02623 Finite Element Method for Differential Equations

Week 2 Exercise Solutions

EXERCISE 2.1
CASE 1:
==========

```
x =         0
         0
         0
         0
    0.2500
    0.2500
    0.2500
    0.2500
    0.5000
    0.5000
    0.5000
    0.5000
    0.7500
    0.7500
    0.7500
    0.7500
    1.0000
    1.0000
    1.0000
    1.0000

y =    1.0000
    0.6667
    0.3333
         0
    1.0000
    0.6667
    0.3333
         0
    1.0000
    0.6667
    0.3333
         0
    1.0000
    0.6667
    0.3333
         0
    1.0000
    0.6667
    0.3333
         0
```

```
elmtab =     1     6     5
     2     6     1
     2     7     6
     3     7     2
     3     8     7
     4     8     3
     5    10     9
     6    10     5
     6    11    10
     7    11     6
     7    12    11
     8    12     7
     9    14    13
    10    14     9
    10    15    14
    11    15    10
    11    16    15
    12    16    11
    13    18    17
    14    18    13
    14    19    18
    15    19    14
    15    20    19
    16    20    15

CASE 2:
=======================================
x =   -2.5000
   -2.5000
   -2.5000
   -2.5000
   -0.6000
   -0.6000
   -0.6000
   -0.6000
    1.3000
    1.3000
    1.3000
    1.3000
    3.2000
    3.2000
    3.2000
    3.2000
    5.1000
    5.1000
    5.1000
    5.1000

y =    1.1000
   -0.8667
   -2.8333
   -4.8000
```

```
   1.1000
  -0.8667
  -2.8333
  -4.8000
   1.1000
  -0.8667
  -2.8333
  -4.8000
   1.1000
  -0.8667
  -2.8333
  -4.8000
   1.1000
  -0.8667
  -2.8333
  -4.8000
```

EXERCISE 2.2

CASE 2.2a:

```
=======================================
delta =    1.8683abc =   -6.5633   -1.9667   -1.9000
    4.9167    1.9667         0
    5.3833         0    1.9000
```

CASE 2.2b:

```
======
Face 1
n1 =     0
n2 =    -1
======
Face 2
n1 =    0.7192
n2 =    0.6948
======
Face 3
n1 =    -1
n2 =     0
```

EXERCISE 2.3

CASE 1:

```
=======================================
B =   -0.6667   -0.3750    1.0417         0         0
   -1.3333   -0.3750    2.0833   -0.3750         0
   -1.3333   -0.3750    2.0833   -0.3750         0
   -0.6667         0    1.0417   -0.3750         0
   -0.6667   -0.7500    2.0833         0   -0.6667
   -1.3333   -0.7500    4.1667   -0.7500   -1.3333
   -1.3333   -0.7500    4.1667   -0.7500   -1.3333
   -0.6667         0    2.0833   -0.7500   -0.6667
   -0.6667   -0.7500    2.0833         0   -0.6667
   -1.3333   -0.7500    4.1667   -0.7500   -1.3333
```

```
   -1.3333   -0.7500    4.1667   -0.7500   -1.3333
   -0.6667         0    2.0833   -0.7500   -0.6667
   -0.6667   -0.7500    2.0833         0   -0.6667
   -1.3333   -0.7500    4.1667   -0.7500   -1.3333
   -1.3333   -0.7500    4.1667   -0.7500   -1.3333
   -0.6667         0    2.0833   -0.7500   -0.6667
         0   -0.3750    1.0417         0   -0.6667
         0   -0.3750    2.0833   -0.3750   -1.3333
         0   -0.3750    2.0833   -0.3750   -1.3333
         0         0    1.0417   -0.3750   -0.6667

d =    -4
    -1
     0
     1
     4

CASE 2:
=========================================
B =   -0.5175   -0.4831    1.0006         0         0
   -1.0351   -0.4831    2.0012   -0.4831         0
   -1.0351   -0.4831    2.0012   -0.4831         0
   -0.5175         0    1.0006   -0.4831         0
   -0.5175   -0.9661    2.0012         0   -0.5175
   -1.0351   -0.9661    4.0024   -0.9661   -1.0351
   -1.0351   -0.9661    4.0024   -0.9661   -1.0351
   -0.5175         0    2.0012   -0.9661   -0.5175
   -0.5175   -0.9661    2.0012         0   -0.5175
   -1.0351   -0.9661    4.0024   -0.9661   -1.0351
   -1.0351   -0.9661    4.0024   -0.9661   -1.0351
   -0.5175         0    2.0012   -0.9661   -0.5175
   -0.5175   -0.9661    2.0012         0   -0.5175
   -1.0351   -0.9661    4.0024   -0.9661   -1.0351
   -1.0351   -0.9661    4.0024   -0.9661   -1.0351
   -0.5175         0    2.0012   -0.9661   -0.5175
         0   -0.4831    1.0006         0   -0.5175
         0   -0.4831    2.0012   -0.4831   -1.0351
         0   -0.4831    2.0012   -0.4831   -1.0351
         0         0    1.0006   -0.4831   -0.5175

d =    -4
    -1
     0
     1
     4
```

```
b =    9.3832
    9.9506
    2.6018
    0.5674
   -0.8996
   -0.4982
  -15.1958
   -6.9474
  -22.1986
  -43.0962
  -57.7938
  -28.2464
  -43.4976
  -85.6942
 -100.3918
  -49.5454
  -17.3824
  -53.0468
  -60.3956
  -43.0132
```

