Improved Prim Algorithm and Its Application in Unmanned Aerial Vehicle Cruise System

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Abstract: Unmanned Aerial Vehicle (UAV) can perform tasks such as military reconnaissance, monitoring, search and target pointing. When a UAV is used to perform a reconnaissance task on a cruise path planning, and certain set of multi-target point, the optimal cruise path should be well scheduled to ensure that the cruise time is minimal. In this paper, an improved Prim algorithm is studied by introducing constraint condition to improve the effect of UAV path planning. Based on the data provided by GPS module, the weight matrix between the target points is obtained by using coordinate transformation, and the optimal cruise sequence is obtained by using the improved Prim algorithm. Simulation results and success application in cruise path planning of UAV show the efficiency of this improved Prim algorithm.

Key Words: UAV; path planning; Prim algorithm; coordinate transformation

1. INTRODUCTION

The recent decade has witnessed the growing concern on unmanned aerial vehicle (UAV), which has been applied in different fields. An important application of UAVs is to implement the reconnaissance mission in forest fire protection, public safety, etc. A good cruise path can not only reduce the cost of executing the reconnaissance task, but also reduce its risk. The cruise path for UAV is usually planned by the methods of static planning or dynamic planning. Static planning refers to the cruise path has been planned before taking off aircraft. Dynamic programming is the process of cruising in accordance with the need to dynamically modify the real-time path planning [1, 2]. Dynamic programming is based on static programming, so static programming is concerned in this paper.

For the problem of path planning, A-Star search algorithm [3], genetic algorithm [4], dynamic programming [5], Dijkstra algorithm [6], particle swarm algorithm [7] and so on are commonly used. A-Star algorithm determines the next path grid by comparing the heuristic function values F of eight neighbors of the current path grid. However, A-Star algorithm can't guarantee the optimal path when there are multiple minimum values. In the case of improper selection of fitness function, genetic algorithm may converge to local optimum, but can't achieve global optimization. In this paper, the UAV path optimization algorithm is based on the Prim algorithm. The minimum spanning tree (MST) is constructed by connecting vertexes one by one, and it can solve the optimal path problem [8, 9]. In this paper, an improved Prim algorithm is proposed for the path optimization problem, which has the same time complexity with Prim algorithm, but it is easier understand and achieve, and significantly superior to original algorithm. Through the validation of a large number of experimental data, the solution obtained by the proposed algorithm is similar as the global solution obtained by traversing all the possible solutions.

In practice, the target coordinates data is usually shown by latitude and longitude of Geodetic Coordinate System, which is also the most common and easiest data collection means. The input data of the system is coordinates in Geodetic Coordinate System. The data of each coordinate point must be converted into the Spatial Cartesian Coordinate system [10]. The significance of coordinate transformation is that the actual distance between any two points can be directly calculated from the Spatial Cartesian Coordinate system, which is the foundation for path planning problem.

The aircraft flight control is based on the integral separation PID algorithm [11, 12]. The accelerometer and the three-axis gyro sensors are used to obtain the acceleration and rotation angular velocity of the aircraft. The pitch angle, roll angle and yaw angle are obtained by 3D attitude calculation [13].The PID algorithm is used to control the three attitude variables of the aircraft， so as to realize the balance control of the aircraft. With the GPS module, the aircraft's own coordinates can be obtained [14]. The UAV can cruise the target point according to the cruise order after path planning. Finally, it can autonomously plan paths and perform reconnaissance tasks.

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2. PRIM ALGORITHM

Prim's algorithm is used to construct the minimum spanning tree of a graph [15-17]. It constructs the minimum spanning tree by connecting vertexes one by one. Basic idea of Primer algorithm is described as follows:

Starting from a vertex u_0 in the connected graph G = (V, {E}), the minimum weight edge(u_0, u_1) associated with it is selected and its vertices are added to the vertex set U of the spanning tree. Each step is then selected from the edges (u, v) whose vertices are included in U and the other vertices are not. The edges are added to the edge set TE of the minimum spanning tree, while the other vertices are added to the set U. Repeat this process until all the vertices are added to the vertex set U of the spanning tree.

