Laborator07

Enunţuri

1.

Să se rezolve următoarele ecuaţii de ordin n:

$$
\begin{aligned}
&a)\ x^{IV}=t+2,\ x(0)=1,\ x^{I}(0)=2,\ x^{II}(0)=-1, x^{III}(0)=0\\
&b)\ x^{II}=t\cdot sint,\ x(0)=1,\ x^{I}(0)=2\\
&c)\ x^{III}=sint+cost,\ x(0)=1,\ x^{I}(0)=2,\ x^{II}(0)=3\\
&d)\ x^{II}=\frac{1}{t},\ x(1)=1,\ x^{I}(1)=2\\
&e)\ x^{III}=lnt,\ x(1)=2,\ x^{I}(1)=1,\ x^{II}(1)=0
\end{aligned}
$$

2.

$$
\begin{aligned}
&a) e^{x^{II}}-(x^{II})^2=t+1\\
&b) x^{II}-\sqrt{x^{II}}=t+3\\
&c) x^{II}+lnx^{II}=t-5
\end{aligned}
$$

3.

$$
\begin{aligned}
&a)\ x^{III}=\sqrt{1+x^{II}},\ x(0)=x^{I}(0)=x^{II}(0)=0\\
&b)\ x^{II}+x^{I}\cdot tgt=sin2t\\
&c)\ t^2\cdot x^{II}+2(x^{I})^2=0,\ x(1)=2,\ x^{I}(1)=3\\
&d)\ x^{III}-x^{II}=t,\ x(1)=1,\ x^{I}(1)=-1,\ x^{II}(1)=2\\
&e) t^2\cdot x^{II}+t\cdot x^{I}=1\\
&f)\ t\cdot x^{III}+x^{II}=1+t\\
&g)\ (1+t^2)\cdot x^{II}-2\cdot t\cdot x^{I}=0,\ x(0)=0,\ x^{I}(0)=3\\
&h)\ x^{(5)}+x^{(4)}=0
\end{aligned}
$$

4.

$$a) t^2 \cdot x \cdot x^{II} = (x - t \cdot x^I)^2$$
$$b) t \cdot x \cdot x^{II} + t \cdot (x^I)^2 - x \cdot x^I = 0$$
$$c) t^2 \cdot x \cdot x^{II} + t^2 \cdot (x^I)^2 - 5 \cdot t \cdot x \cdot x^I + 4 \cdot x^2 = 0,\ x(1) = 1,\ x^I(1) = 0$$

5.

$$a)\ x^{II} + x^2 = 0$$
$$b)\ x^{II} + x \cdot x^I = 0$$
$$c)\ x \cdot x^{III} + 3 \cdot x^I \cdot x^{II} = 0$$

Rezolvare

Exerciţiu 1. b) - [Video]

1 b) $x'' = t \sin t \quad , x(0) = 1, x'(0) = 2$

$x'' = f(t)$

$x' = \int x''\, dt = \int \underset{f}{t} \underset{g'}{\sin t}\, dt = -t\cos t + \int \cos t\, dt = -t \cos t + \sin t + C_1$

$f' = 1,\ g = -\cos t$

$x = \int x'\, dt = \int (-t\cos t + \sin t + C_1)\, dt = -\int \underset{f}{t} \underset{g'}{\cos t}\, dt + \int \sin t\, dt + C_1 \int dt$

$f' = 1,\ g = \sin t$

$= -\left(t \sin t - \int \sin t\, dt\right) + \int \sin t\, dt + C_1 \int dt$

$= -t \sin t + 2(-\cos t) + C_1 t + C_2$

$x(t) = -t \sin t - 2\cos t + C_1 t + C_2$

$x(0) = -2 + C_2 = 1 \Rightarrow C_2 = 3$

$x'(0) = C_1 = 2$

$\Rightarrow x_{PC}(t) = -t \sin t - 2\cos t + 2t + 3$

Exerciţiu 1. e) - [Video]

1 e)

$x''' = \ln t \quad , x(1) = 2,\ x'(1) = 1,\ x''(1) = 0$

$x''' = f(t)$

$x'' = \int x'''\, dt = \int \underset{g'}{1} \cdot \underset{f}{\ln t}\, dt = t \ln t - \int \frac{1}{t} \cdot t\, dt = t \ln t - \int dt = t \ln t - t + C_1$

$f' = \frac{1}{t},\ g = t$

$x' = \int x''\, dt = \int (t \ln t - t + C_1)\, dt = \int \underset{g'}{t} \underset{f}{\ln t}\, dt - \int t\, dt + C_1 \int dt = \frac{t^2}{2} \ln t - \int \frac{1}{t} \cdot \frac{t^2}{2}\, dt - \frac{t^2}{2} + C_1 t$

