1. $\int(6x^2 - 4x + 5\,)\,dx$

$= \int \frac{6x^3}{3} - \frac{4x^2}{2} + 5x + c$

$= \boxed{\mathbf{2x^3 - 2x^2 + 5x + c}}$

3. $\int x(\sqrt{x} - 1)dx$

$= \int\left(x\sqrt{x} - x\right)dx$

$= \int x^{\frac{3}{2}}\,dx - \int xdx$

$= \boxed{\mathbf{\frac{2}{5}x^{\frac{5}{2}} - \frac{1}{2}x^2 + c}}$

5. $\int \frac{2x^2+4x-3}{x^2}dx$

$= \int\left(2 + \frac{4}{X} - \frac{3}{X^2}\right)dx$

$= \int 2dx + \int \frac{4}{x}dx - \int \frac{3}{x^2}dx$

$= 2x + 4\int \frac{dx}{x} - \int \frac{3x^{-1}}{-1}dx$

$= \boxed{\mathbf{2x + 4lnx + \frac{3}{x} + c}}$

7. $\int \frac{x^3-8}{x-2}dx$

= Factor, (x-c), c = 2
P(c) = 0 – the (x-c) is the factor
P(c) = 0

```
 2|  1 0 0 -8
      2 4 8
     1 2 4 0
```

$= \frac{(X^2+2X+4)\cancel{(X-2)}}{\cancel{(X-2)}}$

$= \int(x^2 + 2x + 4)dx$

$= \frac{x^3}{3} + \frac{2x^2}{2} + 4x + c$

$= \boxed{\mathbf{\frac{x^3}{3} + x^2 + 4x + c}}$

9. $\int \sqrt{x^4 - 2x^3 + x^2}dx$

$= \int x^2dx - \int 2x^{\frac{2}{3}}dx + \int xdx$

$= \boxed{\mathbf{\frac{x^3}{3} - \frac{6^{\frac{5}{3}}}{5} + \frac{x^2}{2} + C}}$

1. $\int \sqrt{2-3x}\,dx$

Let u = 2 - 3x$\frac{du}{dx} = -3$

$\frac{-du}{3} = dx$

= $\int u^{\frac{1}{2}}(-\frac{du}{3})$

$= -\frac{1}{3}\int u^{\frac{1}{2}} du$

= $-\frac{1}{3}\left[\frac{2x^{\frac{3}{2}}}{3}\right] + c$

$\boxed{= -\frac{\mathbf{2(2-3x)^{\frac{3}{2}}}}{\mathbf{9}} + \boldsymbol{c}}$

3. $\int x^2(2x^3-1)^4 dx$

Let u = $2x^3 - 1$

$\frac{du}{dx} = 6x^2$

$\frac{du}{6} = x^2 dx$

$= \int x^2(2x^3-1)^4 dx$

$= \int (u^4)(\frac{du}{6})$

$= \frac{1}{6}\int (u^4)du$

$= \frac{1}{6}\left[\frac{u^5}{5}\right] + c$

$= \frac{u^5}{30} + c$

$= \boxed{\frac{\mathbf{(2x^3-1)^5}}{\mathbf{30}} + \boldsymbol{c}}$

5. $\int \frac{(2x+3)dx}{x^2+3x+4}$

Let u = $x^2 + 3x + 4\,\frac{du}{dx} = 2x + 3$

$du = (2x+3)dx$

= $\int \frac{du}{u}$

= $lnu + c$

$\boxed{= \mathbf{ln}(\boldsymbol{x^2} + \mathbf{3}\boldsymbol{x} + \mathbf{4}) + \boldsymbol{c}}$

7. $\int \frac{x^2dx}{(x^3-1)^4}$

Let u = $x^3 - 1\,\frac{du}{dx} = 3x^2$

$\frac{du}{3} = x^2dx$

= $\int \frac{\frac{du}{3}}{x^4}$

= $\frac{1}{3}\int u^{-4}$

= $\frac{1}{3}\left[\frac{u^{-3}}{-3}\right] + c$

= $\frac{u^{-3}}{-9} + c$

$\boxed{= -\frac{\mathbf{1}}{\mathbf{9(x^3-1)^3}} + \boldsymbol{c}}$

9. $\int \frac{dx}{xln^2x}$

Let u = $lnx \frac{du}{dx} = \frac{1}{x}$

$du = \frac{dx}{x}$

= $\int \frac{1}{ln^2x}(\frac{dx}{x})$

= $\int \frac{1}{u^2} du$

= $\int u^{-2} du$

= $\frac{u^{-1}}{-1} + c$

= $\boxed{-\frac{\mathbf{1}}{\boldsymbol{lnx}} + \boldsymbol{c}}$

11. $\int \frac{dx}{e^{x-1}}$

Let u = $e^x \quad du = e^x dx$

= $\int (\frac{1}{u-1} - \frac{1}{u})du$

= $\int \frac{1}{u-1} du - \int \frac{1}{u} du$

Let v = $u - 1$

$\frac{dv}{u} = du$

= $\int \frac{1}{u} dv - \int \frac{1}{u} dv$

= $[lnu - lnu] + c$

$u = u - 1 \quad ; u = e^x$

= $[\ln|u - 1| - \ln|e^x|] + c$

= $\ln(1 - e^x)$

= $\boxed{\mathbf{ln}(\mathbf{1} - \boldsymbol{e^x}) - \boldsymbol{x} + \boldsymbol{c}}$

13. $\int \cos^4 x \sin x dx$

Let u = $\cos x \quad \frac{du}{dx} = -\sin x$

$du = -\sin x\, dx$

= $\int u^4(-du)$

= -$\int u^4 du$

= $-\frac{u^5}{5} + c$

= $\boxed{-\frac{\mathbf{cos^5}\boldsymbol{x}}{\mathbf{5}} + \boldsymbol{c}}$

15. $\int \sqrt{1 + 2\sin 3x}\, cos3xdx$

Let u = $3x$

$\frac{du}{dx} = 3$

$\frac{du}{3} = dx$

= $\int \sqrt{1 + 2\sin u}\, cosu\, (\frac{du}{3})$

= $\frac{1}{3} \int \sqrt{1 + 2\sin u}\, cosudu$

Let v =$1 + 2\sin u$

$\frac{dv}{du} = 2cosu\,; \frac{dv}{2} = cosudu$

= $\int \sqrt{1 + 2\sin 3x}\, cos3xdx$

= $\frac{1}{3}[\int v^{\frac{1}{2}} (\frac{dv}{2})]$

= $\frac{1}{6}\left[\frac{2v^{\frac{3}{2}}}{3}\right] + c$

= $\boxed{\frac{(\mathbf{1}+\mathbf{2}\boldsymbol{sinx})^{\frac{3}{2}}}{\mathbf{9}} + \boldsymbol{c}}$

17. $\int \frac{sec^2 xdx}{a+b\ tanx}$

Let u = $a + b\ tanx$

$\frac{du}{dx} = b \frac{\sin x}{cosx}$; $\frac{du}{b} = sec^2 xdx$

=$\int \frac{\frac{du}{b}}{u}$

= $\frac{1}{b}\int \frac{du}{u}$

= $\boxed{\frac{1}{b}\ln|a + btanx| + c}$

19. $\int \sqrt{tan3x}\, sec^2 3xdx$

Let u = $tan3x$

$\frac{du}{dx} = 3sec^2 3x$; $\frac{du}{3} = sec^2 3xdx$

= $\int u^{\frac{1}{2}}(\frac{du}{3})$

= $\frac{1}{3}[\frac{2u^{\frac{3}{2}}}{3}] + c$

= $\frac{1}{3}\left(\frac{2\tan 3x^{\frac{3}{2}}}{3}\right) + c$

= $\boxed{\frac{2(\tan 3x)^{\frac{3}{2}}}{9} + c}$

21. $\int \frac{3x^2+14x+14}{x+4} dx$

=$\int \frac{f\ (x)}{g\ (x)}$ = $\int Q(x)d(x) + \int \frac{R\ (x)}{g\ (x)} d(x)$

* using synthetic division

-4| 3 14 13

-12 -8

3 2 5 R(x)

$Q(x) = 3x + 2$

$x + 4 = denominator\ g(x)$

= $\int (3x + 2)\, dx + \int \frac{5}{x+4} dx$

For the second integral :

$let\ u = x + 4$; $\frac{dy}{dx} = 1$; $du = dx$

=$\int (3x + 2)dx + 5\int \frac{du}{u}$

= $[\frac{3x^2}{2} + 2x + 5lnu + c]$

= $\boxed{\frac{3x^2}{2} + 2x + 5\ln(x + 4) + c}$

23. $\int \frac{x^5-2x^3-2x}{x^2+1}dx$

$$\begin{array}{r|l} & x^3 - 3x \\ \hline x^2+1 & x^5 - 2x^3 - 2x \\ & x^5 + x^3 \\ \hline & \quad -3x^3 - 2x \\ & \quad -3x^3 - 3x \\ \hline & \qquad\qquad x \end{array}$$

$\frac{f(x)}{g(x)}$dx = $\int Q(x)dx + \int \frac{R(x)}{g(x)}dx$

=$\int (x^3 - 3x)dx + \int \frac{x}{x^2+1}dx$

=$\frac{x^4}{4} - \frac{3x^2}{2}$+$\int \frac{x}{x^2+1}dx$

For the 2nd term

Let u = x^2+1

$$\frac{du}{dx} = 2x$$

$$\frac{du}{2} = xdx$$

=$\frac{x^4}{4} - \frac{3x^2}{2} + \int \frac{\frac{du}{2}}{u}$

= $\boxed{\boldsymbol{\frac{x^4}{4} - \frac{3x^2}{2} + \frac{1}{2}\ln|x^2+1| + c}}$

1. $\int sec5xtan5xdx$

$Let\ u = 5x$

$\frac{du}{dx} = 5 \quad \frac{du}{5} = dx$

$= \int secutanu\left(\frac{du}{5}\right)$

$= \frac{1}{5}\int secutanudu$

$= \frac{1}{5}\int secu + c$

$= \boxed{\frac{1}{5}\int sec5x + c}$

3. $\int \frac{sinx + cosx}{sin^2x}dx$

$= \int \frac{sinx}{sin^2x}dx + \int \frac{cosx}{sin^2x}dx$

$= \int \frac{1}{sinx}dx + \int cotxcscxdx$

$= \int cscdx + \int cotxcscxdx$

$= \boxed{-ln|cscx + cotx| - cscx + c}$

5. $\frac{dx}{\sin\frac{1}{2}x\cot\frac{1}{2}x}$; Let u$= \frac{1}{2}x$

$\frac{du}{dx} = \frac{1}{2} \qquad 2du = dx$

$= \int \frac{2du}{sinucotu}$

$= 2\int \frac{du}{sinu\ cotu}$

$= 2\int \frac{du}{sinu\ \left(\frac{cosu}{sinu}\right)}$

$= 2\int \frac{1}{cosu}(du)$

$= 2\int secudu$

$= \boxed{2ln\ |cscx + cotx| + c}$

7. $\int \frac{\cos^3 xdx}{1-sinx}\cdot\frac{1+sinx}{1+sinx}$

$= \int \frac{(\cos^3 x)(1+sinx)dx}{(1-sinx)(1+sinx)}$

$= \int \frac{(\cos^3 x + \cos^3 xsinx)dx}{1-\sin^2 x}$

$= \int \frac{\cancel{\cos^3 x}(1+sinx)dx}{\cancel{\cos^3 x}}$

$= \int cosx(1 + sinx)dx$

$= sinx + \int cosxsinxdx$

$Let\ u = sinx$

$\frac{du}{dx} = cosx\ ;\ du = cosxdx$

$= sinx + \int udu$

$= sinx + \frac{u^2}{2} + c$

$= \boxed{sinx + \frac{\sin^2 x}{2} + c}$

9. $\int(1 + tanx)^2dx$

$= \int(1 + 2tanx + \tan^2 x)dx$

$= \int[2tanx + (1 + \tan^2 x)]dx$

$= 2\int tanxdx + \int \sec^2 xdx$

$= 2(-ln|cosx|) + tanx + c$

$= \boxed{-2\ ln\ |cosx| + tanx + c}$

11. $\int \frac{\cos 6x dx}{\cos^2 3x}$

Let u = 3x ; 2u = 6x

$\frac{du}{dx} = 3$; $\frac{du}{3} = dx$

$= \int \frac{\cos 2u\left(\frac{du}{3}\right)}{\cos^2 u}$

$= \frac{2}{3}\int \left(\frac{1}{\cos u}\right)\left(\frac{\cos u}{\cos u}\right) du$

$= \frac{2}{3}\int \sec u du$

$= \boxed{\frac{2}{3}\ln|\sec 3x + \tan 3x| + c}$

13. $\int \frac{\sin 2x dx}{2\sin x \cos^2 x}$

$= \int \frac{2\sin x\cos x dx}{(2\sin x\cos x)\cos x}$

$= \int \frac{dx}{\cos x}$

$= \int \frac{1}{\cos x} dx$

$= \int \sec x dx$

$= \boxed{\ln|\sec x + \tan x| + c}$

15. $\int \frac{4\sin^2 x\cos^2 x}{\sin 2x\cos 2x} dx$

$= \int \frac{(4\sin x\cos x)(\sin x\cos x)}{(2\sin x\cos x)\cos 2x} dx$

$= \int \frac{2\sin x\cos x}{\cos 2x} dx$

$= \int \frac{\sin 2x}{\cos 2x} dx$

Let u = 2x

$\frac{du}{dx} = 2$ $\quad \frac{du}{2} = dx$

$\int \frac{\sin u}{\cos u} \cdot \left(\frac{du}{2}\right)$

$= \frac{1}{2}\int \tan u du$

$= \boxed{-\frac{1}{2}\ln|\cos 2x| + c}$

17. $\int \frac{dx}{\sin 3x\tan 3x}$

Let u = 3x

$\frac{du}{dx} = 3$ $\quad \frac{du}{3} = dx$

$= \int \frac{\frac{du}{3}}{\sin u\tan u}$

$= \frac{1}{3}\int \csc u + c$

$= \boxed{-\frac{1}{3}\csc 3x + c}$

1. $\int \frac{dx}{e^{2x}}$

$= \int e^{-2x} dx$

$let\ u = 2x\ ;\ \frac{du}{dx} = -2\ ;\ -\frac{du}{2} = dx$

$= \int e^u (-\frac{du}{2})$

$= -\frac{1}{2}\int e^u du$

$= -\frac{1}{2}(e^u) + c$

$= -\frac{1}{2e^u} + c$

$= \boxed{-\frac{1}{2}(e^{-2x}) + c}$

3. $\int e^{sin\,4x} cos4xdx$

$let\ u = 4x\ ;\ \frac{du}{dx} = 4\ ;\ \frac{du}{4} = dx$

$= \int e^{sinu} cosu(\frac{du}{4})$

$= \frac{1}{4}\int e^{sinu} cosudu$

$let\ v = sinu\ ;\ \frac{dv}{du} = \cos u\ ;\ dv = cosudu$

$= \frac{1}{4}\int e^v dv$

$= \frac{1}{4}(e^v) + c$

$= \boxed{\frac{e^{sin4x}}{4} + c}$

5. $\int \sqrt{e^{3x}}\, dx = \int e^{\frac{3x}{2}} dx$

$let\ u = \frac{3x}{2}\ ;\ \frac{2du}{3} = dx$

$= \int e^u (\frac{2du}{3})$

$= \frac{2}{3}\int e^u du$

$= \frac{2}{3}\left[e^{\frac{3x}{2}}\right] + c$

$= \boxed{\frac{2\sqrt{e^{3x}}}{3} + c}$

7. $\int 5^{3-2x} dx$

$let\ u = 3 - 2x\ ;\ \frac{du}{dx} = -2\ ;\ -\frac{du}{2} = dx$

$= \int 5^u (-\frac{du}{2})$

$= -\frac{1}{2}\int 5^u du$

$= -\frac{1}{2}\left(\frac{5^{3-2x}}{ln5}\right) + c$

$= \boxed{-\frac{5^{3-2x}}{ln25} + c}$

9. $\int 3^x 2^x dx$

$a^x b^x = (ab)^x$

$= \int 6^x dx$

$= \boxed{\frac{6^x}{ln6} + c}$

1. $\int sinh(3x-1)dx$

Let $u = 3x-1$

$\frac{du}{dx} = 3\,;\, dx = \frac{du}{3}$

$= \int sinh\,u\,(\frac{du}{3})$

$= \frac{1}{3}\int sinh\,udu$

$= \frac{1}{3}cosh\,udu + c$

$= \boxed{\frac{1}{3}\boldsymbol{cosh(3x-1)+c}}$

3. $\int csch^2\,(1-x^2)xdx$

Let $u=1-x^2$

$\frac{du}{dx} = -2x$

$-\frac{du}{2} = xdx$

$=\int csch^2u(-\frac{du}{2})$

$=-\frac{1}{2}\int csch^2udu$

$=-\frac{1}{2}(-cothu+c)$

$= \boxed{\frac{1}{2}\boldsymbol{coth(1-x^2)+c}}$

5. $\int \frac{sech^2(lnx)dx}{x}$

$let\ u = lnx\ \ ;\ \frac{du}{dx} = \frac{1}{x}\ \ ; du = \frac{dx}{x}$

$= \int sech^2udu$

$= tanhu + c$

$= \boxed{\boldsymbol{tanh(lnx)+c}}$

7. $\int csch\frac{1}{2}x\,coth\frac{1}{2}xdx$

Let u $=\frac{1}{2}x\ ;\ \frac{du}{dx} = \frac{1}{2}\ ;\ 2du = dx$

$= 2\int cschu\ cothudu$

$= 2(-cschu + c)$

$= \boxed{\boldsymbol{-2csch\frac{1}{2}x+c}}$

1. Given slope $3x^2 + 4$

$$\frac{dy}{dx} = 3x^2 + 4$$

$$dy = (3x^2 + 4)dx$$

$$\int dy = \int (3x^2 + 4)\,dx$$

$$y = \frac{3x^3}{3} + 4x + c$$

$$\boxed{\boldsymbol{y = 3x^2 + 4x + c}}$$

3. Given slope $\frac{x+1}{y-1}$

$$\frac{dy}{dx} = \frac{x+1}{y-1}$$

$$\int (y-1)dy = \int (x+1)dx$$

$$\left(y^2 - 2y = \frac{x^2}{2} + x + c\right)2$$

$$y^2 - 2y = x^2 + 2x + 2c$$

$$\boxed{\boldsymbol{x^2 - y^2 + 2y + 2x + 2c = 0}}$$

5. Given slope $\frac{1}{xy}$

$$\frac{dy}{dx} = \frac{1}{xy}$$

$$\int ydy = \int \frac{dx}{x}$$

$$\left(\frac{y^2}{\cancel{2}} = \frac{\ln x^2}{\cancel{2}} + c\right)2$$

$$\boxed{\boldsymbol{y^2 = lnx^2 + 2c}}$$

7. Given slope $\frac{y^2}{x}$, through (1,4)

$$\frac{dy}{dx} = \frac{y^2}{x}$$

$$\int \frac{dy}{y^2} = \int \frac{dx}{x}$$

$$-\frac{1}{4} = lnx + c$$

$$-\ln x - \frac{1}{4} = c$$

$$-\ln 1 \ -\frac{1}{4} = c$$

$$c = -\frac{1}{4}$$

$$\left(-\ln x - \frac{1}{y} + \frac{1}{4} = 0\right)4y$$

$$-4y\ln x - 4 + y = 0$$

$$\boxed{\boldsymbol{4y\ln x - y + 4 = 0}}$$

9. Given slope $\sqrt{y}$, through (1,1)

$$\frac{dy}{dx} = \sqrt{y}$$

$$\int y^{-\frac{1}{2}}dy = \int dx$$

$$\frac{y^{\frac{1}{2}}}{\frac{1}{2}} = x + c$$

$$2y^{1/2} = x + c$$

When $x = 1$, $y = 1$

$$2(1) = 1 + c \ ; \ c = 1$$

$$\left(2y^{1/2} = x + c\right)^2$$

$$\boxed{\boldsymbol{4y = (x+1)^2}}$$

11. Given slope x^{-2}, through (1,2)

$$\frac{dy}{dx} = \frac{1}{x^2}$$

$$\int dy = \int \frac{dx}{x^2}$$

$$y = -\frac{1}{x} + c$$

$$2 = -\frac{1}{1} + c$$

$$2 = -1 + c$$

$$c = 3$$

$$\left(y = -\frac{1}{x} + 3\right)x$$

$$xy = -1 + 3x$$

$$\boxed{\boldsymbol{xy - 3x + 1 = 0}}$$

13.

a=-32 ft/sec^2

a=-2

$$\frac{dy}{dt} = -32$$

$$\int dv = \int -32dt$$

v=-32t+c

$$\frac{ds}{dt} = -32t + c_1$$

$$\int ds = \int (-32t + c_1)dt$$

s=16t^2 + c_1t + c_2

when t = 0, v = v_o

v=-32t + c_1

v_o= -32(0) + c_1

v_o =c_1

v = -32t + v_o

when t = 1 sec, s=h=48ft

h=-16t^2+ v_ot + c_1

48 = -16(1)^2 + v_o(1) + c_2

64 - v_o = c_2

When t = 0, s = 0, c_2 = 0

s = -16t^2 + v_ot

when t = 1 sec, s = 48

s = -16t^2 + c_1t

48 = -16(1)^2 + c_1(1)

c_1=64

s=-16t^2 + 64t

v = -32t + 64

@ max, v = 0

0 = -32t + 64

32t=64

t = 2 sec

s = -16t^2 + 64t

s = -16(2)^2 + 64(2)

$\boxed{\textbf{s = 64ft}}$

15.

a = 32ft/sec^2

a = 32

$$\frac{dv}{dt} = 32$$

$$\int dv = \int 32dt$$

v = 32t + c_1

$$\frac{ds}{dt} = 32t + c_1$$

$$\int ds = \int (32t + \ c_1)dt$$

S = 16t^2 + c_1 + c_2

when t = 0, v = 0

c_1 = 0

v = 32t

when t = 0 , s = 0

c_2 = 0

s = 16t^2

$$t = \sqrt{\frac{400}{16}}$$

$$t = \frac{20}{4}$$

t = 5 sec

v = v_t

*since it is a free falling body, its velocity is (-)

v_t = -32t

v_t = -32(5)

$\boxed{\mathbf{v_t = -160\ ft/sec}}$

1. $\int \sin 5x \sin x \, dx$

$= \int 2 \sin u \sin v \, dx$

$= \int [\cos(u - x) - \cos(u + v)]dx$

$u = 5x \qquad v = x$

$= \frac{1}{2} \int [\cos(5x - x) - \cos(5x + x)]dx$

$= \frac{1}{2} \int [\cos 4x - \cos 6x]dx$

$= \frac{1}{2} [\int \cos 4x dx - \int \cos 6x dx$

$= \frac{1}{2} [\frac{1}{4} \sin 4x - \frac{1}{6} \sin 6x] + C$

$= \boxed{\boldsymbol{\frac{\sin 4x}{8} - \frac{\sin 6x}{12} + C}}$

3. $\int \sin(9x - 3) \cos(x + 5)dx$

$= \frac{1}{2} \int [\sin(9x - 3 + x + 5) + \sin(9x - 3 - x - 5)dx$

$= \frac{1}{2} \int [\sin(5x + 2) + \sin(3x - 8)]dx$

$let\ z = 5x + 2 \quad ; \quad let\ w = 3x - 8$

$\frac{dz}{dx} = 5 \quad ; \quad \frac{dw}{dx} = 3$

$\frac{dz}{5} = dx \quad ; \quad \frac{dw}{3} = dx$

$= \frac{1}{2} [- \cos z \left(\frac{1}{5}\right) - \frac{1}{3} cos w] + C$

$= \boxed{\boldsymbol{-\frac{1}{10} cos(5x + 2) - \frac{1}{6} cos(3x - 8) + C}}$

5. $\int \cos(3x - 2\pi) \cos(x + \pi)dx$

$= \frac{1}{2} \int [\cos(u + v) + \cos(u - v)]dx$

$let\ u = 3x - 2\pi$

$v = x + \pi$

- $u + v = (3x - 2\pi) + (x + \pi)$

  $= 4x - \pi$

- $(u - v) = (3x - 2\pi) - (x + \pi)$

  $= 2x - 3\pi$

$= \frac{1}{2} \int [\cos(4x - \pi) + \cos(2x - 3\pi)]dx$

$for \cos 4x - \pi = \cos 4x cos\pi + \cancel{sin 4x sin\pi}$

$= -cos 4x$

$for \cos 2x - 3\pi = \cos 2x cos\, 3\pi + \sin 2x sin\, 3\pi$

$= - \cos 2x$

$= \frac{1}{2} \int (\cos 4x - \cos 2x)dx$

$= \frac{1}{2} [- \frac{1}{4} \sin 4x - \frac{1}{2} \sin 2x] + C$

$= \boxed{\boldsymbol{-\frac{1}{8} \sin 4x - \frac{1}{4} \sin 2x + C}}$

7. $\int 4\, sin\, 8x\, cos 3x dx$

$= 2\int[\sin(8x+3x) + sinx(8x-3x)dx$

$= 2\int[sin 11x + \sin 5x]dx$

$let\ u = 11x \quad ; \quad let\ v = 5x$

$\frac{du}{dx} = 11 \quad ; \quad \frac{dv}{dx} = 5$

$\frac{du}{11} = dx \quad ; \quad \frac{dv}{5} = dx$

$= 2[-\frac{1}{11}\cos 11x - \frac{1}{5}\cos 5x\,] + C$

$= \boxed{-\frac{\mathbf{2}}{\mathbf{11}}\mathbf{cos\, 11}\boldsymbol{x} - \frac{\mathbf{2}}{\mathbf{5}}\boldsymbol{cos}\mathbf{5}\boldsymbol{x} + \boldsymbol{C}}$

9. $\int 5\, sin\left(4x+\frac{\pi}{3}\right) sin\left(2x-\frac{\pi}{6}\right) dx$

$= \frac{5}{2}\int[\cos(u-v) - \cos(u+v)]dx$

$let\ u = 4x + \frac{\pi}{3} \quad ;$

- $(u-v) = \left(4x+\frac{\pi}{3}\right) - \left(2x-\frac{\pi}{6}\right)$
  $= 2x + \pi/2$
  $v = 2x - \frac{\pi}{6}$
- $(u+v) = \left(4x+\frac{\pi}{3}\right) + \left(2x-\frac{\pi}{6}\right)$
  $= 6x + \pi/6$

$= \frac{5}{2}\int[cos\left(2x+\frac{\pi}{2}\right) - cos\left(6x+\frac{\pi}{6}\right)]dx$

- $for\ \cos\left(2x+\frac{\pi}{2}\right)$
  $= \cos 2x cos\frac{\pi}{2} - \sin 2x \sin\frac{\pi}{2}$
  $= -sin 2x$
- $for\ \cos\left(6x+\frac{\pi}{6}\right)$
  $= cos\, 6x cos\frac{\pi}{6} - sin\, 6x\, sin\frac{\pi}{6}$
  $= \frac{\sqrt{3}}{2} cos\, 6x - \frac{1}{2} sin\, 6x$

$= \frac{5}{2}\int[-\,sin\, 2x - \frac{\sqrt{3}}{2} cos\, 6x + \frac{1}{2} sin\, 6x\,]dx$

$= \frac{5}{2}[\frac{1}{2} cos\, 2x - \frac{\sqrt{3}}{12} sin\, 6x - \frac{1}{12} six\ 6x + C$

$= \boxed{\frac{\mathbf{5}}{\mathbf{4}}\boldsymbol{cos\, 2x} - \frac{\mathbf{5\sqrt{3}}}{\mathbf{24}}\boldsymbol{sin\, 6x} - \frac{\mathbf{5}}{\mathbf{12}}\boldsymbol{sin\, 6x} + \boldsymbol{C}}$

