

VECTORS

Assignment 1

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Abstract—This document provides solution for the problem 2.13 in the gvv_ncert_vectors.pdf

1 POINTS AND VECTORS

1.1. Show that the points are collinear, and find the ratio in which B divides AC.

$$\mathbf{A} = \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix} \quad (1.1.1)$$

Solution:

let

$$\mathbf{A} = \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix} \quad (1.1.2)$$

Then

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix}, \mathbf{C} - \mathbf{A} = \begin{pmatrix} 10 \\ 5 \\ 15 \end{pmatrix} \quad (1.1.3)$$

and

$$\mathbf{M} = (\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^T \quad (1.1.4)$$

$$\mathbf{M} = \begin{pmatrix} 4 & 2 & 6 \\ 10 & 5 & 15 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - 2.5R_1} \begin{pmatrix} 4 & 2 & 6 \\ 0 & 0 & 0 \end{pmatrix} \quad (1.1.5)$$

$\Rightarrow \text{rank}(\mathbf{M}) = 1. \therefore$ Thus, the points are collinear as can be verified in Fig 1.1

let B divide AC in k:1 then

$$\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} = \frac{k \begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}}{k+1} = \frac{\begin{pmatrix} 11k+1 \\ 3k-2 \\ 7k-8 \end{pmatrix}}{k+1}$$

we get

$$\frac{3k-2}{k+1} = 0$$

$$\Rightarrow 3k-2=0$$

$$\Rightarrow k = \frac{2}{3} \Rightarrow \text{B divides AC in the ratio 2:3}$$

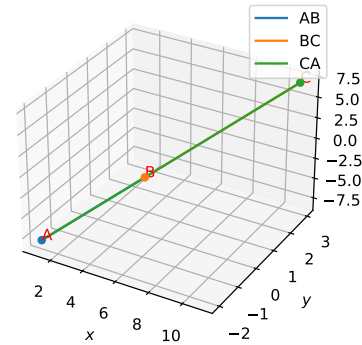


Fig. 1.1. The given points are collinear