7. $\alpha = (\neg l \vee (\delta \wedge \neg \lambda)) \leftrightarrow (l \vee (\neg \delta \rightarrow (\lambda \rightarrow \neg l)))$

$\boxed{7}$

- SGF:

$l, \delta, \lambda, \neg l, \neg \delta, \neg \lambda, \delta \wedge \neg \lambda, \neg l \vee (\delta \wedge \neg \lambda), \lambda \rightarrow \neg l,$
$\neg \delta \rightarrow (\lambda \rightarrow \neg l), l \vee (\neg \delta \rightarrow (\lambda \rightarrow \neg l))$ și $\alpha(\ldots)$.

- Tabel de adevăr.

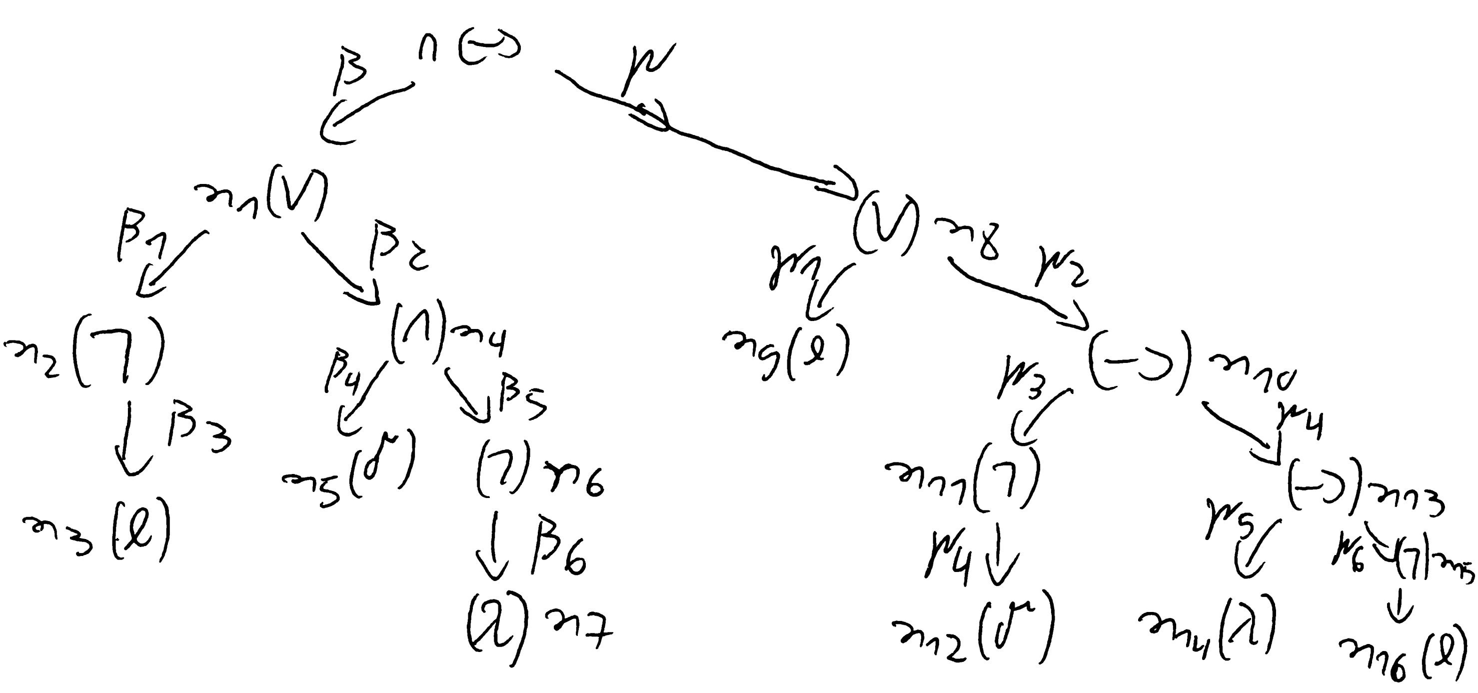
l	δ	λ	$\delta \wedge \neg \lambda$	$\neg l \vee (\delta \wedge \neg \lambda)$	$\lambda \rightarrow \neg l$	$\neg \delta \rightarrow (\lambda \rightarrow \neg l)$	$l \vee (\neg \delta \rightarrow (\lambda \rightarrow \neg l))$	α
T	T	T	F	F	F	T	T	F
T	T	F	T	T	T	T	T	T
T	F	T	F	F	F	F	T	F
T	F	F	F	F	T	T	T	F
F	T	T	F	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

- Substituția.

