A single-pixel terahertz imaging system based on compressed sensing

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We describe a terahertz imaging system that uses a single pixel detector in combination with a series of random masks to enable high-speed image acquisition. The image formation is based on the theory of compressed sensing, which permits the reconstruction of a N-by-N pixel image using much fewer than N^2 measurements. This approach eliminates the need for raster scanning of the object or the terahertz beam, while maintaining the high sensitivity of a single-element detector. We demonstrate the concept using a pulsed terahertz time-domain system and show the reconstruction of both amplitude and phase-contrast images. The idea of compressed sensing is quite general and could also be implemented with a continuous-wave terahertz source. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2989126]

Over the past several years, the wide applicability of terahertz imaging in areas such as detection of foam insulation defects,[1] illicit drug detection,[2] and package inspection[3] has driven the development of high-speed terahertz imaging systems.[4] Most existing terahertz imaging systems use a raster scan to move an object in front of a single pixel detector. This mechanical scanning significantly limits the acquisition speed.[1–3,5] With state-of-the-art technology, it takes about 6 min to scan a $100 \times 100\ \text{mm}^2$ area at 0.25 mm resolution.[5] Real-time terahertz imaging has been demonstrated using focal-plane detector arrays.[6,7] However, these systems tend to have higher complexity and operational cost. For example, available array detectors, such as microbolometer arrays, are relatively insensitive to terahertz radiation, so a bright terahertz source is needed.[6] Single-shot electro-optic sensing also allows video-rate terahertz imaging,[7] but this method requires a large and costly amplified femtosecond laser system. Even though interferometric or tomographic approaches have significantly reduced the number of required measurements by, for example, nonuniform sampling in the Fourier domain, the acquisition speed of such systems are still limited by raster scanning unless a full detector array is used.[8–10] Terahertz reciprocal imaging can achieve high-speed imaging with a single-pixel detector but requires an unconventional source array, with each source element modulated at a different frequency.[11]

For practical, time-critical applications, a terahertz imaging system should not require raster scanning of the object or the terahertz beam. In addition, one would like to preserve the superior detection sensitivity of a single-point detector such as photoconductive antennas (rather than the lower sensitivity provided by existing multipixel arrays) and the simplicity and spatial coherence of a point-source transmitter. Here, we describe a single-pixel terahertz imaging system based on an advanced signal processing theory called compressed sensing (CS),[12,13] which enables both of these objectives. In contrast to our earlier work on terahertz Fourier imaging using CS and phase retrieval,[14] this system does not require mechanical scanning of the terahertz receiver on the image plane.

The rationale behind the improved acquisition speed of the single-pixel terahertz camera is twofold. First, this camera performs compression simultaneously with image sampling by modulating the spatial profile of the terahertz beam with a set of random patterns, a technique enabled by CS. This imaging scheme requires significantly fewer samples than the total number of image pixels to fully reconstruct an image, thus, speeding up the acquisition process.[12,13] Second, the speed of most existing terahertz imaging systems is limited by the need to mechanically raster scan the object (or the terahertz beam).[4] Our method replaces this mechanical scanning with the spatial modulation of the free-space terahertz beam, which can in principle be much faster.

The principle behind the design of CS imaging systems can be summarized in the equation $y=\Phi x$, where y is a $M \times 1$ column vector of measurements, x is an image with N^2 pixels ordered in a $N^2 \times 1$ vector, and the *measurement matrix* Φ is $M \times N^2$. Using CS, we acquire a much smaller number of measurements than the number of pixels in the image, i.e., $M < N^2$, and can still reconstruct the object perfectly through an optimization procedure as described in the references.[12,13] Our previous work on terahertz Fourier imaging using CS and phase retrieval uses a random subset of the Fourier basis as rows of Φ.[14] Here, we choose a set of vectors, whose entries are randomly picked to be 1 or 0 with equal probability, as rows of Φ.[15] In other words, for each row in Φ, only a random subset (approximately half) of the pixels are set to unity (100% transmission), while the remainder is set to zero (no transmission). Thus, the terahertz beam traveling from the object to the detector is filtered by randomly blocking a subset of the spatial wave front.

