

# Laborator08

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Exercițiu 4. c)

## Ecuatii

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Să se rezolve următoarele ecuații diferențiale liniare de ordin  $n$  omogene cu coeficienți constanți.

I

- a)  $x^{III} - 3x^{II} + 2x^I = 0$ ,  $x(0) = 1$ ,  $x^I(0) = 2$ ,  $x^{II}(0) = 0$
- b)  $2x^{III} - 3x^{II} + x^I = 0$ ,  $x(0) = -1$ ,  $x^I(0) = 2$ ,  $x^{II}(0) = 1$
- c)  $x^{III} - 7x^{II} + 14x^I - 8x = 0$ ,  $x(0) = 1$ ,  $x^I(0) = 0$ ,  $x^{II}(0) = 1$
- d)  $x^{II} - 4x^I + 3x = 0$ ,  $x(0) = 2$ ,  $x^I(0) = 4$
- e)  $x^{III} - x^I = 0$

II

- a)  $x^{II} + 2x^I + x = 0$ ,  $x(0) = 1$ ,  $x^I(0) = 3$
- b)  $x^{IV} - 5x^{II} + 4x = 0$
- c)  $x^{III} - 6x^{II} + 12x^I - 8x = 0$
- d)  $x^{IV} - 2x^{II} + x = 0$
- e)  $x^{IV} + 2x^{III} + x^{II} = 0$
- f)  $x^{(6)} - x^{(5)} - 4x^{(4)} + 2x^{III} + 5x^{II} - x^I - 2x = 0$
- g)  $x^{(7)} + 3x^{(6)} + 3x^{(5)} + x^{(4)} = 0$

III

- a)  $x^{II} + x = 0$ ,  $x(0) = 3$ ,  $x^I(0) = 5$
- b)  $x^{IV} + 4x = 0$
- c)  $x^{IV} + 8x^{II} + 16x = 0$
- d)  $x^{II} + 4x^I + 13x = 0$
- e)  $x^{II} + 4x^I + 5x = 0$

## IV

- a)  $x^{IV} + 2x^{III} + 4x^{II} - 2x^I - 5x = 0$   
 b)  $x^{III} - 3x^{II} + 9x^I + 13x = 0$   
 c)  $x^{(7)} - x^{(6)} + x^{(5)} - x^{(4)} = 0$   
 d)  $x^{III} - 5x^{II} + 17x^I - 13x = 0$   
 e)  $x^V + 4x^{IV} + 3x^{III} - 6x^I - 2x = 0$

## Rezolvare

### Exercițiu 1. a)

I a)  $x''' - 3x'' + 2x' = 0$ ,  $x(0) = 1$ ,  $x'(0) = 2$ ,  $x''(0) = 0$

$$\begin{aligned} x &= e^{\lambda t} \\ x' &= \lambda e^{\lambda t} \\ x'' &= \lambda^2 e^{\lambda t} \\ x''' &= \lambda^3 e^{\lambda t} \end{aligned} \Rightarrow \lambda^3 e^{\lambda t} - 3\lambda^2 e^{\lambda t} + 2\lambda e^{\lambda t} = 0 \quad | : e^{\lambda t}$$

$$\lambda^3 - 3\lambda^2 + 2\lambda = 0$$

$$\lambda(\lambda^2 - 3\lambda + 2) = 0$$

$$\lambda_1 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\Delta = 9 - 8 = 1 \Rightarrow \lambda_{2,3} = \frac{3 \pm 1}{2} \Rightarrow \lambda_2 = 2, \lambda_3 = 1$$

$e^{0 \cdot t}, e^{2t}, e^{1 \cdot t}$  - sist. fundam. de sol.

$$x(t) = c_1 + c_2 e^{2t} + c_3 e^t$$

$$x(0) = c_1 + c_2 + c_3 = 1$$

$$x'(t) = c_2 \cdot 2e^{2t} + c_3 e^t$$

$$x'(0) = 2c_2 + c_3 = 2$$

$$x''(t) = 2c_2 \cdot 2e^{2t} + c_3 e^t$$

$$x''(0) = 4c_2 + c_3 = 0$$

$$\begin{cases} c_1 + c_2 + c_3 = 1 \\ 2c_2 + c_3 = 2 \\ 4c_2 + c_3 = 0 \end{cases} \Rightarrow \begin{cases} 2c_2 = -2 \Rightarrow c_2 = -1 \\ c_3 = 4 \end{cases}$$

