School of Computer Science and Applied Mathematics
UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

APPM4058A & COMS7238A: Digital Image Processing
Assignment 2

Hand-out date: May 31, 10:00
Hand-in date: June 15, 23:59

Instructions

- This is an individual assignment.
- Hand in the electronic files and source code on Ulwazi. See more instructions in Section **Hand-Ins**.
- You may use built in image processing functions, unless otherwise specified where image processing algorithms must be coded from scratch.
- The total marks available for this assignment is 50 for Hons students, and 68 for MSc students.

Objectives

- Implement and apply various morphological image processing, image segmentation, and image transformation techniques.
- Understand and apply binary and grayscale image morphology, image segmentation, image transformation and dimensionality reduction.

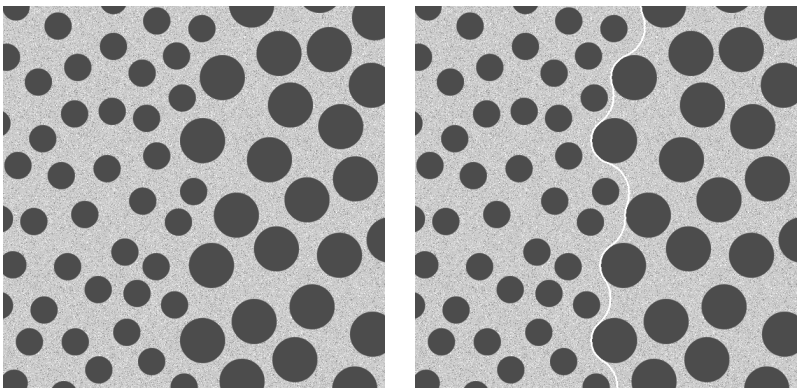
Problems

1. Image **blobs.tif** contains two texture regions — circular blobs of smaller size on the left, and circular blobs of larger size on the right (see Figure 1, (a)). Find the boundary between the two regions using suitable morphological approaches we learned. The result should appear like the one in Figure 1, (b). [6]

2. image **page.png** contains parts of the first page of a paper. Binarize (segment) the image using a thresholding based approach. [6]

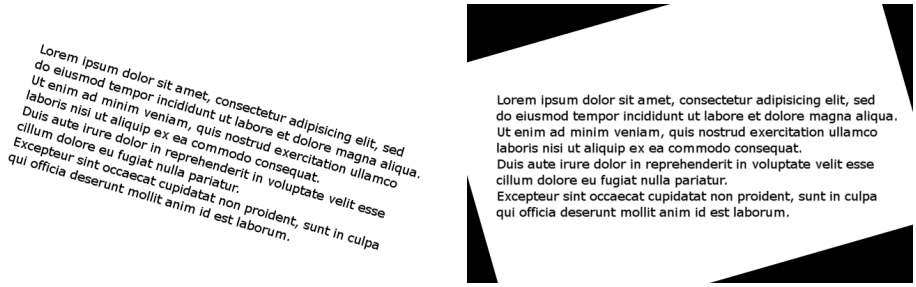
3. Using watershed segmentation and other image processing techniques where necessary to segment (separate) the coins in image **coins_color.jpg**. [12]

4. The text in image **skewed_text.png** is skewed. Propose a Hough transform-based approach that could correct the skewness of the text in the image. In the corrected image, the text should appear aligned along the horizontal direction (see Figure 2 (b)). Implement and test your proposed approach. [8]



(a) (b)

Figure 1: (a) Original image; (b) Two texture regions are separated.



(a) (b)

Figure 2: (a) Original image with skewed text; (b) After correction.

5. Folder **att_faces_aligned** contains facial images of 40 classes (persons), where each class has 10 images. Compute the eigenfaces and perform face identification based on following steps. [18]

   (a) Compute the eigenfaces

   i. Obtain the face images $I_1, I_2, \cdots, I_M$ (training faces)

   ii. Represent every image I_i as a vector Γ_i

   iii. Compute the average (or mean) face vector Ψ:

$$\Psi = \frac{1}{M}\sum_{i=1}^{M}\Gamma_i \tag{1}$$

   Display the mean face image.

   iv. Subtract the mean face:

$$\Phi_i = \Gamma_i - \Psi \tag{2}$$

   v. Compute the covariance matrix C:

$$C = \frac{1}{M}\sum_{n=1}^{M}\Phi_n\Phi_n^{\mathrm{T}} = \mathbf{A}\mathbf{A}^{\mathrm{T}} \quad (N^2 \times N^2 \text{ matrix}) \tag{3}$$

   where $\mathbf{A} = [\Phi_1\Phi_2\cdots\Phi_M] \quad (N^2 \times M \text{ matrix})$.

vi. To compute the eigenvectors $\mathbf{u}_i$ of $\mathbf{A}\mathbf{A}^{\mathrm{T}}$, we compute the matrix $\mathbf{A}^{\mathrm{T}}\mathbf{A}$, then compute the eigenvectors $\mathbf{v}_i$ of $\mathbf{A}^{\mathrm{T}}\mathbf{A}$.

vii. Compute the M best eigenvectors of $\mathbf{A}\mathbf{A}^{\mathrm{T}}$: $\mathbf{u}_i = \mathbf{A}\mathbf{v}_i$ (normalize $\mathbf{u}_i$ such that $|\mathbf{u}_i| = 1$)

viii. Keep only K eigenvectors corresponding to the K largest eigenvalues.

ix. Represent each face (minus the mean) Φ_i in the training set as a linear combination of the best K eigenvectors:

$$\Phi_i = \sum_{j=1}^{K} \mathbf{w}_j \mathbf{u}_j, \ (\mathbf{w}_j = \mathbf{u}_j^{\mathrm{T}} \Phi_i) \tag{4}$$

($\mathbf{u}_j$'s are called eigenfaces.)

x. Each training face Φ_i (normalized) is represented in this basis by a vector:

$$\Omega_i = \begin{bmatrix} w_1^i \\ w_2^i \\ \vdots \\ w_k^i \end{bmatrix}, \quad i = 1, 2, \ldots, M \tag{5}$$

Display the top ten eigenfaces in two rows with each row containing 5 of them.

(b) Face identification

i. Given a test face image Γ, subtract the mean face, the result is denoted by Φ.

ii. Project Φ on the eigenspace. $\hat{\Phi} = \sum_{i=1}^{K} \mathbf{w}_i \mathbf{u}_i$ $(\mathbf{w}_i = \mathbf{u}_i^{\mathrm{T}} \Phi)$

iii. Represent Φ as the coefficient vector Ω obtained in eigenspace. $\Omega = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_k \end{bmatrix}$.

iv. Find $e_r = \min_l \left\| \Omega - \Omega^l \right\|$.

v. Then Γ is recognized as face l from the training set. (You may also consider setting a threshold here.)

(c) Split the data into 50% training and 50% testing data, and measure the accuracy of face identification for the test dataset by varying K (e.g., $K = \{20, 40, 60, 80, 100\}$). Plot the accuracies obtained with regard to K, i.e., the number of eigenfaces.

Additional Problems for MSc Students

1. Implement SLIC algorithm given in slides `chap6_4`. Test your implementation for image **iceberg.tif** to produce results similar to those on Pg.23 in slides `chap6_4`. [10]

2. Perform image segmentation in RGB color space for image **jupiter-moon-closeup.tif** based on the method given on Pg.38 in `chap7` slides . The result expected is similar to those on Pg. 39 in `chap7` slides. [8]

Hand-Ins

Hand in your solutions for each question, together with brief description of the methods being used, comments, and discussions, in a single Jupyter notebook file or an equivalent of it in Matlab.