Laborator08

Ecuații

Să se rezolve următoarele ecuații diferențiale liniare de ordin n omogene cu coeficienți constanți.

I

$$
\begin{aligned}
&a)\ x^{III} - 3x^{II} + 2x^{I} = 0,\ x(0) = 1,\ x^{I}(0) = 2,\ x^{II}(0) = 0\\
&b)\ 2x^{III} - 3x^{II} + x^{I} = 0,\ x(0) = -1,\ x^{I}(0) = 2,\ x^{II}(0) = 1\\
&c)\ x^{III} - 7x^{II} + 14x^{I} - 8x = 0,\ x(0) = 1,\ x^{I}(0) = 0,\ x^{II}(0) = 1\\
&d)\ x^{II} - 4x^{I} + 3x = 0,\ x(0) = 2,\ x^{I}(0) = 4\\
&e)\ x^{III} - x^{I} = 0
\end{aligned}
$$

II

$$
\begin{aligned}
&a)\ x^{II} + 2x^{I} + x = 0,\ x(0) = 1,\ x^{I}(0) = 3\\
&b)\ x^{IV} - 5x^{II} + 4x = 0\\
&c)\ x^{III} - 6x^{II} + 12x^{I} - 8x = 0\\
&d)\ x^{IV} - 2x^{II} + x = 0\\
&e)\ x^{IV} + 2x^{III} + x^{II} = 0\\
&f)\ x^{(6)} - x^{(5)} - 4x^{(4)} + 2x^{III} + 5x^{II} - x^{I} - 2x = 0\\
&g)\ x^{(7)} + 3x^{(6)} + 3x^{(5)} + x^{(4)} = 0
\end{aligned}
$$

III

$$
\begin{aligned}
&a)\ x^{II} + x = 0,\ x(0) = 3,\ x^{I}(0) = 5\\
&b)\ x^{IV} + 4x = 0\\
&c)\ x^{IV} + 8x^{II} + 16x = 0\\
&d)\ x^{II} + 4x^{I} + 13x = 0\\
&e)\ x^{II} + 4x^{I} + 5x = 0
\end{aligned}
$$

IV

$$a)\ x^{IV} + 2x^{III} + 4x^{II} - 2x^{I} - 5x = 0$$
$$b)\ x^{III} - 3x^{II} + 9x^{I} + 13x = 0$$
$$c)\ x^{(7)} - x^{(6)} + x^{(5)} - x^{(4)} = 0$$
$$d)\ x^{III} - 5x^{II} + 17x^{I} - 13x = 0$$
$$e)\ x^{V} + 4x^{IV} + 3x^{III} - 6x^{I} - 2x = 0$$

Rezolvare

Exerciţiu 1. a)

I a) $x''' - 3x'' + 2x' = 0$, $x(0)=1$, $x'(0)=2$, $x''(0)=0$

$x = e^{rt}$, $x' = re^{rt}$, $x'' = r^2 e^{rt}$, $x''' = r^3 e^{rt}$

$\Rightarrow r^3 e^{rt} - 3r^2 e^{rt} + 2re^{rt} = 0 \quad | \cdot e^{rt}$

$r^3 - 3r^2 + 2r = 0$

$r(r^2 - 3r + 2) = 0$

$r_1 = 0$

$r^2 - 3r + 2 = 0$

$\Delta = 9 - 8 = 1 \Rightarrow r_{2,3} = \frac{3 \pm 1}{2}$ < $r_2 = 2$, $r_3 = 1$

$e^{0\cdot t}, e^{2t}, e^{1\cdot t}$ — sist. fundam. de sol.

$x(t) = c_1 + c_2 e^{2t} + c_3 e^{t}$

$x(0) = \boxed{c_1 + c_2 + c_3 = 1}$

$x'(t) = c_2 \cdot 2e^{2t} + c_3 e^{t}$

$x'(0) = \boxed{2c_2 + c_3 = 2}$

$x''(t) = 2c_2 \cdot 2e^{2t} + c_3 e^{t}$

$x''(0) = \boxed{4c_2 + c_3 = 0}$

$\begin{cases} c_1 + c_2 + c_3 = 1 \\ 2c_2 + c_3 = 2 \\ 4c_2 + c_3 = 0 \end{cases} \Rightarrow 2c_2 = -2 \Rightarrow \boxed{c_2 = -1}$

$\Rightarrow \boxed{c_3 = 4}$

$c_1 = 1 - c_2 - c_3 = 1 + 1 - 4 = \boxed{-2 = c_1}$

$x_{PC} = -2 - e^{2t} + 4e^{t}$

Exerciţiu 1. d) - Video

I. d) $x'' - 4x' + 3x = 0$, $x(0) = 2$, $x'(0) = 4$

$x = e^{rt}$, $x' = re^{rt}$, $x'' = r^2 e^{rt}$

$r^2 e^{rt} - 4re^{rt} + 3e^{rt} = 0 \quad | : e^{rt}$

$r^2 - 4r + 3 = 0$

$\Delta = 16 - 12 = 4 \Rightarrow r_{1,2} = \frac{4 \pm 2}{2}$ < 3, 1

$e^{3t}, e^{1\cdot t}$ — sistem fundam. de sol.

