Laborator 07 - TEMĂ Petrulescu Mihai-Silviu.

1. Să se rezolve următoarele ecuații de ordin n:

a) $x^{IV} = t + 2,\ x(0) = 1,\ x'(0) = 2,\ x''(0) = -1,\ x'''(0) = 0$

$x^{IV} = f(t)$

• $x''' = \int x^{IV} dt = \int t\,dt + 2\int dt = \frac{t^2}{2} + 2t + C_1$

• $x'' = \int x''' dt = \frac{1}{2}\int t^2 dt + 2\int t\,dt + C_1\int dt = \frac{t^3}{6} + t^2 + C_1 t + C_2$

• $x' = \int x'' dt = \int \frac{t^3}{6} dt + \int t^2 dt + \int C_1 t\,dt + C_2\int dt$

$= \frac{1}{6}\cdot\frac{t^4}{4} + \frac{t^3}{3} + C_1\frac{t^2}{2} + C_2 t + C_3 = \frac{t^4}{24} + \frac{t^3}{3} + C_1\frac{t^2}{2} + C_2 t + C_3$

• $x = \int x' dt = \frac{1}{24}\int t^4 dt + \frac{1}{3}\int t^3 dt + \frac{C_1}{2}\int t^2 dt + C_2\int t\,dt + C_3\int dt$

$= \frac{1}{24}\cdot\frac{t^5}{5} + \frac{1}{3}\cdot\frac{t^4}{4} + \frac{C_1}{2}\cdot\frac{t^3}{3} + C_2\cdot\frac{t^2}{2} + C_3 t + C_4$

$$\boxed{x = \frac{t^5}{120} + \frac{t^4}{12} + C_1\frac{t^3}{6} + C_2\frac{t^2}{2} + C_3 t + C_4}$$

• $x(0) = \boxed{C_4 = 1}$

• $x'(0) = \boxed{C_3 = 2}$

• $x''(0) = \boxed{C_2 = -1}$

• $x'''(0) = \boxed{C_1 = 0}$

$\Rightarrow \boxed{x_{PC} = \frac{t^5}{120} + \frac{t^4}{12} - \frac{t^2}{2} + 2t + 1}$

c) $x''' = \sin t + \cos t,\ x(0) = 1,\ x'(0) = 2,\ x''(0) = 3.$

$x''' = f(t)$

• $x'' = \int x''' dt = \int \sin t\,dt + \int \cos t\,dt = -\cos t + \sin t + C_1$

• $x' = \int x'' dt = -\int \cos t\,dt + \int \sin t\,dt + C_1\int dt$

$= -\sin t - \cos t + C_1 t + C_2$

• $x = \int x' dt = -\int \sin t\,dt - \int \cos t\,dt + C_1\int t\,dt + C_2\int dt$

$$\boxed{x = \cos t - \sin t + C_1\frac{t^2}{2} + C_2 t + C_3}$$

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Să se rezolve următoarele ecuații de ordin n:

b) $x'' + x \cdot x' = 0$

$F(x, x', x'') = 0$

- $\boxed{y = \frac{dx}{dt}} = x' \Leftrightarrow x'' = y' = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = y \cdot \frac{dy}{dx}$

- $y \cdot \frac{dy}{dx} + x \cdot y = 0 \Leftrightarrow y \cdot \frac{dy}{dx} = -xy \mid : y \Leftrightarrow \frac{dy}{dx} = -x$

- $dy = -x\,dx \Leftrightarrow \int dy = -\int x\,dx \Leftrightarrow \underline{y = -\frac{x^2}{2} + C}$

- $y = \frac{dx}{dt} \Leftrightarrow -\frac{x^2}{2} + C = \frac{dx}{dt} \Leftrightarrow \frac{dx}{-\frac{x^2}{2} + C} = dt \Leftrightarrow \int \frac{1}{\frac{-x^2}{2} + C}\,dx = \int dt \Leftrightarrow$

