Integration by Parts

EXERCISE 10.10

Evaluate each of the following:

1. $\int x\cos x\,dx$
2. $\int e^{x}\cos x\,dx$
3. $\int e^{-x}\cos 2x\,dx$
4. $\int \text{Arcsin}\,4x\,dx$
5. $\int \text{Arctan}\,2x\,dx$
6. $\int x\ln x\,dx$
7. $\int \sec^3 x\,dx$
8. $\int \ln^2 x\,dx$
9. $\int x\cos^2 2x\,dx$
10. $\int (x + e^{x})^2\,dx$
11. $\int (e^{x} + 2x)^2\,dx$
12. $\int \dfrac{x\,\text{Arcsin}\,x}{\sqrt{1-x^2}}\,dx$
13. $\int \sin x \ln(1 + \sin x)\,dx$
14. $\int e^{x}\sin x\cos x\,dx$
15. $\int \dfrac{xe^{x}\,dx}{(1 + x)^2}$
16. $\int x\sin^2 x\,dx$
17. $\int x^2\,\text{Arcsin}\,x\,dx$
18. $\int \dfrac{x^3\,dx}{\sqrt{4-x^2}}$
19. $\int x^3\sqrt{1 + x^2}\,dx$
20. $\int x(2x + 1)^{10}\,dx$

Problem:

$$\int x\cos(x)\,dx$$

Integrate by parts: $\int fg' = fg - \int f'g$

$$f = x,\quad g' = \cos(x)$$
$$\downarrow \text{steps} \qquad \downarrow \text{steps}$$
$$f' = 1,\quad g = \sin(x):$$

$$= x\sin(x) - \int \sin(x)\,dx$$

Now solving:

$$\int \sin(x)\,dx$$

This is a standard integral:

$$= -\cos(x)$$

Plug in solved integrals:

$$x\sin(x) - \int \sin(x)\,dx$$
$$= x\sin(x) + \cos(x)$$

The problem is solved:

$$\int x\cos(x)\,dx$$
$$= x\sin(x) + \cos(x) + C$$

Problem:

$$\int e^{x}\cos(x)\,dx$$

We will integrate by parts twice in a row: $\int fg' = fg - \int f'g$.

First time:

$$f = \cos(x),\quad g' = e^{x}$$
$$\downarrow \text{steps} \qquad \downarrow \text{steps}$$
$$f' = \boxed{-\sin(x)},\quad g = \boxed{e^{x}}:$$

$$= e^{x}\cos(x) - \int -e^{x}\sin(x)\,dx$$

Second time:

$$f = \boxed{-\sin(x)},\quad g' = \boxed{e^{x}}$$
$$\downarrow \text{steps} \qquad \downarrow \text{steps}$$
$$f' = -\cos(x),\quad g = e^{x}:$$

$$= e^{x}\cos(x) - \left(-e^{x}\sin(x) - \int -e^{x}\cos(x)\,dx\right)$$

Apply linearity:

$$= e^{x}\cos(x) - \left(-e^{x}\sin(x) + \int e^{x}\cos(x)\,dx\right)$$

The integral $\int e^{x}\cos(x)\,dx$ appears again on the right side of the equation, we can solve for it:

$$= \frac{e^{x}\sin(x) + e^{x}\cos(x)}{2}$$

The problem is solved:

$$\int e^{x}\cos(x)\,dx$$
$$= \frac{e^{x}\sin(x) + e^{x}\cos(x)}{2} + C$$

Rewrite/simplify:

$$= \frac{e^{x}\cdot(\sin(x) + \cos(x))}{2} + C$$

Problem:

$$\int e^{-x}\cos(2x)\,dx$$

We will integrate by parts twice in a row: $\int fg' = fg - \int f'g$.

First time:

$$f = \cos(2x), \quad g' = e^{-x}$$
$$\downarrow \text{steps} \qquad \downarrow \text{steps}$$
$$f' = \boxed{-2\sin(2x)},\ g = \boxed{-e^{-x}}:$$

$$= -e^{-x}\cos(2x) - \int 2e^{-x}\sin(2x)\,dx$$

Second time:

$$f = \boxed{-2\sin(2x)},\ g' = \boxed{-e^{-x}}$$
$$\downarrow \text{steps} \qquad \downarrow \text{steps}$$
$$f' = -4\cos(2x),\ g = e^{-x}:$$

$$= -e^{-x}\cos(2x) - \left(-2e^{-x}\sin(2x) - \int -4e^{-x}\cos(2x)\,dx\right)$$

Apply linearity:

$$= -e^{-x}\cos(2x) - \left(-2e^{-x}\sin(2x) + 4\int e^{-x}\cos(2x)\,dx\right)$$

The integral $\int e^{-x}\cos(2x)\,dx$ appears again on the right side of the equation, we can solve for it:

$$= \frac{2e^{-x}\sin(2x) - e^{-x}\cos(2x)}{5}$$

The problem is solved:

$$\int e^{-x}\cos(2x)\,dx$$
$$= \frac{2e^{-x}\sin(2x) - e^{-x}\cos(2x)}{5} + C$$

Rewrite/simplify:

$$= \frac{e^{-x}\cdot(2\sin(2x) - \cos(2x))}{5} + C$$

Problem:

$$\int \arcsin(4x)\,dx$$

Substitute $u = 4x \longrightarrow du = 4\,dx$ (steps):

$$= \frac{1}{4}\int \arcsin(u)\,du$$

Now solving:

$$\int \arcsin(u)\,du$$

Integrate by parts: $\int fg' = fg - \int f'g$

$$f = \arcsin(u),\ g' = 1$$
$$\downarrow \text{steps} \qquad \downarrow \text{steps}$$
$$f' = \frac{1}{\sqrt{1-u^2}},\ g = u:$$

$$= u\arcsin(u) - \int \frac{u}{\sqrt{1-u^2}}\,du$$

Now solving:

$$\int \frac{u}{\sqrt{1-u^2}}\,du$$

Substitute $v = 1 - u^2 \longrightarrow dv = -2u\,du$ (steps):

$$= -\frac{1}{2}\int \frac{1}{\sqrt{v}}\,dv$$

Now solving:

$$\int \frac{1}{\sqrt{v}}\,dv$$

Apply power rule:

$$\int v^n\,dv = \frac{v^{n+1}}{n+1} \text{ with } n = -\frac{1}{2}:$$
$$= 2\sqrt{v}$$

Plug in solved integrals:

$$-\frac{1}{2}\int \frac{1}{\sqrt{v}}\,dv$$
$$= -\sqrt{v}$$

Undo substitution $v = 1 - u^2$:

$$= -\sqrt{1-u^2}$$

Plug in solved integrals:

$$u\arcsin(u) - \int \frac{u}{\sqrt{1-u^2}}\,du$$
$$= u\arcsin(u) + \sqrt{1-u^2}$$

Plug in solved integrals:

$$\frac{1}{4}\int \arcsin(u)\,du$$
$$= \frac{u\arcsin(u)}{4} + \frac{\sqrt{1-u^2}}{4}$$

Undo substitution $u = 4x$:

$$= x\arcsin(4x) + \frac{\sqrt{1-16x^2}}{4}$$

The problem is solved:

$$\int \arcsin(4x)\,dx$$
$$= x\arcsin(4x) + \frac{\sqrt{1-16x^2}}{4} + C$$

Problem:

$$\int \arctan(2x)\,dx$$

Substitute $u = 2x \longrightarrow du = 2\,dx$ (steps):

$$= \frac{1}{2}\int \arctan(u)\,du$$

Now solving:

$$\int \arctan(u)\,du$$

Integrate by parts: $\int fg' = fg - \int f'g$

$$f = \arctan(u),\ g' = 1$$

$$\downarrow \text{steps} \qquad \downarrow \text{steps}$$

$$f' = \frac{1}{u^2+1}, \quad g = u:$$

$$= u\arctan(u) - \int \frac{u}{u^2+1}\,du$$

Now solving:

$$\int \frac{u}{u^2+1}\,du$$

Substitute $v = u^2+1 \longrightarrow dv = 2u\,du$ (steps):

$$= \frac{1}{2}\int \frac{1}{v}\,dv$$

Now solving:

$$\int \frac{1}{v}\,dv$$

This is a standard integral:

$$= \ln(v)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{v}\,dv$$

$$= \frac{\ln(v)}{2}$$

Undo substitution $v = u^2+1$:

$$= \frac{\ln(u^2+1)}{2}$$

Plug in solved integrals:

$$u\arctan(u) - \int \frac{u}{u^2+1}\,du$$

$$= u\arctan(u) - \frac{\ln(u^2+1)}{2}$$

Plug in solved integrals:

$$\frac{1}{2}\int \arctan(u)\,du$$

$$= \frac{u\arctan(u)}{2} - \frac{\ln(u^2+1)}{4}$$

Undo substitution $u = 2x$:

$$= x\arctan(2x) - \frac{\ln(4x^2+1)}{4}$$

The problem is solved:

$$\int \arctan(2x)\,dx$$

$$= x\arctan(2x) - \frac{\ln(4x^2+1)}{4} + C$$

Problem:

$$\int x\ln(x)\,dx$$

Integrate by parts: $\int fg' = fg - \int f'g$

$$f = \ln(x),\ g' = x$$

$$\downarrow \text{steps} \qquad \downarrow \text{steps}$$

$$f' = \frac{1}{x}, \quad g = \frac{x^2}{2}:$$

$$= \frac{x^2\ln(x)}{2} - \int \frac{x}{2}\,dx$$

Now solving:

$$\int \frac{x}{2}\,dx$$

Apply linearity:

$$= \frac{1}{2}\int x\,dx$$

Now solving:

$$\int x\,dx$$

Apply power rule:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \text{ with } n = 1:$$

$$= \frac{x^2}{2}$$

Plug in solved integrals:

$$\frac{1}{2}\int x\,dx$$

$$= \frac{x^2}{4}$$

Plug in solved integrals:

$$\frac{x^2\ln(x)}{2} - \int \frac{x}{2}\,dx$$

$$= \frac{x^2\ln(x)}{2} - \frac{x^2}{4}$$

The problem is solved:

$$\int x\ln(x)\,dx$$

$$= \frac{x^2\ln(x)}{2} - \frac{x^2}{4} + C$$

Rewrite/simplify:

$$= \frac{x^2\cdot(2\ln(x)-1)}{4} + C$$

Problem:

$$\int \sec^3(x)\,dx$$

Apply reduction formula:

$$\int \sec^n(x)\,dx = \frac{n-2}{n-1}\int \sec^{n-2}(x)\,dx + \frac{\sec^{n-2}(x)\tan(x)}{n-1}$$

with $n = 3$:

$$= \frac{\sec(x)\tan(x)}{2} + \frac{1}{2}\int \sec(x)\,dx$$

Now solving:

$$\int \sec(x)\,dx$$

This is a standard integral:

(full steps)

$$= \ln(\tan(x) + \sec(x))$$

Plug in solved integrals:

$$\frac{\sec(x)\tan(x)}{2} + \frac{1}{2}\int \sec(x)\,dx$$

$$= \frac{\ln(\tan(x)+\sec(x))}{2} + \frac{\sec(x)\tan(x)}{2}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \sec^3(x)\,dx$$

$$= \frac{\ln(|\tan(x)+\sec(x)|)}{2} + \frac{\sec(x)\tan(x)}{2} + C$$

Rewrite/simplify:

$$= \frac{\ln(|\tan(x)+\sec(x)|) + \sec(x)\tan(x)}{2} + C$$

Problem:

$$\int \ln^2(x)\,dx$$

Integrate by parts: $\int fg' = fg - \int f'g$

$$f = \ln^2(x),\quad g' = 1$$

↓ steps ↓ steps

$$f' = \frac{2\ln(x)}{x},\ g = x:$$

$$= x\ln^2(x) - \int 2\ln(x)\,dx$$

Now solving:

$$\int 2\ln(x)\,dx$$

Apply linearity:

$$= 2\int \ln(x)\,dx$$

Now solving:

$$\int \ln(x)\,dx$$

This is a standard integral:

(full steps)

$$= x\ln(x) - x$$

Plug in solved integrals:

$$2\int \ln(x)\,dx$$

$$= 2x\ln(x) - 2x$$

Plug in solved integrals:

$$x\ln^2(x) - \int 2\ln(x)\,dx$$

$$= x\ln^2(x) - 2x\ln(x) + 2x$$

The problem is solved:

$$\int \ln^2(x)\,dx$$

$$= x\ln^2(x) - 2x\ln(x) + 2x + C$$

Rewrite/simplify:

$$= x\cdot\left(\ln^2(x) - 2\ln(x) + 2\right) + C$$

Problem:

$$\int x\cos^2(2x)\,dx$$

Apply product-to-sum formulas:

$$\sin(x)\sin(y) = \tfrac{1}{2}(\cos(y-x) - \cos(y+x)),\ \sin^2(x) = \tfrac{1}{2}(1-\cos(2x)),$$
$$\cos(x)\cos(y) = \tfrac{1}{2}(\cos(y+x) + \cos(y-x)),\ \cos^2(x) = \tfrac{1}{2}(\cos(2x)+1),$$
$$\sin(x)\cos(y) = \tfrac{1}{2}(\sin(y+x) - \sin(y-x)),\ \cos(x)\sin(x) = \tfrac{1}{2}\sin(2x)$$

$$= \int x\cdot\left(\frac{\cos(4x)}{2} + \frac{1}{2}\right)dx$$

Expand:

$$= \int\left(\frac{x\cos(4x)}{2} + \frac{x}{2}\right)dx$$

Apply linearity:

$$= \frac{1}{2}\int x\cos(4x)\,dx + \frac{1}{2}\int x\,dx$$

Now solving:

$$\int x\cos(4x)\,dx$$

Integrate by parts: $\int fg' = fg - \int f'g$

$$f = x,\quad g' = \cos(4x)$$

↓ steps ↓ steps

$$f' = 1,\quad g = \frac{\sin(4x)}{4}:$$

$$= \frac{x\sin(4x)}{4} - \int \frac{\sin(4x)}{4}\,dx$$

Now solving:

$$\int \frac{\sin(4x)}{4}\,dx$$

Substitute $u = 4x \longrightarrow du = 4\,dx$ (steps):

$$= \frac{1}{16}\int \sin(u)\,du$$

Now solving:

$$\int \sin(u)\,du$$

This is a standard integral:

$$= -\cos(u)$$

Plug in solved integrals:

$$\frac{1}{16}\int \sin(u)\,du$$

$$= -\frac{\cos(u)}{16}$$

Undo substitution $u = 4x$:

$$= -\frac{\cos(4x)}{16}$$

Plug in solved integrals:

$$\frac{x\sin(4x)}{4} - \int \frac{\sin(4x)}{4}\,dx$$

$$= \frac{x\sin(4x)}{4} + \frac{\cos(4x)}{16}$$

Now solving:

$$\int x\,dx$$

Apply power rule:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \text{ with } n = 1:$$

$$= \frac{x^2}{2}$$

Plug in solved integrals:

$$\frac{1}{2}\int x\cos(4x)\,dx + \frac{1}{2}\int x\,dx$$

$$= \frac{x\sin(4x)}{8} + \frac{\cos(4x)}{32} + \frac{x^2}{4}$$

The problem is solved:

$$\int x\cos^2(2x)\,dx$$

$$= \frac{x\sin(4x)}{8} + \frac{\cos(4x)}{32} + \frac{x^2}{4} + C$$

Rewrite/simplify:

$$= \frac{4x\cdot(\sin(4x) + 2x) + \cos(4x)}{32} + C$$

Problem:

$$\int (e^x + x)^2 \, dx$$

Expand:

$$= \int \left(e^{2x} + 2xe^x + x^2\right) dx$$

Apply linearity:

$$= \int e^{2x} \, dx + 2 \int xe^x \, dx + \int x^2 \, dx$$

Now solving:

$$\int e^{2x} \, dx$$

Substitute $u = 2x \longrightarrow du = 2 \, dx$ (steps):

$$= \frac{1}{2} \int e^u \, du$$

Now solving:

$$\int e^u \, du$$

Apply exponential rule:

$$\int a^u \, du = \frac{a^u}{\ln(a)} \text{ with } a = e:$$

$$= e^u$$

Plug in solved integrals:

$$\frac{1}{2} \int e^u \, du$$

$$= \frac{e^u}{2}$$

Undo substitution $u = 2x$:

$$= \frac{e^{2x}}{2}$$

Now solving:

$$\int xe^x \, dx$$

Integrate by parts: $\int fg' = fg - \int f'g$

$$f = x, \quad g' = e^x$$

$$\downarrow \text{steps} \quad \downarrow \text{steps}$$

$$f' = 1, \quad g = e^x:$$

$$= xe^x - \int e^x \, dx$$

Now solving:

$$\int e^x \, dx$$

Use previous result:

$$= e^x$$

Plug in solved integrals:

$$xe^x - \int e^x \, dx$$

$$= xe^x - e^x$$

Now solving:

$$\int x^2 \, dx$$

Apply power rule:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} \text{ with } n = 2:$$

$$= \frac{x^3}{3}$$

Plug in solved integrals:

$$\int e^{2x} \, dx + 2 \int xe^x \, dx + \int x^2 \, dx$$

$$= \frac{e^{2x}}{2} + 2xe^x - 2e^x + \frac{x^3}{3}$$

The problem is solved:

$$\int (e^x + x)^2 \, dx$$

$$= \frac{e^{2x}}{2} + 2xe^x - 2e^x + \frac{x^3}{3} + C$$

Rewrite/simplify:

$$= \frac{3e^{2x} + 2\left(6(x-1)e^x + x^3\right)}{6} + C$$

Problem:

$$\int (e^x + 2x)^2 \, dx$$

Expand:

$$= \int \left(e^{2x} + 4xe^x + 4x^2\right) dx$$

Apply linearity:

$$= \int e^{2x} \, dx + 4 \int xe^x \, dx + 4 \int x^2 \, dx$$

Now solving:

$$\int e^{2x} \, dx$$

Substitute $u = 2x \longrightarrow du = 2 \, dx$ (steps):

$$= \frac{1}{2} \int e^u \, du$$

Now solving:

$$\int e^u \, du$$

Apply exponential rule:

$$\int a^u \, du = \frac{a^u}{\ln(a)} \text{ with } a = e:$$

$$= e^u$$

Plug in solved integrals:

$$\frac{1}{2} \int e^u \, du$$

$$= \frac{e^u}{2}$$

Undo substitution $u = 2x$:

$$= \frac{e^{2x}}{2}$$

Now solving:

$$\int xe^x \, dx$$

Integrate by parts: $\int fg' = fg - \int f'g$

$$f = x, \quad g' = e^x$$

$$\downarrow \text{steps} \quad \downarrow \text{steps}$$

$$f' = 1, \quad g = e^x:$$

$$= xe^x - \int e^x \, dx$$

Now solving:

$$\int e^x \, dx$$

Use previous result:

$$= e^x$$

Plug in solved integrals:

$$xe^x - \int e^x \, dx$$

$$= xe^x - e^x$$

Now solving:

$$\int x^2 \, dx$$

Apply power rule:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} \text{ with } n = 2:$$

$$= \frac{x^3}{3}$$

Plug in solved integrals:

$$\int e^{2x} \, dx + 4 \int xe^x \, dx + 4 \int x^2 \, dx$$

$$= \frac{e^{2x}}{2} + 4xe^x - 4e^x + \frac{4x^3}{3}$$

The problem is solved:

$$\int (e^x + 2x)^2 \, dx$$

$$= \frac{e^{2x}}{2} + 4xe^x - 4e^x + \frac{4x^3}{3} + C$$

Rewrite/simplify:

$$= \frac{3e^{2x} + 8\left(3(x-1)e^x + x^3\right)}{6} + C$$

Problem:

$$\int \frac{x\arcsin(x)}{\sqrt{1-x^2}}\,\mathrm{d}x$$

Integrate by parts: $\int \mathtt{f}\mathtt{g}' = \mathtt{f}\mathtt{g} - \int \mathtt{f}'\mathtt{g}$

$$\mathtt{f} = \arcsin(x),\ \mathtt{g}' = \frac{x}{\sqrt{1-x^2}}$$

↓ steps ↓ steps

$$\mathtt{f}' = \frac{1}{\sqrt{1-x^2}},\ \mathtt{g} = -\sqrt{1-x^2}:$$

$$= -\sqrt{1-x^2}\arcsin(x) - \int -1\,\mathrm{d}x$$

Now solving:

$$\int -1\,\mathrm{d}x$$

Apply linearity:

$$= -\int 1\,\mathrm{d}x$$

Now solving:

$$\int 1\,\mathrm{d}x$$

Apply constant rule:

$$= x$$

Plug in solved integrals:

$$-\int 1\,\mathrm{d}x$$

$$= -x$$

Plug in solved integrals:

$$-\sqrt{1-x^2}\arcsin(x) - \int -1\,\mathrm{d}x$$

$$= x - \sqrt{1-x^2}\arcsin(x)$$

The problem is solved:

$$\int \frac{x\arcsin(x)}{\sqrt{1-x^2}}\,\mathrm{d}x$$

$$= x - \sqrt{1-x^2}\arcsin(x) + C$$

Problem:

$$\int \sin(x)\ln(\sin(x)+1)\,\mathrm{d}x$$

Integrate by parts: $\int \mathtt{f}\mathtt{g}' = \mathtt{f}\mathtt{g} - \int \mathtt{f}'\mathtt{g}$

$$\mathtt{f} = \ln(\sin(x)+1),\ \mathtt{g}' = \sin(x)$$

↓ steps ↓ steps

$$\mathtt{f}' = \frac{\cos(x)}{\sin(x)+1},\quad \mathtt{g} = -\cos(x):$$

$$= -\cos(x)\ln(\sin(x)+1) - \int -\frac{\cos^2(x)}{\sin(x)+1}\,\mathrm{d}x$$

Now solving:

$$\int -\frac{\cos^2(x)}{\sin(x)+1}\,\mathrm{d}x$$

Apply linearity:

$$= -\int \frac{\cos^2(x)}{\sin(x)+1}\,\mathrm{d}x$$

Now solving:

$$\int \frac{\cos^2(x)}{\sin(x)+1}\,\mathrm{d}x$$

Rewrite/simplify using trigonometric/hyperbolic identities:

$$= \int (1-\sin(x))\,\mathrm{d}x$$

... or choose an alternative:

Skip step

Apply linearity:

$$= \int 1\,\mathrm{d}x - \int \sin(x)\,\mathrm{d}x$$

Now solving:

$$\int 1\,\mathrm{d}x$$

Apply constant rule:

$$= x$$

Now solving:

$$\int \sin(x)\,\mathrm{d}x$$

This is a standard integral:

$$= -\cos(x)$$

Plug in solved integrals:

$$\int 1\,\mathrm{d}x - \int \sin(x)\,\mathrm{d}x$$

$$= \cos(x) + x$$

Plug in solved integrals:

$$-\int \frac{\cos^2(x)}{\sin(x)+1}\,\mathrm{d}x$$

$$= -\cos(x) - x$$

Plug in solved integrals:

$$-\cos(x)\ln(\sin(x)+1) - \int -\frac{\cos^2(x)}{\sin(x)+1}\,\mathrm{d}x$$

$$= -\cos(x)\ln(\sin(x)+1) + \cos(x) + x$$

The problem is solved:

$$\int \sin(x)\ln(\sin(x)+1)\,\mathrm{d}x$$

$$= -\cos(x)\ln(\sin(x)+1) + \cos(x) + x + C$$

Rewrite/simplify:

$$= x - \cos(x)\left(\ln(\sin(x)+1) - 1\right) + C$$

Problem:

$$\int e^x \cos(x)\sin(x)\,dx$$

Rewrite/simplify:

$$= \int \frac{e^x \sin(2x)}{2}\,dx$$

... or choose an alternative:

Skip simplification

Apply linearity:

$$= \frac{1}{2}\int e^x \sin(2x)\,dx$$

Now solving:

$$\int e^x \sin(2x)\,dx$$

We will integrate by parts twice in a row: $\int fg' = fg - \int f'g$.