$\tau = \{(l \rightarrow n)|w, \neg \eta | y, (w \rightarrow z)|x\}$.

$\alpha\tau = \alpha$

- Arbore de structură



2. a) $\vdash (\neg\partial \vee x) \to ((\partial \to x) \vee (\neg x \to \neg\partial))$ $\boxed{2.\,a).}$

$\cancel{\alpha_8}$ $\cdot\ \alpha = (\neg\partial \vee x)$

$\beta = (\neg x \to \neg\partial)$

$\delta_1 = \alpha_8 \{\neg\partial \vee x / a,\ \neg x \to \neg\partial / b\}$.

$= \left(((\neg\partial \vee x) \vee (\neg x \to \neg\partial)) \leftrightarrow ((\partial \wedge \neg x) \to (\neg x \to \neg\partial)) \right)$

$\delta_2 = ((\partial \wedge \neg x) \to (\neg x \to \neg\partial)) \to [(\neg\partial \vee x) \vee (\neg x \to \neg\partial)]$, $\frac{\delta_1}{\delta_2}$ EI.

$\delta_3 = (\partial \wedge \neg x) \to ((\neg\partial \vee x) \to (\neg x \to \neg\partial))$ A_1. 1.4.5.

$\delta_4 = (\neg\partial \vee x) \to ((\partial \wedge \neg x) \to (\neg x \to \neg\partial))$ $\frac{\delta_3}{\delta_4}$ PP.

$\delta_5 = (\neg\partial \vee x) \in H$.

$\delta_6 = ((\partial \vee \neg x) \to (\neg x \to \neg\partial))$, $\frac{\delta_5, \delta_4}{\delta_6}$ MP.

$\delta_7 = ((\neg\partial \vee x) \vee (\neg x \to \neg\partial)) \leftrightarrow (\dots) = \delta_1$.

$\delta_8 = \delta_2$ $\frac{\delta_7}{\delta_8}$ EI.

$\delta_9 = (\neg\partial \vee x) \vee (\neg x \to \neg\partial)$ $\frac{\delta_6, \delta_8}{\delta_9}$ MP.

$\Rightarrow \underbrace{\{\alpha\} \vdash (\alpha \vee \beta)}_{\text{de indomnit}}$ din care rezultă $\underbrace{(\alpha \to (\alpha \vee \beta))}_{\text{de indomnit.}}$

SCHIȚA - B.48

3.

3. GENTZEN.

$$S = \{(\neg\vartheta \vee \lambda), (\vartheta \vee (\theta \wedge \omega))\} \Rightarrow \{(\neg\lambda \Rightarrow (\theta \wedge \omega))\}.$$

$G_8: n_1 = \{(\neg\vartheta \vee \lambda), (\vartheta \vee (\theta \wedge \omega)), \neg\lambda\} \Rightarrow \{\theta \wedge \omega\}.$

$G_1: n_2 = \{\neg\vartheta \vee \lambda, \vartheta \vee (\theta \wedge \omega)\} \Rightarrow \{\lambda, \theta \wedge \omega\}.$

$\cancel{G_6: n_3 = \{\neg\vartheta \vee \lambda, \vartheta \vee (\theta \wedge \omega)\} \Rightarrow \{\lambda, \theta\}}$

$\cancel{n_4 = \{\neg\vartheta \vee \lambda, \vartheta \vee (\theta \wedge \omega)\} \Rightarrow \{\lambda, \omega\}}$

$G_3: n_3 = \{\neg\vartheta \vee \lambda, \vartheta\} \Rightarrow \{\lambda, \theta \wedge \omega\}$

$n_4 = \{\neg\vartheta \vee \lambda, \theta \wedge \omega\} \Rightarrow \{\lambda, \theta \wedge \omega\}.$

$G_3: n_5 = \{\neg\vartheta, \vartheta\} \Rightarrow \{\lambda, \theta \wedge \omega\}$

$n_6 = \{\lambda, \vartheta\} \Rightarrow \{\lambda, \theta \wedge \omega\}.$ este axiom

$G_7: n_7 = \{\vartheta\} \Rightarrow \{\vartheta, \lambda, \theta \wedge \omega\}$ este axiom.

