COLLEGE ALGEBRA
Learning Module #5

Topic	**Simplifying, Addition, Multiplication & Division of Fractions**
Duration	**3 hours**

Lesson Proper

I. Simplifying Fractions

A. Lecture:

- ✓ A fraction is said to be in simplified form if the numerator and denominator have no common factor except 1.
- ✓ If a common factor appears in the numerator and denominator such can be renowned by division using the property of real numbers.

$$\frac{ac}{bc} = \frac{a}{b} \qquad b \neq 0\,, \quad c \neq 0$$

$$\frac{(\boldsymbol{x-y})}{(\boldsymbol{x-y})} = \mathbf{1} \quad \boldsymbol{and} \quad \frac{(\boldsymbol{x-y})}{\boldsymbol{y-x}} = \frac{(\boldsymbol{x-y})}{-(\boldsymbol{x-y})} = -\mathbf{1}$$

B. Illustrations:

Problem #1: $\frac{x^2-4}{2x-4}$

Solution:

Factoring the polynomial in the Numerator and Denominator

$$\frac{(x+2)(x-2)}{2(x-2)} \qquad \frac{\to \text{ Difference of Two Squares}}{\to \text{ Common Monomial Factor}}$$

Take note: $(\boldsymbol{x}-\mathbf{2})$ resulted to be the Common Factor

Making Use of Cancellation Method: $\frac{(x+2)\cancel{(x-2)}}{2\cancel{(x-2)}} = \frac{\boldsymbol{x}+\mathbf{2}}{\mathbf{2}}$

Problem #2: $\frac{\mathbf{2}\boldsymbol{x}^{\mathbf{2}}-\mathbf{3}\boldsymbol{x}+\mathbf{1}}{\mathbf{2}\boldsymbol{x}^{\mathbf{2}}+\boldsymbol{x}-\mathbf{1}}$

Solution: $\frac{\cancel{(2x-1)}(x-1)}{\cancel{(2x-1)}(x+1)} = \frac{(\boldsymbol{x}-\mathbf{1})}{(\boldsymbol{x}+\mathbf{1})}$

Take note: Numerator and Denominator are both factorable by other Types of factoring.

Problem #3: $\dfrac{\boldsymbol{-x^2-1}}{\boldsymbol{1-x^4}}$

Solution:
Arrange polynomial in the Denominator in Standard Form $ax^2 + bx + c$

$$\frac{-x^2 - 1}{-x^4 + 1} \quad = \quad \text{Factoring} - 1$$

$$\frac{-(x^2+1)}{-(x^4-1)} \quad = \quad \frac{-\cancel{(x^2+1)}}{-\cancel{(x^2+1)}(x^2-1)} \quad = \frac{1}{x^2-1} = \frac{\mathbf{1}}{(\boldsymbol{x}+\mathbf{1})(\boldsymbol{x}-\mathbf{1})}$$

C. Self-Evaluation:

Solve the following problems:

1. $\frac{5x-6}{10x-12}$ 2. $\frac{4cd-xcd}{8-2x}$ 3. $\frac{x^2-16}{x^2+4x}$ 4. $\frac{3+2x-x^2}{1+5x+4x^2}$

5. . $\frac{6abc-18ab}{3a^2bc-9a^2b}$ 6. $\frac{2x+4y}{x^2+2xy}$ 7. $\frac{x^2-1}{x^2-x}$ 8. $\frac{x^2-x-6}{x^2-5x+6}$

II. Addition and Subtraction of Fractions

A. Lecture:

- ✓ Addition of Fractions depends on the kinds of denominator the fraction have.

1. To add two fractions with the same denominator, use

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad , \quad c \neq 0$$

Illustrations:

a. $\frac{\boldsymbol{x-3}}{\boldsymbol{x-2}} + \frac{\mathbf{1}}{\boldsymbol{x-2}} = \frac{\boldsymbol{x-3+1}}{\boldsymbol{x-2}} = \frac{\boldsymbol{x-2}}{\boldsymbol{x-2}} = \mathbf{1}$

b. $\frac{\boldsymbol{2x}}{\boldsymbol{x-1}} + \frac{\mathbf{2}}{\boldsymbol{1-x}} = \frac{\boldsymbol{2x}}{\boldsymbol{x-1}} + \frac{\mathbf{2}}{-(\boldsymbol{x-1})} = \frac{\boldsymbol{2x}}{\boldsymbol{x-1}} - \frac{\mathbf{2}}{\boldsymbol{x-1}} = \frac{\boldsymbol{2x-2}}{\boldsymbol{x-1}} = \frac{\mathbf{2}(\boldsymbol{x-1})}{(\boldsymbol{x-1})} = \mathbf{2}$

2. To add two fractions with different denominators

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \qquad , \quad b \neq 0 \quad , \quad d \neq 0$$

Illustrations:

a. $\frac{\mathbf{2}}{\mathbf{3}} + \frac{\mathbf{5}}{\mathbf{4}}$

Find the LCD which is 12. Then find the equivalent fraction

$\frac{2}{3} = \frac{8}{12}$ $\qquad$ $\frac{5}{4} = \frac{15}{12}$

÷ $\qquad$ 3

$\frac{2}{3} + \frac{5}{4}$ **is equivalent to** $\frac{8+15}{12} = \frac{23}{12}$

b. $\frac{2x}{x-1} + \frac{2}{1-x} = \frac{2x}{x-1} + \frac{2}{-(x-1)} = \frac{2x-2}{x-1} = 2$

