Tetulescu Mihai - Tiberiu
Număr 46

S1. Fie sist. de vectori $S=\{\vec{x_1}=(-1, n-30, 0), \vec{x_2}=(0,-1,n-35), \vec{x_3}=(1,-n+34,0)\}$

a) $S=\{\vec{x_1}=(-1,16,0), \vec{x_2}=(0,-1,11), \vec{x_3}=(1,-12,0)\}$

b) Vom aplica regula pivotului

$x=(30-n, 0, -35+n)$
$=(-16, 0, 11)$

	$\vec{x_1}$	$\vec{x_2}$	$\vec{x_3}$	$\vec{x}$
$\vec{e_1}$	$\boxed{-1}$	0	1	-16
$\vec{e_2}$	16	-1	-12	0
$\vec{e_3}$	0	11	0	11
$\vec{x_1}$	1	0	-1	16
$\vec{e_2}$	0	$\boxed{-1}$	4	-256
$\vec{e_3}$	0	11	0	-5
$\vec{x_1}$	1	0	-1	16
$\vec{x_2}$	0	1	-4	256
$\vec{e_3}$	0	0	$\boxed{44}$	2827
$\vec{x_1}$	1	0	0	4/11
$\vec{x_2}$	0	1	0	64/11
$\vec{x_3}$	0	0	1	2827/44

S-a obținut $I_3 \Rightarrow S$ - bază, iar coord lui x în baza S sunt $\vec{X}_S=\left(\frac{4}{11}, \frac{64}{11}, \frac{2827}{44}\right)$

c) Fie $\alpha=(\widehat{\vec{x_1},\vec{x_2}}) \Rightarrow \cos\alpha=\frac{\langle\vec{x_1},\vec{x_2}\rangle}{\|\vec{x_1}\|\cdot\|\vec{x_2}\|}$

$\langle\vec{x_1},\vec{x_2}\rangle=(-1)\cdot 0+16\cdot(-1)+11\cdot 0=-16$

$\|\vec{x_1}\|=\sqrt{\langle\vec{x_1},\vec{x_1}\rangle}=\sqrt{(-1)^2+16^2+0^2}=\sqrt{257}$

$\|\vec{x_2}\|=\sqrt{\langle\vec{x_2},\vec{x_2}\rangle}=\sqrt{0+(-1)^2+11^2}=\sqrt{122}$

$\cos\alpha=\frac{-16}{\sqrt{257}\cdot\sqrt{122}}=\frac{-16\sqrt{257}\cdot\sqrt{122}}{257\cdot 122}=\frac{-8\sqrt{257}\cdot\sqrt{122}}{61\cdot 257}$

$\Rightarrow \alpha=\arccos\frac{-8\sqrt{257}\cdot\sqrt{122}}{61\cdot 257}$

$\langle\vec{x_1},\vec{x_2}\rangle=-16\neq 0 \Rightarrow \vec{x_1},\vec{x_2}$ nu sunt perpendiculari.

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S1. d) Pentru a stabili dacă 2 vect sunt coliniari, calc. prod. vect. dintre cei doi: $\vec{x}_1 \times \vec{x}_3 = \begin{vmatrix} i & j & k \\ -1 & 16 & 0 \\ 1 & -12 & 0 \end{vmatrix} = 12k - 16k = -4k.$

Deoarece $\vec{x}_1 \times \vec{x}_3 \neq \vec{0} \Rightarrow \vec{x}_1, \vec{x}_3$ nu sunt coliniari

e) Pentru a stabili dacă vectorii sunt coplanari, calc. produsul mixt dintre cei 3: $(\vec{x}_1, \vec{x}_2, \vec{x}_3) = \begin{vmatrix} -1 & 16 & 0 \\ 0 & -1 & 11 \\ 1 & -12 & 0 \end{vmatrix}$ $= 16 \cdot 11 - (-12) \cdot 11 \cdot (-1)$

$= 176 - 132 = 44.$

Deoarece $(\vec{x}_1, \vec{x}_2, \vec{x}_3) \neq 0 \Rightarrow$ vectorii nu sunt coplanari.