$\cancel{G_3: n_8 = \{\neg\vartheta, \theta \wedge \omega\}}$

$G_2: n_8 = \{\neg\vartheta \vee \lambda, \theta, \omega\} \Rightarrow \{\lambda, \theta \wedge \omega\}.$

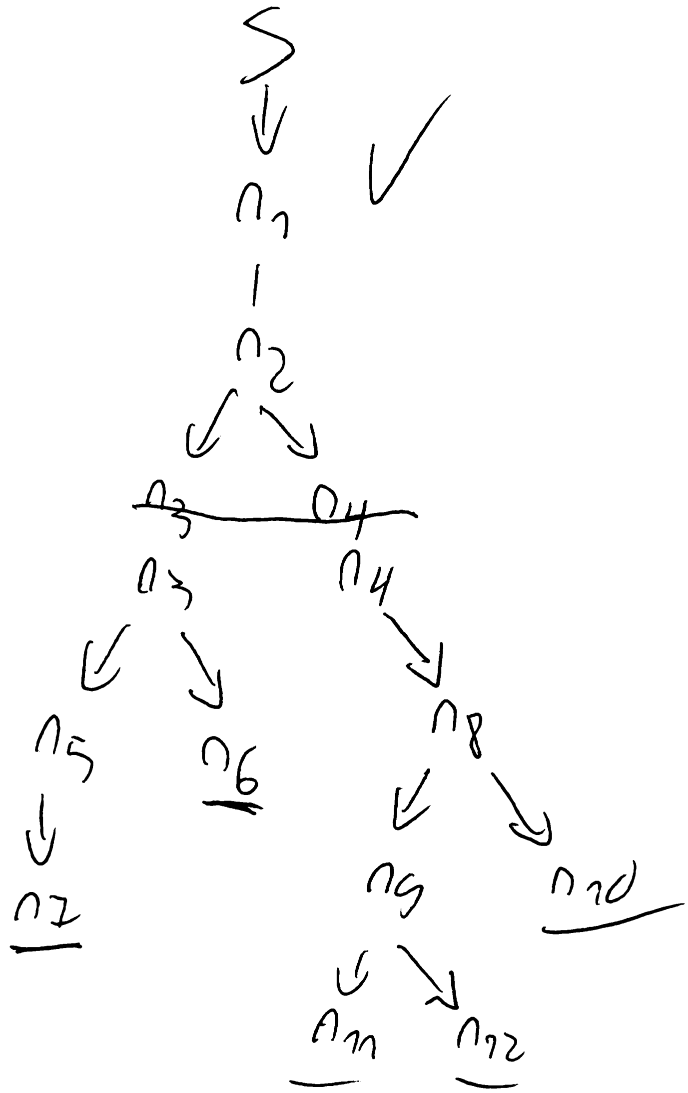
$G_3: n_9 = \{\neg\vartheta, \theta, \omega\} \Rightarrow \{\lambda, \theta \wedge \omega\}$

$n_{10} = \{\underline{\lambda}, \theta, \omega\} \Rightarrow \{\underline{\lambda}, \theta \wedge \omega\}.$ este axiom

$G_6: n_{11} = \{\neg\vartheta, \theta, \omega\} \Rightarrow \{\lambda, \theta\}$

$n_{12} = \{\neg\vartheta, \theta, \omega\} \Rightarrow \{\lambda, \omega\}$ este axiom.

3.



Petculescu Mihai-Silviu

4. $S(\alpha) = \{q \vee \neg\theta \vee \neg\beta \vee a,\ \neg a \vee \beta \vee \theta \vee q,\ \neg q \vee a \vee \neg\beta,\ \theta \vee \neg q \vee \beta\}$

• Pentru $\lambda = \neg q$

$\alpha_\lambda^+ = \{\neg q \vee a \vee \neg\beta,\ \theta \vee \neg q \vee \beta\}$

$\alpha_\lambda^- = \{q \vee \neg\theta \vee \neg\beta \vee a,\ \neg a \vee \beta \vee \theta \vee q\}$

$\alpha_\lambda^0 = \emptyset$

$POS_\lambda(\alpha) = \{a \vee \neg\beta,\ \theta \vee \beta\}$

$NEG_\lambda(\alpha) = \{\neg\theta \vee \neg\beta \vee a,\ \neg a \vee \beta \vee \theta\}$

$REZ_\lambda(\alpha) = \{a \vee \neg\beta \vee \neg\theta \vee a,\ a \vee \neg\beta \vee \neg a \vee \beta \vee \theta,\ \theta \vee \beta \vee \neg\theta \vee \neg\beta \vee a,\ \theta \vee \beta \vee \neg a \vee \beta \vee \theta\}$

Eliminăm tautologiile la $REZ_\lambda(\alpha)$ și obținem.

