POLS/CS&SS 503: Advanced Quantitative Political Methodology

TRANSFORMATIONS

May 5, 2015

Jeffrey B. Arnold

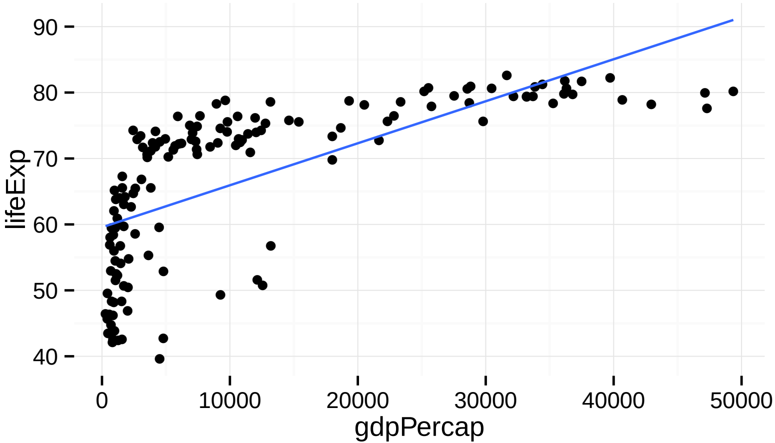
UNIVERSITY of WASHINGTON
DEPARTMENT OF POLITICAL SCIENCE

CENTER for STATISTICS and the SOCIAL SCIENCES

Overview

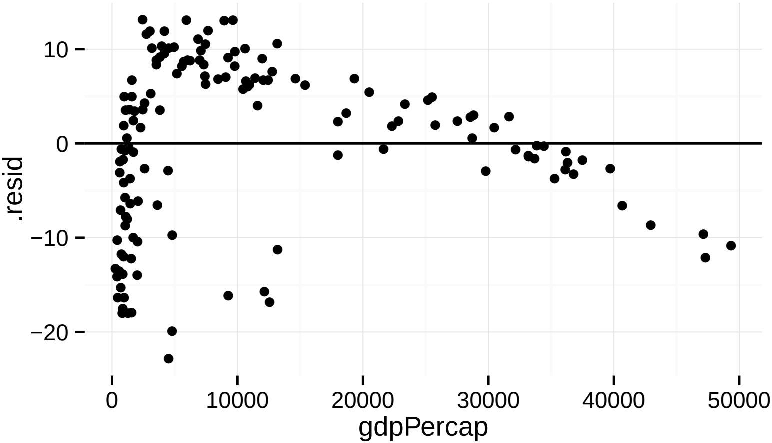
Residuals and Misspecification

Example



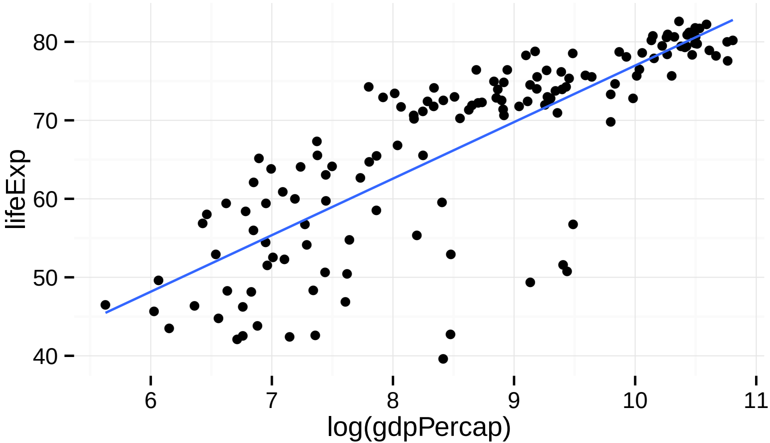
Residuals and Misspecification

Example



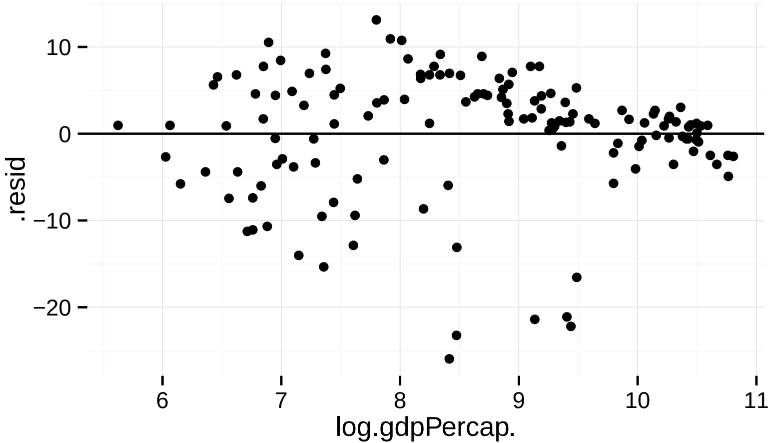
Residuals and Misspecification

Example



Residuals and Misspecification

Example



Logarithms and Power Transformations

Linear Transformations of Regressions

Interpreting Logarithms

How would you interpret the following?

