POLS/CS&SS 503:
Advanced Quantitative Political Methodology

MODEL SPECIFICATION AND FIT

May 12, 2015

Jeffrey B. Arnold

UNIVERSITY of WASHINGTON
DEPARTMENT OF POLITICAL SCIENCE

CENTER for STATISTICS
and the SOCIAL SCIENCES

Overview

How To Choose Among Different Models?

- Depends on your purpose
- Some tools
  - Internal model validation: residuals, outliers
  - Overall model Fit statistics: out of sample is preferred

Measures of Model Fit

Various measure of how the model fits the data, both *in-sample* and *out-of-sample*

The Coefficient of Determination, R^2

$$R^2 = \frac{\text{Explained sum of squares}}{\text{Total sum of squares}} = 1 - \frac{\text{Residual sum of squares}}{\text{Total sum of squares}}$$

$$= \frac{\sum(\hat{y} - \bar{y})^2}{\sum(\hat{y} - \bar{y})^2}$$

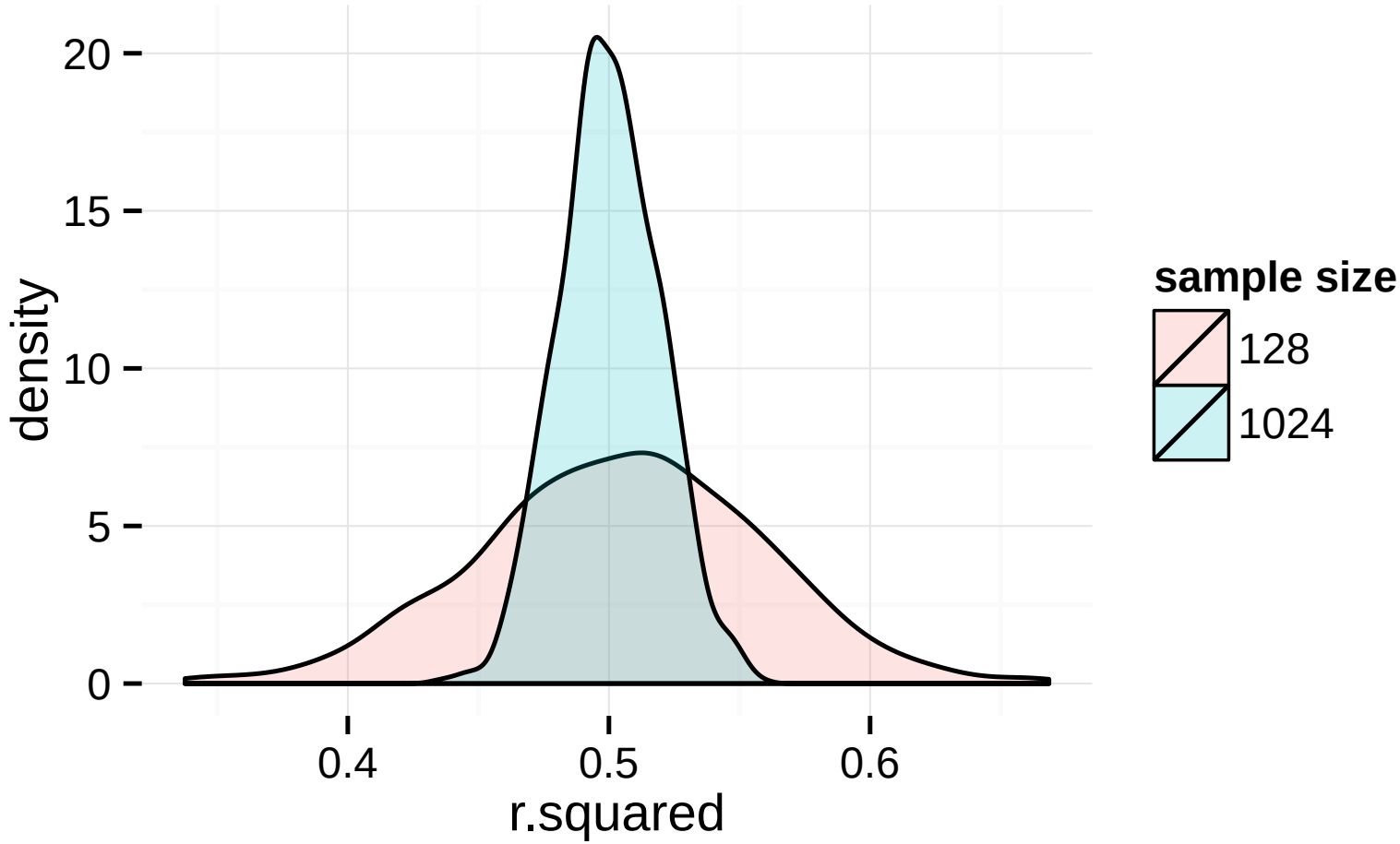
$$= 1 - \frac{\sum \hat{\epsilon}^2}{\sum(\hat{y} - \bar{y})^2}$$