First time:

$$f = \sin(2x), \quad g' = e^x$$

$\downarrow$ steps $\quad \downarrow$ steps

$$f' = \boxed{2\cos(2x)},\ g = \boxed{e^x}:$$

$$= e^x \sin(2x) - \int 2e^x \cos(2x)\,dx$$

Second time:

$$f = \boxed{2\cos(2x)}, \quad g' = \boxed{e^x}$$

$\downarrow$ steps $\quad \downarrow$ steps

$$f' = -4\sin(2x),\ g = e^x:$$

$$= e^x \sin(2x) - \left(2e^x \cos(2x) - \int -4e^x \sin(2x)\,dx\right)$$

Apply linearity:

$$= e^x \sin(2x) - \left(2e^x \cos(2x) + 4\int e^x \sin(2x)\,dx\right)$$

The integral $\int e^x \sin(2x)\,dx$ appears again on the right side of the equation, we can solve for it:

$$= \frac{e^x \sin(2x) - 2e^x \cos(2x)}{5}$$

Plug in solved integrals:

$$\frac{1}{2}\int e^x \sin(2x)\,dx$$

$$= \frac{e^x \sin(2x) - 2e^x \cos(2x)}{10}$$

The problem is solved:

$$\int \frac{e^x \sin(2x)}{2}\,dx$$

$$= \frac{e^x \sin(2x) - 2e^x \cos(2x)}{10} + C$$

Rewrite/simplify:

$$= \frac{e^x \cdot (\sin(2x) - 2\cos(2x))}{10} + C$$

Problem:

$$\int \frac{xe^x}{(x+1)^2}\,dx$$

Integrate by parts: $\int fg' = fg - \int f'g$

$$f = xe^x, \quad g' = \frac{1}{(x+1)^2}$$

$\downarrow$ steps $\quad \downarrow$ steps

$$f' = xe^x + e^x,\ g = -\frac{1}{x+1}:$$

$$= -\frac{xe^x}{x+1} - \int \frac{-xe^x - e^x}{x+1}\,dx$$

Now solving:

$$\int \frac{-xe^x - e^x}{x+1}\,dx$$

Rewrite:

$$= \int -e^x\,dx$$

Apply linearity:

$$= -\int e^x\,dx$$

Now solving:

$$\int e^x\,dx$$

Apply exponential rule:

$$\int a^x\,dx = \frac{a^x}{\ln(a)} \text{ with } a = e:$$

$$= e^x$$

Plug in solved integrals:

$$-\int e^x\,dx$$

$$= -e^x$$

Plug in solved integrals:

$$-\frac{xe^x}{x+1} - \int \frac{-xe^x - e^x}{x+1}\,dx$$

$$= e^x - \frac{xe^x}{x+1}$$

The problem is solved:

$$\int \frac{xe^x}{(x+1)^2}\,dx$$

$$= e^x - \frac{xe^x}{x+1} + C$$

Rewrite/simplify:

$$= \frac{e^x}{x+1} + C$$

Problem:

$$\int x\sin^2(x)\,dx$$

Apply product-to-sum formulas:

$$\sin(x)\sin(y)=\frac{1}{2}(\cos(y-x)-\cos(y+x)),\ \sin^2(x)=\frac{1}{2}(1-\cos(2x)),$$
$$\cos(x)\cos(y)=\frac{1}{2}(\cos(y+x)+\cos(y-x)),\ \cos^2(x)=\frac{1}{2}(\cos(2x)+1),$$
$$\sin(x)\cos(y)=\frac{1}{2}(\sin(y+x)-\sin(y-x)),\ \cos(x)\sin(x)=\frac{1}{2}\sin(2x)$$

$$=\int x\cdot\left(\frac{1}{2}-\frac{\cos(2x)}{2}\right)dx$$

Expand:

$$=\int\left(\frac{x}{2}-\frac{x\cos(2x)}{2}\right)dx$$

Apply linearity:

$$=\frac{1}{2}\int x\,dx-\frac{1}{2}\int x\cos(2x)\,dx$$

Now solving:

$$\int x\,dx$$

Apply power rule:

$$\int x^n\,dx=\frac{x^{n+1}}{n+1}\text{ with }n=1:$$

$$=\frac{x^2}{2}$$

Now solving:

$$\int x\cos(2x)\,dx$$

Integrate by parts: $\int fg'=fg-\int f'g$

$$f=x,\ \ g'=\cos(2x)$$

↓ steps ↓ steps

$$f'=1,\ \ g=\frac{\sin(2x)}{2}:$$

$$=\frac{x\sin(2x)}{2}-\int\frac{\sin(2x)}{2}\,dx$$

Now solving:

$$\int\frac{\sin(2x)}{2}\,dx$$

Substitute $u=2x\longrightarrow du=2\,dx$ (steps):

$$=\frac{1}{4}\int\sin(u)\,du$$

Now solving:

$$\int\sin(u)\,du$$

This is a standard integral:

$$=-\cos(u)$$

Plug in solved integrals:

$$\frac{1}{4}\int\sin(u)\,du$$

$$=-\frac{\cos(u)}{4}$$

Undo substitution $u=2x$:

$$=-\frac{\cos(2x)}{4}$$

Plug in solved integrals:

$$\frac{x\sin(2x)}{2}-\int\frac{\sin(2x)}{2}\,dx$$

$$=\frac{x\sin(2x)}{2}+\frac{\cos(2x)}{4}$$

Plug in solved integrals:

$$\frac{1}{2}\int x\,dx-\frac{1}{2}\int x\cos(2x)\,dx$$

$$=-\frac{x\sin(2x)}{4}-\frac{\cos(2x)}{8}+\frac{x^2}{4}$$

The problem is solved:

$$\int x\sin^2(x)\,dx$$

$$=-\frac{x\sin(2x)}{4}-\frac{\cos(2x)}{8}+\frac{x^2}{4}+C$$

Rewrite/simplify:

$$=-\frac{2x\cdot(\sin(2x)-x)+\cos(2x)}{8}+C$$

Problem:

$$\int x^2\arcsin(x)\,dx$$

Integrate by parts: $\int fg'=fg-\int f'g$

$$f=\arcsin(x),\ g'=x^2$$

↓ steps ↓ steps

$$f'=\frac{1}{\sqrt{1-x^2}},\ g=\frac{x^3}{3}:$$

$$=\frac{x^3\arcsin(x)}{3}-\int\frac{x^3}{3\sqrt{1-x^2}}\,dx$$

Now solving:

$$\int\frac{x^3}{3\sqrt{1-x^2}}\,dx$$

Apply linearity:

$$=\frac{1}{3}\int\frac{x^3}{\sqrt{1-x^2}}\,dx$$

Now solving:

$$\int\frac{x^3}{\sqrt{1-x^2}}\,dx$$

Substitute $u=1-x^2\longrightarrow du=-2x\,dx$ (steps):

$$=\frac{1}{2}\int\frac{u-1}{\sqrt{u}}\,du$$

... or choose an alternative:

Substitute x^2

Don't substitute

Now solving:

$$\int\frac{u-1}{\sqrt{u}}\,du$$

Expand:

$$=\int\left(\sqrt{u}-\frac{1}{\sqrt{u}}\right)du$$

Apply linearity:

$$=\int\sqrt{u}\,du-\int\frac{1}{\sqrt{u}}\,du$$

Now solving:

$$\int\sqrt{u}\,du$$

Apply power rule:

$$\int u^n\,du=\frac{u^{n+1}}{n+1}\text{ with }n=\frac{1}{2}:$$

$$=\frac{2u^{\frac{3}{2}}}{3}$$

Now solving:

$$\int\frac{1}{\sqrt{u}}\,du$$

Apply power rule with $n=-\frac{1}{2}$:

$$=2\sqrt{u}$$

Plug in solved integrals:

$$\int\sqrt{u}\,du-\int\frac{1}{\sqrt{u}}\,du$$

$$=\frac{2u^{\frac{3}{2}}}{3}-2\sqrt{u}$$

Plug in solved integrals:

$$\frac{1}{2}\int\frac{u-1}{\sqrt{u}}\,du$$

$$=\frac{u^{\frac{3}{2}}}{3}-\sqrt{u}$$

Undo substitution $u=1-x^2$:

$$=\frac{(1-x^2)^{\frac{3}{2}}}{3}-\sqrt{1-x^2}$$

Plug in solved integrals:

$$\frac{1}{3}\int\frac{x^3}{\sqrt{1-x^2}}\,dx$$

$$=\frac{(1-x^2)^{\frac{3}{2}}}{9}-\frac{\sqrt{1-x^2}}{3}$$

Plug in solved integrals:

$$\frac{x^3\arcsin(x)}{3}-\int\frac{x^3}{3\sqrt{1-x^2}}\,dx$$

$$=\frac{x^3\arcsin(x)}{3}-\frac{(1-x^2)^{\frac{3}{2}}}{9}+\frac{\sqrt{1-x^2}}{3}$$

The problem is solved:

$$\int x^2\arcsin(x)\,dx$$

$$=\frac{x^3\arcsin(x)}{3}-\frac{(1-x^2)^{\frac{3}{2}}}{9}+\frac{\sqrt{1-x^2}}{3}+C$$

Rewrite/simplify:

$$=\frac{3x^3\arcsin(x)+\sqrt{1-x^2}\,(x^2+2)}{9}+C$$

Problem:

$$\int \frac{x^3}{\sqrt{4-x^2}}\,dx$$

Substitute $u = 4 - x^2 \longrightarrow du = -2x\,dx$ (steps):

$$= \frac{1}{2}\int \frac{u-4}{\sqrt{u}}\,du$$

... or choose an alternative:

Substitute x^2

Don't substitute

Now solving:

$$\int \frac{u-4}{\sqrt{u}}\,du$$

Expand:

$$= \int \left(\sqrt{u} - \frac{4}{\sqrt{u}}\right) du$$

Apply linearity:

$$= \int \sqrt{u}\,du - 4\int \frac{1}{\sqrt{u}}\,du$$

Now solving:

$$\int \sqrt{u}\,du$$

Apply power rule:

$$\int u^n\,du = \frac{u^{n+1}}{n+1} \text{ with } n = \frac{1}{2}:$$

$$= \frac{2u^{\frac{3}{2}}}{3}$$

Now solving:

$$\int \frac{1}{\sqrt{u}}\,du$$

Apply power rule with $n = -\frac{1}{2}$:

$$= 2\sqrt{u}$$

Plug in solved integrals:

$$\int \sqrt{u}\,du - 4\int \frac{1}{\sqrt{u}}\,du$$

$$= \frac{2u^{\frac{3}{2}}}{3} - 8\sqrt{u}$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{u-4}{\sqrt{u}}\,du$$

$$= \frac{u^{\frac{3}{2}}}{3} - 4\sqrt{u}$$

Undo substitution $u = 4 - x^2$:

$$= \frac{\left(4-x^2\right)^{\frac{3}{2}}}{3} - 4\sqrt{4-x^2}$$

The problem is solved:

$$\int \frac{x^3}{\sqrt{4-x^2}}\,dx$$

$$= \frac{\left(4-x^2\right)^{\frac{3}{2}}}{3} - 4\sqrt{4-x^2} + C$$

Rewrite/simplify:

$$= -\frac{\sqrt{4-x^2}\left(x^2+8\right)}{3} + C$$

Problem:

$$\int x^3\sqrt{x^2+1}\,dx$$

Substitute $u = x^2 + 1 \longrightarrow du = 2x\,dx$ (steps):

$$= \frac{1}{2}\int \left(u^{\frac{3}{2}} - \sqrt{u}\right) du$$

... or choose an alternative:

Substitute x^2

Don't substitute

Now solving:

$$\int \left(u^{\frac{3}{2}} - \sqrt{u}\right) du$$

Apply linearity:

$$= \int u^{\frac{3}{2}}\,du - \int \sqrt{u}\,du$$

Now solving:

$$\int u^{\frac{3}{2}}\,du$$

Apply power rule:

$$\int u^n\,du = \frac{u^{n+1}}{n+1} \text{ with } n = \frac{3}{2}:$$

$$= \frac{2u^{\frac{5}{2}}}{5}$$

Now solving:

$$\int \sqrt{u}\,du$$

Apply power rule with $n = \frac{1}{2}$:

$$= \frac{2u^{\frac{3}{2}}}{3}$$

Plug in solved integrals:

$$\int u^{\frac{3}{2}}\,du - \int \sqrt{u}\,du$$

$$= \frac{2u^{\frac{5}{2}}}{5} - \frac{2u^{\frac{3}{2}}}{3}$$

Plug in solved integrals:

$$\frac{1}{2}\int \left(u^{\frac{3}{2}} - \sqrt{u}\right) du$$

$$= \frac{u^{\frac{5}{2}}}{5} - \frac{u^{\frac{3}{2}}}{3}$$

Undo substitution $u = x^2 + 1$:

$$= \frac{\left(x^2+1\right)^{\frac{5}{2}}}{5} - \frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$$

The problem is solved:

$$\int x^3\sqrt{x^2+1}\,dx$$

$$= \frac{\left(x^2+1\right)^{\frac{5}{2}}}{5} - \frac{\left(x^2+1\right)^{\frac{3}{2}}}{3} + C$$

Rewrite/simplify:

$$= \frac{\left(x^2+1\right)^{\frac{3}{2}}\left(3x^2-2\right)}{15} + C$$

Problem:

$$\int x \cdot (2x+1)^{10} \, dx$$

Substitute $u = 2x+1 \longrightarrow du = 2\,dx$ (steps):

$$= \frac{1}{4} \int (u-1)\, u^{10} \, du$$

... or choose an alternative:

Don't substitute

Now solving:

$$\int (u-1)\, u^{10} \, du$$

Expand:

$$= \int \left(u^{11} - u^{10}\right) du$$

Apply linearity:

$$= \int u^{11} \, du - \int u^{10} \, du$$

Now solving:

$$\int u^{11} \, du$$

Apply power rule:

$\int u^{n} \, du = \frac{u^{n+1}}{n+1}$ with $n = 11$:

$$= \frac{u^{12}}{12}$$

Now solving:

$$\int u^{10} \, du$$

Apply power rule with $n = 10$:

$$= \frac{u^{11}}{11}$$

Plug in solved integrals:

$$\int u^{11} \, du - \int u^{10} \, du$$

$$= \frac{u^{12}}{12} - \frac{u^{11}}{11}$$

Plug in solved integrals:

$$\frac{1}{4} \int (u-1)\, u^{10} \, du$$

$$= \frac{u^{12}}{48} - \frac{u^{11}}{44}$$

Undo substitution $u = 2x+1$:

$$= \frac{(2x+1)^{12}}{48} - \frac{(2x+1)^{11}}{44}$$

The problem is solved:

$$\int x \cdot (2x+1)^{10} \, dx$$

$$= \frac{(2x+1)^{12}}{48} - \frac{(2x+1)^{11}}{44} + C$$

Algebraic Substitutions

1. $\int \frac{dx}{x - x^{2/3}}$

2. $\int \frac{\sqrt[3]{x^2}\, dx}{\sqrt[3]{x^2} + 4}$

3. $\int \frac{(x^{1/3} - x^{1/4})\, dx}{4x^{1/2}}$

4. $\int \frac{x^{3/2}\, dx}{x + 1}$

5. $\int \frac{dx}{(x + 2)^{3/4} - (x + 2)^{1/2}}$

6. $\int \frac{(x + 4)\, dx}{(x + 2)\sqrt{x + 5}}$

7. $\int \sqrt{4 + \sqrt{x}}\, dx$

8. $\int \frac{x\,dx}{(2x + 1)^{4/3}}$

9. $\int x\,(x + 4)^{1/3}\, dx$

10. $\int \frac{dx}{(x + 1)\sqrt{x + 3}}$

11. $\int \frac{(4 - \sqrt{2x + 1})dx}{1 - 2x}$

12. $\int \frac{dx}{x^2 (4 + x^3)^{2/3}}$

13. $\int x^5 \sqrt{4 + x^3}\, dx$

14 $\int \frac{dx}{x^3 (2 + x^3)^{1/3}}$

15. $\int x^3 (4 + x^2)^{\frac{3}{2}}\, dx$

16. $\int x^8 \sqrt{1 + x^3}\, dx$

17. $\int \frac{dx}{x^4 \sqrt{1 + x^2}}$

18. $\int \frac{x^2\, dx}{(1 - x^3)^{\frac{3}{2}}}$

19. $\int \frac{dx}{x^2\, (81 + x^4)^{\frac{3}{4}}}$

20. $\int \frac{dx}{x\sqrt{4x - x^2}}$, Let $x = \frac{1}{z}$

21. $\int \frac{(x - x^3)^{1/3}\, dx}{x^4}$, Let $x = \frac{1}{z}$

22. $\int \frac{dx}{x^2 \sqrt{x^2 + 1}}$, Let $x = \frac{1}{z}$

23. $\int \frac{dx}{x\sqrt{x^2 + 2x - 1}}$, Let $x^2 + 2x - 1 = (z - x)^2$

24. $\int \frac{dx}{x\sqrt{x^2 - 2x + 5}}$, Let $x^2 - 2x + 5 = (z - x)^2$

25. $\int \frac{dx}{\sqrt{6 - x - x^2}}$, Let $6 - x - x^2 = (3 + x)^2 z^2$

26. $\int \frac{dx}{x\sqrt{2 + x - x^2}}$, Let $2 + x - x^2 = (x + 1)^2 z^2$

Problem:

$$\int \frac{1}{x - x^{\frac{2}{3}}}\, dx$$

Expand fraction by $\frac{1}{x^{\frac{2}{3}}}$:

$$= \int \frac{1}{(\sqrt[3]{x} - 1)\, x^{\frac{2}{3}}}\, dx$$

Substitute $u = \sqrt[3]{x} - 1 \longrightarrow du = \frac{1}{3x^{\frac{2}{3}}}\, dx$ (steps):

$$= 3\int \frac{1}{u}\, du$$

Now solving:

$$\int \frac{1}{u}\, du$$

This is a standard integral:

$$= \ln(u)$$

Plug in solved integrals:

$$3\int \frac{1}{u}\, du$$

$$= 3\ln(u)$$

Undo substitution $u = \sqrt[3]{x} - 1$:

$$= 3\ln(\sqrt[3]{x} - 1)$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{1}{x - x^{\frac{2}{3}}}\, dx$$

$$= 3\ln(|\sqrt[3]{x} - 1|) + C$$

Problem:

$$\int \frac{x^{\frac{2}{3}}}{x^{\frac{2}{3}}+4}\,\mathrm{d}x$$

Write $x^{\frac{2}{3}}$ as $x^{\frac{2}{3}}+4-4$ and split:

$$= \int \left(\frac{x^{\frac{2}{3}}+4}{x^{\frac{2}{3}}+4} - \frac{4}{x^{\frac{2}{3}}+4}\right)\mathrm{d}x$$

$$= \int \left(1 - \frac{4}{x^{\frac{2}{3}}+4}\right)\mathrm{d}x$$

Apply linearity:

$$= \int 1\,\mathrm{d}x - 4\int \frac{1}{x^{\frac{2}{3}}+4}\,\mathrm{d}x$$

Now solving:

$$\int 1\,\mathrm{d}x$$

Apply constant rule:

$$= x$$

Now solving:

$$\int \frac{1}{x^{\frac{2}{3}}+4}\,\mathrm{d}x$$

Substitute $u = \sqrt[3]{x} \longrightarrow \mathrm{d}u = \frac{1}{3x^{\frac{2}{3}}}\,\mathrm{d}x$ (steps), use:

$$x^{\frac{2}{3}} = u^2$$

$$= 3\int \frac{u^2}{u^2+4}\,\mathrm{d}u$$

Now solving:

$$\int \frac{u^2}{u^2+4}\,\mathrm{d}u$$

Write u^2 as u^2+4-4 and split:

$$= \int \left(\frac{u^2+4}{u^2+4} - \frac{4}{u^2+4}\right)\mathrm{d}u$$

$$= \int \left(1 - \frac{4}{u^2+4}\right)\mathrm{d}u$$

Apply linearity:

$$= \int 1\,\mathrm{d}u - 4\int \frac{1}{u^2+4}\,\mathrm{d}u$$

Now solving:

$$\int 1\,\mathrm{d}u$$

Use previous result:

$$= u$$

Now solving:

$$\int \frac{1}{u^2+4}\,\mathrm{d}u$$

Substitute $v = \frac{u}{2} \longrightarrow \mathrm{d}v = \frac{1}{2}\,\mathrm{d}u$ (steps):

$$= \int \frac{2}{4v^2+4}\,\mathrm{d}v$$

Simplify:

$$= \frac{1}{2}\int \frac{1}{v^2+1}\,\mathrm{d}v$$

Now solving:

$$\int \frac{1}{v^2+1}\,\mathrm{d}v$$

This is a standard integral:

$$= \arctan(v)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{v^2+1}\,\mathrm{d}v$$

$$= \frac{\arctan(v)}{2}$$

Undo substitution $v = \frac{u}{2}$:

$$= \frac{\arctan\left(\frac{u}{2}\right)}{2}$$

Plug in solved integrals:

$$\int 1\,\mathrm{d}u - 4\int \frac{1}{u^2+4}\,\mathrm{d}u$$

$$= u - 2\arctan\left(\frac{u}{2}\right)$$

Plug in solved integrals:

$$3\int \frac{u^2}{u^2+4}\,\mathrm{d}u$$

$$= 3u - 6\arctan\left(\frac{u}{2}\right)$$

Undo substitution $u = \sqrt[3]{x}$:

$$= 3\sqrt[3]{x} - 6\arctan\left(\frac{\sqrt[3]{x}}{2}\right)$$

Plug in solved integrals:

$$\int 1\,\mathrm{d}x - 4\int \frac{1}{x^{\frac{2}{3}}+4}\,\mathrm{d}x$$

$$= x - 12\sqrt[3]{x} + 24\arctan\left(\frac{\sqrt[3]{x}}{2}\right)$$

The problem is solved:

$$\int \frac{x^{\frac{2}{3}}}{x^{\frac{2}{3}}+4}\,\mathrm{d}x$$

$$= x - 12\sqrt[3]{x} + 24\arctan\left(\frac{\sqrt[3]{x}}{2}\right) + C$$

Problem:

$$\int \frac{\sqrt[3]{x}-\sqrt[4]{x}}{4\sqrt{x}}\,\mathrm{d}x$$

Expand:

$$= \int \left(\frac{1}{4\sqrt[6]{x}} - \frac{1}{4\sqrt[4]{x}}\right)\mathrm{d}x$$

Apply linearity:

$$= \frac{1}{4}\int \frac{1}{\sqrt[6]{x}}\,\mathrm{d}x - \frac{1}{4}\int \frac{1}{\sqrt[4]{x}}\,\mathrm{d}x$$

