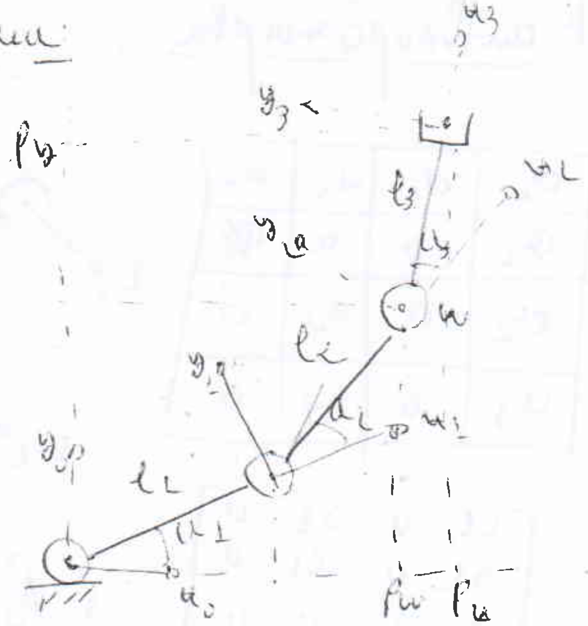
Solution TD: Robotique

1/ Robot planaire

Segment	θ_i	d_i	a_i	α_i
(1)	θ_1	0	l_1	0
(2)	θ_2	0	l_2	0
(3)	θ_3	0	l_3	0

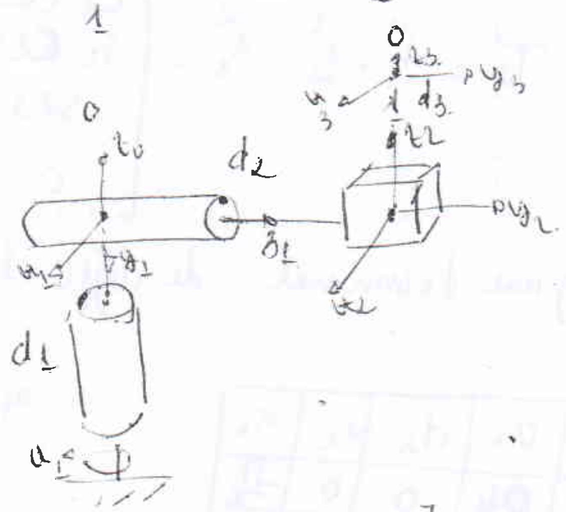


toutes les articulations sont rotoïdes, donc:

$$A_i^{i-1}(\theta_i) = \begin{bmatrix} C_i & -S_i & 0 & a_i C_i \\ S_i & C_i & 0 & a_i S_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1^0 \cdot A_2^1 \cdot A_3^2 = \begin{bmatrix} C_{123} & -S_{123} & 0 & a_1C_1 + a_2C_{12} + a_3C_{123} \\ S_{123} & C_{123} & 0 & a_1S_1 + a_2S_{12} + a_3S_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \end{bmatrix}$$

2/ Robot sphérique



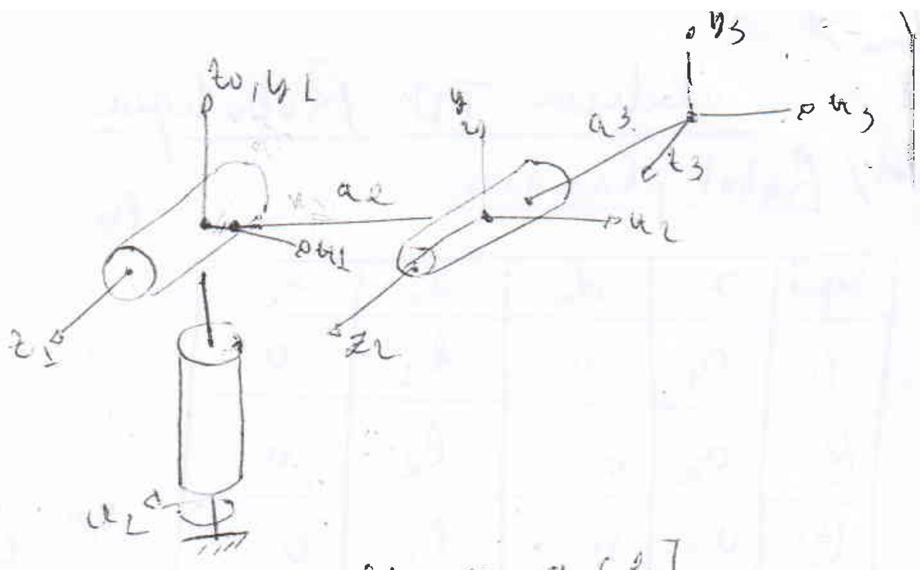
Seg	θ_i	d_i	a_i	α_i
(1)	θ_1	0	0	$-\frac{\pi}{2}$
(2)	θ_2	d_2	0	$\pi/2$
(3)	0	d_3	0	0