$REZ_\lambda(\alpha) = \{a \vee \neg\beta \vee \neg\theta,\ \theta \vee \beta \vee \neg a\}$.

• Pentru $\lambda = a$

$\alpha_\lambda^+ = \{q \vee \neg\theta \vee \neg\beta \vee a,\ \neg q \vee a \vee \neg\beta\}$

$\alpha_\lambda^- = \{\neg a \vee \beta \vee \theta \vee q\}$

$\alpha_\lambda^0 = \{\theta \vee \neg q \vee \beta\}$

$POS_\lambda(\alpha) = \{\theta \vee \neg q \vee \beta,\ q \vee \neg\theta \vee \neg\beta,\ \neg q \vee \neg\beta\}$

$NEG_\lambda(\alpha) = \{\theta \vee \neg q \vee \beta,\ \beta \vee \theta \vee q\}$

$REZ_\lambda(\alpha) = \{\theta \vee \neg q \vee \beta,\ \beta \vee \theta \vee q \vee \neg\theta \vee \neg\beta,\ \beta \vee \theta \vee q \vee \neg q \vee \neg\beta\}$.

Eliminând tautologiile obținem. $REZ_\lambda(\alpha) = \{\theta \vee \neg q \vee \beta\}$.

• Algoritm bazat pe rezoluție.

Inițializare: $\gamma \leftarrow \{q \vee \neg\theta \vee \neg\beta \vee a,\ \neg a \vee \beta \vee \theta \vee q,\ \neg q \vee a \vee \neg\beta,\ \theta \vee \neg q \vee \beta\}$
sw $\leftarrow$ false

Iterația 1: Nu există clauze unitare și nici literali puri

alegem $\lambda = q$ literal

$\lambda \leftarrow REZ_q(\gamma) = \{a \vee \neg\beta \vee \neg\theta,\ \theta \vee \beta \vee \neg a\}$

(după eliminarea tautologiilor)

Iterația 2: Nu există clauze unitare și nici literali puri

alegem $\lambda = a$ literal.

$\lambda \leftarrow REZ_a(\gamma) = \{\neg\beta \vee \neg\theta \vee \beta \vee \theta\}$

Eliminăm tautologiile

$\lambda \leftarrow \emptyset$

Iterația 3: $\gamma = \emptyset \Rightarrow$ write "Validabilă", sw $\leftarrow$ true $\Rightarrow$ STOP.

• Davis – Putnam.

Inițializare: $\gamma \leftarrow \{q \vee \neg\theta \vee \neg\beta \vee a,\ \neg a \vee \beta \vee \theta \vee q,\ \neg q \vee a \vee \neg\beta,\ \theta \vee \neg q \vee \beta\}$
sw $\leftarrow$ false, $T \leftarrow \emptyset$

Iterația 1: Nu există literal pur sau clauză unitară

alegem $\lambda = q$ literal.

$\gamma \leftarrow NEG_\lambda(\gamma) = \{a \vee \neg\beta,\ \theta \vee \beta\}$

$T \leftarrow POS_\lambda(\gamma) = \{\neg\theta \vee \neg\beta \vee a,\ \neg a \vee \beta \vee \theta\}$.

Iterația 2: $\lambda = a$ literal pur; $\gamma \leftarrow NEG_a(\gamma) = \{\theta \vee \beta\}$

Iterația 3: $\lambda = \theta$ literal pur; $\gamma \leftarrow NEG_\theta(\gamma) = \emptyset$.

Iterația 4: $\gamma = \emptyset$; $\gamma \leftarrow T = \{\neg\theta \vee \neg\beta \vee a,\ \neg a \vee \beta \vee \theta\}$.

Iterația 5: Nu există literal pur sau clauză unitară.
alegem $\lambda = a$ literal.
$\gamma \leftarrow NEG_a(\gamma) = \{\neg a \vee \beta \vee \theta\}$
$T \leftarrow POS_a(\gamma) = \{\neg\theta \vee \neg\beta \vee a\}$.

Iterația 6: $\lambda = \beta$ literal pur
$\gamma \leftarrow NEG_\beta(\gamma) = \emptyset$

Iterația 7: $\gamma = \emptyset$, $\gamma \leftarrow T$; $T = \emptyset$.