$x(t) = c_1 e^{3t} + c_2 e^{t}$

$x(0) = \boxed{c_1 + c_2 = 2}$

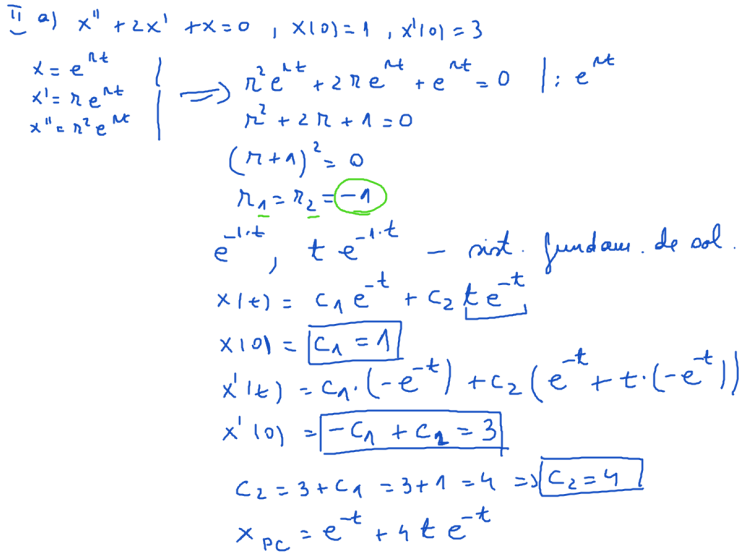
$x'(t) = c_1 \cdot 3e^{3t} + c_2 e^{t}$

$x'(0) = \boxed{3c_1 + c_2 = 4}$

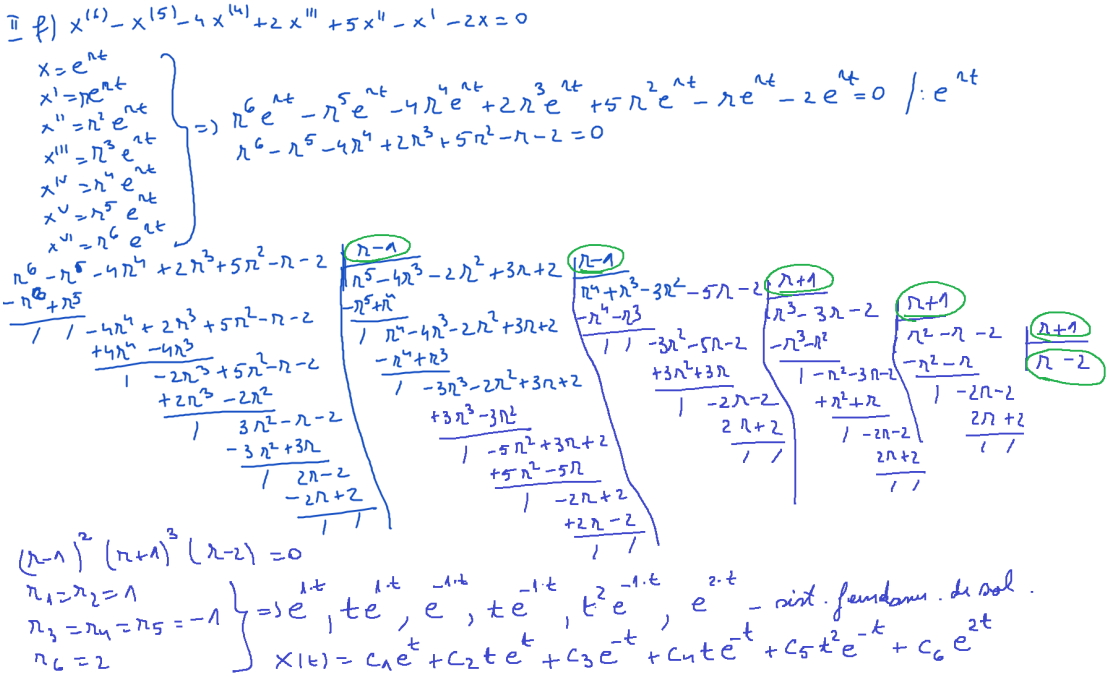
$\begin{cases} c_1 + c_2 = 2 \\ 3c_1 + c_2 = 4 \end{cases}$