$$c_1 = 1 - c_2 - c_3 = 1 + 1 - 4 = -2 = c_1$$

$$x_{pc} = -2 - e^{2t} + 4e^t$$

### Exercițiu 1. d) - Video

I. d)  $x'' - 4x' + 3x = 0$ ,  $x(0) = 2$ ,  $x'(0) = 4$

$$\begin{aligned} x &= e^{\lambda t} \\ x' &= \lambda e^{\lambda t} \\ x'' &= \lambda^2 e^{\lambda t} \end{aligned} \Rightarrow \lambda^2 e^{\lambda t} - 4\lambda e^{\lambda t} + 3e^{\lambda t} = 0 \quad | : e^{\lambda t}$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$\Delta = 16 - 12 = 4 \Rightarrow \lambda_{1,2} = \frac{4 \pm 2}{2} \Rightarrow \lambda_1 = 3, \lambda_2 = 1$$

$e^{3t}, e^{1 \cdot t}$  - sist. fundam. de sol.

$$x(t) = c_1 e^{3t} + c_2 e^t$$

$$x(0) = c_1 + c_2 = 2$$

$$x'(t) = c_1 \cdot 3e^{3t} + c_2 e^t$$

$$x'(0) = 3c_1 + c_2 = 4$$

$$\begin{cases} c_1 + c_2 = 2 \\ 3c_1 + c_2 = 4 \end{cases} \Rightarrow \begin{cases} 2c_1 = 2 \Rightarrow c_1 = 1 \\ c_2 = 1 \end{cases} \Rightarrow x_{pc} = e^{3t} + e^t$$

## Exercițiu 2. a) - Video

II a)  $x'' + 2x' + x = 0$ ,  $x(0) = 1$ ,  $x'(0) = 3$

$$\begin{cases} x = e^{\lambda t} \\ x' = \lambda e^{\lambda t} \\ x'' = \lambda^2 e^{\lambda t} \end{cases} \Rightarrow \lambda^2 e^{\lambda t} + 2\lambda e^{\lambda t} + e^{\lambda t} = 0 \quad | : e^{\lambda t}$$

$$\lambda^2 + 2\lambda + 1 = 0$$

$$(\lambda + 1)^2 = 0$$

$$\lambda_1 = \lambda_2 = -1$$

$e^{-t}$ ,  $t e^{-t}$  - sist. fundam. de sol.

$$x(t) = c_1 e^{-t} + c_2 t e^{-t}$$

$$x(0) = c_1 = 1$$

$$x'(t) = c_1 (-e^{-t}) + c_2 (e^{-t} + t(-e^{-t}))$$

$$x'(0) = -c_1 + c_2 = 3$$

$$c_2 = 3 + c_1 = 3 + 1 = 4 \Rightarrow c_2 = 4$$

$$x_{pc} = e^{-t} + 4t e^{-t}$$

## Exercițiu 2. f)

II f)  $x^{(6)} - x^{(5)} - 4x^{(4)} + 2x''' + 5x'' - x' - 2x = 0$

$$\begin{cases} x = e^{\lambda t} \\ x' = \lambda e^{\lambda t} \\ x'' = \lambda^2 e^{\lambda t} \\ x''' = \lambda^3 e^{\lambda t} \\ x^{(4)} = \lambda^4 e^{\lambda t} \\ x^{(5)} = \lambda^5 e^{\lambda t} \\ x^{(6)} = \lambda^6 e^{\lambda t} \end{cases} \Rightarrow \lambda^6 e^{\lambda t} - \lambda^5 e^{\lambda t} - 4\lambda^4 e^{\lambda t} + 2\lambda^3 e^{\lambda t} + 5\lambda^2 e^{\lambda t} - \lambda e^{\lambda t} - 2e^{\lambda t} = 0 \quad | : e^{\lambda t}$$

$$\lambda^6 - \lambda^5 - 4\lambda^4 + 2\lambda^3 + 5\lambda^2 - \lambda - 2 = 0$$

