Temă - Curs 5 Petculescu Mihai - Silviu

1. Să se determine inversele următoarelor aplicații liniare:

a) $T: \mathbb{R}^2 \to \mathbb{R}^2$, $T(x_1, x_2) = (2x_1 - x_2, x_1 + x_2)$

$A_T = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$; $\det A_T = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 + 1 = 3 \neq 0 \Rightarrow A_T$ inversabilă

$\Rightarrow T$ inversabil

$A_T^t = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$; $A_T^* = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$

$(A_T)^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1/3 & 1/3 \\ -1/3 & 2/3 \end{pmatrix} \Rightarrow T^{-1}_{(\vec{x})} = \left(\frac{1}{3}x_1 + \frac{1}{3}x_2, \frac{-1}{3}x_1 + \frac{2}{3}x_2\right)$

b) $T: \mathbb{R}^3 \to \mathbb{R}^3$, $T(x_1 + x_2, x_2 - x_3, x_1 + x_2 + x_3)$

$A_T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$; $\det A_T = 1 - 1 + 1 = 1 \neq 0 \Rightarrow A_T$ inversabil

$\Rightarrow T$ inversabil

	$\vec{v_1}$	$\vec{v_2}$	$\vec{v_3}$	$\vec{e_1}$	$\vec{e_2}$	$\vec{e_3}$
$\vec{e_1}$	$\boxed{1}$	1	0	1	0	0
$\vec{e_2}$	0	1	-1	0	1	0
$\vec{e_3}$	1	1	1	0	0	1
$\vec{v_1}$	1	1	0	1	0	0
$\vec{e_2}$	0	$\boxed{1}$	-1	0	1	0
$\vec{e_3}$	0	0	1	-1	0	1
$\vec{v_1}$	1	0	1	1	-1	0
$\vec{v_2}$	0	1	-1	0	1	0
$\vec{e_3}$	0	0	$\boxed{1}$	-1	0	1
$\vec{v_1}$	1	0	0	2	-1	-1
$\vec{v_2}$	0	1	0	-1	1	1
$\vec{v_3}$	0	0	1	-1	0	1

A_T^{-1}

$\Rightarrow A_T^{-1} = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix} \Rightarrow T^{-1}_{(\vec{x})} = (2x_1 - x_2 - x_3, -x_1 + x_2 + x_3, -x_1 + x_3)$