# MCEN90017: Advanced Motion Control Assignment 2
# Modelling and Control of an Active Suspension System

**Report Due Date: 11:59:59 PM on Saturday May 16, 2020**

## 1 Introduction

In this assignment you will identify a model and design a model-based controller for an Active Suspension System. You will be working individually and will be required to submit a report via Canvas of not more than 10 pages on the findings related to the tasks given in the assignment.

The University of Melbourne takes plagiarism seriously. Each person is required to submit a report that complies with the university policies on plagiarism. It is your responsibility to read and understand the policies as detailed in https://academichonesty.unimelb.edu.au/.

## 2 Learning objectives

- Experience of mathematical modelling of a control plant
- Design and implementation of model-based controllers

## 3 Before you start

Active suspension is a technology that allows achieving greater degree of ride comfort and car handling. A demonstration of such a system in action can be seen here:

https://www.youtube.com/watch?v=3KPYIaks1UY

In this lab you are tasked to design a controller for a quarter car active suspension system.

You are to design a controller that actively drives the suspension so that it maximises three system and vehicle performance metrics, where possible, subject to tradeoffs between them. The performance metrics may be described qualitatively as follows:

- *Ride Comfort* is related to vehicle body motion sensed by the passengers. A measure for the Ride Comfort is the velocity of the sprung mass in the quarter car model.

- *Suspension Travel* refers to relative displacement between the vehicle body and the tyre. It is constrained within an allowable workspace and excessive suspension travel can lead to premature ageing of the system. In the vehicle, relative displacement between the sprung mass and the unsprung mass represents Suspension Travel.

- *Road Handling* is associated with the contact forces between the road surface and the vehicle tyres. These forces provide the friction between the road and the tyres in a real car. They depend on the tyre deflection. In a quarter car model, the displacement between the unsprung mass and the road represents the tyre deflection.

Different vehicle types place different importance on these metrics. For example, a sports car will typically sacrifice Ride Comfort to achieve better Road Handling, whilst the opposite is often true in a luxury vehicle.

## Part 1 Mathematical modelling

The mass-spring damper model is as illustrated in Figure 1. With this model, the two inputs to the system are considered to be active suspension control command $F_c$ and the road surface position $z_r$.

The generalised coordinate $x_1$ represents the tyre displacement (unsprung mass in quarter car model) and $x_2$ represents the vehicle body displacement (sprung mass in the quarter car model) all with respect to the points where the springs are relaxed. The positive directions are upwards.

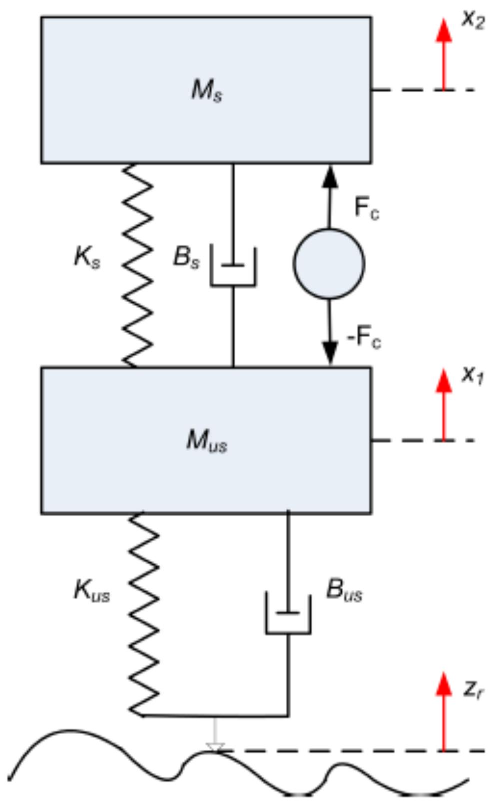



**Figure 1: Double mass-spring-damper model used to model the quarter car.**

| Parameter | Name | Unit |
|---|---|---|
| $M_s$ | Sprung mass | (Kg) |
| $M_{us}$ | Unsprung mass | (Kg) |
| $K_s$ | Suspension stiffness | (N/m) |
| $K_{us}$ | Tyre stiffness | (N/m) |
| $B_s$ | Suspension inherent damping coefficient | (Ns/m) |
| $B_{us}$ | Tyre inherent damping coefficient | (Ns/m) |

**Table 1: Quarter car parameters**

On the vehicle, inertial measurement units (IMUs) provide (noisy) measurements from the suspension system. The available measurements are the suspension deflection/travel, $(x_2 - x_1)$; the vehicle body velocity, $\dot{x}_2$; the tyre deflection, $(x_1 - z_r)$; and the tyre velocity, $\dot{x}_1$.

