POLS/CS&SS 503:
Advanced Quantitative Political Methodology

LINEAR REGRESSION ESTIMATOR

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Overview

Regression Coefficient Anatomy

Coeficients of a simple regression

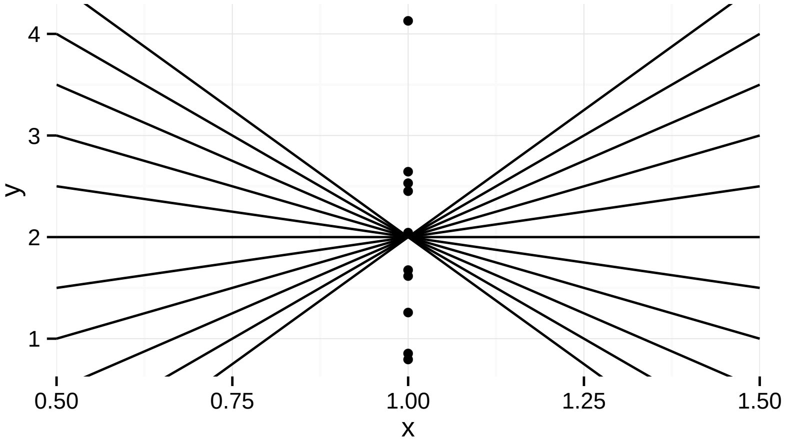
$$Y_i = A + BX_i + E_i$$

The least squares coefficients are

$$A = \bar{Y} + B\bar{X}$$
$$B = \frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_i \left(X_i - \bar{X}\right)^2}$$

- State B in terms of covariance of X and Y and variance?
- State B in terms of correlation of X and Y and standard deviations?
- What values can B take if sd(X) $=$ sd(Y) $= 1$?
- What is $\hat{Y}$ for $X = \bar{X}$?
- What happens to B as V(X) decreases? V(Y) decreases? If V$(X) = 0$

Least squares when V $X = 0$



Least squares coefficients are unidentified if V $x = 0$

- If V $x = 0$ then least squares solution is unidentified
- There is no unique value of A, B that $\arg\min_{A,B} \sum_i E_i^2$

```
y <- c(1, 2, 3, 4, 5)
x <- 1
ybar <- mean(y)
ybar
```

```
## [1] 3
```

```
# A = 2, B = 1
sum((y - 2 - 1 * x) ^ 2)
```

```
## [1] 10
```

```
# A = -7, B = 10
sum((y + 7 - 10 * x) ^ 2)
```

```
## [1] 10
```

Coefficients of a multiple regression

$$\vec{Y} = \boldsymbol{X}\boldsymbol{b} + \boldsymbol{e}$$

- $\boldsymbol{b} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}$. *Not that intuitive!*
- Coefficient $\boldsymbol{b}_j$ is

$$\boldsymbol{b}_k = \frac{\mathsf{C}(\boldsymbol{y}, \tilde{\boldsymbol{x}}_j)}{\mathsf{V}(\tilde{\boldsymbol{x}}_k)}$$

- Where $\tilde{\boldsymbol{x}}_j$ are the residulals of $\boldsymbol{x}_j$ on all X_h where $h \neq j$

$$\tilde{X}_{j,i} = X_{j,i} - \tilde{A} - \sum_{h \neq j} \tilde{B}_h X_h$$

Regression example

See `multiple_regression_anatomy.R`