UNIVERSITY OF CALOOCAN CITY
Brgy. 173, Congressional Road, Caloocan City
COLLEGE OF LIBERAL ARTS AND SCIENCES
INTEGRAL CALCULUS

STANDARD INTEGRATION FORMULAS:

A. Integration of Algebraic Functions

1. $\int du = u + c$
2. $\int (du + dv + dw) = \int du \int dv \int dw$
3. $\int adu = a\int du = au + c$
4. $\int u^n du = \frac{u^{n+1}}{n+1} + c \ where\ n \neq -1$

B. Integration of Logarithmic and Exponential Functions

5. $\int \frac{du}{u} = ln\, u + c$
6. $\int a^u du = \frac{a^u}{ln^a} + c$
7. $\int e^u du = e^u + c$

C. Integration of Trigonometric Functions

8. $\int sin\, u\, du = -cos\, u + c$
9. $\int cos\, u\, du = sin\, u + c$
10. $\int tan\, u\, du = -ln\, |cos\, u| + c$
11. $\int cot\, u\, du = ln\, |sin\, u| + c$
12. $\int sec\, u\, du = ln\, |sec\, u + tan\, u| + c$
13. $\int csc\, u\, du = -ln\, |csc\, u + cot\, u| + c$
14. $\int sec^2\, u\, du = tan\, u + c$
15. $\int csc^2\, u\, du = -cot\, u + c$
16. $\int sec\, u\, tan\, u\, du = sec\, u + c$
17. $\int csc\, u\, cot\, u\, du = -csc\, u + c$

D. Integration of Inverse Trigonometric Functions

18. $\int \frac{du}{\sqrt{a^2+u^2}} = Arc\sin\frac{u}{a} + C$
19. $\int \frac{du}{a^2+u^2} = \frac{1}{a} Arc\tan\frac{u}{a} + C$
20. $\int \frac{du}{u\sqrt{u^2+a^2}} = \frac{1}{a} Arc\sec\frac{u}{a} + C$

Activity #1

1) $\int (6x^2 - 4x + 5)dx$
2) $\int (2x-1)(3x+4)dx$
3) $\int x(\sqrt{x} - 1)dx$
4) $\int \frac{(x+4)dx}{\sqrt{x}}$
5) $\int \frac{x^3-8dx}{x-2}$

Activity #2

1) $\int \sqrt{e^{3x}}dx$
2) $\int \frac{(e^x+1)^2}{e^x}$
3) $\int \sqrt[3]{4^{2x}}dx$
4) $\int \frac{(2x-5)}{x^2-5x+3}$
5) $\int \frac{x^3-x-3}{x-1}$

Activity #3

1) $\int \frac{sec^2(3x)}{1+4tan^2(3x)}$
2) $\int csc\left(\frac{1}{2}\right) cot\left(\frac{1}{2}\right) dx$
3) $\int \frac{cos(ln\, x)dx}{x}$
4) $\int [1 - tan(x)]^2 dx$
5) $\int \frac{sin^3(x)}{1-cos(x)} dx$

Activity #4

1) $\int \frac{dx}{\sqrt{5-9x^2}}$
2) $\int \frac{e^x}{1+e^{2x}} dx$
3) $\int \frac{(x^2-4)\, dx}{x^2-4}$
4) $\int \frac{dx}{\sqrt{5+4x-x^2}}$
5) $\int \frac{\sec(x)\tan(x)\, dx}{1+4sec^2(x)}$

Integral Calculus is the inverse of differentiation or simply known as anti-derivative. Integral Calculus is supposed of two major areas

1) The Indefinite Integrals such as direct methods of integrations and techniques & integration and

2) The Definite Integral

<u>INDEFINITE INTEGRAL</u>

$$\int f(x)dx = F(x) + c$$

$Where:$

$\int = integral\ sign$

$f(x) = integrand$

$dx = variable\ of\ integration$

$F(x) + c = \ values\ of\ the\ indifinite\ intergral$

$c = constant\ of\ integration$

<u>DEFINITE INTEGRAL</u>

$$\int_a^b f(x)dx = [FX]_a^b$$

$\boldsymbol{Where}:$

$\boldsymbol{b} = Upperlimit$

$\boldsymbol{a} = Towerlimit$

<u>STANDARD INTEGRATION FORMULAS:</u>

A. Integration of Algebraic Functions

1. $\int \boldsymbol{du = u + c}$
2. $\int \boldsymbol{(du + dv + dw) = \int du \int dv \int dw}$
3. $\int \boldsymbol{adu = a \int du = au + c}$
4. $\int \boldsymbol{u^n du = \frac{u^{n+1}}{n+1} + c\ where\ n \neq -1}$