$f' = \frac{1}{t},\ g = \frac{t^2}{2}$

$= \frac{t^2}{2} \ln t - \frac{t^2}{4} - \frac{t^2}{2} + C_1 t + C_2$

$= \frac{t^2}{2} \ln t - \frac{3t^2}{4} + C_1 t + C_2$

$x = \int x'\, dt = \int \left(\frac{t^2}{2} \ln t - \frac{3t^2}{4} + C_1 t + C_2\right) dt = \frac{1}{2} \int \underset{g'}{t^2} \underset{f}{\ln t}\, dt - \frac{3}{4} \int t^2\, dt + C_1 \int t\, dt + C_2 \int dt$

$g = \frac{t^3}{3},\ f' = \frac{1}{t}$

$= \frac{1}{2}\left(\frac{t^3}{3} \ln t - \int \frac{1}{t} \cdot \frac{t^3}{3}\, dt\right) - \frac{3}{4} \cdot \frac{t^3}{3} + C_1 \cdot \frac{t^2}{2} + C_2 t = \frac{t^3}{6} \ln t - \frac{1}{6} \cdot \frac{t^3}{3} - \frac{t^3}{4} + C_1 \frac{t^2}{2} + C_2 t + C_3$

$x(1) = -\frac{1}{18} - \frac{1}{4} + \frac{C_1}{2} + C_2 + C_3 = 2$

$x'(1) = -\frac{3}{4} + C_1 + C_2 = 1$

$x''(1) = -1 + C_1 = 0$

$C_1 = 1$

$C_2 = 1 + \frac{3}{4} - 1 = \frac{3}{4}$

$C_3 = 2 + \frac{1}{18} + \frac{1}{4} \underbrace{- \frac{1}{2} - \frac{3}{4}}_{-1} = 1 + \frac{1}{18} = \frac{19}{18}$

$\Rightarrow x_{PC} = \frac{t^3}{6} \ln t - \frac{t^3}{18} - \frac{t^3}{4} + \frac{t^2}{2} + \frac{3t}{4} + \frac{19}{18}$

Exerciţiu 2. b) - Video

2 b) $x'' - \sqrt{x''} = t+3$

$F(t, x^{(n)}) = 0$

$F(t, x'') = 0$

$x'' = y$

$y - \sqrt{y} = t+3 \Rightarrow t = y - \sqrt{y} - 3 = \varphi(y)$

$x' = \int x''\,dt = \int y\left(1 - \frac{1}{2\sqrt{y}}\right)dy = \int\left(y - \frac{y}{2\sqrt{y}}\right)dy = \int y\,dy - \frac{1}{2}\int\sqrt{y}\,dy = \frac{y^2}{2} - \frac{1}{2}\int y^{\frac{1}{2}}dy = \frac{y^2}{2} - \frac{1}{2}\cdot\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1}$

$x' = \frac{y^2}{2} - \frac{1}{2}\cdot\frac{y^{3/2}}{\frac{3}{2}} + C_1 = \frac{y^2}{2} - \frac{1}{2}\cdot y^{3/2}\cdot\frac{2}{3} + C_1 = \frac{y^2}{2} + \frac{1}{3}y^{\frac{3}{2}} + C_1$

$x = \int x'\,dt = \int\left(\frac{y^2}{2} + \frac{1}{3}y^{\frac{3}{2}} + C_1\right)\left(1 - \frac{1}{2y^{1/2}}\right)dy = \int\left(\frac{y^2}{2} - \frac{y^2}{2}\cdot\frac{1}{2y^{1/2}} + \frac{1}{3}y^{\frac{3}{2}} - \frac{1}{3}y^{\frac{3}{2}}\cdot\frac{1}{2y^{1/2}} + C_1 - \frac{C_1}{2y^{\frac{1}{2}}}\right)dy$

$= \frac{1}{2}\int y^2\,dy - \frac{1}{4}\int y^{2-\frac{1}{2}}dy + \frac{1}{3}\int y^{\frac{3}{2}}dy - \frac{1}{6}\int y^{\frac{3}{2}-\frac{1}{2}}dy + C_1\int dy - \frac{C_1}{2}\int y^{-\frac{1}{2}}dy$

$= \frac{1}{2}\cdot\frac{y^3}{3} - \frac{1}{4}\int y^{\frac{3}{2}}dy + \frac{1}{3}\cdot\frac{y^{\frac{3}{2}+1}}{\frac{3}{2}+1} - \frac{1}{6}\int y\,dy + C_1 y - \frac{C_1}{2}\,\frac{y^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$