$\Leftrightarrow t + C_1 = \frac{1}{\sqrt{C}} \cdot \sqrt{2}\, \operatorname{arctg}\left(\frac{x}{\sqrt{C}\sqrt{2}}\right) + C_2$

$$\boxed{t + C_1 = \int \frac{1}{\frac{-x^2}{2} + C}\,dx}$$

a) $x \cdot x''' + 3x'x'' = 0$

$F(x, x', x'', x''') = 0$

- $\boxed{y = \frac{dx}{dt}} = x' \Leftrightarrow \begin{cases} x'' = y' = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = y \cdot \frac{dy}{dx} \\ x''' = y'' = \frac{dy}{d^2t} = \frac{dy}{d^2x} \cdot \frac{d^2x}{d^2t} = y^2 \frac{dy}{d^2x} \end{cases}$

- $x \cdot y^2 \frac{dy}{d^2x} + 3 \cdot y \cdot y \frac{dy}{dx} = 0 \mid : y^2 \frac{dy}{dx}$

$x \cdot \frac{1}{dx} + 3 = 0 \Leftrightarrow \int \frac{1}{x}\,dx = \frac{-1}{3} \Leftrightarrow \underline{\ln|x| + C = \frac{-1}{3}} \Leftrightarrow x = e^{\frac{-1}{3} + C}$

$\Leftrightarrow x = C e^{\frac{-1}{3}}$

- $X(0) = 1 + C_3 = 1 \Rightarrow \boxed{C_3 = 0}$
- $X'(0) = -1 + C_2 = 2 \Rightarrow \boxed{C_2 = 3}$
- $X''(0) = -1 + C_1 = 3 \Rightarrow \boxed{C_1 = 4}$

$\Rightarrow \boxed{x_{PC} = \cos t - \sin t + 2t^2 + 3t}$

d) $x'' = \frac{1}{t}$, $x(1) = 1$, $x'(1) = 2$

$x'' = f(t)$

- $x' = \int x'' \, dt = \int \frac{1}{t} \, dt = \ln|t| + C_1$
- $x = \int x' \, dt = \int \ln|t| \, dt + C_1 \int dt = \int 1 \cdot \ln|t| \, dt + C_1 \int dt$

$g = t \quad f' = \frac{1}{t}$

$= t\ln(|t|) - \int 1 \, dt + C_1 \int dt = t\ln|t| - t + C_1 t + C_2$

$$\boxed{x = t\ln|t| - t + C_1 t + C_2}$$

- $x'(1) = \boxed{C_1 = 2}$
- $x(1) = -1 + 2 + C_2 = 1 \Leftrightarrow \boxed{C_2 = 0}$

$\Rightarrow \boxed{x_{PC} = t\ln|t| + t}$

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2. Să se rezolve următoarele ecuații de ordin n:

a) $e^{x''} - (x'')^2 = t + 1$.

$F(t, x'') = 0$

• Facem schimbarea de funcție $x'' = y$.

$e^y - y^2 = t + 1 \Leftrightarrow e^y - y^2 - 1 = t = \varphi(y) \Leftrightarrow dt = (e^y - 2y)\,dy$

• $x' = \int x''\,dt = \int y(e^y - 2y)\,dy = \int (y e^y)\,dy - \int (2y^2)\,dy = y e^y - e^y - 2\frac{y^3}{3} + C_1 = e^y(y-1) - \frac{2y^3}{3} + C_1$

• $x = \int x'\,dt = \int \left(e^y(y-1) - \frac{2y^3}{3} + C_1\right)(e^y - 2y)\,dy$

$= (e^y)^2(y-1) - 2y e^y(y-1) - \frac{2y^3}{3}\cdot e^y + 2y\cdot\frac{2y^3}{3} + C_1 e^y - C_1\cdot 2y$

$$\boxed{x = \frac{(2y-3)e^{2y}}{4} - (2y^2 - 6y + 6)e^y - \frac{(2y^3 - 6y^2 + 12y - 12)e^y}{3} + \frac{4y^5}{15} + C_1 e^y + C_1 y^2 + C_2, \quad t = e^y - y^2 - 1}$$

c) $x'' + \ln x'' = t - 5$

$F(t, x'') = 0$

• Facem schimbarea de funcție $x'' = y$

$y + \ln y = t - 5 \Leftrightarrow y + \ln y + 5 = t = \varphi(y) \Leftrightarrow dt = \left(1 + \frac{1}{y}\right)dy$

