## POLS/CS&SS 503: Advanced Quantitative Political Methodology

## **MISSING DATA**

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### Overview

What's the Problem?

Methods of Dealing with Missing Data

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Methods of Dealing with Missing Data

## Types of Missingness

- MCAR Missingness completely at random
- MAR Missingness at random
- MNAR Missingness that depends on unobserved variables, or NI Non-ignorable missingness

Funamental Problem with Missing Data

Cannot tell from data alone whether missingness is MAR or MNAR.

## What we will cover and not cover

- · Covering: MCAR
  - Missing values in X
  - Methods: listwise-deletion, multiple imputation
- · Not-covering: MNAR models
  - · Selection models
  - Censoring, Truncation

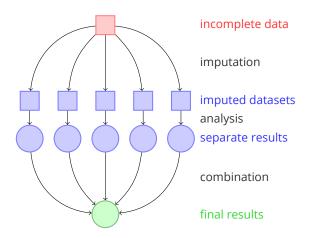
What's the Problem?

Methods of Dealing with Missing Data

### Methods

- Complete case (Listwise deletion)
  - Consistent and valid inferences when MCAR (or MAR but missingness does not depend on the dependent variable)
  - · Even in MCAR, inefficient
- Available case (pairwise deletion):
  - E.g. Covariance matrix. Calculate  $\sum_i (x_{i,j} \bar{x}_j)(x_{i,k} \bar{x}_k)$  for all obs in which  $x_{i,j}, x_{i,k}$  are not-missing, regardless of missingness of other variables.
  - Does not work for all analyses
  - · Can result in nonsensical results
- Unconditional Mean Imputation (Mean substitution)
  - preserves mean of variables; reduced variance
  - attenuates relationships between variables
  - overstates certainty—increases "effective" sample size and distorts inference

## Overview of Multiple Imputation



### When is Listwise Deletion Preferable to MI?

- 1. All of the following need to hold
  - ullet Analysis model is conditional on X and correctly specified
  - ullet There is NI missingness in X
  - Missingness in X is not a function of Y, and unobserved variable affecting Y do not exist
  - · Number of observations after deletion is large
- 2. Know X well enough that we don't trust it to impute, but trust it enough to analyze Y
- 3. Rarely would you prefer listwise deletion to multiple imputation

# Multiple Imputation Estimator Combines Individual Estimates

Given  $B_j^{(1)},\dots,B_j^{(g)}$ , and  $\mathrm{SE}(B_j^{(1)}),\dots,\mathrm{SE}(B_j^{(g)})$  from g imputations: Estimate for **single coefficients is:** 

$$\tilde{\beta}_j = \frac{\sum_{l=1}^g B_j^{(l)}}{g}$$
 Std. Error. 
$$\tilde{\text{SE}}\left(\tilde{\beta}_j\right) = \sqrt{V_j^{(W)} + \frac{g+1}{g}V_j^{(B)}}$$
 Within Imputation Variance 
$$V_j^{(W)} = \frac{1}{g}\sum_{l=1}^g \text{V}(B_j^{(l)})$$
 Between Imputation Variance 
$$V_j^{(B)} = \frac{1}{g-1}\sum_{l=1}^g \left(B_j^{(l)} - \tilde{\beta}_j\right)^2$$

where  $\tilde{eta}_j$  distributed t with complicated d.f. (see Fox, 564)

# Why we don't need to run many imputations

Relative efficiency of multiple imputation

$$RE(\tilde{\beta}_j) = V(\tilde{\beta}_j^{MLE})/V(\tilde{\beta}_j^{MI}) = \frac{g}{g + \gamma_j}$$

where  $\gamma_j$  is the relative rate of missing information

$$\gamma_j = \frac{R_j}{R_j + 1} \qquad \qquad R_j = \frac{g + 1}{g} \times \frac{V_j^{(B)}}{V_j^{(W)}}$$

Main point!

Suppose  $R_j=\gamma$  , then with g=5 iterations, then efficiency is

$$\frac{5}{5+0.5} = 0.91$$

## Advice on Missing Data

- Include all relevant variables in the imputation; at least all used in the estimation, including the dependent variable.
- Even if data are not multivariate normal, multivariate normal works okay.
- Transform data to approximate normality (Amelia has options)
- See TSCS extensions in Amelia
- Post-hoc adjustments okay. Impute naively and adjust, e.g. round to integers, or 0/1.
- If need to save time, work with a single iteration until "final" analysis.
- · Potential problems: complex interactions between variables
- Try default methods; they often work.
- If not ...
  - · Multiple Chained Equations: mice, mi packages
  - Hot-deck imputation
  - · Full Bayesian models

What's the Problem?

Methods of Dealing with Missing Data

- Gelman and Hill, Ch. 25 "Missing Data Imputation"
- Fox, Ch 20 "Missing Data in Regression Models"
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