Laborator13

Enunţ

$$
\begin{aligned}
x' &= x + y + e^t \cdot cost \\
y' &= -x + y - e^t \cdot sint
\end{aligned}
$$

Rezolvare

$$\begin{cases} x' = x+y+e^t\cos t \\ y' = -x+y-e^t\sin t \end{cases} \qquad f(t) = \begin{pmatrix} e^t\cos t \\ -e^t\sin t \end{pmatrix} \qquad \begin{matrix} \gamma = 1 \\ \beta = 1 \end{matrix}$$

$\Rightarrow \gamma + i\beta = 1 + i$ (este răd. de ordin 1 a ec. caracteristice)

$$f(t) = \begin{pmatrix} e^{1\cdot t}(1\cdot\cos 1\cdot t + 0\cdot\sin 1\cdot t) \\ e^{1\cdot t}(0\cdot\cos 1\cdot t - 1\cdot\sin 1\cdot t) \end{pmatrix}$$

$m_1 = p_1 = m_2 = p_2 = 0, \quad k = 1$

$\Rightarrow m = \max\{m_1, m_2, p_1, p_2\} = 0.$

$$X_p = \begin{pmatrix} e^t[(A_1t+A_0)\cos t + (B_1t+B_0)\sin\beta t] \\ e^t[(C_1t+C_0)\cos t + (D_1t+D_0)\sin\beta t] \end{pmatrix}$$

$$\begin{cases} x' = x+y \\ y' = -x+y \end{cases}$$

$x = \alpha_1 e^{rt}, \ y = \alpha_2 e^{rt} \Rightarrow x' = \alpha_1 r e^{rt}, \ y' = \alpha_2 r e^{rt}$

$$\begin{cases} \alpha_1 r = \alpha_1 + \alpha_2 \\ \alpha_2 r = -\alpha_1 + \alpha_2 \end{cases} \Rightarrow \begin{cases} (1-r)\alpha_1 + \alpha_2 = 0 \\ -\alpha_1 + (1-r)\alpha_2 = 0 \end{cases} \quad (*)$$

$$\Delta = \begin{vmatrix} 1-r & 1 \\ -1 & 1-r \end{vmatrix} = (1-r)^2 + 1 = 0$$

$(1-r)^2 = -1 \Rightarrow 1-r = \pm i \Rightarrow r = 1 \mp i \quad (\gamma \pm i\beta, \ \gamma = 1, \ \beta = 1)$

Pt. $r = 1+i$

$$(*) \Rightarrow \begin{cases} -i\alpha_1 + \alpha_2 = 0 \\ -\alpha_1 - i\alpha_2 = 0 \end{cases} \Rightarrow \alpha_2 = i\alpha_1, \quad \alpha_1 \in \mathbb{C}$$

Fie $\alpha_1 = 1 \Rightarrow \alpha_2 = i \Rightarrow \alpha_r = \begin{pmatrix} 1 \\ i \end{pmatrix} \Rightarrow X_r = \begin{pmatrix} e^t(\cos t + i\sin t) \\ ie^t(\cos t + i\sin t) \end{pmatrix} = \begin{pmatrix} e^t\cos t + ie^t\sin t \\ ie^t\cos t - e^t\sin t \end{pmatrix} = \underbrace{\begin{pmatrix} e^t\cos t \\ -e^t\sin t \end{pmatrix}}_{\tilde{X}_r} + i\underbrace{\begin{pmatrix} e^t\sin t \\ e^t\cos t \end{pmatrix}}_{\tilde{\tilde{X}}_r}$

$$X_O = C_1\tilde{X}_r + C_2\tilde{\tilde{X}}_r = C_1\begin{pmatrix} e^t\cos t \\ -e^t\sin t \end{pmatrix} + C_2\begin{pmatrix} e^t\sin t \\ e^t\cos t \end{pmatrix} = \begin{pmatrix} C_1e^t\cos t + C_2e^t\sin t \\ -C_1e^t\sin t + C_2e^t\cos t \end{pmatrix}$$

$x_p = e^t[(A_1t+A_0)\cos t + (B_1t+B_0)\sin t]$

$y_p = e^t[(C_1t+C_0)\cos t + (D_1t+D_0)\sin t]$

$x_p' = e^t[(A_1t+A_0)\cos t + (B_1t+B_0)\sin t] + e^t[A_1\cos t + (A_1t+A_0)(-\sin t) + B_1\sin t + (B_1t+B_0)\cos t]$

$\quad = e^t(A_1t\cos t + A_0\cos t + B_1t\sin t + B_0\sin t + A_1\cos t - A_1t\sin t - A_0\sin t + B_1\sin t + B_1t\cos t + B_0\cos t)$

