SEPARATION OF TORQUE COMPONENTS USING FROZEN PERMEABILITY AND MAXWELL STRESS TENSOR

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***Abstract*—Local force distribution in motors has been widely studied to analyze the origins of torque ripple and vibration. In most cases, there are multiple force waves which need to be identified and analyzed individually. While separation of these forces and the corresponding fields is simpler if the core is below saturation limits, special techniques are required if significant portion of the core is in saturation. In this paper, Frozen Permeability and Maxwell Stress Tensor are used to separate components of air-gap forces in an Interior Permanent Magnet Synchronous Motor. The combined use of Frozen Permeability and Maxwell Stress Tensor is existing in literature. However, the approach proposed in this work allows to view reaction torque as sum of two components that have dissimilar ripple content. Nature of torque-ripple due to these two components is studied by varying the slot-design.**

I. Introduction

Traditional lumped parameters based analysis of motors does not include the effects of stator slots, saturation, mmf-harmonics etc. It approximates motor as a linear system and considers only fundamental magnetic field waves in the air-gap. While the average output of the motor is decided by the fundamental field waves, undesired phenomenon like torque ripple, vibration etc., are caused by spatial harmonics in magnetic field distributions. To study these effects it is necessary to use a field based method. Finite Element Method (FEM) is considered as a robust method in this regard, and can include several practical aspects of motor design. Harmonic fields due to both material non-linearity and slot-effects can be analyzed using FEM.

To analyze ripple and vibration, often, it becomes necessary to separate various components of fields and their corresponding forces. While this task is relatively simple for operating conditions where the core is within saturation limits, additional techniques are required to separate components when core is operating in saturation. Frozen Permeability (FP) method is one such technique that is proposed to separate the contributions of magnetic-field sources present in stator and rotor. This method was proposed in [1], and its experimental verification had been presented in [2]. In a Permanent Magnet Synchronous Motor (PMSM), FP can be used to estimate the effective contributions of PMs and armature excitation towards the resultant air-gap fields. In this work, Maxwell Stress Tensor (MST) and FP are used together to segregate air-gap fields and the corresponding tangential and radial forces in an Interior Permanent Magnet Synchronous Motor (IPMSM).

MST and FP are used in [3] and [4], to separate cogging torque from average synchronous torque. In previous publications that have addressed separation of torque components [3]–[6] the focus was on reaction torque, reluctance torque and cogging torque. The approach used in this paper is based on a simple decomposition of radial and tangential fields into contributions due to armature excitation and the magnets. This method allows to view reaction torque as sum of two components and analyze their contribution to torque ripple.

II. Decomposition of Air-gap fields

Field produced in the air-gap due to armature excitation consists of both radial (B_r) and tangential (B_t) components. Similarly, due to the slotted structure of stator, field created by radially magnetized PMs contains both tangential and radial components in air-gap. Hence, the net field components of air-gap can be decomposed as

$$\begin{aligned} B_r &= B_{rr} + B_{sr} \\ B_t &= B_{rt} + B_{st} \end{aligned} \tag{1}$$

In the above equations, on the right hand side, first letter of subscript denotes the source of field component and second letter corresponds to its orientation (tangential/radial). For example, B_{rr} corresponds to radial component of field produced due to rotor excitation (in IPMSM, source in rotor is PM). Dependency of field components on space and time is not denoted in above equations.

In study of motors, tangential field component is often neglected. This is due to relatively low magnitude of B_{rt} and B_{st} compared to radial components. However, tangential field strength, specifically B_{st} is necessary to develop tangential forces on rotor [7] [8]. In this work, force components are computed without neglecting the effects of both B_{st} and B_{rt}. Using Maxwell Stress Tensor (MST) [9], torque is expressed as

$$\mathrm{T}(t) = R\frac{1}{\mu_0}\oint_S B_r B_t \,\mathrm{d}A \tag{2}$$

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Where R is the rotor outer radius, μ_o is permeability of air and S is the surface of integration considered close to rotor surface. By substituting equation (1) in the above equation, it can be noted that there are four tangential force terms that decide the instantaneous value of torque, as shown below

$$\mathrm{T}(t) = R\frac{1}{\mu_0}\oint_S \left(B_{rr}B_{st} + B_{sr}B_{rt} + B_{sr}B_{st} + B_{rr}B_{rt}\right) \mathrm{d}A \tag{3}$$

Values obtained after integrating each term on the right hand side of above equation will be referred as torque components and denoted as T_1, T_2, T_3 and T_4. For example,

$$T_1(t) = R\frac{1}{\mu_0}\oint_S B_{rr}B_{st}\,\mathrm{d}A$$

Therefore, from equation (3), instantaneous torque can be expressed as

$$\mathrm{T}(t) = T_1(t) + T_2(t) + T_3(t) + T_4(t) \tag{4}$$

Conventionally, T_1 and T_2 are associated with reaction torque, since they are the result of interaction between PM field and armature field. T_3 and T_4 are called as reluctance torque and cogging torque respectively.

