University of Caloocan City

College of Liberal Arts & Sciences

Mathematics Department

COLLEGE ALGEBRA
Learning Module #4

Topic	**FACTORING**
Duration	**3 hours**

Lesson Proper

Factoring of Polynomials is simply the reverse process of the special product formulas. Thus, we shall be adapting the reverse process of the special product formulas to factor polynomials.

A polynomial with integral exponents is no longer factorable if:
1. the coefficients have no common factor, and
2. it cannot be expressed as the product of two polynomials of lower degree.

Types of Factoring:

1. **Common Monomial Factor:** $ax + ay = a(x + y)$

   Example #1: $24x^2 - 18x^3 = 6x^2(4 - 3x)$

2. **Difference of Two Squares:** $x^2 - y^2 = (x + y)(x - y)$

   Example #2: $9a^2 - 25b^2 = (3a)^2 - (5b)^2$
   $= (3a + 5b)(3a - 5b)$

   Example #3: $49b^4 - 16a^8 = \left(7b^2\right)^2 - (4a^4)^2$
   $= (7b^2 + 4a^4)(7b^2 - 4a^4)$

3. **Sum and Difference of Two Cubes:** $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
   $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

   Example #4: $8x^3 - 125y^3 = (2x)^3 - (5y)^3$
   $= (2x - 5y)\left(4x^2 + 10xy + 25y^2\right)$

   Example #5: $27a^6 + 64b^9 = \left(3a^2\right)^3 + \left(4b^3\right)^3$
   $= \left(3a^2 + 4b^3\right)\left(9a^4 - 12a^2b^3 + 16b^2\right)$

4. **Perfect Square Trinomial:** $x^2 + 2xy + y^2 = (x + y)^2$
   $x^2 - 2xy + y^2 = (x - y)^2$

   Example #6: $4x^2 - 12xy + 9y^2 = (2x - 3y)^2$

   Example #7: $25x^4 + 80x^2y^4 + 64y^8 = (5x^2 + 8y^4)^2$

Example Exercises: Factor the following completely:

a. $x^2 - 6x + 9$
b. $2x^3 + 4x^2 + 2x$
c. $4x^2 - 12xy + 9y^2$
d. $81x^6 - 18x^4 + x^2$

Other Types of Factoring

5. Other Trinomials : $\boldsymbol{x^2 + (a+b)x + ab = (x+a)(x+b)}$

Example #8: $\boldsymbol{x^2 - x - 6}$

Solution: The set of factors of x^2 are x and x. While that of -6 are ± 2 and ∓ 3, or ± 1 and ∓ 6. The correct choice depends on the sum of products Of the means and the extremes.

Thus, $\boldsymbol{x^2 - x - 6 = (x+2)(x-3)}$

6. Factoring by Grouping

Sometimes proper grouping of terms is necessary to make the given polynomial factorable. This type of factoring is usually applied to algebraic expressions consisting of at least four terms.

Example #9: $\boldsymbol{ax + ay - bx - by = a(x+y) - b(x+y)}$

$$\boldsymbol{= (x+y)(a-b)}$$

Example #10: $\boldsymbol{x^2 - y^2 + 2y - 1 = x^2 - (y^2 - 2y + 1)}$

$$\boldsymbol{= x^2 - (y-1)^2}$$
$$\boldsymbol{= [x + (y-1)][x - (y-1)]}$$
$$\boldsymbol{= (x + y - 1)(x - y + 1)}$$

7. Addition and Subtraction of Suitable Terms

This type of factoring is usually applied to polynomials of degree 4 with two terms being perfect squares and both preceded by positive sign. Through the addition and subtraction of suitable terms, the given will always lead to the difference of two squares.

Example #11: $\boldsymbol{x^4 + 4}$

Solution: Add and subtract $\boldsymbol{4x^2}$ **based on** $\boldsymbol{(x^2+2)^2}$

$$\boldsymbol{x^4 + 4 = x^4 + 4x^2 + 4 - 4x^2}$$
$$\boldsymbol{= (x^2+2)^2 - (2x)^2}$$
$$\boldsymbol{= (x^2 + 2x + 1)(x^2 - 2x + 2)}$$

8. Sum and Difference of Two Odd Primes

$$\boldsymbol{x^n + y^n = (x+y)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 - \cdots + y^{n-1})}$$
$$\boldsymbol{x^n - y^n = (x-y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 - \cdots + y^{n-1})}$$

Example #12: $\boldsymbol{x^5 + y^5 = (x+y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)}$

Exercises

Factor the following completely. Tell what type of factoring is be used.

1. $x^5 + 32y^5$
2. $x^4 - 14x^2 + 25$
3. $x^3 - x$
4. $a^2 - 121$
5. $x(m+n) - (m+n)$
6. $xy^4 + x^4y$
7. $x^2 - 6x + 9$
8. $25x^2 - 10x + 1$
9. $6x^2 - 9x - 15$
10. $x^3 + x^2 - x - 1$