1. $\int sin^3 xcos^4xdx;\ by\ Case\ I$

$= \int sin^4xcos^4 xsinxdx$

$= \int(1-cos^2 x)^2cos^4xsinxdx$

$= \int(1-2cos^2x+cos^4 x)cos^4xsinxdx$

$= \int(cos^4x-2cos^6x+cos^8 x)sinxdx$

Let u = cosx

$\frac{du}{dx} = -sinx$

$-du = sinxdx$

$= -\int(u^4-2u^6+u^8)du$

$= \left[\frac{2u^6}{7}-\frac{u^5}{5}-\frac{u^9}{9}\right]+C$

$= \boxed{\frac{2}{7}cos^7x-\frac{1}{5}cos^5x-\frac{1}{9}cos^9x+C}$

3. $\int sin^4 3xcos^3 3xdx\ ;by\ Case\ II$

$= \int sin^4 3xcos^2 3xcos3xdx$

$= \int sin^4 3x(1-sin^2 3x)cos3xdx$

$= \int(sin^4 3x-sin^6 3x)cos3xdx$

Let u = sin3x

$\frac{du}{dx} = 3cos3x\ ;\quad \frac{du}{3} = cos3xdx$

$= \int(u^4-u^6)\frac{du}{3}$

$= \frac{1}{3}\left(\frac{u^5}{5}-\frac{u^7}{7}\right)+C$

$= \frac{1}{15}u^5-\frac{1}{21}u^7+C$

$= \boxed{\frac{1}{15}sin^5 3x-\frac{1}{21}sin^7 3x+C}$

5. $\int sin^4 xcos^2xdx$

$= \int(sin^2 x)^2cos^2xdx$

$= \int(\frac{1-cos2x}{2})^2\left(\frac{1+cos2x}{2}\right)dx$

$= \int\frac{1-2cos2x+\left(\frac{1}{4}\right)(cos^2 2x}{4}\left(\frac{1+cos2x}{2}\right)dx$

$= \int\left(\frac{1}{4}\right)-(\frac{1}{2}cos2x+\frac{1}{4}cos^2 2x)\left(\frac{1}{2}+\frac{cos2x}{2}\right)dx$

$= \frac{1}{8}\int(1-2cos2x+cos^2 2x)(1+cos2x)dx$

$= \frac{1}{8}\int(1-2cos2x+cos^2x+cos2x-2cos^2 2x+cos^3 2x)dx$

$= \frac{1}{8}\int(1-cos2x-cos^2 2x+cos^3 2x)dx$

$= \frac{1}{8}\int dx-\int cos2xdx-\int cos^2 2xdx+\int cos^3 2xdx$

$= \frac{1}{8}\ [x-\frac{1}{2}sin2x-(\frac{1}{2}x+\frac{1}{8}sin4x+\frac{1}{2}sin2x-\frac{1}{6}sin^3 2x]$

$= \boxed{\frac{x}{16}-\frac{sin4x}{64}-\frac{sin^3 2x}{48}+C}$

7. $\int(\sqrt{sinx}+cosx)^2dx$

$= \int (sinx + 2\sqrt{sinx}\ cosx + cos^2x)dx$

$= \int sinxdx + 2\int sin^{\frac{1}{2}} cosxdx + \int cos^2 xdx$

$= \int sinxdx + 2\int sin^{\frac{1}{2}} cosxdx + \int (\frac{1+cos2x}{2})dx$

Let u = sinx

$\frac{du}{dx} = cosx$

$du = cosxdx$

$= \int sinxdx + 2\int u^{\frac{1}{2}} du + \int \frac{1}{2}dx + \int \frac{cos2x}{2}dx$

$= -cosx + 2\left(\frac{2}{3}u^{\frac{3}{2}}\right) + \frac{x}{2} + \frac{sin\,2x}{4} + C$

$= \boxed{\boldsymbol{-cosx + \frac{4}{3}sin^{\frac{3}{2}} + \frac{x}{2} + \frac{sin2x}{4} + C}}$

9. $\int(sin3x + cos2x)^2dx$

$=\int(sin^2 3x + 2sin3xcos2x + cos^2 2x)dx$

$=\int sin^2 3x\ dx + 2\int(sin3xcos2x)dx + \int(cos^2 2x)dx$

$=\frac{x}{2} - \frac{1}{2}sin6x - \frac{1}{5}cos\,5x - cosx + \frac{x}{2} + \frac{1}{8}sin4x + c$

$= \boxed{\boldsymbol{x - \frac{sin6x}{12} - \frac{cos5x}{5} - cosx + \frac{sin4x}{8} + c}}$

11. $\int cos^2 4x\ dx$

$= \int \left(\frac{1 + cos8x}{2}\right) dx$

$= \frac{1}{2}\int(1 + cos8x)\ dx$

$= \boxed{\boldsymbol{\frac{x}{2} + \frac{sin8x}{16} + c}}$

13. $\int sin^3 2x\ dx$

$= \int sin^2 2x\ \ sin2x\ dx$

$= \int (1 - cos^2 2x)sin2x\ dx$

$let\ u = cos\ 2x \quad ; \quad Du = -2sin2x\ dx$

$= \int (1 - u^2)\left(-\frac{du}{2}\right)$

$= \frac{-1}{2}\left[u - \frac{u^3}{3}\right] + c$

$= \boxed{\boldsymbol{\frac{-1}{2}cos2x + \frac{1}{6}cos^3 2x + c}}$

15. $\int sin^7 x\,cos^2 x\ dx$

$= \int sin^7 x\,cos^2 x\,cosx\ dx$

$= \int sin^7 x(1 - sin^2 x)cosxdx$

$= \int(sin^7 x - sin^9 x)cosx\ dx$ u=sinx du=cosxdx

$= \int (u^7 - u^9)\ du$

$= \frac{u^8}{8} - \frac{u^{10}}{10} + c$

$= \boxed{\boldsymbol{\frac{1}{8}sin^8 x - \frac{1}{10}sin^{10} x + c}}$

1. $\int tan^2 2xsec^4 2xdx$

$= \int tan^2 2xsec^2 2xsec^2 2xdx$

$= \int tan^2 2x(1 + tan^2 2x)sec^2 2xdx$

$= \int (tan^2 2x + tan^4 2x)sec^2 2xdx$

$let\ u = tan2x\ ;\ \ \frac{du}{dx} = 2sec^2 2x$

$\frac{du}{2} = sec^2 2xdx$

$= \int (u^2 + u^4)(\frac{du}{2})$

$= \frac{1}{2}\int (u^2 + u^4)du$

$= \frac{1}{2}\left(\frac{u^3}{3} + \frac{u^5}{5}\right) + c$

$= \boxed{\left(\frac{tan^3 2x}{6} + \frac{tan^5 2x}{10}\right) + c}$

3. $\int \sqrt{tanx}\, sec^6 xdx\ ; CASE\ I$

$= \int tan^{\frac{1}{2x}} sec^4 xsec^2 xdx$

$= \int tan^{\frac{1}{2}} x(1 + tan^2 x)^2 sec^2 xdx$

$= \int tan^{\frac{1}{2}} x(1 + 2tan^2 x + tan^4 x)sec^2 xdx$

$= \int (tan^{\frac{1}{2}} x + 2tan^{\frac{5}{2}} x + tan^{\frac{9}{2}} x)sec^2 xdx$

$let\ u = tanx\ ;\ \frac{du}{dx} = sec^2 x\ ;\ du = sec^2 x\ \ dx$

$= \int (u^{\frac{1}{2}} x + 2u^{\frac{5}{2}} x + u^{\frac{9}{2}} x)du$

$= \left(\frac{2u^{\frac{3}{2}}}{3} + \frac{4u^{\frac{7}{2}}}{7} + \frac{2\,u^{\frac{11}{2}}}{11}\right) + c$

$= \boxed{\left(\frac{2tan^{\frac{3}{2}} x}{3} + \frac{4tan^{\frac{7}{2}} x}{7} + \frac{2\,tan^{\frac{11}{2}} x}{11}\right) + c}$

5. $\int \_\_\frac{1}{2} xdx\ \rightarrow ans.y = \frac{2}{3}tan\frac{3x}{2} - 2tan\frac{x}{2} + x + c$

Find the missing term:

$\frac{dy}{dx} = tan^2\frac{x}{2}sec^2\frac{x}{2} - sec^2\frac{x}{2} + 1$

$= tan^2\frac{x}{2}sec^2\frac{x}{2} - (sec^2\frac{x}{2} - 1)$

$= tan^2\frac{x}{2}sec^2\frac{x}{2} - tan^2\frac{x}{2}$

$= tan^2\frac{x}{2}(sec^2\frac{x}{2} - 1)$

$= tan^2\frac{x}{2}(tan^2\frac{x}{2})$

$\int dy = \int tan^4\frac{x}{2}dx$

$= \boxed{\textbf{therefore, the missing term is "}tan^4\textbf{"}}$

7. $\int (secx + tan\,x)^2\, dx$

$= \int (sec^2 x + 2secx\,tan\,x + tan^2 x)\, dx$

$= \int tanx + 2secx + \int tan^2 x\, dx$

$= \int tanx + 2secx + \int (sec^2 x - 1)\, dx$

$= tanx + 2secx + tanx - x + c$

$= \boxed{2tanx + 2secx - x + c}$

9. $\int (\frac{sec\,3x}{tan\,3x})^4 dx$

$=\int \left(\frac{sec^4 3x}{tan^4 3x}\right) dx$

$=\int sec^4 3x tan^{-4} 3x dx$

$=\int sec^2 3x sec^2 3x tan^{-4} 3x dx$

$=\int sec^2 3x(1+tan^2 3x)tan^{-4} 3x dx$

$=\int (tan^{-4} 3x + tan^{-2} 3x) sec^2 3x dx$

$let\ u = tan\,3x\ \ ;\frac{du}{dx} = 3sec^2\,3x$

$\frac{du}{3} = \ sec^2 3x dx$

$=\frac{1}{3}\int (u^{-4} + u^{-2})\,du$

$=\frac{1}{3}\left[\frac{u^{-3}}{-3} - \frac{u^{-2}}{3}\right] + c$

$=\frac{1}{3}\left[\frac{tan^{-3} 3x}{-3} - \frac{tan^{-1} 3x}{3}\right] + c$

$=\boxed{-\frac{\boldsymbol{cot^3 3x}}{\boldsymbol{9}} - \frac{\boldsymbol{cot 3x}}{\boldsymbol{3}} + \boldsymbol{c}}$

11. $\int \frac{tan^3 x}{\sqrt{secx}} dx$

$=\int tan^3 x sec^{-\frac{1}{2}} x dx$

$=\int tan^2 x tanx sec^{-\frac{3}{2}} x secx dx$

$=\int (sec^2 x - 1) tanx sec^{-\frac{3}{2}} x secx dx$

$=\int (sec^{\frac{1}{2}} x - sec^{-\frac{3}{2}} x) tanx secx dx$

$let\ u = sec\,x\ ;\frac{du}{dx} = secxtanx$

$du = secxtanxdx$

$=\int (u^{\frac{1}{2}} - u^{-\frac{3}{2}}) du$

$=\frac{2u^{\frac{3}{2}}}{3} - 2u^{-\frac{1}{2}} + c$

$=\boxed{\frac{\boldsymbol{2sec^{\frac{3}{2}}x}}{\boldsymbol{3}} - \frac{\boldsymbol{2}}{\boldsymbol{sec^{\frac{1}{2}}x}} + \boldsymbol{c}}$

1. $\int cot^4 x csc^4 x dx$

$= \int cot^4 x(1 + cot^2 x)csc^2 x dx$

$= \int (cot^4 x + cot^6 x)csc^2 x dx$

$let\ u = cot\ x\ ; \frac{du}{dx} = -csc^2 x\ ; du = -csc^2 x dx$

$= -\int (u^4 + u^6)du$

$= -\frac{u^5}{5} + \frac{u^7}{7} + c$

$= \boxed{-\frac{cot^5 x}{5} + \frac{cot^7 x}{7} + c}$

3. $\int cot^5 4x dx$

$= \int cot^3 4x cot^2 4x dx$

$= \int cot^3 4x(csc^2 4x - 1)dx$

$= \int (cot^3 4x csc^2 4x - cot^3 4x)dx$

$= \int [cot^3 4x csc^2 4x - (csc^2 4x - 1)cot 4x]dx$

$= \int cot^3 4x csc^2 4x dx - \int cot 4x csc^2 4x dx - \int cot 4x dx$

$let\ u = cot\ 4x\ ; \frac{du}{-4} = csc^2 4x dx$

$= -\frac{1}{4}\int u^3 du - \frac{1}{4}\int u du + \frac{1}{4} ln(cos 4x)$

$= -\frac{1}{4}\left(\frac{u^4}{4} dx - \frac{u^2}{2}\right) + \frac{1}{4} ln(cos 4x) + c$

$= \boxed{-\frac{cot^4 4x}{16} + \frac{cot^2 4x}{8} + \frac{1}{4} ln(cos 4x) + c}$

5. $\int \sqrt{cos\ 3x}\ csc^4\ 3x\ dx$

$= cot^{\frac{1}{2}} 3x\ csc^2\ 3x\ csc^2\ 3x\ dx$

$= cot^{\frac{1}{2}} 3x\ (1 + cot^2\ 3x)\ csc^2\ 3x\ dx$

$= \left(cot^{\frac{1}{2}} 3x + cot^{\frac{5}{2}} 3x\right) csc^2\ 3x\ dx$

$let\ u = cot\ 3x$

$\frac{du}{dx} = -3\ csc^2\ 3x \frac{d(x)}{dx}$

$-\frac{du}{3} = csc^2\ 3x\ dx$

$= -\frac{1}{3}\int \left(u^{\frac{1}{2}} + u^{\frac{5}{2}}\right) du$

$= -\frac{1}{3}\left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + \frac{u^{\frac{7}{2}}}{\frac{7}{2}}\right) + c$

$= \boxed{-\frac{2}{9} cot^{\frac{3}{2}} 3x - \frac{2}{21} cot^{\frac{7}{2}} 3x + c}$

7. $\int \frac{\cos^5 2x dx}{\sin^8 2x} = \int \frac{\cos^5 2x dx}{\sin^5 2x}\left(\frac{1}{\sin^3 2x}\right) dx$

$= \int \cot^5 2x \csc^3 2x\, dx \mid$

$= \int \cot^4 2x \csc^2 2x \csc 2x \cot 2x\, dx$

$= \int (\csc^2 2x - 1)^2 \csc^2 2x \csc 2x \cot 2x\, dx$

$= \int (\csc^4 2x - 2\csc^2 2x + 1) \csc^2 2x \csc 2x \cot 2x\, dx$

$= \int (\csc^6 2x - 2\csc^4 2x + \csc^2 2x) \csc 2x \cot 2x\, dx$

$let\ u = \csc 2x$

$\frac{du}{dx} = -2\csc 2x \cot 2x \frac{d(x)}{dx}$

$-\frac{du}{2} = \csc 2x \cot 2x\, dx$

$= -\frac{1}{2}\int (u^6 - 2u^4 + u^2)\, du$

$= -\frac{1}{2}\left(\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3}\right) + c$

$= \boxed{-\frac{\boldsymbol{\csc^7 2x}}{\mathbf{14}} + \frac{\boldsymbol{\csc^5 2x}}{\mathbf{5}} - \frac{\boldsymbol{\csc^3 2x}}{\mathbf{6}} + \boldsymbol{c}}$

9. $\int \frac{\csc^4 x}{\cot^6 x} dx$

$= \int \cot^{-6} x \csc^2 x \csc^2 x\, dx$

$= \int \cot^{-6} x\,(1 + \cot^2 x) \csc^2 x\, dx$

$= \int (\cot^{-6} x + \cot^{-4} x) \csc^2 x\, dx$

$let: u = \cot x$

$\frac{du}{dx} = -\csc^2 x\, dx$

$-du = \csc^2 x dx$

$= -1\int (u^{-6} + u^{-4}) dx$

$= -1\left(-\frac{u^{-5}}{5} - \frac{u^{-3}}{3}\right) + c$

$= \frac{\cot^{-5} x}{5} + \frac{\cot^{-3} x}{3} + c$

$= \boxed{\frac{\boldsymbol{\tan^5 x}}{\mathbf{5}} + \frac{\boldsymbol{\tan^3 x}}{\mathbf{3}} + \boldsymbol{c}}$

1. $\int \frac{x^2dx}{\sqrt{4-x^2}}$

$x = u\ ;\ a = 2\ ;\ \ 2^2 - x^2$

$u = a\,sin\,\theta$

$sin\,\theta = \dfrac{x}{2}$

$x = 2sin\,\theta$

$dx = 2cos\theta d\theta$

$$= \int \frac{x^2dx}{\sqrt{4-x^2}}$$

$$= \int \frac{4\,sin\,\theta\,2cos\theta d\theta}{2cos\theta}$$

$$= 4\int sin^2\theta d\theta$$

$$= 4\int \frac{1-cos2\theta d\theta}{2}$$

$$= 2\int d\theta - \frac{1}{2}(sin2\theta) + C\ ;\ \theta = sin^{-}(\frac{x}{2})$$

$$= 2[arcsin\tfrac{x}{2} - \tfrac{1}{2}(2sin\theta cos\theta)]^2 + C$$

$$= \boxed{\boldsymbol{2arcsin\frac{x}{2} - 2\left(\frac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right) + C}}$$

3. $\int \frac{dx}{x\sqrt{9x^2+4}}$;

$u = 3x$

$a = 2$

$u = atan\theta$

$3x = 2tan\theta$

$x = \dfrac{2}{3}tan\theta\ ;\ tan\theta = \dfrac{3x}{2}$

$dx = \dfrac{2}{3}sec^2\theta d\theta$

$sec\theta = \dfrac{\sqrt{9x^2+4}}{2}$

$2sec\theta = \sqrt{9x^2+4}$

$$= \int \frac{dx}{x\sqrt{9x^2+4}}$$

$$= \int \frac{\frac{2}{3}sec^2\theta d\theta}{\frac{2}{3}tan\theta 2sec\theta}$$

$$= \int \frac{sec\theta d\theta}{2tan\theta}$$

$$= \frac{1}{2}\int \frac{\frac{1}{cos\theta}}{\frac{sin\theta}{cos\theta}}d\theta$$

$$= \frac{1}{2}\int \frac{1}{sin\theta}d\theta$$

$$= \frac{1}{2}\int csc\theta d\theta$$

$$= \frac{1}{2}[-ln|csc\theta + cot\theta|] + C$$

$$= \boxed{\boldsymbol{-\frac{1}{2}\left[ln\left|\frac{\sqrt{9x^2+4}}{3x}\right| - \frac{2}{3x}\right] + C}}$$

5. $\int \frac{x^2dx}{(9-x^2)^{\frac{3}{2}}}$

$= \int \frac{x^2dx}{\sqrt{(9-x^2)^3}}$

$= \int \frac{x^2dx}{(9-x^2)\sqrt{(9-x^2)}}$

$u = x \; ; \; a = 3$

$u = asin\theta$

$x = 3sin\theta$

$dx = 3cos\theta$

$= \int \frac{(3sin\theta)^2(3\,cos\,\theta)}{9-(3sin\theta)^2(3\,cos\,\theta)}$

$= \int \frac{9sin^2\theta}{1-sin^2\theta}$

$= \int \frac{sin^2\theta}{(1-sin^2\theta)}$

$= \frac{sin^2\theta}{cos^2\theta}d\theta$

$= \int tan^2\theta d\theta - \int sec^2\theta d\theta$

$= (tan\theta) - \theta$

$= \boxed{\boldsymbol{\frac{x}{\sqrt{9-x^2}} - Arcsin\frac{x}{3} + c}}$

7. $\int \frac{\sqrt{9-4x^2}}{x^2}dx$

$a = 3\,; \; u = 2x$

$u = asin\theta \quad ; \quad 2x = 3sin\theta$

$x = \frac{3}{2}sin\theta$

$\frac{2x}{3} = sin\theta$

$dx = \frac{3}{2}cos\theta d\theta$

$\theta = sin^{-1}(\frac{2x}{3})$

$= \int \frac{3cos\theta\,(\frac{3}{2}cos\theta d\theta\,)}{(\frac{3}{2}\,sin\theta\,)^2}$

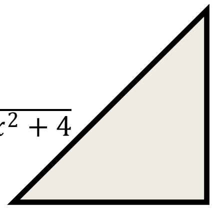
$= \int \frac{3cos\theta\,(3cos\theta d\theta\,)}{2(\frac{3}{2}sin\theta\,)^2}$

$= \int \frac{9cos^2\theta d\theta}{2(\frac{9}{4sin^2\theta})}$

9. $\int \frac{dx}{(x^2+4)^2}$; $where: u = x\ , a = 2$

$x = 2\,tan\theta$; $tan\theta = \dfrac{x}{2}$

$Dx = 2\,sec^2\theta d\theta$; $\theta = arctan\frac{x}{2}$



$$sec\theta = \frac{\sqrt{x^2+4}}{2}$$

$$\left(2sec\theta = \sqrt{x^2+4}\right)^2$$

$$4\,sec^2\theta = x^2 + 4$$

$$= \int \frac{2\,sec^2\theta d\theta}{(4\,sec^2\theta)^2}$$

$$= \int \frac{2\,sec^2\theta d\theta}{16\,sec^4\theta} = \int \frac{d\theta}{8\,sec^2\theta} = \frac{1}{8}\int \frac{d\theta}{sec^2\theta}$$

$$= \frac{1}{8}\int cos^2\theta d\theta$$

$$= \frac{1}{8}\int \frac{1+cos2\vartheta}{2}d\theta$$

$$= \frac{1}{8}\left[\int \frac{1}{2}d\theta + \int \frac{cos2\theta}{2}d\theta\right]$$

$$= \frac{1}{8}\left[\frac{1}{2}\theta + \frac{1}{4}(2)sin\theta cos\theta\right] + c$$

$$= \boxed{\boldsymbol{\frac{1}{16}\theta + \frac{1}{16}sin\theta cos\theta + c}}$$

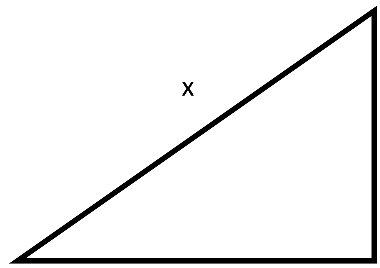
11. $\int \frac{dx}{x\sqrt{x^2-9}}$

$a = 3$; $u = x$

$u = asec\theta$

$x = 3sec\theta$; $dx = 3sec\theta tan\theta d\theta$

$sec\theta = \dfrac{x}{3}$; $\theta = Arcsec\dfrac{x}{3}$



$tan\theta = \dfrac{\sqrt{x^2-9}}{3}$; $3tan\theta = \sqrt{x^2-9}$

$$= \int \frac{dx}{x\sqrt{x^2-9}} = \int \frac{3sec\theta tan\theta d\theta}{3sec\theta(3tan\theta)}$$

$$= \int \frac{d\theta}{3}$$

$$= \frac{1}{3}\theta$$

$$= \boxed{\boldsymbol{\frac{1}{3}Arcsec\frac{x}{3} + c}}$$

13. $\int (\frac{(x^2-16)^{\frac{3}{2}}}{x^3})$

$u = x; a = 4$

$u = sec\emptyset\ ; x = 4sec\emptyset\ ; sec\emptyset = \frac{x}{4}$

$\emptyset = arcsec\emptyset\frac{x}{4}$

$x^3 = 64\sec^3\emptyset;\ dx = 4sec\emptyset tan\emptyset d\emptyset$

$tan\emptyset = \frac{(\sqrt{x^2-16})}{4}; 4tan\emptyset = \sqrt{x^2-16}$

$= \int \frac{((4tan\emptyset)^3(4sec\emptyset tan\emptyset d\emptyset))}{(64\sec^3)}$

$= 4\int \frac{\tan^4\emptyset d\emptyset}{\sec\text{^}2\emptyset}$

$= 4\int \frac{(\sec^2 -1)\text{^}2d\emptyset}{\sec^2\emptyset}$

$= 4\int \frac{\sec^4 -2\sec^2\emptyset + 1}{\sec^2\emptyset}$

$= 4\int \frac{\sec^4\emptyset - 2\sec^2\emptyset + 1}{\sec^2\emptyset}$

$= 4\int \sec^2\emptyset - 2 + 1/\sec^2\emptyset d\emptyset$

$= 4(tan\emptyset - 2\emptyset + \frac{1}{2}\emptyset + sin\emptyset cos\emptyset$

$= \boxed{\sqrt{\boldsymbol{x^2-16}} - \mathbf{6\,arcsec}\frac{\boldsymbol{x}}{\mathbf{4}} + \frac{\mathbf{8}\sqrt{\boldsymbol{x^2-16}}}{\boldsymbol{x^2}} + \boldsymbol{c}}$

15. $\int \frac{dx}{(2x-3)\sqrt{5-12x+4x^2}}$

$5 - 12x + 4x^2 = 2x - 9 - 4$

$a = 2\ ; u = 2x - 3$

$u = asec\emptyset\ ; 2x - 3 = 2sec\emptyset$

$2x - 3 = 2sec\emptyset\ ; 2x = 2sec\emptyset + 3$

$2dx = 2sec\emptyset tan\emptyset$

$dx = sec\emptyset tan\emptyset$

$sec\emptyset = \frac{2x-3}{2}$

$\emptyset = arcsec\frac{2x-3}{2}$

$tan\emptyset = \frac{\sqrt{(2x-3)^2-4}}{2}$

$2tan\emptyset = \sqrt{(2x-3)^2-4}$

$= \frac{\int(sec\emptyset tan\emptyset)}{2sec\emptyset 2tan\emptyset}$

$= \frac{1}{4}\int \frac{sec\emptyset tan\emptyset}{sec\emptyset tan\emptyset}$

$= \frac{1}{4}\int d\emptyset$

$= \frac{1}{4}\emptyset; \emptyset = arcsec\ \frac{(2x-3)}{2}$

$= \boxed{\frac{\mathbf{1}}{\mathbf{4}}\ \boldsymbol{arcsec}\frac{\mathbf{2}\boldsymbol{x}-\mathbf{3}}{\mathbf{2}} + \boldsymbol{c}}$

1. $\int \frac{dx}{x^2+25}$

$Let: u = x$

$a = 5$

$du = dx$

$= \boxed{\frac{1}{5} Arctan \frac{x}{5} + c}$

3. $\int \frac{xdx}{\sqrt{1-x^4}}$

$Let: u = x^2$

$a = 1$

$\frac{du}{2} = dx$

$= \frac{1}{2} Arcsin \frac{x^2}{1} + c$

$= \boxed{\frac{1}{2} Arcsinx^2 + c}$

5. $\int \frac{dx}{49-25x^2}$

$Let: a = 7$

$u = 5x$

$\frac{du}{2} = dx$

$= \boxed{\frac{1}{70} ln \left|\frac{5x-7}{5x+7}\right| + c}$

7. $\int \sqrt{36-9x^2}\, dx$

$Let: a = 6$

$u = 3x$

$\frac{du}{3} = dx$

$= \frac{1}{3}\left[\frac{3x}{2}\sqrt{36-9x^2}\right] + \frac{1}{3}\left[8Arcsin\frac{3x}{6}\right] + c$

$= \frac{1}{3}\left[\frac{3x}{2}\sqrt{36-9x^2}\right] + \frac{1}{3}\left[8Arcsin\frac{x}{2}\right] + c$

$= \boxed{\frac{x}{2}\sqrt{36-9x^2} + 6Arcsin\frac{x}{2} + c}$

9. $\int \sqrt{16x^2+25}dx$

$Let: a = 5$

$u = 4x$

$\frac{du}{4} = dx$

$= \frac{1}{4}\left[\frac{4x}{2}\sqrt{16x^2+25}\right] + \frac{1}{4}\left(\frac{5^2}{2}\right) ln\left|4x + \sqrt{16x^2+25}\right| + c$