$$A_1^0 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad A_2^1 = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_3^0 = A_1^0 \cdot A_2^1 \cdot A_3^2 = \begin{bmatrix} C_1C_2 & -S_1 & C_1S_2 & C_1S_2d_3 - S_1d_2 \\ S_1C_2 & C_1 & S_1S_2 & S_1S_2d_3 + C_1d_2 \\ -S_2 & 0 & C_2 & C_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3/ Robot anthropomorphe

Seg	θ_i	d_i	a_i	α_i
(1)	θ_1	0	0	$\pi/2$
(2)	θ_2	0	a_2	0
(3)	θ_3	0	a_3	0



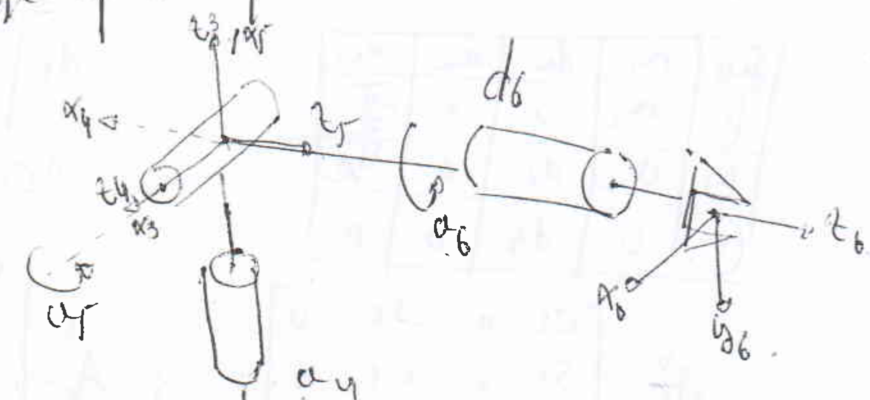
$$A_1^0 = \begin{bmatrix} C1 & 0 & S1 & 0 \\ S1 & 0 & -C1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} , \quad A_2^1 = \begin{bmatrix} C2 & -S2 & 0 & a_2 C2 \\ S2 & C2 & 0 & a_2 S2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} C3 & -S3 & 0 & a_3 C3 \\ S3 & C3 & 0 & a_3 S3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1^0 \cdot A_2^1 \cdot A_3^2 = \begin{bmatrix} C_1 C23 & -C1 S23 & S1 & C1(a_2 C_2 + a_3 C23) \\ S_1 C23 & -S1 S23 & -C1 & S_1(a_2 C_2 + a_3 C23) \\ S23 & C23 & 0 & a_2 S2 + a_3 S23 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4/ organe terminal de type sphérique