• $x' = \int x''\,dt = \int y\left(1 + \frac{1}{y}\right)dy = \int (y+1)\,dy = \frac{y^2}{2} + y + C_1$

• $x = \int x'\,dt = \int \left(\frac{y^2}{2} + y + C_1\right)\left(1 + \frac{1}{y}\right)dy$

$= \int \left(\frac{y^2}{2} + \frac{y}{2} + y + 1 + C_1 + C_1\frac{1}{y}\right)dy$

$= \frac{1}{2}\cdot\frac{y^3}{3} + \frac{1}{2}\cdot\frac{y^2}{2} + \frac{y^2}{2} + y + C_1 y + C_1\cdot\ln|y| + C_2$

$$\boxed{x = \frac{y^3}{6} + \frac{y^2}{4} + \frac{y^2}{2} + y + C_1 y + C_1\ln|y| + C_1, \quad t = y + \ln y + 5}$$

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3. Să se rezolve următoarele ecuații de ordin n:

b) $x'' + x' tg\, t = \sin 2t$

$F(t, x', x'') = 0$;

$\boxed{x' = y} \Leftrightarrow x'' = y' \Leftrightarrow y' + y \underbrace{tg\, t}_{A(t)} = \underbrace{\sin 2t}_{B(t)}$ (ec. dif. afină)

Etapa 1: $y' + y\, tg\, t = 0$

$\frac{dy}{dt} = -y\, tg\, t \Leftrightarrow \frac{1}{y} dy = (-tg\, t) dt \Leftrightarrow \int \frac{1}{y} dy = -\int tg\, t\, dt$

$\Leftrightarrow \ln|y| = \ln|\cos t| + C \Rightarrow \boxed{y_0 = C \cdot \cos t}$

Etapa 2: $\boxed{\varphi_0 = C(t) \cdot \cos t}$

$(C(t) \cdot \cos t)' + (C(t) \cdot \cos t)\, tg\, t = \sin 2t$

$C'(t) \cos t + \cancel{C(t) \cdot (\cos t)'} + \cancel{C(t) \cdot \cos t \cdot \frac{\sin t}{\cos t}} = \sin 2t$

$C'(t) \cos t = \sin 2t$

$C'(t) = \frac{\sin 2t}{\cos t} \Leftrightarrow C(t) = \int \frac{\sin 2t}{\cos t} dt \Leftrightarrow C(t) = -2\cos t + C_1$;

- $\varphi_0 = (-2\cos t + C_1) \cdot \cos t$
- $y_0 = C \cos t$

$\Rightarrow y = \varphi_0 + y_0 = -2\cos^2 t + C \cos t$

- $x' = y = 2\cos^2 t + C \cos t$
- $x = \int (2\cos^2 t + C \cos t) dt \Leftrightarrow \boxed{x = (\cos t + C) \sin t + t + C_1}$

c) $t^2 x'' + 2(x')^2 = 0$, $x(1) = 2$, $x'(1) = 3$

$F(t, x', x'') = 0$;

$\boxed{x' = y} \Leftrightarrow x'' = y' \Leftrightarrow t^2 y' + 2y^2 = 0 \mid t^2$

$y' + \frac{2}{t^2} y^2 = 0$

(ec. cu variabile separabile)

$y' = \frac{-2}{t^2} y^2 \Leftrightarrow \frac{dy}{dt} = \frac{-2}{t^2} y^2 \Leftrightarrow \frac{1}{y^2} dy = \frac{-2}{t^2} dt \Leftrightarrow \int \frac{1}{y^2} dy = \int \frac{-2}{t^2} dt \Leftrightarrow$