Figure 1 illustrates our imaging setup, which consists of a terahertz transmitter/receiver pair (fiber-coupled photoconductive antennas), a planar screen with a random pattern of blocked pixels, and two lenses. The terahertz beam, after passing through the object mask and the random pattern, is collected at the receiver. The positions of the screen, the focusing lens, and the receiver along the beam path are calculated according to the lens law in order to efficiently focus

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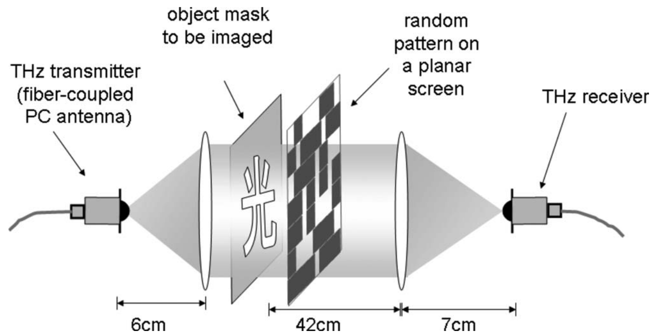

FIG. 1. The terahertz compressive imaging setup. An approximately collimated beam from the terahertz transmitter illuminates an object mask and is partially (~50%) transmitted through a random pattern of opaque pixels. The random patterns, the focusing lens and the receiver, are placed in order to most efficiently focus the terahertz beam onto the receiver antenna. We collect one complete time-domain waveform for each random pattern.

the terahertz beam onto the receiver antenna. The object mask is made of (opaque) copper tape on a transparent plastic plate. In this paper, our object mask has a hole shaped as a Chinese character which means "light," 1.5 cm both by height and width, as shown in Fig. 2(a). For our screens, we use a set of six hundred random patterns printed in copper on standard printed-circuit boards (PCBs). Printed on a uniform grid on the PCBs, each pattern contains 32×32 pixels. The size of each pixel is $1 \times 1\ \mathrm{mm}^2$. A "copper" pixel corresponds to pixel value zero on the random pattern, while a pixel without copper corresponds to the value one, since the PCB material is fairly transparent to the terahertz beam. To ensure accurate alignment as we change from one random pattern to another, we use an automatic translation stage. For each random pattern, we measure one terahertz waveform consisting of the superposition of the radiation transmitted through all of the unmetallized pixels.

In CS, every row of the measurement matrix Φ (that is, every random pattern) is used to form only one measurement, consisting of a complete time-domain waveform. Therefore, for each random pattern, we extract the magnitude of the detected terahertz waveform at a particular frequency to obtain one value. For this demonstration, we chose a frequency of 100 GHz ($\lambda = 3$ mm), because near this frequency the incident terahertz beam is large enough to illuminate the entire object and has a high spectral amplitude. Figures 2(b) and 2(c) shows the CS reconstruction results, 32×32 images of the object in Fig. 2(a) with $1 \times 1\ \mathrm{mm}^2$ pixel size, using 300 and 600 measurements respectively. Our reconstruction algorithm uses minimization of the total variation (min-TV),[13] which takes less than 10 s to compute in Matlab on a standard personal computer. Our system accurately reconstructs both the size of the object and its millimeter-scale

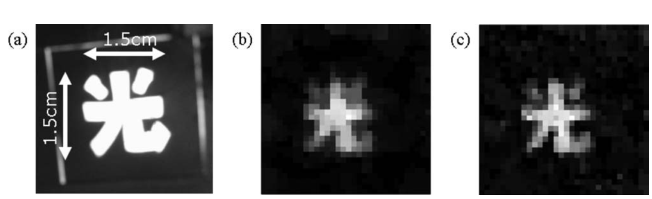

FIG. 2. (a) White-light image of object mask shaped as the Chinese character light. Terahertz images reconstructed via CS using (b) 300 and (c) 600 magnitude measurements, which are respectively about 30% and 60% of the total number of image pixels. Both figures display a 32×32 *image a*nd the pixel size is $1 \times 1\ \mathrm{mm}^2$.

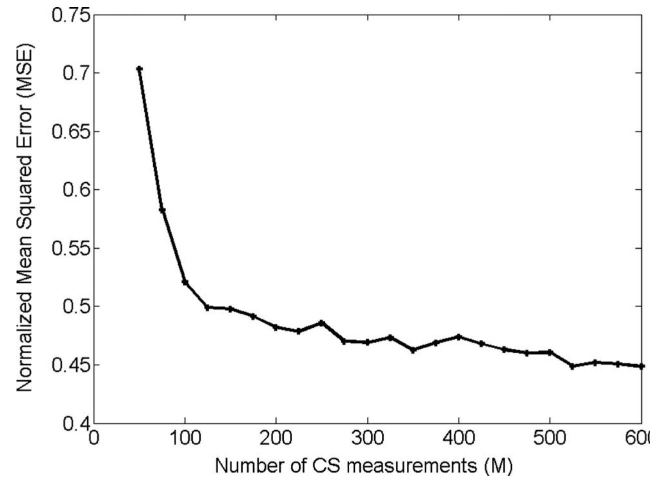

FIG. 3. As the number of measurements (M) used in CS increases, the MSE between the magnitudes of the reconstructed image and the reference image for the object in Fig. 2 normalized by the squared energy of the reference image decreases. The MSE decay is fast from 50 to 100 CS measurements. After the number of CS measurements exceeds the "sparsity" of the object, the decay flattens out.

features. Reconstruction using more measurements yields a sharper image but also adds some artifacts. Sources of noise include laser power fluctuation and alignment errors between patterns.