$= \boxed{\frac{1}{2x}\sqrt{16x^2+25} + \frac{25}{8} ln\left|4x + \sqrt{16x^2+25}\right| + c}$

1. $\int \frac{dx}{x^2-3x+2}$

completing the square

$x^2 - 3x = -2$

$x^2 - 3x = \frac{9}{4} = -2 + \frac{9}{4}$

$\left(x - \frac{3}{2}\right)^2 = \frac{1}{4}$

$\left(x - \frac{3}{2}\right)^2 - \frac{1}{4}$

$= \int \frac{dx}{(x - \frac{3}{2})^2 - \frac{1}{4}}$

$u = x - \frac{3}{2}$

$a = \frac{1}{2}$

$= \int \frac{du}{u^2 - a^2} = \frac{1}{2a} ln \left|\frac{u-a}{u+a}\right| + c$

$= \frac{1}{2\left|\frac{1}{2}\right|} ln \left|\frac{x - \frac{3}{2} - \frac{1}{2}}{x - \frac{3}{2} + \frac{1}{2}}\right| + c$

$= \boxed{\boldsymbol{ln \left|\frac{x-2}{x-1}\right| + c}}$

3. $\int \frac{dx}{2x^2 - 2x + 1}$

$= \int \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \frac{1}{4}}$

$= \int \frac{du}{u^2 + a^2}$

$= \frac{1}{2} arctan \frac{u}{a} + c$

$a = \frac{1}{2}, \ u = x - \frac{1}{2}$

$= \frac{1}{2} arctan \frac{x - \frac{1}{2}}{\frac{1}{2}} + c$

$= \boxed{\boldsymbol{\frac{1}{2} arctan 2x - 1 + c}}$

5. $\int \sqrt{3 - 2x - x^2}$

$= \int \sqrt{4 - (x+1)^2}$

$u = x + 1, a = 2$

$= \int \sqrt{a^2 - u^2} = \frac{u}{a}\sqrt{a^2 - u^2} + \frac{a^2}{2} arcsin \frac{u}{a} +$

$= \frac{x+1}{2}\sqrt{3 - 2x - x^2} + \frac{4}{2} arcsin \frac{x+1}{2} + c$

$= \boxed{\boldsymbol{\frac{x+1}{2}\sqrt{3 - 2x - x^2} + 2arcsin \frac{x+1}{2} + c}}$

7. $\int \frac{dx}{x^2 - 8x + 7}$

Completing the square

$x^2 - 8x = -7$

$x^2 - 8x + 16 = -7 + 16$

$(x-4)^2 = 9$

$(x-4)^2 - 9 = 0$

$= \int \frac{dx}{(x-4)^2 + 9}$

$a = 3 \quad ; u = x - 4$

$= \int \frac{du}{u^2 - a^2}$

$= \frac{1}{2a} ln \left|\frac{u-a}{u+a}\right| + c$

$= \frac{1}{6} ln \left|\frac{x-4-3}{x-4+3}\right| + c$

$= \boxed{\boldsymbol{\frac{1}{6} ln \left|\frac{x-7}{x-1}\right| + c}}$

9. $\int \frac{3+2x}{x^2+9} dx$

$= \int \frac{3dx}{x^2+9} + \int \frac{2xdx}{x^2+9}$

$= 3\int \frac{dx}{x^2+9} + 2\int \frac{xdx}{x^2+9}$

$let\ u = x^2 + 9 ; du = 2xdx$

$= 3\frac{1}{3} Arctan\frac{x}{3} + 2\frac{\frac{du}{2}}{u}$

$= \boxed{\boldsymbol{Arctan\frac{x}{3} + ln|x^2 + 9| + c}}$

11. $\int \frac{2x-3dx}{4x^2-1}$

$= \int \frac{2xdx}{4x^2-1} - \int \frac{3dx}{4x^2-1}$

$= 2\int \frac{xdx}{4x^2-1} - 3\int \frac{dx}{4x^2-1}$

$let\ u = 4x^2 - 1 ; \frac{du}{8} = xdx$

$= 2\int \frac{\frac{du}{8}}{u} - 3[\frac{1}{2} ln \left|\frac{4x^2-1}{4x^2+1}\right| + c]$

$= \boxed{\boldsymbol{\frac{1}{4} ln|4x^2 - 1| - \frac{3}{4} ln \left|\frac{2x-1}{2x+1}\right| + c}}$

13. $\int \frac{(2x+7)dx}{x^2+2x+5}$

$= \int \frac{(2x+2)+5dx}{x^2+2x+5}$

$= \int \frac{(2x+2)}{x^2+2x+5} + 5\int \frac{dx}{(x+1)^2+4}$

$let\ u = x^2 + 2x + 5 ; du = (2xt2)dx$

$= \int \frac{du}{u} + \frac{1}{2} Arctan\frac{x+1}{2} + c$

$= \boxed{\boldsymbol{ln|x^2 + 2x + 5| + \frac{1}{2} Arctan\frac{x+1}{2} + c}}$

15. $\int \frac{(x-3)dx}{\sqrt{4x-x^2}}$

$= \int \frac{(x-2)-1dx}{\sqrt{4x-x^2}}$

$=\int \frac{(x-2)dx}{\sqrt{4x-x^2}} - \int \frac{dx}{\sqrt{4x-x^2}}$

$let\ u = \sqrt{4x-x^2}\ ; du = \frac{4-2x}{2\sqrt{4x-x^2}}dx$

$du = \frac{-2(X-2)dx}{2\sqrt{4x-x^2}}\ ; \sqrt{4x-x^2} = \sqrt{4-(2-x)^2}$

$=\int du - \int \frac{du}{\sqrt{4-(2-x)^2}}$

$= \boxed{-\sqrt{4x-x^2} - Arcsin\frac{2-x}{2} + c}$

17. $\int \frac{(x+3)dx}{\sqrt{8x-x^2}}$

$= \int \frac{(x-4)+7dx}{\sqrt{8x-x^2}}$

$=\int \frac{(x-4)dx}{\sqrt{8x-x^2}} + 7\int \frac{dx}{\sqrt{8x-x^2}}$

$let\ u = \sqrt{8x-x^2}\ ;\ du = \frac{8-2x}{2\sqrt{8x-x^2}}dx$

$du = \frac{-2(x-4)dx}{2\sqrt{8x-x^2}}; \sqrt{8x-x^2}$

$= \sqrt{16-(4-x)^2}$

$=-\int du + 7\int \frac{du}{\sqrt{16-(4-x)^2}}$

$= \boxed{-\sqrt{8x-x^2} + 7Arcsin\frac{4-x}{4} + c}$

19. $\int \frac{(4x+9)dx}{x^2-4x+20} = \int \frac{2(2x+4+17)dx}{x^2-4x+20}$

$= 2\left[\int \frac{2x+4dx}{x^2-4x+20} + \frac{17}{2}\int \frac{dx}{x^2-4x+20}\right]$

$let\ u = x^2-4x+20\ ; du = (2x-4)dx$

$x^2-4x+20 = (x-2)^2+16$

$= 2[\int \frac{du}{u} + \frac{17}{2}\int \frac{dx}{(x-2)^2+16}]$

$= 2[ln|x^2-4x+20| + \frac{17}{2}(\frac{1}{4})Arctan\frac{x-2}{4} + c]$

$= \boxed{2\ ln|x^2-4x+20| + \frac{17}{4}Arctan\frac{x-2}{4} + c}$

1. $\int \frac{dx}{x-x^{\frac{2}{3}}}$

$z = \sqrt[3]{x}$

$= 3\int \frac{z^2 dz}{z^3 - z^2}$

$= 3\int \frac{dz}{z-1}$

$u = z - 1$

$du = dz$

$= 3\int \frac{du}{u}$

$= 3\,ln|u| + c$

$= 3\,ln\,|z-1\,| + c$

$= \boxed{\boldsymbol{3\,ln\,|\sqrt[3]{x}-1\,| + c}}$

3. $\int \frac{(x^{\frac{1}{3}}-x^{\frac{1}{4}}dx}{4x^{\frac{1}{2}}}$

$z = \sqrt[12]{x}$

$dx = 12z^{11}dz$

$= 3\int \frac{(z^4-z^3)\,\cancel{z^{11}}^{5}dz}{\cancel{z^6}}$

$= 3\int (z^9 - z^8)\,dx$

$= 3[\int z^9 dz - \int z^8 dz]$

$= 3[\frac{z^{10}}{10} - \frac{z^9}{9} + c]$

$= \frac{3z^{10}}{10} - \frac{z^9}{3} + c$

$= \frac{3x^{\frac{5}{6}}}{10} - \frac{x^{\frac{7}{4}}}{3} + c$

$= \frac{9x^{\frac{5}{6}}-10x^{\frac{3}{4}}}{30} + c$

$= \boxed{\boldsymbol{\frac{x^{\frac{1}{2}}(9x^{\frac{1}{3}} - 10x^{\frac{1}{4}})}{30} + c}}$

5. $\int \frac{dx}{(x+2)^{\frac{3}{4}}-(x+2)^{\frac{1}{2}}}$

$z = \sqrt[4]{x+2}$

$z^4 = x + 2$

$x = 2 - z^4$

$dx = -4z^3 dz$

$= -4\int \frac{z^3 dz}{z^3 - z^2}$

$= -4\int \frac{zdz}{z-1}$

$u = z - 1$

$du = dz$

$z = u + 1$

$= -4\int \frac{(u+1)du}{u}$

$= -4[\int \frac{u}{u}du + \int \frac{du}{u}]$

$= -4[u + ln|u| + c$

$= \boxed{\boldsymbol{-4[z - 1 + ln|z-1| + c]}}$

7. $\int \sqrt{4+\sqrt{x}}dx$;

$z = (4+\sqrt{x})^{1/2}$

$z^2 - 4 = \sqrt{x} z^4 - 8z^2 + 16 = x$

$z = \sqrt[12]{x}(z^2 - 4)^2 = xdx = (4z^3 - 16z)dz$

$= \int z(4z^3 - 16z)dz$

$= \int (4z^4 - 16z^2)dz$

$= 4\left[\frac{z^5}{5}\right] - 16\left(\frac{z^3}{3}\right) + C$

$= \frac{4}{5}(4+\sqrt{x})^{5/2} - \frac{16}{3}(4+\sqrt{x})^{3/2} + C$

$= (4+\sqrt{x})^{\frac{3}{2}}\left[\left(\frac{4}{5}(4+\sqrt{x})\right) - \frac{16}{3}\right] + C$

$= (4+\sqrt{x})^{\frac{3}{2}}\left[\frac{12(4+\sqrt{x}) - 80}{15}\right] + C$

$= (4+\sqrt{x})^{\frac{3}{2}}\left[\frac{48 + 12\sqrt{x} - 80}{15}\right] + C$

$= \frac{4}{15}(4+\sqrt{x})^{\frac{3}{2}}[12 + 3\sqrt{x} - 20] + C$

$= \boxed{\mathbf{\frac{4}{5}(4+\sqrt{x})^{\frac{3}{2}}(3\sqrt{x} - 8) + C}}$

9. $\int x(x+4)^{\frac{1}{3}} dx$

$z = (x+4)^{\frac{1}{3}}$; $z^3 = (x+4)$

$x = z^3 - 4$; $dx = 3z^3 dz$

$= \int (z^3 - 4)(z)3z^2 dz$

$= 3\int z^6 dz - 4\int z^3 dz$

$= \frac{3z^{\frac{7}{3}}}{7} - 3z^{\frac{4}{3}} + c$

$= \frac{3(x+4)^{\frac{7}{3}}}{7} - 3(x+4)^{\frac{4}{3}} + c$

$= \frac{3(x+4)^{\frac{4}{3}}}{7}(x + 4 - 7) + c$

$\int x(x+4)^{\frac{1}{3}} dx = \boxed{\mathbf{\frac{3(x+4)^{\frac{4}{3}}(x-3)}{7} + c}}$

11. $\int \frac{(4-\sqrt{2x+1})}{1-2x} dx$

$z = \sqrt{2x+1}\,;\ z^2 = 2x+1$

$2x = z^2 - 1\ ;\ x = \dfrac{z^2-1}{2}\ ;\ dx = zdz$

$= \int \dfrac{(4-z)zdz}{1-2\left(\frac{z^2-1}{2}\right)}$

$= \int \dfrac{4z - z^2dz}{2-z^2}$

$= \int 1 + \dfrac{4z - 2dz}{-(z^2-2)}$

$= \int 1 - \dfrac{4z - 2dz}{z^2-2}$

$= \int dz - 2\int \dfrac{(2z-1)dz}{z^2-2}$

$= z - 2\left(\int \dfrac{2zdz}{z^2-2} - \int \dfrac{dz}{z^2-2}\right)$

$= z - 2\,ln|z^2-2| + \frac{1}{\sqrt{2}} ln\left|\frac{\sqrt{2xt1}-\sqrt{2}}{\sqrt{2x+1}+\sqrt{2}}\right| + c$

$= \boxed{\boldsymbol{\sqrt{2x+1} - 2\,ln|2x-1| + \frac{1}{\sqrt{2}} ln\left|\frac{\sqrt{2xt1}-\sqrt{2}}{\sqrt{2x+1}+\sqrt{2}}\right| + c}}$

13. $\int x^5\sqrt{4+x^3}\,dx$

$z = \sqrt{4+x^3} z^2 = 4 + x^3\ ;\ \ x = \sqrt[3]{4-z^2}$

$dx = \dfrac{1}{3}(4-z^2)^{\frac{2}{3}}(-2zdz)$

$= -\dfrac{2zdz}{3(4-z^2)^{\frac{2}{3}}}$

$= \int x^5\sqrt{4+x^3}\,dx$

$= \int \left(\sqrt[3]{4-z^2}\right)^5 (z)\left(\dfrac{-2zdz}{3(4-z^2)^{\frac{2}{3}}}\right)$

$= \int \dfrac{(4-z^2)(z)(-2zdz)}{3}$

$= \int \dfrac{-8z^2 + 2z^4dz}{3}$

$= \dfrac{1}{3}\int 2z^4dz - 8z^2dz$

$= \dfrac{1}{3}\left[\dfrac{2z^5}{5} - \dfrac{8z^3}{3}\right] + c$

$= \dfrac{2z^5}{15} - \dfrac{8z^3}{9} + c$

$= \dfrac{6(\sqrt{4+x^3}) - 40\sqrt{4-x^3}}{45} + c$

$= \dfrac{\sqrt{4+x^3}(6(4+x^3) - 40)}{45} + c$

$= \dfrac{\sqrt{4+x^3}(24 + 6x^3 - 40)}{45} + c$

$= \frac{\sqrt{4+x^3}(-16+6x^3)}{45} + c$

$= \boxed{\boldsymbol{\dfrac{2\sqrt{4+x^3}(3x^3-8)}{45} + c}}$

15. $\int x^3(4+x^2)^{\frac{3}{2}}dx$

$$\frac{3+1}{2}=2$$

$Z=\sqrt{4+\ x^2}$

$z^2\ =(4+x^2)$

$X=\sqrt{4-z^2}$

$dx=\frac{1}{2}(4-z^2)\ -^{\frac{1}{2}}(\text{-}2zdz)$

$=\frac{zdz}{(4-z^2)^{\frac{1}{2}}}$

$=\int(4-z^2)(z^3)(-zdz)$

$$=\int -4z^4+z^6dz$$

$$=\int -\frac{4z^5}{5}+\frac{z^7}{7}+C$$

$$=\frac{28z^5+5z^7}{35}+C$$

$=\frac{-28(\sqrt{4+x^2}\ )^5+5(\sqrt{4+x^2})^7}{35}+C$

$$=\frac{\left(\sqrt{4+x^2}\right)^5(-28+5(4+x^2))}{35}+C$$

$=\frac{\left(\sqrt{4+x^2}\right)^5(-28+20+5x^2)}{35}+C$

$$=\boxed{\frac{\left(\sqrt{\mathbf{4}+\boldsymbol{x}^{\mathbf{2}}}\right)^{\mathbf{5}}(\mathbf{5}\boldsymbol{x}^{\mathbf{2}}-\mathbf{8})}{\mathbf{35}}+\boldsymbol{C}}$$

17. $\int\frac{1}{x^4\sqrt{x^2+1}}dx$

$x=tan\,u\quad ;\quad dx=sec^2\,u\,du$

$then\ \sqrt{x^2+1}=\ \sqrt{tan^2\,u+1}=sec\,u\ \&\ u$
$=tan^{-1}\,x$

$=\int cot^3\,u\,csc\,u\,du$

$=\int cot\,u\ \ csc\,u\,(csc^2(\,u)-1)du$

$=subs.s=csc(u)\ and\ ds=-(cot(u)\,csc(u))du$

$=-\int(s^2-1)ds$

$=\int 1ds-\int s^2ds$

$$=\int s-\frac{s^3}{3}$$

$=csc(u)-\frac{csc^3(u)}{3}+C$

$$=\boxed{\frac{\sqrt{\boldsymbol{x}^{\mathbf{2}}+\mathbf{1}}(\mathbf{2}\boldsymbol{x}^{\mathbf{2}}-\mathbf{1})}{\mathbf{3}\boldsymbol{x}^{\mathbf{3}}}+\boldsymbol{C}}$$

19. $\int (\frac{dx}{x^2(81+x^4)})$

$x = \frac{1}{z}\ ; dx = -\frac{1}{z^2}dz$

$\int \frac{\frac{-dz}{z^2}}{\frac{1}{z^2}\left(81+\frac{1}{z^4}\right)}$

$\int \frac{\frac{-dz}{z^2}}{\frac{81z^4+1}{z^6}}$

$= \int \frac{z^3}{(81z^4+1)^{\frac{3}{4}}}$

$let\ u = 81z^4+1\ ; du = 324z^3dz$

$= \frac{1}{324}\int \frac{du}{u^{\frac{3}{4}}}$

$= \frac{-1}{81}(\ 81z^4+1)^{\frac{1}{4}} + c$

$= \boxed{-\frac{\mathbf{1}}{\mathbf{81}}\left(\frac{\mathbf{81}+\boldsymbol{x^4}}{\boldsymbol{x^4}}\right) + \boldsymbol{c}}$

21. $\int \frac{(x-x^3)^{1/3}}{x^4}dx$

$let\ x = \frac{1}{z^2},\qquad z = \frac{1}{x}$

$= \int \frac{\left(\frac{1}{z}-\frac{1}{x^3}\right)\left(-\frac{dz}{z^2}\right)}{\frac{1}{z^4}}$

$= \int \frac{\left(\frac{z^2-1}{z^3}\right)^{\frac{1}{3}}\left(-\frac{dz}{z^2}\right)}{1/z^4}$

$= \int \frac{\left(\frac{(z^2-1)}{z}\right)^{\frac{1}{3}}\ (-\frac{dz}{z^2})}{z^4}$

$= -\int (z^2-1)^{\frac{1}{3}}zdz$

$let\ u = z^2-1$

$= -\frac{1}{2}\int u^{\frac{1}{3}}du$

$= -\frac{1}{2}(\frac{u^{\frac{4}{3}}}{\frac{4}{3}}) + c$

$= -\frac{3}{8}(z^2-1)^{\frac{4}{3}} + c$

$= -\frac{3}{8}\left(\frac{1}{x^2}-1\right)^{\frac{4}{3}} + c$

$= \boxed{-\frac{3}{8}\left(\frac{1\text{-}x^2}{x^2}\right)^{\frac{4}{3}} + \mathbf{c}}$

1. $\int \frac{Dx}{1+cosx}$

$= \int \frac{\frac{2\,Dz}{1+z^2}}{1+\frac{1-z^2}{1+z^2}}$

$= \int \frac{\frac{2\,dz}{1+z^2}}{\frac{1+z^2+1-z^2}{1+z^2}}$

$= \int \frac{2dz}{2}$

$= \int dz = z + c$

$= \boxed{\boldsymbol{tan\frac{x}{2} + c}}$

3. $\int \frac{dx}{4+2\,sinx} = \int \frac{\frac{2\,dz}{1+z^2}}{4+2\left(\frac{2z}{1+z^2}\right)}$

$= \int \frac{\frac{2dz}{1+z^2}}{4+\frac{4z}{1+z^2}} = \int \frac{\frac{2dz}{1+z^2}}{\frac{4+4z^2+4z}{1+z^2}}$

$= \int \frac{2\,dz}{4z^2+4z+4}$; $where: u = z + \frac{1}{2}$; $a = \frac{\sqrt{3}}{2}$

$= 2\int \frac{dz}{\left(z+\frac{1}{2}\right)^2+\frac{3}{4}} = 2\int \frac{du}{u^2+a^2} = \frac{2}{a}arctan\frac{u}{a} + c$

$= \frac{2}{\frac{\sqrt{3}}{2}}\,arctan\frac{z+\frac{1}{2}}{\frac{\sqrt{3}}{2}} + c = \frac{1}{\sqrt{3}}arctan\frac{2z+1}{\sqrt{3}} + c$

$= \boxed{\boldsymbol{\frac{1}{\sqrt{3}}arctan\frac{2\,tan\frac{x}{2}+1}{\sqrt{3}} + c}}$

5. $\int \frac{dx}{sinx + cosx + 3} = \int \frac{\frac{2dz}{1+z^2}}{\frac{2z}{1+z^2}+\frac{1-z^2}{1+z^2}+3}$

$= \int \frac{\frac{2dz}{1+z^2}}{\frac{1+2z-2z^2}{1+z^2}+3} = \int \frac{\frac{2dz}{1+z^2}}{\frac{1+2z-2z^2+3+3z^2}{1+z^2}}$

$= \int \frac{2dz}{4+2z+2z^2} = \int \frac{2dz}{\left(z+\frac{1}{2}\right)^2+\frac{7}{4}}$

$= 2\int \frac{du}{u^2+a^2}$: $where\ a = \frac{\sqrt{7}}{2}, u = z + \frac{1}{2}$

$= \frac{2}{a}arctan\frac{u}{a} + c = \frac{2}{\frac{\sqrt{7}}{2}}arctan\frac{z+\frac{1}{2}}{\frac{\sqrt{7}}{2}} + c$

$= \frac{1}{\sqrt{7}}arctan\frac{2z+1}{\sqrt{7}} + C$

$= \boxed{\boldsymbol{\frac{1}{\sqrt{7}}arctan\frac{2tan\frac{x}{2}+1}{\sqrt{7}} + c}}$

7. $\int secxdx = \int \frac{1+z^2}{1-z^2}\cdot\frac{2dz}{1+z^2}$

$= \int \frac{2dz}{1-z^2} = 2\int \frac{du}{a^2-u^2}$ $where\ a = 1, u = z$

$= \frac{2}{2a}ln\left|\frac{a+u}{a-u}\right| + c = ln\left|\frac{1+z}{1-z}\right| + C$

$= \boxed{\boldsymbol{\frac{2}{2a}ln\left|\frac{a+u}{a-u}\right| + c = ln\left|\frac{1+tan\frac{x}{2}}{1-tan\frac{x}{2}}\right| + c}}$

1. $\int xcosxdx \; = \; \int xcosxdx \quad ; \; let \; dv = coxdx \quad u = x$

$v = sinx \quad du = dx$

$= \; xsinx - \int sinxdx$

$= \boxed{\boldsymbol{xsinx + cosx + C}}$

3. $\int e^{-x} cos2xdx \; ; \; u = cos2x \quad ; \quad dv = e^{-x}dx \quad ; \; \overline{u} = sin2x \quad ; dv = e^{-x}dx$

$du = sin2xdx \; ; \quad v = \text{-}e^{-x} \quad ; \; \overline{du} = 2cos2xdx \quad ; \; v = \text{-}e^{-x}$

$= \; \text{-}e^{x}cos2x - \int 2e^{-x}sin2xdx$

$= \; \text{-}e^{x}cos2x - 2\int e^{-x}sin2xdx$

$= \; \text{-}e^{-x}cos2x - 2[\text{-}e^{-x}sin2x - \int \text{-}e^{-x}2cos2xdx$

$= \text{-}e^{-x}cos2x + 2e^{-x}sin2x - 4\int e^{-x}cos2xdx$

$add \; 4\int e^{-x}cos2xdx \; to \; both \; sides$

$= \frac{2e^{-x}sin\,2x - e^{-x}cos\,2x}{5} + C$

$= \boxed{\boldsymbol{\frac{e^{-x}}{5}(sin2x - cox2x) + C}}$

5. $\int arctan2xdx \; ; \; dv = dx \; ; \quad u = arctan2x$

$v = x \quad ; \quad du = \frac{2dx}{1+x^2}$

$= \; xarctan2x - \int \frac{2xdx}{1+4x^2}$

$= xarctan2x - 2\int \frac{dx}{1+4x^2}$

$= xarctan2x - \frac{1}{4}\int \frac{du}{u}$

$= \boxed{\boldsymbol{xarctan2x - \frac{1}{4}ln|1 + 4x^2| + C}}$

7. $\int sec^3xdx$; $dv = sec^2xdx$; $u = secx$

$v = tanx$ $\qquad du = secxtanx$

$= \int sec^2xsecxdx$

$= secxtanx - \int (sec^2x - 1)secxdx$

$= secxtanx + ln|secxtanx| - sec^3xdx$; $add \int sec^3xdx$ $on\ both\ sides$

$2\int sec^3xdx = secxtanx + ln|secx + tanx|$

$\int sec^3xdx = \boxed{\boldsymbol{\frac{1}{2}(secxtanx + ln|secx + tanx|) + C}}$

9. $\int xcos^2 2xdx$; $dv = d2xdx$; $u = x$

$v = \frac{1}{2}x + \frac{1}{8}sin4x \qquad du = x$

$= x\left(\frac{1}{2}x + \frac{1}{8}sin4x\right) - \int (\frac{1}{2}x + \frac{1}{8}sin4x)dx$

$= \frac{x^2}{2} + \frac{1}{8}xsin4x - \frac{1}{4}x^2 + \frac{1}{32}cos4x + C$

$= \boxed{\boldsymbol{\frac{1}{4}x^2 + \frac{1}{8}sin4x + \frac{1}{32}cos4x + C}}$

11. $\int \frac{xarcsinxdx}{\sqrt{1-x^2}}$; $dv = \frac{xdx}{\sqrt{1-x^2}}$; $u = arcsinx$

$v = -\sqrt{1-x^2}$; $du = \frac{dx}{\sqrt{1-x^2}}$

$= -\sqrt{1-x^2}\ arcsinx - \int (-\sqrt{1-x^2})(\frac{dx}{\sqrt{1-x^2}})$

$= -\sqrt{1-x^2}\ arcsinx + \int dx$

$= \boxed{\boldsymbol{x - \sqrt{1-x^2}\ arcsinx + C}}$

13. $\int sinxln(1+sinx)dx$; $u = ln(1+sinx)$; $dv = sinxdx$

$du = \left(\frac{1}{1+sinx}\right)cosxdx$; $v = cosx$

$= \text{-}cos\, ln|1+sinx| + \int \frac{cos^2xdx}{1+sinx}$

$= \text{-}cos\, ln|1+sinx| + \int \frac{1-sin^2xdx}{1+sinx}$

$= \text{-}cos\, ln|1+sinx| + \int(1-sinx)dx$

$= \text{-}cos\, ln|1+sinx| + x + cosx + C$

$= \boxed{\boldsymbol{-cos\, ln|1+sinx| + x + cosx + C}}$

15. $\int \frac{e^xxdx}{(x+1)^2}$; $u = e^xx$; $dv = \frac{1}{(x+1)^2}dx$

$du = e^x(x+1)dx$; $v = \text{-}\frac{1}{x+1}$

$= \text{-}\frac{e^xx}{x+1} + \int e^xdx \quad = \boxed{\boldsymbol{\frac{e^x}{x+1} + C}}$

17. $\int x^2arcsinxdx$; $u = arcsinx$; $du = \frac{dx}{\sqrt{1-x^2}}$; $dv = x^2dx$; $v = \frac{x^3}{3}$

