Petculescu Mihai-Silviu

Temă - Curs 3

1. Să se verifice dacă următoarele sisteme sunt liniar independente, sisteme de generatori, baze.

a) $B_1 = \{(1,0,5)^T, (3,1,2)^T, (2,3,1)^T\} \subset \mathbb{R}^3$

card $B_1 = 3$

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 0 & 1 & 3 \\ 5 & 2 & 1 \end{pmatrix}; \quad \det A = 1+0+45-10-6-0 = 30 \neq 0$$

$\Rightarrow$ rang $A = 3 \Rightarrow B_1$ l.i. $\}$ $\Rightarrow B_1$ bază.

$\dim(\mathbb{R}^3) = 3 = $ rang $A \Rightarrow B_1$ - sistem de gen.

b) $B_2 = \{(-1,5,0)^T, (-3,2,1)^T, (-2,1,3)^T\} \subset \mathbb{R}^3$

card $B_2 = 3$

$$A = \begin{pmatrix} -1 & -3 & -2 \\ 5 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}; \quad \det A = -6-10-0+0+1+45 = 30 \neq 0$$

$\Rightarrow$ rang $A = 3 \Rightarrow B_2$ l.i. $\}$ $\Rightarrow B_2$ bază.

$\dim(\mathbb{R}^3) = 3 = $ rang $A \Rightarrow B_2$ - sist. de gen

c) $B_3 = \{(1,0,7)^T, (5,1,3)^T, (4,1,-4)^T\} \subset \mathbb{R}^3$

card $B_3 = 3$

$$A = \begin{pmatrix} 1 & 5 & 4 \\ 0 & 1 & 1 \\ 7 & 3 & -4 \end{pmatrix}; \quad \det A = -4+0+35-28-3+0 = 0$$

$D_1 = \begin{vmatrix} 1 & 5 \\ 0 & 1 \end{vmatrix} = 1 \neq 0 \Rightarrow$ rang $A = 2 \Rightarrow B_3$ - l.d. $\}$ $\Rightarrow B_3$ nu e bază.

$\dim(\mathbb{R}^3) = 3 \neq 2 = $ rang $A \Rightarrow B_3$ - nu e sist. de gen

d) $B_4 = \{(1,1,3)^T, (2,3,7)^T\} \subset \mathbb{R}^3$

card $B_4 = 2$

$$A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \\ 3 & 7 \end{pmatrix} \qquad D_1 = 3-2 = 1 \neq 0 \Rightarrow \text{rang } A = 2 \Rightarrow B_4 \text{ - l.i.}$$

$\Rightarrow B_4$ nu e bază.

$\dim(\mathbb{R}^3) = 3 \neq$ rang $A \Rightarrow B_4$ nu e sist. de gen.

e) $B_5 = \{(2,1,5)^T, (-3,1,7)^T\} \subset \mathbb{R}^3$

card $B_5 = 2$

$A = \begin{pmatrix} \boxed{\begin{matrix} 2 & -3 \\ 1 & 1 \end{matrix}} \\ \begin{matrix} 5 & 7 \end{matrix} \end{pmatrix}$ $D_1 = \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = 2+3 = 5 \neq 0 \Rightarrow$ rang $A = 2 \Rightarrow B_5$ - l.i.

$\dim(\mathbb{R}^3) = 3 \neq$ rang $A \Rightarrow B_5$ nu e sist. de gen.

$\Rightarrow$

$\Rightarrow B_5$ nu e bază.

2. Să se determine coordonatele vectorului $\vec{x} = (-9,4,-1)^T$ în baza

$B = \{\underbrace{(3,1,2)^T}_{\vec{v_1}}, \underbrace{(-1,2,1)^T}_{\vec{v_2}}, \underbrace{(4,-1,1)^T}_{\vec{v_3}}\}$

$\vec{x} = \alpha_1 \vec{v_1} + \alpha_2 \vec{v_2} + \alpha_3 \vec{v_3}$

$= \alpha_1(3,1,2) + \alpha_2(-1,2,1) + \alpha_3(4,-1,1)$

$= (3\alpha_1, \alpha_1, 2\alpha_1) + (-\alpha_2, 2\alpha_2, \alpha_2) + (4\alpha_3, -\alpha_3, \alpha_3)$

