Laboratorul 2
Săptămâna 23.03.2020-27.03.2020

1. Noţiuni teoretice

Definiţia 1.0.1 *Fie $S(\alpha)$ o reprezentare clauzală liberă de tautologii şi λ literal. Fie submulţimile de clauze,*

$$\begin{aligned}
\alpha_\lambda^+ &= \{k \mid k \in S(\alpha), k \langle\lambda\rangle\}, \\
\alpha_\lambda^- &= \{k \mid k \in S(\alpha), k \langle(\neg\lambda)\rangle\}, \\
\alpha_\lambda^0 &= \{k \mid k \in S(\alpha), k \rangle\lambda\langle,\ k \rangle(\neg\lambda)\langle\}, \\
POS_\lambda(\alpha) &= \alpha_\lambda^0 \cup \left\{k \setminus \lambda \ \mid k \in \alpha_\lambda^+\right\}, \\
NEG_\lambda(\alpha) &= \alpha_\lambda^0 \cup \left\{k \setminus (\neg\lambda) \ \mid k \in \alpha_\lambda^-\right\}.
\end{aligned}$$

Exemplul 1.0.1 Fie $S(\alpha) = \{k_1, k_2, k_3, k_4, k_5, k_6\}$ unde

$$\begin{aligned}
k_1 &= (\neg p) \vee o, \\
k_2 &= (\neg p) \vee (\neg c), \\
k_3 &= (\neg m) \vee c \vee i, \\
k_4 &= m, \\
k_5 &= p, \\
k_6 &= (\neg i).
\end{aligned}$$

Pentru $\lambda = (\neg p)$,

$$\begin{aligned}
\alpha_\lambda^+ &= \{(\neg p) \vee o, (\neg p) \vee (\neg c)\}, \\
\alpha_\lambda^- &= \{p\}, \\
\alpha_\lambda^0 &= \{(\neg m) \vee c \vee i, m, (\neg i)\}, \\
POS_\lambda(\alpha) &= \{(\neg m) \vee c \vee i, m, (\neg i)\} \cup \{o, (\neg c)\}, \\
NEG_\lambda(\alpha) &= \{(\neg m) \vee c \vee i, m, (\neg i)\} \cup \{\square\}.
\end{aligned}$$

Exemplul 1.0.2 *Să se calculeze mulţimile α_λ^+, α_λ^-, α_λ^0, $POS_\lambda(\alpha)$, $NEG_\lambda(\alpha)$ unde $\lambda = \beta$ respectiv $\lambda = \neg\delta$ iar*

$$S(\alpha) = \{\neg\beta \vee \delta \vee \gamma,\ \beta \vee \eta \vee \neg\delta,\ \delta \vee \beta \vee \theta,\ \neg\delta,\ \gamma \vee \neg\eta \vee \neg\delta, \delta, \beta \vee \eta \vee \theta,\ \neg\beta \vee \neg\delta\}.$$

Soluţie
Fie $\lambda = \beta$. În acest caz, obţinem mulţimile:

$$\begin{aligned}
\alpha_\beta^+ &= \{\beta \vee \eta \vee \neg\delta,\ \delta \vee \beta \vee \theta,\ \beta \vee \eta \vee \theta\} \\
\alpha_\beta^- &= \{\neg\beta \vee \delta \vee \gamma,\ \neg\beta \vee \neg\delta\} \\
\alpha_\beta^0 &= \{\neg\delta,\ \gamma \vee \neg\eta \vee \neg\delta, \delta, \} \\
POS_\beta\,(\alpha) &= \{\neg\delta,\ \gamma \vee \neg\eta \vee \neg\delta, \delta, \} \cup \{\eta \vee \neg\delta,\ \delta \vee \theta,\ \eta \vee \theta\} = \\
&= \{\neg\delta,\ \gamma \vee \neg\eta \vee \neg\delta, \delta, \eta \vee \neg\delta,\ \delta \vee \theta,\ \eta \vee \theta\} \\
NEG_\beta\,(\alpha) &= \{\neg\delta,\ \gamma \vee \neg\eta \vee \neg\delta, \delta, \} \cup \{\delta \vee \gamma,\ \neg\delta\} = \\
&= \{\neg\delta,\ \gamma \vee \neg\eta \vee \neg\delta, \delta,\ \delta \vee \gamma,\ \neg\delta\}.
\end{aligned}$$