$= \frac{1}{3}arcsin - \frac{1}{3}\int \frac{x^3}{\sqrt{1-x^2}}dx$

$= \frac{1}{3}x^3arcsinx + (\frac{1}{3}cos\, e - \frac{cos^3e}{9}) + C$

$= \frac{1}{3}x^3arcsinx + \left(\frac{3-\sqrt{1-x^2}}{9}\right) + C$

$= \frac{1}{3}x^3arcsin + \frac{\sqrt{1-x^2}(2+x^2)}{9} + C$

consider; $\frac{1}{3}\int \frac{x^3}{\sqrt{1-x^2}}dx$; $a = 1$; $v = x$; $x = sin\, e$; $dx = cos\, e\, de$; $\sqrt{1-x^2} = cose$

$= \frac{1}{3}\int \frac{sin^3e}{cose}(cosede)$

$= \frac{1}{3}\int sin^2e(sinede)$

$= \boxed{\boldsymbol{\frac{1}{3}(-cose + \frac{cos^3e}{3}) + C}}$

1. $\int \frac{12x+18}{(x+2)(x+4)(x-1)}$

$$\frac{12x+18}{(x+2)(x+4)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x+4)} + \frac{C}{(x-1)}$$

$$12x+18 = A(x+4)(x-1) + B(x+2)(x-1) + C(x+2)(x+4)$$

$$12x+18 = A(x^2+3x-4) + B(x^2+x-2) + C(x^2+6x+8)$$

$$12x+18 = Ax^2+3Ax-4A+Bx^2+B-2B+Cx^2+6Cx+8C$$

$$Ax^2+Bx^2+Cx^2 = 0$$
$$3Ax+Bx+6Cx = 12x$$
$$4A+B+8C = 18$$
$$A = 1$$
$$B = -3$$
$$C = 2$$

$$= \int \frac{dx}{(x+2)} + \int \frac{-3dx}{(x+4)} + \int \frac{2dx}{(x-1)}$$

$$= \boxed{\boldsymbol{ln|x+2| - 3\,ln|x+4| + 2ln|x-1|}}$$

3. $\int \frac{dx}{(x-1)(x-4)}$

$$1 = \frac{A}{(x-1)} + \frac{B}{(x-4)}$$

$$1 = A(x-4) + B(x-1)$$

$$1 = Ax-4A+Bx-B$$

$$A+B = 0$$
$$-4A-B = 1$$
$$A = -B$$
$$B = \frac{1}{3}$$

$$= \int \frac{\frac{-1}{3}}{(x-1)}dx + \int \frac{\frac{1}{3}}{(x-4)}dx$$

$$= -\frac{1}{3}ln|x-1| + \frac{1}{3}ln|x-4| + C \quad = \boxed{\frac{\mathbf{1}}{\mathbf{3}}\left[\frac{\boldsymbol{ln|x-4|}}{\boldsymbol{ln|x-1|}}\right]\mathbf{+C}}$$

5. $\int \frac{6x^2+23x-9\,dx}{(x^3+2x^2-3x)}$

$$\int \frac{6x^2+23x-9\,dx}{x(x+3)(x-2)}$$

$$6x^2+23x-9=\frac{A}{x}+\frac{B}{(x+3)}+\frac{C}{(x-1)}$$

$$6x^2+23x-9=A(x+3)(x-1)+B(x)(x-1)+C(x)(x+3)$$

$$6x^2+23-9=A(x^2+2X-3)+B(x^2-x)+C(x^2+3x)$$

$$A+B+C=6$$
$$2A-B+3C=23$$
$$-3A+0B+0C=-9$$
$$A=3$$
$$B=-2$$
$$C=5$$

$=3\int\frac{dx}{x}-2\int\frac{dx}{(x+3)}+5\int\frac{dx}{(x-1)}$

$= \boxed{\boldsymbol{3ln|x|-2ln|x+3|+5ln|x-1|+C}}$

7. $\int \frac{x^3+5x^2+9x+7}{x^2+5x+4}dx$

$$\int \frac{x^3+5x^2+9x+7}{(x+4)(x+1)}dx$$

By division of polynomials,

$$\frac{5x+7}{(x+4)(x+1)}=\frac{A}{(x+4)}+\frac{B}{(x+1)}$$

$$5x+7=A(x+1)+B(x+4)$$

$$if\ x=4,$$
$$A=\frac{13}{3}$$
$$if\ x=-1$$
$$B=\frac{2}{3}$$

$$=\int xdx+\int\frac{\frac{13}{3}dx}{(x+4)}+\int\frac{\frac{2}{3}dx}{(x+1)}$$

$= \boxed{\boldsymbol{\frac{x^2}{2}+\frac{13}{3}ln|x+4|+\frac{2}{3}ln|x+1|+C}}$

9. $\int \frac{2x+1}{(x-2)(x-3)^2}$

$$2x+1=\frac{A}{(x-2)}+\frac{B}{(x-3)}+\frac{C}{(x-3)^2}$$

$$2x+1=A(x-3)^2+B(x-3)(x-2)+C(x-2)$$

$$2x+1=A(x^2-6x+9)+B(x^2-5x+6)+C(x-2)$$

$$A+B=0$$

$$-6A-5B+C=2$$

$$9A+6B-2C=1$$

$$A=5$$

$$B=-5$$

$$C=7$$

$$=\int\frac{5dx}{(x-2)}+\int\frac{-5dx}{(x-3)}+\int\frac{7dx}{(x-3)^2}$$

$$=5ln|x-2|-5ln|x-3|+\frac{7}{x-3}$$

$$=\boxed{\boldsymbol{5ln\left|\frac{x-2}{x-3}\right|+\frac{7}{(x-3)}}}$$

11. $\int \frac{2x-5\,dx}{x(x-1)}$

$$\frac{2x-5}{x(x-1)} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} + \frac{D}{(x-1^3}$$

$$2x-5 = A(x-1)^3 + bx(x-1)^2 + cx(x-1) + Dx$$

$$2x-5 = Ax^3 - 3Ax^2 + 3Ax - A + Bx^3 - 2Bx^2 + Bx + Cx^2 - Cx + Dx$$

$$2x-5 = Ax^3 = 3Ax^2 - 2Bx^2 + Bx + Cx^2 - Cx + Dx$$

$$2x-5 = Ax^3 + Bx^2 - 3Ax^2 - 2Bx^2 + Cx^2 + 3Ax + Bx - Cx + Dx - A$$

$$2x-5 = (A+B)x^3 + (-3A-2B+c)x^2 + (3A+B-C+D)x - A$$

$$A + B = 0$$

$$-3A - 2B + C = 0$$

$$3A + B - C + D = 2$$

$$-A = -5$$

$$A = 5$$

$$B = -5$$

$$C = 5$$

$$D = -3$$

$$= \int \frac{5dx}{x} + \int \frac{-5dx}{(x-1)} + \int \frac{5dx}{(x-1)^2} + \int \frac{-3dx}{(x-1)^3}$$

$$= 5\int \frac{dx}{x} - 5\int \frac{dx}{(x-1)} + 5\int \frac{dx}{(x-1)^2} - 3\int \frac{dx}{(x-1)^3}$$

$$= 5ln|x| - 5ln|x-1| - \frac{5}{(x-1)} + \frac{3}{2(x-1)^2} + C$$

$$= \boxed{\boldsymbol{5ln\left|\frac{x}{x-1}\right| - \frac{5}{(x-1)} + \frac{3}{2(x-1)^2} + C}}$$

13. $\int \frac{3x^2+17x+32}{x^3+8x^2+16x}$

$$\int \frac{3x^2+17x+32}{x(x+4)^2}$$

$$\frac{3x^2+17x+32}{x(x+4)^2} = \frac{A}{x} + \frac{B}{(x+4)} + \frac{C}{(x+4)^2}$$

$$A + B = 3$$
$$8A + 4B + C = 17$$
$$16A = 32$$
$$A = 2$$
$$B = 1$$
$$C = 3$$

$$= \int \frac{2dx}{x} + \int \frac{dx}{(x+4)} + \int \frac{-3dx}{(x+4)^2}$$
$$= \boxed{\boldsymbol{2ln|x| + ln|x+4| + \frac{3}{x+4}}}$$

15. $\int \frac{2x+1}{(3x-1)(x^2+2x+2)}$

$$\frac{2x+1}{(3x-1)(x^2+2x+2)} = \frac{A}{(3x-1)} + \frac{B(2x+2)+C}{x^2+2x+2}$$

$$2x+1 = A(x^2+2x+2) + B(2x+2)(3x-1) + C(3x-1)$$

$$2x+1 = A(x^2+2x+2) + B(6x^2+4x+2) + C(3x-1)$$

$$A + B = 0$$
$$2A + 4B + 3C = 2$$
$$2A + 2B - C = 1$$
$$A = -\frac{5}{2}$$
$$B = \frac{5}{2}$$
$$C = -1$$

$$= -\frac{5}{2}\int \frac{dx}{(3x-1)} + \frac{5}{2}\int \frac{(2x+2)\,dx}{x^2+2x+2} - \int \frac{dx}{x^2+2x+2}$$
$$= -\frac{5}{2} ln|3x-1| + \frac{5}{2} ln|x^2+2x+2| - ln|x^2+2x+2|$$

$$= \boxed{\boldsymbol{\frac{5}{2} ln\left|\frac{x^2+2x+2}{3x-1}\right| - ln|x^2+2x+2|}}$$

17. $\int \frac{5x^2-x+17}{(x+2)(x^2+9)}dx$

$$\frac{5x^2-x+17}{(x+2)(x^2+9)}=\frac{A}{x+2}+\frac{B(2x)+C}{x^2+9}$$

$$5x^2-x+17=A(x^2+9)+(2Bx+C)(x+2)$$

$$5x^2-x+17=Ax^2+9A+2Bx^2+4Bx+Cx+2C$$

$$5x^2-x+17=Ax^2+2Bx^2+4Bx+Cx+9A+2C$$

$$5x^2-x+17=(A+2B)x^2+(4B+C)x+9A+2C$$

$$x^2=A+2B=5$$

$$x=4B+C=-1$$

$$c=9A+2C=17$$

$$(A+2B=5)-2 \quad =-2A-4B=-10$$

$$4B+C=-1 \quad =\frac{4B+C=-1}{-2A+C=-11}$$

$$(-2A+C=-1)-2 \quad =4A-2C=22$$

$$9A+2C=17 \quad =\frac{9A+2C=17}{13A \ =39}$$

A=3

9(3)+2C=17	*4B-5=-1*
27+2C=17	*4B=-1+5*
2C=17-27	*4B=4*
2C=-10	*B=1*
C=-5	

$$=\int \frac{3}{x+2}+\frac{(1)2x-5}{x^2+9}dx$$

$$=3\int\frac{dx}{x+2}+2\int\frac{xdx}{x^2+9}-5\int\frac{dx}{x^2+9}$$

$$=\boxed{\boldsymbol{3ln[x+2]+ln[x^2+9]-\frac{5}{3}Arctan\,\frac{x}{3}+C}}$$

19. $\int \frac{(4x^2+21x+54)}{x^2+6x+13}$

$$\frac{4-(3x-2)}{x^2+6x+13}$$

$$\frac{A(2x+6)+B}{x^2+6x+13}$$

$$A(2x+6)+B=3x-2$$

$$2A+B=3$$

$$B=-11$$

$$A=\frac{3}{2}$$

$$=\int 4-[\frac{3}{2}\int\frac{(2x+6)dx}{x^2+\ 6x+13}+(-11\int\frac{dx}{x^2}+6x+13)]$$

$$=-11\int\frac{dx}{(x^2+6x+9)+(13-9)}$$

$$=-11\int\frac{dx}{(x+3)^2+(13-9)^2}$$

$$=-11(\tfrac{1}{2}arctan\tfrac{x+3}{2})$$

$$=4x-\left(\tfrac{3}{2}ln|x^2+6x+13|-\tfrac{11}{2}arctan\tfrac{x+3}{2}\right)$$

$$=\boxed{\mathbf{4x-\tfrac{3}{2}ln|x^2+6x+13|+\tfrac{11}{2}arctan\tfrac{x+3}{2}+C}}$$

21. $\int \frac{x^3+7x^2+25x+35}{x^2+5x+6}$

$$\int x+2+\frac{9x+23}{x^2+5x+6}dx$$

$$\frac{9x+23}{(x+3)(x+2)}=\frac{A}{x+3}+\frac{B}{x+2}$$

$$9x+23=A(x+2)+B(x+3)$$

x=-3

9(-3)+23= A(-3+2)+B(-3+3)

-27+23=A(-1)+B(0)

-4=-A

A=4

If x=-2

9(-2)+23= A(-2+2)+B(-2+3)

-18+23=A(0)+B

5=B

B=5

$$=\int x+2+\frac{-2}{x+3}+\frac{5}{x+2}dx$$

$$=\int xdx+2\int dx-4\int\frac{dx}{x+3}+5\int\frac{dx}{x+2}$$

$$=\boxed{\boldsymbol{\frac{x^2}{2}+2x-4ln[x+3]+5ln[x+2]+c}}$$

23. $\int \frac{x^2-x-8}{(2x-3)(x^2+2x+2)}$

$$\frac{A}{(2x-3)}+\frac{B(2x-2)+C}{x^2+2x+2}$$

$$A(x^2+2x+2)+B(2x+2)(2x-3)+C(2x-3)$$

$$A(x^2+2x+2)+B(4x^2-2x-6)+C(2x-3)$$

$$A+4B=1$$

$$2A\text{-}2B+2C=\text{-}1$$

$$2A\text{-}6B\text{-}3C=\text{-}8$$

$$A=\frac{1}{2}$$

$$A=\frac{1}{2}$$

$$C=1$$

$$-1\int\frac{dx}{(2x-3)}+\frac{1}{2}\int\frac{(2x+2)dx}{x^2+2x+2}+\int\frac{dx}{x^2+2x+2}$$

$$-ln(2x-3)+\frac{1}{2}ln\left|x^2+2x+2\right|+\int\frac{dx}{(x+1)^2+1^2}$$

$$=-\frac{1}{2}-ln(2x-3)+ln\left|x^2+2x+2\right|+arctan(x+1)+C$$

$$=\boxed{\boldsymbol{\frac{1}{2}ln\left|\frac{x^2+2x+2}{2x-3}\right|+arctan(x+1)+c}}$$

25. $\int \frac{(x^5+2x^3-3x)}{(x^2+1)^3}$

$$= \int \frac{x^5 + 2x^3 - 3x}{x^6 + 3x^4 + 2x^2 + 1}$$

$$= \left[\frac{A(2x) + B}{x^2 + 1} + \frac{C(2x) + D}{(x^2 + 1)^2} + \frac{E(2x) + F}{(x^2 + 1)^3}\right](x^2 + 1)^3$$

$$= A(2x)(x^2 + 1)^2 + B(x^2 + 1)^2 + C(2x)(x^2 + 1) + D(x^2 + 1) + E(2x) + F$$

$$= A(2x)(x^4 + 2x^2 + 1) + B(x^4 + 2x^2 + 1) + C(2x^3 + 2x) + D(x^2 + 1) + E(2x) + F$$

$$= A(2x^5 + 4x^3 + 2x) + B(x^4 + 2x^2 + 1) + C(2x^3 + 2x) + D(x^2 + 1) + E(2x) + F$$

$x^5: 2A = 1$; $A = \frac{1}{2}$

$x^4: B = 0$; $B = 0$

$x^3: 4A + 2C = 2$; $C = 0$

$x^2: 2B + D = 0$; $D = 0$

$x: 2A + 2C + 2E = -3$; $E = 0$

$c: B + D + F = 0$; $F = 0$

$$= \boxed{\boldsymbol{\frac{1}{2} ln|x^2 + 1| + \frac{1}{(x^2 + 1)^2} + C}}$$

27.

$$\frac{x^4 + 2x^3 + 11x^2 + 8x + 16}{x(x^2+4)^2}$$

$$[\frac{A}{X} + \frac{B(2x)+C}{(x^2+4)} + \frac{D(2x)+E}{(x^2+4)^2}][(x^2+4)^2]$$

$$A(x^2+4)^2 + B(2x)(x)(x^2+4) + C(x^2+4)(x) + D(2x)(x) + E(x)$$

$$A(x^4 + 8x^2 + 16) + B(2x^4 + 8x^2) + C(x^3 + 4x) + D2x^2 + Ex$$

x^4: $A + 2B = 1$ $\quad$ $A = 1$

x^3: $C = 2$ $\quad$ $B = 0$

x^2: $8A + 8B + 2D = 11$ $\quad$ $C = 2$

X: $4C + E = 8$ $\quad$ $D = 3/2$

C : $16A = 16$ $\quad$ $E = 0$

$$= \int \frac{dx}{x} + \frac{2dx}{x^2+4} + \frac{3}{2}\int \frac{2xdx}{(x^2+4)^2}$$

$$= \ ln x \ + \ 2\left(\frac{1}{2}\ arctan\frac{x}{2}\right) - \frac{3}{2(x^2+4)} + C$$

$$= \boxed{\boldsymbol{ln x \ + \ arctan\frac{x}{2} - \frac{3}{2(x^2+4)} + C}}$$

$* n = 10$

1. $\sum_{i=1}^{n} 12i^3$

$= 12 \sum_{i=1}^{n=10} i3$

$= 12\left(\frac{10^2(10+1)^2}{4}\right)$

$= 3(100(121))$

$= \boxed{\mathbf{36300}}$

3. $\sum_{i=1}^{n=10} (12i^2 + 4i)$

$= 12 \sum_{i=1}^{n=10} i^2 + 4 \sum_{i=1}^{n=10} i$

$= 12\left(\frac{10(10+1)(2(10)+1)}{6}\right) + 4\left(\frac{10(10+1)}{2}\right)$

$= 2(110)(21) + 2(110)$

$= \boxed{\mathbf{4840}}$

5. $\sum_{i=1}^{n=10} i(i-1)(i+1)$

$= \sum_{i=1}^{n} i^3 - i$

$= \sum_{i=1}^{n=10} i^3 - i$

$= \sum_{i=1}^{n=10} i^3 + \sum_{i=1}^{n=10} i$

$= \frac{10^2(10+1)^2}{4} - \frac{10(10+1)}{2}$

$= \boxed{\mathbf{2970}}$

7. $\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}=\mathbf{10}} (\mathbf{3}\boldsymbol{i} + \mathbf{1})^{\mathbf{2}}$

$= \sum_{i=1}^{n=10} 9i^2 + 6i + 1$

$= 9\sum i^2 + 6\sum i + \sum 1$

$= 9\left[\frac{10(10+1)(2(10)+1)}{6}\right] + 6\left[\frac{10(10+1)}{2}\right] + 10$

$= \boxed{\mathbf{3805}}$

9. $(\boldsymbol{a_1} - \boldsymbol{b_1}) + (\boldsymbol{a_2} - \boldsymbol{b_2}) + \cdots + (\boldsymbol{a_n} - \boldsymbol{b_n})$

$= \boxed{\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}} \boldsymbol{a_i} - \boldsymbol{b_i}}$

11. $f(x_1)\Delta x_1 + f(x_2)\Delta x_2 + \cdots + f(x_n)\Delta x_n$

$$=\boxed{\sum_{i=1}^{n} \boldsymbol{f(x_i)\ \Delta x_i}}$$

13. $1^4 + 2^4 + 3^4 + \cdots + n^4$

$$=\boxed{\sum_{i=1}^{n} \boldsymbol{i^4}}$$

15. $a_1 b_{1+} a_2 b_{2+} a_3 b_3 + \cdots + a_n b_n$

$$=\boxed{\sum_{i=1}^{n} \boldsymbol{a_i b_i}}$$

17. $u_1^3 + u_2^3 + u_3^3 + \cdots + u_n^3$

$$=\boxed{\sum_{i=1}^{n} \boldsymbol{u_i^3}}$$

1. $\int_1^2 3x^2 dx$

$a = 0 \,; b = 2$

$\Delta x = \dfrac{2-0}{n}$

$= \dfrac{2}{n}$

$Zi = a + i\Delta x$

$= 0 + i\left(\dfrac{2}{n}\right)$

$= \dfrac{2i}{n}$

$= \lim_{n=\infty} \sum_{i=1}^{3} \dfrac{2i}{n}\left(\dfrac{2}{n}\right)$

$= \lim_{n\to\infty} 3\sum \dfrac{4i^2}{n^2}\left(\dfrac{2}{n}\right)$

$= \lim_{n\to\infty} 3\sum \dfrac{8i^2}{n^3}$

$= \lim_{n\to\infty} 24\left(\dfrac{n(n+1)(2n+1)}{6}\right)\dfrac{1}{n^3}$

$= \lim_{n\to\infty} 24\left[\dfrac{(n^2+1)(2n+1)}{6\,n^3}\right]$

$= \lim_{n\to\infty} 24\left[\dfrac{\overset{1}{\cancel{2n^3}}+\overset{0}{\cancel{n^2}}+\overset{0}{\cancel{2n^2}}+\overset{0}{\cancel{n}}}{6n^3}\right]$

$= \boxed{\mathbf{8}}$

3. $\int_0^1 2x\,(x-1)dx$

$a = 0 \quad ; \quad b = 1$

$\Delta x = \dfrac{1-0}{n} \quad ; \quad Zi = \dfrac{i}{n}$

$= \lim_{n\to\infty} 2\sum x^2 - xdx$

$= \{\lim_{n\to\infty} 2\,[\sum(\frac{i^2}{2})\left(\frac{1}{n}\right)] - \sum\frac{i}{n^3}\}$

$= \left\{\lim_{n\to\infty} 2\left[\frac{1}{n^2}\left(\frac{n(n+1)(2n+)}{6}\right)\left(\frac{1}{n}\right)\right] - \frac{\overset{1}{\cancel{n}}-\overset{0}{\cancel{n}}}{n^2}\right\}$

$= \left\{\lim_{n\to\infty} 2\left[\dfrac{1}{n^3}\left(\dfrac{2n^3+n^2+2n^2+n}{6}\right)\right] - 1\right\}$

$= \frac{2}{3} - 1$

$= \boxed{-\dfrac{\mathbf{1}}{\mathbf{3}}}$

5. $\int_1^5 2x + 3dx$

$\Delta x = \dfrac{5-1}{n} \quad ; \quad Zi = 1 + i\left(\dfrac{4}{n}\right)$

$= \dfrac{4}{n}$

$= \lim_{n=\infty} \sum(1+\frac{4i}{n})\cdot\frac{4}{n} + 3n$

$= \lim_{n=\infty} \sum\frac{4}{n} + \sum\frac{16i}{n^2} + \sum 3n$

$= \lim_{n=\infty} \frac{4n}{n} + \frac{16}{n^2} + \left(\frac{n(n+1)}{2}\right) + 3n$

$= \boxed{\mathbf{36}}$

7. $\int_0^2 x^3 dx$

$\Delta x = \frac{2}{n}\ ; Zi = \frac{2i}{n}$

$= \lim_{n\to\infty} \sum \left(\frac{2i}{n}\right)^3 \left(\frac{2}{n}\right)$

$= \lim_{n\to\infty} \sum \left(\frac{8i^3}{n^3}\right)\left(\frac{2}{n}\right)$

$= \lim_{n\to\infty} \frac{16}{n^4}\left(\frac{n^2(n+1)^2}{4}\right)\sum i^3$

$= \lim_{n\to\infty} \frac{4}{n^4}(n^2(n^2+2n+1)$

$= lim_{n\to\infty} \frac{4\cancel{n^4}}{\cancel{n^4}} + \frac{8}{n^3} + \frac{4n^2}{n^3}$

$= 4 + 0 + 0$

= $\boxed{\mathbf{4}}$

1. $\int_1^2 3x^2 - 2x + 1\, dx$

$= \left(\frac{3x^3}{3} + \frac{2x^2}{2} + x\right)$

$= 8 - 4 + 2 - 1 + 1 - 1$

$= \boxed{\mathbf{5}}$

3. $\int_1^3 3x^2 + \frac{4}{x^2}\, dx$

$= \left(\frac{3x^3}{3} + \frac{4}{x}\right)$

$= 27 - \frac{4}{3} - 1 + 4$

$= \boxed{\frac{\mathbf{86}}{\mathbf{3}}}$

5. $\int_0^{\sqrt{7}} \sqrt[3]{1 + x^2}\, dx$

$u = 1 + x^2$

$du = 2xdx$

$= \frac{1}{2}\left(\frac{3(1+x^2)^{\frac{4}{3}}}{4}\right)$

$= \boxed{\frac{\mathbf{45}}{\mathbf{8}}}$

7. $\int_2^3 \frac{xdx}{x^2 + 1}$

$u = x^2 + 1$

$du = 2xdx$

$= \frac{1}{2}\int_2^3 \frac{du}{u}$

$= \frac{1}{2}\, ln|10| - ln|5|$

$= \boxed{\mathbf{0.347}}$

9. $\int_{-1}^{0} \frac{dy}{\sqrt{-(x^2+2x-1)}}$

$= \int_{-1}^{0} \frac{dy}{\sqrt{-(x + 2x + 1 - 1 - 1)}}$

$= \int_{-1}^{0} \frac{dy}{\sqrt{-[(x + 1)^2 + 2]}}$

$= \int_{-1}^{0} \frac{dy}{\sqrt{-(x + 1)^2 + 2}}$

$= \int_{-1}^{0} \frac{dy}{\sqrt{2 - (x + 1)^2}}$

$let\ a = \sqrt{2} \quad ; \quad u = (x + 1)$

$= \left[Arcsin\ \frac{x+1}{\sqrt{2}}\right] + c$

$= \boxed{\frac{\boldsymbol{\pi}}{\mathbf{4}}}$

11. $\int_0^e \frac{xdx}{x^2+e}$

$let\ u = x^2 + e\ ; du = 2xdx\ ; \frac{du}{2} = xdx$

$= \int_o^e \frac{\frac{du}{2}}{u}$

$= \frac{1}{2}\int_o^e \frac{du}{u}$

$= \frac{1}{2}[lnu]_0^e$

$= \frac{1}{2}[\ln(x^2+e)]_0^e$

$= \frac{1}{2}[\ln(e^2+e) - ln(0+e)]\ ; lna - lnb = ln\frac{a}{b}$

$= \frac{1}{2}ln\frac{e^2+e}{e} = \frac{1}{2}ln\frac{e(e+1)}{e}$

$= \frac{1}{2}ln(e+1) = ln(e+1)^{\frac{1}{2}}$

$= \boxed{\boldsymbol{ln\sqrt{e+1}}}$

13. $\int_0^2 \frac{dx}{x^2+4}$ *u= x; du=dx; a=2*

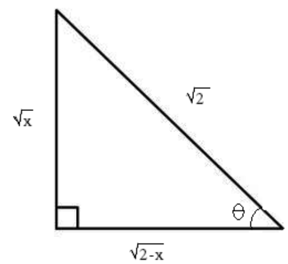
$= \left[\frac{1}{2}arctan\frac{x}{2}\right]$

$= \frac{1}{2}arctan\,1$

$= \boxed{\boldsymbol{\frac{\pi}{8}}}$

15. $\int_0^1 \sqrt{2x-x^2}\ dx$

$\int_0^1 (\sqrt{x}\cdot\sqrt{2-x})\ dx$



$\cos\theta = \frac{\sqrt{2-x}}{\sqrt{2}}; \sqrt{2}cos\theta = \sqrt{2-x}$

$sin\theta = \frac{\sqrt{x}}{\sqrt{2}}\ ; \sqrt{2}sin\theta = \sqrt{x}$

$x = 2sin^2\theta\ ; dx = 4sin\theta cos\theta d\theta$

$At\ x = 1, \theta = {}^{\pi}/_{4};\quad x = 0, \theta = 0$

$= \int_o^{\pi/4} \sqrt{2}cos\theta \cdot \sqrt{2}sin\theta \cdot 4sin\theta cos\theta d\theta$

$= 8\int_0^{\pi/4} sin^2\theta cos^2\theta\, d\theta$

$= 8\int_0^{\pi/4} \left(\frac{1-cos2\theta}{2}\right)\left(\frac{1+cos2\theta}{2}\right) d\theta$