Ex. Evaluate the integral of the following:

1. $\int \boldsymbol{5xdx}$

$$= 5\int xdx$$
$$= 5\left(\frac{x^2}{2}\right) + c$$
$$= \frac{5}{2}x^2 + c$$

2. $\int \boldsymbol{(8x^4 - 5x^2 + 2)dx}$

$$= 8\int x^4dx - 5\int x^2dx + 2\int dx$$
$$= 8\left(\frac{x^5}{5}\right) - 5\left(\frac{x^3}{3}\right) + 2x + c$$
$$= \frac{8}{5}x^5 - \frac{5}{3}x^3 + 2x + c$$

3. $\int \boldsymbol{\frac{7}{x^5}dx}$

$$= 7\int \frac{dx}{x^5}$$
$$= 7\int x^{-5}dx$$
$$= 7\left(\frac{x^{-4}}{-4}\right) + c$$
$$= -\frac{7}{4x^4} + c$$

4. $\int \boldsymbol{\frac{dx}{(3x-5)^2}}$

$\boldsymbol{let\ du = 3x - 5}$

$\boldsymbol{IF = \ Integration\ Factor}\ [\ dont\ remove\ IF\ ]$

$$IF = \frac{1}{3}$$
$$= \frac{1}{3}\int \frac{3dx}{(3x-5)^2}$$
$$= \frac{1}{3}\int (3x-5)^{-2} * 3dx\ \{replace\ (3dx)\ to\ c\ \}$$
$$= \frac{1}{3}\left[\frac{(3x-5)^{-1}}{-1}\right] + c$$
$$= -\frac{1}{3(3x-5)} + c$$
$$= -\frac{1}{9x-15} + c$$

$\boldsymbol{Check}:$

$$= -\frac{1}{3}\ (\frac{1}{3x-5}) + c$$
$$= \frac{1}{3}\left(\frac{-1}{3x-5}\right) + 3dx\ \{\ rule\ 10\ diff.calc.\}$$
$$= \frac{dx}{3x-5^2}$$

5. $\int(2-x)(1+3x)dx$ **{ *FOIL Method* }**

$$= \int (2+6x-x-3x^2)dx$$
$$= \int (2+5x-3x^2)dx$$
$$= 2\int dx 5\int xdx - 3\int x^2dx$$
$$= 2x + \frac{5}{2}x^2 - x^3 + c$$

Exercise:

1) $\int(6x^2-4x+5)dx$
2) $\int(2x-1)(3x+4)dx$
3) $\int x(\sqrt{x-1})dx$
4) $\int \frac{(x+4)dx}{\sqrt{x}}$
5) $\int \frac{2x^2+4x-3\,dx}{x^2}$
6) $\int(4\sqrt[3]{x}-2x\sqrt{x})dx$
7) $\int \frac{x^3-8dx}{x-2}$
8) $\int \frac{(1+\sqrt[3]{x})^2dx}{\sqrt[3]{x}}$
9) $\int \sqrt{x^4-2x^3+x^2}\,dx$
10) $\int\left(\frac{5}{\sqrt{x}}-\frac{3}{x^2}+\frac{2}{x^4}\right)dx$

1) $\int(6x^2-4x+5)dx$

$$= 6\int x^2dx - 4\int xdx + 5\int dx$$
$$= 6\int \frac{x^3}{3}dx - 4\int\frac{x^2}{2}dx + 5\int dx$$
$$= \mathbf{2x^3 - 2x^2 + 5x + c}$$

2) $\int(2x-1)(3x+4)dx$

$$= \int 6x^2dx + 5xdx - 4dx$$
$$= 6\int x^2dx + 5\int xdx - 4\int dx$$
$$= 6\int\frac{x^3}{3}dx + 5\int\frac{x^2}{2}dx - 4\int dx$$
$$= \mathbf{2x^3 - \frac{5x^2}{2} - 4x + c}$$

