g) $x^{(7)} + 3x^{(6)} + 3x^{(5)} + x^{(4)} = 0$

$$\left.\begin{array}{l} x = e^{nt} \\ x' = n e^{nt} \\ x'' = n^2 e^{nt} \\ x''' = n^3 e^{nt} \\ x^{(4)} = n^4 e^{nt} \\ x^{(5)} = n^5 e^{nt} \\ x^{(6)} = n^6 e^{nt} \\ x^{(7)} = n^7 e^{nt} \end{array}\right\} \Rightarrow n^7 e^{nt} + 3 \cdot n^6 e^{nt} + 3n^5 e^{nt} + n^4 e^{nt} = 0 \quad \Big| : e^{nt}$$

$$n^7 + 3n^6 + 3n^5 + n^4 = 0$$

$$n^4(n^3 + 3n^2 + 3n + 1) = 0 \Rightarrow n_1 = n_2 = n_3 = n_4 = 0$$

$$n^3 + 3n^2 + 3n + 1 = 0 \Leftrightarrow (n+1)^3 = 0$$

$$\Leftrightarrow n_5 = n_6 = n_7 = -1.$$

- $e^{0t}, \; t e^{0t}, \; t^2 e^{0t}, \; t^3 e^{0t}, \; e^{-t}, \; t e^{-t}, \; t^2 e^{-t}$

  – sist. fundam. de sol.

- $x(t) = c_1 + c_2 t + c_3 t^2 + c_4 t^3 + c_5 e^{-t} + c_6 t e^{-t} + c_7 t^2 e^{-t}$

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III b) $x^{IV} + 4x = 0$

$$\left.\begin{aligned} x &= e^{rt} \\ x' &= r e^{rt} \\ x'' &= r^2 e^{rt} \\ x''' &= r^3 e^{rt} \\ x^{IV} &= r^4 e^{rt} \end{aligned}\right\} \Rightarrow r^4 e^{rt} + 4e^{rt} = 0 \;|: e^{rt}$$

$$r^4 + 4 = 0$$

$$r^4 + 4 = (r^2 + 2r + 2)(r^2 - 2r + 2) = 0$$

$$\Delta = 4 - 4 \cdot 2 = -4$$

$$r_{1,2} = \frac{-2 \pm 2i}{2} = -1 \pm i \quad (\alpha \pm \beta, \alpha = -1, \beta = 1)$$

$$r_{3,4} = \frac{2 \pm 2i}{2} = 1 \pm i \quad (\alpha \pm \beta, \alpha = 1, \beta = 1)$$

- $e^{-1t} \cos t, e^{-1t} \sin t, e^{1t} \cos t, e^{1t} \sin t$ – sist. fund de soluții
- $x(t) = C_1 e^{-t} \cos t + C_2 e^{-t} \sin t + C_3 e^{t} \cos t + C_4 e^{t} \sin t$

d) $x'' + 4x' + 13x = 0$

$$\left.\begin{aligned} x &= e^{rt} \\ x' &= r e^{rt} \\ x'' &= r^2 e^{rt} \end{aligned}\right\} \Rightarrow r^2 e^{rt} + 4r e^{rt} + 13 e^{rt} = 0 \;|: e^{rt}$$

$$r^2 + 4r + 13 = 0$$

$$\Delta = 16 - 4 \cdot 13 = -36$$

$$r_{1,2} = \frac{-4 \pm 6i}{2} = -2 \pm 3i \quad (\alpha \pm \beta, \alpha = -2, \beta = 3)$$

- $e^{-2t} \cos 3t, e^{-2t} \sin 3t$ – sist. fund. de soluții
- $x(t) = C_1 e^{-2t} \cos 3t + C_2 e^{-2t} \sin 3t$

e) $x'' + 4x' + 5x = 0$

$$\left.\begin{aligned} x &= e^{rt} \\ x' &= r e^{rt} \\ x'' &= r^2 e^{rt} \end{aligned}\right\} \Rightarrow r^2 e^{rt} + 4r e^{rt} + 5 e^{rt} = 0 \;|: e^{rt}$$

$$r^2 + 4r + 5 = 0$$

$$\Delta = 16 - 4 \cdot 5 = -4$$

$$r_{1,2} = \frac{-4 \pm 2i}{2} = -2 \pm i \quad (\alpha \pm \beta, \alpha = -2, \beta = 1)$$

• $e^{-2t}\cos t$, $e^{-2t}\sin t$ – sist. fund. de sol.