$= 2\int_0^{\pi/4}(1 - cos^2 2\theta)\, d\theta$

$= \boxed{\boldsymbol{{}^{\pi}/_{4}}}$

17. $\int_0^1 xe^x\, dx$

$u = x \quad ; \quad dv = e^x\, dx$
$du = dx \quad ; \quad v = e^x$

$= xe^x - \int_0^1 e^x dx = (xe^x - e^x)$

$= (1 - 1 + 0 - 1) = \boxed{\mathbf{1}}$

19. $\int_0^{\frac{\pi}{2}} sin^2\, xcoxdx$

$let\ u = sinx\ ;\ du = cosxdx$

$= \int_o^{\frac{\pi}{2}} u^2 du$

$= \left(\frac{u^3}{3}\right)$

$= \left(\frac{sin^3 x}{3}\right)$

$= \boxed{\frac{\mathbf{1}}{\mathbf{3}}}$

21. $\int_o^{\frac{\pi}{2}} sin^6\, xcos^4 x\ \ dx$

$= \frac{(6-1)(6-3)(6-5)(4-1)(4-3)(\frac{\pi}{2})}{(6+4)(6+4-2)(6+4-4)(6+4-6)(6+4-8)}$

$= \boxed{\frac{\mathbf{3\pi}}{\mathbf{512}}}$

23. $\int_o^{\frac{\pi}{2}} sin^7\, x$

$= \frac{(4-1)(7-3)(7-5)}{7(7-2)(7-4)(7-6)}$

$= \boxed{\frac{\mathbf{16}}{\mathbf{35}}}$

25. $\int_0^{\pi} sin^6 \frac{x}{2} cos^2 \frac{x}{2} dx$

$u = \frac{x}{2}\ \ ;\ \ du = \frac{dx}{2}$

$= 2\int_o^{\pi} sin^6\, u\, cos^2\, u\, du$

$= 2\left(\frac{(6-1)(6-3)(6-5)(2-1)}{(6+2)(6+2-2)(6+2-4)(6+2-6)}\right)\frac{\pi}{2}$

$= \boxed{\frac{\mathbf{5\pi}}{\mathbf{128}}}$

27. $\int_8^{\frac{\pi}{4}} sin^2\, 4x\, cos^2\, 2x\ dx$

$= \frac{(2-1)(2-1)}{(2+2)(2+2-2)}$

$= \frac{1}{4(2)}\left(\frac{\pi}{2}\right)$

$= \boxed{\frac{\boldsymbol{\pi}}{\mathbf{16}}}$

29. $\int_0^2 (4-x^2)^{\frac{3}{2}}\ dx\ ; let\ x = 2sin\emptyset$

$dx = 2cos\emptyset sin\emptyset$

$= \int_0^2 (4-(2sin\emptyset)^2)^{\frac{3}{2}}\ \ (2cos\emptyset d\emptyset)$

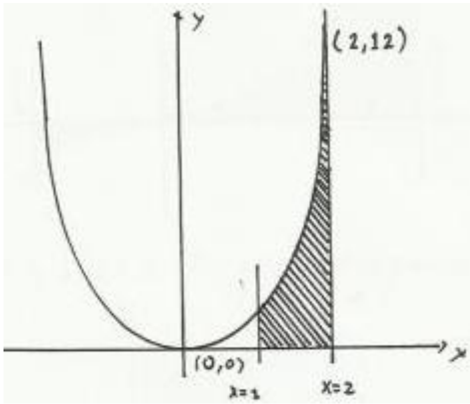
$= \int_0^2 (4\,cos^2\,\emptyset)^{\frac{3}{2}}\ \ 2cos\emptyset cos\emptyset d\emptyset$

$= \int_0^2 8\,cos^3\,\emptyset\, 2cos\emptyset\ d\emptyset$

$= (\frac{(4-1)((4-3))}{4(4-2)}\left(\frac{\pi}{2}\right)$

$= \boxed{\frac{\mathbf{3\pi}}{\mathbf{16}}}$

1. $y = 3x^2$; $from\ x = 1\ to\ x = 2$



$$A = \int_1^2 ydx$$

$A = \int_1^2 3x^2 dx$

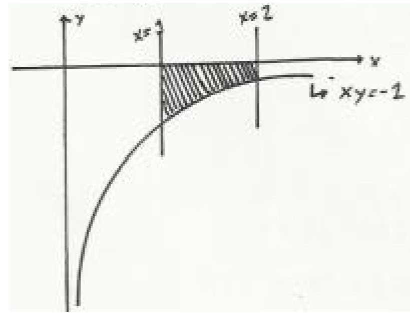
$A = [x^3]_1^2$

$A = (2)^3 - (1)^3$

$\boxed{\boldsymbol{A = 7\ sq.units}}$

3. $xy = -1$; $from\ x = 1\ to\ x = 2$

$$y = -\frac{1}{x}$$



$$A = \int_1^2 ydx$$

$A = \int_1^2 -\frac{1}{x} dx$

$A = [-\ln x]_1^2$

$A = \{[-\ln 2] - [-\ln 1]\}$

$A = -\ln 2;$ *but there is no negative area,*

hence,

$\boxed{\boldsymbol{A = \ln 2\ sq.units}}$

5. $y = 3lnx, x = 2 \; to \; y = 4$

$$\int_0^a dA = \int ydx$$

$$A = 3\int_2^4 lnxdx$$

$$= 3[xlnx - x]$$

$$= 3[4\,ln\,4 - 4] - 3[2\,ln\,2 - 2]$$

$$= 3[4\,ln\,4 - 4 - 2ln\,2 + 2]$$

$$= 3[8ln2 - 2ln2 - 2]$$

$$= 3[6ln2 - 2]$$

$$= 6[3ln2 - 1]$$

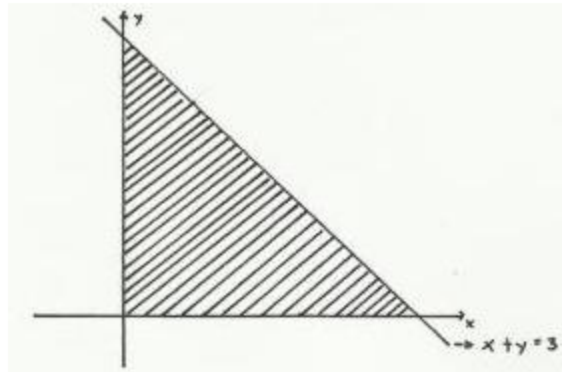
$$\boxed{\boldsymbol{A = 6[ln8 - 1] \; sq.units}}$$

7. $y = 9 - x^2 \quad ; \quad x = -3 \; to \; x = 3$

$A = \int_{-3}^{3}(4 - x^2)dx$

$$\boxed{\boldsymbol{A = 6 \; square \; units}}$$

9. $x + y = 3 \; \& \; the \; coordinate \; axes$



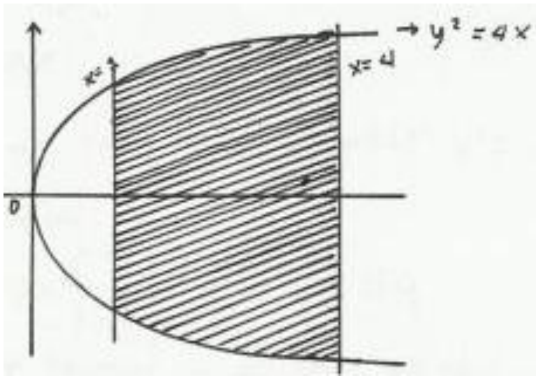
$A = \int_0^3 (3 - x)dx$

$$A = \left[3x - \frac{x^3}{2}\right]_0^3$$

$$A = \left[3(3) - \frac{(3)^2}{2}\right]$$

$$\boxed{\boldsymbol{A = \frac{9}{2} \; sq.units}}$$

11. $y^2 = 4x, \quad x = 1 \; and \; x = 4$



$$A = \int_1^4 \sqrt{4x}dx$$

$$A = \int_1^4 4x^{\frac{1}{2}}dx$$

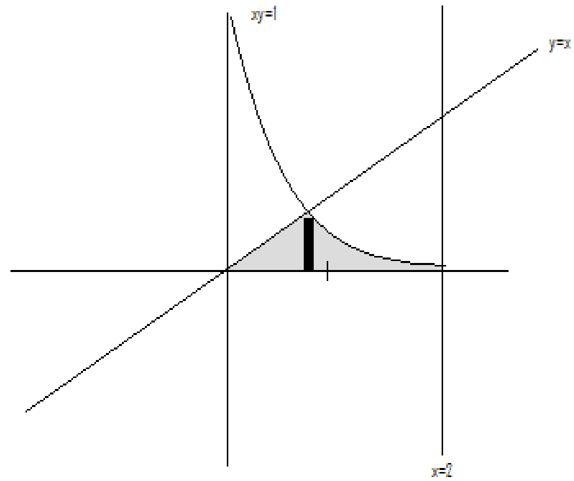
$$A = \left[\frac{8}{3}x^{\frac{3}{4}}\right]$$

$$A = \frac{8(4)^{3/2}}{3} - \frac{8(1)^{3/2}}{3}$$

$$A = \frac{64}{3} - \frac{8}{3}$$

$$\boxed{\boldsymbol{A = \frac{56}{3} \; sq.units}}$$

13. $xy = 1, y = x, x = 2, y = 0$



$$xy = 1; \; y = x$$

$$x(x) = 1$$

$$x = 1 \;\; ; \;\; y = 1 \quad ; \quad (1,1)$$

$$A1 = \int_1^2 \frac{1}{x}dx$$

$$= \; (ln \; x)$$

$$= ln \; 2 - ln \; 1$$

$$A1 = ln \; 2 \; sq.units$$

$$A2 = \frac{1}{2}bh$$

$$= \frac{1}{2}(1)(1)$$

$$A2 = \frac{1}{2}sq.units$$

$$At \; = A1 \;\; + \; A2$$

$$\boxed{\boldsymbol{A = (ln\; 2 + \frac{1}{2})sq.units}}$$

1. $y = x^2\,; y = 2x + 3$
$y = 2x + 3$
$x^2 = 2x + 3$
$x^2 - 2x - 3 = 0$
$(x - 3)(x + 1) = 0$
$x = 3, x = -1$

$A = \int_{-1}^{3}[(2x + 3) - x^2]dx$

$= [x^2 + 3x - \frac{x^3}{3}]_{-1}^{3}$

$= \left[3^2 + 3(3) - \frac{(3)^3}{3}\right] - \left[(-1)^2 + 3(-1) - \frac{(-1)^3}{3}\right]$

$=\left[9 + \frac{5}{3}\right]$

$\boxed{\boldsymbol{A = \frac{32}{3}\, sq.units}}$

3. $x^2 = y - 1 \quad ; \; x = y - 3$

$Y_1 = Y_2$
$(y - 3)^2 = y - 1$
$y^2 - 6y + 9 = y - 1$
$(y - 5)(y - 2) = 0$
$y = 5 \;, y = 2$
$x = 5 - 3 = 2$

$A = \int_{-1}^{2}[(x + 3) - (x^2 + 1)]dx$

$= \int_{-1}^{2}[x + 2 - x^2]dx$

$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$

$=\left[\frac{2^2}{2} + 2(2) - \frac{2^3}{3}\right] - \left[\frac{(-1)^2}{2} + 2(-1) - \frac{(-1)^3}{3}\right]$

$= \frac{10}{3} + \frac{7}{6} \quad = A = \frac{27}{6} \quad = \boxed{\boldsymbol{\frac{9}{2}\, sq.units}}$

5. $y = x^2 \quad ; y = 2 - x^2$

$\frac{dy}{dx} = 2x \quad ; \quad (0,0)$

$x = 0\,, y = 0$

$\frac{d^2y}{dx^2} = 2\;(concave\ upward)$

$point\ of\ intersection$:

$y_1 = y_2$

$x^2 = 2 - x^2$

$x^2 - 2 + x^2 = 0$

$(2x + 2)(x - 1)$

$2x + 2 = 0x - 1 = 0$

$2x = -\frac{2}{2}x = 1$

$x = -1\; y = 1$

$dA = [Y1 - Y2]dx$

$\int dA = \int_{-1}^{1}(2 - x^2 - x^2)\,dx$

$= \int_{-1}^{1}(2 - 2x^2)\,dx$

$= \left[2x - \frac{2x^3}{3}\right] = \left[2 - \frac{2}{3}\right] - [-2 + \frac{2}{3}]$

$= 2 - \frac{2}{3} + 2 - \frac{2}{3} \;=\; \frac{12-4}{3}$

$= \boxed{\boldsymbol{\frac{8}{3}\, sq.units}}$

7. $y = sinx\ ; x = cosx\ ; x = \frac{\pi}{4}\ and\ x = \frac{\pi}{2}$

x	y
0	0
90	1
180	0
270	-1
360	0

x	y
0	1
90	0
180	-1
270	0
360	1

$A_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} sinxdx = [-cosx]$

$= [-cos\frac{\pi}{4}] - [-cos\frac{\pi}{2}] = \frac{\sqrt{2}}{2}$

$A_1 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} cosxdx = [sinx] = \left[sinx\frac{\pi}{2}\right] - \left[sin\frac{\pi}{4}\right]$

$= 1 - \frac{\sqrt{2}}{2}$

$\boldsymbol{A_2 - A_1} = \boxed{\boldsymbol{\sqrt{2} - 1\ sq.units}}$

9. $x^2 = 4y\ , y = \frac{8}{x^2+4}$

$y = \frac{x^2}{4}$

$x^2(x^2+4) = 32$

$A = \int_{-2}^{2} \left(\frac{8}{x^2+4}\right) - \left(\frac{x^2}{4}\right) dx$

$\boxed{A = 4.95}$

11. $y = x^3\ , y = 8, x = 0$

$\frac{dy}{dx} = 3x^2 \quad , \quad 0 = 3x^2$

$y = 0\ , x = 0$

$\frac{d^2y}{dx^2} = 6x(concave\ upward)$

$point\ of\ intersection:$

$y_1 = y_2$

$x^3 = 8$

$x^3 - 8 = 0$

$x^3 = 8$

$x = \sqrt[3]{8}$

$x = 2$

$when\ x = 2$

$y = 8\ , (2,8)$

$when\ x = -2$

$y = (-2)^3\ , y = -8$

(-2,-8)

$dA = [Y1 - Y2]dx$

$\int dA = \int_0^2 (8 - x^3)\, dx$

$= [\mathbf{16 - 4}] = \boxed{\boldsymbol{12\ sq.units}}$

13. $y = 2x + 1 \quad , y = 7 - x \ , \ x = 8$

$A = \int_2^8 [(2x + 1) - (7 - x)]dx$

$= \int_2^8 [2x + 1 - 7 + x]dx$

$= \int_2^8 [3x - 6]dx$

$= \left[\frac{3x^2}{2} - 6x\right]_2^8$

$= \left[\frac{3(8)^2}{2} - 6(8)\right] - \left[\frac{3(2)^2}{2} - 6(2)\right]$

$= \boxed{\mathbf{54\ sq\ units}}$

15. $y = lnx^3, y = lnx; x = e$

$A = \int_1^e (y_2 - y_1)dx$

$= \int_1^e [(lnx^3) - (lnx)]dx$

$= \int_1^e lnx^3 - \int_1^e lnx$

$u = lnx^3 \quad ; \quad v = x \quad ; \ u = lnx \quad ; \quad dv = dx$

$du = \frac{3x^2}{x^3} dv = dx \quad ; \quad du = \frac{dx}{x} \quad ; \quad v = x$

$= xlnx^3 - \int_1^e x(\frac{3x^2}{x^3})$

$= \left[xlnx^3 - \int 3x\right]_1^e - [xlnx - \int x(\frac{dx}{x})]_1^e$

$= [xlnx^3 - 3x]_1^e - [xlnx - x]_1^e$

$= \boxed{\mathbf{2\ sq.units}}$

17. $y^2 = 2ax\ , y^2 = 4ax - a^2$

$y^2 = 2axy^2 = 4ax - a^2$

$x = \frac{y^2}{2a}\ \ ; x = \frac{y^2+a^2}{4a}$

$\frac{dx}{dy} = \frac{2y}{2a}$

$\frac{dx}{dy} = \frac{y}{a}$

$0 = 0\ ; (0{,}0)$

$\frac{d^2x}{dy^2} = \frac{1}{a}(open\ to\ the\ right)$

$point\ of\ intersection$:

$X_1 = X_2$

$\frac{y^2}{2a} = \frac{y^2 + a^2}{4a}$

$4ay^2 = 2ay^2 + 2a^3$
$4ay^2 - 2ay^2 - 2a^3 = 0$
$2ay^2 - 2a^3 = 0$
$2ay^2 = 2a^3$
$y^2 = \frac{2a^3}{2a}$
$y^2 = a^2$
$y = \sqrt{a^2}$
$y = \pm a$
$X_1 = X_2 = \frac{a^2}{2a} = \frac{a}{2}$

$when\ x = 4a$

$x = \frac{(4a)^2}{2a}$

$= \frac{16a^2}{2a}$

$= 8a$

$when\ x = -4a$

$x = (-4a)^2$

$= \frac{16a^2}{2a}$

$\boxed{= 8a}$

$\int_o^A dA = \int_{-a}^{a} [\frac{y^2 + a^2}{4a} - \frac{y^2}{2a}]dy$

$= \int_{-a}^{a} (\frac{y^2+a^2-y^2}{4a})dz$

$= \left[\frac{y^3}{12a} + \frac{a^2y}{4a} - \frac{2y^3}{12a}\right]_{\text{-a}}^{\text{a}}$

$= \left[\frac{a^3}{12a} + \frac{a^2a}{4a} - \frac{2a^3}{12a}\right] - \left[\frac{(-a)^3}{12a} + \frac{a^2(-a)}{4a} - \frac{2(-a)^3}{12a}\right]$

$= \frac{a^3 - 2a^3 + a^3 - 2a^3}{12a} + \frac{a^3 + a^3}{4a}$

$= \frac{-2a^3}{12a} + \frac{2a^3}{4a} \quad = \frac{-2a^3+6a^3}{12a}$

$= \frac{4a^3}{12a}$

$\boxed{\boldsymbol{A= \frac{a^2}{3} sq.\ units}}$

19. $y^2 = x + 1 \quad ; \quad y = 1 - x$

$v_{1=}y^2 - 1; \quad yx = 1$

$\frac{dx}{dy} = 2y \quad ; \quad x_2 = 1 - y$

$x = 0; \quad y = 0$

$\frac{d^2x}{dy^2} = 2 \quad (concave\ to\ the\ right)$

$point\ of\ intersection$

$x_{1=}y_2\ ;\ y^2 - 1 = \frac{1}{y}$

$y^2 + y - 2 = 0$

$(y-1)(y+2)$

$y - 1 = 0 \quad | \quad y + 2 = 0$

y=1 | *y=-2*

$v = 0 \quad | \quad y = 3$

$when\ x = 1, y = \sqrt{2}$

$when\ x = 2, y = \sqrt{5}$

$when\ y = 1, x = 0$

$when\ y = 2, x = 3$

$when\ y = 3, x = 8$

$then;$

$dA = [X_2 - X_1]dy$

$$\int_{-2}^{1} dA = \int_{-2}^{1} (1 - y) - (y^2 - 1)dy$$

$A = [1 - y - y^2 + 1]_{-2}^{1}$

$A = [2 - y - y^2]_{-2}^{1}$

$$A = \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_{-2}^{1}$$

$$A = \left[2(1) - \frac{(1)^2}{2} - \frac{(1)^3}{3}\right] \quad - A = \left[2(-2) - \frac{(-2)^2}{2} - \frac{(-2)^3}{3}\right]$$

$A = 2 - \frac{1}{2} - \frac{1}{3} + 4 + 2 - \frac{8}{3}$

$$\boxed{\boldsymbol{A = \frac{9}{2} sq.units}}$$

21. $y^2 = 4x \quad ; \quad y = 4x - 4$

$4x = y^2 2x = y + 4$

$x = \frac{y^2}{4} x = \frac{y+4}{2}$

$\frac{dx}{dy} = \frac{1}{4} 2y$

$0 = \frac{1}{4} 2y$

$0 = 0$

$(0,0)$

$\frac{d^2x}{dy^2} =$ *(concave to the right)*

$point\ of\ intersection$

$\frac{y^2}{4} = \frac{y+4}{2}$

$2y^2 - 4y + 4(4)$

$2y^2 - 4y - 16 = 0$

$(2y - 8)(y + 2)$

$2y - 8 = 0y + 2 = 0$

$y = 4; x = 4(1,2)$

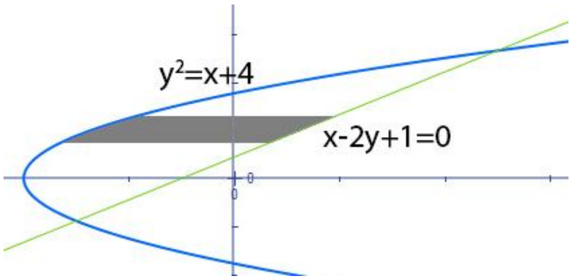
$(4,4)$

$dA = (x_2 - x_1)dy$

$A = \int_{-2}^{4} \left(\frac{y+4}{2} - \frac{y^2}{4}\right) dy$

$$\boxed{\boldsymbol{A = 9\ sq.units}}$$

23. $y^2 = x + 4\,, x - 2y + 1 = 0$



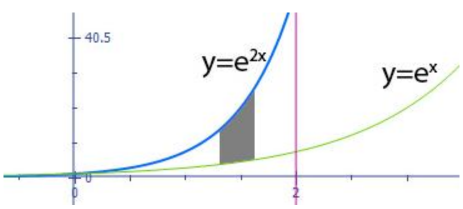
$A = \int_{-1}^{3} \left((2y - 1) - (y^2 - 4)\right) dy$

$= \int_{-1}^{3} (3 + 2y - y^2) dy$

$= \left[3y + y^2 - \frac{y^3}{3}\right]_{-1}^{3}$

$= \boxed{\mathbf{\frac{32}{3}}}$

25. $y = e^{2x}\,, y = e\,, x = 2$



$A = \int_{0}^{2} (e^{2x} - e^x) dx$

$= \left[\frac{e^{2x}}{2} - e^x\right]_0^2$

$= \frac{e^4}{2} - e^2 - \frac{1}{2} + 1$

$= \boxed{\mathbf{{}^1/_2\,(e^2 - 1)^2}}$

$\mathbf{1.}\ y = x^2 - 2x\ , x - axis, about\ the\ x - axis$

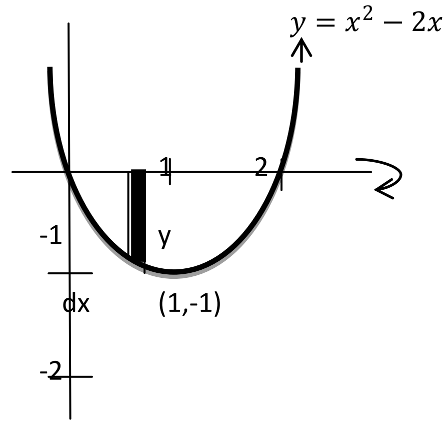
$\frac{dy}{dx} = 2x - 2\ , equate\ to\ zero$

$0 = 2x - 2 \quad ; \quad y = 1^2 - 2(1)$

$x = 1 \qquad ; \quad y = -1$

$\frac{d^{2y}}{dx^2} = 2$

$v(1, -1)$



x	0	1	2	3
y	0	-1	0	3

$dv = (\pi y^2)dx$

$dv = \pi(x^2 - 2x)^2 dx$

$\int dv = \pi \int (x^4 - 4x^3 + 4x^2)dx$

$v = \pi \left[\frac{x^5}{5} - \frac{\cancel{4}x^4}{\cancel{4}} + \frac{4x^3}{3}\right]$

$= \pi \left\{\left[\frac{1}{5}(2)^5 - 2^4 + \frac{4}{3}(2)^3\right] - [0]\right\}$

$= \pi \left[\frac{3^2}{5} - 16 + \frac{32}{3}\right]$

$= \pi \left[\frac{96 - 240 + 160}{15}\right]$

$= \pi \left(\frac{16}{15}\right)$

$$\boxed{\boldsymbol{V = \frac{16\pi}{15} units^3}}$$

3. $x + y = 5\ ; y = 0\ ; x = 0\ ; about\ y = 0$

$when\ x = 0\ ;\quad y = 5$

$when\ y = 0\ ;\quad x = 5$

$dv = \pi y^2 dx \quad ;\quad but\ y = 5 - x$

$y^2 = (5 - x)^2$

$dv = \pi(5 - x^2)dx$

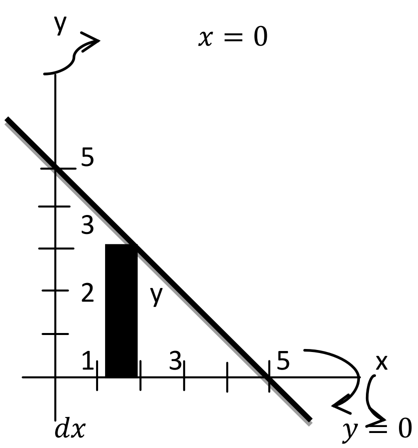
$$\int_0^v dv = \pi \int_0^5 (25 - 10x + x^2)\,dx$$

$$V = \pi \left[25x - \frac{10x^2}{2} + \frac{x^3}{3}\right]$$

$$V = \pi \left\{\left[\left(25(5) - 5(5)^2 + \frac{1}{3}(5)^3\right)\right] - [0]\right\}$$

$$V = \pi \left[125 - 125 + \frac{125}{3}\right] - 0$$

$$\boxed{\boldsymbol{V = \frac{125\pi}{3}\ units^3}}$$



5. $x + y = 6\ ; y = 3\ ; x = 0\ ; about\ y - axis$

$x = (6 - y)$

$dv = \pi x^2 dy$

$dv = \pi(6 - y)dy$

$dv = \pi(36 - 12y + y^2)dy$

$$\int_0^v dv = \int_0^3 (36 - 12y + y^2)dy$$

$$V = \pi \left[36y - 12\left(\frac{y^2}{2}\right) + \frac{y^3}{3}\right]$$

$$V = \left\{\left[(36)(3) - 6(3)^2 + \frac{1}{3}(3)^2\right] - [0]\right\}$$

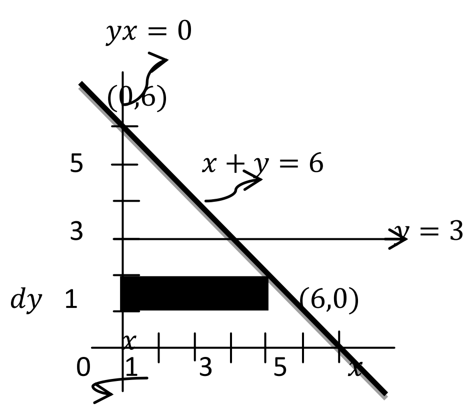
$$V = \pi \left[(36)(3) - 6(9) + \frac{1}{3(27)}\right]$$

$$V = \pi[(36)(3) - 6(9) + 9]$$

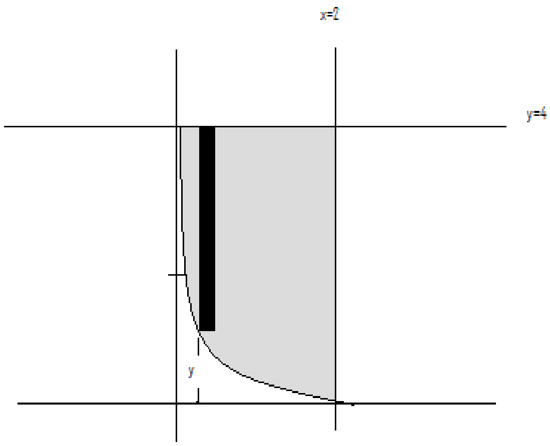
$$V = \pi(9)(12 - 6 + 1)$$

$$V = \pi(9)(7)$$

$$\boxed{\boldsymbol{V = 63\pi\ units^3}}$$



7. $xy = 4, x = 2, y = 4;\ about\ y = 4$



$V = \pi r^2 h$

$V = \pi(4-y)^2 dx$

$V = \pi\left(4-\frac{4}{x}\right)^2 dx$

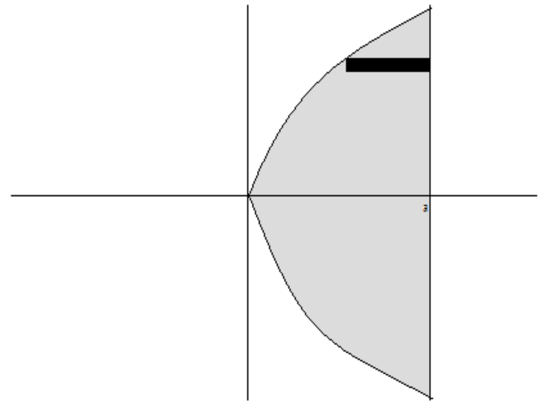
$\int_0^v dv = \pi \int_0^2 (16 - \frac{32}{x} + \frac{16}{x^2})\, dx$