III. Extraction of Torque Components

In this section, FEM based procedure to extract torque components is detailed. For demonstrating the proposed approach, an IPMSM is considered. Stator of the case study IPMSM consists a three phase single layer distributed winding and a six-pole IPM rotor (Fig. 1). Details of the motor are given in Table I.

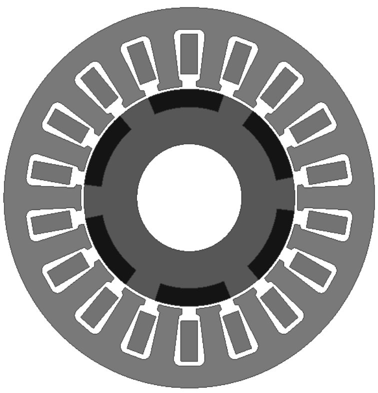


Fig. 1: Three phase 18 slots 6 pole IPMSM.

A. Core in linear region

Following two simulations are performed to obtain the field components for operating conditions where core is within saturation limits.

TABLE I: Specifications of IPMSM

Power	5 kW
Torque	22 N.m
Speed	2000 rpm
Voltage	48 V
Poles	6
Stator slots	18
Rotor outer diameter	60 mm
Stator outer diameter	120 mm
Air gap	0.75 mm
Magnet Thickness	5 mm
Remanent Flux Density	1.09 T

- Fields solution in the air-gap is obtained by providing only PM excitation. From this solution, values of B_{rr} and B_{rt} at every time step are recorded.
- Field solution with only armature excitation is obtained. Using this, values of B_{sr} and B_{st} are recorded.

An additional reference solution is obtained to ensure that superposition principle used in equation (1) is valid. In this reference simulation, fields solution is obtained with both armature excitation and PMs. Using this, B_r and B_t are extracted at every time step. These waveforms have matched with the waveforms obtained by adding the fields of individual sources. The error goes to a maximum of 2% for both net radial and tangential field components. As suggested in [4], similar mesh with triangular elements is used in all three simulations to reduce the errors due to finite element discretization.

B. Core operating in saturation

If the core is in saturation then Frozen Permeability method can be used to estimate the contributions of individual field sources towards air-gap field components. In Frozen Permeability method, to segregate the fields due to armature excitation and PMs, the permeability values across the motor model are fixed to those values that would have occurred if both PM and armature are excited together.

To implement FP method any FEM software that allows to import the permeability values from a pre-solved similar model can be used. In this work 2D magnetostatic solver of Ansys Maxwell (FEM Software) is used. A scripting interface compatible with Ansys is used to obtain the field solutions for different rotor positions.

Procedure to implement FP method in FEM software is given below.

Reference solution: Both armature and PMs are excited. B_r and B_t are recorded.

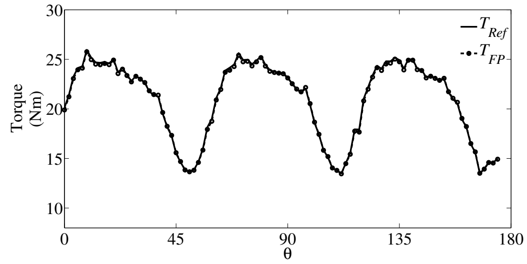
FP solution:

1. In this step, PMs are the only field source in the motor model. Field distribution is obtained by keeping μ values at every node same as that of reference solution.
2. In this step, armature excitation is the only field source. Fields are obtained by keeping μ values at every node same as that of reference solution.

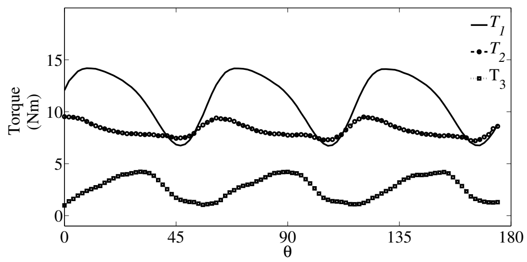
The net field components obtained in reference solution and the sum of respective field components obtained using FP solutions are compared. They are in good agreement and a maximum error of 5% is seen near the stator-teeth corners. Torque waveform obtained from the fields of reference solution (T_{Ref}) and using FP method (T_{FP}) is shown in Fig. 2a. Torque components obtained using equation (3) are shown in Fig. 2b and Fig. 2c.