Now solving:

$$\int \frac{1}{\sqrt[6]{x}}\,\mathrm{d}x$$

Apply power rule:

$$\int x^{\mathrm{n}}\,\mathrm{d}x = \frac{x^{\mathrm{n}+1}}{\mathrm{n}+1} \text{ with } \mathrm{n} = -\frac{1}{6}:$$

$$= \frac{6x^{\frac{5}{6}}}{5}$$

Now solving:

$$\int \frac{1}{\sqrt[4]{x}}\,\mathrm{d}x$$

Apply power rule with $\mathrm{n} = -\frac{1}{4}$:

$$= \frac{4x^{\frac{3}{4}}}{3}$$

Plug in solved integrals:

$$\frac{1}{4}\int \frac{1}{\sqrt[6]{x}}\,\mathrm{d}x - \frac{1}{4}\int \frac{1}{\sqrt[4]{x}}\,\mathrm{d}x$$

$$= \frac{3x^{\frac{5}{6}}}{10} - \frac{x^{\frac{3}{4}}}{3}$$

The problem is solved:

$$\int \frac{\sqrt[3]{x}-\sqrt[4]{x}}{4\sqrt{x}}\,\mathrm{d}x$$

$$= \frac{3x^{\frac{5}{6}}}{10} - \frac{x^{\frac{3}{4}}}{3} + C$$

Problem:

$$\int \frac{x^{\frac{3}{2}}}{x+1}\,dx$$

Substitute $u = \sqrt{x} \longrightarrow du = \frac{1}{2\sqrt{x}}\,dx$ (steps), use:

$$x^2 = u^4$$
$$x = u^2$$

$$= 2\int \frac{u^4}{u^2+1}\,du$$

Now solving:

$$\int \frac{u^4}{u^2+1}\,du$$

Perform polynomial long division:

$$= \int \left(\frac{1}{u^2+1} + u^2 - 1\right) du$$

Apply linearity:

$$= \int \frac{1}{u^2+1}\,du + \int u^2\,du - \int 1\,du$$

Now solving:

$$\int \frac{1}{u^2+1}\,du$$

This is a standard integral:

$$= \arctan(u)$$

Now solving:

$$\int u^2\,du$$

Apply power rule:

$$\int u^n\,du = \frac{u^{n+1}}{n+1} \text{ with } n = 2:$$

$$= \frac{u^3}{3}$$

Now solving:

$$\int 1\,du$$

Apply constant rule:

$$= u$$

Plug in solved integrals:

$$\int \frac{1}{u^2+1}\,du + \int u^2\,du - \int 1\,du$$

$$= \arctan(u) + \frac{u^3}{3} - u$$

Plug in solved integrals:

$$2\int \frac{u^4}{u^2+1}\,du$$

$$= 2\arctan(u) + \frac{2u^3}{3} - 2u$$

Undo substitution $u = \sqrt{x}$:

$$= \frac{2x^{\frac{3}{2}}}{3} - 2\sqrt{x} + 2\arctan(\sqrt{x})$$

The problem is solved:

$$\int \frac{x^{\frac{3}{2}}}{x+1}\,dx$$

$$= \frac{2x^{\frac{3}{2}}}{3} - 2\sqrt{x} + 2\arctan(\sqrt{x}) + C$$

Rewrite/simplify:

$$= \frac{2\left((x-3)\sqrt{x} + 3\arctan(\sqrt{x})\right)}{3} + C$$

Problem:

$$\int \frac{1}{(x+2)^{\frac{3}{4}} - \sqrt{x+2}}\,dx$$

Substitute $u = -x-2 \longrightarrow du = -dx$ (steps):

$$= -\int \frac{1}{(-u)^{\frac{3}{4}} - \sqrt{-u}}\,du$$

Now solving:

$$\int \frac{1}{(-u)^{\frac{3}{4}} - \sqrt{-u}}\,du$$

Substitute $v = \sqrt[4]{-u} \longrightarrow dv = -\frac{1}{4(-u)^{\frac{3}{4}}}\,du$ (steps):

$$= -4\int \frac{v}{v-1}\,dv$$

Now solving:

$$\int \frac{v}{v-1}\,dv$$

Substitute $w = v-1 \longrightarrow dw = dv$ (steps):

$$= \int \frac{w+1}{w}\,dw$$

... or choose an alternative:

Don't substitute

Expand:

$$= \int \left(\frac{1}{w} + 1\right) dw$$

Apply linearity:

$$= \int \frac{1}{w}\,dw + \int 1\,dw$$

Now solving:

$$\int \frac{1}{w}\,dw$$

This is a standard integral:

$$= \ln(w)$$

Now solving:

$$\int 1\,dw$$

Apply constant rule:

$$= w$$

Plug in solved integrals:

$$\int \frac{1}{w}\,dw + \int 1\,dw$$

$$= \ln(w) + w$$

Undo substitution $w = v-1$:

$$= v + \ln(v-1) - 1$$

Plug in solved integrals:

$$-4\int \frac{v}{v-1}\,dv$$

$$= -4v - 4\ln(v-1) + 4$$

Undo substitution $v = \sqrt[4]{-u}$:

$$= -4\sqrt[4]{-u} - 4\ln\left(\sqrt[4]{-u} - 1\right) + 4$$

Plug in solved integrals:

$$-\int \frac{1}{(-u)^{\frac{3}{4}} - \sqrt{-u}}\,du$$

$$= 4\sqrt[4]{-u} + 4\ln\left(\sqrt[4]{-u} - 1\right) - 4$$

Undo substitution $u = -x-2$:

$$= 4\ln\left(\sqrt[4]{x+2} - 1\right) + 4\sqrt[4]{x+2} - 4$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{1}{(x+2)^{\frac{3}{4}} - \sqrt{x+2}}\,dx$$

$$= 4\ln\left(\left|\sqrt[4]{x+2} - 1\right|\right) + 4\sqrt[4]{x+2} - 4 + C$$

Rewrite/simplify:

$$= 4\left(\ln\left(\left|\sqrt[4]{x+2} - 1\right|\right) + \sqrt[4]{x+2}\right) + C$$

Problem:

$$\int \frac{x+4}{(x+2)\sqrt{x+5}}\,dx$$

Substitute $u = \sqrt{x+5} \longrightarrow du = \frac{1}{2\sqrt{x+5}}\,dx$ (steps):

$$= 2\int \frac{u^2-1}{u^2-3}\,du$$

... or choose an alternative:

Substitute $x+5$

Substitute $x+2$

Now solving:

$$\int \frac{u^2-1}{u^2-3}\,du$$

Write u^2-1 as u^2-3+2 and split:

$$= \int \left(\frac{u^2-3}{u^2-3} + \frac{2}{u^2-3}\right) du$$

$$= \int \left(\frac{2}{u^2-3} + 1\right) du$$

Apply linearity:

$$= 2\int \frac{1}{u^2-3}\,du + \int 1\,du$$

Now solving:

$$\int \frac{1}{u^2-3}\,du$$

Factor the denominator:

$$= \int \frac{1}{\left(u-\sqrt{3}\right)\left(u+\sqrt{3}\right)}\,du$$

Perform partial fraction decomposition:

$$= \int \left(\frac{1}{2\sqrt{3}\left(u-\sqrt{3}\right)} - \frac{1}{2\sqrt{3}\left(u+\sqrt{3}\right)}\right) du$$

Apply linearity:

$$= \frac{1}{2\sqrt{3}}\int \frac{1}{u-\sqrt{3}}\,du - \frac{1}{2\sqrt{3}}\int \frac{1}{u+\sqrt{3}}\,du$$

Now solving:

$$\int \frac{1}{u-\sqrt{3}}\,du$$

Substitute $v = u-\sqrt{3} \longrightarrow dv = du$ (steps):

$$= \int \frac{1}{v}\,dv$$

This is a standard integral:

$$= \ln(v)$$

Undo substitution $v = u-\sqrt{3}$:

$$= \ln\left(u-\sqrt{3}\right)$$

Now solving:

$$\int \frac{1}{u+\sqrt{3}}\,du$$

Substitute $v = u+\sqrt{3} \longrightarrow dv = du$ (steps):

$$= \int \frac{1}{v}\,dv$$

Use previous result:

$$= \ln(v)$$

Undo substitution $v = u+\sqrt{3}$:

$$= \ln\left(u+\sqrt{3}\right)$$

Plug in solved integrals:

$$\frac{1}{2\sqrt{3}}\int \frac{1}{u-\sqrt{3}}\,du - \frac{1}{2\sqrt{3}}\int \frac{1}{u+\sqrt{3}}\,du$$

$$= \frac{\ln\left(u-\sqrt{3}\right)}{2\sqrt{3}} - \frac{\ln\left(u+\sqrt{3}\right)}{2\sqrt{3}}$$

Now solving:

$$\int 1\,du$$

Apply constant rule:

$$= u$$

Plug in solved integrals:

$$2\int \frac{1}{u^2-3}\,du + \int 1\,du$$

$$= -\frac{\ln\left(u+\sqrt{3}\right)}{\sqrt{3}} + \frac{\ln\left(u-\sqrt{3}\right)}{\sqrt{3}} + u$$

Plug in solved integrals:

$$2\int \frac{u^2-1}{u^2-3}\,du$$

$$= -\frac{2\ln\left(u+\sqrt{3}\right)}{\sqrt{3}} + \frac{2\ln\left(u-\sqrt{3}\right)}{\sqrt{3}} + 2u$$

Undo substitution $u = \sqrt{x+5}$:

$$= -\frac{2\ln\left(\sqrt{x+5}+\sqrt{3}\right)}{\sqrt{3}} + \frac{2\ln\left(\sqrt{x+5}-\sqrt{3}\right)}{\sqrt{3}} + 2\sqrt{x+5}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{x+4}{(x+2)\sqrt{x+5}}\,dx$$

$$= \frac{2\ln\left(\left|\sqrt{x+5}-\sqrt{3}\right|\right)}{\sqrt{3}} - \frac{2\ln\left(\sqrt{x+5}+\sqrt{3}\right)}{\sqrt{3}} + 2\sqrt{x+5} + C$$

Rewrite/simplify:

$$= \frac{2\ln\left(\left|\sqrt{x+5}-\sqrt{3}\right|\right) - 2\ln\left(\sqrt{x+5}+\sqrt{3}\right)}{\sqrt{3}} + 2\sqrt{x+5} + C$$

Problem:

$$\int \sqrt{\sqrt{x}+4}\,dx$$

Substitute $u = \sqrt{x}+4 \longrightarrow du = \frac{1}{2\sqrt{x}}\,dx$ (steps):

$$= 2\int \left(u^{\frac{3}{2}} - 4\sqrt{u}\right) du$$

... or choose an alternative:

Substitute $\sqrt{x}$

Now solving:

$$\int \left(u^{\frac{3}{2}} - 4\sqrt{u}\right) du$$

Apply linearity:

$$= \int u^{\frac{3}{2}}\,du - 4\int \sqrt{u}\,du$$

Now solving:

$$\int u^{\frac{3}{2}}\,du$$

Apply power rule:

$$\int u^{n}\,du = \frac{u^{n+1}}{n+1} \text{ with } n = \frac{3}{2}:$$

$$= \frac{2u^{\frac{5}{2}}}{5}$$

Now solving:

$$\int \sqrt{u}\,du$$

Apply power rule with $n = \frac{1}{2}$:

$$= \frac{2u^{\frac{3}{2}}}{3}$$

Plug in solved integrals:

$$\int u^{\frac{3}{2}}\,du - 4\int \sqrt{u}\,du$$

$$= \frac{2u^{\frac{5}{2}}}{5} - \frac{8u^{\frac{3}{2}}}{3}$$

Plug in solved integrals:

$$2\int \left(u^{\frac{3}{2}} - 4\sqrt{u}\right) du$$

$$= \frac{4u^{\frac{5}{2}}}{5} - \frac{16u^{\frac{3}{2}}}{3}$$

Undo substitution $u = \sqrt{x}+4$:

$$= \frac{4\left(\sqrt{x}+4\right)^{\frac{5}{2}}}{5} - \frac{16\left(\sqrt{x}+4\right)^{\frac{3}{2}}}{3}$$

The problem is solved:

$$\int \sqrt{\sqrt{x}+4}\,dx$$

$$= \frac{4\left(\sqrt{x}+4\right)^{\frac{5}{2}}}{5} - \frac{16\left(\sqrt{x}+4\right)^{\frac{3}{2}}}{3} + C$$

Rewrite/simplify:

$$= \frac{4\left(\sqrt{x}+4\right)^{\frac{3}{2}}\left(3\sqrt{x}-8\right)}{15} + C$$