3) $\int x(\sqrt{x-1})dx$

Apple u - substitution: $\int(u+1)(\sqrt{u})du$

$$= \int u^{\frac{3}{2}}\left(u^{\frac{1}{2}}\right)du$$
$$= \int u^{\frac{3}{2}}du\int u^{\frac{1}{2}}du$$
$$= \int \frac{u^{\frac{3}{2}+1}}{\frac{3}{2}+1}du\int\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}du$$
$$= \int\frac{u^{\frac{5}{2}}}{\frac{5}{2}}du\int\frac{u^{\frac{3}{2}}}{\frac{3}{2}}du$$
$$\int\frac{u^{\frac{5}{2}}}{\frac{5}{2}}du = \frac{2}{5}u^{\frac{5}{2}}$$
$$\int\frac{u^{\frac{3}{2}}}{\frac{3}{2}}du = \frac{2}{3}u^{\frac{3}{2}}$$
$$= \frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}$$

Substitute u

$$= \mathbf{\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}}}$$

B. *Integration of Logarithmic and Exponential Functions*

4) $\int \frac{du}{u} = \boldsymbol{ln\, u + c}$

5) $\int a^u du = \frac{a^u}{ln^a} + c$

6) $\int e^u du = e^u + c$

Rational function $\int \frac{}{u} dx$ [*but not all*]
$\boldsymbol{u = is\ x\ with\ the\ number}$
$\boldsymbol{a = is\ natural\ base\ (number)}$

Evaluate the integrals of the following:

1) $\int \frac{4}{3x+2} dx = 4\int \frac{dx}{3x+2}$

$let\ u = 3x + 2$
$du = 3dx$
$IF = \frac{1}{3}$
$= \frac{4}{3}\int \frac{3dx}{3x+2}$ [$3dx$ < *will be remove*]
$= \frac{4}{3} ln(3x + 2) + c$
$= \boldsymbol{ln(3x+2)^{\frac{4}{3}} + c}$

2) $\int \frac{(2x+3)}{x^2+3x+1} dx$

$let\ u = x^2 + 3x + 1$
$du = 2xdx + 3dx$
$\quad = (2x + 3)dx$
$= \boldsymbol{ln(x^2 + 3x + 1) + c}$

3) $\int 10^{2x} dx = \frac{10^{2x}}{ln(10)} + c$

$let\ u = 2x$
$du = 2dx$
$IF = \frac{1}{2}$
$= \frac{1}{2}\int 10^{2x}\, dx$
$= \frac{1}{2}\left(\frac{10^{2x}}{ln(10)}\right) + c$
$= \frac{10^{2x}}{2ln(10)} + c$
$= \frac{10^{2x}}{ln(10)^2} + c$
$= \boldsymbol{\frac{10^{2x}}{ln(100)} + c}$

4) $\int \frac{dx}{e^x} = \int e^{-x} dx$

$let\ u = -x$
$du = -dx$
$IF = -1$ (*reciprocal is* -1 *also*)
$= -\int e^{-x} \cdot -dx$
$= -e^{-x} + c$
$= -\frac{1}{e^x} + c$

5) $\int e^{5x} dx$

$let\ u = 5x$
$du = 5dx$
$IF = \frac{1}{5}$
$= \frac{1}{5}\int e^{5x} \cdot 5dx$
$= \boldsymbol{\frac{1}{5} e^{5x} + c}$

Supplementary Problem:
Evaluate the integrals of the following:

1. $\int \frac{2}{1-5x}dx$
2. $\int \frac{x}{x^2-4}dx$
3. $\int e^{-2x}dx$
4. $\int 5^{3-2x}dx$
5. $\int (e^{3x}+1)^2 dx$

1. $\int \frac{2}{1-5x}dx = 2\int \frac{1}{1-5x}dx$

$let\ u = 1-5x$
$du = 5dx$
$IF = \frac{1}{5}$
$= \frac{2}{5}\int \frac{1}{1-5x}\cdot 5dx$
$= \frac{2}{5}ln(1-5x)+c$
$= \boldsymbol{ln(1-5x)^{\frac{2}{5}}+c}$

2. $\int \frac{x}{x^2-4}dx$

$let\ u = x^2-4$
$du = 2xdx$
$IF = \frac{1}{2}$
$= \frac{1}{2x}\int \frac{x}{x^2-4}\cdot 2xdx$
$= \frac{x}{2x}\int \frac{1}{x^2-4}\cdot 2xdx$
$= \frac{x}{2x}ln(x^2-4)+c$
$= \boldsymbol{ln(x^2-4)^{\frac{x}{2x}}+c}$