- GDP per $\text{cap}_i = \alpha + \beta \log(\text{school})_i$
- log GDP per $\text{cap}_i = \alpha + \beta(\text{school})_i$
- log GDP per $\text{cap}_i = \alpha + \beta \log(\text{school})_i$

Linearizing Functions

Can you linearize these with logarithms?

Exponential

$$y_i = e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 \epsilon_i}$$

Gravity Equation

$$\text{trade}_{ij} = \frac{\alpha \text{GDP}_i^{\beta_1} \text{GDP}_j^{\beta_2}}{\delta d_{ij}^{\gamma}}$$

Cobb-Douglas

$$y = \alpha (x^{\delta \gamma} x^{(1-\delta)})^{\gamma}$$

CES Production Function

$$y = \alpha (\delta x^{\rho} + (1-\delta) x^{\rho})^{\gamma / \rho}$$

Interpretating Logarithms

Why use natural log for regression

- Note: $\log(1+r) \approx r$ when r small
- $$\log(x) - \log(x(1+r)) = \log(1+r) \approx r = \%\Delta x/100$$
- Only holds for natural logarithm

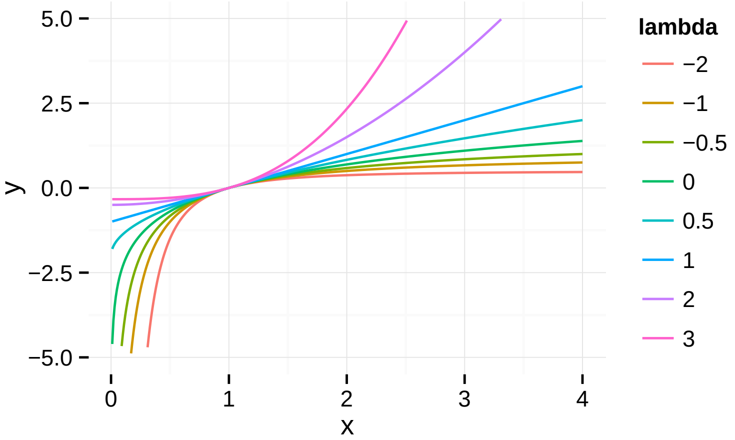
Converting between bases

To convert $\log_e$ to $\log_{10}$

$$\log_{10}(x) = \frac{\log_e(x)}{\log_e(10)}$$

Box-Cox Family of Transformations

```
## Warning in loop_apply(n, do.ply): Removed 272
rows containing missing values (geom_path).
```



Plot for $\lambda = 0.25, 0.5, 0, 2, 4, 8$ for $x = (0, 4]$

Box-Cox Family of Transforms

$$\begin{cases} f(x,\lambda) = \frac{x^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0 \\ f(x,\lambda) = \log x & \text{if } \lambda = 0 \end{cases}$$

- Requires $x > 0$. If negative, use $x + c$ (some problems), or Yeo-Johnson
- Can solve for λ to transform x as close to wrt. Normal skew.
- **car** function: `powerTransform|`, `bcTransform|`.
- `In regression: If know` λ `can transform` y `or` x

Logarithms and Power Transformations

Linear Transformations of Regressions

Linear Transformations of Regression

$$(y_i + a)/b = \alpha + \beta(x_i + d)/e + \epsilon_i$$

$$(y_i + \bar{y})/s_y = \alpha + \beta(x_i + \bar{x})/s_x + \epsilon_i$$

Standardized Coefficients / Regressors

$$y = \alpha + \beta 0 + \beta_1 \frac{x_i - \bar{x}}{\text{SD}(x)} + \epsilon_i$$

- Can be useful for default interpretation (controversial)
- Bad for skewed variables, binary variables? But about same as comparing $X + \text{SD}\, X$ post-estimation.
- Transform regressors, not functions of regressors.
- Gelman: Continuous: divide by 2 SD; Binary: center at mean.
- No need for them for default interpretation. With computational power, simulations better.
- Very important to standardize X in machine learning applications, or anywhere with complicated optimization problems.