Mathematics Department

COLLEGE ALGEBRA
Learning Module #6b

Topic	Fractional Exponents/Radicals
Duration	3 hours
Lesson Proper	We shall extend the definition of a^n to include fractions or rational numbers for n. If m/n is a rational number with positive integer a, then $$\boldsymbol{a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m}$$ The form $\boldsymbol{\sqrt[n]{a^m} = a^{m/n}}$ is called the **principal root** of $\boldsymbol{a^m}$. The numerator **m** indicates a power and the denominator **n** is called the **root or order**. Specifically, $\boldsymbol{a^{1/n}}$ means the **principal nth root of a**. The symbol $\boldsymbol{\sqrt[n]{a}}$ is called a **radica**l, where **a** is called the **radicand**, and **n** is called the **index or order**. **To evaluate radicals**, it is sometimes convenient to express the radical with fractional exponent or apply the rule $\boldsymbol{\sqrt[n]{a^n} = a}$. The following are examples on radicals: 1. $(-64)^{\frac{2}{3}} = (\sqrt[3]{-64})^2 = (-4)^2 = 16$ 2. $(-32)^{-\frac{2}{3}} = \frac{1}{(-32)^{2/5}} = \frac{1}{(-2)^2} = \frac{1}{4}$ 3. $\sqrt[6]{x^{18}} = x^{18/6} = x^3$ 4. $\sqrt[3]{8x^6} = \sqrt[3]{(2x^2)^3} = [(2x^2)^3)]^{\frac{1}{3}} = 2x^2$ 5. $\sqrt[6]{16x^2} = (16x^2)^{1/6} = (2^4x^2)^{1/6} = (2^4x)^{1/3} = \sqrt[3]{4x}$ 6. $\sqrt[3]{-64x^6} = -4x^2$ 7. $\sqrt[5]{-32x^{10}} = -2x^2$ Notice that we impose the condition a >0 in the definition of square root of the number a because it will not always hold if a<0. For example: $$(\sqrt{-7})^2 \neq -7$$

So to avoid conflict we give a stronger definition for the square of a number, i.e.

$$\sqrt{a^2} = |a|, \quad \text{for any number a.}$$

In general, for an even n and any real number a.

$$\sqrt[n]{a^n} = |a| \ .$$

Examples: a. $\sqrt[3]{x^3} = x$ b. $\sqrt[5]{32a^5} = \sqrt[5]{2^5a^5} = 2a$

Laws on Radicals

1. $(\sqrt[n]{a})^n = a, \quad a > 0.$ Example: $(\sqrt[5]{6})^5 = 6$
2. $\sqrt[n]{ab} = \sqrt[n]{a}\ \sqrt{b}, \ a, b > 0$ Example: $\sqrt{18} = \sqrt{9 * 2} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$
3. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, \quad a, b > 0$ Example: $\sqrt[5]{\frac{5}{32}} = \frac{\sqrt[5]{5}}{\sqrt[5]{32}} = \frac{\sqrt[5]{5}}{2}$
4. $\sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a} = \sqrt[m]{\sqrt[n]{a}}$ Example: $\sqrt[6]{4} = \sqrt[3]{\sqrt{4}} = \sqrt[3]{2}$

Simplified Radical Form

1. All exponents in the radicand must be less than the index.
2. Any exponents in the radicand can have no factors in common with the index.
3. No fractions appear under a radical.
4. No radicals appear in the denominator of a fraction.

EXERCISES:

Simplify the following radicals:

a. $\sqrt[4]{x^7}$ **b.** $\sqrt[3]{-16}$ **c.** $\sqrt[5]{-64}$ **d.** $\sqrt[6]{x^{11}}$

e. $\sqrt[3]{\frac{9}{x^{12}}}$ **f.** $\sqrt[8]{x^2}$ **g.** $\sqrt[6]{x^4y^8}$ **h.** $\sqrt[3]{\frac{8}{x^{12}}}$

i. $\sqrt[3]{-64x^6z^{24}}$