$\Leftrightarrow \frac{-1}{y} = \frac{2}{t} + C \Leftrightarrow -1 = \frac{2}{t} y + C \Leftrightarrow \boxed{y = \frac{(-1-C)t}{2}}$

- $x' = y = \frac{(-1-C)t}{2}$
- $x = \int x' dt = \int \left(\frac{(-1-C)t}{2} \right) dt = \frac{(-1-C)t^2}{4} + C_1$

- $x'(1) = \frac{-1-C}{2} = 3 \Leftrightarrow \boxed{C = -7}$
- $x(1) = \frac{6}{4} + C_1 = 2 \Leftrightarrow C_1 = 2 - \frac{6}{4} = \frac{1}{2}$

$x_{PC} = \frac{(-1+7)t^2}{4} + \frac{1}{2} = \frac{6t^2}{4} + \frac{1}{2} = \frac{3t^2+1}{2}$

d) $x''' - x'' = t, \quad x(1) = 1, x'(1) = -1, x''(1) = 2$

$F(t, x'', x''') = 0;$

$\boxed{x'' = y} \Leftrightarrow x''' = y' \Leftrightarrow y' - y = t$ (Ecuație de tip afină)

Etapa 1 $\quad y' - y = 0 \Leftrightarrow \frac{dy}{dt} = y \Leftrightarrow \frac{1}{y} dy = dt \Leftrightarrow \int \frac{1}{y} dy = \int dt \Leftrightarrow$

$\Leftrightarrow \ln|y| = t + C \Leftrightarrow \boxed{y_o = C e^t}$

Etapa 2: $\boxed{\varphi_0 = C(t) e^t}$

$C'(t) e^t + \cancel{C(t) e^t} - \cancel{C(t) e^t} = t \Leftrightarrow C'(t) e^t = t \Leftrightarrow C'(t) = \frac{t}{e^t} \Leftrightarrow$

$\Leftrightarrow C(t) = \int t e^{-t} dt = -(t+1) e^{-t} + C_1$

$\varphi_0 = (-(t+1) e^{-t} + C_1) e^t = -(t+1) + C_1 e^t$

$y = \varphi_0 + y_o = -(t+1) + C_1 e^t + C e^t = -(t+1) + C e^t$

$x'' = y = -(t+1) + C e^t$

$x' = \int x'' dt = \int (-(t+1) + C e^t) dt = C e^t - \frac{t^2}{2} - t + C_1$

$x = \int x' dt = \int (C e^t - \frac{t^2}{2} - t + C_1) dt = C e^t - \frac{t^3}{6} - \frac{t^2}{2} + C_1 t + C_2$

$\bullet\ x''(1) = -2 + Ce = 2 \Rightarrow \boxed{C = \frac{4}{e}}$

$\bullet\ x'(1) = \frac{4}{e}\cdot e - \frac{1}{2} - 1 + C_1 = -1 \Leftrightarrow \boxed{C_1 = -4 + \frac{1}{2} = \frac{-7}{2}}$

$\bullet\ x(1) = \frac{4}{e}\cdot e - \frac{1}{6} - \frac{1}{2} + \frac{-7}{2} + C_2 = 1 \Leftrightarrow \boxed{C_2 = \frac{7}{6}}$

$\Rightarrow x_{PC} = 4e^{t-1} - \frac{t^3}{6} - \frac{t^2}{2} - \frac{7}{2}t + \frac{7}{6}$

f) $t x''' + x'' = 1 + t$

$F(t, x'', x''') = 0$

$\boxed{x'' = y} \Leftrightarrow x''' = y' \Leftrightarrow t y' + y = 1 + t \,|:t \Leftrightarrow y' + \frac{1}{t}y = 1 + \frac{1}{t}$

(ec. de tip afină)

<u>Etapa 1</u> $y' + \frac{1}{t}y = 0 \Leftrightarrow y' = \frac{-1}{t}y \Leftrightarrow \frac{dy}{dt} = \frac{-1}{t}y \Leftrightarrow \frac{1}{y}dy = \frac{-1}{t}dt \Leftrightarrow$