Problem:

$$\int \frac{x}{(2x+1)^{\frac{4}{3}}}\,dx$$

Substitute $u = 2x+1 \longrightarrow du = 2\,dx$ (steps):

$$= \frac{1}{4}\int \frac{u-1}{u^{\frac{4}{3}}}\,du$$

Now solving:

$$\int \frac{u-1}{u^{\frac{4}{3}}}\,du$$

Expand:

$$= \int \left(\frac{1}{\sqrt[3]{u}} - \frac{1}{u^{\frac{4}{3}}}\right) du$$

Apply linearity:

$$= \int \frac{1}{\sqrt[3]{u}}\,du - \int \frac{1}{u^{\frac{4}{3}}}\,du$$

Now solving:

$$\int \frac{1}{\sqrt[3]{u}}\,du$$

Apply power rule:

$$\int u^{n}\,du = \frac{u^{n+1}}{n+1} \text{ with } n = -\frac{1}{3}:$$

$$= \frac{3u^{\frac{2}{3}}}{2}$$

Now solving:

$$\int \frac{1}{u^{\frac{4}{3}}}\,du$$

Apply power rule with $n = -\frac{4}{3}$:

$$= -\frac{3}{\sqrt[3]{u}}$$

Plug in solved integrals:

$$\int \frac{1}{\sqrt[3]{u}}\,du - \int \frac{1}{u^{\frac{4}{3}}}\,du$$

$$= \frac{3u^{\frac{2}{3}}}{2} + \frac{3}{\sqrt[3]{u}}$$

Plug in solved integrals:

$$\frac{1}{4}\int \frac{u-1}{u^{\frac{4}{3}}}\,du$$

$$= \frac{3u^{\frac{2}{3}}}{8} + \frac{3}{4\sqrt[3]{u}}$$

Undo substitution $u = 2x+1$:

$$= \frac{3(2x+1)^{\frac{2}{3}}}{8} + \frac{3}{4\sqrt[3]{2x+1}}$$

The problem is solved:

$$\int \frac{x}{(2x+1)^{\frac{4}{3}}}\,dx$$

$$= \frac{3(2x+1)^{\frac{2}{3}}}{8} + \frac{3}{4\sqrt[3]{2x+1}} + C$$

Rewrite/simplify:

$$= \frac{3\,(2x+3)}{8\sqrt[3]{2x+1}} + C$$

Problem:

$$\int x\sqrt[3]{x+4}\,dx$$

Substitute $u = x+4 \longrightarrow du = dx$ (steps):

$$= \int \left(u^{\frac{4}{3}} - 4\sqrt[3]{u}\right) du$$

Apply linearity:

$$= \int u^{\frac{4}{3}}\,du - 4\int \sqrt[3]{u}\,du$$

Now solving:

$$\int u^{\frac{4}{3}}\,du$$

Apply power rule:

$$\int u^{n}\,du = \frac{u^{n+1}}{n+1} \text{ with } n = \frac{4}{3}:$$

$$= \frac{3u^{\frac{7}{3}}}{7}$$

Now solving:

$$\int \sqrt[3]{u}\,du$$

Apply power rule with $n = \frac{1}{3}$:

$$= \frac{3u^{\frac{4}{3}}}{4}$$

Plug in solved integrals:

$$\int u^{\frac{4}{3}}\,du - 4\int \sqrt[3]{u}\,du$$

$$= \frac{3u^{\frac{7}{3}}}{7} - 3u^{\frac{4}{3}}$$

Undo substitution $u = x+4$:

$$= \frac{3(x+4)^{\frac{7}{3}}}{7} - 3(x+4)^{\frac{4}{3}}$$

The problem is solved:

$$\int x\sqrt[3]{x+4}\,dx$$

$$= \frac{3(x+4)^{\frac{7}{3}}}{7} - 3(x+4)^{\frac{4}{3}} + C$$

Rewrite/simplify:

$$= \frac{3\,(x-3)\,(x+4)^{\frac{4}{3}}}{7} + C$$

Problem:

$$\int \frac{1}{(x+1)\sqrt{x+3}}\,\mathrm{d}x$$

Substitute $u=\sqrt{x+3} \longrightarrow \mathrm{d}u=\frac{1}{2\sqrt{x+3}}\,\mathrm{d}x$ (steps):

$$=2\int \frac{1}{u^2-2}\,\mathrm{d}u$$

... or choose an alternative:

Substitute $x+3$

Substitute $x+1$

Now solving:

$$\int \frac{1}{u^2-2}\,\mathrm{d}u$$

Factor the denominator:

$$=\int \frac{1}{\left(u-\sqrt{2}\right)\left(u+\sqrt{2}\right)}\,\mathrm{d}u$$

Perform partial fraction decomposition:

$$=\int \left(\frac{1}{2^{\frac{3}{2}}\left(u-\sqrt{2}\right)}-\frac{1}{2^{\frac{3}{2}}\left(u+\sqrt{2}\right)}\right)\mathrm{d}u$$

Apply linearity:

$$=\frac{1}{2^{\frac{3}{2}}}\int \frac{1}{u-\sqrt{2}}\,\mathrm{d}u-\frac{1}{2^{\frac{3}{2}}}\int \frac{1}{u+\sqrt{2}}\,\mathrm{d}u$$

Now solving:

$$\int \frac{1}{u-\sqrt{2}}\,\mathrm{d}u$$

Substitute $v=u-\sqrt{2} \longrightarrow \mathrm{d}v=\mathrm{d}u$ (steps):

$$=\int \frac{1}{v}\,\mathrm{d}v$$

This is a standard integral:

$$=\ln(v)$$

Undo substitution $v=u-\sqrt{2}$:

$$=\ln\left(u-\sqrt{2}\right)$$

Now solving:

$$\int \frac{1}{u+\sqrt{2}}\,\mathrm{d}u$$

Substitute $v=u+\sqrt{2} \longrightarrow \mathrm{d}v=\mathrm{d}u$ (steps):

$$=\int \frac{1}{v}\,\mathrm{d}v$$

Use previous result:

$$=\ln(v)$$

Undo substitution $v=u+\sqrt{2}$:

$$=\ln\left(u+\sqrt{2}\right)$$

Plug in solved integrals:

$$\frac{1}{2^{\frac{3}{2}}}\int \frac{1}{u-\sqrt{2}}\,\mathrm{d}u-\frac{1}{2^{\frac{3}{2}}}\int \frac{1}{u+\sqrt{2}}\,\mathrm{d}u$$

$$=\frac{\ln\left(u-\sqrt{2}\right)}{2^{\frac{3}{2}}}-\frac{\ln\left(u+\sqrt{2}\right)}{2^{\frac{3}{2}}}$$

Plug in solved integrals:

$$2\int \frac{1}{u^2-2}\,\mathrm{d}u$$

$$=\frac{\ln\left(u-\sqrt{2}\right)}{\sqrt{2}}-\frac{\ln\left(u+\sqrt{2}\right)}{\sqrt{2}}$$

Undo substitution $u=\sqrt{x+3}$:

$$=\frac{\ln\left(\sqrt{x+3}-\sqrt{2}\right)}{\sqrt{2}}-\frac{\ln\left(\sqrt{x+3}+\sqrt{2}\right)}{\sqrt{2}}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{1}{(x+1)\sqrt{x+3}}\,\mathrm{d}x$$

$$=\frac{\ln\left(\left|\sqrt{x+3}-\sqrt{2}\right|\right)}{\sqrt{2}}-\frac{\ln\left(\sqrt{x+3}+\sqrt{2}\right)}{\sqrt{2}}+C$$

Rewrite/simplify:

$$=\frac{\ln\left(\left|\sqrt{x+3}-\sqrt{2}\right|\right)-\ln\left(\sqrt{x+3}+\sqrt{2}\right)}{\sqrt{2}}+C$$

Problem:

$$\int \frac{4-\sqrt{2x+1}}{1-2x}\,\mathrm{d}x$$

Rewrite/simplify:

$$=\int \frac{\sqrt{2x+1}-4}{2x-1}\,\mathrm{d}x$$

... or choose an alternative:

Skip simplification

Substitute $u=\sqrt{2x+1} \longrightarrow \mathrm{d}u=\frac{1}{\sqrt{2x+1}}\,\mathrm{d}x$ (steps):

$$=\int \frac{(u-4)\,u}{u^2-2}\,\mathrm{d}u$$

... or choose an alternative:

Substitute $2x+1$

Substitute $2x-1$

Perform polynomial long division:

$$=\int \left(\frac{2-4u}{u^2-2}+1\right)\mathrm{d}u$$

Apply linearity:

$$=\int 1\,\mathrm{d}u-2\int \frac{2u-1}{u^2-2}\,\mathrm{d}u$$

Now solving:

$$\int 1\,\mathrm{d}u$$

Apply constant rule:

$$=u$$

Now solving:

$$\int \frac{2u-1}{u^2-2}\,\mathrm{d}u$$

Expand:

$$=\int \left(\frac{2u}{u^2-2}-\frac{1}{u^2-2}\right)\mathrm{d}u$$

Apply linearity:

$$=2\int \frac{u}{u^2-2}\,\mathrm{d}u-\int \frac{1}{u^2-2}\,\mathrm{d}u$$

Now solving:

$$\int \frac{u}{u^2-2}\,\mathrm{d}u$$

Substitute $v=u^2-2 \longrightarrow \mathrm{d}v=2u\,\mathrm{d}u$ (steps):

$$=\frac{1}{2}\int \frac{1}{v}\,\mathrm{d}v$$

Now solving:

$$\int \frac{1}{v}\,\mathrm{d}v$$

This is a standard integral:

$$=\ln(v)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{v}\,\mathrm{d}v$$

$$=\frac{\ln(v)}{2}$$

Undo substitution $v=u^2-2$:

$$=\frac{\ln(u^2-2)}{2}$$

Now solving:

$$\int \frac{1}{u^2-2}\,\mathrm{d}u$$

Factor the denominator:

$$=\int \frac{1}{\left(u-\sqrt{2}\right)\left(u+\sqrt{2}\right)}\,\mathrm{d}u$$

Perform partial fraction decomposition:

$$=\int \left(\frac{1}{2^{\frac{3}{2}}\left(u-\sqrt{2}\right)}-\frac{1}{2^{\frac{3}{2}}\left(u+\sqrt{2}\right)}\right)\mathrm{d}u$$

Apply linearity:

$$=\frac{1}{2^{\frac{3}{2}}}\int \frac{1}{u-\sqrt{2}}\,\mathrm{d}u-\frac{1}{2^{\frac{3}{2}}}\int \frac{1}{u+\sqrt{2}}\,\mathrm{d}u$$

Now solving:

$$\int \frac{1}{u-\sqrt{2}}\,\mathrm{d}u$$

Substitute $v=u-\sqrt{2} \longrightarrow \mathrm{d}v=\mathrm{d}u$ (steps):

$$=\int \frac{1}{v}\,\mathrm{d}v$$

Use previous result:

$$=\ln(v)$$

Undo substitution $v=u-\sqrt{2}$:

$$=\ln\left(u-\sqrt{2}\right)$$

Now solving:

$$\int \frac{1}{u+\sqrt{2}}\,\mathrm{d}u$$

Substitute $v=u+\sqrt{2} \longrightarrow \mathrm{d}v=\mathrm{d}u$ (steps):

$$=\int \frac{1}{v}\,\mathrm{d}v$$

Use previous result:

$$=\ln(v)$$

Undo substitution $v=u+\sqrt{2}$:

$$=\ln\left(u+\sqrt{2}\right)$$

Plug in solved integrals:

$$\frac{1}{2^{\frac{3}{2}}}\int \frac{1}{u-\sqrt{2}}\,\mathrm{d}u-\frac{1}{2^{\frac{3}{2}}}\int \frac{1}{u+\sqrt{2}}\,\mathrm{d}u$$

$$=\frac{\ln\left(u-\sqrt{2}\right)}{2^{\frac{3}{2}}}-\frac{\ln\left(u+\sqrt{2}\right)}{2^{\frac{3}{2}}}$$

Plug in solved integrals:

$$2\int \frac{u}{u^2-2}\,\mathrm{d}u-\int \frac{1}{u^2-2}\,\mathrm{d}u$$

$$=\ln(u^2-2)+\frac{\ln\left(u+\sqrt{2}\right)}{2^{\frac{3}{2}}}-\frac{\ln\left(u-\sqrt{2}\right)}{2^{\frac{3}{2}}}$$

Plug in solved integrals:

$$\int 1\,\mathrm{d}u-2\int \frac{2u-1}{u^2-2}\,\mathrm{d}u$$

$$=-2\ln(u^2-2)-\frac{\ln\left(u+\sqrt{2}\right)}{\sqrt{2}}+\frac{\ln\left(u-\sqrt{2}\right)}{\sqrt{2}}+u$$

