

## 02623 Finite Element Method for Differential Equations

### Week 2 Exercise Solutions

#### EXERCISE 2.1

##### CASE 1:

=====

x = 0

0

0

0

0.2500

0.2500

0.2500

0.2500

0.5000

0.5000

0.5000

0.5000

0.7500

0.7500

0.7500

0.7500

1.0000

1.0000

1.0000

1.0000

y = 1.0000

0.6667

0.3333

0

1.0000

0.6667

0.3333

0

1.0000

0.6667

0.3333

0

1.0000

0.6667

0.3333

0

1.0000

0.6667

0.3333

0

```

elmtab = 1 6 5
2 6 1
2 7 6
3 7 2
3 8 7
4 8 3
5 10 9
6 10 5
6 11 10
7 11 6
7 12 11
8 12 7
9 14 13
10 14 9
10 15 14
11 15 10
11 16 15
12 16 11
13 18 17
14 18 13
14 19 18
15 19 14
15 20 19
16 20 15

```

CASE 2:

=====

x = -2.5000

```

-2.5000
-2.5000
-2.5000
-0.6000
-0.6000
-0.6000
-0.6000
1.3000
1.3000
1.3000
1.3000
3.2000
3.2000
3.2000
3.2000
5.1000
5.1000
5.1000
5.1000

```

y = 1.1000

```

-0.8667
-2.8333
-4.8000

```

1.1000  
-0.8667  
-2.8333  
-4.8000  
1.1000  
-0.8667  
-2.8333  
-4.8000  
1.1000  
-0.8667  
-2.8333  
-4.8000  
1.1000  
-0.8667  
-2.8333  
-4.8000

## EXERCISE 2.2

CASE 2.2a:

```
=====
delta =  1.8683abc = -6.5633 -1.9667 -1.9000
      4.9167  1.9667    0
      5.3833    0  1.9000
```

CASE 2.2b:

```
=====
Face 1
n1 =  0
n2 = -1
=====
Face 2
n1 =  0.7192
n2 =  0.6948
=====
Face 3
n1 = -1
n2 =  0
```

## EXERCISE 2.3

CASE 1:

```
=====
B = -0.6667 -0.3750  1.0417    0    0
    -1.3333 -0.3750  2.0833 -0.3750    0
    -1.3333 -0.3750  2.0833 -0.3750    0
    -0.6667    0  1.0417 -0.3750    0
    -0.6667 -0.7500  2.0833    0 -0.6667
    -1.3333 -0.7500  4.1667 -0.7500 -1.3333
    -1.3333 -0.7500  4.1667 -0.7500 -1.3333
    -0.6667    0  2.0833 -0.7500 -0.6667
    -0.6667 -0.7500  2.0833    0 -0.6667
    -1.3333 -0.7500  4.1667 -0.7500 -1.3333
```

```

-1.3333 -0.7500  4.1667 -0.7500 -1.3333
-0.6667    0  2.0833 -0.7500 -0.6667
-0.6667 -0.7500  2.0833    0 -0.6667
-1.3333 -0.7500  4.1667 -0.7500 -1.3333
-1.3333 -0.7500  4.1667 -0.7500 -1.3333
-0.6667    0  2.0833 -0.7500 -0.6667
    0 -0.3750  1.0417    0 -0.6667
    0 -0.3750  2.0833 -0.3750 -1.3333
    0 -0.3750  2.0833 -0.3750 -1.3333
    0    0  1.0417 -0.3750 -0.6667

```

d = -4

```

-1
0
1
4

```

CASE 2:

=====

```

B = -0.5175 -0.4831  1.0006    0    0
-1.0351 -0.4831  2.0012 -0.4831    0
-1.0351 -0.4831  2.0012 -0.4831    0
-0.5175    0  1.0006 -0.4831    0
-0.5175 -0.9661  2.0012    0 -0.5175
-1.0351 -0.9661  4.0024 -0.9661 -1.0351
-1.0351 -0.9661  4.0024 -0.9661 -1.0351
-0.5175    0  2.0012 -0.9661 -0.5175
-0.5175 -0.9661  2.0012    0 -0.5175
-1.0351 -0.9661  4.0024 -0.9661 -1.0351
-1.0351 -0.9661  4.0024 -0.9661 -1.0351
-0.5175    0  2.0012 -0.9661 -0.5175
-0.5175 -0.9661  2.0012    0 -0.5175
-1.0351 -0.9661  4.0024 -0.9661 -1.0351
-1.0351 -0.9661  4.0024 -0.9661 -1.0351
-0.5175    0  2.0012 -0.9661 -0.5175
    0 -0.4831  1.0006    0 -0.5175
    0 -0.4831  2.0012 -0.4831 -1.0351
    0 -0.4831  2.0012 -0.4831 -1.0351
    0    0  1.0006 -0.4831 -0.5175

```

d = -4

```

-1
0
1
4

```

b = 9.3832

9.9506  
2.6018  
0.5674  
-0.8996  
-0.4982  
-15.1958  
-6.9474  
-22.1986  
-43.0962  
-57.7938  
-28.2464  
-43.4976  
-85.6942  
-100.3918  
-49.5454  
-17.3824  
-53.0468  
-60.3956  
-43.0132

## EXERCISE 2.4

### CASE 1:

=====

B =     0     0   1.0000     0     0

0	0	1.0000	0	0
0	0	1.0000	0	0
0	0	1.0000	0	0
0	0	1.0000	0	0
-1.3333	-0.7500	4.1667	0	0
-1.3333	0	4.1667	-0.7500	0
0	0	1.0000	0	0
0	0	1.0000	0	0
-1.3333	-0.7500	4.1667	0	-1.3333
-1.3333	0	4.1667	-0.7500	-1.3333
0	0	1.0000	0	0
0	0	1.0000	0	0
0	-0.7500	4.1667	0	-1.3333
0	0	4.1667	-0.7500	-1.3333
0	0	1.0000	0	0
0	0	1.0000	0	0
0	0	1.0000	0	0
0	0	1.0000	0	0
0	0	1.0000	0	0

d = -4

-1  
0  
1  
4

b = 1.0000

1.0000  
1.0000  
1.0000  
1.0000  
2.0833  
2.0833  
1.0000  
1.0000  
0.7500  
0.7500  
1.0000  
1.0000  
2.0833  
2.0833  
1.0000  
1.0000  
1.0000  
1.0000  
1.0000

CASE 2:

```
=====
B =      0      0  1.0000      0      0
      0      0  1.0000      0      0
      0      0  1.0000      0      0
      0      0  1.0000      0      0
-1.0351 -0.9661  4.0024      0      0
-1.0351      0  4.0024 -0.9661      0
      0      0  1.0000      0      0
      0      0  1.0000      0      0
-1.0351 -0.9661  4.0024      0 -1.0351
-1.0351      0  4.0024 -0.9661 -1.0351
      0      0  1.0000      0      0
      0      0  1.0000      0      0
      0 -0.9661  4.0024      0 -1.0351
      0      0  4.0024 -0.9661 -1.0351
      0      0  1.0000      0      0
      0      0  1.0000      0      0
      0      0  1.0000      0      0
      0      0  1.0000      0      0
      0      0  1.0000      0      0
```

d = -4

-1  
0  
1  
4

```
b = -22.2900
-10.4572
 9.1111
36.4150
-0.4020
-11.7107
16.9886
23.5520
 0.5480
-42.5668
-26.5414
32.3490
21.7140
95.6645
220.9047
103.9600
104.2500
154.9441
213.3738
279.5390
```

EXERCISE 2.6. Hint: Check slide 30 in lecture 4 on Mesh Generation. Signed distance functions are available in the DistMesh package.

To find the DistMesh package, look for the paper and the matlab files via the link.  
Persson, P.-O. and Strang, G. 2004 A simple mesh generator in Matlab. SIAM Review.  
Download scripts at: <http://persson.berkeley.edu/distmesh/>