- Commonly used
- Ranges between
- Why can it never be less than 0?
- What happens when you add a variable?
- What is the case when $R^2 = 1$
- Bivariate case: $\text{Cor}(y, x)^2$
- General case: $\text{Cor}(y, \hat{y})^2$

What R^2 does and doesn't say

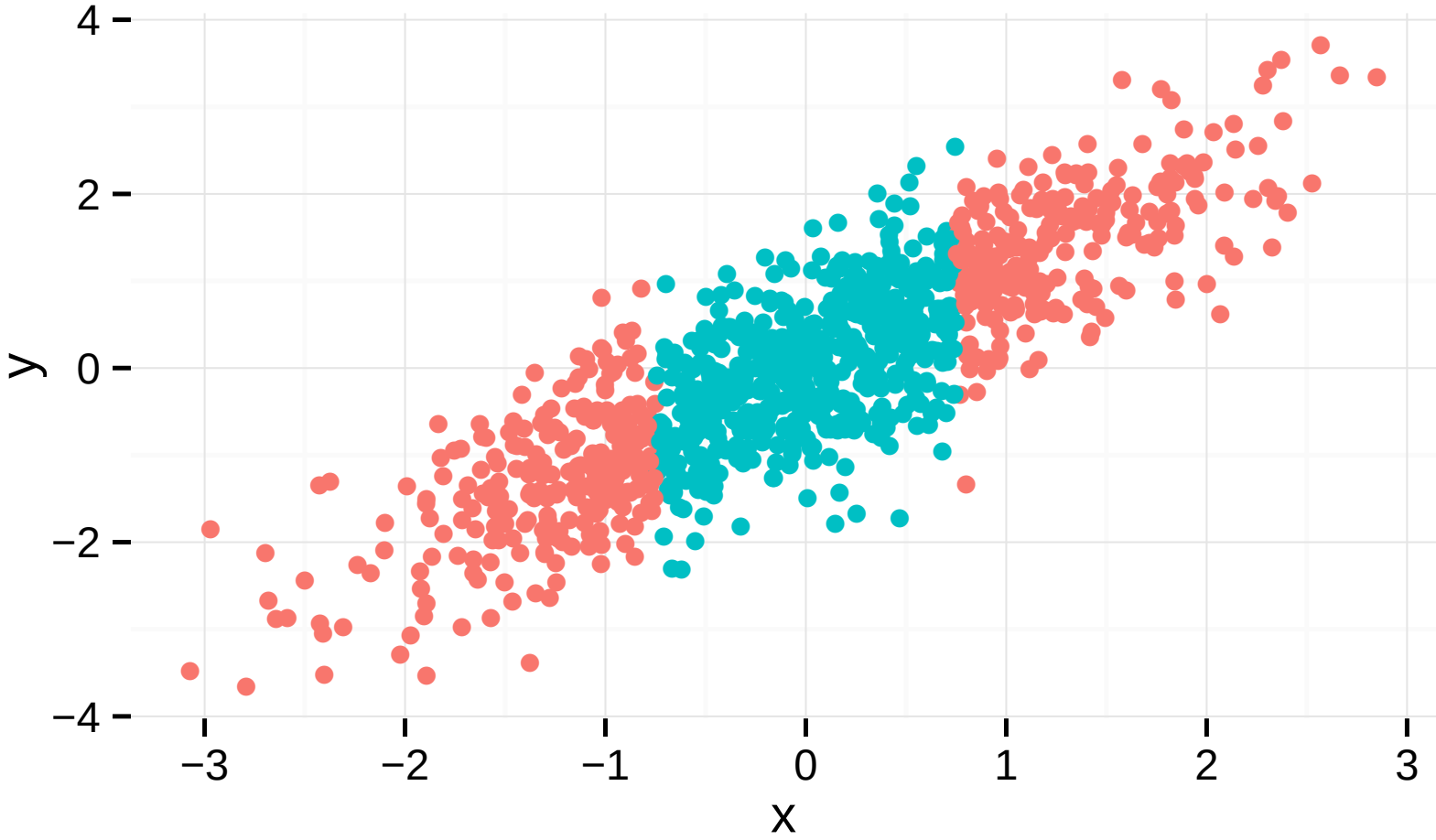
- Indirectly reports scatter around the regression line
- Only *in sample*
- Maximizing R^2 perverse:
  - Not usually interesting for explanation. Y regressed on itself, vote choice on vote intention.
  - Not usually best for prediction
- Not an estimate

