Seminar03 - Rezolvare

Exerciţiu01

1) $$\begin{cases} R\cdot q'(t) + \dfrac{q(t)}{C} = E \\ q(0) = 0 \end{cases}$$

$$R q'(t) + \frac{q(t)}{C} = E \quad |:R$$

$$q'(t) + \frac{q(t)}{RC} = \frac{E}{R}$$

$$q' + \underbrace{\frac{1}{RC}}_{a(t)} q = \underbrace{\frac{E}{R}}_{b(t)} \quad (\text{ec. afină})$$

Etapa 1

$$q' + \frac{1}{RC} q = 0 \Leftrightarrow \frac{dq}{dt} = -\frac{1}{RC} q$$

$$\frac{1}{q} dq = -\frac{1}{RC} dt \Leftrightarrow \int \frac{1}{q} dq = -\int \frac{1}{RC} dt$$

$$\ln q = -\frac{1}{RC} t + K$$

$$q = e^{-\frac{t}{RC} + K}$$

$$\Rightarrow \boxed{q_0 = K \cdot e^{-\frac{t}{RC}}}$$

Etapa 2

$$\varphi_0 = K(t) \cdot e^{-\frac{t}{RC}}$$

$$\left(K(t) \cdot e^{-\frac{t}{RC}}\right)' + \frac{1}{RC} \cdot K(t) e^{-\frac{t}{RC}} = \frac{E}{R}$$

$$K'(t) \cdot e^{-\frac{t}{RC}} + \cancel{K(t) \cdot e^{-\frac{t}{RC}} \cdot \left(-\frac{1}{RC}\right)} + \cancel{\frac{1}{RC} K(t) e^{-\frac{t}{RC}}} = \frac{E}{R}$$

$$K'(t) \cdot e^{-\frac{t}{RC}} = \frac{E}{R} \quad | \cdot e^{t/RC}$$

$$K'(t) = \frac{E}{R} \cdot e^{t/RC}$$

$$K(t) = \int \frac{E}{R} e^{t/RC} dt = \frac{E}{R} \cdot \frac{e^{\frac{t}{RC}}}{\frac{1}{RC}} = \frac{E}{\cancel{R}} \cdot e^{\frac{t}{RC}} \cdot \frac{\cancel{R}C}{1}$$

$$K(t) = CE e^{t/RC}$$

$$\Rightarrow \varphi_0 = CE \cdot e^{t/RC} \cdot e^{-\frac{t}{RC}} \Rightarrow \boxed{\varphi_0 = CE}$$

$$q = q_0 + \varphi_0 = K \cdot e^{-t/RC} + CE$$

$$q(0) = K + CE = 0 \Rightarrow K = -CE \Rightarrow q_{RC} = CE - CE e^{-\frac{t}{RC}}$$

$$v(t) = \frac{q(t)}{C}$$

$$\Rightarrow v(t) = \frac{CE\left(1 - e^{-\frac{t}{RC}}\right)}{C} = E\left(1 - e^{-\frac{t}{RC}}\right)$$

$$i(t) = q'(t) = -\cancel{C}E \cdot e^{-\frac{t}{RC}} \cdot \left(-\frac{1}{R\cancel{C}}\right) = \frac{E}{R} e^{-\frac{t}{RC}}$$

Exerciţiu02

② $L \cdot i'(t) + R \cdot i(t) = -mSB \cos\omega t \quad | : L$

$$i'(t) + \underbrace{\frac{R}{L}}_{A(t)} i(t) = \underbrace{-\frac{mSB}{L}\cos\omega t}_{B(t)}$$

Etapa 1

$$i'(t) + \frac{R}{L} i(t) = 0$$

$$\frac{di}{dt} = -\frac{R}{L} i$$

$$\frac{di}{i} = -\frac{R}{L} dt$$

$$\int \frac{1}{i} di = -\int \frac{R}{L} dt$$

$$\ln i = -\frac{R}{L} t + C$$

$$i = e^{-\frac{R}{L}t + C} \Rightarrow \boxed{i_0 = C \cdot e^{-\frac{R}{L}t}}$$