$\Leftrightarrow \int \frac{1}{y}dy = \int \frac{-1}{t}dt \Leftrightarrow \ln|y| = -\ln|t| + C \Leftrightarrow \ln|y| = \ln|C\cdot t^{-1}|$

$\boxed{y_0 = Ct^{-1}}$

<u>Etapa 2</u>: $\boxed{\varphi_0 = C(t)t^{-1}}$

$C'(t)t^{-1} + \cancel{C(t)\cdot\frac{-1}{t^2}} + \cancel{\frac{1}{t}\cdot C(t)\cdot\frac{1}{t}} = 1 + \frac{1}{t}$

$C'(t) = t + 1 \Leftrightarrow C(t) = \int(t+1)dt = \frac{t^2}{2} + t + C_1$

$\varphi_0 = \left(\frac{t^2}{2} + t + C_1\right)t^{-1}$

$y = \varphi_0 + y_0 = \left(\frac{t^2}{2} + t + C_1\right)t^{-1} + Ct^{-1} = \frac{t}{2} + 1 + Ct^{-1}$

$\bullet\ x'' = y = \frac{t}{2} + 1 + Ct^{-1}$

$\bullet\ x' = \int x''dt = C\ln|t| + \frac{t^2}{4} + t + C_1$

$\bullet\ x = \int x'dt = Ct\ln|t| + \frac{t^3}{12} + \frac{t^2}{2} + C_1t + C_2$

g) $(1+t^2)x'' - 2tx' = 0$, $x(0) = 0$, $x'(0) = 3$

$F(t, x', x'') = 0$

$\boxed{x' = y} \Leftrightarrow x'' = y' \Leftrightarrow (1+t^2)y' - 2ty = 0$. (ec. cu var. separabile)

$(1+t^2)y' = 2ty \Leftrightarrow \frac{dy}{dt}(1+t^2) = 2ty \Leftrightarrow \frac{1}{y}dy = \frac{2t}{1+t^2}dt \Leftrightarrow$

$\Leftrightarrow \int \frac{1}{y}dy = \int \frac{2t}{1+t^2}dt \Leftrightarrow \ln|y| = \ln(t^2+1) + C \Rightarrow y = C(t^2+1)$

- $x' = y = C(t^2+1)$
- $x = \int x' dt = \int (C(t^2+1))dt = C(\frac{t^3}{3}+t) + C_1$

- $x'(0) = C(0+1) = 3 \Leftrightarrow \boxed{C=3}$
- $x(0) = 3(\frac{0}{3}+0) + C_1 \Leftrightarrow \boxed{C_1 = 0}$

$\Rightarrow x_{PC} = t^3 + 3t$

h) $x^{(5)} + x^{(4)} = 0$

$F(t, x^{(4)}, x^{(5)}) = 0$

$\boxed{x^{(4)} = y} \Leftrightarrow x^{(5)} = y' \Leftrightarrow y' + y = 0.$ (ec. cu var. separabile)

$y' = -y \Leftrightarrow \frac{dy}{dt} = -y \Leftrightarrow \frac{1}{y}dy = -1dt \Leftrightarrow \int \frac{1}{y}dy = \int -1dt \Leftrightarrow$

$\Leftrightarrow \ln|y| = -t + C \Leftrightarrow y = e^{(-t+C)} = Ce^{-t} \Leftrightarrow \boxed{y = Ce^{-t}}$

- $x^{(4)} = y = Ce^{-t}$
- $x^{(3)} = \int x^{(4)}dt = \int Ce^{-t}dt = -Ce^{-t} + C_1$
- $x^{(2)} = \int x^{(3)}dt = \int(-Ce^{-t} + C_1)dt = Ce^{-t} + C_1 t + C_2$
- $x^{(1)} = \int x^{(2)}dt = \int(Ce^{-t} + C_1 t + C_2)dt = -Ce^{-t} + C_1\frac{t^2}{2} + C_2 t + C_3$
- $x = \int x^{(1)}dt = \int(-Ce^{-t} + C_1\frac{t^2}{2} + C_2 t + C_3) =$

