

# Laborator02

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## Laborator02

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e)

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## Enunțuri

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1. Să se rezolve următoarele ecuații diferențiale direct integrabile:

$$\begin{aligned} a) \quad & \begin{cases} x'(t) = t^2 + 2, & x, t \in R \\ x(1) = 1 \end{cases} \\ b) \quad & \begin{cases} x'(t) = \frac{1}{t^2-1}, & t \in (-1, 1) \\ x \in R \end{cases} \\ c) \quad & \begin{cases} x'(t) = \frac{1}{1+t^2}, & t, x \in R \\ x(-1) = -2 \end{cases} \\ d) \quad & \begin{cases} x'(t) = \sin t + 4t^2 \\ x(\frac{\pi}{3}) = -\frac{1}{3} \end{cases} \\ e) \quad & \begin{cases} x'(t) = \frac{\sqrt{\ln t}}{t}, & t > 1 \\ x(e) = \frac{5}{3} \end{cases} \end{aligned}$$

2. Să se rezolve următoarele ecuații diferențiale cu variabile separate:

$$\begin{aligned} a) \quad & \frac{1}{1+t^2} dt + \frac{1}{x} dx = 0, \quad x > 0, t \in R \\ b) \quad & dx + \frac{1}{t^2-9} dt = 0, \quad t > 3, x \in R \\ c) \quad & \sqrt{x} dx = \sqrt{t} dt, \quad t > 0, x > 0 \\ & x(1) = 1 \\ d) \quad & \sin t dt - \cos x dx = 0, \quad x \in [0, \pi] \\ & x(0) = \frac{\pi}{2} \\ e) \quad & \frac{x}{1-x^2} dx = \frac{1}{1-t} dt, \quad t < 1, x \in (0, 1) \\ & x(0) = \frac{1}{2} \end{aligned}$$

3. Să se rezolve următoarele ecuații diferențiale cu variabile separabile:

$$\begin{aligned}
 a) & (t+1) \cdot x'(t) = 2x - 3 \\
 b) & (t^2 - 1) \cdot x'(t) + 2tx^2 = 0 \\
 c) & x'(t) = \frac{-t}{\sqrt{1+t^2}} \cdot \frac{\sqrt{1+x^2}}{x}, \quad x < 0, t \in \mathbb{R} \\
 d) & \frac{dx}{dt} = \frac{t}{1+t}(1-x), \quad t > -1, x > 1 \\
 & x(0) = 5
 \end{aligned}$$

## Rezolvare

### Exercitiul 1

a) - [Video](#)

$$① a) \begin{cases} x'(t) = t^2 + 2 \\ x(1) = 1 \end{cases}, t, x \in \mathbb{R}$$

$$x(t) = \underbrace{t^2 + 2}_{f(t)} \quad (\text{ec. direct integrabilă})$$

$$x(t) = \int (t^2 + 2) dt = \int t^2 dt + \int 2 dt = \frac{t^3}{3} + 2t + C$$

$$x(1) = \frac{1^3}{3} + 2 \cdot 1 + C = \frac{1}{3} + 2 + C = 1 \Rightarrow C = 1 - \frac{1}{3} - 2 = \frac{3-1-6}{3} = \frac{-4}{3} \Rightarrow \boxed{C = -\frac{4}{3}}$$

$$\Rightarrow x_{pc}(t) = \frac{t^3}{3} + 2t - \frac{4}{3}$$

e)

$$① e) \begin{cases} x'(t) = \frac{\sqrt{\ln t}}{t} \\ x(e) = \frac{5}{3} \end{cases}, t > 1, x \in \mathbb{R}$$

$$x'(t) = \underbrace{\frac{\sqrt{\ln t}}{t}}_{f(t)} \quad (\text{ec. direct integrabilă})$$

$$\begin{aligned}
 x(t) &= \int \frac{\sqrt{\ln t}}{t} dt = \int \sqrt{\ln t} \cdot \frac{1}{t} dt = \int \sqrt{u} \cdot u' dt = \int u^{\frac{1}{2}} \cdot u' dt = \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C \\
 &= \frac{(\ln t)^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3} \sqrt{(\ln t)^3} + C \\
 u(t) &= \ln t \\
 u'(t) &= \frac{1}{t}
 \end{aligned}$$

$$x(e) = \frac{2}{3} \sqrt{\underbrace{(\ln e)^3}_1} + C = \frac{2}{3} + C = \frac{5}{3} \Rightarrow C = \frac{5}{3} - \frac{2}{3} = \frac{3}{3} = 1 \Rightarrow \boxed{C = 1}$$

$$x_{pc}(t) = \frac{2}{3} \sqrt{(\ln t)^3} + 1$$

## Exercitiul 2

### c) - Video

$$\textcircled{2} \text{ c) } \begin{cases} \sqrt{x} dx = \sqrt{t} dt, & t > 0 \\ x(1) = 1 \end{cases}$$

