Seminar12

Enunţ

$$b)\begin{cases} x' &= 2x + y \\ y' &= 2y + 4z \\ z' &= x - z \end{cases}$$
$$c)\begin{cases} x' &= x + y - 2z \\ y' &= 4x + y \\ z' &= 2x + y - z \end{cases}$$

Rezolvare

c) $\begin{cases} x' = x + y - 2z \\ y' = 4x + y \\ z' = 2x + y - z \end{cases}$

$x = \alpha_1 e^{rt}, \quad y = \alpha_2 e^{rt}, \quad z = \alpha_3 e^{rt}$

$x' = \alpha_1 r e^{rt}, \quad y' = \alpha_2 r e^{rt}, \quad z' = \alpha_3 r e^{rt}$

$\begin{cases} \alpha_1 r = \alpha_1 + \alpha_2 - 2\alpha_3 \\ \alpha_2 r = 4\alpha_1 + \alpha_2 \\ \alpha_3 r = 2\alpha_1 + \alpha_2 - \alpha_3 \end{cases} \Rightarrow \begin{cases} (1-r)\alpha_1 + \alpha_2 - 2\alpha_3 = 0 \\ 4\alpha_1 + (1-r)\alpha_2 = 0 \\ 2\alpha_1 + \alpha_2 + (-1-r)\alpha_3 = 0 \end{cases} \quad (*)$

$\Delta = \begin{vmatrix} 1-r & 1 & -2 \\ 4 & 1-r & 0 \\ 2 & 1 & -1-r \end{vmatrix} \underline{\underline{C_2\cdot(2)+C_1}} \begin{vmatrix} 1-r & 1 & 0 \\ 4 & 1-r & 2(1-r) \\ 2 & 1 & 1-r \end{vmatrix} = (1-r) \begin{vmatrix} 1-r & 1 & 0 \\ 4 & 1-r & 2 \\ 2 & 1 & 1 \end{vmatrix}$

$= (1-r)\left[(1-r)^2 + \cancel{4} - 2(1-r) - \cancel{4}\right] = (1-r)^2(1-r-2) = (1-r)^2(-1-r) = 0$

$-1-r = 0 \Rightarrow r_1 = -1$

$(1-r)^2 = 0 \Rightarrow r_2 = r_3 = 1$

Pt. $r_1 = -1$

$(*) \Rightarrow \begin{cases} 2\alpha_1 + \alpha_2 - 2\alpha_3 = 0 \\ 4\alpha_1 + 2\alpha_2 = 0 \\ 2\alpha_1 + \alpha_2 = 0 \end{cases} \begin{matrix} \Rightarrow \cancel{2\alpha_1} - \cancel{2\alpha_1} - 2\alpha_3 = 0 \Rightarrow \alpha_3 = 0 \\ \\ \Rightarrow \alpha_2 = -2\alpha_1, \quad \alpha_1 \in \mathbb{R} \end{matrix}$

Fie $\alpha_1 = 1 \Rightarrow \alpha_2 = -2 \Rightarrow \alpha_{r_1} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \Rightarrow X_{r_1} = \begin{pmatrix} e^{-t} \\ -2e^{-t} \\ 0 \end{pmatrix}$

Pt. $r_2 = r_3 = 1$

$(*) \Rightarrow \begin{cases} \alpha_2 - 2\alpha_3 = 0 \\ 4\alpha_1 = 0 \\ 2\alpha_1 + \alpha_2 - 2\alpha_3 = 0 \end{cases} \begin{matrix} \Rightarrow \alpha_1 = 0 \\ \alpha_2 = 2\alpha_3, \quad \alpha_3 \in \mathbb{R}. \end{matrix}$

Fie $\alpha_3 = 1 \Rightarrow \alpha_2 = 2 \Rightarrow \alpha_{r_{2,3}} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \Rightarrow$ un singur vector l.i.

Pt. $r = 1$ considerăm $x = (A_1 t + A_0)e^t, \quad y = (B_1 t + B_0)e^t, \quad z = (C_1 t + C_0)e^t$

$x' = A_1 e^t + (A_1 t + A_0)e^t, \quad y' = B_1 e^t + (B_1 t + B_0)e^t, \quad z = C_1 e^t + (C_1 t + C_0)e^t$

$\begin{cases} A_1 + \cancel{A_1 t} + \cancel{A_0} = \cancel{A_1 t} + \cancel{A_0} + \underline{B_1 t} + B_0 - \underline{2C_1 t} - 2C_0 \\ B_1 + \cancel{B_1 t} + \cancel{B_0} = \underline{4A_1 t} + 4A_0 + \cancel{B_1 t} + \cancel{B_0} \\ C_1 + \underline{C_1 t} + C_0 = \underline{2A_1 t} + A_0 + \underline{B_1 t} + B_0 - \underline{C_1 t} - C_0 \end{cases} \Leftrightarrow \begin{cases} B_1 - 2C_1 = 0 \Rightarrow \boxed{B_1 = 2C_1} \\ A_1 = B_0 - 2C_0 \to B_0 = 2C_0 \\ 4A_1 = 0 \to \boxed{A_1 = 0} \\ B_1 = 4A_0 \\ C_1 = 2A_1 + B_1 - C_1 \Rightarrow 1 + B_1 = 2C_1 \Rightarrow B_1 = 2C_1 \\ C_1 + C_0 = A_0 + B_0 - C_0 \Rightarrow C_1 = A_0 + B_0 - 2C_0 \end{cases}$

$B_1 = 2C_1 = 2A_0$

$B_0 = 2C_0$ $\quad A_0, C_0 \in \mathbb{R}$

$A_1 = 0$

$C_1 = A_0$

Fie $A_0 = 1 \Rightarrow B_1 = 2,\ C_1 = 1,\ C_0 = 1 \Rightarrow B_0 = 2$

$$x_{n_{2,3}} = \begin{pmatrix} e^t \\ (2t+2)e^t \\ (t+1)e^t \end{pmatrix} = \begin{pmatrix} e^t \\ 2t e^t + 2e^t \\ te^t + e^t \end{pmatrix} = \underbrace{\begin{pmatrix} e^t \\ 2e^t \\ e^t \end{pmatrix}}_{x_{n_2}} + \underbrace{\begin{pmatrix} 0 \\ 2te^t \\ te^t \end{pmatrix}}_{x_{n_3}}$$

$$x = C_1 x_{n_1} + C_2 x_{n_2} + C_3 x_{n_3} =$$

$$= C_1 \begin{pmatrix} e^{-t} \\ -2e^{-t} \\ 0 \end{pmatrix} + C_2 \begin{pmatrix} e^t \\ 2e^t \\ e^t \end{pmatrix} + C_3 \begin{pmatrix} 0 \\ 2te^t \\ te^t \end{pmatrix} = \begin{pmatrix} c_1 e^{-t} + c_2 e^t \\ -2c_1 e^{-t} + 2c_2 e^t + 2c_3 te^t \\ c_2 e^t + c_3 te^t \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow$$

$x = c_1 e^{-t} + c_2 e^t$

$y = -2c_1 e^{-t} + 2c_2 e^t + 2c_3 t e^t$

$z = c_2 e^t + c_3 t e^t$