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S_2. Se consideră punctele $A(0,1,n-35)$, $B(30-n,2,0)$, $C(-1,40-n,3)$

a) $A(0,1,11)$, $B(-16,2,0)$, $C(-1,-6,3)$.

b) $\overrightarrow{BC}=(x_C-x_B, y_C-y_B, z_C-z_B)=(-1-(-16), -6-2, 3-0)=(15,-8,3)$

$BC: \dfrac{x-x_B}{x_C-x_B}=\dfrac{y-y_B}{y_C-y_B}=\dfrac{z-z_B}{z_C-z_B} \Leftrightarrow \dfrac{x+16}{-1+16}=\dfrac{y-2}{-6-2}=\dfrac{z-0}{3}$

$\Leftrightarrow \dfrac{x+16}{15}=\dfrac{y-2}{-8}=\dfrac{z}{3}$ (ecuații carteziene)

$\dfrac{x+16}{15}=\dfrac{y-2}{-8}=\dfrac{z}{3}=t \Leftrightarrow \begin{cases} x=15t-16 \\ y=-8t+2 \\ z=3t \end{cases}$ (ec. parametrice scalare)

c) ecuația planului P

$$\begin{vmatrix} x & y & z & 1 \\ x_A & y_A & z_A & 1 \\ x_B & y_B & z_B & 1 \\ x_C & y_C & z_C & 1 \end{vmatrix} = x\begin{vmatrix} y_A & z_A & 1 \\ y_B & z_B & 1 \\ y_C & z_C & 1 \end{vmatrix} - y\begin{vmatrix} x_A & z_A & 1 \\ x_B & z_B & 1 \\ x_C & z_C & 1 \end{vmatrix} + z\begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix} -$$

$$- \begin{vmatrix} x_A & y_A & z_A \\ x_B & y_B & z_B \\ x_C & y_C & z_C \end{vmatrix} = x\begin{vmatrix} 1 & 11 & 1 \\ 2 & 0 & 1 \\ -6 & 3 & 1 \end{vmatrix} - y\begin{vmatrix} 0 & 11 & 1 \\ -16 & 0 & 1 \\ -1 & 3 & 1 \end{vmatrix} + z\begin{vmatrix} 0 & 1 & 1 \\ -16 & 2 & 1 \\ -1 & -6 & 1 \end{vmatrix} - \begin{vmatrix} 0 & 1 & 11 \\ -16 & 2 & 0 \\ -1 & -6 & 3 \end{vmatrix}$$

$= -85\cdot x - 117y + 113z - 1126$

d) $D(n-30, n-20, 0) \Rightarrow D(16,26,0)$.

$d(D,P)=\dfrac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}=\dfrac{|-85\cdot 16-117\cdot 26-1126|}{\sqrt{85^2+117^2+113^2}}=$

$=\dfrac{5528}{\sqrt{33683}}$.

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S2. e) măsura unghiurilor AC și BC.

$BC = (15, -8, -3)$

$AC = (x_C - x_A, y_C - y_A, z_C - z_A) = (\overset{-1}{\cancel{3-11}}, -6-1, 3-11) = (-1, -7, -8).$

$m(\widehat{AC, BC}) = \alpha.$

$$\cos\alpha = \frac{\langle \vec{AC}, \vec{BC} \rangle}{\|\vec{AC}\| \cdot \|\vec{BC}\|}$$

$\langle \vec{AC}, \vec{BC} \rangle = 15 \cdot (-1) + (-7) \cdot (-8) + (-3) \cdot (-8) = 65.$

$\|\vec{AC}\| = \sqrt{\langle \vec{AC}, \vec{AC} \rangle} = \sqrt{1+49+64} = \sqrt{114}$