$(-) \quad 2c_1 = 2 \Rightarrow \boxed{c_1 = 1} \Rightarrow \boxed{c_2 = 1} \Rightarrow x_{PC} = e^{3t} + e^{t}$

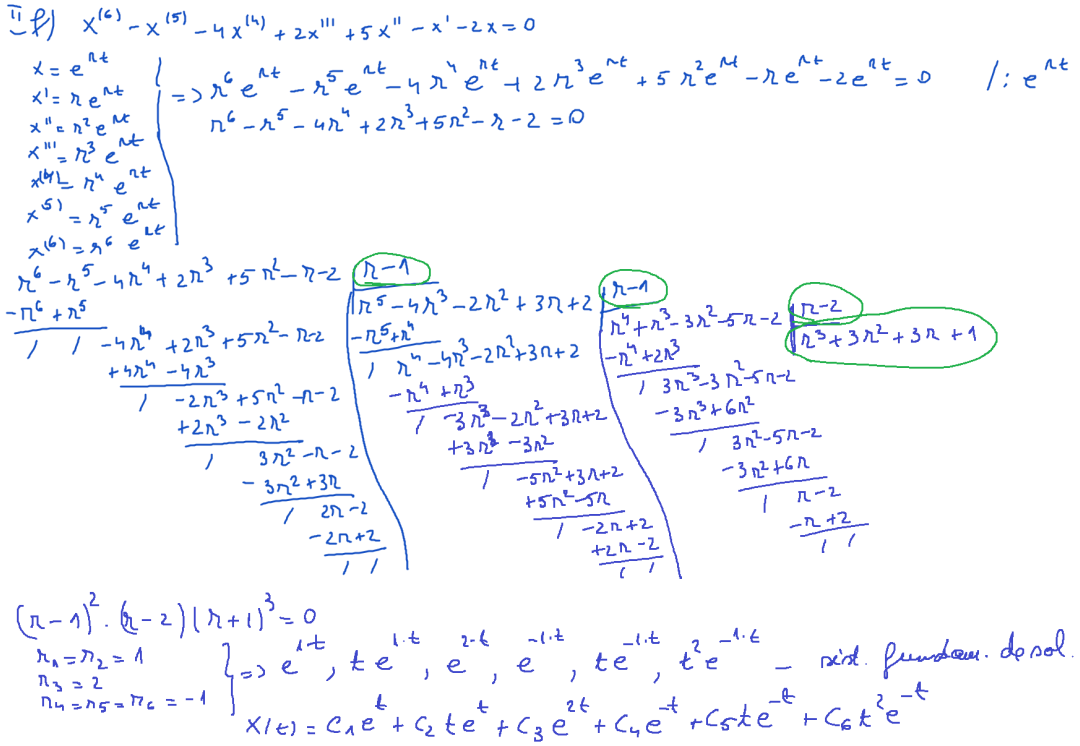
Exerciţiu 2. a) - Video



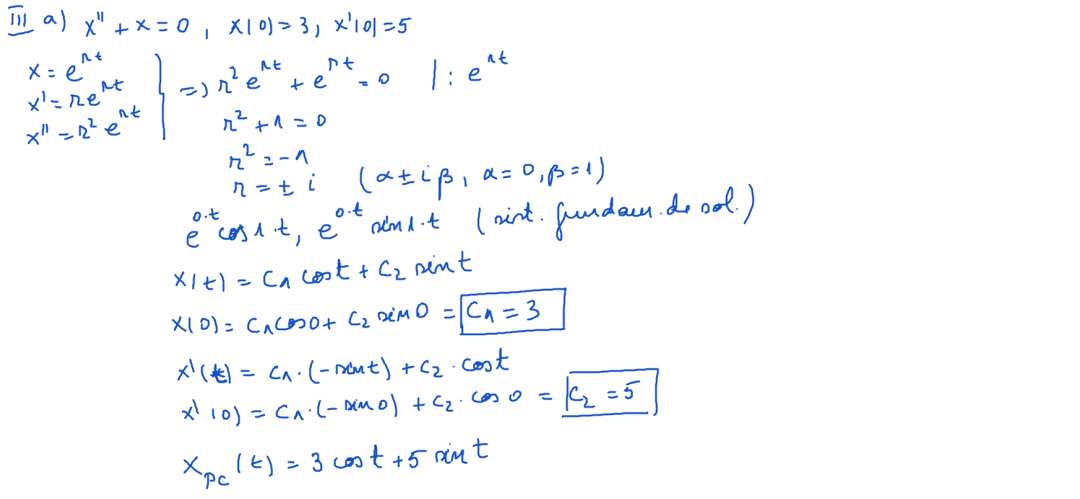
Exerciţiu 2. f)