Iterația 8: $\lambda = a$ literal pur
$\gamma \leftarrow NEG_a(\gamma) = \emptyset$

Iterația 9: $\gamma = \emptyset \Rightarrow$ write("Validabila"), sw $\leftarrow$ true

5. $\varphi = (d \vee \neg p) \rightarrow ((\neg d \vee q) \rightarrow (\neg p \vee q))$ $\boxed{5.}$

FNC = CNF.

T_2: $\neg(d \vee \neg p) \vee (\neg(\neg d \vee q) \vee (\neg p \vee q))$

T_3: $(\neg d \wedge p) \vee (d \wedge \neg q) \vee (\neg p \vee q)$

T_5 $(\neg d \wedge p) \vee [(d \vee \neg p \vee q) \wedge (\underbrace{\neg q \vee \neg p \vee q}_{T})]$.

T_5: $\cancel{(\neg d \wedge p) \vee (d \vee \neg p \vee q)}$

$\cancel{\neg d \vee d \vee}$

$[(\neg d \wedge p) \vee (d \vee \neg p \vee q)] \wedge [(\neg d \wedge p) \vee (\neg q \vee \neg p \vee q)]$.

$(\underline{\neg d \vee d} \vee \neg p \vee q) \wedge (\neg p \vee d \vee \neg p \vee q) \quad \wedge \quad (\neg d \vee \neg q \vee \neg p \vee q) \wedge (p \vee \neg q \vee \neg p \vee q)$

Fără tautologii: $(\neg p \vee d \vee q) := CNF$.

$I(\varphi) = I(d \vee \neg p) \rightarrow (I(\neg d \vee q) \rightarrow I(\neg p \vee q))$

$= \neg(I(d) \vee \neg I(p)) \vee (\neg(\neg I(d) \vee I(q)) \vee (\neg I(p) \vee I(q)))$

$= (\neg I(d) \wedge I(p)) \vee ((I(d) \wedge \neg I(q)) \vee (\neg I(p) \vee I(q)))$

$= (\dots) \vee [(I(d) \vee \neg I(p) \vee I(q)) \wedge (\underbrace{\neg I(q) \vee \neg I(p) \vee I(q)}_{T})]$

$= (\dots) \vee$ ✓ de continuat pe verso.

Petculescu Mihai-Eduard

CNF:

T_2: $\neg(\alpha \vee \neg\mu) \vee (\neg(\neg\alpha \vee \eta) \vee (\neg\mu \vee \eta))$

T_3: $(\neg\alpha \wedge \mu) \vee (\alpha \wedge \neg\eta) \vee (\neg\mu \vee \eta)$

T_5 $(\neg\alpha \wedge \mu) \vee ((\alpha \vee \neg\eta \vee \eta) \wedge (\neg\eta \vee \neg\mu \vee \eta))$

$\cancel{(\neg\alpha \wedge \mu) \vee (\alpha \vee \neg\mu \vee \eta)}$

$\cancel{(\neg\alpha \vee \alpha \vee \neg\mu \vee \eta) \wedge (\mu}$

$((\neg\alpha \wedge \mu) \vee (\alpha \vee \neg\mu \vee \eta)) \wedge ((\neg\alpha \wedge \mu) \vee (\neg\eta \vee \neg\mu \vee \eta))$

$\underline{\neg\mu \vee \alpha \vee \eta}$

$$I(\varphi) = I(\alpha \vee \neg\mu) \to (I(\neg\alpha \vee \eta) \to I(\neg\mu \vee \eta))$$

$$= \neg(I(\alpha) \vee \neg I(\mu)) \vee (\neg(\neg I(\alpha) \vee I(\eta)) \vee (\neg I(\mu) \vee I(\eta)))$$

$$= (\neg I(\alpha) \wedge I(\mu)) \vee ((I(\alpha) \vee \neg I(\mu) \vee I(\eta)) \wedge \underbrace{(\neg I(\eta) \vee \neg I(\mu) \vee I(\eta))}_{T})$$

$$= (\neg I(\alpha) \wedge I(\mu)) \vee I(\alpha) \vee \neg I(\mu) \vee I(\eta)$$

$$= (\underline{\neg I(\alpha) \vee I(\alpha)} \vee \neg I(\mu) \vee I(\eta)) \wedge$$
$$(\underline{I(\mu)} \vee I(\alpha) \vee \underline{\neg I(\mu)} \vee I(\eta))$$

$$= T \wedge T = T.$$