$= \frac{y^3}{6} - \frac{1}{4}\,\frac{y^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{1}{3}\,\frac{y^{\frac{5}{2}}}{\frac{5}{2}} - \frac{1}{6}\,\frac{y^2}{2} + C_1 y - \frac{C_1}{2}\cdot\frac{y^{\frac{1}{2}}}{\frac{1}{2}} + C_2$

$x = \frac{y^3}{6} - \frac{1}{4_2}\cdot\frac{2}{5}y^{\frac{5}{2}} + \frac{1}{3}\cdot\frac{2}{5}y^{5/2} - \frac{1}{6}\,\frac{y^2}{2} + C_1 y - \frac{C_1}{\cancel{2}}\cdot\cancel{2}\,y^{1/2} + C_2, \quad t = y - \sqrt{y} - 3$

Exerciţiu 3. a) - Video

3 a) $x''' = \sqrt{1 + x''}$, $x(0) = x'(0) = x''(0) = 0$

$F(t, x^{(k)} \ldots x^{(n)}) = 0$

$F(t, x'', x''') = 0$

$x'' = y \Rightarrow x''' = y'$

$y' = \sqrt{1+y}$

$\frac{dy}{dt} = \sqrt{1+y}$

$\frac{dy}{\sqrt{1+y}} = dt$

$2\int\frac{1}{2\sqrt{1+y}}dy = \int dt$

$2\sqrt{1+y} = t + C$

$\sqrt{1+y} = \frac{t}{2} + C$

$1 + y = \left(\frac{t}{2} + c\right)^2$

$y = \left(\frac{t}{2} + c\right)^2 - 1$

$x'' = \left(\frac{t}{2} + c\right)^2 - 1$

$x' = \int x''\,dt = \int\left[\left(\frac{t}{2} + c\right)^2 - 1\right]dt = \int\left(\frac{t^2}{4} + tc + c^2 - 1\right)dt$

$= \frac{t^3}{12} + c\cdot\frac{t^2}{2} + c^2 t - t + C_1$

$x = \int x'\,dt = \int\left(\frac{t^3}{12} + \frac{c}{2}t^2 + c^2 t - t + C_1\right)dt$

$= \frac{t^4}{48} + \frac{c}{2}\cdot\frac{t^3}{3} + c^2\cdot\frac{t^2}{2} - \frac{t^2}{2} + C_1 t + C_2$

$x(0) = \boxed{C_2 = 0}$

$x'(0) = \boxed{C_1 = 0}$

$x''(0) = c^2 - 1 = 0 \Rightarrow c^2 = 1 \Rightarrow c = \pm 1$

$x_{pc} = \frac{t^4}{48} \pm \frac{t^3}{6} + \cancel{\frac{t^2}{2}} - \cancel{\frac{t^2}{2}} = \frac{t^4}{48} \pm \frac{t^3}{6}$

Exerciţiu 3. e) - Video

③ e) $t^2 x'' + t x' = 1$

$F(t, x', x'') = 0$.

$\boxed{x' = y} \Rightarrow x'' = y'$

$t^2 y' + t y = 1 \quad | : t^2$

$y' + \underbrace{\frac{1}{t}}_{A(t)} y = \underbrace{\frac{1}{t^2}}_{B(t)}$ (ec. dif. afină)

Etapa 1 $y' + \frac{1}{t} y = 0$

$\frac{dy}{dt} = -\frac{1}{t} y$

$\frac{dy}{y} = -\frac{1}{t} dt$

$\int \frac{1}{y} dy = -\int \frac{1}{t} dt$

$\ln|y| = -\ln|t| + C$

$\ln|y| = -\ln|t| + \ln C$.

$\boxed{y_0 = \frac{c}{t}}$

Etapa 2

$\varphi_0 = \frac{c(t)}{t}$

$\left(\frac{c(t)}{t}\right)' + \frac{1}{t} \cdot \frac{c(t)}{t} = \frac{1}{t^2}$

$\frac{c'(t) \cdot t - c(t)}{t^2} + \frac{c(t)}{t^2} = \frac{1}{t^2}$

$\frac{c'(t)}{t} = \frac{1}{t^2} \quad | \cdot t$

$c'(t) = \frac{1}{t}$

$c(t) = \int \frac{1}{t} dt = \ln t + C_1$

$\boxed{\varphi_0(t) = \frac{\ln t + C_1}{t}}$

$y = y_0 + \varphi_0$

$\boxed{y = \frac{c}{t} + \frac{\ln t}{t}}$

$x' = \frac{c}{t} + \frac{\ln t}{t}$

$x = \int \left(\frac{c}{t} + \frac{\ln t}{t}\right) dt$

$= c \int \frac{1}{t} dt + \int \frac{1}{t} \cdot \ln t \, dt$

$x(t) = c \ln t + \frac{\ln^2 t}{t} + C_1$