Temă - Curs 1 Petculescu Mihai-Iulian

1. Să se verifice dacă următoarele mulțimi sunt spații vectoriale:

a) $\underset{V}{\overset{\parallel}{\mathbb{R}}} / \underset{K}{\overset{\parallel}{\mathbb{Q}}}$ $\quad +: \mathbb{R}\times\mathbb{R}\to\mathbb{R} \quad (\mathbb{Q},+,\cdot)$ - corp. com.

$\cdot : \mathbb{Q}\times\mathbb{R}\to\mathbb{R}. \quad (\mathbb{R},+)$ - grup. com.

1) $\alpha(\vec{x}+\vec{y}) = \alpha\vec{x}+\alpha\vec{y},\ \forall \alpha\in\mathbb{Q},\ \vec{x},\vec{y}\in\mathbb{R}$ (A).

2) $(\alpha+\beta)\vec{x} = \alpha\vec{x}+\beta\vec{x},\ \forall \alpha,\beta\in\mathbb{Q},\ \vec{x}\in\mathbb{R}$ (A)

3) $\alpha(\beta\vec{x}) = \alpha\beta\vec{x},\ \forall \alpha,\beta\in\mathbb{Q},\ \vec{x}\in\mathbb{R}$ (A)

4) $1_{\mathbb{Q}}\vec{x} = \vec{x},\ \forall \vec{x}\in\mathbb{R},\ 1_{\mathbb{Q}}$ - el. neutru în $\mathbb{Q} \Rightarrow \mathbb{R}/\mathbb{Q}$ e spațiu vectorial.

b) $\underset{V}{\overset{\parallel}{\mathbb{R}}} / \underset{K}{\overset{\parallel}{\mathbb{Z}}}$ $\quad (\mathbb{Z},+,\cdot)$ nu e corp. comutativ

$\Rightarrow \mathbb{R}/\mathbb{Z}$ nu e spațiu vectorial.

c) $\underset{V}{\overset{\parallel}{\mathbb{C}}} / \underset{K}{\overset{\parallel}{\mathbb{Q}}}$ $\quad +: \mathbb{C}\times\mathbb{C}\to\mathbb{C}$; $(\mathbb{Q},+,\cdot)$ - corp. com.

$\cdot : \mathbb{Q}\times\mathbb{C}\to\mathbb{C} \quad (\mathbb{C},+)$ - grup. com.

1) $\alpha(\vec{x}+\vec{y}) = \alpha\vec{x}+\alpha\vec{y},\ \forall \alpha\in\mathbb{Q},\ \vec{x},\vec{y}\in\mathbb{C}$ (A)

2) $(\alpha+\beta)\vec{x} = \alpha\vec{x}+\beta\vec{x},\ \forall \alpha,\beta\in\mathbb{Q},\ \vec{x}\in\mathbb{C}$ (A)

3) $\alpha(\beta\vec{x}) = \alpha\beta\vec{x},\ \forall \alpha,\beta\in\mathbb{Q},\ \vec{x}\in\mathbb{C}$ (A)

4) $1_{\mathbb{Q}}\vec{x} = \vec{x},\ \forall \vec{x}\in\mathbb{C},\ 1_{\mathbb{Q}}$ - el. neutru în $\mathbb{Q}$

$\Rightarrow \mathbb{C}/\mathbb{Q}$ e spațiu vect.

d) $\underset{V}{\overset{\parallel}{\mathbb{C}}} / \underset{K}{\overset{\parallel}{\mathbb{R}}}$ $\quad$ Analog c) $\Rightarrow \mathbb{C}/\mathbb{R}$ spațiu vectorial

e) $\underset{V}{\overset{\parallel}{\mathbb{Q}}} / \underset{K}{\overset{\parallel}{\mathbb{Z}}}$ $\quad (\mathbb{Z},+,\cdot)$ nu e corp. comutativ

$\Rightarrow \mathbb{Q}/\mathbb{Z}$ nu e spațiu vectorial

2. Să se arate că următoarele mulțimi nu sunt spații vectoriale:

a) $\underset{V}{\overset{\parallel}{\mathbb{Q}}} / \underset{K}{\overset{\parallel}{\mathbb{R}}}$; $(\mathbb{R},+,\cdot)$ corp. com ; $+: \mathbb{Q}\times\mathbb{Q}\to\mathbb{Q}$ (A) $\Rightarrow \mathbb{Q}/\mathbb{R}$ nu e spațiu vectorial.

$(\mathbb{Q},+)$ grup. com ; $\cdot : \mathbb{R}\times\mathbb{Q}\to\mathbb{Q}$ (F)

b) $\underset{V}{\overset{\parallel}{\mathbb{Z}}} / \underset{K}{\overset{\parallel}{\mathbb{R}}}$; $(\mathbb{Z},+)$ grup com. ; $+: \mathbb{Z}\times\mathbb{Z}\to\mathbb{Z}$ (A) $\Rightarrow \mathbb{Z}/\mathbb{R}$ nu e spațiu vect.