C. Torque ripple

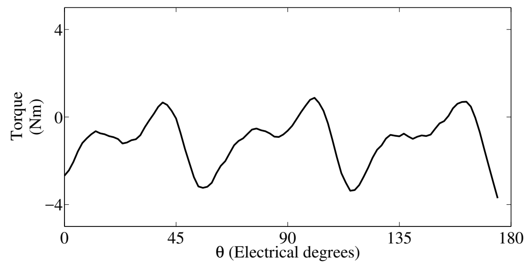
In previous publications on torque ripple [3]–[6] , reaction torque is not considered as a sum of two components. Fig. 2b shows that T_1 and T_2 have dissimilar trends, though both ripples are caused due to variation in effective air-gap permeance. Change in the ripple of T_1, T_2 and T_3 with slot opening width is shown in Fig. 3. Where slot opening width (in %) is defined as $\frac{Slot\,Width}{Slot\,Pitch} \times 100$. T_1 increases with slot opening, while T_2 is nearly constant.



(a)



(b)



(c)

Fig. 2: (a) Net Torque (b) Torque components T_1, T_2, T_3 (c) T_4

In other case studies performed by the authors, with different slot-pole combinations, it is observed that T_2 becomes significant only when the stator-teeth are saturated and armature reaction is comparable to PM flux. Average value of T_2 is majorly dependent on slot-opening width. If slot-opening width becomes comparable to air-gap thickness, the presence of B_{rt} in the air-gap is negligible and hence T_2 is relatively less. Therefore, the average value of T_2 relative to T_1 decreases with increasing slot number.

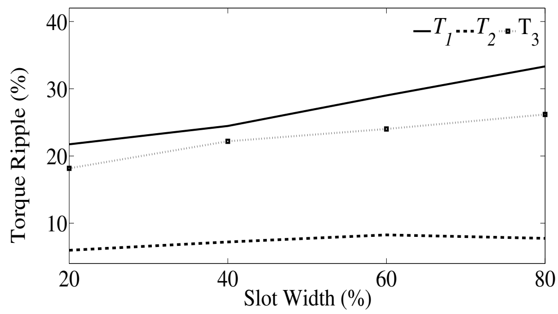


Fig. 3: Torque ripple vs. Slot opening width

The negative average value of cogging torque is due to synchronously rotating saliency seen by the PM flux. The rotating saliency of stator is due to saturation of core, and this phenomenon is detailed in [3].

D. Radial force distribution

From the information of segregated field components, distribution of different air-gap radial force waves can be obtained. Using MST, radial force intensity in the air-gap is given as

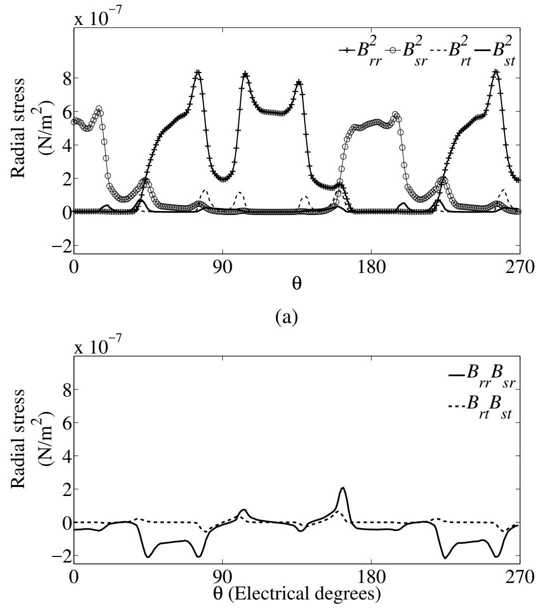
$$\begin{aligned} f_r =& \frac{1}{2\mu_0}(B_r^2 - B_t^2) \\ =& \frac{1}{2\mu_0}(B_{rr}^2 + B_{sr}^2 - B_{rt}^2 - B_{st}^2 + 2(B_{rr}B_{sr} - B_{st}B_{rt})) \end{aligned} \quad (5)$$

Variation of each term in equation (5) for a quarter section of the motor is shown in Fig. 4. By considering only significant terms, net local radial stress in the air-gap can be approximated as

$$f_r = \frac{1}{2\mu_0}(B_{rr}^2 + B_{sr}^2 - B_{rt}^2 + 2B_{rr}B_{sr}) \quad (6)$$

Radial stress obtained from the above equation is shown in Fig. 5. In the figure, positive sign indicates that rotor experiences radial forces of expansion. In literature, MST is extensively used to estimate the radial force intensity across the air-gap. However, when net intensity is viewed as sum of the individual components, as given in equation (6), the causes of vibration can be analyzed to a greater extent.