$V = \pi\left[16(2) - 32ln2 - \frac{16}{2} - 0\right]$

$V = 8\pi(4 - 4\ln 2 - 1)$

$\boxed{\boldsymbol{V = 8\pi[3 - 4\ln 2] cu.units}}$

9. $y^2 = 4ax, x = a;\ about\ x = a$



$V = \pi r^2 h$

$V = \pi(a-x)^2 h$

$\int_0^v dv = \int_{-2a}^{2a} \pi(a - \frac{y^2}{4a})^2 dy$

$V = \pi \int_{-2a}^{2a} (a^2 - \frac{y^2}{2} + \frac{y^4}{16a^2})\, dy$

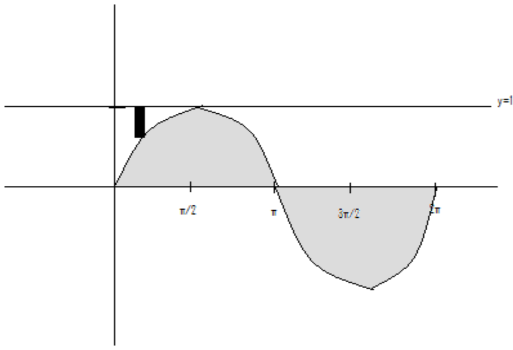
$V = \pi\left(a^2 y - \frac{y^3}{6} + \frac{y^5}{16(5)a^2}\right)$

$V = \left[a^2(2a) - \frac{2a^3}{6} + \frac{2a^5}{16(5)a^2}\right] - \left[a^2(-2a) - \frac{-2a^3}{16} + \frac{-2a^5}{16(5)a^2}\right]$

$V = 4a^3\pi\left[1 - \frac{2}{3} + \frac{1}{5}\right]$

$\boxed{\boldsymbol{V = \frac{32a^3\pi}{15} cu.units}}$

$\mathbf{11}. y = \sin x, x = 0, y = 1;\ about\ y = 1$



$V = \pi r^2 h$

$V = \pi(1-y)^2 dx$

$\int_0^v v = \int_0^{\frac{\pi}{2}} \pi\,(1-\sin x\,)^2 dx$

$V = \pi \int_0^{\frac{\pi}{2}} (1 - 2sinx + sin^2 x)dx$

$V = \pi[x + 2\cos x + \frac{x}{2} - \frac{sin2x}{4}]$

$V = \pi\left[\frac{3x}{2} + 2\cos x + \frac{x}{2} - \frac{\sin 2x}{4}\right]$

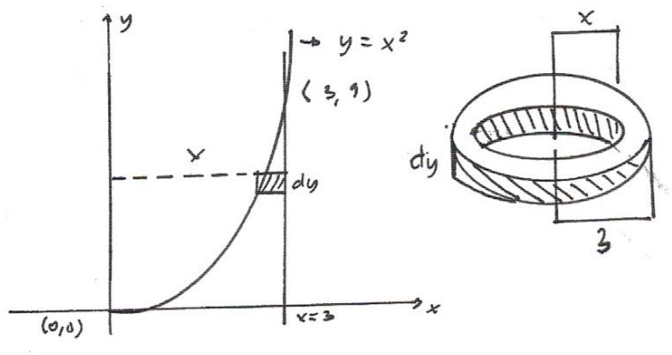
$V = \pi\left[\frac{3x}{2} + 2\cos x - \frac{\sin 2x}{4}\right]$

$V = \pi\left[\frac{3\pi}{4} + 0 - 4(0)\right] - [0 + 2 + 0]$

$V = \frac{3\pi^2}{4} - 2\pi$

$$\boxed{\boldsymbol{V = \frac{\pi}{4}(3\pi - 8) cu.units}}$$

1. $y = x^2, x = 3, y = 0; about\ the\ y - axis$



$$V = \pi \int_0^9 (3^2 - x^2)dy$$

$$V = \pi \int_0^9 (9 - y)dy$$

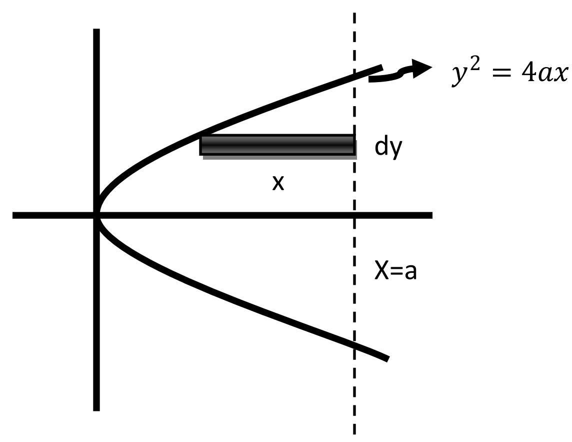
$$V = \pi \left[9y - \frac{y^2}{2}\right]_0^9$$

$$V = \pi \left[9(9) - \frac{(9)^2}{2}\right]_0^9$$

$$\boxed{\boldsymbol{V = \frac{81\pi}{2}\ CUBIC\ UNITS}}$$

3. $y^2 = 4ax, x = a; about\ the\ y - axis$

x	y
0	0
a	2a



$$V = \pi \int_{-2a}^{2a} (a^2 - x^2)dy$$

$$= \pi \int_{-2a}^{2a} (a^2 - \left(\frac{y^2}{4a}\right)^2)dy$$

$$= \pi \int_{-2a}^{2a} \left(a^2 - \frac{y^4}{16a^2}\right)dy$$

$$= \pi \left[a^2 y - \frac{y^5}{80a^2}\right]_{-2a}^{2a}$$

$$= \pi \left[(2a^3 - \frac{32a^5}{80a^2}) - (-2a^3 + \frac{32a^5}{80a^2})\right]$$

$$= \pi \left[(2a^3 - \frac{2a^3}{5}) - (2a^3 + \frac{2a^3}{5})\right]$$

$$\boxed{\boldsymbol{V = \frac{16\pi a^3}{5}}}$$

5. $x^2+y^2 = a^2, x = b$

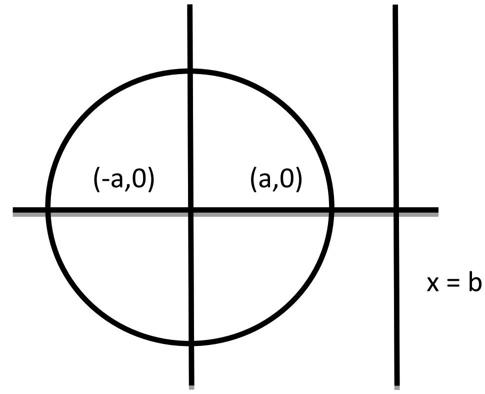
$V = 4\pi \int_0^a (a^2 - y^2 + b)dy$

$V = 4\pi \left[a^2y - \frac{y^3}{3} + by\right]$ a

$V = 4\pi \left[a^3 - \frac{a^3}{3} - ab\right]$ o

$V = 4\pi \left[\frac{2a^3}{3} - ab\right]$

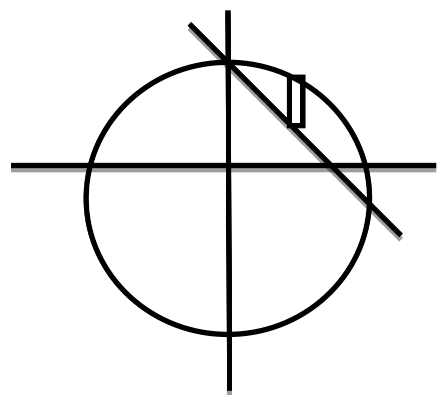
$$\boxed{\boldsymbol{V = \frac{8\pi a^3}{3}}}$$



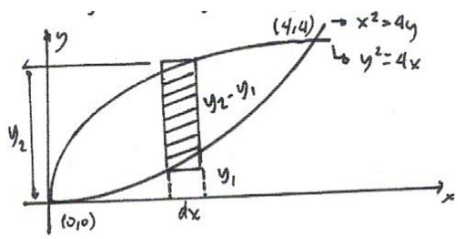
7. $x^2 + y^2 = 25\ , x + y = 5\ ; y = 0$

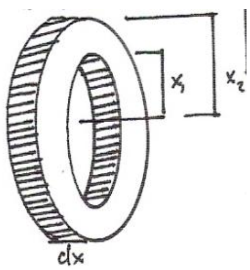
$V = \pi \int_0^5 [(25 - x^2) - (5 - x)^2]\, dx$

$$\boxed{\boldsymbol{V = \frac{125\pi}{3}\ cu.units}}$$



9. $y^2 = 4x,\ x^2 = 4y; about\ the\ x - axis$





$y_2 = y_1$

$\left(\sqrt{4x} = \frac{x^2}{4}\right)^2$

$64x = x^3$

$x = 4, y = 4: POI\ (4,4)$

$$V = \pi \int_0^4 \left[\left(\sqrt{4x}\right)^2 - \left(\frac{x^2}{4}\right)^2\right] dx$$

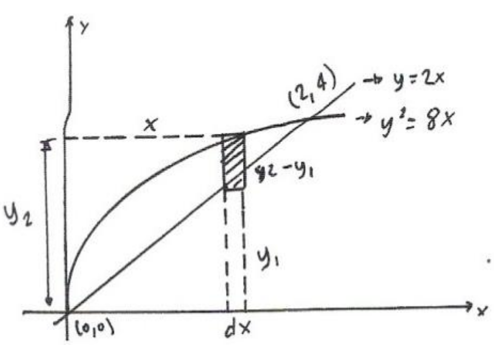
$$V = \pi \int_0^4 \left(4x - \frac{x^4}{16}\right) dx$$

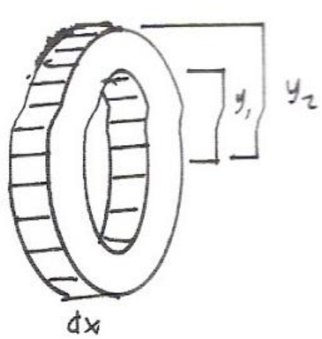
$$V = \pi \left[2x^2 - \frac{x^5}{80}\right]_0^4$$

$$V = \pi \left(2(4)^2 + \frac{(4)^5}{80}\right)$$

$$\boxed{\boldsymbol{V = \frac{96\pi}{5}\ CUBIC\ UNITS}}$$

11. $y^2 = 8x,\ Y = 2x; about\ y = 4$





$y_2 = y_1$

$\left(\sqrt{8x} = 2x\right)^2$

$8x = 4x^2$

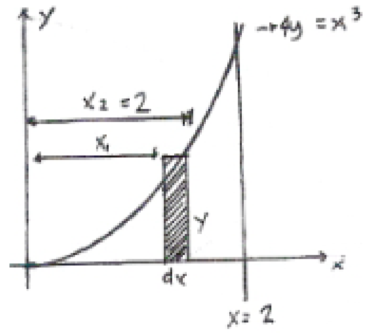
$x = 2, y = 4: POI\ (2,4)$

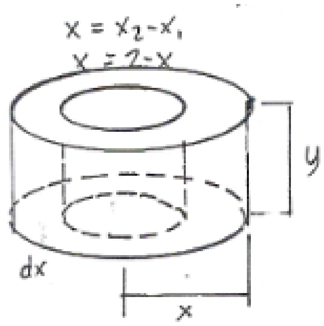
$$V = \pi \left[4x^2 - \frac{4x^3}{3}\right]_0^2$$

$$V = \pi \left(4(2)^2 + \frac{4(2)^3}{3}\right)$$

$$\boxed{\boldsymbol{V = \frac{16\pi}{3}\ CUBIC\ UNITS}}$$

1. $4y = x^3, y = 0, x = 2,; \; about \; x = 2$





$V = 2\pi \int_0^2 xydx$

$V = 2\pi \int_0^2 (2 - x)\frac{x^3}{4}\, dx$

$V = 2\pi \int_0^2 [\frac{x^2}{2} - \frac{x^4}{4}]dx$

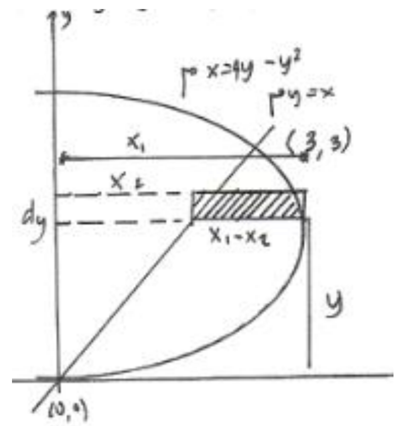
$V = 2\pi[\left[\frac{x^4}{4} - \frac{x^5}{20}\right]\begin{matrix}2\\0\end{matrix}$

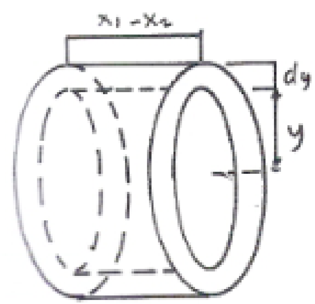
$V = 2\pi \left[\frac{(2)^4}{4} - \frac{(2)^5}{20}\right]\begin{matrix}2\\0\end{matrix}$

$V = 2\pi\left[\frac{3}{5}\right]$

$\boxed{V = \frac{4\pi}{5} \text{ cubic units}}$

3. $x = 4y - y^2, y = x, about \; y = 0$





$V = 2\pi\int_0^3 xydy$

$V = 2\pi\int_0^3[(4y - y^2) - y]ydy$

$V = 2\pi\int_0^3[4y^2 - y^3 - y^2]dy$

$V = 2\pi \left[\frac{4}{3}y^2 - \frac{1}{4}y^4 - \frac{1}{3}y^3\right]\begin{matrix}3\\0\end{matrix}$

$V = 2\pi\left[y^3 - \frac{y^4}{4}\right]\begin{matrix}3\\0\end{matrix}$

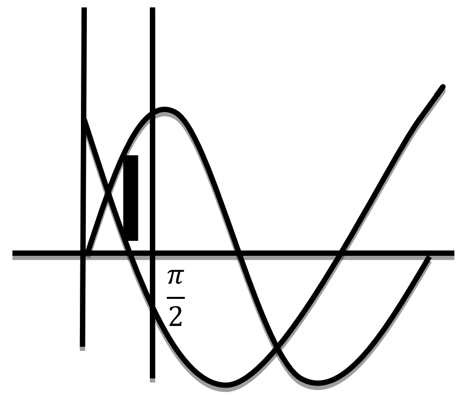
$V = 2\pi\left[(3)^3 - \frac{(3)^4}{4}\right]\begin{matrix}3\\0\end{matrix}$

$\boxed{V = \frac{27\pi}{2} \text{ cubic units}}$

5. $y = sinx, y = cosx, x = \frac{\pi}{2}$

$V = 2\pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x(sinx - cosx)dx$

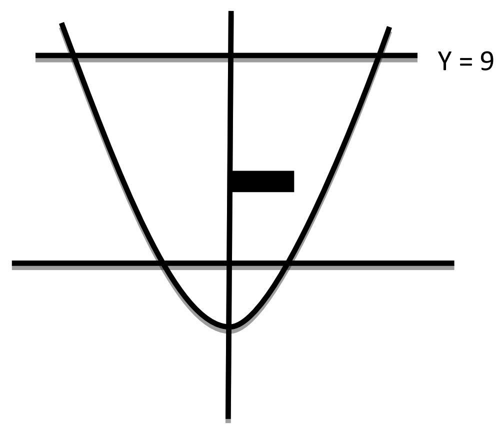
$$\boxed{V = \frac{\pi}{2}(4 + \sqrt{2}\pi - 2\pi) cu.units}$$



7. $x = 2\sqrt{y} \quad , x = 0 \ , y = 0$

$V = 2\pi \int_0^9 (9 - y)(2\sqrt{y})dy$

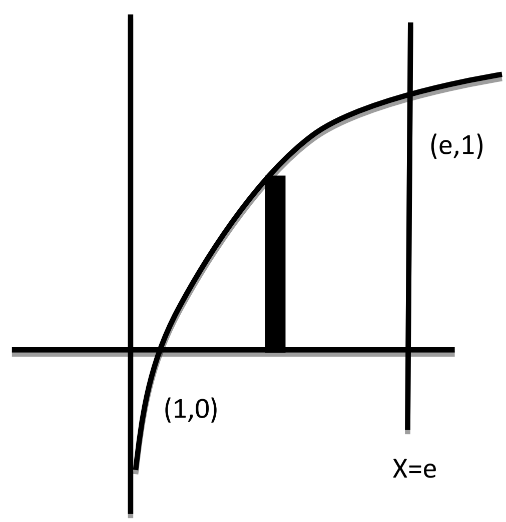
$$\boxed{V = \frac{1296\pi}{5} \ cu.units}$$



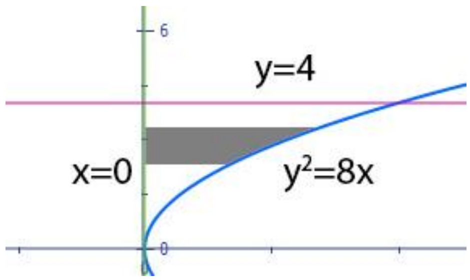
9. $y = lnx , x = e , y = 0$

$V = 2\pi \int_1^e x(lnx)dx$

$$\boxed{V = 13.77 \ cu.units}$$



11. $y^2 = 8x$, $x = 0$, $y = 4$; about $y = 4$



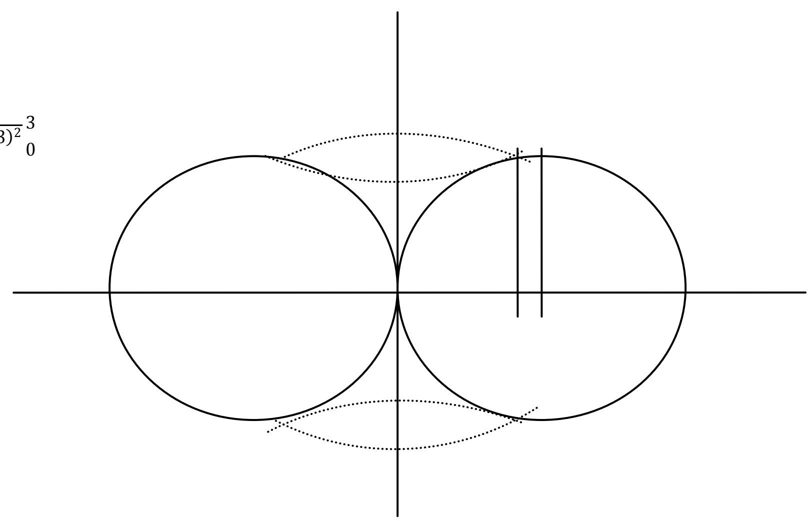
$$V = 2\pi \int_0^4 (4-y)\left(\frac{y^2}{8}\right) dy$$

$$= \frac{\pi}{4}\int_0^4 (4y^2 - y^3)dy$$

$$= \frac{\pi}{4}\left[\frac{4y^3}{3} - \frac{y^4}{4}\right]_0^4$$

$$= \boxed{\frac{\mathbf{16\pi}}{\mathbf{3}}}$$

13. $(x - 3)^2 + y^2 = 9$; *about the* y – *axis.*



$$V = 8\pi \int_0^3 \mathrm{x}\sqrt{9-(x-3)^2}\ \mathrm{dx}$$

$$V = 8\pi\left(\frac{-(9-(\mathrm{x}-3)2}{3}\right)^{\frac{3}{2}} + \frac{27}{2}\frac{sinx - 3}{3} + \frac{9}{2}(x-3)(\sqrt{9-(x-3)^2}\Big|_0^3$$

$$V = 8\pi(27sin\theta - \frac{27}{2}sin - 1)$$

$$V = 8\pi(\frac{27}{2})(-sin - 1 + sin\theta)$$

$$V = 108\pi(\frac{\pi}{2})$$

$$\boxed{\boldsymbol{V = 54\pi^2}}$$

15. $x^2 + y^2 = a^2$; $about\ x = b\ (b > a)$

$$V = \pi \int_{-a}^{a} (b - x)^2 - (b - x)^2\, dy$$

$$V = \pi \int_{-a}^{a} [(b^2 - bx + x^2) - (b^2 - bx + x^2)]\, dy$$

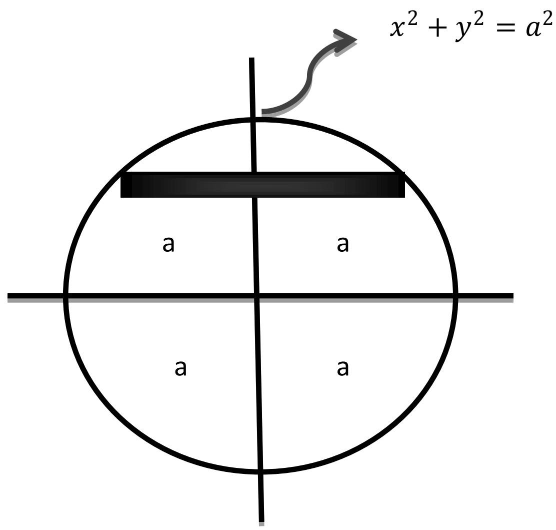
$$V = \pi \int_{-a}^{a} 4bxdy$$

$note{:}\ x^2 + y^2 = a^2 \quad = x = \sqrt{y^2 - a^2}$

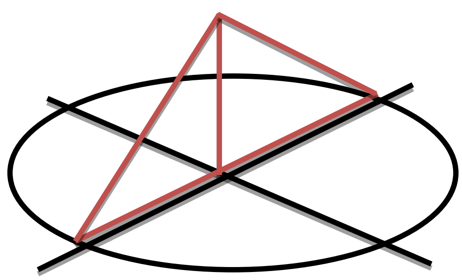
$$V = 4b\pi \int_{-a}^{a} \sqrt{y^2 - a^2}dy$$

$$V = 4b\pi \left[\frac{y}{2}\sqrt{y^2 - a^2} - \frac{a^2}{2}\ln\left|y + \sqrt{y^2 - a^2}\right| + c\right]_{-a}^{a}$$

$$\boxed{\boldsymbol{V = 2\pi^2 a^2 b}}$$



1. $x^2 + y^2 = 36$



$A(x) = \frac{S^2}{2}$, $S = 2y$

$A(x) = 2y^2$, $y = \sqrt{36 - x^2}$

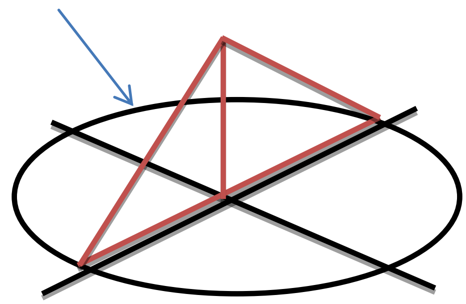
$v = \int_{-6}^{6} A(x)dx$

$v = \int_{-6}^{6} 2x^2 dx$

$v = \int_{-6}^{6} 2(3x - x^2)dx$

$\boxed{\boldsymbol{v = 576\ cu.units}}$

3. $9x^2 + 16y^2 = 144$



$A(x) = \frac{1}{2}bh$

$A(x) = \frac{1}{2}(2y)(y)$

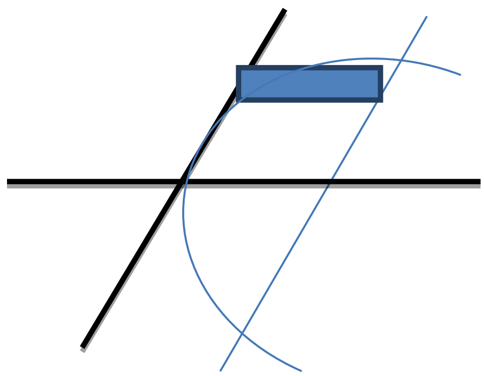
$A(x) = y^2$

$V = 2\int_{0}^{8} y^2\, dx$

$V = 2\int_0^8 \left(\frac{144-9x^2}{16}\right) dx$

$\boxed{\boldsymbol{V = 48\ cu.units}}$

5.



