Ecuații diferențiale și cu derivate parțiale
Curs 04
30.10.2020

4. f) Seminar 03

$(t-x-1)+(x-t+2)x'=0$

$x'(-t+x+2)=-t+x+1$

$x'=\frac{(-t+x)+1}{(-t+x)+2}=f\left(\frac{a_1t+b_1x+c_1}{a_2t+b_2x+c_2}\right)$

$D=\begin{vmatrix}-1 & 1\\ -1 & 1\end{vmatrix}=0$

$\boxed{u=-t+x} \Rightarrow u'=-1+x' \Rightarrow x'=u'+1$

$u'+1=\frac{u+1}{u+2} \Rightarrow u'=\frac{u+1}{u+2}-\frac{u+2}{1}=\frac{u+1-u-2}{u+2}=\frac{-1}{u+2}$

$\frac{du}{dt}=\frac{-1}{u+2}$

$(u+2)du=-dt \Leftrightarrow \int(u+2)du=-\int dt$

$\frac{u^2}{2}+2u=-t+C \quad |\cdot 2 \Leftrightarrow u^2+4u+2t=C$

$(x-t)^2+4(x-t)+2t=C$

$(x-t)^2+4x-2t=C$

4) c) Seminar 04

$\underbrace{(2tx-t)}_{P(t,x)}dt+\underbrace{(x^2+x+2t^2)}_{Q(t,x)}dx=0\ |$

$\left.\begin{aligned}\frac{\partial P}{\partial x}(t,x)=2t\\ \frac{\partial Q}{\partial t}(t,x)=4t\end{aligned}\right\}\ \frac{\partial P}{\partial x}\neq\frac{\partial Q}{\partial t}$

Pp. $\mu=\mu(t)$

$$\underbrace{\mu(t)(2tx-t)dt}_{p^*(t,x)} + \underbrace{\mu(t)(x^2+x+2t^2)}_{q^*(t,x)}dx = 0$$

$$\frac{\partial p^*}{\partial x} = \mu(t)\cdot 2t \ ; \quad \frac{\partial q^*}{\partial t} = \mu'(t)(x^2+x+2t^2) + \mu(t)\cdot 4t$$

$$\mu'(t)(x^2+x+2t^2) + 4t\cdot\mu(t) = 2t\,\mu(t)$$

$$\mu'(t)(x^2+x+2t^2) = -2t\,\mu(t) \quad \text{(nu e bine!)}$$

P.P $\mu = \mu(x)$

$$\underbrace{\mu(x)(2tx-t)dt}_{p^*(t,x)} + \underbrace{\mu(x)(x^2+x+2t^2)}_{q^*(t,x)}dx = 0$$

$$\frac{\partial p^*}{\partial x} = \mu'(x)(2tx-t) + \mu(x)\cdot 2t$$

$$\frac{\partial q^*}{\partial t} = \mu(t)\cdot 4t.$$

$$\mu'(x)(2tx-t) + 2t\,\mu(x) = 4t\,\mu(x)$$

$$\mu'(x)\cdot t(2x-1) = 2t\,\mu(x)$$

$$\mu'(x)(2x-1) = 2\mu(t)$$

$$\frac{d\mu}{dx}(2x-1) = 2\mu \ ; \quad \frac{d\mu}{\mu} = \frac{2}{2x-1}dx$$

$$\ln|\mu| = \ln|2x-1| + C \iff \ln|\mu| = \ln C|2x-1|$$

Fie $C = 1 \Rightarrow \mu(x) = 2x-1.$

$$\underbrace{(2x-1)(2tx-1)dt}_{p^*(t,x)} + \underbrace{(2x-1)(x^2+x+2t^2)}_{q^*(t,x)}dx = 0$$

$$F(t,x) = c$$

$$F(t,x) = \int_0^t p^*(z,0)\,dz + \int_0^x q^*(t,\xi)\,d\xi$$

$$= \int_0^t (-1)(-z)\,dz + \int_0^x (2\xi-1)(\xi^2+\xi+2t^2)\,d\xi.$$

$$= \int_0^t \xi\,d\xi + \int_0^x (2\xi^3 + 2\xi^2 + 4\xi t^2 - \xi^2 - \xi - 2t^2)\,d\xi$$

$$= \frac{z^2}{2}\Big|_0^t + \left(2\cdot\frac{\xi^4}{4} + 2\cdot\frac{\xi^3}{3} + 4t^2\cdot\frac{\xi^2}{2} - \frac{\xi^3}{3} - \frac{\xi^2}{2} - 2t^2\xi\right)\Big|_0^x$$

$$= \dots$$

3. a. Lab. 4.

$$\underbrace{\frac{t}{x^2}}_{P(t,x)} dt + \underbrace{\frac{x^2-t^2}{x^3}}_{Q(t,x)} dx = 0.$$

$$\left.\begin{aligned} \frac{\partial P}{\partial x} &= \frac{-t \cdot 2x}{x^4} = \frac{-2t}{x^3} \\ \frac{\partial Q}{\partial t} &= \left(\frac{1}{x} - \frac{t^2}{x^3}\right)'_t = -\frac{2t}{x^3} \end{aligned}\right\} \Rightarrow \frac{\partial P}{\partial x} = \frac{\partial Q}{\partial t}$$