Output to Exercise 2.4
CASE 1:
==========

elmtab =

```
    5     1     6
    2     6     1
    6     2     7
    3     7     2
    7     3     8
    4     8     3
    9     5    10
    6    10     5
   10     6    11
    7    11     6
   11     7    12
    8    12     7
   13     9    14
   10    14     9
   14    10    15
   11    15    10
   15    11    16
   12    16    11
   17    13    18
   14    18    13
   18    14    19
   15    19    14
```

```
   19   15   20
   16   20   15


B =

         0         0    1.0000         0         0
         0         0    1.0000         0         0
         0         0    1.0000         0         0
         0         0    1.0000         0         0
         0         0    1.0000         0         0
   -1.3333   -0.7500    4.1667         0         0
   -1.3333         0    4.1667   -0.7500         0
         0         0    1.0000         0         0
         0         0    1.0000         0         0
   -1.3333   -0.7500    4.1667         0   -1.3333
   -1.3333         0    4.1667   -0.7500   -1.3333
         0         0    1.0000         0         0
         0         0    1.0000         0         0
         0   -0.7500    4.1667         0   -1.3333
         0         0    4.1667   -0.7500   -1.3333
         0         0    1.0000         0         0
         0         0    1.0000         0         0
         0         0    1.0000         0         0
         0         0    1.0000         0         0
         0         0    1.0000         0         0


d =

   -4
   -1
    0
    1
    4


b =

   1.0000
   1.0000
   1.0000
   1.0000
   1.0000
   2.0833
   2.0833
   1.0000
```

```
   1.0000
   0.7500
   0.7500
   1.0000
   1.0000
   2.0833
   2.0833
   1.0000
   1.0000
   1.0000
   1.0000
   1.0000
```

full(A(1:13,1:13)) =

ans =

```
   1.0000         0         0         0         0         0         0         0         0         0         0         0         0
        0    1.0000         0         0         0         0         0         0         0         0         0         0         0
        0         0    1.0000         0         0         0         0         0         0         0         0         0         0
        0         0         0    1.0000         0         0         0         0         0         0         0         0         0
        0         0         0         0    1.0000         0         0         0         0         0         0         0         0
        0         0         0         0         0    4.1667   -0.7500         0         0   -1.3333         0         0         0
        0         0         0         0         0   -0.7500    4.1667         0         0         0   -1.3333         0         0
        0         0         0         0         0         0         0    1.0000         0         0         0         0         0
        0         0         0         0         0         0         0         0    1.0000         0         0         0         0
        0         0         0         0         0   -1.3333         0         0         0    4.1667   -0.7500         0         0
        0         0         0         0         0         0   -1.3333         0         0   -0.7500    4.1667         0         0
        0         0         0         0         0         0         0         0         0         0         0    1.0000         0
        0         0         0         0         0         0         0         0         0         0         0         0    1.0000
```

CASE 2:
==========

B =

```
        0         0    1.0000         0         0
        0         0    1.0000         0         0
        0         0    1.0000         0         0
        0         0    1.0000         0         0
        0         0    1.0000         0         0
   -1.0351   -0.9661    4.0024         0         0
   -1.0351         0    4.0024   -0.9661         0
        0         0    1.0000         0         0
        0         0    1.0000         0         0
   -1.0351   -0.9661    4.0024         0   -1.0351
   -1.0351         0    4.0024   -0.9661   -1.0351
```

```
     0      0   1.0000       0       0
     0      0   1.0000       0       0
     0  -0.9661   4.0024       0  -1.0351
     0      0   4.0024  -0.9661  -1.0351
     0      0   1.0000       0       0
     0      0   1.0000       0       0
     0      0   1.0000       0       0
     0      0   1.0000       0       0
     0      0   1.0000       0       0
```

d =

```
   -4
   -1
    0
    1
    4
```

b =

```
  -22.2900
  -10.4572
    9.1111
   36.4150
   -0.4020
  -11.7107
   16.9886
   23.5520
    0.5480
  -42.5668
  -26.5414
   32.3490
   21.7140
   95.6645
  220.9047
  103.9600
  104.2500
  154.9441
  213.3738
  279.5390
```

EXERCISE 2.6. Hint: Check slide 30 in lecture 4 on Mesh Generation. Signed distance functions are available in the DistMesh package.

To find the DistMesh package, look for the paper and the matlab files via the link.
Persson, P.-O. and Strang, G. 2004 A simple mesh generator in Matlab. SIAM Review.
Download scripts at: http://persson.berkeley.edu/distmesh/