

# Laborator01

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## Exerciții

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1. Să se calculeze următoarele primitive:

$$1) \int (x^2 + 3x - 2) dx$$

$$2) \int x^2 \ln x dx$$

$$3) \int \frac{3x^2 + 4}{x^3 + 4x} dx$$

2. Să se determine primitiva  $F : \mathbb{R} \rightarrow \mathbb{R}$  a funcției  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = \frac{x}{x^2+1}$  cu proprietatea că  $F(0) = 3$ .

3. Să se determine primitivele următoarelor funcții, care îndeplinesc condițiile precizate:

$$a) \int \ln x dx, \quad F(1) = 3$$

$$b) \int \frac{5}{x^2 + 3x + 2} dx, \quad F(0) = 5$$

$$c) \int \sin^2 x dx, \quad F(0) = 1$$

$$d) \int \frac{3}{x^2 + 4x + 5} dx, \quad F(-2) = 2$$

## Rezolvare

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### Exercitiul 1

①

$$1) \int (x^2 + 3x - 2) dx = \int x^2 dx + 3 \int x dx - 2 \int dx = \frac{x^3}{3} + 3 \cdot \frac{x^2}{2} - 2x + C$$

$$2) \int \frac{x^3 \ln x}{x^4} dx = \frac{x^3}{3} \ln x - \int \frac{1}{3} \cdot \frac{x^3}{3} dx = \frac{x^3}{3} \ln x - \frac{1}{9} \int x^3 dx$$

$$f' = \frac{1}{x}, g = \frac{x^3}{3}$$

$$= \frac{x^3}{3} \ln x - \frac{1}{9} \cdot \frac{x^4}{4} + C$$

$$= \frac{x^3}{3} \ln x - \frac{x^4}{36} + C$$

$$3) \int \frac{3x^2 + 4}{x^3 + 4x} dx = \int \frac{u'}{u} dx = \ln|u| = \ln|x^3 + 4x| + C$$

$$u(x) = x^3 + 4x$$

$$u'(x) = 3x^2 + 4$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int c \cdot f dx = c \int f dx$$

$$\int (f+g) dx = \int f dx + \int g dx$$

$$\int f \cdot g' dx = f \cdot g - \int f' \cdot g dx$$

## Exercitiul 2

②

$$\int f(x) dx = \underbrace{F(x)}_{f(x)} + C, F' = f$$

$$F(0) = 3$$

$$\frac{1}{2} \int \frac{2x}{x^2 + 1} dx = \frac{1}{2} \int \frac{u'}{u} dx = \frac{1}{2} \ln|u| = \frac{1}{2} \ln(x^2 + 1) + C$$

$$u(x) = x^2 + 1$$

$$u' = 2x$$

$$F(0) = \frac{1}{2} \ln(0+1) + C = \frac{1}{2} \ln 1 + C = \boxed{C=3}$$

$$F(x) = \frac{1}{2} \ln(x^2 + 1) + 3$$

## Exercitiul 3

3

$$a) \int \underbrace{1}_{g'} \cdot \underbrace{\ln x}_f dx = x \ln x - \int \frac{1}{x} \cdot x dx = x \ln x - \int dx = \underbrace{x \ln x - x + C}_{F(x)}$$

$$f' = \frac{1}{x}, g = x$$

$$F(1) = 1 \cdot \ln 1 - 1 + C = -1 + C = 3 \Rightarrow \boxed{C=4} \Rightarrow F(x) = x \ln x - x + 4$$

$$b) \int \frac{5}{x^2+3x+2} dx = 5 \int \frac{1}{x^2+3x+2} dx$$

$$ax^2+bx+c=0, x_1, x_2 \text{ rad} \Rightarrow ax^2+bx+c=a(x-x_1)(x-x_2) \quad !$$

$$x^2+3x+2=0 \\ \Delta = 9-8=1 \Rightarrow x_{1,2} = \frac{-3 \pm 1}{2} \begin{pmatrix} -1 \\ -2 \end{pmatrix} \Rightarrow x^2+3x+2 = (x+1)(x+2)$$

$$\frac{1}{x^2+3x+2} = \frac{\frac{x+2}{x+2} A}{x+1} + \frac{\frac{x+1}{x+2} B}{x+2} = \frac{Ax+2A+Bx+B}{(x+1)(x+2)} = \frac{(A+B)x+2A+B}{(x+1)(x+2)}$$

$$\begin{cases} A+B=0 \\ 2A+B=1 \end{cases} \xrightarrow{(-)} \begin{matrix} A=1 \\ B=-1 \end{matrix}$$

$$5 \int \left( \frac{1}{x+1} - \frac{1}{x+2} \right) dx = \underbrace{5 (\ln|x+1| - \ln|x+2|)}_{F(x)} + C$$

$$F(0) = 5$$

$$F(0) = 5 (\ln 1 - \ln 2) + C = -5 \ln 2 + C = 5 \Rightarrow \boxed{C = 5 + 5 \ln 2}$$

$$\Rightarrow F(x) = 5 (\ln|x+1| - \ln|x+2|) + 5 + 5 \ln 2 \\ = 5 (\ln|x+1| - \ln|x+2| + 1 + \ln 2)$$

d)

$$\int \frac{3}{x^2+4x+5} dx = 3 \int \frac{1}{x^2+4x+5} dx$$

$$x^2+4x+5=0$$

$$\Delta = 16 - 20 = -4 < 0 \Rightarrow \text{ec. nu are sol. reale} \Rightarrow x^2+4x+5 = x^2+4x+4+1 = (x+2)^2+1$$

$$3 \int \frac{1}{(x+2)^2+1} dx = 3 \underbrace{\arctg(x+2)}_{F(x)} + C$$

$$\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \arctg \frac{x}{a} + C$$

$$F(-2) = 3 \underbrace{\arctg 0}_0 + C = \boxed{C=2} \Rightarrow F(x) = 3 \arctg(x+2) + 2$$