Laborator 07 - TEMA Petculescu Mihai - Silviu

3. Să se rezolve următoarele ecuații de ordin n:

b) $x'' + x' tg\,t = \sin 2t$

$F(t, x', x'') = 0;$

$\boxed{x' = y} \Leftrightarrow x'' = y' \Leftrightarrow y' + y \underbrace{tg\,t}_{A(t)} = \underbrace{\sin 2t}_{B(t)}$ (ec. dif. afină)

Etapa 1: $y' + y\,tg\,t = 0$

$\frac{dy}{dt} = -y\,tg\,t \Leftrightarrow \frac{1}{y} dy = (-tg\,t) dt \Leftrightarrow \int \frac{1}{y} dy = -\int tg\,t\,dt$

$\Leftrightarrow \ln|y| = \ln|\cos t| + C \Rightarrow \boxed{y_0 = C \cdot \cos t}$

Etapa 2: $\boxed{\varphi_0 = C(t) \cdot \cos t}$

$(C(t) \cdot \cos t)' + (C(t) \cdot \cos t)\,tg\,t = \sin 2t$

$C'(t)\cos t + \cancel{C(t) \cdot (\cos t)'} + \cancel{C(t) \cdot \cos t \cdot \frac{\sin t}{\cos t}} = \sin 2t$

$C'(t) \cos t = \sin 2t$

$C'(t) = \frac{\sin 2t}{\cos t} \Leftrightarrow C(t) = \int \frac{\sin 2t}{\cos t} dt \Leftrightarrow C(t) = -2\cos t + C_1;$

- $\varphi_0 = (-2\cos t + C_1) \cdot \cos t$
- $y_0 = C \cos t$

$\Rightarrow y = \varphi_0 + y_0 = -2\cos^2 t + C \cos t$

- $x' = y = 2\cos^2 t + C\cos t$
- $x = \int (2\cos^2 t + C\cos t) dt \Leftrightarrow \boxed{x = (\cos t + C)\sin t + t + C_1}$

c) $t^2 x'' + 2(x')^2 = 0, \quad x(1) = 2, \quad x'(1) = 3$

$F(t, x', x'') = 0;$

$\boxed{x' = y} \Leftrightarrow x'' = y' \Leftrightarrow t^2 y' + 2y^2 = 0 \mid t^2$

$y' + \frac{2}{t^2} y^2 = 0$

(ec. cu variabile separabile)

$$y' = \frac{-2}{t^2} y^2 \Leftrightarrow \frac{dy}{dt} = \frac{-2}{t^2} y^2 \Leftrightarrow \frac{1}{y^2} dy = \frac{-2}{t^2} dt \Leftrightarrow \int \frac{1}{y^2} dy = \int \frac{-2}{t^2} dt \Leftrightarrow$$

$$\Leftrightarrow \frac{-1}{y} = \frac{2}{t} + C \Leftrightarrow -1 = \frac{2}{t} y + C \Leftrightarrow \boxed{y = \frac{(-1-C)t}{2}}$$

- $x' = y = \frac{(-1-C)t}{2}$
- $x = \int x' dt = \int \left(\frac{(-1-C)t}{2}\right) dt = \frac{(-1-C)t^2}{4} + C_1$

- $x'(1) = \frac{-1-C}{2} = 3 \Leftrightarrow \boxed{C = -7}$
- $x(1) = \frac{6}{4} + C_1 = 2 \Leftrightarrow C_1 = 2 - \frac{6}{4} = \frac{1}{2}$

$$x_{PC} = \frac{(-1+7)t^2}{4} + \frac{1}{2} = \frac{6t^2}{4} + \frac{1}{2} = \frac{3t^2+1}{2}$$

d) $x''' - x'' = t, \quad x(1) = 1, x'(1) = -1, x''(1) = 2$

$F(t, x'', x''') = 0;$

$\boxed{x'' = y} \Leftrightarrow x''' = y' \Leftrightarrow y' - y = t$ (ecuație de tip afină)

Etapa 1 $\quad y' - y = 0 \Leftrightarrow \frac{dy}{dt} = y \Leftrightarrow \frac{1}{y} dy = dt \Leftrightarrow \int \frac{1}{y} dy = \int dt \Leftrightarrow$