3. $\int e^{-2x}dx$

$let\ u = -2x$
$du = -2dx$
$IF = -\frac{1}{2}$
$= -\frac{1}{2}\int e^{-2x}\cdot -2dx$
$= -\frac{1}{2}e^{-2x}+c$
$= \boldsymbol{-\frac{1}{2e^{2x}}+c}$

4. $\int 5^{3-2x}dx$

$let\ u = 3-2x$
$du = 2dx$
$IF = \frac{1}{2}$
$= \frac{1}{2}\int 5^{3-2x}\cdot 2dx$
$= \frac{1}{2}\left(\frac{5^{3-2x}}{ln(5)}\right)+c$
$= \frac{5^{3-2x}}{2ln(5)}+c$
$= \frac{5^{3-2x}}{ln(5)^2}+c$
$= \boldsymbol{\frac{5^{3-2x}}{ln(25)}+c}$

5. $\int (e^{3x}+1)^2 dx$

$= \int (e^{6x}+2e^{3x}+1)dx$
$= \int e^{6x}dx + 2\int e^{3x}dx + 1\int dx$

$let\ u = 6x$	$let\ u = 3x$
$du = 6dx$	$du = 3dx$
$IF = \frac{1}{6}$	$IF = \frac{1}{3}$

$= \frac{1}{6}\int e^{6x}\cdot 6dx + \frac{2}{3}\int e^{3x}\cdot 3dx + 1\int dx$
$= \boldsymbol{\frac{1}{6}e^{6x}+\frac{2}{3}e^{3x}+x+c}$

C. Integration of Trigonometric Functions

8. $\int \boldsymbol{\sin u\, du = -\cos u + c}$
9. $\int \boldsymbol{\cos u\, du = \sin u + c}$
10. $\int \boldsymbol{\tan u\, du = -\ln|\cos u| + c}$
11. $\int \boldsymbol{\cot u\, du = \ln|\sin u| + c}$
12. $\int \boldsymbol{\sec u\, du = \ln|\sec u + \tan u| + c}$
13. $\int \boldsymbol{\csc u\, du = -\ln|\csc u + \cot u| + c}$
14. $\int \boldsymbol{\sec^2 u\, du = \tan u + c}$
15. $\int \boldsymbol{\csc^2 u\, du = -\cot u + c}$
16. $\int \boldsymbol{\sec u \tan u\, du = \sec u + c}$
17. $\int \boldsymbol{\csc u \cot u\, du = -\csc u + c}$

1. $\int \cos\left(\frac{1}{4}x\right) dx$

$let\ u = \frac{1}{4}x$

$du = \frac{1}{4}dx$

$IF = 4$

$= 4\int \cos\left(\frac{1}{4}x\right) \cdot \frac{1}{4}dx$

$= \boldsymbol{4\sin\left(\frac{1}{4}x\right) + c}$

2. $\int 5\sec^2(2x)\, dx$

$= 5\int sec^2(2x)\, dx$

$let\ u = 2x$

$du = 2dx$

$IF = \frac{1}{2}$

$= \frac{5}{2}\int sec^2(2x) \cdot 2dx$

$= \boldsymbol{\frac{5}{2}\tan(2x) + c}$

3. $\int \sec(x)\tan(x)\, dx$

$= \boldsymbol{\sec(x) + c}$

4. $\int \frac{dx}{\cos^2(3x)}$

From Trigonometric Identities
(Reciprocal Identities)#19

$= \frac{1}{cos\ \emptyset} = sec\ \emptyset$

$= \frac{1}{cos^2\emptyset} = sec^2\emptyset$

$= \frac{1}{cos^2(3x)} = sec^2(3x)$

$= \int sec^2(3x)\, dx$

$let\ u = 3x$

$du = 3dx$

$IF = \frac{1}{3}$

$= \frac{1}{3}\int sec^2(3x) \cdot 3dx$

$= \boldsymbol{\frac{1}{3}\tan(3x) + c}$

or

$= \boldsymbol{\frac{\tan(3x)}{3} + c}$

5. $\int \frac{[\cos(x) - \sin(x)]}{\sin(x)} dx$

$= \int \frac{cos(x)}{sin(x)} dx - \int \frac{\cancel{sin(x)}}{\cancel{sin(x)}} dx$

$= \int cot(x)\, dx - \int dx$

$= \boldsymbol{\ln|\sin(x)| - x + c}$

Supplementary Problem:

Evaluate the integrals of the following:

1. $\int \boldsymbol{sin(5x)dx}$
2. $\int \frac{1}{3} \boldsymbol{csc^2\left(\frac{1}{2}x\right)dx}$
3. $\int \boldsymbol{e^{3x}cot(e^{3x})dx}$
4. $\int \boldsymbol{sec^2(x)\ tan^2(x)dx}$
5. $\int \boldsymbol{\frac{csc^2(3x)}{1-cot(3x)}dx}$

1) $\int \boldsymbol{sin(5x)dx}$

$$let\ u = 5x$$
$$du = 5dx$$
$$IF = \frac{1}{5}$$
$$= \frac{1}{5}\int sin(5x) \cdot 5dx$$
$$= \boldsymbol{\frac{1}{5}cos(5x) + c}$$

2) $\int \boldsymbol{\frac{1}{3}csc^2\left(\frac{1}{2}x\right)dx}$

$$let\ u = \frac{1}{2}x$$
$$du = \frac{1}{2}dx$$
$$IF = 2$$
$$= 2 \cdot \frac{1}{3}\int csc^2\left(\frac{1}{2}x\right) \cdot \frac{1}{2}dx$$
$$= \frac{2}{3}\int csc^2\left(\frac{1}{2}x\right) \cdot \frac{1}{2}dx$$
$$= \boldsymbol{\frac{2}{3}cot\left(\frac{1}{2}x\right) + c}$$

3) $\int \boldsymbol{e^{3x}cot(e^{3x})dx}$

$$let\ u = e^{3x}$$
$$du = e^{3x} \cdot 3dx$$
$$= 3e^{3x}dx$$
$$IF = \frac{1}{3}$$
$$= \frac{1}{3}\int e^{3x}cot(e^{3x}) \cdot 3e^{3x}dx$$
$$= \frac{1}{3}ln|sin(e^{3x})| + c$$
$$= \boldsymbol{ln\left|sin(e^{3x})^{\frac{1}{3}}\right| + c}$$

4) $\int \boldsymbol{sec^2(x) \cdot tan^2(x)dx}$

$$= \int tan^2(x) \cdot sec^2(x)\ dx$$
$$= \int [tan(x)]^2 \cdot sec^2(x)\ dx$$
$$= \int u^n dx$$
$$let\ u = \tan(x)$$
$$du =\cdot sec^2(x)\ dx$$

By power Formula

$$= \boldsymbol{\frac{tan^3(x)}{3} + c}$$

5) $\int \boldsymbol{\frac{csc^2(3x)}{1-cot(3x)}dx}$

$$= \int \frac{du}{u}$$
$$let\ u = 1 - cot(3x)$$
$$du = -\left(-csc^2(3x)\right) \cdot 3dx$$
$$= 3csc^2(3x)dx$$
$$IF = \frac{1}{3}$$
$$= \frac{1}{3}\int \frac{3csc^2(3x)dx}{1 - cot(3x)}dx$$
$$= \frac{1}{3}ln\left(1 - cot(3x)\right) + c$$
$$= \boldsymbol{ln\left(1 - cot(3x)\right)^{\frac{1}{3}} + c}$$

D. *Integration of Inverse Trigonometric Functions*

21. $\int \frac{du}{\sqrt{a^2+u^2}} = \boldsymbol{Arc\sin\frac{u}{a} + C}$

22. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\boldsymbol{Arc\tan\frac{u}{a} + C}$

23. $\int \frac{du}{u\sqrt{u^2+a^2}} = \frac{1}{a}\boldsymbol{Arc\sec\frac{u}{a} + C}$

$\boldsymbol{Ex.\,Evaluate\ the\ integrals\ of\ the\ following}$:

1. $\int \frac{dx}{\sqrt{16-x^2}} = \boldsymbol{Arc\sin\frac{x}{4} + C}$

$a^2 = 16\,; a = 4$

$u^2 = x^2\,; u = x$

$du = dx$

$If\ not\ the\ same\ its\ not\ rule\ 18$

$\boldsymbol{Arc\sin\frac{x}{4} + C}$

2. $\int \frac{dx}{3+2x^2}$

$a^2 = 3\,; a = \sqrt{3}$

$u^2 = 2x^2\,; u = \sqrt{2}x$

$du = \sqrt{2}dx$

$IF = \frac{1}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}}\int \frac{\sqrt{2}dx}{3+2x^2}$$

$$\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{3}}Arc\tan\left(\frac{\sqrt{2}x}{\sqrt{3}}\right)\right) + C$$

