EDP!

S

1. Să se determine sol. generală a sistemelor simetrice:

a) $\dfrac{dx}{x} = -\dfrac{dy}{2y} = \dfrac{dz}{-z}$, $x \neq 0, y \neq 0, z \neq 0$

R: $\begin{cases} \varphi_1(x,y,z) = x\sqrt{y} = C_1 \\ \varphi_2(x,y,z) = xz = C_2 \end{cases}$

b) $\dfrac{dx}{z-y} = \dfrac{dy}{x-z} = \dfrac{dz}{y-x}$; $x \neq y \neq z$

R: $\begin{cases} x+y+z = C_1 \\ x^2+y^2+z^2 = C_2 \end{cases}$

c) $\dfrac{dx}{xy^2} = \dfrac{dy}{x^2y} = \dfrac{dz}{z(x^2+y^2)}$, $x \neq 0$, $y \neq 0$, $z \neq 0$

R: $\begin{cases} x^2 - y^2 = C_1 \\ \dfrac{xy}{z} = C_2 \end{cases}$

d) $\dfrac{dx}{2y(2-x)} = \dfrac{dy}{x^2-z^2-y^2-4x} = \dfrac{dz}{-2yz}$, $x > 2$, $y \neq 0$, $z > 0$

R: $\begin{cases} \dfrac{x-2}{y} = C_1 \\ \dfrac{x^2+y^2+z^2}{z} = C_2 \end{cases}$

e) $\dfrac{dx}{x(x+y)} = \dfrac{dy}{-y(x+y)} = \dfrac{dz}{(y-x)(2x+2y+z)}$; $|x| = |y| \neq 0$,

R: $\begin{cases} xy = C_1 \\ (x+y)(x+y+z) = C_2 \end{cases}$

f) $\dfrac{dx}{2xz} = \dfrac{dy}{2yz} = \dfrac{dz}{x^2-y^2}$; $|x| \neq |y| \neq 0$

R: $\begin{cases} \dfrac{y}{x} = C_1 \\ 2z^2 - x^2 + y^2 = C_2 \end{cases}$

2. Să se rezolve următoarele sisteme cu ajutorul integralelor prime

a) $\begin{cases} x' = \dfrac{y}{x-y} \\ y' = \dfrac{x}{x-y} \end{cases}$ $x \neq y$

R: $\begin{cases} x - y + t = C_1 \\ x^2 - y^2 = C_2 \end{cases}$

b) $\begin{cases} x' = y \\ y' = -\dfrac{y^2+1}{x} \end{cases}$, $x \neq 0$

R: $\begin{cases} y^2(z^2+1) = C_1 \\ yz + x = C_2 \end{cases}$

c) $\begin{cases} x' = 2zy \\ y' = 4xz \\ z' = xy \end{cases}$, $x \neq 0$, $y \neq 0$, $z \neq 0$

R: $\begin{cases} \dfrac{y^2}{2} - 2z^2 = C_1 \\ \dfrac{x^2}{2} - z^2 = C_2 \\ x^2 + \dfrac{y^2}{2} - 2z^2 = C_3 \end{cases}$

d) $\begin{cases} x' = x \\ y' = y \\ z' = -2xy \end{cases}$ $x \neq 0$, $y \neq 0$ $\quad R: \begin{cases} \frac{x}{y} = C_1 \\ \ln x - t = C_2 \\ xy - z = C_3 \end{cases}$ $\Rightarrow$ $x = Ce^t$, $y = \frac{C}{C_1}e^t = C_4 e^t$, $z = CC_4 e^{2t} - C_3$

e) $\begin{cases} x' = y \\ y' = x \\ z' = x - y \end{cases}$ $y \neq x \neq 0$ $\quad R: \begin{cases} x^2 - y^2 = C_1 \\ x - y + z = C_2 \\ y - x - e^{-t} = C_3 \end{cases}$ $\Rightarrow x - y = C_2 - z$

f) $\begin{cases} x' = y + xy \\ y' = x + yx \\ z' = z^2 - 1 \end{cases}$ $x \neq y$, $|z| > 1$ $\quad R: \begin{cases} (x-y)\sqrt{z^2-1} = C_1 \\ \ln(x-y) + t = C_2 \\ \frac{1}{2}\ln(z^2-1) - t = C_3 \end{cases}$

g) $\begin{cases} x' = xy \\ y' = -y^2 \\ z' = -x(1+x^2) \end{cases}$ $x > 0$, $y > 0$ $\quad R: \begin{cases} xy = C_1 \\ \frac{1}{y} - t = C_2 \\ \frac{x^3}{3} + \frac{x^5}{5} + xyz = C_3 \end{cases}$