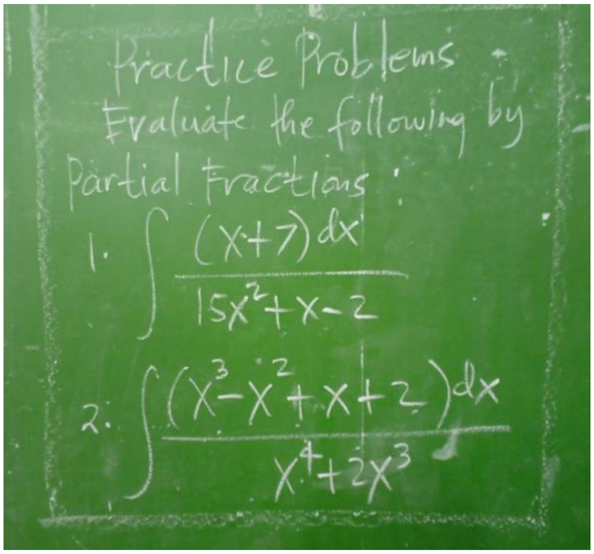

Problem:

$$\int \frac{x^3-x^2+x+2}{x^4+2x^3}\,\mathrm{d}x$$

Factor the denominator:

$$=\int \frac{x^3-x^2+x+2}{x^3\cdot(x+2)}\,\mathrm{d}x$$

Perform partial fraction decomposition:

$$=\int\left(\frac{3}{2(x+2)}-\frac{1}{2x}+\frac{1}{x^3}\right)\mathrm{d}x$$

Apply linearity:

$$=\frac{3}{2}\int\frac{1}{x+2}\,\mathrm{d}x-\frac{1}{2}\int\frac{1}{x}\,\mathrm{d}x+\int\frac{1}{x^3}\,\mathrm{d}x$$

Now solving:

$$\int\frac{1}{x+2}\,\mathrm{d}x$$

Substitute $u=x+2\longrightarrow \mathrm{d}u=\mathrm{d}x$ (steps):

$$=\int\frac{1}{u}\,\mathrm{d}u$$

This is a standard integral:

$$=\ln(u)$$

Undo substitution $u=x+2$:

$$=\ln(x+2)$$

Now solving:

$$\int\frac{1}{x}\,\mathrm{d}x$$

Use previous result:

$$=\ln(x)$$

Now solving:

$$\int\frac{1}{x^3}\,\mathrm{d}x$$

Apply power rule:

$$\int x^{\mathrm{n}}\,\mathrm{d}x=\frac{x^{\mathrm{n}+1}}{\mathrm{n}+1}\text{ with }\mathrm{n}=-3:$$

$$=-\frac{1}{2x^2}$$

Plug in solved integrals:

$$\frac{3}{2}\int\frac{1}{x+2}\,\mathrm{d}x-\frac{1}{2}\int\frac{1}{x}\,\mathrm{d}x+\int\frac{1}{x^3}\,\mathrm{d}x$$

$$=\frac{3\ln(x+2)}{2}-\frac{\ln(x)}{2}-\frac{1}{2x^2}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int\frac{x^3-x^2+x+2}{x^4+2x^3}\,\mathrm{d}x$$

$$=\frac{3\ln(|x+2|)}{2}-\frac{\ln(|x|)}{2}-\frac{1}{2x^2}+C$$

Rewrite/simplify:

$$=\frac{3\ln(|x+2|)-\ln(|x|)-\frac{1}{x^2}}{2}+C$$

Problem:

$$\int\frac{x+7}{15x^2+x-2}\,\mathrm{d}x$$

Write $x+7$ as $\frac{1}{30}(30x+1)+\frac{209}{30}$ and split:

$$=\int\left(\frac{30x+1}{30(15x^2+x-2)}+\frac{209}{30(15x^2+x-2)}\right)\mathrm{d}x$$