$$x = Ce^{-t} + C_1\frac{t^3}{6} + C_2\frac{t^2}{2} + C_3 t + C_4$$

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4. Să se rezolve următoarele ecuații de ordin n:

a) $t^2 \cdot x \cdot x'' = (x - tx')^2$

$F(t, x, x', x'') = 0$

$F(t, mx, mx', mx'') = t^2 mx\, mx'' - (mx - tmx')^2$

$= m^2(t^2 x x'') - m^2(x - tx')^2$

$= m^2(t^2 x x'' - (x - tx')) = m^2 \cdot F(t, x, x', x'')$

• $\boxed{x' = xy} \Rightarrow x'' = x'y + xy' = xy^2 + xy'$

$t^2 x(xy^2 + xy') - (x - txy)^2 = 0$

$t^2x^2y^2 + t^2x^2y' - (x^2 - 2tx^2y + t^2x^2y^2) = 0$

$\cancel{t^2x^2y^2} + t^2x^2y' - x^2 + 2tx^2y - \cancel{t^2x^2y^2} = 0 \quad |: x^2$

$t^2y' - 1 + 2ty = 0 \;|: t^2 \Leftrightarrow y' - \frac{1}{t^2} + \frac{2y}{t} = 0 \Leftrightarrow y' + \frac{2}{t}y = \frac{1}{t^2}$ (ec. afină)

Etapa 1: $y' = \frac{-2}{t}y \Leftrightarrow \frac{dy}{dt} = \frac{-2}{t}y \Leftrightarrow \frac{1}{y}dy = \frac{-2}{t}dt \Leftrightarrow \int \frac{1}{y}dy = \int \frac{-2}{t}dt \Leftrightarrow$

$\Leftrightarrow \ln|y| = -2\ln|t| + c \Rightarrow \boxed{y_0 = ct^{-2}}$

Etapa 2: $\varphi_0 = c(t)t^{-2}$

$c'(t)t^{-2} + \cancel{c(t)(-2t^{-3})} + \cancel{\frac{2}{t}ct^{-2}} = \frac{1}{t^2}$

$c'(t)\cancel{t^{-2}} = \cancel{\frac{1}{t^2}} \Leftrightarrow c'(t) = 1 \Leftrightarrow c(t) = \int 1\,dt = t + c_1$

$\varphi_0 = (t + c_1)t^{-2} = t^{-1} + c_1t^{-2}$

$y = y_0 + \varphi_0 = ct^{-2} + t^{-1} + c_1t^{-2} = t^{-1} + ct^{-2}$

• Dar $y = \frac{x'}{x} \Leftrightarrow t^{-1} + ct^{-2} = \frac{x'}{x} \Leftrightarrow \frac{dx}{x} = (t^{-1} + ct^{-2})dt \Leftrightarrow$

$\Leftrightarrow \int \frac{dx}{x} = \int (t^{-1} + ct^{-2})dt \Leftrightarrow \ln|x| = \ln|t| - \frac{c}{t} + c_1 \Rightarrow x = c_1 t \cdot e^{\frac{-c}{t}}$

$\boxed{x = c_1 t e^{\frac{-c}{t}}}$

b) $txx'' + t(x')^2 - xx' = 0$

$F(t,x,x',x'') = 0$

$F(t, mx, mx', mx'') = tmx\,mx'' + tm^2(x')^2 - mx\,mx'$

$= tm^2xx'' + tm^2(x')^2 - m^2xx'$

$= m^2(txx'' + t(x')^2 - xx') = m^2 \cdot F(t,x,x',x'')$

$\boxed{x' = xy} \Rightarrow x'' = x'y + xy' = xy^2 + xy'$

$tx(xy^2 + xy') + t(xy)^2 - x(xy) = 0$

$tx^2y^2 + tx^2y' + tx^2y^2 - x^2y = 0 \mid : x^2$

$ty^2 + ty' + ty^2 - y = 0 \Leftrightarrow 2ty^2 + ty' - y = 0 \mid : t \Leftrightarrow y' + 2y^2 - \frac{1}{t}y = 0$

