Data Structure – Mo's Algorithm (I)

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Example: CodeForces 86D Powerful array

- An array of positive integers $a_1, a_2, \ldots, a_n$ is given. Let us consider its arbitrary subarray $a_l, a_{l+1}, \ldots, a_r$, where $1 \leq l \leq r \leq n$. For every positive integer s denote by K_s the number of occurrences of s into the subarray. We call the power of the subarray the sum of products $K_s \cdot K_s \cdot s$ for every positive integer s. The sum contains only finite number of nonzero summands as the number of different values in the array is indeed finite. You should calculate the power of t given subarrays.

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- $1 \leq n, t \leq 200000, 1 \leq a_i \leq 10^6$
- Time Limit: 5s

Example: CodeForces 617E XOR and Favorite Number

- Bob has a favorite number k and a_i of length n. Now he asks you to answer m queries. Each query is given by a pair l_i and r_i and asks you to count the number of pairs of integers i and j, such that $l \leq i \leq j \leq r$ and the xor of the numbers $a_i, a_{i+1}, \ldots, a_j$ is equal to k.

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- $1 \leq n, m \leq 10^5, 0 \leq a_i, k \leq 10^6$

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- Time Limit: 1.5s

Example: CodeForces 375D Tree and Queries

- You have a rooted tree consisting of n vertices. Each vertex of the tree has some color. We will assume that the tree vertices are numbered by integers from 1 to n. Then we represent the color of vertex v as c_v. The tree root is a vertex with number 1.

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- In this problem you need to answer to m queries. Each query is described by two integers v_j, k_j. The answer to query v_j, k_j is the number of such colors of vertices x, that the subtree of vertex v_j contains at least k_j vertices of color x.

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- Time Limit: 1s

Example: CodeForces 852I Dating

- This story is happening in a town named BubbleLand. There are n houses in BubbleLand. In each of these n houses lives a boy or a girl. People there really love numbers and everyone has their favorite number f. That means that the boy or girl that lives in the i-th house has favorite number equal to f_i. The houses are numerated with numbers 1 to n. The houses are connected with $n - 1$ bidirectional roads and you can travel from any house to any other house in the town. There is exactly one path between every pair of houses. A new dating had agency opened their offices in this mysterious town and the citizens were very excited. They immediately sent q questions to the agency and each question was of the following format:

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- $1 \leq n, q \leq 100000, 1 \leq a_i \leq 10^9$

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- 2) You are given two integers p and x. Change a_p to x.
- $1 \leq n, q \leq 100000, 1 \leq a_i \leq 10^9$
- Time Limit: 4s

Example: CodeForces 1767F Two Subtrees

- You are given a rooted tree consisting of n vertices. The vertex 1 is the root. Each vertex has an integer written on it; this integer is val_i for the vertex i. You are given q queries to the tree. The i-th query is represented by two vertices, u_i and v_i. To answer the query, consider all vertices w that lie in the subtree of u_i or v_i (if a vertex is in both subtrees, it is counted twice). For all vertices in these two subtrees, list all integers written on them, and find the integer with the maximum number of occurrences. If there are multiple integers with maximum number of occurrences, the minimum among them is the answer.

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- $1 \leq n, q, val_i \leq 200000$

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- $1 \leq n, q, val_i \leq 200000$
- Time Limit: 9s

Example: CodeForces 840D Destiny

- Once, Leha found in the left pocket an array consisting of n integers, and in the right pocket q queries of the form l r k. If there are queries, then they must be answered. Answer for the query is minimal x such that x occurs in the interval l r strictly more than $\frac{r-l+1}{k}$ times or -1 if there is no such number. Help Leha with such a difficult task.

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- $1 \leq n, q \leq 300000, 1 \leq a_i \leq n, 2 \leq k \leq 5$
- Time Limit: 2.5s

Thanks!