Figure 3 plots the normalized mean-squared error (MSE) of the reconstructed images of the object in Fig. 2 using different numbers of masks (CS measurements). Due to the reduced rate of convergence of the min-TV reconstruction algorithm for number of CS measurements below 250, we use the minimization of the $l1$-norm in the wavelet basis[12] as our reconstruction method for the results in Fig. 3. The normalized MSE of each reconstructed image is computed as in $\Sigma_i[X_r(i) - X(i)]^2 / \Sigma_i X(i)^2$, where the summations are across all image pixels, X_r is the reconstructed image and X is a 32×32 grayscale reference image downsampled from a digital photograph of the object. The MSE sharply decreases from 50 to 100 CS measurements and then decays more slowly afterwards. The shape of this decay curve is dependent on the sparsity of the object in the reconstruction basis and on the system noise.[16] In general, this decay is fast until the number of measurements reaches the sparsity level of the object. Our experimental result is consistent with this trend.

We emphasize that this reconstruction from only the amplitude measurements at a single frequency is equivalent to imaging using a continuous-wave (cw) terahertz source. If we apply this method directly to cw terahertz systems, we could use a more sensitive single-pixel detector, such as a Schottky diode, and thus reduce the source power requirement dramatically compared to imaging with existing multipixel detector arrays.

There have been very few implementations of CS systems capable of acquiring and reconstructing complex image data. Since pulsed terahertz systems are well-known for providing spectroscopic phase information, we demonstrate this capability in our single-pixel system, using CS algorithms specifically designed to reconstruct complex images.[17] The object to be imaged consists of a simple rectangular hole, half of which is covered by a transparent plastic plate. For each random pattern, we determine *both* the magnitude and phase of the terahertz radiation at a single frequency (again,

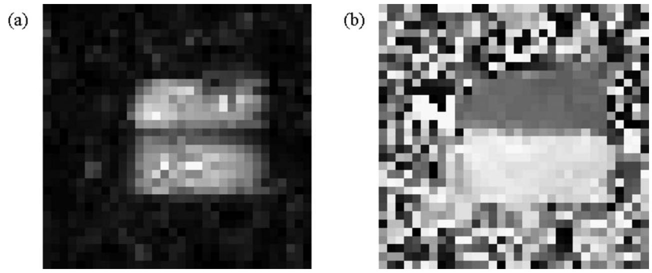

FIG. 4. CS reconstructions of (a) the image amplitude and (b) the phase using 400 (complex) measurements. Here, the object is a rectangular hole in an opaque screen, covered with a (transparent) plastic plate. The plate covering the upper half of the hole is thicker than the lower half. The reconstructed phase image exhibits this contrast much better than the amplitude image.

100 GHz). CS reconstruction with 400 measurements using the SPGL1 algorithm (see Ref. 17) yields the image amplitude and phase shown in Fig. 4. While the amplitude image shows almost no contrast, the phase image shows a sharp contrast between the upper and lower halves of the rectangular aperture. The thickness difference (Δd) can be estimated from the measured phase difference ($\Delta\theta$) according to $\Delta\theta=\omega(n-1)\Delta d/c$, which is valid since the plastic is essentially dispersionless. From our experimental data, when the number of CS measurements increases from 200 to 600, Δd estimates averaged across 100–200 GHz rapidly converges to the true Δd measured with a micrometer. To capture complex terahertz data in our system, acquisition of the entire terahertz waveform is necessary. In this case, we obtain hyperspectral and phase information at the expense of lower imaging speed due to the mechanical movements of the delay line.

To conclude, the single-pixel, pulsed terahertz camera described in this paper does not rely on raster scanning or a source/detector array, but uses random patterns for imaging. Based on the theory of CS, the system is capable of recovering a 32×32 image of a rather complicated object with only 300 measurements ($\sim$30%). This significant reduction in the number of measurements used for CS image reconstruction can speed up the acquisition tremendously compared to traditional raster scan systems. Unlike its optical counterpart which measures only intensity,[15] our pulsed terahertz camera can reconstruct complex images. Currently, the major limitation of our setup is the slow translation of one random pattern to another. However, it is clear that other schemes for binary spatial modulation of a terahertz beam, driven either optically or electrically, can operate extremely rapidly and with no mechanical moving parts. This should allow the acquisition of sufficient information for image reconstruction at a rate comparable with video imaging.

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