$$\begin{array}{r} \lambda^6 - \lambda^5 - 4\lambda^4 + 2\lambda^3 + 5\lambda^2 - \lambda - 2 \\ -\lambda^6 + \lambda^5 \\ \hline -4\lambda^4 + 2\lambda^3 + 5\lambda^2 - \lambda - 2 \\ +4\lambda^4 - 4\lambda^3 \\ \hline -2\lambda^3 + 5\lambda^2 - \lambda - 2 \\ +2\lambda^3 - 2\lambda^2 \\ \hline 3\lambda^2 - \lambda - 2 \\ -3\lambda^2 + 3\lambda \\ \hline -\lambda - 2 \\ -2\lambda - 2 \\ \hline 1 \end{array} \quad \begin{array}{r} \lambda^5 - 4\lambda^3 - 2\lambda^2 + 3\lambda + 2 \\ -\lambda^5 + \lambda^4 \\ \hline \lambda^4 - 4\lambda^3 - 2\lambda^2 + 3\lambda + 2 \\ -\lambda^4 + \lambda^3 \\ \hline -3\lambda^3 - 2\lambda^2 + 3\lambda + 2 \\ +3\lambda^3 - 3\lambda^2 \\ \hline -5\lambda^2 + 3\lambda + 2 \\ +5\lambda^2 - 5\lambda \\ \hline -2\lambda + 2 \\ +2\lambda - 2 \\ \hline 1 \end{array} \quad \begin{array}{r} \lambda^4 + \lambda^3 - 3\lambda^2 - 5\lambda - 2 \\ -\lambda^4 + \lambda^3 \\ \hline -3\lambda^2 - 5\lambda - 2 \\ +3\lambda^2 + 3\lambda \\ \hline -2\lambda - 2 \\ 2\lambda + 2 \\ \hline 1 \end{array} \quad \begin{array}{r} \lambda^3 - 3\lambda - 2 \\ -\lambda^3 + \lambda^2 \\ \hline -3\lambda - 2 \\ +\lambda^2 + \lambda \\ \hline -2\lambda - 2 \\ 2\lambda + 2 \\ \hline 1 \end{array} \quad \begin{array}{r} \lambda^2 - \lambda - 2 \\ -\lambda^2 + \lambda \\ \hline -\lambda - 2 \\ 2\lambda + 2 \\ \hline 1 \end{array} \quad \begin{array}{r} \lambda + 1 \\ -\lambda^2 - \lambda \\ \hline -\lambda - 2 \\ 2\lambda + 2 \\ \hline 1 \end{array}$$

$$(\lambda - 1)^2 (\lambda + 1)^3 (\lambda - 2) = 0$$

$$\begin{cases} \lambda_1 = \lambda_2 = 1 \\ \lambda_3 = \lambda_4 = \lambda_5 = -1 \\ \lambda_6 = 2 \end{cases} \Rightarrow e^t, t e^t, e^{-t}, t e^{-t}, t^2 e^{-t}, e^{2t} - \text{sist. fundam. de sol.}$$

$$x(t) = c_1 e^t + c_2 t e^t + c_3 e^{-t} + c_4 t e^{-t} + c_5 t^2 e^{-t} + c_6 e^{2t}$$

## Alfa - Descompunere

$$(1) f) x^{(6)} - x^{(5)} - 4x^{(4)} + 2x''' + 5x'' - x' - 2x = 0$$

$$\begin{cases} x = e^{\lambda t} \\ x' = \lambda e^{\lambda t} \\ x'' = \lambda^2 e^{\lambda t} \\ x''' = \lambda^3 e^{\lambda t} \\ x^{(4)} = \lambda^4 e^{\lambda t} \\ x^{(5)} = \lambda^5 e^{\lambda t} \\ x^{(6)} = \lambda^6 e^{\lambda t} \end{cases} \Rightarrow \lambda^6 e^{\lambda t} - \lambda^5 e^{\lambda t} - 4\lambda^4 e^{\lambda t} + 2\lambda^3 e^{\lambda t} + 5\lambda^2 e^{\lambda t} - \lambda e^{\lambda t} - 2e^{\lambda t} = 0 \quad / : e^{\lambda t}$$

$$\lambda^6 - \lambda^5 - 4\lambda^4 + 2\lambda^3 + 5\lambda^2 - \lambda - 2 = 0$$

$$\begin{array}{r} \lambda^6 - \lambda^5 - 4\lambda^4 + 2\lambda^3 + 5\lambda^2 - \lambda - 2 \\ -\lambda^6 + \lambda^5 \\ \hline -4\lambda^4 + 2\lambda^3 + 5\lambda^2 - \lambda - 2 \\ +4\lambda^4 - 4\lambda^3 \\ \hline -2\lambda^3 + 5\lambda^2 - \lambda - 2 \\ +2\lambda^3 - 2\lambda^2 \\ \hline 3\lambda^2 - \lambda - 2 \\ -3\lambda^2 + 3\lambda \\ \hline -\lambda - 2 \\ +\lambda + 2 \\ \hline 0 \end{array} \quad \begin{array}{r} \lambda^5 - 4\lambda^3 - 2\lambda^2 + 3\lambda + 2 \\ -\lambda^5 + \lambda^4 \\ \hline \lambda^4 - 4\lambda^3 - 2\lambda^2 + 3\lambda + 2 \\ -\lambda^4 + \lambda^3 \\ \hline -3\lambda^3 - 2\lambda^2 + 3\lambda + 2 \\ +3\lambda^3 - 3\lambda^2 \\ \hline -5\lambda^2 + 3\lambda + 2 \\ +5\lambda^2 - 5\lambda \\ \hline -2\lambda + 2 \\ +2\lambda - 2 \\ \hline 0 \end{array} \quad \begin{array}{r} \lambda^4 + \lambda^3 - 3\lambda^2 - 5\lambda - 2 \\ -\lambda^4 + 2\lambda^3 \\ \hline 3\lambda^3 - 3\lambda^2 - 5\lambda - 2 \\ -3\lambda^3 + 6\lambda^2 \\ \hline 3\lambda^2 - 5\lambda - 2 \\ -3\lambda^2 + 6\lambda \\ \hline \lambda - 2 \\ -\lambda + 2 \\ \hline 0 \end{array}$$