$\Leftrightarrow \ln|y| = t + C \Leftrightarrow \boxed{y_0 = Ce^t}$

Etapa 2: $\boxed{\varphi_0 = C(t)e^t}$

$C'(t)e^t + \cancel{C(t)e^t} - \cancel{C(t)e^t} = t \Leftrightarrow C'(t)e^t = t \Leftrightarrow C'(t) = \frac{t}{e^t} \Leftrightarrow$

$\Leftrightarrow C(t) = \int te^{-t} dt = -(t+1)e^{-t} + C_1$

$\varphi_0 = (-(t+1)e^{-t} + C_1)e^t = -(t+1) + C_1 e^t$

$y = \varphi_0 + y_0 = -(t+1) + C_1 e^t + Ce^t = -(t+1) + Ce^t$

$x'' = y = -(t+1) + Ce^t$

$x' = \int x'' dt = \int (-(t+1) + Ce^t) dt = Ce^t - \frac{t^2}{2} - t + C_1$

$x = \int x' dt = \int \left(Ce^t - \frac{t^2}{2} - t + C_1\right) dt = Ce^t - \frac{t^3}{6} - \frac{t^2}{2} + C_1 t + C_2$

- $x''(1) = -2 + Ce = 2 \Rightarrow \boxed{C = \frac{4}{e}}$

- $x'(1) = \frac{4}{e} \cdot e - \frac{1}{2} - 1 + C_1 = -1 \Leftrightarrow \boxed{C_1 = -4 + \frac{1}{2} = \frac{-7}{2}}$

- $x(1) = \frac{4}{e} \cdot e - \frac{1}{6} - \frac{1}{2} + \frac{-7}{2} + C_2 = 1 \Leftrightarrow \boxed{C_2 = \frac{7}{6}}$

$\Rightarrow x_{PC} = 4e^{t-1} - \frac{t^3}{6} - \frac{t^2}{2} - \frac{7}{2}t + \frac{7}{6}$

f) $t x''' + x'' = 1 + t$

$F(t, x'', x''') = 0$

$\boxed{x'' = y} \Leftrightarrow x''' = y' \Leftrightarrow t y' + y = 1 + t \,|: t \Leftrightarrow y' + \frac{1}{t} y = 1 + \frac{1}{t}$

(ec. de tip afină)

<u>Etapa 1</u> $y' + \frac{1}{t} y = 0 \Leftrightarrow y' = \frac{-1}{t} y \Leftrightarrow \frac{dy}{dt} = \frac{-1}{t} y \Leftrightarrow \frac{1}{y} dy = \frac{-1}{t} dt \Leftrightarrow$

$\Leftrightarrow \int \frac{1}{y} dy = \int \frac{-1}{t} dt \Leftrightarrow \ln|y| = -\ln|t| + C \Leftrightarrow \ln|y| = \ln|C \cdot t^{-1}|$

$\boxed{y_0 = C t^{-1}}$

<u>Etapa 2</u>: $\boxed{\varphi_0 = C(t) t^{-1}}$

$C'(t) t^{-1} + \cancel{C(t) \cdot \frac{-1}{t^2}} + \cancel{\frac{1}{t} \cdot C(t) \cdot \frac{1}{t}} = 1 + \frac{1}{t}$

$C'(t) = t + 1 \Leftrightarrow C(t) = \int (t+1) dt = \frac{t^2}{2} + t + C_1$

$\varphi_0 = \left(\frac{t^2}{2} + t + C_1\right) t^{-1}$

$y = \varphi_0 + y_0 = \left(\frac{t^2}{2} + t + C_1\right) t^{-1} + C t^{-1} = \frac{t}{2} + 1 + C t^{-1}$

- $x'' = y = \frac{t}{2} + 1 + C t^{-1}$
- $x' = \int x'' dt = C \ln|t| + \frac{t^2}{4} + t + C_1$
- $x = \int x' dt = C t \ln|t| + \frac{t^3}{12} + \frac{t^2}{2} + C_1 t + C_2$

g) $(1+t^2) x'' - 2t x' = 0$, $x(0) = 0$, $x'(0) = 3$

$F(t, x', x'') = 0$

$\boxed{x' = y} \Leftrightarrow x'' = y' \Leftrightarrow (1+t^2) y' - 2t y = 0$. (ec. cu var. separabile)