The process of constructing the minimum spanning tree using Prim algorithm is shown in Fig.1, where V1toV6 are the target points, and the number between the two target points is the weight between the two target points. An undirected weighted connected network G is shown in Fig.1. (a). Fig.1. (b)- (f) illustrates the constructing of the minimum spanning tree. Without loss of generality, take V1 as the starting point. Find the smallest weight value from all the edges connected V1 with other points, and connect the two points as shown in Fig.1. (b). Then find the adjacent points with the smallest weight, connect them to get Fig.1. (c).Along this way, until the formation of n-1 edges, the spanning tree is constructed as shown in Fig.1. (f).

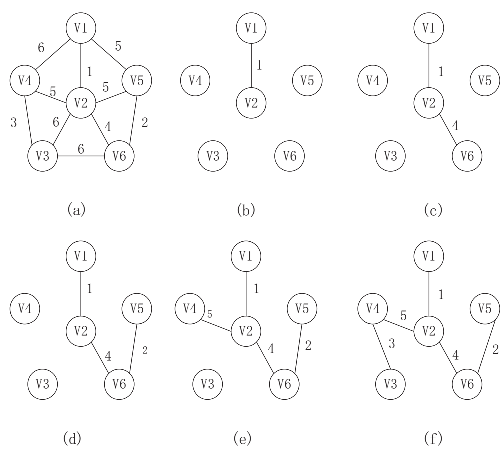

Fig.1. Constructing Process of MST based Prim Algorithm

The process of constructing the minimum spanning tree using Prim algorithm is shown in Fig.1. A more specific description to constructing of the minimum spanning tree can be described as follows.

Step1: Initialize $U = \{u_0\}(u_0 \in V), TE = \{\}$.

Step2: From the constructed edges $(u, v) \in E$ constituted by any $u \in U, v \in V - U$, an edge (u_0, v_0) with the least weight value can be found, then add it to TE , and take v_0 into U.

Step3: If $U = V$, then turn to Step4, otherwise go to Step2.

Step4: Therefore, in the spanning tree $T = (V, (TE))$, there must be an edge-set TE including $n-1$ edges, T is the minimum spanning tree of the connected graph G.

3. IMPROVED PRIM ALGORITHM

In this sub-section, the Prim algorithm is improved to solve the problem of UAV path planning. There are two aspects that need to be improved. First, from the Prim algorithm described above, the uncertainty of the initial point selection bring about the spanning tree is not unique. In practice, we want to get the spanning tree which is the unique optimal one. Therefore, Prim algorithm must be improved by setting a necessary search starting point, a search limiting conditions and a search order, to make the original uncertainty of the process into a certain process, and retain the advantages of the Prim algorithm. Second, although the minimum spanning tree path can be used directly to construct the UAV cruise path, but in reality we hope that the UAV cruise path is a non-reciprocating path. However, the minimum spanning tree will inevitably lead to some edges of the vertex degree more than 2, which will form a reciprocating path and an increase of cruising costs. In view of these two aspects of the improvement requirements, the improved algorithm is described as follows.

Step1: Traverse all edges to find the edge (u_i, u_j) with smallest weight, add the vertex u_i and u_j into the set of vertices U , $U = \{u_i, u_j\}$ where $(u_i, u_j) \in V$, and so the set of edge $TE = \{(u_i, u_j)\}$.

Step2: Taking the vertex u_i and u_j in the set $U = \{u_i, u_j\}$ as the search starting point, and the minimum spanning tree T is obtained by the Prim algorithm.

Step3: If there is any vertex degree of the minimum spanning tree T is greater than 2, deletes all edges connected to the vertex except for the two edges with the smallest weight values. Otherwise, directly turn to Step4.

Step4: Find an edge that is connected by vertices with degrees less than 2, which has the least weight. Before adding this edge to the set TE , check whether a loop is formed. If a loop is generated, the edge is discarded. Then continue to find the next smaller edge. Otherwise perform the fifth step.