$x(t) = C_1 e^{-2t}\cos t + C_2 e^{-2t}\sin t$

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IV. b) $x''' - 3x'' + 9x' + 13x = 0$

$$\left.\begin{array}{l} x = e^{rt} \\ x' = r e^{rt} \\ x'' = r^2 e^{rt} \\ x''' = r^3 e^{rt} \end{array}\right\} \Rightarrow r^3 e^{rt} - 3r^2 e^{rt} + 9r e^{rt} + 13 e^{rt} = 0 \quad \Big| : e^{rt}$$

$$r^3 - 3r^2 + 9r + 13 = 0$$

$$(r+1)(r^2 - 4r + 13) = 0 \Rightarrow r_1 = -1.$$

$$\begin{array}{l|l} r^3 - 3r^2 + 9r + 13 & r+1 \\ \cline{2-2} -r^3 - r^2 & r^2 - 4r + 13 \\ \cline{1-1} / \quad -4r^2 + 9r + 13 & \\ \quad\;\; 4r^2 + 4r & \\ \cline{1-1} \quad / \quad\;\; +13r + 13 & \\ \quad\quad\;\; -13r - 13 & \\ \cline{1-1} \quad\quad\quad / \quad / & \end{array}$$

$r^2 - 4r + 13 = 0$

$\Delta = 16 - 4 \cdot 13 = -36$

$r_{2,3} = \frac{4 \pm 6i}{2} = 2 \pm 3i$

$(\alpha \pm \beta, \alpha = 2, \beta = 3)$

- e^{-t}, $e^{2t}\cos 3t$, $e^{2t}\sin 3t$
- $x(t) = C_1 e^{-t} + C_2 e^{2t}\cos 3t + C_3 e^{2t}\sin 3t$

d) $x''' - 5x'' + 17x' - 13x = 0$

$$\left.\begin{array}{l} x = e^{rt} \\ x' = r e^{rt} \\ x'' = r^2 e^{rt} \\ x''' = r^3 e^{rt} \end{array}\right\} \Rightarrow r^3 e^{rt} - 5r^2 e^{rt} + 17r e^{rt} - 13 e^{rt} = 0 \quad \Big| : e^{rt}$$

$$r^3 - 5r^2 + 17r - 13 = 0$$

$$(r-1)(r^2 - 4r + 13) = 0 \Rightarrow r_1 = 1$$

$$\begin{array}{l|l} r^3 - 5r^2 + 17r - 13 & r-1 \\ \cline{2-2} -r^3 + r^2 & r^2 - 4r + 13 \\ \cline{1-1} / \quad -4r^2 + 17r - 13 & \\ \quad\;\; +4r^2 - 4r & \\ \cline{1-1} \quad / \quad\;\; +13r - 13 & \\ \quad\quad\;\; -13r + 13 & \\ \cline{1-1} \quad\quad\quad / \quad / & \end{array}$$

- $r^2 - 4r + 13 = 0$

$\Delta = 16 - 4 \cdot 13 = -36$

$r_{2,3} = \frac{+4 \pm 6i}{2} = 2 \pm 3i$

$(\alpha \pm \beta, \alpha = 2, \beta = 3)$

• e^{1t}, $e^{2t}\cos 3t$, $e^{2t}\sin 3t$ – sist. fund. de sol.

$$x(t) = c_1 e^{t} + c_2 e^{2t}\cos 3t + c_3 e^{2t}\sin 3t$$

e) $x^{V} + 4x^{IV} + 3x''' - 6x' - 2x = 0$

$$\left.\begin{aligned} x &= e^{nt} \\ x' &= n e^{nt} \\ x'' &= n^2 e^{nt} \\ x''' &= n^3 e^{nt} \\ x^{IV} &= n^4 e^{nt} \\ x^{V} &= n^5 e^{nt} \end{aligned}\right\} \Rightarrow n^5 e^{nt} + 4n^4 e^{nt} + 3n^3 e^{nt} - 6n e^{nt} - 2e^{nt} = 0 \;\Big|\, : e^{nt}$$

