Laborator01

Exerciţii

1. Să se calculeze următoarele primitive:

$$1) \int (x^2 + 3x - 2)\ dx$$
$$2) \int x^2 lnx\ dx$$
$$3) \int \frac{3x^2 + 4}{x^3 + 4x}\ dx$$

2. Să se determine primitiva $F : R \rightarrow R$ a funcţiei $f : R \rightarrow R,\ f(x) = \frac{x}{x^2+1}$ cu proprietatea că $F(0) = 3$.
3. Să se determine primitivele următoarelor funcţii, care îndeplinesc condiţiile precizate:

$$\begin{aligned} a) \int lnx\ dx, &\qquad F(1) = 3 \\ b) \int \frac{5}{x^2 + 3x + 2}\ dx, &\qquad F(0) = 5 \\ c) \int sin^2x\ dx, &\qquad F(0) = 1 \\ d) \int \frac{3}{x^2 + 4x + 5}\ dx, &\qquad F(-2) = 2 \end{aligned}$$

Rezolvare

Exercitiul 1

1

1) $\int (x^2+3x-2)\,dx = \int x^2\,dx + 3\int x\,dx - 2\int dx = \frac{x^3}{3} + 3\cdot\frac{x^2}{2} - 2x + C$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

$\int c\cdot f\,dx = c\int f\,dx$

$\int (f+g)\,dx = \int f\,dx + \int g\,dx$

$\int f\cdot g'\,dx = f\cdot g - \int f'\cdot g\,dx$

2) $\int \underbrace{x^2}_{g'}\underbrace{\ln x}_{f}\,dx = \frac{x^3}{3}\ln x - \int \frac{1}{\cancel{x}}\cdot\frac{\cancel{x^3}x^2}{3}\,dx = \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2\,dx$

$f' = \frac{1}{x}$, $g = \frac{x^3}{3}$

$= \frac{x^3}{3}\ln x - \frac{1}{3}\cdot\frac{x^3}{3} + C$

$= \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$

3) $\int \frac{3x^2+4}{x^3+4x}\,dx = \int \frac{u'}{u}\,dx = \ln|u| = \ln|x^3+4x| + C$

$u(x) = x^3+4x$

$u'(x) = 3x^2+4$

Exercitiul 2

2

$\int f(x)\,dx = \underbrace{F(x)+C}_{F(x)}.\quad F' = f$

$F(0) = 3$

$\frac{1}{2}\int \frac{2x}{x^2+1}\,dx = \frac{1}{2}\int \frac{u'}{u}\,dx = \frac{1}{2}\ln|u| = \underbrace{\frac{1}{2}\ln(x^2+1) + C}_{F(x)}$

$u(x) = x^2+1$

$u' = 2x$

$F(0) = \frac{1}{2}\ln(0+1) + C = \frac{1}{2}\underbrace{\ln 1}_{0} + C = \boxed{C = 3}$

$F(x) = \frac{1}{2}\ln(x^2+1) + 3$

Exercitiul 3

③

a) $\int \underbrace{1}_{g'}\cdot\underbrace{\ln x}_{f}\,dx = x\ln x - \int \frac{1}{\cancel{x}}\cdot\cancel{x}\,dx = x\ln x - \int dx = \underbrace{x\ln x - x + C}_{F(x)}$

$f' = \frac{1}{x},\ g = x$

$F(1) = 1\cdot\underbrace{\ln 1}_{0} - 1 + C = -1 + C = 3 \Rightarrow \boxed{C=4} \Rightarrow F(x) = x\ln x - x + 4$

b) $\int \frac{5}{x^2+3x+2}\,dx = 5\int \frac{1}{x^2+3x+2}\,dx$

$\boxed{ax^2+bx+c=0,\ x_1,x_2 \text{ răd} \Rightarrow ax^2+bx+c = a(x-x_1)(x-x_2)}$!

$x^2+3x+2=0$

$\Delta = 9-8=1 \Rightarrow x_{1,2} = \frac{-3\pm 1}{2}$ $\begin{cases} -1 \\ -2 \end{cases}$ $\Rightarrow x^2+3x+2 = (x+1)(x+2)$

$\frac{1}{x^2+3x+2} = \frac{^{x+2)}A}{x+1} + \frac{^{x+1)}B}{x+2} = \frac{Ax+2A+Bx+B}{(x+1)(x+2)} = \frac{(A+B)x+2A+B}{(x+1)(x+2)}$

$\begin{cases} A+B=0 \\ 2A+B=1 \end{cases} \xRightarrow{(-)} A=1,\ B=-1$

$5\int\left(\frac{1}{x+1}-\frac{1}{x+2}\right)dx = \underbrace{5\left(\ln|x+1|-\ln|x+2|\right)+C}_{F(x)}$

$F(0)=5$

$F(0)=5(\ln 1-\ln 2)+C = -5\ln 2+C = 5 \Rightarrow \boxed{C=5+5\ln 2}$

$\Rightarrow F(x) = 5(\ln|x+1|-\ln|x+2|)+5+5\ln 2$

$= 5(\ln|x+1|-\ln|x+2|+1+\ln 2)$

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d)

$$\int \frac{3}{x^2+4x+5}\,dx = 3\int \frac{1}{x^2+4x+5}\,dx$$

$x^2+4x+5=0$

$\Delta = 16-20=-4<0 \Rightarrow$ ec. nu are sol. reale $\Rightarrow x^2+4x+5 = x^2+4x+4+1$

$= (x+2)^2+1$

$$3\int \frac{1}{(x+2)^2+1}\,dx = \underbrace{3\,\text{arctg}(x+2)+C}_{F(x)}$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\,\text{arctg}\,\frac{x}{a}+C$$

$F(-2) = 3\underbrace{\text{arctg}\,0}_{0}+C = \boxed{C=2} \Rightarrow F(x) = 3\,\text{arctg}(x+2)+2$