$(1+t^2)y' = 2ty \Leftrightarrow \frac{dy}{dt}(1+t^2) = 2ty \Leftrightarrow \frac{1}{y}dy = \frac{2t}{1+t^2}dt \Leftrightarrow$

$\Leftrightarrow \int \frac{1}{y}dy = \int \frac{2t}{1+t^2}dt \Leftrightarrow \ln|y| = \ln(t^2+1)+c \Rightarrow y = c(t^2+1)$

- $x' = y = c(t^2+1)$
- $x = \int x'dt = \int (c(t^2+1))dt = c\left(\frac{t^3}{3}+t\right)+c_1$

- $x'(0) = c(0+1) = 3 \Leftrightarrow \boxed{c=3}$
- $x(0) = 3\left(\frac{0}{3}+0\right)+c_1 \Leftrightarrow \boxed{c_1=0}$

$\Rightarrow x_{PC} = t^3+3t$

h) $x^{(5)} + x^{(4)} = 0$

$F(t, x^{(4)}, x^{(5)}) = 0$

$\boxed{x^{(4)} = y} \Leftrightarrow x^{(5)} = y' \Leftrightarrow y' + y = 0$. (ec. cu var. separabile)

$y' = -y \Leftrightarrow \frac{dy}{dt} = -y \Leftrightarrow \frac{1}{y}dy = -1dt \Leftrightarrow \int \frac{1}{y}dy = \int -1dt \Leftrightarrow$

$\Leftrightarrow \ln|y| = -t+c \Leftrightarrow y = e^{(-t+c)} = ce^{-t} \Leftrightarrow \boxed{y = ce^{-t}}$

- $x^{(4)} = y = ce^{-t}$
- $x^{(3)} = \int x^{(4)}dt = \int ce^{-t}dt = -ce^{-t}+c_1$
- $x^{(2)} = \int x^{(3)}dt = \int(-ce^{-t}+c_1)dt = ce^{-t}+c_1t+c_2$
- $x^{(1)} = \int x^{(2)}dt = \int(ce^{-t}+c_1t+c_2)dt = -ce^{-t}+c_1\frac{t^2}{2}+c_2t+c_3$
- $x = \int x^{(1)}dt = \int\left(-ce^{-t}+c_1\frac{t^2}{2}+c_2t+c_3\right) =$

$$x = ce^{-t}+c_1\frac{t^3}{6}+c_2\frac{t^2}{2}+c_3t+c_4$$

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4. Să se rezolve următoarele ecuații de ordin n:

a) $t^2 \cdot x \cdot x'' = (x - t x')^2$

$F(t, x, x', x'') = 0$

$F(t, mx, mx', mx'') = t^2 mx\, mx'' - (mx - t m x')^2$

$= m^2 (t^2 x x'') - m^2 (x - t x')^2$

$= m^2 (t^2 x x'' - (x - t x')) = m^2 \cdot F(t, x, x', x'')$

• $\boxed{x' = xy} \Rightarrow x'' = x'y + xy' = xy^2 + xy'$

$t^2 x (xy^2 + xy') - (x - txy)^2 = 0$

$t^2 x^2 y^2 + t^2 x^2 y' - (x^2 - 2tx^2 y + t^2 x^2 y^2) = 0$

$\cancel{t^2 x^2 y^2} + t^2 x^2 y' - x^2 + 2tx^2 y - \cancel{t^2 x^2 y^2} = 0 \quad | : x^2$

$t^2 y' - 1 + 2ty = 0 \quad | : t^2 \Leftrightarrow y' - \frac{1}{t^2} + \frac{2y}{t} = 0 \Leftrightarrow y' + \frac{2}{t} y = \frac{1}{t^2}$ (ec. afină)

Etapa 1: $y' = \frac{-2}{t} y \Leftrightarrow \frac{dy}{dt} = \frac{-2}{t} y \Leftrightarrow \frac{1}{y} dy = \frac{-2}{t} dt \Leftrightarrow \int \frac{1}{y} dy = \int \frac{-2}{t} dt \Leftrightarrow$

$\Leftrightarrow \ln|y| = -2 \ln|t| + c \Rightarrow \boxed{y_0 = c t^{-2}}$

Etapa 2: $\varphi_0 = c(t) t^{-2}$

$c'(t) t^{-2} + \cancel{c(t)(-2t^{-3})} + \cancel{\frac{2}{t} c t^{-2}} = \frac{1}{t^2}$