$\|\vec{BC}\| = \sqrt{\langle \vec{BC}, \vec{BC} \rangle} = \sqrt{225+64+9} = \sqrt{298}.$

$$\cos\alpha = \frac{65}{\sqrt{114} \cdot \sqrt{298}} = \frac{65 \cdot \sqrt{114} \cdot \sqrt{298}}{114 \cdot 298}$$

$$\alpha = \arccos\left(\frac{65 \cdot \sqrt{114} \cdot \sqrt{298}}{114 \cdot 298}\right)$$

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$\bar{5}_3$. Se consideră conica $\Gamma = x^2 - 4xy + y^2 + 6nx - 2ny + 1 = 0$

a) $\Gamma = x^2 - 4xy + y^2 + 276x - 92y + 1 = 0$

$a_{11} = 1$, $a_{12} = -2$, $a_{22} = 1$, $a_{10} = 138$, $a_{20} = -46$, $a_{00} = 1$

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{10} \\ a_{12} & a_{22} & a_{20} \\ a_{10} & a_{20} & a_{00} \end{vmatrix} = \begin{vmatrix} 1 & -2 & 138 \\ -2 & 1 & -46 \\ 138 & -46 & 1 \end{vmatrix} = 1 + 46 \cdot 2 \cdot 138 + 138 \cdot 2 \cdot 46 - 138^2 - 46^2 - 4$$

$= 1 + 138(92 \cdot 2 - 138) - 46^2 - 4$

$= -3 + 138 \cdot 46 - 46^2$

$= 92 \cdot 46 - 3 = 4229$

$$\delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = 1 - 4 = -3.$$

$I = a_{11} + a_{22} = 1 + 1 = 2.$

Cum $D \neq 0 \Rightarrow$ conică nedegenerată

Cum $\delta \neq 0 \Rightarrow$ conică cu centru

Cum $\delta < 0 \Rightarrow$ conică de tip hiperbolic.

c) $A = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$

$$A - \lambda I_2 = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} 1-\lambda & -2 \\ -2 & 1-\lambda \end{pmatrix}$$

$\det(A - \lambda I_2) = (1-\lambda)(1-\lambda) - 4 = 1 - 2\lambda + \lambda^2 - 4 = \lambda^2 - 2\lambda - 3.$

$\lambda^2 - 2\lambda - 3 = 0 \Leftrightarrow \lambda^2 + \lambda - 3\lambda - 3 = 0 \Leftrightarrow \lambda(\lambda+1) - 3(\lambda+1) = 0$

$\Leftrightarrow (\lambda - 3)(\lambda + 1) = 0 \Rightarrow \begin{cases} \lambda_1 = 3 \\ \lambda_2 = -1 \end{cases}$

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Continuare S_3. c)

• Pt $\lambda = 3$ avem:

$$(A - 3I_2)\cdot \vec{u}_{\lambda_1} = \begin{pmatrix}-2 & -2\\ -2 & -2\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix} = \begin{pmatrix}-2u_1-2u_2\\ -2u_1-2u_2\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}$$

$-2u_1 - 2u_2 = 0 \Rightarrow -u_1 = u_2$

Fie $u_1 = 1 \Leftrightarrow u_2 = -1 \Rightarrow u_{\lambda_1} = \begin{pmatrix}1\\ -1\end{pmatrix}$; $\|\vec{u}_{\lambda_1}\| = \sqrt{2}$.

• Pt $\lambda = -1$ avem

$$(A + I_2)\cdot \vec{u}_{\lambda_2} = \begin{pmatrix}2 & -2\\ -2 & 2\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix} = \begin{pmatrix}2u_1-2u_2\\ -2u_1+2u_2\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}$$

$2u_1 - 2u_2 = 0 \Leftrightarrow u_1 = u_2$

Fie $u_1 = 1 \Leftrightarrow u_2 = 1 \Rightarrow u_{\lambda_2} = \begin{pmatrix}1\\ 1\end{pmatrix}$; $\|\vec{u}_{\lambda_2}\| = \sqrt{2}$.