$F(t,x) = C$, $F(t,x) = \int_0^t P(\tau, x_0)\, d\tau + \int_{x_0}^x Q(t,\tau)\, d\tau$

$$= \int_0^t \frac{\tau}{x_0^2}\, d\tau + \int_{x_0}^x \frac{\tau^2 - t^2}{\tau^3}\, d\tau$$

$$= \frac{1}{x_0^2} \cdot \frac{\tau^2}{2}\Big|_0^t + \int_{x_0}^x \left(\frac{1}{\tau} - \frac{t^2}{\tau^3}\right) d\tau$$

$$= \frac{t^2}{x_0^2} + \ln|\tau|\Big|_{x_0}^x - t^2 \int_{x_0}^x \tau^{-3}\, d\tau$$

$$= \frac{t^2}{x_0^2} + \ln x - \ln x_0 - t^2 \frac{\tau^{-2}}{-2}\Big|_{x_0}^x$$

$$F(t,x) = \frac{t^2}{2x_0^2} + \ln|x| - \ln|x_0| + \frac{t^2}{2x^2} - \frac{t^2}{2x_0^2}$$

$$\ln|x| + \frac{t^2}{2x^2} = C$$

2. a) Laborator 04

$$\underbrace{x' + x^2}_{A(t)} - \underbrace{2x \sin t}_{B(t)} + \underbrace{\sin^2 t - \cos t = 0}_{C(t)}, \quad \underset{L(\sin)}{\varphi_0(t) = \sin t}$$

$x = y + \varphi_0 \Rightarrow x = y + \sin t \Rightarrow x' = y' + \cos t$

$y' + \cancel{\cos t} + (y + \sin t)^2 - 2 \sin t (y + \sin t) + \sin^2 t - \cancel{\cos t} = 0$

$y' + \cos t + (y + \sin t)^2 - 2 \sin t (y + \sin t) + \sin^2 t - \cos t = 0$

$y + y^2 + 2y \sin t + \sin^2 t - 2y \sin t - 2 \sin^2 t + \sin^2 t = 0$

$y' + y^2 = 0$

$$\frac{dy}{dt} = -y^2$$

$$\frac{dy}{-y^2} = dt \Rightarrow \int -\frac{1}{y^2}\,dy = \int dt \Leftrightarrow \frac{1}{y} = t + C \Rightarrow y = \frac{1}{t+C},\ t, C \neq 0.$$

$$\Rightarrow x = \frac{1}{t+C} + \sin t$$

2. d). Lab 03

$$\begin{cases} \frac{dx}{dt} = x - t^2 \\ x(1) = 2 \end{cases}$$

$$\frac{dx}{dt} = \underbrace{x}_{A(t)} - \underbrace{t^2}_{B(t)}$$

<u>Etapa 1</u>

$$\frac{dx}{dt} = x;\ \frac{dx}{x} = dt \Leftrightarrow \int \frac{1}{x}\,dx = \int dt \Leftrightarrow \ln|x| = t + C.$$

$$x_0 = C \cdot e^t$$

<u>Etapa 2</u>: $Y_0 = C(t)e^t$

$$(C(t)e^t)' = C(t)e^t - t^2$$

$$C'(t)\cdot e^t + C(t)\cdot e^t = C(t)e^t - t^2$$

$$C'(t)e^t = -t^2 \quad |\cdot e^{-t} \Rightarrow C'(t) = -t^2 e^{-t}$$

$$C(t) = -\int t^2 e^{-t}\,dt = \int t^2 (e^{-t})' = t^2 e^{-t} - \int 2t e^{-t}\,dt$$

$$\downarrow f \quad \downarrow g' \qquad f' = 2t$$

$$= t^2 e^{-t} - 2\int t \cdot e^{-t}\,dt$$

$$= t^2 e^{-t} + 2\int t\,(e^{-t})'\,dt$$

$$\downarrow f \quad \downarrow g'$$

$$= t^2 e^{-t} + 2\left(t e^{-t} - \int e^{-t}\,dt\right)$$

$$= t^2 e^{-t} + 2t e^{-t} + 2e^{-t} + C_1 = (t^2 + 2t + 2)e^{-t} + C_1$$

$$\overline{Y_0} = \left((t^2 + 2t + 2)e^{-t} + C_1\right)e^{t} = t^2 + 2t + 2 + C_1 e^t$$

$$x(t) = Ce^t + t^2 + 2t + 2$$

$$x(1) = Ce + 1 + 2 + 2 = Ce + 5 = 2 \Rightarrow Ce = -3 \Rightarrow C = \frac{-3}{e} \Rightarrow x_{PC} = -3e^{t-1} + t^2 + 2t + 2.$$