 |

$I_3$ $\qquad$ $M_{B_1,B_2}$

c) $B_1 = \{(1,1,2)^T, (1,0,3)^T, (3,-1,2)^T\}$

$B_2 = \{(-1,4,2)^T, (1,3,-2)^T, (0,7,5)^T\}$

| | $\vec{v_1}$ | $\vec{v_2}$ | $\vec{v_3}$ | $\vec{w_1}$ | $\vec{w_2}$ | $\vec{w_3}$ |
|---|---|---|---|---|---|---|
| $\vec{e_1}$ | $\boxed{1}$ | 1 | 3 | -1 | 1 | 0 |
| $\vec{e_2}$ | 1 | 0 | -1 | 4 | 3 | 7 |
| $\vec{e_3}$ | 2 | 3 | 2 | 2 | -2 | 5 |
| $\vec{v_1}$ | 1 | 1 | 3 | -1 | 1 | 0 |
| $\vec{e_2}$ | 0 | $\boxed{-1}$ | -4 | 5 | 2 | 7 |
| $\vec{e_3}$ | 0 | 1 | -4 | 4 | -4 | 5 |
| $\vec{v_1}$ | 1 | 0 | -1 | 4 | 3 | -7 |
| $\vec{v_2}$ | 0 | 1 | 4 | -5 | -2 | -7 |
| $\vec{e_3}$ | 0 | 0 | $\boxed{-8}$ | 9 | -2 | 12 |
| $\vec{v_1}$ | 1 | 0 | 0 | -23/8 | 13/4 | 17/2 |
| $\vec{v_2}$ | 0 | 1 | 0 | -4/8 | -3 | -1 |
| $\vec{v_3}$ | 0 | 0 | 1 | -9/8 | 1/4 | 3/2 |

$I_3$ $\qquad$ $M_{B_1 B_2}$

d) $B_1 = \{(1,3,2)^T, (0,1,5), (1,2,4)^T\}$

$B_2 = \{(1,2,4)^T, (2,4,5)^T, (4,-1,2)^T\}$

| | $\vec{v_1}$ | $\vec{v_2}$ | $\vec{v_3}$ | $\vec{w_1}$ | $\vec{w_2}$ | $\vec{w_3}$ |
|---|---|---|---|---|---|---|
| $\vec{e_1}$ | $\boxed{1}$ | 0 | 1 | 1 | 2 | 4 |
| $\vec{e_2}$ | 3 | 1 | 2 | 2 | 4 | -1 |
| $\vec{e_3}$ | 2 | 5 | 4 | 4 | 5 | 2 |
| $\vec{v_1}$ | 1 | 0 | 1 | 1 | 2 | 4 |
| $\vec{e_2}$ | 0 | $\boxed{1}$ | -1 | -1 | -2 | -13 |
| $\vec{e_3}$ | 0 | 5 | 2 | 2 | 1 | -6 |
| $\vec{v_1}$ | 1 | 0 | 1 | 1 | 2 | 4 |
| $\vec{v_2}$ | 0 | 1 | -1 | -1 | -2 | -13 |
| $\vec{e_3}$ | 0 | 0 | $\boxed{7}$ | 7 | 8 | 59 |
| $\vec{v_1}$ | 1 | 0 | 0 | 0 | 6/7 | -31/7 |
| $\vec{v_2}$ | 0 | 1 | 0 | 0 | -6/7 | 150/7 |
| $\vec{v_3}$ | 0 | 0 | 1 | 1 | 8/7 | 59/7 |

$\Big\} M_{B_1B_2}$

e) $B_1 = \{(1,4)^T, (2,-3)^T\}$ ; $B_2 = \{(3,-1)^T, (4,2)^T\}$.

| | $\vec{v_1}$ | $\vec{v_2}$ | $\vec{w_1}$ | $\vec{w_2}$ |
|---|---|---|---|---|
| $\vec{e_1}$ | $\boxed{1}$ | 2 | 3 | 4 |
| $\vec{e_2}$ | 4 | -3 | -1 | 2 |
| $\vec{v_1}$ | 1 | 2 | 3 | 4 |
| $\vec{e_2}$ | 0 | $\boxed{-11}$ | -13 | -14 |
| $\vec{v_1}$ | 1 | 0 | 7/11 | 16/11 |
| $\vec{v_2}$ | 0 | 1 | 13/11 | 14/11 |

$\Big\} M_{B_1B_2}$

f) $B_1 = \{(4,-3)^T, (1,5)^T\}$ ; $B_2 = \{(3,2)^T, (4,-3)^T\}$

| | $\vec{v_1}$ | $\vec{v_2}$ | $\vec{w_1}$ | $\vec{w_2}$ |
|---|---|---|---|---|
| $\vec{e_1}$ | $\boxed{4}$ | 1 | 3 | 4 |
| $\vec{e_2}$ | -3 | 5 | 2 | -3 |
| $\vec{v_1}$ | 1 | 1/4 | 3/4 | 1 |
| $\vec{e_2}$ | 0 | 23/4 | 17/4 | 0 |
| $\vec{v_1}$ | 1 | 0 | 13/23 | 1 |
| $\vec{v_2}$ | 0 | 1 | 17/23 | 0 |