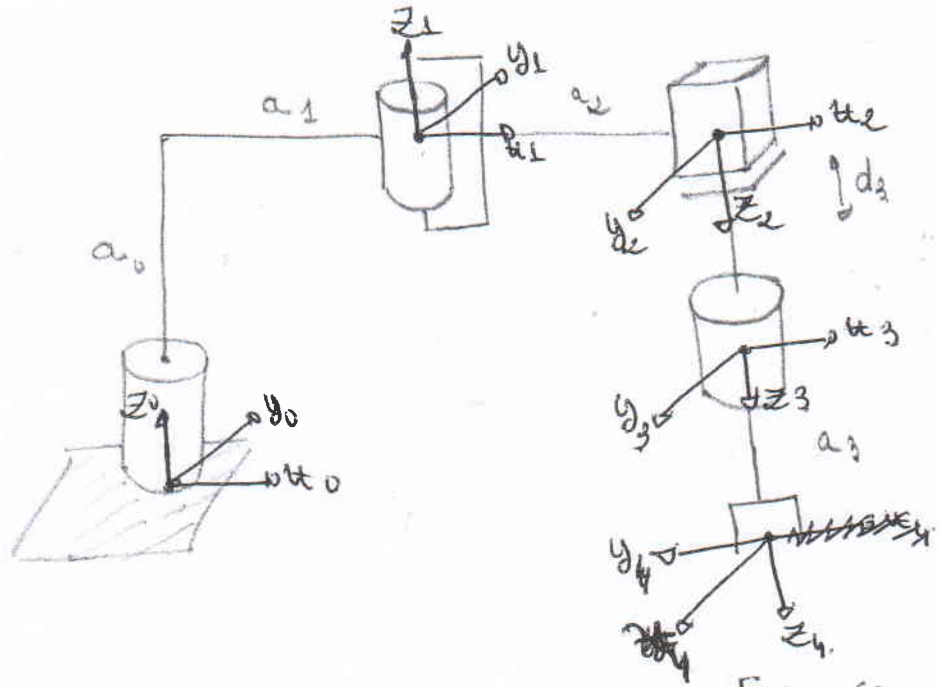
Seg	θ_i	d_i	a_i	α_i
4	θ_4	0	0	$-\pi/2$
5	θ_5	0	0	$\pi/2$
6	θ_6	$d6$	0	0



$$A_4^3 = \begin{bmatrix} C4 & 0 & -S4 & 0 \\ S4 & 0 & C4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} , \quad A_5^4 = \begin{bmatrix} C5 & 0 & S5 & 0 \\ S5 & 0 & -C5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad A_6^5 = \begin{bmatrix} C6 & -S6 & 0 & 0 \\ S6 & C6 & 0 & 0 \\ 0 & 0 & 1 & d6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = A_4^3 \cdot A_5^4 \cdot A_6^5 = \begin{bmatrix} C4C5C6 - S4S6 & -C4C5S6 - S4C6 & C4S5 & C4S5 d6 \\ S4C5C6 + C4S6 & -S4C5S6 + C4C6 & S4S5 & S4S5 d6 \\ -S5C6 & S5S6 & C5 & C5 \cdot d6 \end{bmatrix}$$

Solution au Robot SCARA.



Seg	a_i	α_i	d_i	θ_i
1	a_1	0	a_0	θ_1
2	a_2	π	0	θ_2
3	0	0	d_3	0
4	[illegible]	0	a_3	[illegible]

$$A_0^1 = \begin{bmatrix} C1 & -S1 & 0 & a_1 C1 \\ S1 & C1 & 0 & a_1 S1 \\ 0 & 0 & 1 & a_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_1^2 = \begin{bmatrix} C2 & S2 & 0 & a_2 C2 \\ S2 & -C2 & 0 & a_2 S2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3^4 = \begin{bmatrix} C3 & -S3 & 0 & 0 \\ S3 & C3 & 0 & 0 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = A_0^1 \cdot A_1^2 \cdot A_2^3 \cdot A_3^4 .$$

$$= \begin{bmatrix} C12 & S12 & 0 & a_2 C12 + a_1 C1 \\ S12 & -C12 & 0 & a_2 S12 + a_1 S1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C3 & -S3 & 0 & 0 \\ S3 & C3 & 0 & 0 \\ 0 & 0 & 1 & a_3 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} C12\,C3 + S12\,S3 & -C12\,S3 + S12\,C3 & 0 & a_2 C12 + a_1 C1 \\ S12\,C3 - C12\,S3 & -S12\,S3 + C12\,C3 & 0 & a_2 S12 + a_1 S1 \\ 0 & 0 & -1 & -(a_3 + d_3) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$