R^2 varies between samples



R^2 of samples drawn from a linear model with a population $R^2 = 0.5$.

R^2 is a function of variation in X



- Complete sample (red + blue): $R^2 = 0.72$, $\hat{\sigma} = 0.65$
- Restricted sample (blue only): $R^2 = 0.29$, $\hat{\sigma} = 0.66$

Adjusted R^2

What's adjusted?

$$\begin{aligned}\tilde{R}^2 &= 1 - \frac{S_E^2}{S_Y^2} \\ &= 1 - \frac{n-1}{n-k-1} \times \frac{RSS}{TSS}\end{aligned}$$

- Where n is number of obs, k is number of variables.
- Unlike R^2, treat squared error terms as estimates of populatio, not sample statistics.
- How adjusted R^2 change with respect to n? With respect to k?
- But it is an ad hoc adjustment

Standard Error of the Regression

The standard error of the regression is the estimate of the population σ:

$$\hat{\sigma}_\epsilon = S_E = \sqrt{\frac{\sum E_i^2}{n-k-1}} = \sqrt{\frac{\sum_i (y_i - \hat{y}_i)^2}{n-k-1}}$$

- S_E is at least as useful to report as R^2
- S_E: on average, how much does the fitted value miss the actual value.
- On the same scale as y. Easier for interpretation and substantive importance.

Likelihood Function

- Likelihood is the probability of observing the data given a statistical model.
- The **likelihood** of a linear model with normal errors:

$$
\begin{aligned}
L(\hat{\beta}, \hat{\sigma}_\epsilon) = p(y|\hat{\beta}, \hat{\sigma}) &= \prod_i N(y_i|X_i\hat{\beta}, \hat{\sigma}_\epsilon^2) \\
&= \left(\frac{1}{\hat{\sigma}_\epsilon\sqrt{2\pi}}\right)^n \prod_i \exp\left(-\frac{(y_i - x_i'\hat{\beta})^2}{2\hat{\sigma}_\epsilon^2}\right) \\
&= \left(\frac{1}{\hat{\sigma}_\epsilon\sqrt{2\pi}}\right)^n \prod_i \exp\left(-\frac{\hat{\epsilon}_i^2}{2\hat{\sigma}_\epsilon^2}\right)
\end{aligned}
$$

- For computational stability (the product of probabilities is a small number), the **log likelihood** is usually used

$$
\log L(\hat{\beta}, \hat{\sigma}_\epsilon) = -n \log \hat{\sigma}_\epsilon - \frac{1}{2}\log 2\pi - \frac{1}{2\hat{\sigma}_\epsilon^2}\sum_i \hat{\epsilon}_i^2
$$