$c'(t) \cancel{t^{-2}} = \cancel{\frac{1}{t^2}} \Leftrightarrow c'(t) = 1 \Leftrightarrow c(t) = \int 1\, dt = t + c_1$

$\varphi_0 = (t + c_1) t^{-2} = t^{-1} + c_1 t^{-2}$

$y = y_0 + \varphi_0 = c t^{-2} + t^{-1} + c_1 t^{-2} = t^{-1} + c t^{-2}$

• Dar $y = \frac{x'}{x} \Leftrightarrow t^{-1} + c t^{-2} = \frac{x'}{x} \Leftrightarrow \frac{dx}{x} = (t^{-1} + c t^{-2}) dt \Leftrightarrow$

$\Leftrightarrow \int \frac{dx}{x} = \int (t^{-1} + c t^{-2}) dt \Leftrightarrow \ln|x| = \ln|t| - \frac{c}{t} + c_1 \Rightarrow x = c_1 t \cdot e^{\frac{-c}{t}}$

$$\boxed{x = c_1 t e^{\frac{-c}{t}}}$$

b) $t x x'' + t(x')^2 - x x' = 0$

$F(t, x, x', x'') = 0$

$F(t, mx, mx', mx'') = t\,mx\,mx'' + t m^2 (x')^2 - mx\,mx'$

$= t m^2 x x'' + t m^2 (x')^2 - m^2 x x'$

$= m^2 (t x x'' + t(x')^2 - x x') = m^2 \cdot F(t, x, x', x'')$

$\boxed{x' = xy} \Rightarrow x'' = x'y + xy' = xy^2 + xy'$

$tx(xy^2 + xy') + t(xy)^2 - x(xy) = 0$

$tx^2y^2 + tx^2y' + tx^2y^2 - x^2y = 0 \,|: x^2$

$ty^2 + ty' + ty^2 - y = 0 \Leftrightarrow 2ty^2 + ty' - y = 0 \,|: t \Leftrightarrow y' + 2y^2 - \frac{1}{t}y = 0$

• $y' = -2y^2 + \frac{1}{t}y$ (ec. de tip Ricatti)

$A(t) = -2$, $B(t) = \frac{1}{t}$, $f(t) = 0$

• Verificăm $\frac{1}{t}$: $\frac{-1}{t^2} = -2 \cdot \frac{1}{t^2} + \frac{1}{t} \cdot \frac{1}{t} \Leftrightarrow \frac{-1}{t^2} = \frac{-2}{t^2} + \frac{1}{t^2}$ (A)

$\boxed{y = z + \frac{1}{t}} \Leftrightarrow y' = z' - \frac{1}{t^2}$

$z' - \frac{1}{t^2} = -2\left(z + \frac{1}{t}\right)^2 + \frac{1}{t}\left(z + \frac{1}{t}\right) \Leftrightarrow z' - \frac{1}{t^2} = -2\left(z^2 + 2z\frac{2}{t} + \frac{1}{t^2}\right) + \frac{z}{t} + \frac{1}{t^2}$

$z' - \frac{1}{t^2} = -2z^2 - 4\frac{z}{t} - \frac{2}{t^2} + \frac{z}{t} + \frac{1}{t^2}$

$z' = \underbrace{-2}_{A(t)} z^2 \underbrace{-3}_{B(t)} \frac{z}{t}$ (ec. de tip Bernoulli cu $\alpha = 2$)

$\frac{z'}{z^2} = -2 - 3(zt)^{-1} \Leftrightarrow z' \cdot z^{-2} = -2 - 3 \cdot z^{-1} \cdot t^{-1}$

$\boxed{u = z^{-1}} \Rightarrow u' = -z^{-2} \cdot z' \Rightarrow u' = 2 + 3ut^{-1}$ (ec. de tip afină)

Etapa 1: $u' = 3ut^{-1} \Leftrightarrow \frac{du}{dt} = \frac{3ut^{-1}}{1} \Leftrightarrow \frac{1}{u}du = 3t^{-1}dt \Leftrightarrow \int \frac{1}{u}du = \int 3t^{-1}dt \Leftrightarrow$