$= (3\alpha_1 - \alpha_2 + 4\alpha_3, \alpha_1 + 2\alpha_2 - \alpha_3, 2\alpha_1 + \alpha_2 + \alpha_3) = (-9,4,-1)$

$\begin{cases} 3\alpha_1 - \alpha_2 + 4\alpha_3 = -9 \\ \alpha_1 + 2\alpha_2 - \alpha_3 = 4 \\ 2\alpha_1 + \alpha_2 + \alpha_3 = -1 \end{cases}$; $D = \begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} = 6+4+2-16+3+1 = 0$

$D_1 = \begin{vmatrix} 3 & -1 \\ 1 & 2 \end{vmatrix} = 6+1 = 7 \neq 0$

$D_c = \begin{vmatrix} 3 & -1 & -9 \\ 1 & 2 & 4 \\ 2 & 1 & -1 \end{vmatrix} = -6-9-8+36-12-1 = 0$

$\Rightarrow$ sistem comp. nedeterminat

Notăm $\alpha_3 = \beta$

$\begin{cases} 3\alpha_1 - \alpha_2 = -9 - 4\beta \mid \cdot 2 \\ \alpha_1 + 2\alpha_2 = 4 + \beta \end{cases} \Leftrightarrow \begin{cases} 6\alpha_1 - 2\alpha_2 = -18 - 8\beta \\ \alpha_1 + 2\alpha_2 = 4 + \beta \quad \text{"+"} \end{cases}$

$7\alpha_1 = -14 - 7\beta$

$\alpha_1 = \frac{-14-7\beta}{7} = -2 - \beta.$

$\alpha_1 + 2\alpha_2 = 4 + \beta \Leftrightarrow -2 - \beta + 2\alpha_2 = 4 + \beta \Leftrightarrow 2\alpha_2 = 6 + 2\beta \Rightarrow \alpha_2 = 3 + \beta.$

$\Rightarrow$ Soluție: $(-2-\beta, 3+\beta, \beta)$, $\beta \in \mathbb{R}$, coord unice $\Rightarrow$ imposibil.

3. Să se verifice dacă sistemul de vectori $B=\{(1,0,-2)^T,(3,-1,2)^T,(-1,2,1)^T\}\subset \mathbb{R}^3$ formează o bază și în caz afirmativ să se determine coord. vect $\vec{x}=(-6,4,-5)^T$ în această bază.

$\text{card } B = 3$

$$A=\begin{pmatrix} 1 & 3 & -1 \\ 0 & -1 & 2 \\ -2 & 2 & 1 \end{pmatrix};\quad \det A = -1-0-12+2-4-0 = -15 \neq 0$$

$\Rightarrow \text{rang } A = 3 \Rightarrow B$ - l.i.

$\dim(\mathbb{R}^3)=3=\text{rang } A \Rightarrow B$ sist. de generatori

$\Rightarrow B$ e bază

$\det A = -15 \neq 0$

$$\alpha_1 = \frac{D\alpha_1}{\det A} = \frac{-15}{-15} = 1$$

$$D\alpha_1 = \begin{vmatrix} -6 & 3 & -1 \\ 4 & -1 & 2 \\ -5 & 2 & 1 \end{vmatrix} = 6-8-30+5+24-12 = -15.$$

$$\alpha_2 = \frac{D\alpha_2}{\det A} = \frac{30}{-15} = -2$$

$$D\alpha_2 = \begin{vmatrix} 1 & -6 & -1 \\ 0 & 4 & 2 \\ -2 & -5 & 1 \end{vmatrix} = 4+0+24-8+10+0 = 30$$

$$\alpha_3 = \frac{D\alpha_3}{\det A} = \frac{-15}{-15} = 1$$

$$D\alpha_3 = \begin{vmatrix} 1 & 3 & -6 \\ 0 & -1 & 4 \\ -2 & 2 & -5 \end{vmatrix} = 5-0-24+12-8+0 = -15$$

$\Rightarrow \vec{x}_B = (1,-2,1)$