

02623 Finite Element Method for Differential Equations

Week 2 Exercise Solutions

EXERCISE 2.1

CASE 1:

```
=====
```

x = 0
0
0
0
0.2500
0.2500
0.2500
0.2500
0.5000
0.5000
0.5000
0.5000
0.7500
0.7500
0.7500
0.7500
0.7500
1.0000
1.0000
1.0000
1.0000

y = 1.0000
0.6667
0.3333
0
1.0000
0.6667
0.3333
0
1.0000
0.6667
0.3333
0
1.0000
0.6667
0.3333
0
1.0000
0.6667
0.3333
0

```
elmtab = 1 6 5
 2 6 1
 2 7 6
 3 7 2
 3 8 7
 4 8 3
 5 10 9
 6 10 5
 6 11 10
 7 11 6
 7 12 11
 8 12 7
 9 14 13
10 14 9
10 15 14
11 15 10
11 16 15
12 16 11
13 18 17
14 18 13
14 19 18
15 19 14
15 20 19
16 20 15
```

CASE 2:

```
=====
x = -2.5000
```

```
-2.5000
-2.5000
-2.5000
-0.6000
-0.6000
-0.6000
-0.6000
1.3000
1.3000
1.3000
1.3000
3.2000
3.2000
3.2000
3.2000
5.1000
5.1000
5.1000
5.1000
```

```
y = 1.1000
```

```
-0.8667
-2.8333
-4.8000
```

1.1000
-0.8667
-2.8333
-4.8000
1.1000
-0.8667
-2.8333
-4.8000
1.1000
-0.8667
-2.8333
-4.8000
1.1000
-0.8667
-2.8333
-4.8000
1.1000
-0.8667
-2.8333
-4.8000

EXERCISE 2.2

CASE 2.2a:

```
=====
delta = 1.8683abc = -6.5633 -1.9667 -1.9000
  4.9167  1.9667    0
  5.3833      0  1.9000
```

CASE 2.2b:

```
=====
Face 1
n1 =  0
n2 = -1
=====
Face 2
n1 =  0.7192
n2 =  0.6948
=====
Face 3
n1 = -1
n2 =  0
```

EXERCISE 2.3

CASE 1:

```
=====
B = -0.6667 -0.3750  1.0417    0    0
 -1.3333 -0.3750  2.0833 -0.3750    0
 -1.3333 -0.3750  2.0833 -0.3750    0
 -0.6667     0  1.0417 -0.3750    0
 -0.6667 -0.7500  2.0833    0 -0.6667
 -1.3333 -0.7500  4.1667 -0.7500 -1.3333
 -1.3333 -0.7500  4.1667 -0.7500 -1.3333
 -0.6667     0  2.0833 -0.7500 -0.6667
 -0.6667 -0.7500  2.0833    0 -0.6667
 -1.3333 -0.7500  4.1667 -0.7500 -1.3333
```

```

-1.3333 -0.7500  4.1667 -0.7500 -1.3333
-0.6667      0  2.0833 -0.7500 -0.6667
-0.6667 -0.7500  2.0833      0 -0.6667
-1.3333 -0.7500  4.1667 -0.7500 -1.3333
-1.3333 -0.7500  4.1667 -0.7500 -1.3333
-0.6667      0  2.0833 -0.7500 -0.6667
      0 -0.3750  1.0417      0 -0.6667
      0 -0.3750  2.0833 -0.3750 -1.3333
      0 -0.3750  2.0833 -0.3750 -1.3333
      0      0  1.0417 -0.3750 -0.6667

