



Mathematics Department

COLLEGE ALGEBRA

Learning Module #2

Topic	POLYNOMIALS
Duration	3 hours
Lesson Proper	<p>II. POLYNOMIALS</p> <ul style="list-style-type: none"> ➤ Expressions combining numbers and letters involving at least one of the basic operations. <p>Definitions of Basic Terms in Polynomials</p> <ul style="list-style-type: none"> ➤ A constant is a symbol that assumes one specific value. ➤ A variable is a symbol that assumes many values. ➤ An algebraic expression, or simply expression, is a collection of constants and variables involving at least one of the basic operations in mathematics ➤ A term is an expression preceded by plus or minus sign one term: r^2; $\frac{2}{x}$; $\frac{a+b}{a-b}$; \sqrt{y}; $(x-y)^4$; $\frac{(a+b+c)}{3}$; $x \div y$ two terms: $ab - 3$; $\frac{x-abc}{3} - 1$; $\frac{1}{x} - \frac{1}{y}$; $x - \frac{x}{x+1}$; $a + x \div y$ ➤ A monomial is a term involving only the product of a real number and variables with nonnegative integral exponents Monomial: 6; $3b$; $15xyz^2$; x^2y^4; $\frac{xy}{3}$; abc; $\sqrt{2}$; abx Not monomial: $\frac{1}{x}$; $\frac{a}{b} - 5$; \sqrt{x}; $x + 2$; $\frac{1}{a+b}$; $x + y$ ➤ A polynomial is a sum of finite number of monomials. The general polynomial in one variable of degree n is of the form $a_nx^n + \cdots + a_1x + a_0$ A binomial is a polynomial consisting of exactly two terms A trinomial is a polynomial consisting of exactly three terms If a monomial is expressed as a product of two or more symbols, each of the symbol is called the coefficient of the rest of the product. Ex: $2xy - 2$ is called the numerical coefficient and xy is called the literal coefficient ➤ Two monomials (or two terms) are similar if they have the same literal coefficient

Addition of Expressions or Polynomials

Rule1: To add two or more monomials with the same literal coefficients, add only their numerical coefficients, and affix the literal coefficients:

Ex. 1: $-8x + 15x = (-8 + 15)x = 7x$

Rule 2: To add two or more polynomials, add similar or like terms together. **Ex. 2:** $3x^2 - 4x - 4y$; $7x^2 - 2y - 2$ and $-4x^2 + x - y - 7$

Ex. 3: Subtract $4x - y - 3$ from $2x - y - 4$

Rule 3: To remove a grouping symbol preceded by a:

- minus sign, change the sign of each of the terms;
- plus sign, no further change is done;
- Factor, use the distributive law

Ex. 4: $4x - 2y - 5 - 2(8x - 7y) - (3x - 4y - 1)$

Rule 4: When one symbol of grouping is within another symbol of grouping, the innermost symbol must be removed first.

Ex. 4: $- \{ - 2x - y - [3x - (4x + y - 3) - y] - 7 \}$

Exercises: Simplify the following:

- a. $- \{ 12x - 3y - 8 - [6x - (x + 7y - 3)] \}$ (ans. $- 7x - 4y + 11$)
b. $- [-3x - y - (4y - z)] - [8y + 7x - (5x - 9z)]$ (ans. $x - 3y - 10z$)

Powers with Positive Integral Exponents

Laws of Exponents:

1. The Product of Powers: $a^m * a^n = a^{m+n}$ **Ex. 5:** $x^5 * x^4 = x^9$
2. The Quotient of Powers: $\frac{a^m}{a^n} = a^{m-n}$ **Ex. 6:** $\frac{x^7}{x^3} = x^4$; $\frac{x^4}{x^6} = \frac{1}{x^2}$
3. The Power of a Power: $(a^m)^n = a^{mn}$ **Ex. 7:** $(x^2)^4 = x^8$
4. The Power of a Product: $(ab)^m = a^m b^m$ **Ex. 8:** $(2a^2b^4)^3 = 8a^6b^{12}$
5. The Power of a Quotient: $\left[\frac{a}{b}\right]^m = \frac{a^m}{b^m}$ **Ex. 9:** $\left[\frac{-2x}{3}\right]^4 = \frac{16x^4}{81}$

Exercises: Simplify the following:

- a. -4^2 b. $- \left[\frac{-3}{2}\right]^4$ c. $\frac{(a-b)^3}{a-b}$ d. $\frac{8^{3x}}{4^{4x}}$
e. $\frac{25^{100}}{125^{50}}$ f. $\frac{(x^4y^8z)^3}{(x^5y^8zw)^2}$ g. $\left[\frac{12x^3y^4z}{18x^2y^6}\right]^2$

Product of Polynomials

Rule 1: To multiply two monomials, use commutative, associative and the laws of exponents in multiplication. **Ex. 10:** $(3x^2y^4z^2w^9)(-4xyz^4wv)$

Rule 2: To multiply two polynomials, use the distributive law and apply Rule

1. **Ex. 11:** $(2x - 3y)(4x + 5y)$

Exercises: Find the products of the following:

- $(5x^4 - 8y^3)(5x^4 + 8y^3)$
- $(5x^4 - 8y^3)(5x^4 - 8y^3)$
- $(a^2 - b)(a^2 + a^2b + b^2)$
- $(x^7 - x^6 + x^4 - x^3 - 5x + 3)(x^5 - x^2 + 2)$

Simplify the following:

- $-(2x - 5y) - \{4 - [5y - (2y - 9x) - 3] - 6x\}$
- $-\{2x - [4y - 3x - (4 - 3y + 5x)] - 7\}$
- $-[8x - 2y - (5 + 7y)] - \{6x - (2 - 8y - 2x)\}$
- Subtract the sum of the second and third expressions from the first
 $3x - 2y + 8z; 12x - y - 10z; 3x - y + 7z$
- Subtract the first from the sum of the second and the third expressions
 $2x - y - 3; 4x + y - 5; x + 11y - 8$
- $\frac{3x^n - 2x^n}{(x^n)^2}$
- $\frac{2x^4 + 3x^4}{2x^4 \cdot 3x^4}$
- $\frac{64^a}{128^a}$
- $\frac{(2x^4y^8)^3}{(4x^8y^4)^2}$
- $(3x^4y^2z^2)^2(4x^2y^4z^3w)^3$
- $(7x - 8y)(7x - 8y)$
- $(21x^4 - 13y^3)(21x^4 + 13y^3)$
- $(x^5 - 3x^4 - x^3 + x - 3)(x^3 - x + 5)$