1) a) $\dfrac{dx}{x} = \dfrac{dy}{-2y} = \dfrac{dz}{-z}$, $x, y, z \neq 0$.

$\dfrac{dx}{x} = \dfrac{dy}{-2y}$

$\int \frac{1}{x}\,dx = -\frac{1}{2}\int \frac{1}{y}\,dy$

$\ln|x| = -\frac{1}{2}\ln|y| + C_1 \quad |\cdot 2$

$2\ln|x| = -\ln|y| + C_1$

$\ln|x|^2 + \ln|y| = C_1$

$\ln |x|^2 \cdot |y| = C_1$

$\boxed{x^2 y = C_1 = \varphi_1(x, y, z)}$

$\dfrac{dx}{x} = \dfrac{dz}{-z}$

$\int \frac{1}{x}\,dx = -\int \frac{1}{z}\,dz$

$\ln|x| = -\ln|z| + C_2$

$\ln|x| + \ln|z| = C_2$

$\ln(|x| \cdot |z|) = C_2$

$\boxed{x z = C_2 = \varphi_2(x, y, z)}$

Metoda I

Fie z variabila indep.

φ_1, φ_2 indep $\Leftrightarrow \dfrac{D(\varphi_1, \varphi_2)}{D(x, y)} \neq 0 \Leftrightarrow \begin{vmatrix} \frac{\partial \varphi_1}{\partial x} & \frac{\partial \varphi_1}{\partial y} \\ \frac{\partial \varphi_2}{\partial x} & \frac{\partial \varphi_2}{\partial y} \end{vmatrix} = \begin{vmatrix} 2xy & x^2 \\ z & 0 \end{vmatrix} = -zx^2 \neq 0 \Rightarrow \varphi_1 \varphi_2$ indep.

$\Rightarrow$ Sol. în forma implicită este $\begin{cases} x^2 y = C_1 \\ x z = C_2 \end{cases} \longrightarrow x = \dfrac{C_2}{z} \nearrow y = \dfrac{C_1}{x^2} = \dfrac{C_1}{\frac{C_2^2}{z^2}} = \dfrac{C_1 z^2}{C_2^2} \Rightarrow$ Sol. în forma explicită $\begin{cases} x = \frac{C_2}{z} \\ y = \frac{C_1 z^2}{C_2^2} \end{cases}$

Metoda 2

φ_1, φ_2 indep. $\Leftrightarrow$ rang $\left(\dfrac{D(\varphi_1, \varphi_2)}{D(x,y,z)}\right) = 2$

$$\begin{pmatrix} \dfrac{\partial \varphi_1}{\partial x} & \dfrac{\partial \varphi_1}{\partial y} & \dfrac{\partial \varphi_1}{\partial z} \\[2ex] \dfrac{\partial \varphi_2}{\partial x} & \dfrac{\partial \varphi_2}{\partial y} & \dfrac{\partial \varphi_2}{\partial z} \end{pmatrix} = \begin{pmatrix} 2xy & x^2 & 0 \\ z & 0 & x \end{pmatrix}$$

$D_p = \begin{vmatrix} x^2 & 0 \\ 0 & x \end{vmatrix} = x^3 \neq 0 \Rightarrow$ rang $\left(\dfrac{D(\varphi_1, \varphi_2)}{D(x,y,z)}\right) = 2 \Rightarrow \varphi_1, \varphi_2 =$ independente.

$\Rightarrow \begin{cases} x^2 y = c_1 \\ xz = c_2 \end{cases}$ (Sol. în formă implicită)