Laborator5 - Temă - Model 1

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Exerciţiul 1.0.1.

Se consideră formula

$$\alpha = ((\neg a \vee (b \wedge \neg c)) \leftrightarrow (a \vee (\neg b \rightarrow \neg(c \wedge a))))$$

şi substituţia

$$\sigma = \{(x \vee \neg m)|\alpha, (d \wedge \neg t)|a, (q \vee p)|m, a|q\}$$

Să se determine:

- secvenţa generativă formule (SGF) pentru formula α
- tabelul de adevăr pentru formula α
- arborele de structură pentru formula α
- $\alpha\sigma$ rezultatul aplicării substituţiei σ pentru formula α şi arborele de structură asociat lui $\alpha\sigma$

Rezolvare

SGF:

$$a, b, c, \neg a, \neg b, \neg c, b \wedge \neg c, \neg a \vee (b \wedge \neg c), c \wedge a, \neg(c \wedge a), \neg b \rightarrow \neg(c \wedge a), a \vee (\neg b \rightarrow \neg(c \wedge a)),$$
$$((\neg a \vee (b \wedge \neg c)) \leftrightarrow (a \vee (\neg b \rightarrow \neg(c \wedge a)))) = \alpha$$

Tabel de Adevăr:

a	b	c	$b \wedge \neg c$	$\neg a \vee (b \wedge \neg c)$	$\neg(c \wedge a)$	$\neg b \rightarrow \neg(c \wedge a)$	$a \vee (\neg b \rightarrow \neg(c \wedge a))$	α
T	T	T	F	F	F	T	T	F
T	T	F	T	T	T	T	T	T
T	F	T	F	F	F	F	T	F
T	F	F	F	F	T	T	T	F
F	T	T	F	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Arbore de Structură:

$$T(\alpha): \begin{array}{ccc} & r & \\ \swarrow & & \searrow \\ T(\beta) & & T(\gamma) \end{array}, \varphi(r) = \leftrightarrow, \beta = \neg a \vee (b \wedge \neg c), \gamma = a \vee (\neg b \to \neg(c \wedge a))$$

$$T(\beta): \begin{array}{ccc} & n_1 & \\ \swarrow & & \searrow \\ T(\beta_1) & & T(\beta_2) \end{array}, \varphi(n_1) = \vee, \beta_1 = \neg a, \beta_2 = b \wedge \neg c$$

$$T(\beta_1): \begin{array}{c} n_2 \\ \downarrow \\ T(\beta_3) \end{array}, \varphi(n_2) = \neg, \beta_3 = a$$

$$T(\beta_3) = n_3, \varphi(n_3) = a$$

$$T(\beta): \begin{array}{ccc} & n_4 & \\ \swarrow & & \searrow \\ T(\beta_4) & & T(\beta_5) \end{array}, \varphi(n_4) = \wedge, \beta_4 = b, \beta_5 = \neg c$$

$$T(\beta_4) = n_5, \varphi(n_5) = b$$

$$T(\beta_5): \begin{array}{c} n_6 \\ \downarrow \\ T(\beta_6) \end{array}, \varphi(n_6) = \neg, \beta_6 = c$$

$$T(\beta_6) = n_7, \varphi(n_7) = c$$

$$T(\gamma): \begin{array}{ccc} & n_8 & \\ \swarrow & & \searrow \\ T(\gamma_1) & & T(\gamma_2) \end{array}, \varphi(n_8) = \wedge, \gamma_1 = a, \gamma_2 = \neg b \to \neg(c \wedge a)$$

$$T(\gamma_1) = n_9, \varphi(n_9) = a$$

$$T(\gamma_2): \begin{array}{ccc} & n_{10} & \\ \swarrow & & \searrow \\ T(\gamma_3) & & T(\gamma_4) \end{array}, \varphi(n_{10}) = \to, \gamma_3 = \neg b, \gamma_4 = \neg(c \wedge a)$$

