Mathematics Department

COLLEGE ALGEBRA
Learning Module #6c

Topic	**Radicals and its Operations**
Duration	**3 hours**
Lesson Proper	**Three Ways to Simplify Radicals:** 1. Removal of Perfect nth powers ➢ Break down the radicand into perfect and nonperfect nth powers and apply the $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$. 2. Reducing the index to the lowest possible order ➢ Reducing the index is done by applying the property $\sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a} = \sqrt[m]{\sqrt[n]{a}}$ or expressing the radical into its exponential form and simplifying the fractional exponent. 3. Rationalizing the denominator of the radicand ➢ It is the process to remove the radical sign in the denominator, Which is making the radicand nonfrational. Example#1: $$\sqrt[3]{\frac{2}{5x^2}} = \sqrt[3]{\frac{2}{5x^2} * \frac{5^2x}{5^2x}} = \frac{\sqrt[3]{50x}}{5x}$$ Example#2: $$\sqrt[3]{\frac{1}{75}} = \sqrt[3]{\frac{1}{3{\cdot}5^2} * \frac{3^2{\cdot}5}{3^2{\cdot}5}} = \frac{\sqrt[3]{45}}{15}$$ Exercises: Simplify the following: 1. $\sqrt[4]{\frac{1}{192\,x^{11}y^5}}$ 2. $\sqrt[5]{\frac{1}{3xy^3}}$ 3. $\sqrt[3]{\frac{1}{25x^8y^{10}}}$ 4. $\sqrt[5]{\frac{\mathbf{1}}{\mathbf{3xy^3}}}$ 5. $\sqrt[6]{\frac{\mathbf{1}}{\mathbf{72x^2y}}}$

Addition & Subtraction of Radicals

Similar Radicals are radicals of the same order and the same radicand. Similar radicals can be combined into a single radical by the use of the distributive law. Radicals of different indices and radicands are called dissimilar radicals.

Example#1: $\sqrt{18} + \sqrt{8} - \sqrt{50}$

$= \sqrt{9 * 2} + \sqrt{4 * 2} - \sqrt{25 * 2}$

$= 3\sqrt{2} + 2\sqrt{2} - 5\sqrt{2} = (3 + 2 - 5)\sqrt{2} = \mathbf{0}$

Example#2: $\sqrt[3]{-8} - \sqrt[3]{-64} - \sqrt[4]{4} + \sqrt{8}$

$= -2 + 4 - \sqrt{2} + 2\sqrt{2}$

$= (-2 + 4) + \left(-\sqrt{2} + 2\sqrt{2}\right) = \mathbf{2} + \sqrt{\mathbf{2}}$

Multiplication of Radicals

- To multiply radicals of the same order, use the law of radicals

  $\sqrt[n]{a} * \sqrt[n]{b} = \sqrt[n]{ab}$

  Then simplify by the removal of the **n^th^** perfect powers from the radicand

Example#1: $\sqrt[3]{4x^3y^2} * \sqrt[3]{4x^5y^2}$

$= \sqrt[3]{16x^8y^4} = \sqrt[3]{8x^6y^3} * \sqrt[3]{2x^2y} = \mathbf{2x^2y\sqrt[3]{2x^2y}}$

Example#2: $\left(4\sqrt{6} - 5\sqrt{7}\right) * (2\sqrt{6} - 3\sqrt{7})$

$= \left(4\sqrt{6} * 2\sqrt{6}\right) - \left(5\sqrt{7} * 2\sqrt{6}\right) - (4\sqrt{6} * 3\sqrt{7}) + \left(5\sqrt{7} * 3\sqrt{7}\right)$

$= 8\sqrt{36} - 10\sqrt{42} - 12\sqrt{42} + 15\sqrt{49}$

$= 8(6) - 22\sqrt{42} + 15(7) = (48 + 105) - 22\sqrt{42} = \mathbf{153 - 22\sqrt{42}}$

- To multiply two radicals of different order, it is necessary to express them as radicals of the same order.

Example#3: $\sqrt[3]{\mathbf{2}} * \sqrt{\mathbf{3}} = \mathbf{2^{\frac{1}{3}}} * \mathbf{3^{\frac{1}{2}}} = \mathbf{2^{\frac{2}{6}}} * \mathbf{3^{\frac{3}{6}}}$

$= \sqrt[6]{\mathbf{2^2}} * \sqrt[6]{\mathbf{3^3}}$

$= \sqrt[6]{\mathbf{4 * 27}} = \sqrt[6]{\mathbf{108}}$

Example#4: $\sqrt[4]{4}\ \sqrt{2} = \sqrt{\sqrt{4}} * \sqrt{2} = \sqrt{2} * \sqrt{2} = \mathbf{2}$

Division of Radicals

- To divide radicals of the same order, use the law on radicals and rationalize the denominator of the radicand.

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Example#1: $\dfrac{\sqrt[4]{2x}}{\sqrt[4]{6x^3y^3}} = \sqrt[4]{\dfrac{1}{3x^2y^3}}$

$$= \sqrt[4]{\frac{1}{3x^2y^3} * \frac{3^3x^2y}{3^3x^2y}} = \frac{\sqrt[4]{27x^2y}}{3xy}$$

- To divide the radicals of different orders, it is necessary to express them as radicals with the same order.

Example#3: $\dfrac{\sqrt{3}}{\sqrt[3]{2}} = \dfrac{3^{\frac{1}{2}}}{2^{\frac{1}{3}}} = \dfrac{3^{\frac{3}{6}}}{2^{\frac{2}{6}}} = \sqrt[6]{\dfrac{3^3}{2^2} * \dfrac{2^4}{2^4}} = \dfrac{\sqrt[6]{432}}{2}$

- To divide the radicals with denominator of the radicand consisting of at least two terms, again rationalize the denominator.

Example#4: $\sqrt{\dfrac{a-b}{a+b}} = \sqrt{\dfrac{a-b}{a+b} * \dfrac{a+b}{a+b}} = \dfrac{\sqrt{a^2-b^2}}{a+b}$

Example#5: $\dfrac{4}{\sqrt{3}-2} = \dfrac{4}{\sqrt{3}-2} * \dfrac{\sqrt{3}+2}{\sqrt{3}+2} = \dfrac{4(\sqrt{3}+2)}{3-4} = -4(\sqrt{3}+2)$

Activities

Perform the indicated operations:

1. $\sqrt[3]{-16} - \sqrt[3]{-128} - \sqrt[3]{-2}$
2. $\sqrt[3]{\frac{1}{2}} - \sqrt[3]{\frac{1}{4}} - \sqrt[6]{16} - \sqrt[6]{4}$
3. $\sqrt[3]{18} * \sqrt[3]{4}$
4. $(2\sqrt{5}-1)^2$
5. $\sqrt[4]{27x^2y^3} * \sqrt[4]{15x^3y}$
6. $\dfrac{\sqrt[4]{3}}{\sqrt[4]{8y^2z}}$