Etapa 2 $\quad i_p = C(t) \cdot e^{-\frac{R}{L}t}$

$$\left(C(t) \cdot e^{-\frac{R}{L}t}\right)' + \frac{R}{L} \cdot C(t) \cdot e^{-\frac{R}{L}t} = -\frac{mSB}{L}\cos\omega t$$

$$c'(t) \cdot e^{-\frac{R}{L}t} + \cancel{C(t) \cdot e^{-\frac{R}{L}t} \cdot \left(-\frac{R}{L}\right) + \frac{R}{L} C(t) e^{-\frac{R}{L}t}} = -\frac{mSB}{L}\cos\omega t$$

$$c'(t) \cdot e^{-\frac{R}{L}t} = -\frac{mSB}{L}\cos\omega t \quad | \cdot e^{\frac{R}{L}t}$$

$$c'(t) = -\frac{mSB}{L}\cos\omega t \cdot e^{\frac{R}{L}t}$$

$$c(t) = -\frac{mSB}{L}\int \underset{f}{\cos\omega t} \cdot \underbrace{e^{\frac{R}{L}t}}_{g'} dt$$

$$f' = -\omega \sin\omega t, \quad g = \frac{e^{\frac{R}{L}t} \cdot L}{R}$$

$$c(t) = -\frac{mSB}{L}\left(\frac{L}{R} e^{\frac{R}{L}t} \cdot \cos\omega t + L\int \omega \sin\omega t \cdot \frac{e^{\frac{R}{L}t}}{R}\right.$$

$$= -\frac{mSB}{L}\frac{L}{R} e^{\frac{R}{L}t}\cos\omega t - \frac{mSB}{L} \cdot \frac{\omega L}{R}\int \sin\omega t \cdot e^{\frac{R}{L}t} dt$$

$c(t) =$

$$I = \int \underset{f}{\sin\omega t} \cdot \underset{g'}{e^{+\frac{R}{L}t}} dt \qquad = \frac{L}{R} e^{\frac{R}{L}t} \cdot \omega \cos\omega t - \int \omega \frac{L}{R} e^{\frac{R}{L}t} \cos\omega t \, dt$$

$$f = \omega\cos\omega t, \quad g = \frac{L}{R} e^{\frac{R}{L}t}$$

$$= \frac{L}{R} e^{\frac{R}{L}t} \cdot \omega \cos\omega t - \frac{\omega L}{R}\int \underbrace{e^{\frac{R}{L}t}}_{g'} \underset{f}{\cos\omega t} \, dt$$

$$f' = -\omega \sin\omega t$$

$$g = \frac{L}{R} e^{\frac{R}{L}t}$$

$$= \frac{L}{R} e^{\frac{R}{L}t} \cdot \omega \cos\omega t - \frac{\omega L}{R}\left(-\frac{\omega L}{R}\sin\omega t \, e^{\frac{R}{L}t} + \int \frac{\omega L}{R} e^{\frac{L}{L}t} \underbrace{\cos\omega t \, dt}_{I}\right)$$

$$I = \frac{\omega L}{R} e^{\frac{R}{L}t} \cos\omega t + \left(\frac{\omega L}{R}\right)^2 \sin\omega t \, e^{\frac{R}{L}t} - \left(\frac{\omega L}{R}\right)^2 I$$

$$I\left[1 + \left(\frac{\omega L}{R}\right)^2\right] = \frac{\omega L}{R} \cdot e^{\frac{R}{L}t}\left(\cos\omega t + \frac{\omega L}{R}\sin\omega t\right)$$

$$\Rightarrow c(t) = \frac{\omega L}{R} e^{\frac{R}{L}t}\left(\cos\omega t + \frac{\omega L}{R}\sin\omega t\right) \Big/ \left(1 + \left(\frac{\omega L}{R}\right)^2\right) + C_1$$

$$i_p = c(t) \cdot e^{-\frac{R}{L}t} = \frac{\frac{\omega L}{R}\left(\cos\omega t + \frac{\omega L}{R}\sin\omega t\right)}{1 + \left(\frac{\omega L}{R}\right)^2} + C_1 e^{-\frac{R}{L}t}$$

$$i(t) = i_0 + i_p = C e^{-\frac{R}{L}t} + \frac{\frac{\omega L}{R}\left(\cos\omega t + \frac{\omega L}{R}\sin\omega t\right)}{1 + \left(\frac{\omega L}{R}\right)^2}$$