POLS/CS&SS 503:
Advanced Quantitative Political Methodology

MISSING DATA

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Overview

Types of Missingness

- **MCAR** Missingness completely at random
- **MAR** Missingness at random
- **MNAR** Missingness that depends on unobserved variables, or **NI** Non-ignorable missingness

Funamental Problem with Missing Data

Cannot tell from data alone whether missingness is MAR or MNAR.

Examples of Missingness

- **MCAR**

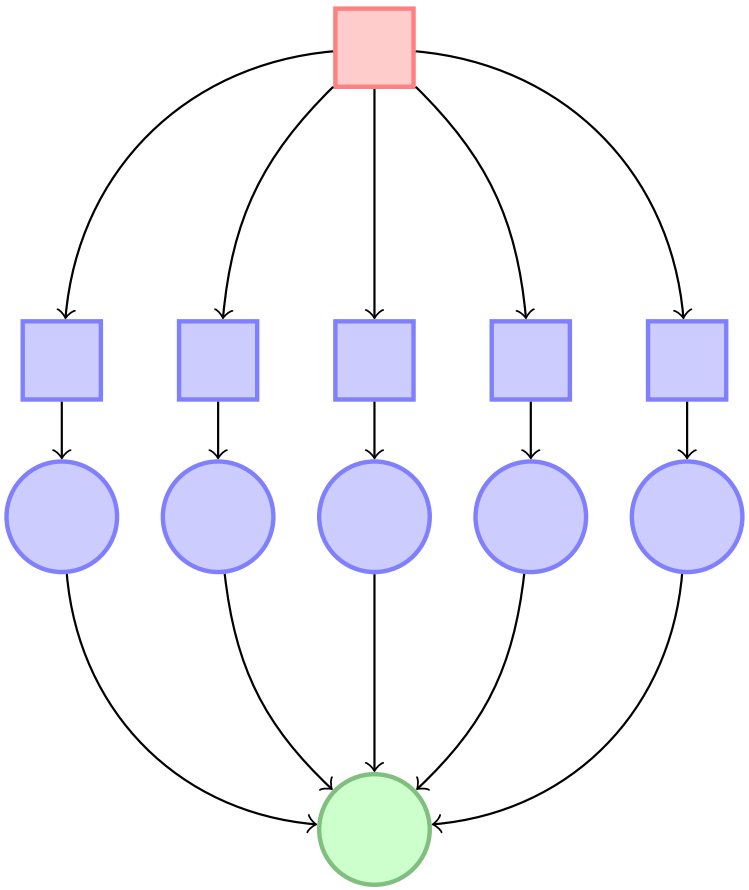
What we will cover and not cover

- Covering: MCAR
  - Missing values in X
  - Methods: listwise-deletion, multiple imputation
- Not-covering: MNAR models
  - Selection models
  - Censoring, Truncation

Methods

- Complete case (Listwise deletion)
  - Consistent and valid inferences when MCAR (or MAR but missingness does not depend on the dependent variable)
  - Even in MCAR, inefficient
- Available case (pairwise deletion):
  - E.g. Covariance matrix. Calculate $\sum_i (x_{i,j} - \bar{x}_j)(x_{i,k} - \bar{x}_k)$ for all obs in which $x_{i,j}, x_{i,k}$ are not-missing, regardless of missingness of other variables.
  - Does not work for all analyses
  - Can result in nonsensical results
- Unconditional Mean Imputation (Mean substitution)
  - preserves mean of variables; reduced variance
  - attenuates relationships between variables
  - overstates certainty—increases "effective" sample size and distorts inference

Overview of Multiple Imputation



incomplete data

imputation

imputed datasets

analysis

separate results

combination

final results

When is Listwise Deletion Preferable to MI?

1. All of the following need to hold
    - Analysis model is conditional on X and correctly specified
    - There is NI missingness in X
    - Missingness in X is not a function of Y, and unobserved variable affecting Y do not exist
    - Number of observations after deletion is large
2. Know X well enough that we don't trust it to impute, but trust it enough to analyze Y
3. Rarely would you prefer listwise deletion to multiple imputation

Multiple Imputation Estimator Combines Individual Estimates

Given $B_j^{(1)}, \ldots, B_j^{(g)}$, and $\mathsf{SE}(B_j^{(1)}), \ldots, \mathsf{SE}(B_j^{(g)})$ from g imputations:
Estimate for **single coefficients is:**

$$\text{Point Estimate} \qquad \tilde{\beta}_j = \frac{\sum_{l=1}^{g} B_j^{(l)}}{g}$$

$$\text{Std. Error.} \quad \tilde{\mathsf{SE}}\left(\tilde{\beta}_j\right) = \sqrt{V_j^{(W)} + \frac{g+1}{g} V_j^{(B)}}$$

$$\text{Within Imputation Variance} \qquad V_j^{(W)} = \frac{1}{g} \sum_{l=1}^{g} \mathsf{V}(B_j^{(l)})$$

$$\text{Between Imputation Variance} \qquad V_j^{(B)} = \frac{1}{g-1} \sum_{l=1}^{g} (B_j^{(l)} - \tilde{\beta}_j)^2$$

where $\tilde{\beta}_j$ distributed t with complicated d.f. (see Fox, 564)

Why we don't need to run many imputations

Relative efficiency of multiple imputation

$$RE(\tilde{\beta}_j) = V(\tilde{\beta}_j^{MLE})/V(\tilde{\beta}_j^{MI}) = \frac{g}{g+\gamma_j}$$

where γ_j is the relative rate of missing information

$$\gamma_j = \frac{R_j}{R_j+1} \qquad\qquad R_j = \frac{g+1}{g} \times \frac{V_j^{(B)}}{V_j^{(W)}}$$

Main point!

Suppose $R_j = \gamma$, then with $g = 5$ iterations, then efficiency is

$$\frac{5}{5+0.5} = 0.91$$

Advice on Missing Data

- Include all relevant variables in the imputation; at least all used in the estimation, including the dependent variable.
- Even if data are not multivariate normal, multivariate normal works okay.
- Transform data to approximate normality (Amelia has options)
- See TSCS extensions in Amelia
- Post-hoc adjustments okay. Impute naively and adjust, e.g.round to integers, or 0/1.
- If need to save time, work with a single iteration until “final” analysis.
- Potential problems: complex interactions between variables
- Try default methods; they often work.
- If not ...
    - Multiple Chained Equations: **mice**, **mi** packages
    - Hot-deck imputation
    - Full Bayesian models

What's the Problem?

Methods of Dealing with Missing Data

References

References

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