Step5: Add the edge found in step4 to the set TE. If the number of edges is less than n-1 (where n is the number of vertices), turn to Step 4. If the number of edges is equal to n-1, the shortest path planning is completed.

The following 10 target points are used as an example to illustrate the effectiveness of the improved Prim algorithm. The coordinates of the target point in the Cartesian coordinate system is shown in Table 1.

The distance matrix between the any two target points is calculated from the listed ten points, as shown in Table 2.

The steps to get the optimal solution using improved Prim algorithm are described as follows.

Step 1: Initialization. Traverse all edges to find the edge with the smallest weight (V_1, V_5), these two points are added to the set U. That is $U = \{V_1, V_5\}$, $TE = \{(V_1, V_5)\}$.

Step 2: Taking the set $U = \{V_1, V_5\}$ as the starting point, use Prim algorithm to get the minimum spanning tree T, as shown in Fig. 2. (a).

Now $U = \{V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9, V_{10}\}$

$TE = \{(V_9, V_1), (V_1, V_5), (V_5, V_6), (V_6, V_3), (V_3, V_2), (V_6, V_4), (V_4, V_8), (V_8, V_{10}), (V_{10}, V_7),\}$

Step 3: It is easy to see from Fig. 2. (a), that only vertices V_4 and V_6 have degrees greater than 2, and the edge with the largest weight in the edge connected to vertex V_4 is edge (V_4, V_8). So delete the edge (V_4, V_8), edge (V_3, V_6) can be deleted in a similar way. $TE = TE \cap \complement_{TE}\{(V_3, V_6), (V_4, V_8)\}$. The results are shown in Fig. 2. (b).Then create a new set L which contains all the vertices with degrees less than 2, $L = \{V_2, V_3, V_7, V_8, V_9\}$

Step 4: In set L, any two points are connected to form an edge set TL. Find the edges with the smallest weight(V_2, V_9) in edge set TL. And add (V_2, V_9) to the set TE, since there is no loop, so the edge is added to the TE, otherwise, discard the edge, as shown in Fig.2. (c). then continue proceed to step 4, find the smallest weight(V_3, V_8), and add it to the edge set TE. Since there is no loop, and the number of edges is equal to n-1.

So the optimized path by improved Prim algorithm is completed. And the optimal path shown in Fig. 2 (d) is $V_7 - V_{10} - V_4 - V_6 - V_5 - V_1 - V_9 - V_2 - V_3 - V_8$ and the total length of the scheduled path is 234.68.

To verify the validity of the algorithm, the global optimal solution by traversing methods is 234.68. The results show that the improved Prim algorithm is just globally optimal.

4. APPLICATION OF IMPROVED PRIM ALGORITHM IN UAV PATH PLANING

4.1 UAV Experimental Platform Description

In this paper, the experimental platform is four-rotor aircraft [18]. The main controller chip for this experimental platform is STM32f104. The experimental platform contains wireless data transmission module, motor drive circuit, GPS module, and so on. In the four-rotor control panel, OLED is used as human-computer interaction interface. The data is transferred to the host Microcontroller Unit (MCU) by the NRF24L01 wireless module. Brushless motor used as the actuator, the test module is used to test whether the aircraft arrived at the designated location. The experimental platform is shown in Fig.3.

Table 1 Coordinate of Target Point

Point	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}
Coordinate	(42,83)	(85,104)	(96,78)	(73,47)	(52,72)	(69,70)	(40,23)	(105,38)	(55,103)	(70,21)

Table 2 Weight Matrix between Target Points

	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}
V_1	0.00	47.85	54.23	47.51	14.87	29.97	60.03	77.42	23.85	68.03
V_2	47.85	0.00	28.23	58.25	45.97	37.58	92.66	68.96	30.02	84.34
V_3	54.23	28.23	0.00	38.60	44.41	28.16	78.49	41.00	48.02	62.65
V_4	47.51	58.25	38.60	0.00	32.65	23.35	40.80	33.24	58.82	26.17
V_5	14.87	45.97	44.41	32.65	0.00	17.12	50.45	62.97	31.14	54.08
V_6	29.97	37.58	28.16	23.35	17.12	0.00	55.23	48.17	35.85	49.01
V_7	60.03	92.66	78.49	40.80	50.45	55.23	0.00	66.71	81.39	30.07
V_8	77.42	68.96	41.00	33.24	62.97	48.17	66.71	0.00	82.01	38.91
V_9	23.85	30.02	48.02	58.82	31.14	35.85	81.39	82.01	0.00	83.36
V_{10}	68.03	84.34	62.65	26.17	54.08	49.01	30.07	38.91	83.36	0.00