Pentru $\lambda = \neg\delta$ obţinem următoarele mulţimi:

$$\begin{aligned}
\alpha_{\neg\delta}^+ &= \{\beta \vee \eta \vee \neg\delta,\ \neg\delta,\ \gamma \vee \neg\eta \vee \neg\delta\} \\
\alpha_{\neg\delta}^- &= \{\neg\beta \vee \delta \vee \gamma,\ \delta \vee \beta \vee \theta,\ \delta, \} \\
\alpha_{\neg\delta}^0 &= \{\beta \vee \eta \vee \theta\} \\
POS_{\neg\delta}\,(\alpha) &= \{\beta \vee \eta \vee \theta, \} \cup \{\beta \vee \eta,\ \square,\ \gamma \vee \neg\eta\} = \{\beta \vee \eta \vee \theta,\ \beta \vee \eta,\ \square,\ \gamma \vee \neg\eta\} \\
NEG_{\neg\delta}\,(\alpha) &= \{\beta \vee \eta \vee \theta, \} \cup \{\neg\beta \vee \gamma,\ \beta \vee \theta,\ \square\} = \{\beta \vee \eta \vee \theta,\ \neg\beta \vee \gamma,\ \beta \vee \theta,\ \square\}.
\end{aligned}$$

Exemplul 1.0.3 *Fie $\lambda = \eta$ şi $S(\alpha) = \{\beta \vee \eta \vee \gamma,\ \neg\beta \vee \eta \vee \theta,\ \neg\eta,\ \gamma \vee \neg\eta,\ \theta \vee \beta \vee \neg\eta\}$. Calculaţi mulţimile α_λ^+, α_λ^-, α_λ^0, $POS_\lambda\,(\alpha)$, $NEG_\lambda\,(\alpha)$.*

Soluţie

$$\begin{aligned}
\alpha_\eta^+ &= \{\beta \vee \eta \vee \gamma,\ \neg\beta \vee \eta \vee \theta\} \\
\alpha_\eta^- &= \{\neg\eta,\ \gamma \vee \neg\eta,\ \theta \vee \beta \vee \neg\eta\} \\
\alpha_\eta^0 &= \emptyset \\
POS_\eta\,(\alpha) &= \emptyset \cup \{\beta \vee \gamma,\ \neg\beta \vee \theta\} = \{\beta \vee \gamma,\ \neg\beta \vee \theta\} \\
NEG_\eta\,(\alpha) &= \emptyset \cup \{\square,\ \gamma,\ \theta \vee \beta\} = \{\square,\ \gamma,\ \theta \vee \beta\}
\end{aligned}$$

2. TEMĂ: Exerciţii

Exerciţiul 1.0.1 *Să se calculeze mulţimile α_λ^+, α_λ^-, α_λ^0, $POS_\lambda\,(\alpha)$, $NEG_\lambda\,(\alpha)$ pentru următoarele reprezentări clauzele:*

a) $S(\alpha) = \{a \vee b \vee \neg c,\ \neg b \vee d \vee \neg a,\ a \vee c \vee \neg b,\ \neg a \vee \neg c \vee b\}$ şi $\lambda_1 = a$, respectiv $\lambda_2 = \neg d$.

b) $S(\alpha) = \{\neg a \vee b \vee c,\ \neg b \vee d \vee \neg e \vee a,\ \neg a \vee \neg c \vee d \vee e,\ b \vee c \vee a \vee e\}$ şi $\lambda_1 = \neg b$, respectiv $\lambda_2 = \neg e$.

Exerciţiul 1.0.2 *Să se calculeze mulţimile α_λ^+, α_λ^-, α_λ^0, $POS_\lambda\,(\alpha)$, $NEG_\lambda\,(\alpha)$ pentru*

a) $S(\alpha) = \{\beta \vee \omega \vee \neg\theta,\ \neg\omega \vee \gamma \vee \neg\beta,\ \beta \vee \theta \vee \neg\omega,\ \neg\beta \vee \neg\theta \vee \omega\}$ şi $\lambda_1 = \beta$, respectiv $\lambda_2 = \neg\gamma$

b) $S(\alpha) = \{\neg\gamma \vee \theta \vee \psi,\ \neg\theta \vee \beta \vee \neg\delta \vee \gamma,\ \neg\gamma \vee \neg\psi \vee \beta \vee \delta,\ \theta \vee \psi \vee \gamma \vee \delta\}$ şi $\lambda_1 = \neg\theta$, respectiv $\lambda_2 = \neg\delta$.