$\sqrt{x} dx = \sqrt{t} dt$  (ec. cu variabile separate)

$$\int \sqrt{x} dx = \int \sqrt{t} dt \Leftrightarrow \int x^{\frac{1}{2}} dx = \int t^{\frac{1}{2}} dt \Leftrightarrow \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$\frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C \Leftrightarrow x^{\frac{3}{2}} = t^{\frac{3}{2}} + C$$

$$\sqrt{x^3} = \sqrt{t^3} + C \Leftrightarrow x^3 = (\sqrt{t^3} + C)^2 \Rightarrow x = \sqrt[3]{(\sqrt{t^3} + C)^2}$$

$$x(1) = 1 \Rightarrow \sqrt[3]{(1+C)^2} = 1 \Rightarrow (1+C)^2 = 1 \Rightarrow 1+C = \pm 1 \begin{cases} C=0 \\ C=-2 \end{cases}$$

$$C=0 \Rightarrow x = \sqrt[3]{(\sqrt{t^3})^2} = \sqrt[3]{t^3} = t \Rightarrow x_{PC}(t) = t$$

$$C=-2 \Rightarrow x_{PC} = \sqrt[3]{(\sqrt{t^3}-2)^2}$$

### e)

$$\textcircled{2} \text{ e) } \begin{cases} \frac{x}{1-x^2} dx = \frac{1}{1-t} dt, & t < 1 \\ x \in (0,1) \\ x(0) = \frac{1}{2} \end{cases}$$

$$\frac{x}{1-x^2} dx = \frac{1}{1-t} dt \quad (\text{ec. cu var. separate})$$

$$-\frac{1}{2} \int \frac{2x}{1-x^2} dx = - \int \frac{1}{1-t} dt$$

$$\begin{aligned} u(x) &= 1-x^2 \\ u'(x) &= -2x \end{aligned}$$

$$\begin{aligned} v(t) &= 1-t \\ v'(t) &= -1 \end{aligned}$$

$$\Leftrightarrow -\frac{1}{2} \int \frac{u'}{u} dx = - \int \frac{v'}{v} dt$$

$$-\frac{1}{2} \ln|u| = -\ln|v| + C \quad / \cdot (-2)$$

$$\ln|1-x^2| = 2\ln|1-t| + C$$

$$\ln(1-x^2) = \ln(1-t)^2 + \ln C$$

$$\ln(1-x^2) = \ln(1-t)^2 \cdot C$$

$$1-x^2 = C(1-t)^2$$

$$x^2 = 1-C(1-t)^2 \Rightarrow x = \sqrt{1-C(1-t)^2}, \quad 1-C(1-t)^2 \geq 0$$

$$\begin{aligned} x(0) &= \sqrt{1-C} = \frac{1}{2} \Rightarrow 1-C = \frac{1}{4} \Rightarrow C = \frac{3}{4} \\ x_{PC} &= \sqrt{1-\frac{3}{4}(1-t)^2}, \quad 1-\frac{3}{4}(1-t)^2 \geq 0 \end{aligned}$$

## Exercitiul 3

### b) - Video

$$\textcircled{3} \text{ b) } (t^2 - 1) \cdot x'(t) + 2tx^2 = 0$$

$$x'(t) = \frac{dx}{dt}$$

$$(t^2 - 1) \cdot \frac{dx}{dt} + 2tx^2 = 0$$

$$\underbrace{(t^2 - 1)}_{A_1(t)} dx + \underbrace{2tx^2}_{A_2(t)B_2(t)} dt = 0 \quad (\text{ec. cu variabile separabile})$$

$$(t^2 - 1) dx = -2tx^2 dt$$

$$-\frac{1}{x^2} dx = \frac{2t}{t^2 - 1} dt$$

$$\int -\frac{1}{x^2} dx = \int \frac{2t}{t^2 - 1} dt \quad (\Rightarrow) \quad \frac{1}{x} = \ln|t^2 - 1| + C \quad \Rightarrow \quad x = \frac{1}{\ln|t^2 - 1| + C}, \quad \ln|t^2 - 1| + C \neq 0$$

$u(t) = t^2 - 1$   
 $u'(t) = 2t$

c)

$$\textcircled{3} \text{ c) } x'(t) = \frac{-t}{\sqrt{1+t^2}} \cdot \frac{\sqrt{1+x^2}}{x}, \quad x < 0, \quad t \in \mathbb{R}$$

$$\frac{dx}{dt} = \frac{-t}{\sqrt{1+t^2}} \cdot \frac{\sqrt{1+x^2}}{x}$$

$$\frac{x}{\sqrt{1+x^2}} dx = -\frac{t}{\sqrt{1+t^2}} dt$$

$$\int \frac{2x}{2\sqrt{1+x^2}} dx = -\int \frac{2t}{2\sqrt{1+t^2}} dt \quad (\Rightarrow) \quad \int \frac{u'}{2\sqrt{u}} dx = -\int \frac{t'}{2\sqrt{t}} dt$$

$$u(x) = 1+x^2$$

$$u'(x) = 2x$$

$$\sqrt{1+x^2} = -\sqrt{1+t^2} + C$$

$$1+x^2 = (C - \sqrt{1+t^2})^2$$

$$x^2 = (C - \sqrt{1+t^2})^2 - 1$$

$$x = -\sqrt{(C - \sqrt{1+t^2})^2 - 1}, \quad (C - \sqrt{1+t^2})^2 - 1 \geq 0$$