```

d = -4

```

-1
 0
 1
 4

```

CASE 2:

```

=====
B = -0.5175 -0.4831  1.0006      0      0
-1.0351 -0.4831  2.0012 -0.4831      0
-1.0351 -0.4831  2.0012 -0.4831      0
-0.5175      0  1.0006 -0.4831      0
-0.5175 -0.9661  2.0012      0 -0.5175
-1.0351 -0.9661  4.0024 -0.9661 -1.0351
-1.0351 -0.9661  4.0024 -0.9661 -1.0351
-0.5175      0  2.0012 -0.9661 -0.5175
-0.5175 -0.9661  2.0012      0 -0.5175
-1.0351 -0.9661  4.0024 -0.9661 -1.0351
-1.0351 -0.9661  4.0024 -0.9661 -1.0351
-0.5175      0  2.0012 -0.9661 -0.5175
-0.5175 -0.9661  2.0012      0 -0.5175
-1.0351 -0.9661  4.0024 -0.9661 -1.0351
-1.0351 -0.9661  4.0024 -0.9661 -1.0351
-0.5175      0  2.0012 -0.9661 -0.5175
      0 -0.4831  1.0006      0 -0.5175
      0 -0.4831  2.0012 -0.4831 -1.0351
      0 -0.4831  2.0012 -0.4831 -1.0351
      0      0  1.0006 -0.4831 -0.5175

```

d = -4

```

-1
 0
 1
 4

```

b = 9.3832
 9.9506
 2.6018
 0.5674
 -0.8996
 -0.4982
 -15.1958
 -6.9474
 -22.1986
 -43.0962
 -57.7938
 -28.2464
 -43.4976
 -85.6942
 -100.3918
 -49.5454
 -17.3824
 -53.0468
 -60.3956
 -43.0132

EXERCISE 2.4

CASE 1:

```
=====
B =   0   0  1.0000   0   0
      0   0  1.0000   0   0
      0   0  1.0000   0   0
      0   0  1.0000   0   0
      0   0  1.0000   0   0
-1.3333 -0.7500  4.1667   0   0
-1.3333     0  4.1667 -0.7500   0
      0   0  1.0000   0   0
      0   0  1.0000   0   0
-1.3333 -0.7500  4.1667   0 -1.3333
-1.3333     0  4.1667 -0.7500 -1.3333
      0   0  1.0000   0   0
      0   0  1.0000   0   0
      0 -0.7500  4.1667   0 -1.3333
      0   0  4.1667 -0.7500 -1.3333
      0   0  1.0000   0   0
      0   0  1.0000   0   0
      0   0  1.0000   0   0
      0   0  1.0000   0   0
      0   0  1.0000   0   0
```

d = -4
 -1
 0
 1
 4

b = 1.0000
1.0000
1.0000
1.0000
1.0000
2.0833
2.0833
1.0000
1.0000
0.7500
0.7500
1.0000
1.0000
2.0833
2.0833
1.0000
1.0000
1.0000
1.0000
1.0000
1.0000

CASE 2:

=====

B =	0	0	1.0000	0	0
	0	0	1.0000	0	0
	0	0	1.0000	0	0
	0	0	1.0000	0	0
	0	0	1.0000	0	0
	-1.0351	-0.9661	4.0024	0	0
	-1.0351	0	4.0024	-0.9661	0
	0	0	1.0000	0	0
	0	0	1.0000	0	0
	-1.0351	-0.9661	4.0024	0	-1.0351
	-1.0351	0	4.0024	-0.9661	-1.0351
	0	0	1.0000	0	0
	0	0	1.0000	0	0
	0	-0.9661	4.0024	0	-1.0351
	0	0	4.0024	-0.9661	-1.0351
	0	0	1.0000	0	0
	0	0	1.0000	0	0
	0	0	1.0000	0	0
	0	0	1.0000	0	0
	0	0	1.0000	0	0

d = -4
-1
0
1
4

```
b = -22.2900
-10.4572
 9.1111
36.4150
-0.4020
-11.7107
16.9886
23.5520
 0.5480
-42.5668
-26.5414
32.3490
21.7140
95.6645
220.9047
103.9600
104.2500
154.9441
213.3738
279.5390
```

EXERCISE 2.6. Hint: Check slide 30 in lecture 4 on Mesh Generation. Signed distance functions are available in the DistMesh package.

To find the DistMesh package, look for the paper and the matlab files via the link.
Persson, P.-O. and Strang, G. 2004 A simple mesh generator in Matlab. SIAM Review.
Download scripts at: <http://persson.berkeley.edu/distmesh/>