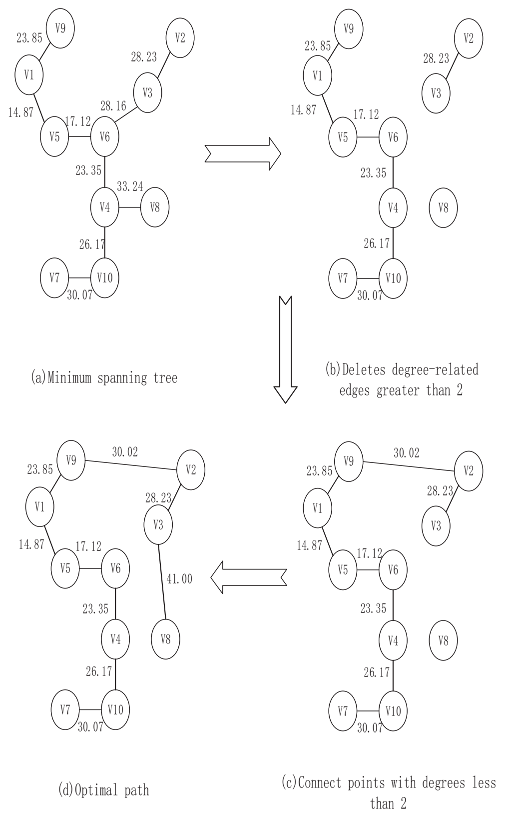

Fig.2. Path Optimization Process of Improved Prim Algorithm

Coordinate data in the geodetic coordinate system of the Specified target point is input to the main MCU before the cruise of the unmanned aerial vehicle. First, the coordinates in the geodetic coordinate system are transformed into the coordinates in Spatial Cartesian Coordinate system. Then the coordinate of Spatial Cartesian Coordinate system is input to the improved Prim algorithm. And the optimal path of UAV cruise is the output of the algorithm. Finally, the UAV perform reconnaissance missions.

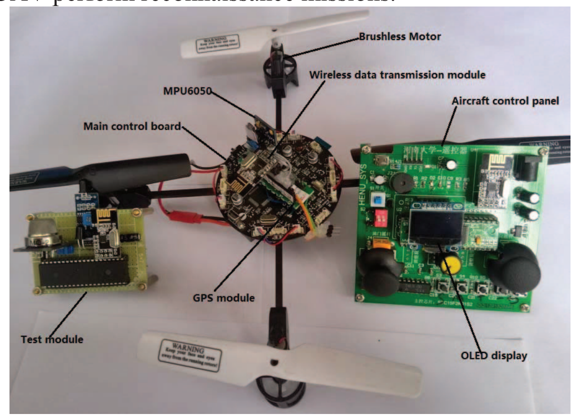

Fig.3. Experimental Platform

Figure 4 is the system framework of the experimental platform we developed. PID controller is used for attitude stabilization control of UAV. Additional GPS module is equipped on the UAV to obtain the coordinates of the aircraft itself. To apply the improved Prim algorithm in unmanned aerial cruise, it is necessary to convert the geodetic coordinate system to spatial Cartesian coordinate system. The algorithm given in Section 3 of this paper is descripted in Cartesian coordinate system.

