Seminar01 - Rezolvare

Exerciţiu01 şi Exerciţiu02

1) $\int \frac{\ln x}{x} dx$

$$= \int \ln x \cdot \frac{1}{x}\, dx = \int u' \cdot u\, dx = \frac{u^2}{2} = \underbrace{\frac{\ln^2 x}{2}}_{F(x)} + \mathcal{C}$$

$u(x) = \ln x$

$u'(x) = \frac{1}{x}$

$F(1) = 2$

$F(1) = \frac{\ln^2 1}{2} + C = \boxed{C = 2} \Rightarrow F(x) = \frac{\ln^2 x}{2} + 2$

2) $\int \underbrace{1}_{g'} \cdot \underbrace{\ln^2 x}_{f}\, dx = x \ln^2 x - \int 2\ln x \cdot \frac{1}{\cancel{x}} \cdot \cancel{x}\, dx = x \ln^2 x - 2\int \underbrace{1}_{g'} \cdot \underbrace{\ln x}_{f}\, dx$

$f' = \frac{1}{x}, g = x$

$f' = 2\ln x \cdot \frac{1}{x}, g = x$

$= x\ln^2 x - 2\left(x \ln x - \int \frac{1}{x} \cdot x\, dx\right) = x \ln^2 x - 2x\ln x + 2x + \mathcal{C}$

$F(1) = 1 \cdot \ln^2 1 - 2 \cdot 1 \cdot \ln 1 + 2 \cdot 1 + C = 2 + C = 1 \Rightarrow \boxed{C = -1} \Rightarrow F(x) = x\ln^2 x - 2x \ln x + 2x - 1$

Exerciţiu03 şi Exerciţiu04

3) $\int \underbrace{x}_{f} \underbrace{e^{x+1}}_{g'} dx = x e^{x+1} - \int e^{x+1} dx = x e^{x+1} - e^{x+1} + \mathcal{C}$

$f' = 1, g = e^{x+1}$

$F(0) = 2$

$F(0) = 0 \cdot e - e + C = -e + C = 2 \Rightarrow \boxed{C = 2 + e} \Rightarrow F(x) = x e^{x+1} - e^{x+1} + 2 + e$

4) $\int (x + x^3) e^{x^2} dx = \frac{1}{2}\int 2x e^{x^2} dx + \int x^3 e^{x^2} dx = \frac{1}{2}\int u' \cdot e^u dx + \int x^3 e^{x^2} dx$

$u(x) = x^2$

$u'(x) = 2x$

$= \frac{1}{2} e^{x^2} + \frac{1}{2}\int \underbrace{x^2}_{f} \cdot \underbrace{2x e^{x^2}}_{g'} dx = \frac{1}{2} e^{x^2} + \frac{1}{2}\left(x^2 e^{x^2} - \int 2x e^{x^2} dx\right)$

$f' = 2x, g = e^{x^2}$

$= \underbrace{\cancel{\frac{1}{2} e^{x^2}} + \frac{1}{2} x^2 e^{x^2} - \cancel{\frac{1}{2} e^{x^2}} + \mathcal{C}}_{F(x)}$

$F(0) = \boxed{C = 3} \Rightarrow F(x) = \frac{1}{2} x^2 e^{x^2} + 3$

Exerciţiu05

$$5)\ \underset{\smile}{I} = \int \left[x + \ln(1+x^2) \right] dx = \int x\,dx + \int \ln(1+x^2)\,dx = \frac{x^2}{2} + \int \ln(1+x^2)\,dx$$

$$\int \underbrace{1}_{g'} \cdot \underbrace{\ln(1+x^2)}_{f}\,dx = x\ln(1+x^2) - \int \frac{2x}{1+x^2}\cdot x\,dx = x\ln(1+x^2) - 2\int \frac{x^2}{1+x^2}\,dx$$

$$f' = \frac{1}{1+x^2}\cdot 2x, \quad g = x$$

$$= x\ln(1+x^2) - 2\int \frac{x^2+1-1}{x^2+1}\,dx$$

$$= x\ln(1+x^2) - 2\left(\int 1\,dx - \int \frac{1}{x^2+1}\,dx\right)$$

$$= x\ln(1+x^2) - 2x + 2\,\mathrm{arctg}\,x$$

$$\underset{\smile}{I} = \underbrace{\frac{x^2}{2} + x\ln(1+x^2) - 2x + 2\,\mathrm{arctg}\,x + C}_{F(x)}$$

$$F(0) = C = 1 \Rightarrow F(x) = \frac{x^2}{2} + x\ln(1+x^2) - 2x + 2\,\mathrm{arctg}\,x + 1$$