Information Criteria

- Information criteria include log Likelihod + a penalty for complexity
- The two Most common are AIC and BIC:

$$AIC = -2 \log L(\hat{\beta}, \hat{\sigma}_\epsilon) + 2k$$
$$BIC = -2 \log L(\hat{\beta}, \hat{\sigma}_\epsilon) + k \log n$$

- Lower is better
- Smaller values = better fit
- See Fox for justifications
- AIC = approx leave one out cross-validation; BIC = a specific k-fold cross-validation

Out of Sample Methods

- Compare models on how well they do on data that was not used to estimate their parameters.
- In practice, serves as a good check against spurious findings
- Even if our goal is explanation, not prediction, scientific models strive for generality
- Usual caveat: best fitting may not be the only criteria for the model

Out of Sample Goodness of Fit

- Method
  1. Split data into training $(X_{\text{training}}, y_{\text{training}})$, test data, $(X_{\text{test}}, y_{\text{test}})$.
  2. Fit model to training data, $(X_{\text{training}}, y_{\text{training}})$, obtain $\hat{\beta}_{\texttt{training}}$
  3. Calcuate fitted $\hat{y}_{\text{test}}$ for the test sample $(X_{\text{test}}, y_{\text{test}})$.
  4. Calculate predicted mean squared error of the **test** data

$$RMSE_{\text{prediction}} = \hat{\sigma}_{\text{test}} = \sqrt{\frac{1}{n_{\text{test}}} \sum_{i \in \text{test}} \hat{\epsilon}_i^2}$$

- Usually MSE of test data lower than MSE of training data. In-sample fit statistics are overly optimistic.
- Good rule of thumb: 70–75% training, 30–25% test
- Can use other prediction statistics to evaulate models

Cross Validation

Reuse data for multiple in-sample and out-of-sample tests. More efficient use of data.

- k-fold cross validation
  1. Select all but $1/k$th of the data: $(y_{\text{training}}, X_{\text{training}})$
  2. Repeat out of sample tests k times
- Leave-one-out (LOO-CV): $k = n$.
- 5– or 10–fold cross-validation; generally the best in terms of bias / variance tradeoff.
- The best model minimizes prediction RMSE
- **Important:** the test and trainining data should be from same "population". Randomly sampled in cross-section. Need to be careful in panel, blocked, or time-series.

Fox on Model Selection

Problems

- Simultaneous inference
- Fallacy of affirming the consequent
- Impact of large samples on hypothesis tests
- Exaggerated precision

Fox on Model Selection

Strategies

- Alternative model-selection criteria (not stat sig)
- Compensating for simulaneous inference
- Avoiding model selection: maximally complex and flexible model.
- Model averaging: select many models.

Fox on Model Selection

General Advice

- It is problematic to use stat. hypoth. tests for model selection. Simultaneous inference, biased results. Complicated models in large n, exaggerated prediction. (p. 6008)
- Most methods maximize *predication* not interpretation
- When purpose is interpretation, simplify based on substantive considerations, even if that includes removing small, but stat sig coefficients. (p. 622)
- **validation**: using separate model choice and inference

Gelman and Hill's Rules for Building a Regression Model for Prediction

- Include all input variables expected to be important in predicting outcome (substantively)
- Not always necessary to include these separately, e.g. indices
- For inputs with large effects, consider including interactions
- Whether to exclude a varaible from prediction based on significance
  - Not stat sig, expected sign: keep. Will not help much, but will not hurt predictions.
  - Not stat sig, not expected sign: consider removing
  - Stat sig, not expected sign: **Think hard** Are there lurking variables?
  - Stat sig, expected sign: keep
- Think hard before the model; but adjust to new information
- Gelman and Hill use *predictaion* differently than Fox.

Gelman and Hill, p. 69

References

- John Fox, *Applied Regression Analysis and Generalized Linear Models*, Ch. 22, “Model Selection, Averaging, and Validation”.
- Christopher Adolph (Spring 2014) “Linear Regression: Specification and Fitting” [Lecture slides]. |http://faculty.washington.edu/cadolph/503/topic5.pw.pdf|.