d) $\begin{cases} \frac{1}{2}\cdot\frac{\partial P}{\partial x} = 0 \\ \frac{1}{2}\cdot\frac{\partial P}{\partial y} = 0 \end{cases} \Leftrightarrow \begin{cases} \frac{1}{2}(2x - 4y + 276) = 0 \\ \frac{1}{2}(-4x + 2y + (-92)) = 0 \end{cases}$

$\Leftrightarrow \begin{cases} x - 2y + 138 = 0 \\ -2x + y - 46 = 0 \end{cases} \Leftrightarrow \begin{cases} x - 2y = -138 \\ -2x + y = 46 \end{cases}$ "+"

$-3x = -46 \Rightarrow x = \frac{46}{3}$

$-2x + y = 46 \Leftrightarrow y = 46 + 2\cdot\frac{46}{3} = \frac{46\cdot 3 + 46\cdot 2}{3} = \frac{230}{3}$.

$\Rightarrow C\left(\frac{46}{3}, \frac{230}{3}\right)$

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53. a) Vectorii ortonormați sunt:

$$\vec{u}_{\lambda_1} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \text{ și } \vec{u}_{\lambda_2} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$T = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Aplicăm rotația $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix}x'\\y'\end{pmatrix} \Rightarrow \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix} \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}} \\ \frac{-x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}} \end{pmatrix} \Rightarrow$

$$\Rightarrow \begin{cases} x = \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}} \\ y = \frac{-x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}} \end{cases}$$

Înlocuim în ecuația conicei:

$$\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right)^2 - 4\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right)\left(\frac{-x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right) + \left(\frac{-x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right)^2 + 276\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right) -$$
$$- 92\left(\frac{-x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right) + 1 = 0$$

$$\left(\frac{(x')^2}{2} + x'y' + \frac{(y')^2}{2}\right) - 4\left(\frac{-(x')^2}{2} + \frac{(y')^2}{2}\right) + \left(\frac{(-x')^2}{2} - x'y' + \frac{(y')^2}{2}\right) +$$
$$+ \frac{276x'}{\sqrt{2}} + \frac{276y'}{\sqrt{2}} + \frac{92x'}{\sqrt{2}} - \frac{92y'}{\sqrt{2}} + 1 = 0.$$

$$(x')^2 + (y')^2 + 2(x')^2 - 2(y')^2 + \frac{368x'}{\sqrt{2}} + \frac{184y'}{\sqrt{2}} + 1 = 0.$$

$$3(x')^2 - (y')^2 + \frac{368x'}{\sqrt{2}} + \frac{184y'}{\sqrt{2}} + 1 = 0.$$

$$3\left((x')^2 + \frac{368 \cdot 3x'}{\sqrt{2}}\right) - \left((y')^2 - \frac{184y'}{\sqrt{2}}\right) + 1 = 0.$$

$$3\left((x')^2 + \frac{2 \cdot 184 \cdot 3x'}{\sqrt{2}} + 92 \cdot 9 \cdot 184\right) - \left((y')^2 - \frac{2 \cdot 92y'}{\sqrt{2}} + 46 \cdot 92\right) + 1 -$$
$$- 92 \cdot 3 \cdot 9 \cdot 184 + 46 \cdot 92 = 0.$$

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Continuare S3. I)

$$3\left(x' + \frac{184\cdot 3}{\sqrt{2}}\right)^2 - \left(y' - \frac{92}{\sqrt{2}}\right)^2 = 92\cdot 3\cdot 9\cdot 184 - 46\cdot 92 - 1$$

$$= 92\cdot 46(4\cdot 27 - 1) - 1.$$

$$= 92\cdot 46\cdot 107 - 1.$$

Facem transformarea: $x = x' + \frac{184\cdot 3}{\sqrt{2}}$

(translația) $y = y' - \frac{92}{\sqrt{2}}$

$$\Rightarrow 3x^2 - y^2 = 92\cdot 46\cdot 107 - 1 = 452823$$

$$\frac{x^2}{150941} - \frac{y^2}{452823} = 1. \text{ (hiperbolă)}.$$