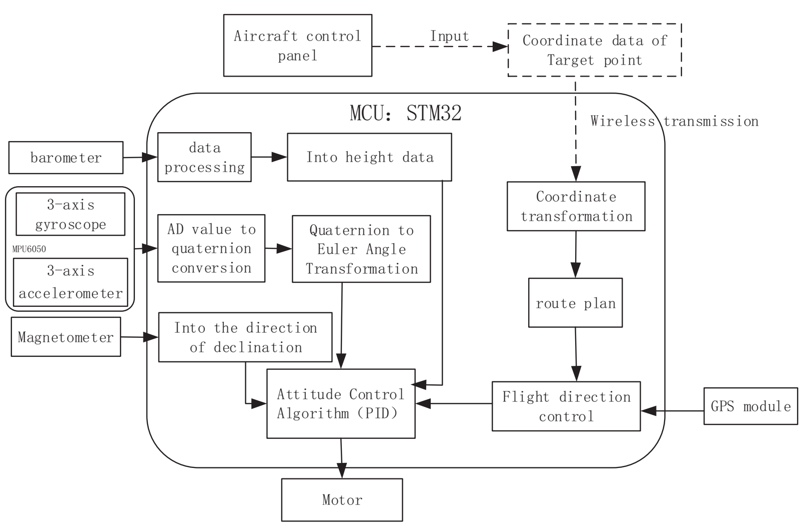

Fig.4.Framework for the Aircraft

4.2 Coordinate System Transformation

For each target point that needs to be cruised, it is very convenient to get the coordinate data in longitude and latitude. Since the path planning algorithm is developed in Spatial Cartesian coordinate system, it is necessary to transform the original data obtained in geodetic coordinate system to Spatial Cartesian coordinate system.

The origin of Spatial Cartesian coordinate system lies in the center of the reference ellipsoid. The Z axis represents the north pole of the reference ellipsoid. The X axis represents the intersection of the starting meridian and the equator. And the Y axis lies on the equatorial plane. According to the right-handed coordinate system, the angle between X and Y axis is 90°.The coordinate of a point in space can be represented by the projection on each coordinate axis of the coordinate system. The Spatial Cartesian coordinate system can be represented by Fig.5.

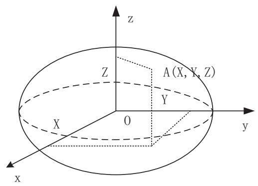

Fig.5. Spatial Cartesian coordinate system

In Geodetic Coordinate System, latitude, longitude and surface height are used to describe the spatial location. Geodetic Coordinate System can be shown in Fig. 6.

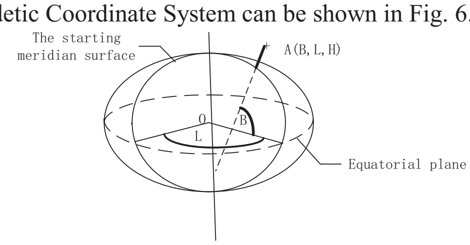

Fig.6. Geodetic Coordinate System

The coordinate transformation process can be described in Eq. (1)

$$\left.\begin{aligned} X &= (N+H)\cos B\cos L \\ Y &= (N+H)\cos B\sin L \\ Z &= [N(1-e^2)+H]\sin B \end{aligned}\right\} \qquad (1)$$

Where $N = \dfrac{a}{\sqrt{1-e\sin^2 B}}$, N is the curvature radius of ellipsoid. $e = \dfrac{c}{a} = \dfrac{\sqrt{a^2+b^2}}{a}$,e is the eccentricity, a is the long axis of the ellipsoid, b is the short axis of ellipsoid. According to the national geodetic coordinate system criterion of 2000, the value of these parameters is listed in Table 3.

Table 3 Value of the Parameter

parameter	value
a	6378.137km
b	6356.7523141km
e	0.081819191042811

Taking Henan University as an example, eleven target points are selected in Henan University. Assume that ground height of the target point is zero. By the above formula, the coordinates of Geodetic Coordinate System(L,B,H) of each target point and the corresponding coordinates of the Spatial Cartesian coordinate system are obtained. The coordinate transformation results are shown in Table 4.