How to find the LCM (Least Common Multiple) of the following sets of expressions:

C. Examples:

Find the least common multiple of the following sets of expressions:

a. x^2 and x^3

b. $uw^4x^3y^9$ and $w^6x^7y^2z$

c. $(x^3-1); (x^2-1); (x^2-2x-1)$

d. $(x+1)(x^2-2x+1);\ (x^2-x-2)^2; (x^2-1)(x-2)^3$

e. $(x^2-1)(x-1);\ x^3-1; (x^2+x+1)^2$

Ans. $(x+1)(x-1)^2(x^2+x+1)^2$

Solutions:

a. The greatest power serves as the LCM. Thus, the LCM of x^2 and x^3 is x^3.

b. Selecting the greatest power for each kind of variable gives the LCM of $uw^4x^3y^9$ and $w^6x^7y^2z$ as $u6x^7y^9z$

c. $x^3-1 = (x-1)(x^2+x+1)$; $x^2-1 = (x+1)(x-1)$; $x^2-2x-1 = (x-1)^2$.

Selecting the greatest power for each kind of factor, we get:

LCM $=(x-1)^2(x^2+x+1)(x+1)$

d. Express the given into its prime factors. For each kind of factor, choose the greatest power to comprise the LCM.

$$\begin{array}{rcl|c|c} (x+1)(x^2-2x+1) & = & (x+1) & (x-1)^2 & \\ (x^2-x-2)^2 & = & (x+1)^2 & & (x-2)^2 \\ (x^2-1)(x-2)^3 & = & (x+1) & (x-1) & (x-2)^3 \end{array}$$

Selecting the greatest power for each kind of factor, we get:

LCM $= (x+1)^2(x-1)^2(x+2)^3$

D. Self-Evaluation:

Solve the following problems:

1. $\frac{7}{36} - \frac{2}{45} + \frac{11}{60}$

2. $\frac{11}{42} - \frac{2}{63} + \frac{5}{72}$

3. $\frac{2}{x-y} - \frac{3}{x+y}$

4. $\frac{2x-1}{x^2(x-1)^2} - \frac{2}{(x^2-1)(x-1)}$

III. Multiplication and Division of Fractions

A. Lecture:

- ✓ To multiply and divide two fractions, make use of the following theories:

$$\frac{a}{b},\frac{c}{d} = \frac{ac}{bd} \qquad where \quad b \neq 0\,, \quad d \neq 0$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc} \qquad where \quad b \neq 0\,, \quad d \neq 0$$

B. Illustrations:

a. $\frac{3}{5}\cdot\frac{10}{21} = \frac{2}{7}$ $\Rightarrow$ Dividing 3 & 21 by a 3 CF · Dividing 5 & 10 by 5

b. $\frac{8x^3}{9y^2} \div \frac{4x^2}{3y} = \frac{8x^3}{9y^2}\cdot\frac{3y}{4x^2} = \frac{2x}{3y}$

c. $\frac{x^2-1}{x-1} \div (x+1) \Rightarrow \frac{(x+1)(x-1)}{(x-1)}\cdot\frac{1}{(x+1)} = 1$

C. Examples:

Problem #1:

$$\frac{x^2y-xy}{y^2-1}\cdot\frac{y^3+y^2}{x^3-x^2} \div \frac{y^2}{y-1}$$

$$= \frac{\cancel{x}y\cancel{(x-1)}}{\cancel{(y+1)}\cancel{(y-1)}}\cdot\frac{\cancel{y^2}\cancel{(y+1)}}{x^{\cancel{2}}\cancel{(x-1)}}\cdot\frac{\cancel{y-1}}{\cancel{y^2}} = \frac{y}{x}$$

Problem #2:

$$\frac{x^4-16}{x^3-64}\cdot\frac{x^2+4x+16}{x^4-5x^2+4} \div \frac{x^3+4x}{x^3-3x^2-4x}$$

$$= \frac{\cancel{(x^2+4)}\cancel{(x+2)}\cancel{(x-2)}}{\cancel{(x-4)}\cancel{(x^2+4x+16)}}\cdot\frac{\cancel{x^2+4x+16}}{\cancel{(x-2)}\cancel{(x+2)}\cancel{(x+1)}(x-1)}\cdot\frac{x\cancel{(x+1)}\cancel{(x-4)}}{x\cancel{(x^2+4)}}$$

$$= \frac{1}{x-1}$$

Problem #3:

$$\frac{y^3-27}{9y-y^3} \div \frac{y^4+3y^3+9y^2}{y^4+3y^3}$$

$$= \frac{(y-3)\cancel{(y^2+3y+9)}}{y(3-y)\cancel{(3+y)}}\cdot\frac{y^3\cancel{(y+3)}}{y^2\cancel{(y^2+3y+9)}} = -1$$

D. Self-Evaluation:

Solve the following problems:

1. $\frac{16a^2b^4c^3}{27a^4c^2} \div \frac{8b^2c^3}{9a^2c^3}$

2. $\frac{xy-x^2}{x^2} \div \frac{xy-y^2}{y^2}$

3. $\frac{4x^3-4x^2+x}{8x^2-4x} \div \frac{4x^3-2x^2}{4x^3}$

4. $\frac{x^4-27x}{x^2-6x+9} \div \frac{x^3+3x^2+9x}{x^2-9}$

5. $\frac{x^2-y^2}{ax+bx-ay-by} \div \frac{ax-2ay+bx-2by}{x^2+3xy+2y^2}$