$\ln|u| = 3\ln|t| + C \Rightarrow \boxed{u_0 = Ct^3}$

Etapa 2: $\varphi_0 = C(t)t^3$

$$C'(t)t^3 - \cancel{C(t)3t^2} - \cancel{3\frac{1}{t}\cdot C(t)t^3} = 2$$

$$C'(t)t^3 = 2 \Leftrightarrow C'(t) = \frac{2}{t^3} \Leftrightarrow C(t) = \int \frac{2}{t^3}dt \Leftrightarrow C(t) = \frac{-1}{t^2} + C_1$$

$$\varphi_0 = \left(\frac{-1}{t^2} + C_1\right)t^3 = -t + C_1 t^3$$

$$\mu = \mu_0 + \varphi_0 = -t + C_1 t^3 + Ct^3 = -t + Ct^3$$

$$\mu = -t + Ct^3$$

- $z = \frac{1}{\mu} \Leftrightarrow z = \frac{1}{-t + Ct^3}$

- $y = z + \frac{1}{t} \Leftrightarrow y = \frac{1}{-t + Ct^3} + \frac{1}{t}$

- $y = \frac{x'}{x} \Leftrightarrow \frac{x'}{x} = \frac{1}{-t + Ct^3} + \frac{1}{t} \Leftrightarrow \frac{dx}{x} = \left(\frac{1}{-t + Ct^3} + \frac{1}{t}\right)dt \Leftrightarrow$

$$\Leftrightarrow \int \frac{dx}{x} \cancel{dx} = \int \left(\frac{1}{-t + Ct^3} + \frac{1}{t}\right)dt$$

$$\ln|x| = \frac{\ln|Ct^2 - 1|}{2} - C_1 = \ln\left|(Ct^2 - 1)^{\frac{1}{2}} \cdot C_1\right|$$

$$\underline{x = C_1 \cdot (Ct^2 - 1)^{\frac{1}{2}}}$$

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1. Să se rezolve următoarele ecuații de ordin n:

a) $x^{IV} = t + 2$, $x(0) = 1$, $x'(0) = 2$, $x''(0) = -1$, $x'''(0) = 0$

$x^{IV} = f(t)$

• $x''' = \int x^{IV} dt = \int t dt + 2\int dt = \frac{t^2}{2} + 2t + C_1$

• $x'' = \int x''' dt = \frac{1}{2}\int t^2 dt + 2\int t dt + C_1 \int dt = \frac{t^3}{6} + t^2 + C_1 t + C_2$

• $x' = \int x'' dt = \int \frac{t^3}{6} dt + \int t^2 dt + \int C_1 t dt + C_2 \int dt$

$= \frac{1}{6} \cdot \frac{t^4}{4} + \frac{t^3}{3} + C_1 \frac{t^2}{2} + C_2 t + C_3 = \frac{t^4}{24} + \frac{t^3}{3} + C_1 \frac{t^2}{2} + C_2 t + C_3$

• $x = \int x' dt = \frac{1}{24}\int t^4 dt + \frac{1}{3}\int t^3 dt + \frac{C_1}{2}\int t^2 dt + C_2 \int t dt + C_3 \int dt$

$= \frac{1}{24} \cdot \frac{t^5}{5} + \frac{1}{3} \cdot \frac{t^4}{4} + \frac{C_1}{2} \cdot \frac{t^3}{3} + C_2 \cdot \frac{t^2}{2} + C_3 t + C_4$

$$\boxed{X = \frac{t^5}{120} + \frac{t^4}{12} + C_1 \frac{t^3}{6} + C_2 \frac{t^2}{2} + C_3 t + C_4}$$

• $x(0) = \boxed{C_4 = 1}$

• $x'(0) = \boxed{C_3 = 2}$

• $x''(0) = \boxed{C_2 = -1}$

• $x'''(0) = \boxed{C_1 = 0}$

$\Rightarrow \boxed{x_{PC} = \frac{t^5}{120} + \frac{t^4}{12} - \frac{t^2}{2} + 2t + 1}$

c) $x''' = \sin t + \cos t$, $x(0) = 1$, $x'(0) = 2$, $x''(0) = 3$.