$A(y) = (1 - x)(2y^2)$

$$V = 2\int_0^2 (1 - x)2y^2 dy$$

$$V = 2\int_0^2 (1 - \frac{y^2}{4})y^2 dy$$

$$V = \frac{64}{15}$$

$$\boxed{\boldsymbol{V = 4.2667\ cu.units}}$$

1. $y = x^{\frac{3}{2}}$ *from* $x = 0$ *to* $x = 5$

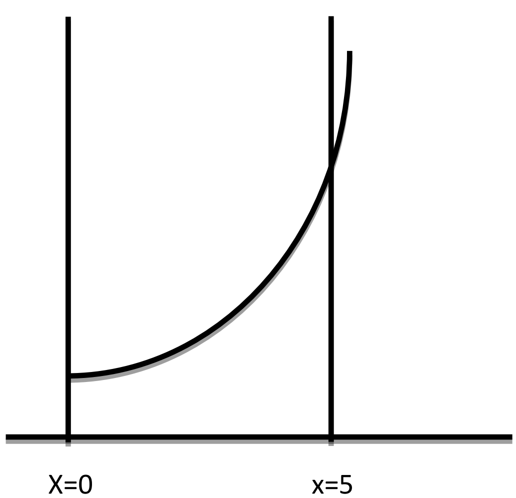
$$y = x^{\frac{3}{2}}$$
$$dy = \frac{3}{2}x^{\frac{1}{2}}dx$$
$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$$

$$s = \int_0^5 \sqrt{1 + (\frac{dy}{dx})^2}\ dx$$

$$= \int_0^5 \sqrt{1 + (\frac{3}{2}x^{\frac{1}{2}})^2}\text{dx}$$

$$= \int_0^5 \sqrt{1 + \frac{9}{4}x\ dx}$$

$$\boxed{\boldsymbol{s = 12.407\ units}}$$



3. *the entire hypocycloid* $x^{\frac{2}{3}} + y^{\frac{2}{3}} + a^{\frac{2}{3}}$

$$S = \int_0^9 \sqrt{1 + \left(-\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}\right)^2}\ dx$$

$$S = \int_0^9 \sqrt{\frac{x^{\frac{2}{3}} + y^{\frac{1}{3}}}{x^{\frac{2}{3}}}}dx$$

$$Note: a^{\frac{2}{3}} = x^{\frac{2}{3}} + y^{\frac{2}{3}}$$

$$S = \int_0^9 \sqrt{\frac{a^{\frac{2}{3}}}{x^{\frac{2}{3}}}}dx$$

$$S = \int_0^9 \frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}}dx$$

$$S = a^{\frac{1}{3}}\left[\frac{3x^{\frac{2}{3}}}{2}\right]_0^9$$

$$S = \frac{3a}{2}$$

$$S = 4\left(\frac{3a}{2}\right)$$

$$\boxed{\boldsymbol{S = 6a}}$$

5. $y = Arcsine^x$, $from\ y = \frac{\pi}{6}$ $to\ y = \frac{\pi}{2}$

$y = Arcsine^x$; $\ln siny = x$

$$\frac{1}{siny} cosydy = dx$$

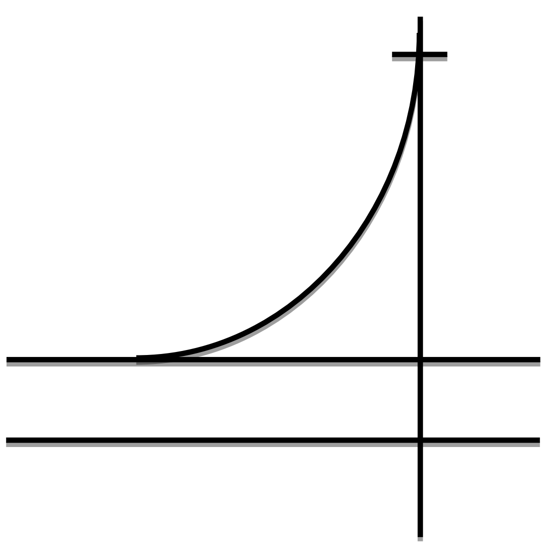
$$\frac{dx}{dy} = \frac{cosy}{siny}$$

$$\frac{dx}{dy} = coty$$

$$S = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{1 + cot^2 y}\ dy$$

$$\boxed{\boldsymbol{S = 1.31696\ units}}$$



7. $one\ area\ of\ the\ cycloid\ x = a(\theta - sin\theta),$
$y = a(1 - cos\theta)$

$x = a(\theta - sin\theta)$ $\qquad$ $y = a(1 - cos\theta)$
$dx = a(d\theta - cos\theta d\theta)$ $\qquad$ $dy = a(sin\theta d\theta)$
$\frac{dx}{d\theta} = a(1 - cos\theta)$ $\qquad$ $\frac{dy}{d\theta} = asin\theta$

$$s = \int_0^{2\pi} \sqrt{a^2(1 - cos\theta)^2 + a^2 \sin^2\theta}$$

$$s = a \int_0^{2\pi} \sqrt{(1 - cos\theta)^2 + \sin^2\theta}$$

$$\boxed{\boldsymbol{s = 8a}}$$

9. $The\ Cardioid\ \ r = 2(1 - cos\theta)$

$r = 2(1 - cos\theta)$
$dr = 2(sin\theta)d\theta$
$\frac{dr}{d\theta} = 2sin\theta$
$r^2 = 4(1 - cos\theta)^2$

$$S = \int_0^{2\pi} \sqrt{4(1 - cos\theta)^2 + 4sin^2\theta}\, d\theta$$

$$S = 2 \int_0^{2\pi} \sqrt{(1 - cos\theta)^2 + sin^2\theta}\, d\theta$$

$$\boxed{\boldsymbol{S = 16\ units}}$$

1. $x^2 + y^2 = 16$; $from\ x = 2\ to\ x = 4$

$$S = 2\pi \int_2^4 yds$$

$$y = \sqrt{16 - x^2}$$

$$\frac{dy}{dx} = \frac{1}{2}(16 - x^2)^{-\frac{1}{2}}(-2x)$$

$$\frac{dy}{dx} = -\frac{x}{\sqrt{16 - x^2}}$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$ds = \sqrt{1 + \frac{x^2}{16 - x^2}}dx$$

$$ds = \sqrt{\frac{16 - x^2 + x^2}{16 - x^2}}dx$$

$$ds = \frac{4}{\sqrt{16 - x^2}}dx$$

$$S = 2\pi \int_2^4 \sqrt{16 - x^2}\frac{4}{\sqrt{16 - x^2}}dx$$

$$S = 2\pi \int_2^4 4dx$$

$$\boxed{\boldsymbol{S = 16\pi\ sq.units}}$$

3. $y^2 = 12x$; $from\ x = 0\ to\ x = 3$

$$S = 2\pi \int_0^3 yds$$

$$y = \sqrt{12x}$$

$$\frac{dy}{dx} = \frac{1}{2}(12x)^{\frac{1}{2}}(12)$$

$$\frac{dy}{dx} = \frac{6}{\sqrt{12x}}$$

$$ds = \sqrt{1 + \frac{36}{12x}}dx$$

$$ds = \frac{\sqrt{12x + 36}}{\sqrt{12x}}dx$$

$$ds = \frac{2\sqrt{3x + 9}}{\sqrt{12x}}dx$$

$$S = 2\pi \int_0^3 \sqrt{12x}\left(\frac{2\sqrt{3x + 9}}{\sqrt{12x}}\right)dx$$

$$S = 4\pi \int_0^3 \sqrt{3x + 9}\,dx$$

$$\boxed{\boldsymbol{S = 137.860\ sq.units}}$$

5. $y = x^3$; $from\ x = 0\ to\ x = 1$

$$\frac{dy}{dx} = 3x^2$$

$$ds = \sqrt{1 + 9x^4}dx$$

$$S = 2\pi \int_0^1 x^3\sqrt{1 + 9x^4}dx$$

$$\boxed{\boldsymbol{S = 3.5631\ sq.units}}$$

7. $x = cos2y$; $from\ y = 0\ to\ y = \frac{\pi}{4}$

$S = 2\pi \int_0^{\frac{\pi}{4}} xds$

$\frac{dx}{dy} = -\sin 2y(2)$

$ds = \sqrt{1 + 4\sin^2 2y}$

$S = 2\pi \int_0^{\frac{\pi}{4}} \cos 2y\, \sqrt{1 + 4\sin^2 2y}\, dy$

$\boxed{\boldsymbol{S = 4.93665\ sq.units}}$

9. $4 - x^2$ $from\ x = 0\ to\ x = 2$

$S = 2\pi \int_0^2 xds$

$\frac{dy}{dx} = -2x$

$ds = \sqrt{1 + 4x\hat{}2}\ dx$

$S = 2\pi \int_0^2 x\sqrt{1 + 4x^2}dx$

$\boxed{\boldsymbol{S = 36.1769\ sq.units}}$

13. $y = mx\ ;\ x = 0\ ;\ x = 1\ ;\ about\ the\ x -$
$axis$

$S\ =\ 2\pi\ \int_0^1 \sqrt{mx \quad 1\ +\ m2 \quad dx}$

$S\ =\ 2\pi\, m\ \sqrt{1\ +\ m2} = \int_0^1 x\ dx$

$S\ =\ 2\pi\, m\ \left[\sqrt{1\ +\ m2}(\,x2/2\,)\right]_0^1$

$S\ =\ 2\pi\, m\ \sqrt{1\ +\ m2}(\,½\,)$

$\boxed{\boldsymbol{S\ =\ \pi\, m \sqrt{1\ +\ m2}}}$

EXERCISE 13.1 | FORCE OF FLUID PRESSURE

1. $F = wA\overline{x}$

$= (62.5lb/ft^3)(96ft^2)(4ft)$

$= 24000lb$

$$P = \frac{F}{A}$$

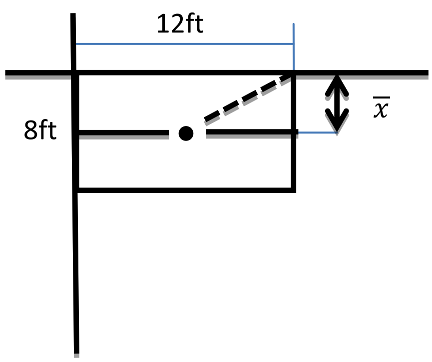
$$P = \frac{wA\overline{x}}{A}$$

$$P = w\overline{x}$$

$$P = (\frac{62.5lb^3}{ft})(4ft)(\frac{1ft^2}{144in^2})$$

$$P = \frac{(625)(4)}{144}$$

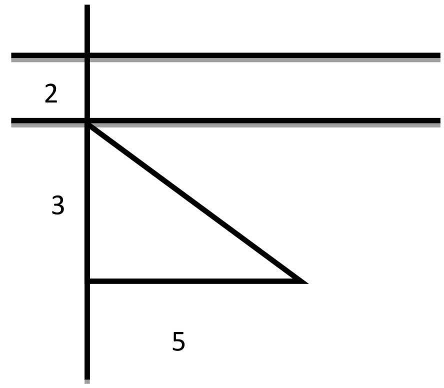
$$\boxed{\boldsymbol{P = 1.74\ psi}}$$



3. $F = wA\overline{x}$

$$F = w\left[\frac{1}{2}(5)(3)\left(2 + (\frac{2}{3})(3)\right)\right]$$

$$\boxed{\boldsymbol{F = 30w\ lb}}$$



5. $F = 50w$

$base = 3ft$

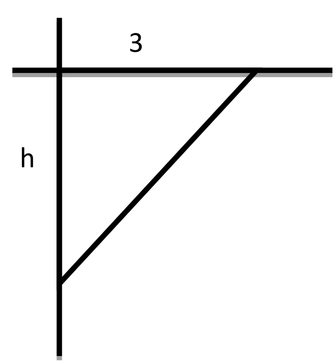
$F = wA\overline{x}$

$$50 = \frac{1}{2}(h)(3)\left(\frac{1}{3}h\right)$$

$$50 = \frac{h^2}{2}$$

$$100 = h^2$$

$$\boxed{\boldsymbol{h = 10ft}}$$

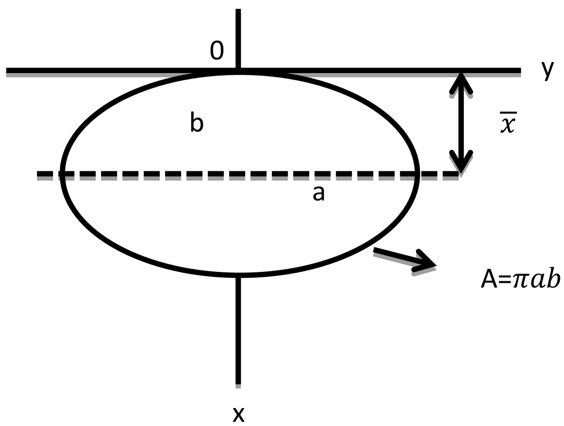


7. $F = wA\overline{x}$

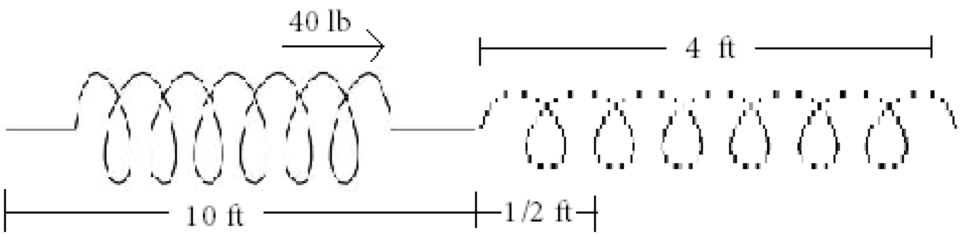
$= w[(\pi)(3)(2)](2)$

$\boxed{\boldsymbol{F = 12\pi\, w}}$

$b = 6$ $=$ major axis
$a = 4$ $= minor\ axis$



1.



$w = \int_a^b f(x)dx$

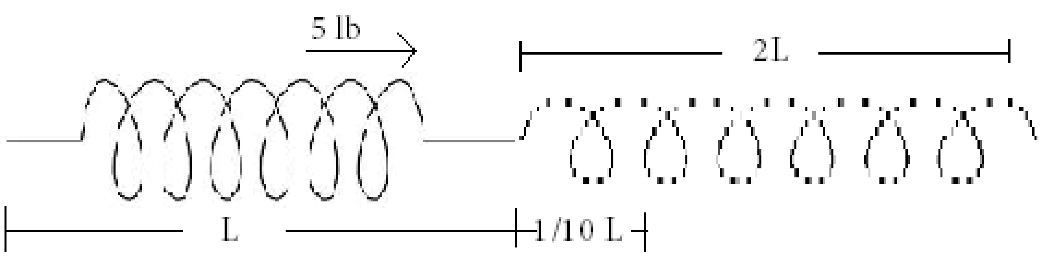
$f(x) = kx$; $where\ x = \frac{1}{2}\ ft,$ $f(x) = 40\ lb$; $a = 0,$ $b = 14 - 10 = 4$

$40\ lb = k\left(\frac{1}{2}\ ft\right), k = 80$

$w = \int_0^4 80x dx$

$\boxed{\boldsymbol{w = 640\ lb - ft}}$

3.



$w = \int_a^b f(x)dx$

$f(x) = kx\ ; where\ x = \frac{1}{10}\ L\ ft,$ $f(x) = 5\ lb$ $a = 0, b = L$

$w = \int_0^L \frac{50}{L} x dx$

$\boxed{\boldsymbol{w = 25L\ ft - lb}}$

5.

$W = FS$

$dw = (w)dv(60 - x)$

$dw = \pi r^2 w(60 - x)dx$

$dw = 9\pi w(60 - x)dx$

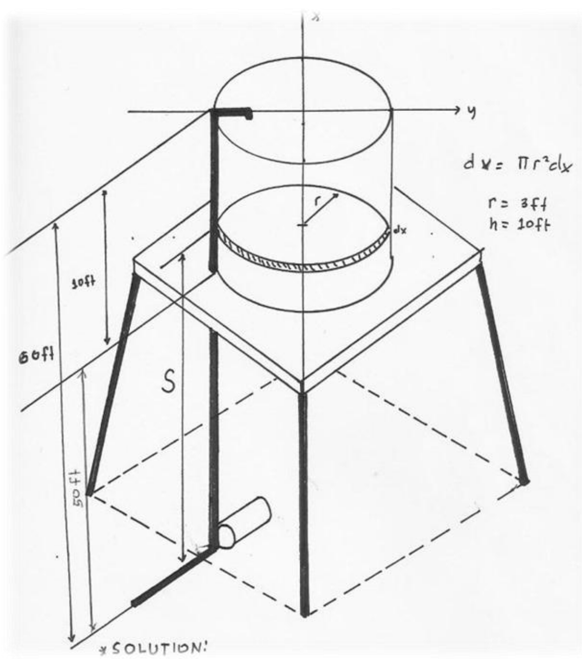
$$\int_0^w dw = 9\pi w \int_0^{10} (60 - x)dx$$

$$w = 9\pi w[60x - x^2]\begin{matrix}10\\0\end{matrix}$$

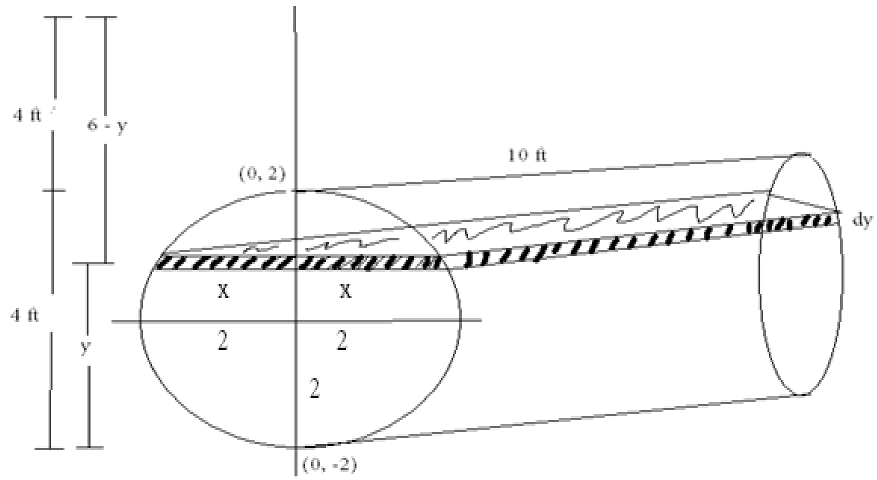
$$w = 9\pi w\left[60x - \frac{x^2}{2}\right]\begin{matrix}10\\0\end{matrix}$$

$w = 9\pi w[600 - 50]$

$$\boxed{\boldsymbol{w = 4950w\pi\ ft.lb}}$$



9.



$$w = w\int_a^b hdV$$

$$V_{rectangle} = l\ x\ w\ x\ h; where\ l = 10\ ft, w = 2x, h = dy$$

$$x^2 + y^2 = r^2\ ; x = \sqrt{r^2 - y^2}\ ; where\ r = 2$$

$$w = \pi\int_{-2}^{2}(6 - y)\,(10\ ft)(2x)dy$$

$$w = 20\pi\int_{-2}^{2}(6 - y)\left(\sqrt{2^2 - y^2}\right)dy$$

$$\boxed{\boldsymbol{w = 240\pi w\ ft - lb}}$$

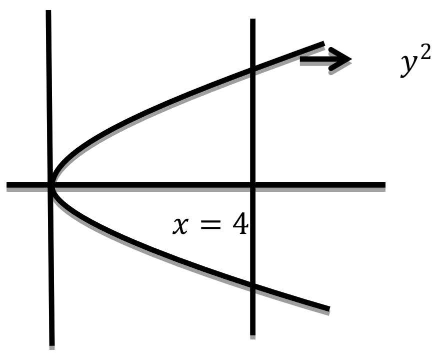
1. $y^2 = 4x, the\ x - axis\ and\ x = 4$

$M_x = \frac{1}{2}\int_0^4 4x dx$

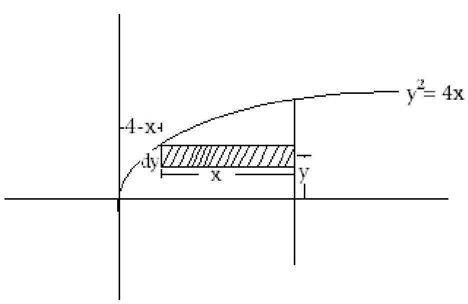
$\boxed{\boldsymbol{M_x = 16}}$

$M_y = \int_0^4 x\sqrt{4x}\, dx$

$\boxed{\boldsymbol{M_y = 25.6}}$



3. $x = 4$



$$M_\lambda = \int_a^b l dA$$

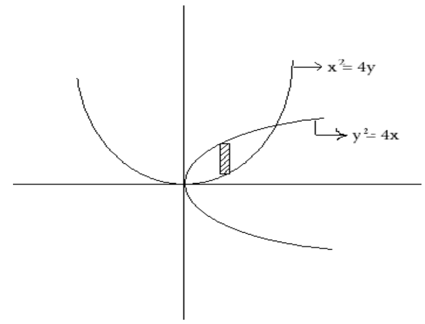
$$M_\lambda = \int_o^4 x(4 - x)\, dy$$

$$M_\lambda = \int_o^4 (4x - x^2)\, dy$$

$$M_\lambda = \int_o^4 \left(y^2 - \frac{y^4}{16}\right) dy$$

$$\boxed{\boldsymbol{M_\lambda = \frac{256}{15}}}$$

5. $y^2 = 4x\ and\ x^2 = 4y$



$$4x = \frac{x^4}{16}$$

$$64x - x^4 = 0$$

$$x(64 - x^3) = 0$$

$$x_1 = 0, x_2 = 4$$

$$M_x = \frac{1}{2}\int_o^4 \left(4x - \frac{x^4}{16}\right) dy$$

$$\boxed{\boldsymbol{M_x = \frac{48}{5}}}$$

7. $M = \int_0^3 (3-y)[\left(\sqrt{9-y^2} - (3-y)\right]dy$

$= \int_0^3 (3-y)\sqrt{(3+y)(3-y)} - (3-y)^2 dy$

$= \int_0^3 (3-y)^{\frac{3}{2}}(3+y)^{\frac{1}{2}} - (3-y)^2 dy$

$= \int_0^3 3\sqrt{9-y^2}dy - \int_0^3 y\sqrt{9-y^2}dy - \int_0^3 (3-y)^2 dy$

$* \; A = 3\int_0^3 \sqrt{9-y^2}\, dy$

$cos\theta = \frac{\sqrt{9-y^2}}{3}$ $\qquad sin\theta = \frac{y}{3}$

$3cos\theta = \sqrt{9-y^2}$ $\qquad 3sin\theta = y;\; \theta = arcsin\frac{y}{3}$

$3cos\theta d\theta = dy$

$y = 3; \theta = \frac{\pi}{2}$

$y = 0;\; \theta = 0$

$= 3\int_0^{\frac{\pi}{2}} 3cos\theta cos\theta d\theta$

$= 27\int_0^{\frac{\pi}{2}} cos^2\theta d\theta = 27\int_0^{\frac{\pi}{2}} \left(\frac{1+cos2\theta}{2}\right) d\theta$

$= 27\left[\frac{\theta}{2} + \frac{sin2\theta}{4}\right]_0^{\frac{\pi}{2}} = 27\left(\frac{\pi}{4}\right) = \frac{27\pi}{4}$

$* \; B = -\int_0^3 y\sqrt{9-y^2}dy$

$u = 9 - y^2$ $\qquad @\, y = 3; u = 0$

$du = -2ydy$ $\qquad y = 0; u = 9$

$-\frac{du}{2} = ydy$

$= \frac{1}{2}\left(\frac{9-y^2}{\frac{3}{2}}\right)|_0^3$

$= \frac{1}{2}\left(\frac{2}{3}\right)[(9-9)^{\frac{3}{2}} - (9-0)^{\frac{3}{2}}]$

$= \frac{1}{3}(-27) = -9$

$* \; C = -\int_0^3 (3-y)^2\, dy$

$u = 3 - y$

$du = -dy$

$= \frac{(3-y)^3}{3}\;|_0^3$

$= \frac{0}{3} - \frac{3^3}{3}$

$= -9$

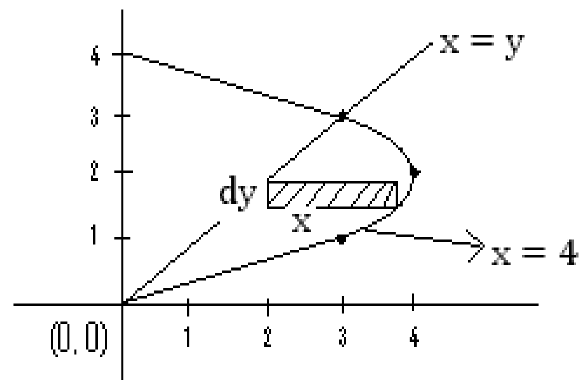
$M = \frac{27\pi}{4} - 9 - 9$

$M = \frac{27\pi}{4} - 18$

$M = \frac{27\pi - 72}{4}$

$\boxed{\boldsymbol{M = \frac{9}{4}[3\pi - 8]}}$

9. $x = 4y - y^2\,, y = x$



$M_y = \frac{1}{2}\int_c^d (x_r^2 - x_l^2)dy$

$M_y = \frac{1}{2}\int_0^3 [(4y-y^2)^2 - y^2]dy$

$\boxed{\boldsymbol{M_y = \frac{54}{5}}}$

1. $x + 2y = 6, x = 0, y = 0$

Solving for A

$dA = ydx$

$\int_0^6 dA = \int_0^6 \left(3 - \frac{x}{2}\right) dx$

$A = [3x - \frac{x^2}{4}]_0^6$

$A = \left[3(6) - \frac{36}{4}\right]$

$\boldsymbol{A = 9\ sq.units}$

Solving for $\bar{x}$

$A\bar{x} = \int_0^6 Xc\ dA$

$A\bar{x} = \int_0^6 x\left(3 - \frac{x^2}{2}\right) dx$

$A\bar{x} = \int_0^6 \left(3x - \frac{x^2}{2}\right) dx$

$A\bar{x} = [\frac{3x^2}{2} - \frac{x^3}{3}]_0^6$

$9\bar{x} = 18$

$\boldsymbol{\bar{x} = 2\ units}$

Solving for $\bar{y}$

$A\bar{y} = \int_0^6 Yc\ dA$

$A\bar{y} = \frac{1}{2}\int_0^6 (3 - \frac{x}{2})\ (3 - \frac{x}{2}) dx$

$A\bar{y} = \frac{1}{2}\int_0^6 (9 - 3x + \frac{x^2}{4}) dx$

$A\bar{y} = \frac{1}{2}[9x - \frac{3}{2}x^2 + \frac{x^3}{12}]_0^6$

$\bar{y} = \frac{1}{3}(3)$

$\boldsymbol{\bar{y} = 1\ unit}$

$\boxed{\textbf{\textit{Centroid: (2,1)}}}$

3. $y = sinx,\ \ y = 0\ from\ \ x = 0 - \pi$

$$A{=}\int ydx$$

$$=\int_0^{\pi} sinxdx$$

$$=[-cosx]$$

$$A{=}2$$

$Mx = \int ycda; yc = \frac{y}{2}$

$= \int_0^{\pi}(\frac{y}{2})ydx$

$= \frac{1}{2}\int_0^{\pi} y^2dx$

$= \frac{1}{2}\int_0^{\pi}(sin^2x)dx$

$= \frac{1}{2}\int_0^{\pi}(\frac{1-cos\,2x}{2})\ dx$

$= \frac{1}{2}(\frac{x}{2} - (2)\left(\frac{sin\,2x}{2}\right) dx$

$= \frac{1}{2}(\frac{x}{2} - \frac{sin\,2x}{4})$

$Mx = \frac{\pi}{4}(2)$

$\bar{x}\ = \frac{\pi}{2}$

$My = \int_0^{\pi} xcda; xc = x$

$=\ \int_0^{\pi} xydA$

$= \int_0^{\pi} xsinxdx$

$u = x\ ;\ dv = sinx$

$du = dx\ \ ;\ \ v = -cosx$

$= -cosx\ - \int -cosxdx$

$=\ [-xcosx\ + sinx]$

$= -\pi cos\pi + sin\pi + 0 - sin0$

$= \pi$

$\bar{y} = (\frac{\pi}{4})(2)$

$= \frac{\pi}{8}$

Centroid: $\boldsymbol{\left(\frac{\pi}{2}, \frac{\pi}{8}\right)}$

7. $y^2 = x^3, y = 2x$

$$A = \int_0^4 (2x - x^{\frac{3}{2}})dx$$

$$A = [x^2 - \frac{2}{5}x^{\frac{5}{2}}]$$

$$A = [16 - \frac{64}{5}]$$

$$A = \frac{16}{5} sq. units$$

$$A\bar{x} = \int_0^4 yxdx$$

$$A\bar{x} = \int_0^4 (2x - x^{\frac{3}{2}})xdx$$

$$A\bar{x} = \int_0^4 (2x^2 - x^{\frac{5}{2}})dx$$

$$A\bar{x} = [\frac{2}{3}x^3 - \frac{2}{7}x^{\frac{7}{2}}]$$

$$\bar{x} = \frac{5}{16}[\frac{2}{3}(4)^3 - \frac{2}{7}(4)^{\frac{7}{2}}$$

$$\bar{x} = \frac{5}{16}[\frac{128}{3} - \frac{257}{7}]$$

$$\bar{x} = \frac{40}{21} units$$

$$A\bar{y} = \frac{1}{2}\int_0^4 y^2\, dx$$

$$A\bar{y} = \frac{1}{2}\int_0^4 [(2x)^2 - x^{\frac{5}{2}}]dx$$

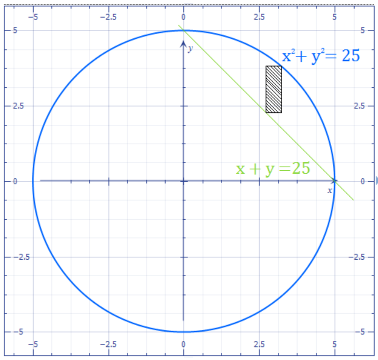
$$A\bar{y} = \frac{1}{2}\int_0^4 (4x^2 - x^3)\, dx$$