$$n^5 + 4n^4 + 3n^3 - 6n - 2 = 0$$

$$(n-1)(n^4 + 5n^3 + 8n^2 + 8n + 2) = 0 \Rightarrow n_1 = 1$$

• $n^5 + 4n^4 + 3n^3 - 6n - 2 \;\big|\; n - 1$

$$\begin{array}{l|l} n^5 + 4n^4 + 3n^3 - 6n - 2 & n-1 \\ \hline -n^5 + n^4 & n^4 + 5n^3 + 8n^2 + 8n + 2 \\ \hline / \;\; +5n^4 + 3n^3 - 6n - 2 & \\ \quad -5n^4 + 5n^3 & \\ \hline \quad / \;\; +8n^3 - 6n - 2 & \\ \qquad -8n^3 + 8n^2 & \\ \hline \qquad / \;\; +8n^2 - 6n - 2 & \\ \qquad\quad -8n^2 + 8n & \\ \hline \qquad\quad / \;\; 2n - 2 & \\ \qquad\qquad -2n + 2 & \\ \hline \qquad\qquad / \quad / & \end{array}$$

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I. Să se rezolve urm. ec. dif. liniare de ordin n omogene cu coeficienți constanți.

b) $2x''' - 3x'' + x' = 0$, $x(0) = -1$, $x'(0) = 2$, $x''(0) = 1$

$$\left.\begin{aligned} x &= e^{rt} \\ x' &= r e^{rt} \\ x'' &= r^2 e^{rt} \\ x''' &= r^3 e^{rt} \end{aligned}\right\} \Rightarrow 2r^3 e^{rt} - 3r^2 e^{rt} + r e^{rt} = 0 \,|\, : e^{rt}$$

$$2r^3 - 3r^2 + r = 0$$

$$r(2r^2 - 3r + 1) = 0 \longrightarrow \boxed{r_1 = 0}$$

$$2r^2 - 3r + 1 = 0$$

$$\Delta = 9 - 8 = 1 \Rightarrow r_{2,3} = \frac{3 \pm 1}{2} \quad \boxed{r_2 = 2} \quad \boxed{r_3 = 1}$$

- $e^{0\cdot t}, e^{2\cdot t}, e^{1\cdot t}$ - sist. fundamental de sol.
- $x(t) = C_1 + C_2 e^{2t} + C_3 e^{t} \Leftrightarrow x(0) = \boxed{C_1 + C_2 + C_3 = -1}$
- $x'(t) = C_2 2 e^{2t} + C_3 e^{t} \Leftrightarrow x'(0) = \boxed{2C_2 + C_3 = 2}$
- $x''(t) = C_2 4 e^{2t} + C_3 e^{t} \Leftrightarrow x''(0) = \boxed{4C_2 + C_3 = 1}$

$$\begin{cases} C_1 + C_2 + C_3 = -1 \Leftrightarrow C_1 = -1 - 3 + \frac{1}{2} = \boxed{\frac{-7}{2} = C_1} \\ \left.\begin{aligned} 2C_2 + C_3 &= 2 \\ 4C_2 + C_3 &= 1 \end{aligned}\right\} \Rightarrow 2C_2 = -1 \Leftrightarrow \boxed{C_2 = \frac{-1}{2}} \Leftrightarrow \boxed{C_3 = 3} \end{cases}$$

- $x_{PC} = \frac{-7}{2} - \frac{1}{2} e^{2t} + 3e^{t}$

---

c) $x''' - 7x'' + 14x' - 8x = 0$, $x(0) = 1$, $x'(0) = 0$, $x''(0) = 1$

$$\left.\begin{aligned} x &= e^{rt} \\ x' &= r e^{rt} \\ x'' &= r^2 e^{rt} \\ x''' &= r^3 e^{rt} \end{aligned}\right\} \Rightarrow r^3 e^{rt} - 7r^2 e^{rt} + 14 r e^{rt} - 8e^{rt} = 0 \,|\, : e^{rt}$$

$$r^3 - 7r^2 + 14r - 8 = 0$$

$$\begin{array}{l|l} r^3 - 7r^2 + 14r - 8 & r - 1 \\ \hline -r^3 + r^2 & r^2 - 6r + 8 \\ \quad -6r^2 + 14r - 8 & \\ \quad +6r^2 - 6r & \\ \quad\quad / \quad -8r - 8 & \end{array}$$

$$r^2 - 6r + 8 = r^2 - 4r - 2r + 8 = (r-4)(r-2)$$

• $(r-1)(r-2)(r-4)=0 \Leftrightarrow \begin{cases} r_1=1 \\ r_2=2 \\ r_3=4 \end{cases}$

• e^{1t}, e^{2t}, e^{4t} – sist. fundam de sol.