$$(\lambda - 1)^2 (\lambda - 2) (\lambda + 1)^3 = 0$$

$$\begin{cases} \lambda_1 = \lambda_2 = 1 \\ \lambda_3 = 2 \\ \lambda_4 = \lambda_5 = \lambda_6 = -1 \end{cases} \Rightarrow e^{1t}, te^{1t}, e^{2t}, e^{-1t}, te^{-1t}, t^2 e^{-1t} \text{ - sist. fundam. de sol.}$$

$$x(t) = C_1 e^t + C_2 t e^t + C_3 e^{2t} + C_4 e^{-t} + C_5 t e^{-t} + C_6 t^2 e^{-t}$$

### Exercițiu 3. a)

$$(11) a) x'' + x = 0, \quad x(0) = 3, \quad x'(0) = 5$$

$$\begin{cases} x = e^{\lambda t} \\ x' = \lambda e^{\lambda t} \\ x'' = \lambda^2 e^{\lambda t} \end{cases} \Rightarrow \lambda^2 e^{\lambda t} + e^{\lambda t} = 0 \quad / : e^{\lambda t}$$

$$\lambda^2 + 1 = 0$$

$$\lambda^2 = -1$$

$$\lambda = \pm i \quad (\alpha \pm i\beta, \alpha = 0, \beta = 1)$$

$$e^{0t} \cos 1t, e^{0t} \sin 1t \quad (\text{sist. fundam. de sol.})$$

$$x(t) = C_1 \cos t + C_2 \sin t$$

$$x(0) = C_1 \cos 0 + C_2 \sin 0 = \boxed{C_1 = 3}$$

$$x'(t) = C_1 \cdot (-\sin t) + C_2 \cdot \cos t$$

$$x'(0) = C_1 \cdot (-\sin 0) + C_2 \cdot \cos 0 = \boxed{C_2 = 5}$$

$$x_{pc}(t) = 3 \cos t + 5 \sin t$$

### Exercițiu 3. c) - [Video](#)

III) c)  $x^{IV} + 8x'' + 16x = 0$

$$\begin{cases} x = e^{\lambda t} \\ x' = \lambda e^{\lambda t} \\ x'' = \lambda^2 e^{\lambda t} \\ x''' = \lambda^3 e^{\lambda t} \\ x^{IV} = \lambda^4 e^{\lambda t} \end{cases} \Rightarrow \lambda^4 e^{\lambda t} + 8\lambda^2 e^{\lambda t} + 16e^{\lambda t} = 0 \quad | : e^{\lambda t}$$

$$\lambda^4 + 8\lambda^2 + 16 = 0$$

$$\text{Not } \lambda = p \Rightarrow p^2 + 8p + 16 = 0$$

$$(p+4)^2 = 0$$

$$\Rightarrow p = -4 \quad (\text{de } 2 \text{ ori})$$

$$\lambda^2 = -4 \quad (\text{de } 2 \text{ ori})$$

$$\lambda = \pm 2i \quad (\text{de } 2 \text{ ori}) \quad (\alpha \pm i\beta, \alpha=0, \beta=2)$$

$e^{0 \cdot t} \cos 2 \cdot t, e^{0 \cdot t} \sin 2 \cdot t, t \cdot e^{0 \cdot t} \cos 2 \cdot t, t \cdot e^{0 \cdot t} \sin 2 \cdot t$  - sist. fundam. de sol.

