University of Caloocan City

College of Liberal Arts & Sciences

Mathematics Department

COLLEGE ALGEBRA
Learning Module #6

Topic	**Complex Fractions and Exponents**
Duration	**3 hours**
Lesson Proper	

I. Complex fraction is a fraction whose numerator or denominator or both contains fractions.

Examples: $\dfrac{2}{\frac{5}{7}} = \dfrac{14}{5}$; $\dfrac{\frac{2}{5}}{7} = \dfrac{2}{35}$; $\dfrac{\frac{3}{4}}{\frac{2}{5}}$;

$\dfrac{\frac{x}{y}}{2}$; $\dfrac{\frac{a+b}{c}}{\frac{a-b}{d}}$; $\dfrac{\frac{x-1}{y+1}}{\frac{a}{b}}$

- Complex fractions can be reduced to lowest form by simplifying the numerator and denominator individually and then apply the division operation.

$$\frac{a/b}{c/d} = \frac{a}{b} * \frac{d}{c} = \frac{ad}{bc}$$

Example #1: $\dfrac{2x/3}{4/9x} = \dfrac{2x}{3} * \dfrac{9x}{4} = \dfrac{3x^2}{2}$

2. $$\frac{\frac{1}{x}+\frac{1}{y}}{\frac{x}{y}+\frac{y}{x}} = \frac{\frac{y+x}{xy}}{\frac{x^2-y^2}{xy}} = \frac{x+y}{xy} * \frac{xy}{(x+y)(x-y)} = \frac{1}{x-y}$$

3. $$\frac{25-\frac{x^2}{y^2}}{5-\frac{x}{y}} = \frac{\frac{25y^2-x^2}{y^2}}{\frac{5y-x}{y}} = \frac{(5y+x)(5y-x)}{y^2} * \frac{y}{5y-x} = \frac{5y+x}{y}$$

Self-evaluation:

a. $\dfrac{\frac{5}{12}+\frac{3}{8}}{\frac{7}{18}-\frac{11}{12}}$ b. $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}}$ c. $\dfrac{\frac{5-2y}{4-y}-1}{\frac{y^2-y+3}{y-4}+1}$

Example #4: Simplify the following:

$$\frac{4}{1-\frac{1}{1-\frac{1}{7}}} = \frac{4}{1-\frac{1}{\frac{7-1}{7}}} = \frac{4}{1-\frac{1}{\frac{6}{7}}} = \frac{4}{1-\frac{7}{6}} = \frac{4}{-\frac{1}{6}} = -24$$

II. Zero and Negative Exponents

The power a^n is defined as: $a^n = \underbrace{a * a \ldots a}_{\text{n factors}}$

$$a^0 = 1,\ a \neq 0$$

$$a^{-n} = \frac{1}{a^n}\ ,\ n\ is\ any\ positive\ integer, a \neq 0$$

Example #5: $(4a)^{-2} = \frac{1}{(4a)^2} = \frac{1}{16a^2}$

- **The rules on positive integral powers applies for negative and rational exponents.**

$$[\frac{a}{b}]^{-n} = \frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n}$$

Example #6: $[\frac{2a}{b}]^{-3} = [\frac{b}{2a}]^3 = \frac{b^3}{8a^3}$

- This means that: 1) a fraction raised to a negative integral exponent can be expressed to its reciprocal raised to a positive integral exponent.
2) Moreover, a factor of the numerator may be moved in the denominator, or vice-versa, if the sign of the exponent is changed.

Exercises: Simplify the following without negative exponents.

1. $-2^0 - 2 + 2^{-1} - 2^{-2} + 2^{-3}$
2. $-2^{-1} - (-2^2)^{-1} - (2^{-2})^0$
3. $\frac{2^{-1} - 3^{-1}}{2^{-1} + 3^{-1}}$
4. $[\frac{b}{a}]^{-9} * [\frac{a^2b^{-2}}{a^{-1}b^4}]^{-3}$

Self-evaluation

a. $-4 - 4^{-1} - 4^0 - 4^{-2}$

b. $(x-y)(x^{-1} - y^{-1})$

c. $\frac{5^{-1} - 5^{-2}}{5^{-1} + 5^{-2}}$

d. $\frac{x^{-1} - y^{-1}}{x - y}$

e. $[\frac{x^{-1} + y^{-1}}{x^{-1}y^2 + x^2y^{-1}}]^{-1}$