Graph Thoery – Connectivity

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2023.3.17

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- If H is a subgraph of G and there does not exists any connected graph F s.t $H \subset F \subseteq G$, then H is a **connected component** of G.

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- 1) Add edge & Delete edge; 2) Ask whether u and v are in the same connected component. *you must handle these operations online*

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- Holm-de Lichtenberg-Thorup Algorithm

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- 1) Edge $u \to [l, r]$ or $[u, v] \to [l, r]$; 2) Ask whether u and v are in the same connected component
- Can not to traverse all edges or all states

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- Analyze properties of the given problem

Related Problems

- CF1681F. Unique Occurences

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- CF787D. Legacy

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- CF1681F. Unique Occurences
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- HDU6072 Logical Chain (You need to know Korasaju algorithm)
  https://acm.hdu.edu.cn/showproblem.php?pid=6072

Thanks!