**Derive the governing differential equation of motion of the nominal (i.e. without measurement noise) quarter car model with active suspension for general parameters $\boldsymbol{M_s}$, $\boldsymbol{M_{us}}$, $\boldsymbol{K_s}$, $\boldsymbol{K_{us}}$ and $\boldsymbol{B_s}$, if it can be reasonably assumed that $\boldsymbol{B_{us} = 0}$.**

Your answer should be expressed in the continuous time LTI form:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du^{\cdot}$$

## Part 2: System Identification

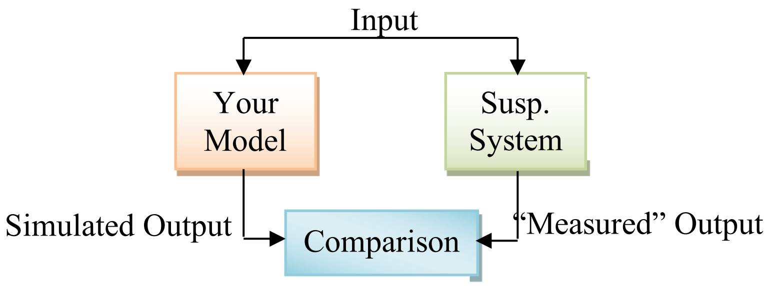
Now that you have a physics-based continuous time model of the system, you need to parameterise it using sampled data. You can do this using the following steps.

1. Your first step is to discretise appropriately the model that you derived in part 1.

2. Now download the Assignment2_2020.slx file from CANVAS. This is serving as a proxy for a hardware-based system, with data generated by the model replacing data that would otherwise be physically measured and collected. This file includes the continuous time model with inputs being the road profile, $z_r$, and applied force, $F_c$. The model is sampled with a period of 0.05 seconds (i.e. 20Hz).

3. Choosing an appropriate road profile trajectory $z_r(t)$ for system identification, collect data points that will allow you to estimate the unknown physical parameters of your model (i.e. those in Table 1). You may choose any trajectory for $F_c$(t).

4. Validate your model by comparing the responses of your identified system and the Simulink model of the system to a different road profile than used in step 3.




## Part 3 Controller design and implementation.

Now that you (hopefully) have a control-oriented model of the system, you can design a feedback controller to automatically generate the active force, $F_c$. Model predictive control (MPC) is a model-based controller, which explicitly incorporates constraints into the feedback controller and uses a discrete time model of the system, and is required for this part of the assignment. The constraints that you should attempt to satisfy at all times are given in Table 2.

**You will now need to design an MPC controller for $\mathbf{F_c}$ to regulate the states of the suspension system to the origin in a manner you deem appropriate given the metrics on page 1.**

1. Formulate a cost function that addresses the metrics, and is able to provide some form of stability guarantee. Use it to set up the online control problem that you will solve.

2. You will now test your controller's ability to deliver on your proposed problem formulation in Step 1. Trial the closed loop performance on a step and two sinusoidal road profiles with amplitude 0.1m and frequencies of 1 rad/s and 5 rad/s. You may assume the road profile is known over the prediction horizon if that improves performance.

3. Investigate the performance of your controller for three different combinations of tuning parameters (i.e. Q, R in a quadratic stage cost formulation and the prediction horizon, N).

| Specification | Constraint |
|---|---|
| Actuator limit (input constraint) | $\lvert F_c \rvert \leq 2.5$ |
| Maximum suspension travel | $\lvert x_2 - x_1 \rvert \leq 0.2$ |
| Maximum vehicle body velocity | $\lvert \dot{x}_2 \rvert \leq 0.4$ |

**Table 2: Performance specifications**

# 4 Assessment

**Report (20 marks)**

Prepare a report to discuss your findings. The report should be limited to 10 pages and:

1. Address and discuss the tasks given above,
2. Display sound understanding of the controllers that have been implemented, and
3. Discuss any other relevant thoughts.

Prepare a report as a pdf file. Use an appropriate engineering report format. A small portion of the assignment mark will be assigned to the formatting of the report

Attach the relevant MATLAB files with your report.