$\cdot\ y' = -2y^2 + \frac{1}{t}y$ (ec. de tip Riccati)

$A(t) = -2,\ B(t) = \frac{1}{t},\ f(t) = 0$

• Verificăm $\frac{1}{t}$: $\frac{-1}{t^2} = -2 \cdot \frac{1}{t^2} + \frac{1}{t} \cdot \frac{1}{t} \Leftrightarrow \frac{-1}{t^2} = \frac{-2}{t^2} + \frac{1}{t^2}$ (A)

$\boxed{y = z + \frac{1}{t}} \Leftrightarrow y' = z' - \frac{1}{t^2}$

$z' - \frac{1}{t^2} = -2\left(z + \frac{1}{t}\right)^2 + \frac{1}{t}\left(z + \frac{1}{t}\right) \Leftrightarrow z' - \frac{1}{t^2} = -2\left(z^2 + z\frac{2}{t} + \frac{1}{t^2}\right) + \frac{z}{t} + \frac{1}{t^2}$

$z' - \frac{1}{t^2} = -2z^2 - 4\frac{z}{t} - \frac{2}{t^2} + \frac{z}{t} + \frac{1}{t^2}$

$z' = \underbrace{-2}_{A(t)}z^2 \underbrace{-3}_{B(t)}\frac{z}{t}$ (ec. de tip Bernoulli cu $\alpha = 2$)

$\frac{z'}{z^2} = -2 - 3(zt)^{-1} \Leftrightarrow z' \cdot z^{-2} = -2 - 3 \cdot z^{-1} \cdot t^{-1}$

$\boxed{u = z^{-1}} \Rightarrow u' = -z^{-2} \cdot z' \Rightarrow u' = 2 + 3ut^{-1}$ (ec. de tip afină)

Etapa 1: $u' = 3ut^{-1} \Leftrightarrow \frac{du}{dt} = \frac{3ut^{-1}}{1} \Leftrightarrow \frac{1}{u}du = 3t^{-1}dt \Leftrightarrow \int \frac{1}{u}du = \int 3t^{-1}dt \Leftrightarrow$

$\ln|u| = 3\ln|t| + C \Rightarrow \boxed{u_0 = Ct^3}$

Etapa 2: $\varphi_0 = C(t)t^3$

$$c'(t)t^3 - \cancel{c(t)3t^2} - \cancel{3\frac{1}{t}\cdot c(t)t^3} = 2$$

$$c'(t)t^3 = 2 \Leftrightarrow c'(t) = \frac{2}{t^3} \Leftrightarrow c(t) = \int \frac{2}{t^3}dt \Leftrightarrow c(t) = \frac{-1}{t^2} + C_1$$

$$y_0 = \left(\frac{-1}{t^2} + C_1\right)t^3 = -t + C_1 t^3$$

$$u = u_0 + y_0 = -t + C_1 t^3 + Ct^3 = -t + Ct^3$$

$$\underline{u = -t + Ct^3}$$

- $z = \frac{1}{u} \Leftrightarrow z = \frac{1}{-t+Ct^3}$
- $y = z + \frac{1}{t} \Leftrightarrow y = \frac{1}{-t+Ct^3} + \frac{1}{t}$
- $y = \frac{x'}{x} \Leftrightarrow \frac{x'}{x} = \frac{1}{-t+Ct^3} + \frac{1}{t} \Leftrightarrow \frac{dx}{x} = \left(\frac{1}{-t+Ct^3} + \frac{1}{t}\right)dt \Leftrightarrow$

$$\Leftrightarrow \int \frac{dx}{x} \cancel{dx} = \int \left(\frac{1}{-t+Ct^3} + \frac{1}{t}\right)dt$$

$$\ln|x| = \frac{\ln|Ct^2-1|}{2} - C_1 = \ln\left|(Ct^2-1)^{\frac{1}{2}} \cdot C_1\right|$$

$$\underline{x = C_1 \cdot (Ct^2-1)^{\frac{1}{2}}}$$