$x''' = f(t)$

• $x'' = \int x''' dt = \int \sin t \, dt + \int \cos t \, dt = -\cos t + \sin t + C_1$

• $x' = \int x'' dt = -\int \cos t \, dt + \int \sin t \, dt + C_1 \int dt$

$= -\sin t - \cos t + C_1 t + C_2$

• $x = \int x' dt = -\int \sin t \, dt - \int \cos t \, dt + C_1 \int t dt + C_2 \int dt$

$$\boxed{x = \cos t - \sin t + C_1 \frac{t^2}{2} + C_2 t + C_3}$$

$\bullet X(0) = 1 + C_3 = 1 \Rightarrow \boxed{C_3 = 0}$

$\bullet X'(0) = -1 + C_2 = 2 \Rightarrow \boxed{C_2 = 3}$

$\bullet X''(0) = -1 + C_1 = 3 \Rightarrow \boxed{C_1 = 4}$

$\Rightarrow \boxed{X_{PC} = \cos t - \sin t + 2t^2 + 3t}$

d) $x'' = \frac{1}{t}, \quad x(1) = 1, \quad x'(1) = 2$

$x'' = f(t)$

$\bullet\ x' = \int x'' \, dt = \int \frac{1}{t} \, dt = \ln|t| + C_1$

$\bullet\ x = \int x' \, dt = \int \ln|t| \, dt + C_1 \int dt = \int \underset{\substack{\downarrow \\ g = t}}{1} \cdot \underset{\substack{\downarrow \\ f' = \frac{1}{t}}}{\ln|t|} \, dt + C_1 \int dt$

$=$

$= t \ln(|t|) - \int 1 \, dt + C_1 \int dt = t \ln|t| - t + C_1 t + C_2$

$$\boxed{x = t \ln|t| - t + C_1 t + C_2}$$

$\bullet\ x'(1) = \boxed{C_1 = 2}$

$\bullet\ x(1) = -1 + 2 + C_2 = 1 \Leftrightarrow \boxed{C_2 = 0}$

$\Rightarrow \boxed{x_{PC} = t \ln|t| + t}$

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2. Să se rezolve următoarele ecuații de ordin n:

a) $e^{x''} - (x'')^2 = t+1$.

$F(t, x'') = 0$

• Facem schimbarea de funcție $x'' = y$.

$e^y - y^2 = t+1 \iff e^y - y^2 - 1 = t = \varphi(y) \iff dt = (e^y - 2y)\,dy$

• $x' = \int x''\,dt = \int y(e^y - 2y)\,dy = \int (y e^y)\,dy - \int (2y^2)\,dy = ye^y - e^y -$
$- 2\frac{y^3}{3} + C_1 = e^y(y-1) - \frac{2y^3}{3} + C_1$

• $x = \int x'\,dt = \int \left(e^y(y-1) - \frac{2y^3}{3} + C_1\right)(e^y - 2y)\,dy$

$= (e^y)^2(y-1) - 2y e^y(y-1) - \frac{2y^3}{3}\cdot e^y + 2y\cdot\frac{2y^3}{3} + C_1 e^y - C_1\cdot 2y$

$$\boxed{x = \frac{(2y-3)e^{2y}}{4} - (2y^2 - 6y + 6)e^y - \frac{(2y^3 - 6y^2 + 12y - 12)e^y}{3} + \frac{4y^5}{15} + C_1 e^y + C_1 y^2 + C_2\,,\quad t = e^y - y^2 - 1}$$

c) $x'' + \ln x'' = t - 5$

$F(t, x'') = 0$

• Facem schimbarea de funcție $x'' = y$

$y + \ln y = t - 5 \iff y + \ln y + 5 = t = \varphi(y) \iff dt = \left(1 + \frac{1}{y}\right)dy$

• $x' = \int x''\,dt = \int y\left(1 + \frac{1}{y}\right)dy = \int (y+1)\,dy = \frac{y^2}{2} + y + C_1$

• $x = \int x'\,dt = \int \left(\frac{y^2}{2} + y + C_1\right)\left(1 + \frac{1}{y}\right)dy$

$= \int \left(\frac{y^2}{2} + \frac{y}{2} + y + 1 + C_1 + C_1\frac{1}{y}\right)dy$

$= \frac{1}{2}\cdot\frac{y^3}{3} + \frac{1}{2}\cdot\frac{y^2}{2} + \frac{y^2}{2} + y + C_1 y + C_1\cdot \ln|y| + C_2$

$$\boxed{x = \frac{y^3}{6} + \frac{y^2}{4} + \frac{y^2}{2} + y + C_1 y + C_1 \ln|y| + C_1\,,\quad t = y + \ln y + 5}$$