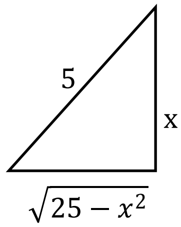
$$A\bar{y} = \frac{1}{2}\left[\frac{4}{3}x^3 - \frac{x^4}{4}\right]_0^4$$

$$\bar{y} = \frac{10}{3} units$$

$$\boxed{\boldsymbol{Centroid}: \left(\frac{\mathbf{40}}{\mathbf{21}}, \frac{\mathbf{10}}{\mathbf{3}}\right)}$$

9. $x^2 + y^2 = 25, \quad x + y = 5$





$$A = \int_0^5 \left[\left(\sqrt{25 - x^2}\right) - (5x)\right] dx$$

$$A = \int_0^5 \sqrt{25 - x^2}dx \;-\; 5\int_0^5 dx + \int_0^5 xdx$$

A B C

A: $\int_0^5 \sqrt{25 - x^2}dx$

$$5\cos\theta = \sqrt{25 - x^2}$$

$$5\sin\theta = x\,; \quad \theta = \arcsin\frac{x}{5}$$

$$5\cos\theta = dx \qquad @x = 5\,;\ \theta = \frac{\pi}{2}$$

$$x = 0\,; \quad \theta = 0$$

$$= \int_0^{\frac{\pi}{2}} 5\cos\theta \cdot 5\cos\theta$$

$$= 25\int_0^{\frac{\pi}{2}} \cos^2\theta d\theta \;\rightarrow \cos^2\theta = \frac{1 + cos2\theta}{2}$$

$$= 25\left[\frac{1}{2}\int_0^{\frac{\pi}{2}} d\theta + \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos 2\theta d\theta\right]$$

$$= 25\left[\frac{\pi}{4} + 0\right]_0^{\Pi/2}$$

$$= \frac{25\pi}{4}$$

B: $-5\int_0^5 dx$

$$= -5x\Big|_0^5$$

$$= -25$$

C: $\left.\frac{x^2}{2}\right|_0^5$

$$= \frac{25}{2}$$

$$\therefore A = \frac{25\pi}{4} - 25 + \frac{25}{2}$$

$$A = 25\pi - \frac{25}{2}$$

$$A = \frac{25\pi - 50}{4}$$

$$A = \frac{25}{4}(\pi - 2)$$

$$M_y = \int_0^5 x\left[\left(\sqrt{25 - x^2}\right) - (5x)\right] dx$$

$$= \int_0^5 x\sqrt{25 - x^2}dx - \int_0^5 5xdx + \int_0^5 x^2dx$$

$$u = 25 - x^2$$

$$du = -2xdx$$

$$= \left[-\frac{1}{2}\frac{(25-x^2)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{5x^2}{2} + \frac{x^3}{3}\right]_0^5$$

$$= -\frac{(25-x^2)^{\frac{3}{2}}}{3} - \frac{5x^2}{2} + \left.\frac{x^3}{3}\right|_0^5$$

$$= \left[-\frac{0}{3} - \frac{125}{2} + \frac{125}{3}\right] - \left[-\frac{125}{3} - 0 + 0\right]$$

$$= -\frac{125}{2} + \frac{250}{3} = -\frac{375 + 500}{6}$$

$$M_y = \frac{125}{6}$$

$$M_x = \frac{1}{2}\int_0^5 \left[\left(\sqrt{25 - x^2}\right)^2 - (5x)^2\right] dx$$

$$= \frac{1}{2}\int_0^5 (25 - x^2)dx - \frac{1}{2}\int_0^5 (5 - x)^2 dx$$

$$= \frac{1}{2}\left(25x - \frac{x^3}{3}\right) \quad -1/2\left(25x + \frac{x^3}{3} - 5x\right)^2$$

$$= \left[\frac{1}{2}\left(125 - \frac{125}{3}\right) - \frac{1}{2}\left(125 + \frac{125}{3} - 125\right)\right] - [0]$$

$$M_x = \frac{125}{6}$$

$\therefore$ *Solving for centroid:*

$$\bar{x} = \frac{M_x}{A} = \frac{\frac{125}{6}}{\frac{25(\pi-2)}{4}} = \frac{125}{6} \cdot \frac{4}{25(\pi - 2)}$$

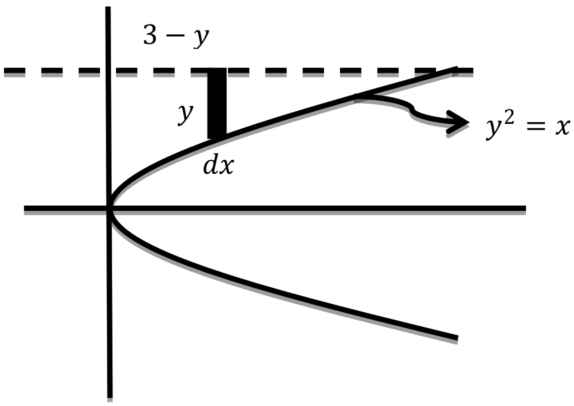
$$\bar{x} = \frac{10}{3(\pi - 2)}$$

$$\bar{y} = \frac{M_x}{A} = \frac{\frac{125}{6}}{\frac{25(\pi-2)}{4}}$$

$$\bar{y} = \frac{10}{3(\pi-2)}$$

$$\boxed{\textbf{Centroid is at } \left(\frac{\mathbf{10}}{\mathbf{3(\pi - 2)}}, \frac{\mathbf{10}}{\mathbf{3(\pi - 2)}}\right)}$$

1. $y^2 = x \;; y = 3 \;; x = 0 \;;\; about\ the\ y - axis$



$$M_{xz} = \int Y_c dV \quad ; \quad \bar{y} = \frac{M_{xz}}{V}$$

x	y
0	0
9	3

$$V = 2\pi \int_0^9 xydx \quad = 2\pi \int_0^9 x(3-y)dx$$

$$M_{xz} = \int Y_c dV \; = 2\pi \int_0^9 \left(\frac{3+y}{2}\right)(x)(\sqrt{x})dx$$

$$M_{xz} = 381.70$$

$$\bar{y} = \frac{M_{xz}}{V} = \frac{381.70}{152.68}$$

$$\bar{y} = 2.5$$

$$\boxed{(\mathbf{0, 2.5, 0})}$$

3. $x^2y = 4, x = 1, x = 4, y = 0\ about\ x - axis$

$$Mxz = \int_a^b Ycdv$$

$$= 2\pi \int_1^4 \frac{y}{2} xydx = \frac{2\pi}{2}\int_1^4 y^2 xdx = \pi \int_1^4 \left(\frac{4}{x^2}\right)^2 xdx$$

$$= \pi \int_1^4 x\left(\frac{16}{x^4}\right)xdx = 16\pi \int_1^4 \left(\frac{x}{x^4}\right)xdx$$

$$= 16\pi \int_1^4 \frac{dx}{x^3} = 16\pi \left[\frac{x^{-2}}{-2}\right]_1^4 = \left[\frac{-8\pi}{x^2}\right]_1^4$$

$$= \frac{15\pi}{2}$$

$$V = \pi \int_a^b l^2 dx$$

$$= \pi \int_1^4 \left(\frac{4}{x^2}\right)^2 dx = \pi \int_1^4 \frac{16}{x^4}dx = 16\pi \int_1^4 \frac{dx}{x^2}$$

$$= 16\pi \int_1^4 x^{-4}dx = 16\pi \left[\frac{x^{-3}}{-3}\right]_1^4 = \left[-\frac{16\pi}{3x^3}\right]_1^4$$

$$V = \frac{21\pi}{4}$$

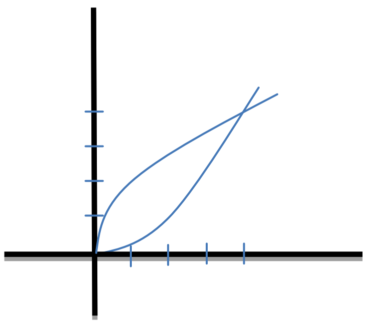
$$\bar{y} = \frac{Mxz}{V} = \frac{\frac{15\pi}{2}}{\frac{21\pi}{4}}$$

$$\boxed{= \left(\mathbf{0}, \tfrac{10}{7}, \mathbf{0}\right)}$$

7. $x^2 = 4y \quad , \quad y^2 = 4x \quad about\ x - axis$

x	y
0	0
1	¼
2	1
4	4

x	y
0	0
1/4	1
1	2
4	4



$$= \pi \int_0^4 \left[\left(\sqrt{4x}\right)^2 - \left(\frac{x^2}{4}\right)^2 \right] dx$$

$$= \pi \int_0^4 \left(4x - \frac{x^4}{16}\right) dx$$

$$= \frac{96\pi}{5}$$

$$Mxz = 2\pi \int_0^4 \left(\frac{\sqrt{4x} + \frac{x^2}{4}}{2}\right)(x)\left(\sqrt{4x} - \frac{x^2}{4}\right) dx$$

$$= \frac{128\pi}{3}$$

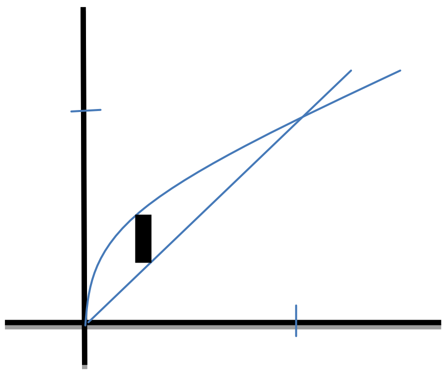
$$\frac{Mxz}{V} = y = \frac{\frac{128\pi}{3}}{\frac{96\pi}{5}}$$

$$\boxed{\boldsymbol{y = \left(0, \frac{20}{9}, 0\right)}}$$

11. $y^2 = 4x, y = x \quad about\ x = 0$

X	*Y*
0	*0*
1/4	*1*
1	*2*
4	*4*

X	*Y*
1	*1*
2	*2*
3	*3*
4	*4*



$$V = 2\pi \int_0^4 X(\sqrt{4X} - X)dx$$

$$V = 26.80829731\ cu.units$$

$$Mxz = 2\pi \int yc\ xdx$$

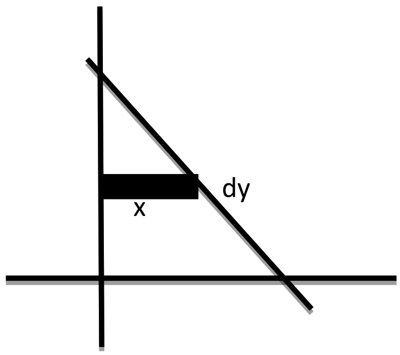
$$Mxz = 2\pi \int_0^4 (\frac{\sqrt{4x} + x}{2})\ x\left(\sqrt{4x} - x\right)dx$$

$$Mxz = 64\pi/3$$

$$y = \frac{Mxz}{V} = 2.5$$

$\boxed{y=(0, 2.5, 0)}$

1. $2x + y = 6\ , x = 0\ , y = 0\ ; about\ x - axis$



x	y
0	6
3	0

$$I_x = \int_0^6 y^2 x dy \ = \int_0^6 y^2 \left(\frac{6-y}{2}\right) dy$$

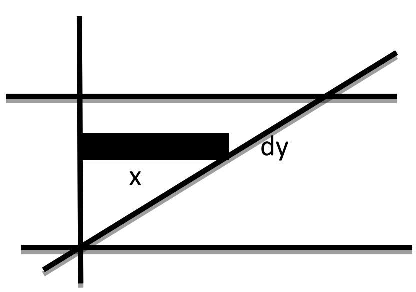
$$= \frac{1}{2}\int_0^6 (6y^2 - y^3) dy$$

$$= \frac{1}{2}\left[2y^3 - \frac{y^4}{4}\right]$$

$$= \frac{1}{2}\left[2(6)^3 - \frac{(6)^4}{4}\right]$$

$$= \boxed{\mathbf{54}}$$

3. $y^3 = x\ \ , x = 8\ , y = 0\ ; with\ respect\ to\ y = 0$

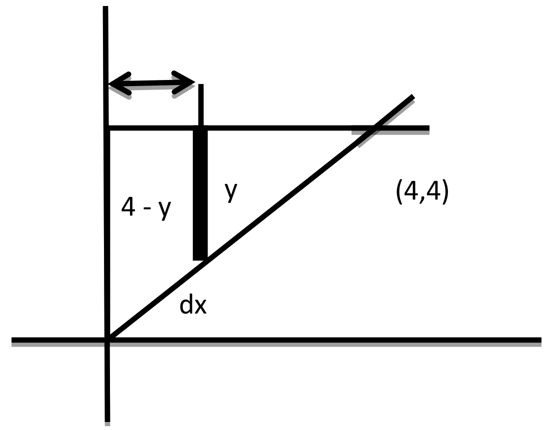


$$I_x = \int_0^2 y^2 x dy \ = \int_0^2 y^2 (y^3) dy$$

$$= \int_0^2 y^5 dy \ = \left[\frac{y^6}{6}\right] \ = \left[\frac{2^6}{6}\right]$$

$$\boxed{\boldsymbol{I_x} = \frac{\mathbf{32}}{\mathbf{3}}}$$

5. $x = 2\sqrt{y}\ , x = 0,\ y = 4$



x	y
0	0
4	4

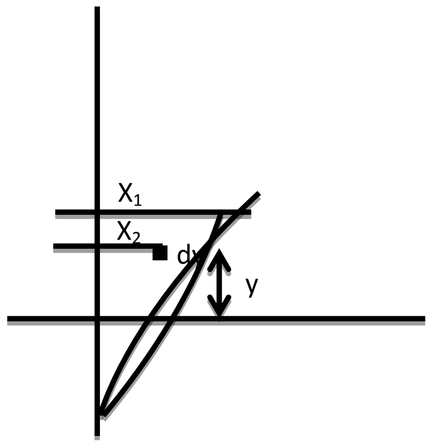
$$I_y = \int_0^4 r^2 dA$$

$$= \int_0^4 x^2 (4 - y) dx$$

$$= \int_0^4 x^2 \left(4 - \frac{x^2}{2}\right) dx$$

$$\boxed{\boldsymbol{I_y} = \frac{\mathbf{512}}{\mathbf{15}}}$$

7. $y^2 = 8x \ , y = 2x$



x	y	x	y
0	0	0	0
1	$2\sqrt{2}$	1	2
2	4	2	4

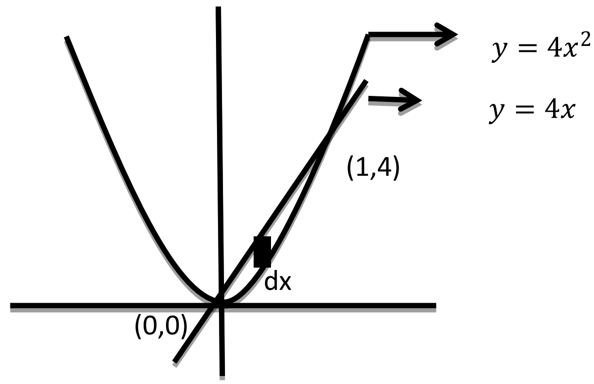
$I_x = \int_0^4 y^2(x_r - x_l)\, dy$

$I_x = \int_0^4 y^2(\frac{y}{2} - \frac{y^2}{8})\, dy$

$I_x = \int_0^4 (\frac{y^3}{2} - \frac{y^4}{8})\, dy$

$\boxed{\boldsymbol{I_x = \frac{32}{5}}}$

9. $y = 4x^2 \ , y = 4x \ ; with\ respect\ to\ y - axis$



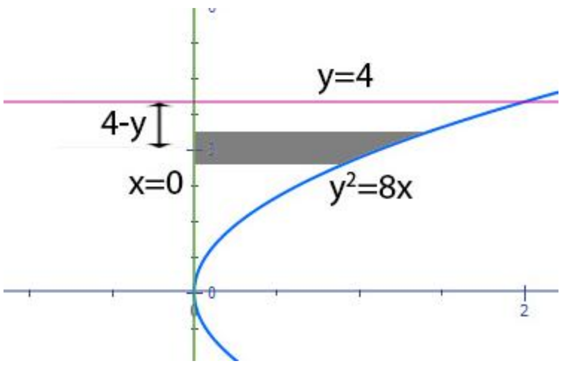
x	y	x	y
0	0	0	0
1	4	1	4

$I_y = \int_a^b x^2(y_u - y_l)\, dx$

$I_y = \int_0^1 x^2(4x - 4x^2)\, dx$

$\boxed{\mathbf{I_y = \frac{1}{5}}}$

11. $y^2 = 8x$, $x = 0$, $y = 4$, with respect to $y = 4$



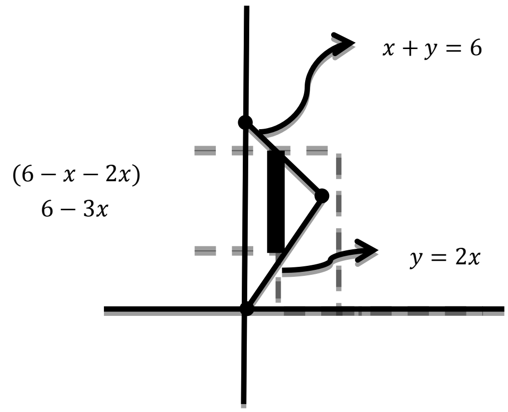
$$I_x = \int_0^4 (4-y)^2 \left(\frac{y^2}{8}\right) dy$$

$$= \frac{1}{8}\int_0^4 (16y^2 - 8y^3 + y^4) dy$$

$$= \frac{1}{8}\left[\frac{16y3}{3} - 2y^4 + \frac{y^5}{5}\right]_0^4$$

$$= \boxed{\mathbf{\frac{64}{15}}}$$

13. $y = x$, $y = 2x$, $x + y = 6$, *with respect to* $x = 0$



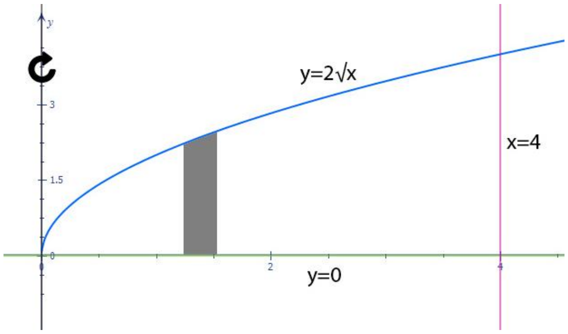
x	y	x	y	x	y
0	0	0	0	0	0
1	1	1	2	1	5
2	2	2	4	2	4

$$\boldsymbol{I_y = \int_a^b x^2 (Y_u - Y_l)\, dx}$$

$$\boldsymbol{I_y = \int_a^b x^2 \left((6-x) - \left(\frac{x}{2}\right)\right) dx}$$

$$\boxed{\boldsymbol{I_y = \frac{19}{2}}}$$

1. $y = 2\sqrt{x}\ , y = 0\ , x = 4\ ; about\, x = 0$



$$I_y = 2\pi \int_0^4 x^3\left(2\sqrt{x} - 0\right)dx$$

$$= 4\pi \int_0^4 x^{7/2} dx$$

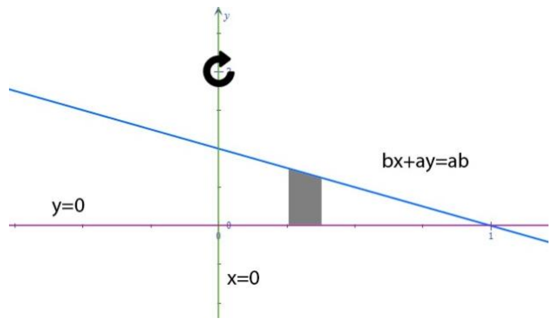
$$= 4\pi \left[\frac{2x^{9/2}}{9}\right]_0^4$$

$$= 4\pi \left[\frac{2(4)^{9/2}}{9} - \frac{2(0)^{9/2}}{9}\right]_0^4$$

$$= 4\pi \left[\frac{1024}{9}\right]$$

$$= \boxed{\boldsymbol{\frac{4096\pi}{9}}}$$

3. $bx + ay = ab\ , x = 0\ , y = 0\ ; about\ the\ y\text{-}axis$



$$I_y = 2\pi \int_0^a x^3 \left(\frac{ab - bx}{a} - 0\right) dx$$

$$= \frac{2b\pi}{a} \int_0^a (x^3 a - x^4) dx$$

$$= \frac{2b\pi}{a}\left(a \int_0^a x^3 dx - \int_0^a x^4 dx\right)$$

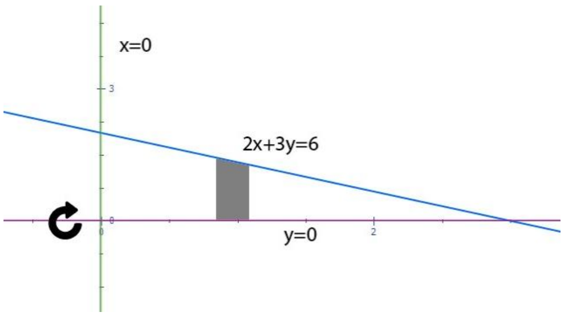
$$= \frac{2b\pi}{a}\left(a \left[\frac{x^4}{4}\right]_0^a - \left[\frac{x^5}{5}\right]_{09}^a\right)$$

$$= \frac{2b\pi}{a}\left(\frac{a^5}{4} - \frac{a^5}{5}\right)$$

$$= \frac{2b\pi}{a}\left(\frac{a^5}{20}\right)$$

$$= \boxed{\boldsymbol{\frac{a^4 b\pi}{10}}}$$

5. $2x + 3y = 6\ , x = 0\ , y = 0\ ;$ *about the x-axis*

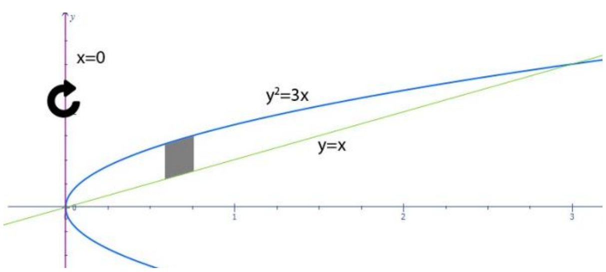


$$I_x = \frac{\pi}{2}\int_0^3 \left(\left(\frac{6-2x}{3}\right)^4 - 0\right)dx$$

$$= \frac{\pi}{2}\int_0^3 \frac{16x^4 - 192x^3 + 864x^2 - 1728x + 1296}{81}\,dx$$

$$= \boxed{\frac{\mathbf{24\pi}}{\mathbf{5}}}$$

7. $y^2 = 3x\ , y = x\ ;$ *about* $x = 0$



$$I_y = 2\pi\int_0^3 x^3\left(x\sqrt{3} - x\right)dx$$

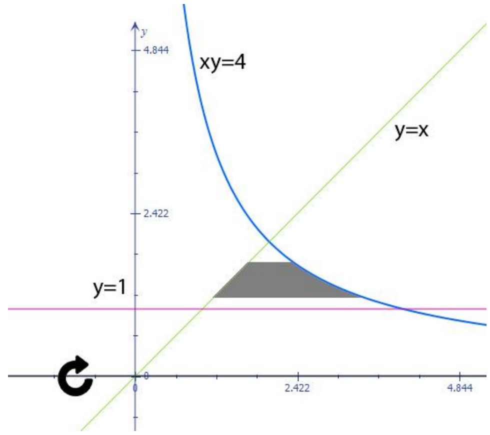
$$= 2\pi\int_0^3 \left(x^{7/2}\sqrt{3} - x^4\right)dx$$

$$= 2\pi\left(\sqrt{3}\int_0^3 x^{7/2}dx - \int_0^3 x^4 dx\right)$$

$$= 2\pi\left(54 - \frac{243}{5}\right)$$

$$= \boxed{\frac{\mathbf{54\pi}}{\mathbf{5}}}$$

9. $xy = 4\ , y = x\ , y = 1\ ;$ *about* $y = 0$



$$I_x = 2\pi\int_1^2 y^3\left(\frac{4}{y} - y\right)dy$$

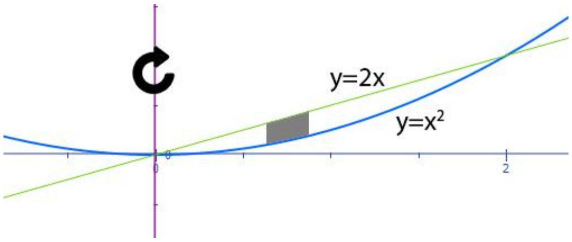
$$= 2\pi\int_1^2 (4y^2 - y^4)dy$$

$$= 2\pi\left(4\left[\frac{y^3}{3}\right]_1^2 - \left[\frac{y^5}{5}\right]_1^2\right)$$

$$= 2\pi\left[\frac{28}{3} - \frac{31}{5}\right]$$

$$= \boxed{\frac{\mathbf{94\pi}}{\mathbf{15}}}$$

11. $y = x^2$, $y = 2x$;*about the y-axis*



$$I_y = 2\pi \int_0^2 x^3(2x - x^2)dx$$

$$= 2\pi \int_0^2 (2x^4 - x^5)dx$$

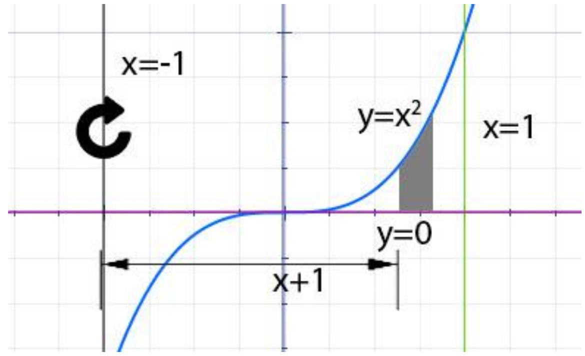
$$= 2\pi \left[2\int_0^2 x^4 dx - \int_0^2 x^5 dx\right]$$

$$= 2\pi \left(2\left[\frac{x^5}{5}\right]_0^2 - \left[\frac{x^6}{6}\right]_0^2\right)$$

$$= 2\pi \left[\frac{64}{5} - \frac{32}{3}\right]$$

$$= \boxed{\frac{\mathbf{64\pi}}{\mathbf{15}}}$$

13. $y = x^3$, $x = 1$, $y = 0$; *about* $x = -1$



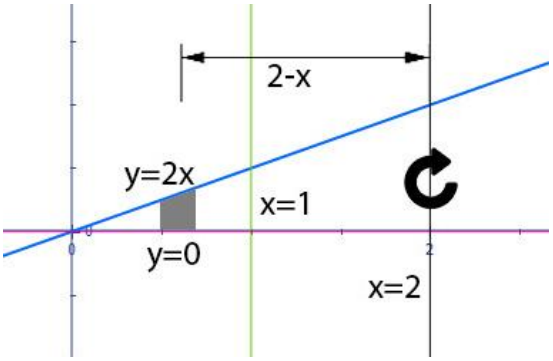
$$I_y = 2\pi \int_0^1 (x+1)^3(x^3 - 0)dx$$

$$= 2\pi \int_0^1 (x^6 + 3x^5 + 3x^4 + x^3)dx$$

$$= 2\pi \left[\frac{x^7}{7} + \frac{x^6}{2} + \frac{3x^5}{5} + \frac{x^4}{4}\right]_0^1$$

$$= \boxed{\frac{\mathbf{209\pi}}{\mathbf{70}}}$$

15. $y = 2x\ , x = 1\ , y = 0\ ;\ about\ x = 2$



$$I_y = 2\pi\int_0^1 (2-x)^3(2x)dx$$

$$= 4\pi\int_0^1 (8x - 12x^2 + 6x^3 - x^4)$$

$$= 4\pi\left[4x^2 - 4x^3 + \frac{3x^4}{2} - \frac{x^5}{5}\right]_0^1$$

$$= \boxed{\frac{\mathbf{26\pi}}{\mathbf{5}}}$$