$\boldsymbol{Based\ on\ laws\ of\ radicals}$ $\boldsymbol{\sqrt{2}\cdot\sqrt{3} = \sqrt{(2)(3)} = \sqrt{6}}$

$$\boldsymbol{\frac{1}{\sqrt{6}}Arc\tan\left(\frac{\sqrt{2}x}{\sqrt{3}}\right) + C}$$

3. $\int \frac{2x\ dx}{\sqrt{1-4x^4}}$

$$2\int \frac{x\,dx}{\sqrt{1-(2x^2)^2}}$$

$a^2 = 1\,; a = 1$

$u^2 = (2x^2)^2\,; u = 2x^2$

$du = 4xdx$

$IF = \frac{1}{4}$

$$\frac{2}{4}\int \frac{4x\,dx}{\sqrt{1-(2x^2)^2}}$$

$$\frac{2}{4}\left(Arc\sin\left(\frac{2x^2}{1}\right) + C\right)$$

$$\boldsymbol{\frac{1}{2}Arc\sin(2x^2) + C}$$

4. $\int \frac{3\,dx}{2x\sqrt{24-1}}$

$$3\int \frac{dx}{2x\sqrt{4x^2-1}}$$

$a^2 = 1\,; a = 1$

$u^2 = 4x^2\,; u = 2x$

$du = 2dx$

$IF = \frac{1}{2}$

$$\frac{1}{2}\cdot 3\int \frac{2\,dx}{2x\sqrt{4x^2-1}}$$

$$\frac{3}{2}\left(\frac{1}{1}\ Arc\sec\left(\frac{2x}{1}\right) + C\right)$$

$$\boldsymbol{\frac{3}{2}Arc\sec(2x) + C}$$

5. $\int \frac{(x-5)\,dx}{x^2+5}$

$$\int \frac{x\,dx}{x^2+5} \qquad - 5\int \frac{dx}{x^2+5}$$

$\boldsymbol{Rule\ 5}$ $\qquad$ $\boldsymbol{Rule\ 19}$

$$\boldsymbol{\int \frac{du}{u}} \qquad \boldsymbol{\int \frac{du}{a^2+u^2}}$$

$let\ u = x^2 + 5$

$du = 2x\ dx$

$IF = \frac{1}{2}$

$a^2 = x^2\,; a = x$

$u^2 = 5\,; u = \sqrt{5}$

$du = dx$

$$\frac{1}{2}\int \frac{2x\,dx}{x^2+5} - 5\int \frac{dx}{x^2+5}$$

$$\frac{1}{2}\ln(x^2+5) - \frac{\cancel{5}\sqrt{5}}{\cancel{5}}Arc\tan\left(\frac{\sqrt{5}}{5}x\right) + C$$

$$\boldsymbol{\ln(x^2+5)^{\frac{1}{2}} - \sqrt{5}\,Arc\tan\left(\frac{\sqrt{5}}{5}x\right) + C}$$

1) $\int \sec(5x)\tan(5x)\,dx$
2) $\int \frac{dx}{\sin(x)\cos(x)}$
3) $\int \frac{\sin(x)+\cos(x)}{\sin^2(x)+}dx$
4) $\int \sec^2(4x-3)dx$
5) $\int \frac{dx}{\sin^{\frac{1}{2}}(x)+\cot^{\frac{1}{2}}(x)}$
6) $\int \frac{dx}{1-\cos(x)}$
7) $\int \frac{\cos^3(x)}{1-\sin(x)}dx$
8) $\int \frac{\cos(4x)}{\sin(2x)}dx$
9) $\int [1+\tan(x)]^2 dx$
10) $\int x^2\cos(4x^3)dx$
11) $\int \frac{\cos(6x)dx}{\cos^2(3x)}$
12) $\int \sin(2x)\sec(x)\,dx$
13) $\int \frac{\sin(2x)dx}{2\sin(\mathrm{x})\cos^2(x)}$
14) $\int [\cot(\mathrm{x})+\tan(\mathrm{x})]^2 dx$
15) $\int \frac{4\sin^2(\mathrm{x})\cos^2(x)}{\sin(2\mathrm{x})\cos(2x)}dx$
16) $\int \frac{dx}{\tan(5x)}$
17) $\int \frac{dx}{\sin(3x)\tan(3x)}$