$\Big\} M_{B_1B_2}$

3. Să se determine cu ajutorul bazei substituției inversele matricelor.

b) $A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 0 & -1 \\ 4 & 1 & 5 \end{pmatrix}$

| | $\vec{v_1}$ | $\vec{v_2}$ | $\vec{v_3}$ | $\vec{e_1}$ | $\vec{e_2}$ | $\vec{e_3}$ |
|---|---|---|---|---|---|---|
| $\vec{e_1}$ | $\boxed{1}$ | 1 | 2 | 1 | 0 | 0 |
| $\vec{e_2}$ | 3 | 0 | -1 | 0 | 1 | 0 |
| $\vec{e_3}$ | 4 | 1 | 5 | 0 | 0 | 1 |
| $\vec{v_1}$ | 1 | 1 | 2 | 1 | 0 | 0 |
| $\vec{e_2}$ | 0 | $\boxed{-3}$ | -7 | -3 | 1 | 0 |
| $\vec{e_3}$ | 0 | -3 | -3 | -4 | 0 | 1 |
| $\vec{v_1}$ | 1 | 0 | -1/3 | 0 | 1/3 | 0 |
| $\vec{v_2}$ | 0 | 1 | 7/3 | 1 | -1/3 | 0 |
| $\vec{e_3}$ | 0 | 0 | $\boxed{4}$ | -1 | -1 | -1 |
| $\vec{v_1}$ | 1 | 0 | 0 | -1/12 | 1/4 | -1/12 |
| $\vec{v_2}$ | 0 | 1 | 0 | 19/12 | 1/4 | -7/12 |
| $\vec{v_3}$ | 0 | 0 | 1 | -1/4 | -1/4 | -1/4 |

$\underbrace{\phantom{xx}}_{I_3}$ $\underbrace{\phantom{xx}}_{A^{-1}}$

c) $A = \begin{pmatrix} 1 & 0 & 5 \\ 6 & 1 & -2 \\ 1 & 1 & 0 \end{pmatrix}$

| | $\vec{v_1}$ | $\vec{v_2}$ | $\vec{v_3}$ | $\vec{e_1}$ | $\vec{e_2}$ | $\vec{e_3}$ |
|---|---|---|---|---|---|---|
| $\vec{e_1}$ | 1 | 0 | 5 | 1 | 0 | 0 |
| $\vec{e_2}$ | 6 | 1 | -2 | 0 | 1 | 0 |
| $\vec{e_3}$ | 1 | 1 | 0 | 0 | 0 | 1 |
| $\vec{v_1}$ | 1 | 0 | 5 | 1 | 0 | 0 |
| $\vec{e_2}$ | 0 | $\boxed{1}$ | -32 | -6 | 1 | 0 |
| $\vec{e_3}$ | 0 | 0 | $\boxed{27}$ | -1 | 0 | 1 |
| $\vec{v_1}$ | 1 | 0 | 5 | 1 | 0 | 0 |
| $\vec{v_2}$ | 0 | 1 | -32 | -6 | 1 | 0 |
| $\vec{e_3}$ | 0 | 0 | $\boxed{27}$ | 5 | -1 | 1 |
| $\vec{v_1}$ | 1 | 0 | 0 | 2/27 | -5/27 | -5/27 |
| $\vec{v_2}$ | 0 | 1 | 0 | -2/27 | -5/27 | 32/27 |
| $\vec{v_3}$ | 0 | 0 | 1 | 5/27 | -1/27 | 1/27 |

$\underbrace{\phantom{xx}}_{I_3}$ $\underbrace{\phantom{xx}}_{A^{-1}}$

d) $A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 6 & 5 \\ 2 & 7 & -1 \end{pmatrix}$

| | $\vec{v_1}$ | $\vec{v_2}$ | $\vec{v_3}$ | $\vec{e_1}$ | $\vec{e_2}$ | $\vec{e_3}$ |
|---|---|---|---|---|---|---|
| $\vec{e_1}$ | $\boxed{1}$ | 3 | 1 | 1 | 0 | 0 |
| $\vec{e_2}$ | 2 | 6 | 5 | 0 | 1 | 0 |
| $\vec{e_3}$ | 2 | 7 | -1 | 0 | 0 | 1 |
| $\vec{v_1}$ | 1 | 3 | 1 | 1 | 0 | 0 |
| $\vec{e_2}$ | 0 | $\boxed{0}$ | 3 | -2 | 1 | 0 |
| $\vec{e_3}$ | 0 | 1 | -3 | -2 | 0 | 1 |
| $\vec{v_1}$ | 1 | 3 | 1 | 1 | 0 | 0 |
| $\vec{e_3}$ | 0 | $\boxed{1}$ | -3 | -2 | 0 | 1 |
| $\vec{e_2}$ | 0 | 0 | 3 | -2 | 1 | 0 |
| $\vec{v_1}$ | 1 | 0 | 9 | 7 | 0 | -3 |
| $\vec{v_2}$ | 0 | 1 | -3 | -2 | 0 | 1 |
| $\vec{e_2}$ | 0 | 0 | $\boxed{3}$ | -2 | 1 | 0 |

| | $\vec{e_1}$ | $\vec{e_2}$ | $\vec{e_3}$ | $\vec{e_1}$ | $\vec{e_2}$ | $\vec{e_3}$ |
|---|---|---|---|---|---|---|
| $\vec{v_1}$ | 1 | 0 | 0 | 13 | -3 | -3 |
| $\vec{v_2}$ | 0 | 1 | 0 | -4 | 1 | 1 |
| $\vec{v_3}$ | 0 | 0 | 1 | -2/3 | 1/3 | 0 |

$\underbrace{\phantom{xx}}_{I_3}$ $\underbrace{\phantom{xx}}_{A^{-1}}$