$$T(\gamma_3): \begin{array}{c} n_{11} \\ \downarrow \\ T(\gamma_5) \end{array}, \varphi(n_{11}) = \neg, \gamma_5 = b$$

$$T(\gamma_5) = n_{12}, \varphi(n_{12}) = b$$

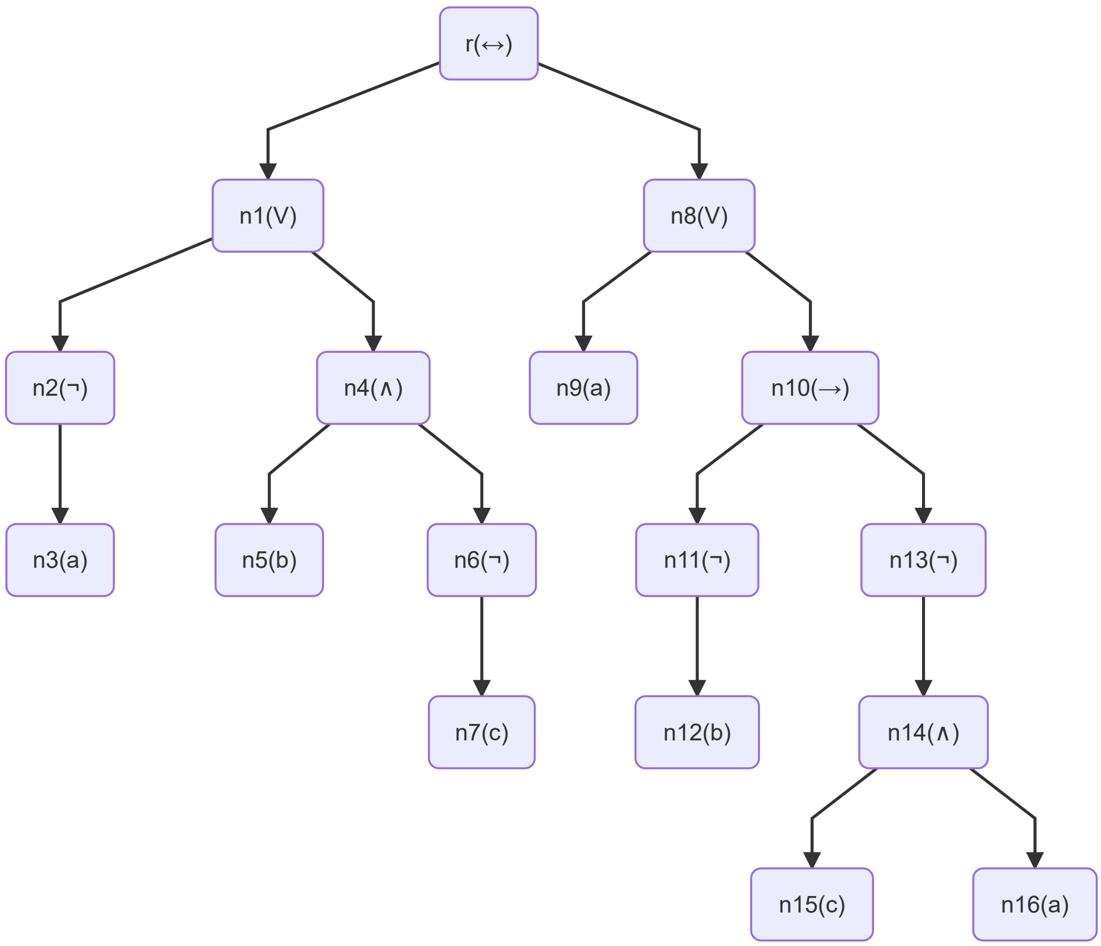
$$T(\gamma_4): \begin{array}{c} n_{13} \\ \downarrow \\ T(\gamma_6) \end{array}, \varphi(n_{13}) = \neg, \gamma_6 = c \wedge a$$