• $x(t)=c_1e^{t}+c_2e^{2t}+c_3e^{4t} \Leftrightarrow x(0)=\boxed{c_1+c_2+c_3=1}$

• $x'(t)=c_1e^{t}+c_2\cdot 2e^{2t}+c_3\cdot 4e^{4t} \Leftrightarrow x'(0)=\boxed{c_1+2c_2+4c_3=0}$

• $x''(t)=c_1e^{t}+c_2\cdot 4e^{2t}+c_3\cdot 16e^{4t} \Leftrightarrow x''(0)=\boxed{c_1+4c_2+16c_3=1}$

$\begin{cases} c_1+c_2+c_3=1 \\ c_1+2c_2+4c_3=0 \\ c_1+4c_2+16c_3=1 \end{cases} \Leftrightarrow \begin{cases} c_2+3c_3=-1 \\ 2c_2+12c_3=1 \end{cases} \Leftrightarrow 6c_3=3 \Leftrightarrow \boxed{c_3=\frac{1}{2}}$

$c_2=-1-\frac{3}{2}=\frac{-2-3}{2}=\boxed{\frac{-5}{2}=c_2}$

$c_1-\frac{5}{2}+\frac{1}{2}=1 \Leftrightarrow c_1=1-\frac{1}{2}+\frac{5}{2}=\frac{7-1}{2}=\boxed{3=c_1}$

• $\underline{x_{PC}=3e^{t}-\frac{5}{2}e^{2t}+\frac{1}{2}e^{4t}}$

e) $x'''-x'=0$

$\left.\begin{aligned} x&=e^{rt} \\ x'&=re^{rt} \\ x''&=r^2e^{rt} \\ x'''&=r^3e^{rt} \end{aligned}\right\} \Leftrightarrow r^3e^{rt}-re^{rt}=0 \mid : e^{rt}$

$r^3-r=0$

$r(r^2-1)=0 \Leftrightarrow r_1=0$

$r^2-1=0 \Leftrightarrow (r+1)(r-1)=0 \Leftrightarrow r_2=-1, r_3=1$

• $e^{0\cdot t}, e^{-1t}, e^{1t}$ – sist. fundamental de sol.

$x(t)=c_1+c_2e^{-t}+c_3e^{t}$

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II b) $x^{IV} - 5x'' + 4x = 0$

$$\left.\begin{aligned} x &= e^{rt} \\ x' &= r e^{rt} \\ x'' &= r^2 e^{rt} \\ x''' &= r^3 e^{rt} \\ x^{IV} &= r^4 e^{rt} \end{aligned}\right\} \Rightarrow \begin{aligned} & r^4 e^{rt} - 5r^2 e^{rt} + 4r e^{rt} = 0 \;\Big|\; : e^{rt} \\ & r^4 - 5r^2 + 4r = 0 \\ & r(r^3 - 5r + 4) = 0 \quad \Leftrightarrow r_1 = 0 \\ & r^3 - r \cdot 5 + 4 = 0 \end{aligned}$$

- $r^3 - 5r + 4 = 0 \Leftrightarrow r(r^2-1) - 4(r-1) = 0 \Leftrightarrow r(r+1)(r-1) - 4(r-1) = 0$

$(r-1)(r(r+1)-4) = 0 \Leftrightarrow \underline{(r-1)}(r^2+r-4) = 0 \Leftrightarrow r_2 = 1$

- $r^2 + r - 4 = 0$

$\Delta = 1 - 4(-4) = 17$

$$r_{3,4} = \frac{-1 \pm \sqrt{17}}{2} \quad \begin{cases} r_3 = \dfrac{-1-\sqrt{17}}{2} \\[2ex] r_4 = \dfrac{-1+\sqrt{17}}{2} \end{cases}$$

- $e^{0t}, e^{1t}, e^{\frac{-1-\sqrt{17}}{2} \cdot t}, e^{\frac{-1+\sqrt{17}}{2} \cdot t}$ - sist. fund. de sol.

