INTEGRAL CALCULUS: Learning Module No. 5

Topic	**METHODS OF INTEGRATION**
Sub-Topic	**Integration By Parts**
Duration	3 hours
Introduction Note: This module may contain copyrighted material. The use of which has not been specifically authorized by the copyright owner. This module is for educational purpose only for online instruction and is not used to generate profit. Thus this constitutes a "Fair Use" of the copyrighted material as provided by virtue of Republic Act No. 8293 otherwise known as Intellectual Property Code of the Philippines.	The method of Integration by Parts is specifically helpful when the integrand is a product of two kinds of functions such as the following: 1. Algebraic and Trigonometric $\int x^2 sinx\, dx$ 2. Algebraic and Logarithmic $\int x^2 lnx\, dx$ 3. Algebraic and Exponential $\int x\, e^x dx$ 4. Exponential and Trigonometric $\int e^x cosx\, dx$
Theories/Concepts/Formulas	**1. Integration by Parts** From: $d(uv) = udv + vdu$ Integrating both sides of the equation: $\int d(uv) = \int udv + \int vdu$ $uv = \int udv + \int vdu$

	$$uv - \int vdu = \int udv$$ Thus, $$\boxed{\int udv = uv - \int vdu}$$
	In choosing u and dv always remember the following: 1. dx is always included in dv. 2. It must be possible to integrate dv directly in some instances. 3. It is best usually to choose the most complicated factor as dv.
YouTube Link/s	https://www.youtube.com/watch?v=bLhxQIdbWW8 https://www.youtube.com/watch?v=dqaDSlYdRcs https://www.youtube.com/watch?v=-5Qv7-nfVjI
Sample Problems	**Ex. Evaluate the integrals of the following functions:** 1. $\int x\, e^x dx$ **Solution**: u = x dv = e^x dx du = dx v = e^x $$\int x\, e^x dx = xe^x - \int e^x dx$$ $$= \boxed{\boldsymbol{xe^x - e^x + C}}$$ or $$= \boxed{\boldsymbol{e^x(x-1) + C}}$$ 2. $\int x^2\, lnx\, dx$ **Solution**: u = ln x dv = x^2 dx du = $\frac{dx}{x}$ v = $\frac{x^3}{3}$

$$= (lnx)\left(\frac{x^3}{3}\right) - \int \left(\frac{x^{\cancel{3}2}}{3}\right)\left(\frac{dx}{\cancel{x}}\right)$$

$$= \frac{x^3}{3} lnx - \frac{1}{3}\int x^2 dx$$

$$= \frac{x^3}{3} lnx - \frac{1}{3}\left(\frac{x^3}{3}\right) + C$$

$$= \boxed{\boldsymbol{\frac{x^3}{3} lnx - \left(\frac{x^3}{9}\right) + C}} \quad \text{or}$$

$$= \boxed{\boldsymbol{\frac{x^3}{3}\left(lnx - \frac{1}{3}\right) + C}}$$

$3.\ Arctan\ 3x\ dx$

Solution:

u = Arctan 3x dv = dx

du = $\frac{3dx}{9x^2+1}$ v = x

$$= (Arctan\ 3x)(x) - \int (x)\left(\frac{3dx}{9x^2+1}\right)$$

$$= x\ Arctan\ 3x - 3\int \frac{xdx}{9x^2+1}$$

Let u = $9x^2 + 1$

du = 18xdx, *if*=$\frac{1}{18}$

$$= x\ Arctan\ 3x - \frac{\cancel{3}}{\cancel{1}8}\int \frac{18xdx}{9x^2+1}$$

$$= x\ Arctan\ 3x - \frac{1}{6}\ln(9x^2+1) + C$$

$$= \boxed{\boldsymbol{x\ Arctan\ 3x - \ln(9x^2+1)^{\frac{1}{6}} + C}}$$

4. $\int x^2 sinx\, dx$

Solution:

u = x^2 dv = sin x dx

du = 2xdx v = -cos x

$$= (x^2)(-cosx) - \int (-\cos x)\,(2xdx)$$

$$= -x^2 \cos x + 2\int x\, cos\, x\, dx$$

u = x dv = cos x dx

du = dx v = sin x

$$= -x^2 \cos x + 2\,[x\, sinx - \int sinx\, dx]$$

$$= -x^2 cosx + 2xsinx - 2\int sinx\, dx$$

$$= -x^2 cosx + 2xsinx + 2cosx + C$$

$$= \boxed{\mathbf{2xsinx + 2cosx - x^2cosx + C}}$$

5. $\int e^x cosx\, dx$

Solution:

u = e^x dv = cos x dx

du = e^x dx v = sin x

$$\int e^x cosx\, dx = e^x sinx - \int (sinx)(e^x dx)$$

$$\int e^x cosx\, dx = e^x sinx - \int (e^x sinx\, dx)$$

u = e^x dv = sin x dx

du = e^x dx v = -cos x

$$\int e^x cosx\, dx = e^x sinx - [e^x(-cosx) - \int (-cosx)(e^x dx)]$$

	$\int e^x cosx\, dx = e^x sinx + e^x cosx - \int e^x cosx\, dx$
	$\int e^x cosx\, dx + \int e^x\, cosx\, dx = e^x sinx + e^x cosx$
	$2\int e^x cosx\, dx = e^x sinx + e^x cosx$
	$\int e^x cosx\, dx = \frac{e^x sinx + e^x cosx}{2} + C$
	$\boxed{\boldsymbol{\int e^x cosx\, dx = \frac{e^x}{2}(sinx + cosx) + C}}$