$$x(t) = C_1 \cos 2t + C_2 \sin 2t + C_3 t \cos 2t + C_4 t \sin 2t$$

## Exercițiu 4. a) - Video

IV)

$$\begin{cases} x = e^{\lambda t} \\ x' = \lambda e^{\lambda t} \\ x'' = \lambda^2 e^{\lambda t} \\ x''' = \lambda^3 e^{\lambda t} \\ x^{IV} = \lambda^4 e^{\lambda t} \end{cases} \Rightarrow \lambda^4 e^{\lambda t} + 2\lambda^3 e^{\lambda t} + 4\lambda^2 e^{\lambda t} - 2\lambda e^{\lambda t} - 5e^{\lambda t} = 0 \quad | : e^{\lambda t}$$

$$\lambda^4 + 2\lambda^3 + 4\lambda^2 - 2\lambda - 5 = 0$$

$$\begin{array}{r} \lambda^4 + 2\lambda^3 + 4\lambda^2 - 2\lambda - 5 \\ -\lambda^4 + \lambda^3 \\ \hline 3\lambda^3 + 4\lambda^2 - 2\lambda - 5 \\ -3\lambda^3 + 3\lambda^2 \\ \hline 7\lambda^2 - 2\lambda - 5 \\ -7\lambda^2 + 7\lambda \\ \hline 5\lambda - 5 \\ -5\lambda + 5 \\ \hline 0 \end{array}$$

$$\begin{array}{r} \lambda^3 + 3\lambda^2 + 7\lambda + 5 \\ -\lambda^3 - \lambda^2 \\ \hline 2\lambda^2 + 7\lambda + 5 \\ -2\lambda^2 - 2\lambda \\ \hline 5\lambda + 5 \\ -5\lambda - 5 \\ \hline 0 \end{array}$$

$$\begin{array}{r} \lambda^2 + 2\lambda + 5 \\ -\lambda^2 - 2\lambda \\ \hline 5 \end{array}$$

$$(\lambda-1)(\lambda+1)(\lambda^2+2\lambda+5) = 0$$

$$\lambda_1 = 1$$

$$\lambda_2 = -1$$

$$\lambda^2 + 2\lambda + 5 = 0$$

$$\Delta = 4 - 20 = -16$$

$$\lambda_{1,2} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$$

$$(\alpha \pm i\beta, \alpha=-1, \beta=2)$$

$e^{-1 \cdot t}, e^{-1 \cdot t} \cos 2 \cdot t, e^{-1 \cdot t} \sin 2 \cdot t$  (sist. fundam. de sol.)

$$x(t) = C_1 e^t + C_2 e^{-t} + C_3 e^{-t} \cos 2t + C_4 e^{-t} \sin 2t$$

## Exercițiu 4. c)

V) c)  $x^{(7)} - x^{(6)} + x^{(5)} - x^{(4)} = 0$

$$\begin{cases} x = e^{\lambda t} \\ x' = \lambda e^{\lambda t} \\ x'' = \lambda^2 e^{\lambda t} \\ x''' = \lambda^3 e^{\lambda t} \\ x^{IV} = \lambda^4 e^{\lambda t} \\ x^{(5)} = \lambda^5 e^{\lambda t} \\ x^{(6)} = \lambda^6 e^{\lambda t} \\ x^{(7)} = \lambda^7 e^{\lambda t} \end{cases} \Rightarrow \lambda^7 e^{\lambda t} - \lambda^6 e^{\lambda t} + \lambda^5 e^{\lambda t} - \lambda^4 e^{\lambda t} = 0 \quad | : e^{\lambda t}$$

$$\lambda^7 - \lambda^6 + \lambda^5 - \lambda^4 = 0$$

$$\lambda^4 (\lambda^3 - \lambda^2 + \lambda - 1) = 0$$

$$\lambda^4 [\lambda^2(\lambda-1) + (\lambda-1)] = 0$$

$$\lambda^4 (\lambda-1)(\lambda^2+1) = 0$$

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$$

$$\lambda - 1 = 0 \Rightarrow \lambda_5 = 1$$

$$\lambda^2 + 1 = 0 \Rightarrow \lambda^2 = -1 \Rightarrow \lambda_{6,7} = \pm i \quad (\alpha \pm i\beta, \alpha=0, \beta=1)$$

$e^{0 \cdot t}, t \cdot e^{0 \cdot t}, t^2 \cdot e^{0 \cdot t}, t^3 \cdot e^{0 \cdot t}, e^{1 \cdot t}, e^{0 \cdot t} \cos 1 \cdot t, e^{0 \cdot t} \sin 1 \cdot t$  (sist. fundam. de sol.)

$$x(t) = C_1 + C_2 t + C_3 t^2 + C_4 t^3 + C_5 e^t + C_6 \cos t + C_7 \sin t$$