In SPM motors, rotor magnetic field contains odd harmonics, whose order is of the form $2n+1$, where n is an integer. Stator fields contains harmonics of the order $2n+3$. However, in motors where slots are present in both rotor and stator, like IPMSM and induction motors, there will be additional even and odd harmonics due to revolving permeances [10]. It is challenging to estimate the magnitude of each of these harmonics using analytical equations, specifically when the stator core has significant saturation. Hence, estimation of torque ripple and vibration analysis becomes difficult. The



(b)

Fig. 4: (a)Stress due to squared terms (b) Stress due to product terms

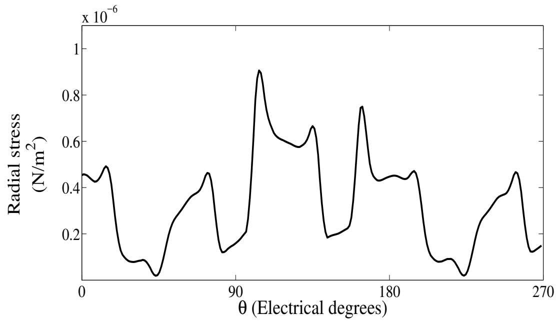


Fig. 5: Net radial stress in the air-gap

method proposed in this paper can be used to segregate the contributions due to individual field sources, when slot effects and saturation effects are dominant. Thus obtained field distributions of each source can be studied for spatial harmonics.

Errors in the field solution due to discretization in FEM, and errors due to the choice of integration surface in the air-gap are identified as limitations to this method. These errors can be minimized by choosing two or three layered uniform mesh for air-gap, and by choosing the integration surface in the middle of the air-gap [9], [11]. Also, if the proposed method is to be applied for motors with higher pole number, simulation time increases significantly. This is because, spatial harmonics of higher order, that make more than around 50 cycles along the air-gap, need finer mesh to estimate their magnitude.

IV. Conclusion

Study of torque components and radial force distribution is necessary to mitigate torque ripple and vibration of electric motors. Such a study requires to identify the contribution of individual field sources towards the resultant field in the air-gap. This can be achieved using simple superposition principle if the core is within saturation. In saturated conditions, superposition does not cause much issue once Frozen Permeability method is applied. With the proposed approach, reaction torque can be viewed as a combination of two components. Study of these components, with respect to time and by changing stator slot-design, has revealed that these components have dissimilar trends in their contribution to net torque ripple. The proposed method of force decomposition, in tandem with Fourier analysis, can be used to identify the sources (armature/magnet) and structures (stator/rotor) that are responsible for a specific frequency in vibration or torque ripple.

References

[1] N. Bianchi and S. Bolognani, "Magnetic models of saturated interior permanent magnet motors based on finite element analysis," *IEEE Industry Applications Conference. Thirty-Third IAS Annual Meeting*, vol. 1, pp. 27-34, Oct. 1998.

[2] J. A. Walker and D. Dorrell, "Verification of the frozen permeabilities method of calculating the interior permanent magnet motor," *Digests of the IEEE International Magnetics Conference*, pp. 715-716, Apr. 2005.

[3] W. Q. Chu and Z. Q. Zhu, "Average torque separation in permanent magnet synchronous machines using frozen permeability," *IEEE Transactions on Magnetics*, vol. 49, no. 3, pp. 1202–1210, Mar. 2013.

[4] W.Q.Chu and Z.Q.Zhu, "On-load cogging torque calculation in permanent magnet machines," *IEEE Transactions on Magnetics*, vol. 49, no. 6, pp. 2982–2989, Jun. 2013.

[5] G. T. de Paula, J. R. B. d. A. Monteiro, T. E. P. de Almeida, M. P. de Santana, W. C. A. Pereira, and C. M. R. Oliveira, "Evaluation of surface mounted pm machine's parameters on load conditions using frozen permeability method," *11th IEEE/IAS International Conference on Industry Applications*, pp. 1-7, Dec. 2014.

[6] D. J. Gmez, A. Tovar-Barranco, A. L. Rodrguez, A. L. de Heredia, and I. Villar, "On-load cogging torque calculation using frozen permeability method and permeance network models," *XXII International Conference on Electrical Machines (ICEM)*, pp. 499–505, Sept. 2016.

[7] C. B. Gray, "Rotor torque distribution in electrical machines via maxwell's stresses," *IEEE Transactions on Education*, vol. 24, no. 4, pp. 283–286, Nov. 1981.

[8] W. Zhu, S. Pekarek, B. Fahimi, and B. J. Deken, "Investigation of force generation in a permanent magnet synchronous machine," *IEEE Transactions on Energy Conversion*, vol. 22, no. 3, pp. 557–565, Sept. 2007.

[9] S. J. Salon, *Finite Element Analysis of electrical Machines*. Springer, 1995.

[10] G. Kron, "Induction motor slot combinations, rules to predetermine crawling, vibration, noise and hooks in the speed-torque curve," *Transactions A.I.E.E.*, Dec. 1931.

[11] S. Salon, S. Bhatia, and D. Burow, "Some aspects of torque calculations in electrical machines," *IEEE Transactions on Magnetics*, vol. 33, no. 2, pp. 2018–2021, Mar. 1997.