$$T(\gamma_6): \begin{array}{ccc} & n_{14} & \\ \swarrow & & \searrow \\ T(\gamma_7) & & T(\gamma_8) \end{array}, \varphi(n_{14}) = \wedge, \gamma_7 = c, \gamma_8 = a$$

$$T(\gamma_7) = n_{15}, \varphi(n_{15}) = c$$

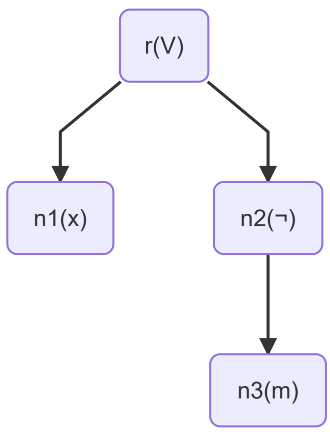
$$T(\gamma_8) = n_{16}, \varphi(n_{16}) = a$$

Final:



Aplicare substituţie $\alpha\sigma$:

$$\alpha\sigma = x \vee \neg m$$



Exerciţiul 1.0.2.

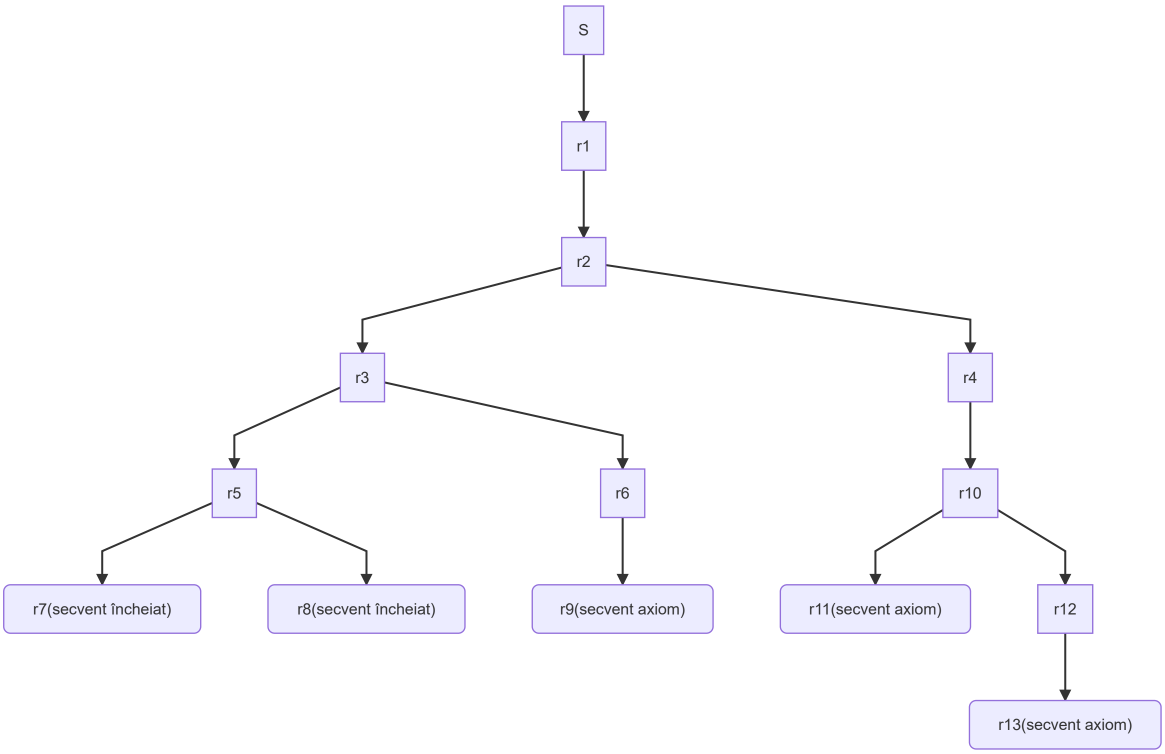
a) Să se verifice dacă următorul secvent este demonstrabil:

$$S = \{(\alpha \vee (\neg\beta)), (\beta \vee (\gamma \wedge \theta))\} \Rightarrow \{\neg\alpha \to (\theta \wedge \gamma)\}$$