$(\mathbb{R},+,\cdot)$ corp com ; $\cdot : \mathbb{R}\times\mathbb{Z}\to\mathbb{Z}$ (F)

c) R/C ; $\begin{matrix} \| & \| \\ V & K \end{matrix}$; $(R,+)$ - grup com. ; $(C,+,\cdot)$ - corp. com ; $+: R\times R \to R$ (A) ; $\cdot: C\times R \to R$ (F) ; $\Rightarrow R/C$ nu e sp. vectorial.

d) Q/C ; $\begin{matrix} \| & \| \\ V & K \end{matrix}$; $(Q,+)$ - grup. com. ; $(C,+,\cdot)$ - corp. com ; $+: Q\times Q \to Q$ (A) ; $\cdot: C\times Q \to Q$ (F) $\Rightarrow Q/C$ nu e spatiu vectorial.

e) Z/Q ; $\begin{matrix} \| & \| \\ V & K \end{matrix}$; $(Z,+)$ - grup. com ; $(Q,+,\cdot)$ - corp. com ; $+: Z\times Z \to Z$ (A) ; $\cdot: Q\times Z \to Z$ (F) $\Rightarrow Z/Q$ nu e sp. vect.

3. Să se verifice dacă urm. mulțimi sunt subspații ?

b) $S_2 = \{(a,0,c) \mid a,c \in R\}$

Fie $\vec{x},\vec{y} \in S_2 \Leftrightarrow \vec{x} = (a_1,0,c_1)$; $\vec{y} = (a_2,0,c_2)$

$\alpha\vec{x}+\beta\vec{y} = \alpha(a_1,0,c_1)+\beta(a_2,0,c_2) = (\alpha a_1,0,\alpha c_1)+(\beta a_2,0,\beta c_2) =$

$= (\underbrace{\alpha a_1+\beta a_2}_{a}, 0, \underbrace{\alpha c_1+\beta c_2}_{c}) \in S_2 \Rightarrow S_2 \in S_s(R^3)$

c) $S_3 = \{(0,b,c) \mid b,c \in R\}$

Fie $\vec{x},\vec{y} \in S_3 \Leftrightarrow \vec{x} = (0,b_1,c_1)$; $\vec{y} = (0,b_2,c_2)$

$\alpha\vec{x}+\beta\vec{y} = \alpha(0,b_1,c_1)+\beta(0,b_2,c_2) = (0,\alpha b_1,\alpha c_1)+(0,\beta b_2,\beta c_2) =$

$= (0,\underbrace{\alpha b_1+\beta b_2}_{b}, \underbrace{\alpha c_1+\beta c_2}_{c}) \in S_3 \Rightarrow S_2 \in S_s(R^3)$

d) $S_4 = \{(a,0,0) \mid a \in R\}$

Fie $\vec{x},\vec{y} \in S_4 \Leftrightarrow \vec{x} = (a_1,0,0)$, $\vec{y} = (a_2,0,0)$

$\alpha\vec{x}+\beta\vec{y} = \alpha(a_1,0,0)+\beta(a_2,0,0) = (\alpha a_1,0,0)+(\beta a_2,0,0) =$

$= (\underbrace{\alpha a_1+\beta a_2}_{a},0,0) \in S_4 \Rightarrow S_4 \in S_s(R^3)$

e) $S_5 = \{(0,b,0) \mid b \in R\}$

Fie $\vec{x},\vec{y} \in S_5 \Leftrightarrow \vec{x} = (0,b_1,0)$; $\vec{y} = (0,b_2,0)$

$\alpha\vec{x}+\beta\vec{y} = \alpha(0,\beta b_2,0) = (0,\underbrace{\alpha b_1+\beta b_2}_{b},0) \in S_5 \Rightarrow S_5 \in S_s(R^3)$

$f)\ S_6 = \{(0,0,z) \mid z \in \mathbb{R}\}$

Fie $\vec{x}, \vec{y} \in S_6 \Leftrightarrow \vec{x} = (0,0,z_1),\ \vec{y} = (0,0,z_2)$

$\alpha \vec{x} + \beta \vec{y} = \alpha(0,0,z_1) + \beta(0,0,z_2) = (0,0,\alpha z_1) + (0,0,\beta z_2) =$

$= (0,0,\underbrace{\alpha z_1 + \beta z_2}_{z}) \in S_6 \Rightarrow S_6 \in S_b(\mathbb{R}^3)$

$g)\ W = \{(\alpha, \beta, \alpha+\beta) \mid \alpha, \beta \in \mathbb{R}\}$

Fie $\vec{x}, \vec{y} \in W \Leftrightarrow \vec{x} = (\alpha_1, \beta_1, \alpha_1+\beta_1),\ \vec{y} = (\alpha_2, \beta_2, \alpha_2+\beta_2)$

$a\vec{x} + b\vec{y} = a(\alpha_1, \beta_1, \alpha_1+\beta_1) + b(\alpha_2, \beta_2, \alpha_2+\beta_2) =$

$= (a\alpha_1, a\beta_1, a(\alpha_1+\beta_1)) + (b\alpha_2, b\beta_2, b(\alpha_2+\beta_2)) =$

$= (\underbrace{a\alpha_1 + b\alpha_2}_{\alpha_1}, \underbrace{a\beta_1 + b\beta_2}_{\alpha_2}, \underbrace{a\alpha_1 + b\alpha_2 + a\beta_1 + b\beta_2}_{\alpha_1+\alpha_2}) \in W \Rightarrow W \in S_b(\mathbb{R}^3)$