Table 4 Coordinate Conversion Results

	longitude	latitude	X	Y	Z
A	114.3194	34.8249	-2185.59	4836.17	3667.23
B	114.3191	34.8253	-2185.55	4836.15	3667.28
C	114.3191	34.8257	-2185.54	4836.13	3667.31
D	114.3193	34.8270	-2185.52	4836.05	3667.43
E	114.3198	34.8277	-2185.55	4835.99	3667.50
F	114.3169	34.8274	-2185.31	4836.12	3667.47
G	114.3177	34.8259	-2185.41	4836.17	3667.33
H	114.3163	34.8246	-2185.33	4836.30	3667.21
I	114.3157	34.8246	-2185.28	4836.32	3667.21
J	114.3155	34.8236	-2185.29	4836.38	3667.11
K	114.3168	34.8230	-2185.41	4836.37	3667.06

Use the eleven target points shown in Table 4 as the test data, the path planning results obtained by the improved algorithm is shown in Fig.7. The total length is 1.3250km. Compared to the original algorithm which is shown in Fig.8 .the total length of original algorithm is 1.4302km. It is obvious that the improved algorithm can obtained a better result.

4.3 Simulation and Analysis of the Proposed Path Planning Algorithm

In order to test the validity of this algorithm, 10 different cities are selected and each city randomly selects multiple target points for analysis. Table 4 lists the geodetic coordinates and the transformed Spatial Cartesian coordinates of city A (Kaifeng city) to be cruised target points. Based on the improved Prim algorithm proposed in this paper, let it converted into C language program and integrated to experimental platform shown in Figure 4.

The coordinates of other N points to be visited in other cities are only used in MATLAB simulation of cruise path planning. The purpose of this section is to test the effectiveness of the path planning method developed in this paper. The effectiveness of the algorithm is evaluated by the total length of the path, the simulation results are shown in Table 5. The complexity of different path planning algorithm is compared in Table 6, easy see that the improved algorithm is comparable with the original algorithm in the sense of time complexity. Although the algorithm execution time is slightly increased, the accuracy of the algorithm solution is significantly improved. The improved algorithm can be regarded as a more effective algorithm.

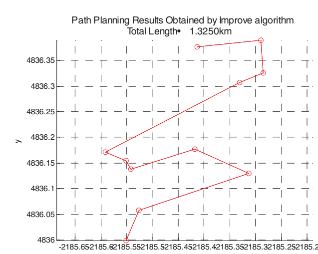

Fig.7. Path Planning Results Obtained by Improve Algorithm

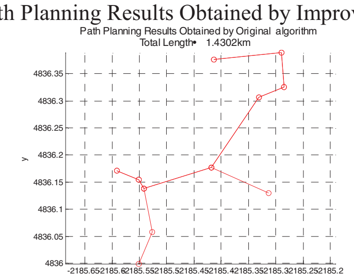

Fig.8. Path Planning Results Obtained by Original Algorithm

5. CONCLUSION

In this paper, an improved Prim algorithm is studied by introducing constraint condition to improve the effect of UAV path planning. Simulation result and success application in cruise path planning of UAV show the efficiency of this algorithm.

Table 5 Algorithm Simulation Results

City where points are located	Number of points	Improve algorithm (km)	original algorithm (km)	Optimal solution (km)	Improved Error rate (%)	Original error rate (%)
Kaifeng	11	1.3250	1.4304	1.2934	2.44	10.59
Datong	12	8.0990	8.8335	8.0990	0	9.07
Nanjing	12	9.1389	9.53994	8.8748	2.97	7.45
Shangqiu	11	6.5667	7.9601	6.5667	0	21.22
Shanghai	8	7.4576	8.6559	7.4576	0	16.07
Suzhou	8	10.2125	12.5889	10.0249	1.87	25.58
Wuxi	11	8.9443	9.49764	8.9443	0	6.19
Xian	13	10.8802	11.6238	10.4092	1.1	11.67
Zhengzhou	9	5.6563	6.1724	5.6563	0	9.12
Chongqing	9	6.3968	7.8972	6.1072	4.74	29.31

Table 6 Complex Comparison of Different Algorithm

The algorithm name	time complexity	Average time of algorithm (s)	Error rate of algorithm (%)
Prim algorithm	$O(n^2)$	1.2694	14.627
Improved Prim algorithm	$O(n^2)$	1.3685	1.312

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