Sistem:

$$
\begin{aligned}
& S=\{(\alpha \vee \neg \beta),(\beta \vee(\gamma \wedge \theta))\} \Rightarrow\{\neg \alpha \rightarrow(\theta \wedge \gamma)\} \\
G8: \ & r1=\{\alpha \vee \neg \beta, \beta \vee(\gamma \wedge \theta), \neg \theta\} \Rightarrow\{\theta \wedge \gamma\} \\
G1: \ & r2=\{\alpha \vee \neg \beta, \beta \vee(\gamma \wedge \theta)\} \Rightarrow\{\theta, \theta \wedge \gamma\} \\
G3: \ & r3=\{\alpha, \beta \vee(\gamma \wedge \theta)\} \Rightarrow\{\theta, \theta \wedge \gamma\} \\
& r4=\{\neg \beta, \beta \vee(\gamma \wedge \theta)\} \Rightarrow\{\theta, \theta \wedge \gamma\} \\
G3: \ & r5=\{\alpha, \beta\} \Rightarrow\{\theta, \theta \wedge \gamma\} \\
& r6=\{\alpha, \gamma \wedge \theta)\} \Rightarrow\{\theta, \theta \wedge \gamma\} \\
G6: \ & r7=\{\alpha, \beta\} \Rightarrow\{\theta\} \ secvent \ incheiat \\
& r8=\{\alpha, \beta\} \Rightarrow\{\theta, \gamma\} \ secvent \ incheiat \\
G2: \ & r9=\{\alpha, \gamma, \theta)\} \Rightarrow\{\theta, \theta \wedge \gamma\} \ secvent \ axiom \\
G1: \ & r10=\{\beta \vee(\gamma \wedge \theta)\} \Rightarrow\{\beta, \theta, \theta \wedge \gamma\} \\
G3: \ & r11=\{\beta\} \Rightarrow\{\beta, \theta, \theta \wedge \gamma\} \ secvent \ axiom \\
& r12=\{\gamma \wedge \theta\} \Rightarrow\{\beta, \theta, \theta \wedge \gamma\} \\
G2: \ & r13=\{\gamma, \theta\} \Rightarrow\{\beta, \theta, \theta \wedge \gamma\} \ secvent \ axiom \\
& S \ nu \ e \ tautologie
\end{aligned}
$$

Schema:



b) Să se calculeze mulţimile α_λ^+, α_λ^-, α_λ^0,POS_λ^α, NEG_λ^α, REZ_λ^α, unde $\lambda=\beta$, respectiv $\lambda=\neg\delta$, iar:

$$
S(\alpha)=\{\neg \gamma \vee \beta \vee \neg \eta, \neg \beta \vee \delta \vee \neg \gamma, \neg \delta, \beta, \theta \vee \beta, \delta \vee \beta \vee \neg \theta, \gamma \vee \eta \vee \neg \delta\}
$$

Pentru $\lambda=\beta$:

$$
\begin{aligned}
\alpha_\lambda^{+} & =\{\neg \gamma \vee \beta \vee \neg \eta, \beta, \theta \vee \beta, \delta \vee \beta \vee \neg \theta\} \\
\alpha_\lambda^{-} & =\{\neg \beta \vee \delta \vee \neg \gamma\} \\
\alpha_\lambda^0 & =\{\neg \delta, \gamma \vee \eta \vee \neg \delta\} \\
POS_\lambda^\alpha & =\{\neg \delta, \gamma \vee \eta \vee \neg \delta, \neg \gamma \vee \neg \eta, \square, \theta, \delta \vee \neg \theta\} \\
NEG_\lambda^\alpha & =\{\neg \delta, \gamma \vee \eta \vee \neg \delta, \delta \vee \neg \gamma\} \\
REZ_\lambda^\alpha & =\{\neg \delta, \gamma \vee \eta \vee \neg \delta, \delta \vee \neg \gamma \vee \neg \eta, \delta \vee \neg \gamma \vee \square, \delta \vee \neg \gamma \vee \theta, \delta \vee \neg \gamma \vee \neg \theta\}
\end{aligned}
$$

Pentru $\lambda=\neg\delta$:

$$\begin{aligned}
\alpha_\lambda^+ &= \{\neg\delta, \gamma \vee \eta \vee \neg\delta\} \\
\alpha_\lambda^- &= \{\neg\beta \vee \delta \vee \neg\gamma, \delta \vee \beta \vee \neg\theta\} \\
\alpha_\lambda^0 &= \{\neg\gamma \vee \beta \vee \neg\eta, \beta, \theta \vee \beta\} \\
POS_\lambda^\alpha &= \{\neg\gamma \vee \beta \vee \neg\eta, \beta, \theta \vee \beta, \Box, \gamma \vee \eta\} \\
NEG_\lambda^\alpha &= \{\neg\gamma \vee \beta \vee \neg\eta, \beta, \theta \vee \beta, \neg\beta \vee \neg\gamma, \beta \vee \neg\theta\} \\
REZ_\lambda^\alpha &= \{\neg\gamma \vee \beta \vee \neg\eta, \beta, \theta \vee \beta, \neg\beta \vee \neg\gamma \vee \Box, \neg\beta \vee \eta, \beta \vee \neg\theta \vee \Box, \beta \vee \neg\theta \vee \gamma \vee \eta\}
\end{aligned}$$

Exerciţiul 1.0.3.

Să se determine forma normală conjunctivă (CNF) şi să se aplice algoritmul bazat pe rezoluţie pentru formula:

$$\alpha = ((b \to (\neg a)) \leftrightarrow (\neg c \to d))$$

CNF:

$$\begin{gathered}
((b \to \neg a) \to (\neg c \to d)) \wedge ((\neg c \to d) \to (b \to \neg a)) \\
(\neg(\neg b \vee \neg a) \vee (c \vee d)) \wedge (\neg(c \vee d) \vee (\neg b \vee \neg a)) \\
((b \wedge a) \vee (c \vee d)) \wedge ((\neg c \wedge \neg d) \vee (\neg b \vee \neg a)) \\
(a \vee c \vee d) \wedge (b \vee c \vee d) \wedge (\neg c \vee \neg b \vee \neg a) \wedge (\neg d \vee \neg b \vee \neg a)
\end{gathered}$$

Soluţie:

$$\begin{aligned}
Initializare: \ & \gamma \leftarrow \{a \vee c \vee d, b \vee c \vee d, \neg c \vee \neg b \vee \neg a, \neg d \vee \neg b \vee \neg a\} \\
Iteratia\ 1: \ & Nu\ exista\ clauze\ unitare\ si\ nici\ literali\ puri \\
& alegem\ \lambda = a\ literal \\
& \gamma \leftarrow REZ_a(\gamma) = \{b \vee c \vee d, \neg b \vee d, \neg b \vee c\} \\
Iteratia\ 2: \ & \lambda = c\ literal\ pur \\
& \gamma \leftarrow NEG_c(\gamma) = \{\neg b \vee d\} \\
Iteratia\ 3: \ & \lambda = d\ literal\ pur \\
& \gamma \leftarrow NEG_d(\gamma) = \emptyset \\
Iteratia\ 4: \ & \gamma = \emptyset \Rightarrow write\ \ ''\ validabila\ '', sw \leftarrow true \\
& \Rightarrow STOP
\end{aligned}$$