- $x(t) = c_1 + c_2 e^t + c_3 e^{\frac{(-1-\sqrt{17})t}{2}} + c_4 e^{\frac{(-1+\sqrt{17})t}{2}}$

c) $x''' - 6x'' + 12x' - 8x = 0$

$$\left.\begin{aligned} x &= e^{rt} \\ x' &= r e^{rt} \\ x'' &= r^2 e^{rt} \\ x''' &= r^3 e^{rt} \end{aligned}\right\} \Leftrightarrow \begin{aligned} & r^3 e^{rt} - 6r^2 e^{rt} + 12r e^{rt} - 8e^{rt} = 0 \;\Big|\; : e^{rt} \\ & r^3 - 6r^2 + 12r - 8 = 0 \end{aligned}$$

- $r^3 - 6r^2 + 12r - 8 \;|\; r-2$

$$\begin{array}{l|l} r^3 - 6r^2 + 12r - 8 & r-2 \\ \underline{-r^3 + 2r^2} & r^2 - 4r + 4 = (r-2)^2 \\ / \quad -4r^2 + 12r - 8 & \\ \quad\;\; \underline{4r^2 - 8r} & \\ \quad\;\; / \quad +4r - 8 & \\ \quad\quad\quad\; \underline{-4r + 8} & \\ \quad\quad\quad\; / \quad / & \end{array}$$

- $(r-2)^3 = 0 \Leftrightarrow r_1 = r_2 = r_3 = 2$
- $e^{2t}, te^{2t}, t^2 e^{2t}$ – sist. fund. de sol.

$x(t) = c_1 e^{2t} + c_2 t e^{2t} + c_3 e^{2t}$

e) $x^{IV} + 2x''' + x'' = 0$

$$\left.\begin{array}{l} x = e^{rt} \\ x' = r e^{rt} \\ x'' = r^2 e^{rt} \\ x''' = r^3 e^{rt} \\ x^{IV} = r^4 e^{rt} \end{array}\right\} \Rightarrow \begin{array}{l} r^4 e^{rt} + 2r^3 e^{rt} + r^2 e^{rt} = 0 \mid : e^{rt} \\ r^4 + 2r^3 + r^2 = 0 \\ r^2(r^2 + 2r + 1) = 0 \Rightarrow r_1 = r_2 = 0 \\ r^2 + 2r + 1 = (r+1)^2 = 0 \Leftrightarrow r_3 = r_4 = -1 \end{array}$$

- $e^{0t}, te^{0t}, e^{-t}, te^{-t}$ – sist. fund. de sol.

$x(t) = c_1 + c_2 t + c_3 e^{-t} + c_4 t e^{-t}$

d) $x^{IV} - 2x'' + x = 0$

$$\left.\begin{array}{l} x = e^{rt} \\ x' = r e^{rt} \\ x'' = r^2 e^{rt} \\ x''' = r^3 e^{rt} \\ x^{IV} = r^4 e^{rt} \end{array}\right\} \Rightarrow \begin{array}{l} r^4 e^{rt} - 2r^2 e^{rt} + r e^{rt} = 0 \mid : e^{rt} \\ r^4 - 2r^2 + r = 0 \\ r(r^3 - 2r + 1) = 0 \Rightarrow r_1 = 0 \\ r^3 - 2r + 1 = 0 \Leftrightarrow (r-1)(r^2 + r - 1) = 0 \Rightarrow r_2 = 1 \end{array}$$

- $r^3 - 2r + 1 \;|\; r - 1$

$-r^3 + r^2 \qquad\qquad r^2 + r - 1$

$/ \; r^2 - 2r + 1$

$-r^2 + r$

$/ \; -r + 1$

$+r - 1$

$/ \quad /$

$r^2 + r - 1 = 0$

$\Delta = 1 - 4(-1) = 5$

$r_{3,4} = \dfrac{-1 \pm \sqrt{5}}{2}$ — $r_3 = \dfrac{-1-\sqrt{5}}{2}$, $r_4 = \dfrac{-1+\sqrt{5}}{2}$

- $e^{0t}, e^{1t}, e^{\frac{-1-\sqrt{5}}{2}t}, e^{\frac{-1+\sqrt{5}}{2}t}$ – sist. fund. de sol.

$x(t) = c_1 + c_2 e^{t} + c_3 e^{\frac{-1-\sqrt{5}}{2}t} + c_4 e^{\frac{-1+\sqrt{5}}{2}t}$