Alfa - Descompunere



Exerciţiu 3. a)



Exerciţiu 3. c) - Video

III c) $x^{IV} + 8x'' + 16x = 0$

$x = e^{rt}$, $x' = re^{rt}$, $x'' = r^2e^{rt}$, $x''' = r^3e^{rt}$, $x^{IV} = r^4e^{rt}$

$\Rightarrow r^4e^{rt} + 8r^2e^{rt} + 16e^{rt} = 0 \quad | : e^{rt}$

$r^4 + 8r^2 + 16 = 0$

Not. $r^2 = p \Rightarrow p^2 + 8p + 16 = 0$

$(p+4)^2 = 0$

$\Rightarrow p = -4$ (de 2 ori)

$r^2 = -4$ (de 2 ori)

$r = \pm 2i$ (de 2 ori) $(\alpha \pm i\beta,\ \alpha = 0,\ \beta = 2)$

$e^{0\cdot t}\cos 2\cdot t,\ e^{0\cdot t}\sin 2\cdot t,\ te^{0\cdot t}\cos 2t,\ te^{0\cdot t}\sin 2t$ – sist. fundam. de sol.

$x(t) = C_1\cos 2t + C_2\sin 2t + C_3 t\cos 2t + C_4 t\sin 2t$

Exerciţiu 4. a) - Video

IV

$x = e^{rt}$, $x' = re^{rt}$, $x'' = r^2e^{rt}$, $x''' = r^3e^{rt}$, $x^{IV} = r^4e^{rt}$

$\Rightarrow r^4e^{rt} + 2r^3e^{rt} + 4r^2e^{rt} - 2re^{rt} - 5e^{rt} = 0 \quad | : e^{rt}$

$r^4 + 2r^3 + 4r^2 - 2r - 5 = 0$

$r^4 + 2r^3 + 4r^2 - 2r - 5 \ \big|\ r - 1$

$-r^4 + r^3$

$3r^3 + 4r^2 - 2r - 5$

$-3r^3 + 3r^2$

$7r^2 - 2r - 5$

$-7r^2 + 7r$

$5r - 5$

$-5r + 5$

0

$r^3 + 3r^2 + 7r + 5 \ \big|\ r + 1$

$-r^3 - r^2$

$2r^2 + 7r + 5$

$-2r^2 - 2r$

$5r + 5$

$-5r - 5$

0

$r^2 + 2r + 5$

$(r-1)(r+1)(r^2+2r+5) = 0$

$r_1 = 1$

$r_2 = -1$

$r^2 + 2r + 5 = 0$

$\Delta = 4 - 20 = -16$

$r_{1,2} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$

$(\alpha \pm i\beta,\ \alpha = -1,\ \beta = 2)$

$e^{1\cdot t},\ e^{-1\cdot t},\ e^{-1\cdot t}\cos 2\cdot t,\ e^{-1\cdot t}\sin 2\cdot t$

(sist. fundam. de sol)

$x(t) = C_1e^t + C_2e^{-t} + C_3e^{-t}\cos 2t + C_4e^{-t}\sin 2t$

Exerciţiu 4. c)

IV c) $x^{(7)} - x^{(6)} + x^{(5)} - x^{(4)} = 0$

$x = e^{rt}$, $x' = re^{rt}$, $x'' = r^2e^{rt}$, $x''' = r^3e^{rt}$, $x^{IV} = r^4e^{rt}$, $x^{(5)} = r^5e^{rt}$, $x^{(6)} = r^6e^{rt}$, $x^{(7)} = r^7e^{rt}$

$\Rightarrow r^7e^{rt} - r^6e^{rt} + r^5e^{rt} - r^4e^{rt} = 0 \quad | : e^{rt}$

$r^7 - r^6 + r^5 - r^4 = 0$

$r^4(r^3 - r^2 + r - 1) = 0$

$r^4[r^2(r-1) + (r-1)] = 0$

$r^4(r-1)(r^2+1) = 0$

$r_1 = r_2 = r_3 = r_4 = 0$

$r - 1 = 0 \Rightarrow r_5 = 1$

$r^2 + 1 = 0 \Rightarrow r^2 = -1 \Rightarrow r_{6,7} = \pm i \quad (\alpha \pm i\beta,\ \alpha = 0,\ \beta = 1)$

$e^{0\cdot t},\ te^{0\cdot t},\ t^2e^{0\cdot t},\ t^3e^{0\cdot t},\ e^{1\cdot t},\ e^{0\cdot t}\cos 1\cdot t,\ e^{0\cdot t}\sin 1\cdot t$ (sist. fundam. de sol.)

$x(t) = C_1 + C_2t + C_3t^2 + C_4t^3 + C_5e^t + C_6\cos t + C_7\sin t$