Apply linearity:

$$=\frac{1}{30}\int\frac{30x+1}{15x^2+x-2}\,\mathrm{d}x+\frac{209}{30}\int\frac{1}{15x^2+x-2}\,\mathrm{d}x$$

Now solving:

$$\int\frac{30x+1}{15x^2+x-2}\,\mathrm{d}x$$

Substitute $u=15x^2+x-2\longrightarrow \mathrm{d}u=(30x+1)\,\mathrm{d}x$ (steps):

$$=\int\frac{1}{u}\,\mathrm{d}u$$

This is a standard integral:

$$=\ln(u)$$

Undo substitution $u=15x^2+x-2$:

$$=\ln(15x^2+x-2)$$

Now solving:

$$\int\frac{1}{15x^2+x-2}\,\mathrm{d}x$$

Factor the denominator:

$$=\int\frac{1}{(3x-1)(5x+2)}\,\mathrm{d}x$$

Perform partial fraction decomposition:

$$=\int\left(\frac{3}{11(3x-1)}-\frac{5}{11(5x+2)}\right)\mathrm{d}x$$

Apply linearity:

$$=\frac{3}{11}\int\frac{1}{3x-1}\,\mathrm{d}x-\frac{5}{11}\int\frac{1}{5x+2}\,\mathrm{d}x$$

Now solving:

$$\int\frac{1}{3x-1}\,\mathrm{d}x$$

Substitute $u=3x-1\longrightarrow \mathrm{d}u=3\,\mathrm{d}x$ (steps):

$$=\frac{1}{3}\int\frac{1}{u}\,\mathrm{d}u$$

Now solving:

$$\int\frac{1}{u}\,\mathrm{d}u$$

Use previous result:

$$=\ln(u)$$

Plug in solved integrals:

$$\frac{1}{3}\int\frac{1}{u}\,\mathrm{d}u$$

$$=\frac{\ln(u)}{3}$$

Undo substitution $u=3x-1$:

$$=\frac{\ln(3x-1)}{3}$$

Now solving:

$$\int\frac{1}{5x+2}\,\mathrm{d}x$$

Substitute $u=5x+2\longrightarrow \mathrm{d}u=5\,\mathrm{d}x$ (steps):

$$=\frac{1}{5}\int\frac{1}{u}\,\mathrm{d}u$$

Now solving:

$$\int\frac{1}{u}\,\mathrm{d}u$$

Use previous result:

$$=\ln(u)$$

Plug in solved integrals:

$$\frac{1}{5}\int\frac{1}{u}\,\mathrm{d}u$$

$$=\frac{\ln(u)}{5}$$

Undo substitution $u=5x+2$:

$$=\frac{\ln(5x+2)}{5}$$

Plug in solved integrals:

$$\frac{3}{11}\int\frac{1}{3x-1}\,\mathrm{d}x-\frac{5}{11}\int\frac{1}{5x+2}\,\mathrm{d}x$$

$$=\frac{\ln(3x-1)}{11}-\frac{\ln(5x+2)}{11}$$

Plug in solved integrals:

$$\frac{1}{30}\int\frac{30x+1}{15x^2+x-2}\,\mathrm{d}x+\frac{209}{30}\int\frac{1}{15x^2+x-2}\,\mathrm{d}x$$

$$=\frac{\ln(15x^2+x-2)}{30}-\frac{19\ln(5x+2)}{30}+\frac{19\ln(3x-1)}{30}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int\frac{x+7}{15x^2+x-2}\,\mathrm{d}x$$

$$=\frac{\ln(|15x^2+x-2|)}{30}-\frac{19\ln(|5x+2|)}{30}+\frac{19\ln(|3x-1|)}{30}+C$$

Rewrite/simplify:

$$=\frac{\ln(|15x^2+x-2|)-19\ln(|5x+2|)+19\ln(|3x-1|)}{30}+C$$

PRACTICE PROBLEMS [PARTIAL FRACTIONS]