$y_p' = e^t[(C_1t+C_0)\cos t + (D_1t+D_0)\sin t] + e^t[C_1\cos t + (C_1t+C_0)(-\sin t) + D_1\sin t + (D_1t+D_0)\cos t]$

$\quad = e^t(C_1t\cos t + C_0\cos t + D_1t\sin t + D_0\sin t + C_1\cos t - C_1t\sin t - C_0\sin t + D_1\sin t + D_1t\cos t + D_0\cos t)$

$$\begin{cases} (A_1t+A_0+A_1+B_1t+B_0)\cos t + (B_1t+B_0-A_1t-A_0+B_1)\sin t = (A_1t+A_0)\cos t + (B_1t+B_0)\sin t + (C_1t+C_0)\cos t \\ \qquad + (D_1t+D_0)\sin t + \cos t \\ (C_1t+C_0+C_1+D_1t+D_0)\cos t + (D_1t+D_0-C_1t-C_0+D_1)\sin t = -(A_1t+A_0)\cos t - (B_1t+B_0)\sin t + (C_1t+C_0)\cos t \\ \qquad + (D_1t+D_0)\sin t - \sin t \end{cases}$$

$$\begin{cases} \cancel{A_1t} + \cancel{A_0} + A_1 + B_1t + B_0 = \cancel{A_1t} + \cancel{A_0} + C_1t + C_0 + 1 \\ \cancel{B_1t} + \cancel{B_0} - A_1t - A_0 + B_1 = \cancel{B_1t} + \cancel{B_0} + D_1t + D_0 \\ \cancel{C_1t} + \cancel{C_0} + C_1 + D_1t + D_0 = -A_1t - A_0 + \cancel{C_1t} + \cancel{C_0} \\ \cancel{D_1t} + \cancel{D_0} - C_1t - C_0 + D_1 = -B_1t - B_0 + \cancel{D_1t} + \cancel{D_0} - 1 \end{cases}$$

$$\begin{cases} B_1 = C_1 \\ A_1 + B_0 = C_0 + 1 \\ -A_1 = D_1 \\ -A_0 + B_1 = D_0 \\ D_1 = -A_1 \ \checkmark \\ C_1 + D_0 = -A_0 \\ -C_1 = -B_1 \ \checkmark \\ -C_0 + D_1 = -B_0 - 1 \end{cases}$$

$\to B_1 = C_1$

$\to D_1 = -A_1$

$\to B_0 = C_0 + 1 - A_1$

$\to D_0 = -A_0 + B_1$

$\to D_0 = -A_0 - C_1$

$\to B_0 = C_0 - D_1 - 1$

$\Rightarrow \cancel{-A_0} + B_1 = \cancel{-A_0} - C_1 \Rightarrow B_1 = -C_1 \Rightarrow C_1 = -C_1$

$\Rightarrow 2C_1 = 0 \Rightarrow \boxed{C_1 = 0} \Rightarrow \boxed{B_1 = 0}$

$\Rightarrow \cancel{C_0} + 1 - A_1 = \cancel{C_0} - D_1 - 1$

$1 - A_1 = A_1 - 1 \Rightarrow 2A_1 = 2 \Rightarrow \boxed{A_1 = 1}$

$\Rightarrow \boxed{D_1 = -1}$

$B_0 = C_0 + 1 - A_1 \Rightarrow \boxed{B_0 = C_0}$, $A_0, C_0 \in \mathbb{R}$

$D_0 = -A_0 + B_1 \Rightarrow \boxed{D_0 = -A_0}$

Fie $A_0 = C_0 = 1 \Rightarrow B_0 = 1, D_0 = -1$

$$x_p = \begin{pmatrix} e^t[(t+1)\cos t + \sin t] \\ e^t[\cos t + (-t-1)\sin t] \end{pmatrix}$$

$$X = X_o + X_p = \begin{pmatrix} c_1 e^t \cos t + c_2 e^t \sin t \\ -c_1 e^t \sin t + c_2 e^t \cos t \end{pmatrix} + \begin{pmatrix} e^t[(t+1)\cos t + \sin t] \\ e^t[\cos t - (t+1)\sin t] \end{pmatrix}$$

$$= \begin{pmatrix} c_1 e^t \cos t + c_2 e^t \sin t + e^t[(t+1)\cos t + \sin t] \\ -c_1 e^t \sin t + c_2 e^t \cos t + e^t[\cos t - (t+1)\sin t] \end{pmatrix}$$

$$\begin{cases} x = c_1 e^t \cos t + c_2 e^t \sin t + e^t[(t+1)\cos t + \sin t] \\ y = -c_1 e^t \sin t + c_2 e^t \cos t + e^t[\cos t - (t+1)\sin t] \end{cases}$$