Undo substitution $u=\sqrt{2x+1}$:

$$=-\frac{\ln\left(\sqrt{2x+1}+\sqrt{2}\right)}{\sqrt{2}}+\frac{\ln\left(\sqrt{2x+1}-\sqrt{2}\right)}{\sqrt{2}}-2\ln(2x-1)+\sqrt{2x+1}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{\sqrt{2x+1}-4}{2x-1}\,\mathrm{d}x$$

$$=\frac{\ln\left(\left|\sqrt{2x+1}-\sqrt{2}\right|\right)}{\sqrt{2}}-2\ln(|2x-1|)-\frac{\ln\left(\sqrt{2x+1}+\sqrt{2}\right)}{\sqrt{2}}+\sqrt{2x+1}+C$$

Rewrite/simplify:

$$=\frac{\ln\left(\left|\sqrt{2x+1}-\sqrt{2}\right|\right)-\ln\left(\sqrt{2x+1}+\sqrt{2}\right)}{\sqrt{2}}-2\ln(|2x-1|)+\sqrt{2x+1}+C$$

Problem:

$$\int \frac{1}{x^2 \cdot (x^3+4)^{\frac{2}{3}}} \,dx$$

Substitute $u = \frac{\sqrt[3]{x^3+4}}{x} \longrightarrow du = \left(\frac{x}{(x^3+4)^{\frac{2}{3}}} - \frac{\sqrt[3]{x^3+4}}{x^2}\right) dx$ (steps):

$$= \int -\frac{1}{4} \,du$$

Apply linearity:

$$= -\frac{1}{4} \int 1 \,du$$

Now solving:

$$\int 1 \,du$$

Apply constant rule:

$$= u$$

Plug in solved integrals:

$$-\frac{1}{4} \int 1 \,du$$

$$= -\frac{u}{4}$$

Undo substitution $u = \frac{\sqrt[3]{x^3+4}}{x}$:

$$= -\frac{\sqrt[3]{x^3+4}}{4x}$$

The problem is solved:

$$\int \frac{1}{x^2 \cdot (x^3+4)^{\frac{2}{3}}} \,dx$$

$$= -\frac{\sqrt[3]{x^3+4}}{4x} + C$$

Problem:

$$\int x^5 \sqrt{x^3+4} \,dx$$

Substitute $u = x^3 + 4 \longrightarrow du = 3x^2 \,dx$ (steps):

$$= \frac{1}{3} \int \left(u^{\frac{3}{2}} - 4\sqrt{u}\right) du$$

... or choose an alternative:

Substitute x^3

Now solving:

$$\int \left(u^{\frac{3}{2}} - 4\sqrt{u}\right) du$$

Apply linearity:

$$= \int u^{\frac{3}{2}} \,du - 4 \int \sqrt{u} \,du$$

Now solving:

$$\int u^{\frac{3}{2}} \,du$$

Apply power rule:

$$\int u^n \,du = \frac{u^{n+1}}{n+1} \text{ with } n = \frac{3}{2}:$$

$$= \frac{2u^{\frac{5}{2}}}{5}$$

Now solving:

$$\int \sqrt{u} \,du$$

Apply power rule with $n = \frac{1}{2}$:

$$= \frac{2u^{\frac{3}{2}}}{3}$$

Plug in solved integrals:

$$\int u^{\frac{3}{2}} \,du - 4 \int \sqrt{u} \,du$$

$$= \frac{2u^{\frac{5}{2}}}{5} - \frac{8u^{\frac{3}{2}}}{3}$$

Plug in solved integrals:

$$\frac{1}{3} \int \left(u^{\frac{3}{2}} - 4\sqrt{u}\right) du$$

$$= \frac{2u^{\frac{5}{2}}}{15} - \frac{8u^{\frac{3}{2}}}{9}$$

Undo substitution $u = x^3 + 4$:

$$= \frac{2(x^3+4)^{\frac{5}{2}}}{15} - \frac{8(x^3+4)^{\frac{3}{2}}}{9}$$

The problem is solved:

$$\int x^5 \sqrt{x^3+4} \,dx$$

$$= \frac{2(x^3+4)^{\frac{5}{2}}}{15} - \frac{8(x^3+4)^{\frac{3}{2}}}{9} + C$$

Rewrite/simplify:

$$= \frac{2(x^3+4)^{\frac{3}{2}}(3x^3-8)}{45} + C$$

Problem:

$$\int \frac{1}{x^3 \sqrt[3]{x^3+2}} \,dx$$

Substitute $u = \frac{\sqrt[3]{x^3+2}}{x} \longrightarrow du = \left(\frac{x}{(x^3+2)^{\frac{2}{3}}} - \frac{\sqrt[3]{x^3+2}}{x^2}\right) dx$ (steps):

$$= \int -\frac{u}{2} \,du$$

Apply linearity:

$$= -\frac{1}{2} \int u \,du$$

Now solving:

$$\int u \,du$$

Apply power rule:

$$\int u^n \,du = \frac{u^{n+1}}{n+1} \text{ with } n = 1:$$

$$= \frac{u^2}{2}$$

Plug in solved integrals:

$$-\frac{1}{2} \int u \,du$$

$$= -\frac{u^2}{4}$$

Undo substitution $u = \frac{\sqrt[3]{x^3+2}}{x}$:

$$= -\frac{(x^3+2)^{\frac{2}{3}}}{4x^2}$$

The problem is solved:

$$\int \frac{1}{x^3 \sqrt[3]{x^3+2}} \,dx$$

$$= -\frac{(x^3+2)^{\frac{2}{3}}}{4x^2} + C$$

Problem:

$$\int x^3 \cdot (x^2+4)^{\frac{3}{2}} \,dx$$

Substitute $u = x^2 + 4 \longrightarrow du = 2x \,dx$ (steps):

$$= \frac{1}{2} \int \left(u^{\frac{5}{2}} - 4u^{\frac{3}{2}}\right) du$$

... or choose an alternative:

Substitute x^2

Don't substitute

Now solving:

$$\int \left(u^{\frac{5}{2}} - 4u^{\frac{3}{2}}\right) du$$

Apply linearity:

$$= \int u^{\frac{5}{2}} \,du - 4 \int u^{\frac{3}{2}} \,du$$

Now solving:

$$\int u^{\frac{5}{2}} \,du$$

Apply power rule:

$$\int u^n \,du = \frac{u^{n+1}}{n+1} \text{ with } n = \frac{5}{2}:$$

$$= \frac{2u^{\frac{7}{2}}}{7}$$

Now solving:

$$\int u^{\frac{3}{2}} \,du$$

Apply power rule with $n = \frac{3}{2}$:

$$= \frac{2u^{\frac{5}{2}}}{5}$$

Plug in solved integrals:

$$\int u^{\frac{5}{2}} \,du - 4 \int u^{\frac{3}{2}} \,du$$

$$= \frac{2u^{\frac{7}{2}}}{7} - \frac{8u^{\frac{5}{2}}}{5}$$

Plug in solved integrals:

$$\frac{1}{2} \int \left(u^{\frac{5}{2}} - 4u^{\frac{3}{2}}\right) du$$

$$= \frac{u^{\frac{7}{2}}}{7} - \frac{4u^{\frac{5}{2}}}{5}$$

Undo substitution $u = x^2 + 4$:

$$= \frac{(x^2+4)^{\frac{7}{2}}}{7} - \frac{4(x^2+4)^{\frac{5}{2}}}{5}$$

The problem is solved:

$$\int x^3 \cdot (x^2+4)^{\frac{3}{2}} \,dx$$

$$= \frac{(x^2+4)^{\frac{7}{2}}}{7} - \frac{4(x^2+4)^{\frac{5}{2}}}{5} + C$$

Rewrite/simplify:

$$= \frac{(x^2+4)^{\frac{5}{2}}(5x^2-8)}{35} + C$$

Problem:

$$\int x^8\sqrt{x^3+1}\,dx$$

Substitute $u = x^3+1 \longrightarrow du = 3x^2\,dx$ (steps), use:

$$x^6 = (u-1)^2$$

$$= \frac{1}{3}\int (u-1)^2\sqrt{u}\,du$$

... or choose an alternative:

Substitute x^3

Now solving:

$$\int (u-1)^2\sqrt{u}\,du$$

Expand:

$$= \int \left(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + \sqrt{u}\right) du$$

Apply linearity:

$$= \int u^{\frac{5}{2}}\,du - 2\int u^{\frac{3}{2}}\,du + \int \sqrt{u}\,du$$

Now solving:

$$\int u^{\frac{5}{2}}\,du$$

Apply power rule:

$$\int u^n\,du = \frac{u^{n+1}}{n+1} \text{ with } n = \frac{5}{2}:$$

$$= \frac{2u^{\frac{7}{2}}}{7}$$

Now solving:

$$\int u^{\frac{3}{2}}\,du$$

Apply power rule with $n = \frac{3}{2}$:

$$= \frac{2u^{\frac{5}{2}}}{5}$$

Now solving:

$$\int \sqrt{u}\,du$$

Apply power rule with $n = \frac{1}{2}$:

$$= \frac{2u^{\frac{3}{2}}}{3}$$

Plug in solved integrals:

$$\int u^{\frac{5}{2}}\,du - 2\int u^{\frac{3}{2}}\,du + \int \sqrt{u}\,du$$

$$= \frac{2u^{\frac{7}{2}}}{7} - \frac{4u^{\frac{5}{2}}}{5} + \frac{2u^{\frac{3}{2}}}{3}$$

Plug in solved integrals:

$$\frac{1}{3}\int (u-1)^2\sqrt{u}\,du$$

$$= \frac{2u^{\frac{7}{2}}}{21} - \frac{4u^{\frac{5}{2}}}{15} + \frac{2u^{\frac{3}{2}}}{9}$$

Undo substitution $u = x^3+1$:

$$= \frac{2(x^3+1)^{\frac{7}{2}}}{21} - \frac{4(x^3+1)^{\frac{5}{2}}}{15} + \frac{2(x^3+1)^{\frac{3}{2}}}{9}$$

The problem is solved:

$$\int x^8\sqrt{x^3+1}\,dx$$

$$= \frac{2(x^3+1)^{\frac{7}{2}}}{21} - \frac{4(x^3+1)^{\frac{5}{2}}}{15} + \frac{2(x^3+1)^{\frac{3}{2}}}{9} + C$$

Rewrite/simplify:

$$= \frac{2(x^3+1)^{\frac{3}{2}}\left(15x^6 - 12x^3 + 8\right)}{315} + C$$

Problem:

$$\int \frac{1}{x^4\sqrt{x^2+1}}\,dx$$

Perform trigonometric substitution:

Substitute $x = \tan(u) \longrightarrow u = \arctan(x), dx = \sec^2(u)\,du$ (steps):

$$= \int \frac{\sec^2(u)}{\tan^4(u)\sqrt{\tan^2(u)+1}}\,du$$

Simplify using $\tan^2(u)+1 = \sec^2(u)$:

$$= \int \frac{\sec(u)}{\tan^4(u)}\,du$$

... or choose an alternative:

Perform hyperbolic substitution

Prepare for substitution, use:

$$\tan(u) = \frac{1}{\cot(u)},$$

$$\sec(u) = \frac{\csc(u)}{\cot(u)},$$

$$\cot^2(u) = \csc^2(u) - 1$$

$$= \int -\cot(u)\csc(u)\left(1 - \csc^2(u)\right) du$$

Substitute $v = \csc(u) \longrightarrow dv = -\cot(u)\csc(u)\,du$ (steps):

$$= -\int \left(v^2 - 1\right) dv$$

... or choose an alternative:

Substitute $\sin(u)$

Now solving:

$$\int \left(v^2 - 1\right) dv$$

Apply linearity:

$$= \int v^2\,dv - \int 1\,dv$$

Now solving:

$$\int v^2\,dv$$

Apply power rule:

$$\int v^n\,dv = \frac{v^{n+1}}{n+1} \text{ with } n = 2:$$

$$= \frac{v^3}{3}$$

Now solving:

$$\int 1\,dv$$

Apply constant rule:

$$= v$$

Plug in solved integrals:

$$\int v^2\,dv - \int 1\,dv$$

$$= \frac{v^3}{3} - v$$

Plug in solved integrals:

$$-\int \left(v^2 - 1\right) dv$$

$$= v - \frac{v^3}{3}$$

Undo substitution $v = \csc(u)$:

$$= \csc(u) - \frac{\csc^3(u)}{3}$$

Undo substitution $u = \arctan(x)$, use:

$$\csc(\arctan(x)) = \frac{\sqrt{x^2+1}}{x}$$

$$= \frac{\sqrt{x^2+1}}{x} - \frac{(x^2+1)^{\frac{3}{2}}}{3x^3}$$

The problem is solved:

$$\int \frac{1}{x^4\sqrt{x^2+1}}\,dx$$

$$= \frac{\sqrt{x^2+1}}{x} - \frac{(x^2+1)^{\frac{3}{2}}}{3x^3} + C$$

Rewrite/simplify:

$$= -\frac{\left(1 - 2x^2\right)\sqrt{x^2+1}}{3x^3} + C$$

Problem:

$$\int \frac{x^2}{(1-x^3)^{\frac{3}{2}}}\,dx$$

Substitute $u = 1 - x^3 \longrightarrow du = -3x^2\,dx$ (steps):

$$= -\frac{1}{3}\int \frac{1}{u^{\frac{3}{2}}}\,du$$

Now solving:

$$\int \frac{1}{u^{\frac{3}{2}}}\,du$$

Apply power rule:

$$\int u^{n}\,du = \frac{u^{n+1}}{n+1} \text{ with } n = -\frac{3}{2}:$$

$$= -\frac{2}{\sqrt{u}}$$

Plug in solved integrals:

$$-\frac{1}{3}\int \frac{1}{u^{\frac{3}{2}}}\,du$$

$$= \frac{2}{3\sqrt{u}}$$

Undo substitution $u = 1 - x^3$:

$$= \frac{2}{3\sqrt{1-x^3}}$$

The problem is solved:

$$\int \frac{x^2}{(1-x^3)^{\frac{3}{2}}}\,dx$$

$$= \frac{2}{3\sqrt{1-x^3}} + C$$

Problem:

$$\int \frac{1}{x^2\cdot(x^4+81)^{\frac{3}{4}}}\,dx$$

Substitute $u = \frac{\sqrt[4]{x^4+81}}{x} \longrightarrow du = \left(\frac{x^2}{(x^4+81)^{\frac{3}{4}}} - \frac{\sqrt[4]{x^4+81}}{x^2}\right)dx$ (steps):

$$= \int -\frac{1}{81}\,du$$

Apply linearity:

$$= -\frac{1}{81}\int 1\,du$$

Now solving:

$$\int 1\,du$$

Apply constant rule:

$$= u$$

Plug in solved integrals:

$$-\frac{1}{81}\int 1\,du$$

$$= -\frac{u}{81}$$

Undo substitution $u = \frac{\sqrt[4]{x^4+81}}{x}$:

$$= -\frac{\sqrt[4]{x^4+81}}{81x}$$

The problem is solved:

$$\int \frac{1}{x^2\cdot(x^4+81)^{\frac{3}{4}}}\,dx$$

$$= -\frac{\sqrt[4]{x^4+81}}{81x} + C$$

Trigonometric Substitution

1. $\displaystyle\int \frac{x^2\,dx}{\sqrt{4-x^2}}$

2. $\displaystyle\int \frac{\sqrt{9-x^2}}{x^2}\,dx$

3. $\displaystyle\int \frac{dx}{x\sqrt{9x^2+4}}$

4. $\displaystyle\int \frac{x^2\,dx}{(x^2+16)^{3/2}}$

5. $\displaystyle\int \frac{x^2\,dx}{(9-x^2)^{3/2}}$

6. $\displaystyle\int x^2\sqrt{9-4x^2}\;dx$

7. $\displaystyle\int \frac{\sqrt{9-4x^2}\;dx}{x^2}$

8. $\displaystyle\int \frac{dx}{x^2\sqrt{x^2+1}}$

9. $\displaystyle\int \frac{dx}{(x^2+4)^2}$

10. $\displaystyle\int x^3\sqrt{4x^2-1}\;dx$

11. $\displaystyle\int \frac{dx}{x\sqrt{x^2-9}}$

12. $\displaystyle\int \frac{dx}{x^3\sqrt{x^2-4}}$

13. $\displaystyle\int \frac{(x^2-16)^{3/2}}{x^3}\,dx$

14. $\displaystyle\int \frac{x^2\,dx}{(1-x^2)^{5/2}}$

15. $\displaystyle\int \frac{dx}{(2x-3)\sqrt{5-12x+4x^2}}$

Problem:

$$\int \frac{x^2}{\sqrt{4-x^2}}\,dx$$

Perform trigonometric substitution:

Substitute $x = 2\sin(u) \longrightarrow u = \arcsin\left(\frac{x}{2}\right)$, $dx = 2\cos(u)\,du$ (steps):

$$= \int \frac{8\cos(u)\sin^2(u)}{\sqrt{4-4\sin^2(u)}}\,du$$

Simplify using $4-4\sin^2(u) = 4\cos^2(u)$:

$$= 4\int \sin^2(u)\,du$$

... or choose an alternative:

Perform hyperbolic substitution

Now solving:

$$\int \sin^2(u)\,du$$

Apply reduction formula:

$$\int \sin^n(u)\,du = \frac{n-1}{n}\int \sin^{n-2}(u)\,du - \frac{\cos(u)\sin^{n-1}(u)}{n}$$

with $n = 2$:

$$= -\frac{\cos(u)\sin(u)}{2} + \frac{1}{2}\int 1\,du$$

... or choose an alternative:

Apply product-to-sum formulas

Now solving:

$$\int 1\,du$$

Apply constant rule:

$$= u$$

Plug in solved integrals:

$$-\frac{\cos(u)\sin(u)}{2} + \frac{1}{2}\int 1\,du$$

$$= \frac{u}{2} - \frac{\cos(u)\sin(u)}{2}$$

Plug in solved integrals:

$$4\int \sin^2(u)\,du$$

$$= 2u - 2\cos(u)\sin(u)$$

Undo substitution $u = \arcsin\left(\frac{x}{2}\right)$, use:

$$\sin\left(\arcsin\left(\frac{x}{2}\right)\right) = \frac{x}{2}$$

$$\cos\left(\arcsin\left(\frac{x}{2}\right)\right) = \sqrt{1-\frac{x^2}{4}}$$

$$= 2\arcsin\left(\frac{x}{2}\right) - x\sqrt{1-\frac{x^2}{4}}$$

The problem is solved:

$$\int \frac{x^2}{\sqrt{4-x^2}}\,dx$$

$$= 2\arcsin\left(\frac{x}{2}\right) - x\sqrt{1-\frac{x^2}{4}} + C$$

Problem:

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx$$

Integrate by parts: $\int fg' = fg - \int f'g$

$$f = \sqrt{9-x^2}, \quad g' = \frac{1}{x^2}$$

$\downarrow$ steps $\quad \downarrow$ steps

$$f' = -\frac{x}{\sqrt{9-x^2}}, \quad g = -\frac{1}{x}:$$

$$= -\frac{\sqrt{9-x^2}}{x} - \int \frac{1}{\sqrt{9-x^2}}\,dx$$

Now solving:

$$\int \frac{1}{\sqrt{9-x^2}}\,dx$$

Substitute $u = \frac{x}{3} \longrightarrow du = \frac{1}{3}\,dx$ (steps):

$$= \int \frac{3}{\sqrt{9-9u^2}}\,du$$

Simplify:

$$= \int \frac{1}{\sqrt{1-u^2}}\,du$$

This is a standard integral:

$$= \arcsin(u)$$

Undo substitution $u = \frac{x}{3}$:

$$= \arcsin\left(\frac{x}{3}\right)$$

Plug in solved integrals:

$$-\frac{\sqrt{9-x^2}}{x} - \int \frac{1}{\sqrt{9-x^2}}\,dx$$

$$= -\frac{\sqrt{9-x^2}}{x} - \arcsin\left(\frac{x}{3}\right)$$

The problem is solved:

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx$$

$$= -\frac{\sqrt{9-x^2}}{x} - \arcsin\left(\frac{x}{3}\right) + C$$

Problem:

$$\int \frac{1}{x\sqrt{9x^2+4}}\,dx$$

Substitute $u = \sqrt{9x^2+4} \longrightarrow du = \frac{9x}{\sqrt{9x^2+4}}\,dx$ (steps):

$$= \int \frac{1}{u^2-4}\,du$$

... or choose an alternative:

Substitute $9x^2+4$

Substitute x^2

Don't substitute

Factor the denominator:

$$= \int \frac{1}{(u-2)(u+2)}\,du$$

Perform partial fraction decomposition:

$$= \int \left(\frac{1}{4(u-2)} - \frac{1}{4(u+2)}\right) du$$

Apply linearity:

$$= \frac{1}{4}\int \frac{1}{u-2}\,du - \frac{1}{4}\int \frac{1}{u+2}\,du$$

Now solving:

$$\int \frac{1}{u-2}\,du$$

Substitute $v = u-2 \longrightarrow dv = du$ (steps):

$$= \int \frac{1}{v}\,dv$$

This is a standard integral:

$$= \ln(v)$$

Undo substitution $v = u-2$:

$$= \ln(u-2)$$

Now solving:

$$\int \frac{1}{u+2}\,du$$

Substitute $v = u+2 \longrightarrow dv = du$ (steps):

$$= \int \frac{1}{v}\,dv$$

Use previous result:

$$= \ln(v)$$

Undo substitution $v = u+2$:

$$= \ln(u+2)$$

Plug in solved integrals:

$$\frac{1}{4}\int \frac{1}{u-2}\,du - \frac{1}{4}\int \frac{1}{u+2}\,du$$

$$= \frac{\ln(u-2)}{4} - \frac{\ln(u+2)}{4}$$

Undo substitution $u = \sqrt{9x^2+4}$:

$$= \frac{\ln\left(\sqrt{9x^2+4}-2\right)}{4} - \frac{\ln\left(\sqrt{9x^2+4}+2\right)}{4}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{1}{x\sqrt{9x^2+4}}\,dx$$

$$= \frac{\ln\left(\left|\sqrt{9x^2+4}-2\right|\right)}{4} - \frac{\ln\left(\sqrt{9x^2+4}+2\right)}{4} + C$$

Rewrite/simplify:

$$= \frac{\ln\left(\left|\sqrt{9x^2+4}-2\right|\right) - \ln\left(\sqrt{9x^2+4}+2\right)}{4} + C$$

Problem:

$$\int \frac{x^2}{(x^2+16)^{\frac{3}{2}}}\,dx$$

Perform trigonometric substitution:

Substitute $x = 4\tan(u) \longrightarrow u = \arctan\left(\frac{x}{4}\right)$, $dx = 4\sec^2(u)\,du$ (steps):

$$= \int \frac{64\sec^2(u)\tan^2(u)}{\left(16\tan^2(u)+16\right)^{\frac{3}{2}}}\,du$$

Simplify using $16\tan^2(u) + 16 = 16\sec^2(u)$:

$$= \int \frac{\tan^2(u)}{\sec(u)}\,du$$

... or choose an alternative:

Perform hyperbolic substitution

Rewrite/simplify using trigonometric/hyperbolic identities:

$$= \int \cos(u)\tan^2(u)\,du$$

Rewrite/simplify using trigonometric/hyperbolic identities:

$$= \int \cos(u)\left(\sec^2(u)-1\right)\,du$$

$$= \int \left(\sec(u)-\cos(u)\right)du$$

Apply linearity:

$$= \int \sec(u)\,du - \int \cos(u)\,du$$

Now solving:

$$\int \sec(u)\,du$$

This is a standard integral:

(full steps)

$$= \ln(\tan(u)+\sec(u))$$

Now solving:

$$\int \cos(u)\,du$$

This is a standard integral:

$$= \sin(u)$$

Plug in solved integrals:

$$\int \sec(u)\,du - \int \cos(u)\,du$$

$$= \ln(\tan(u)+\sec(u)) - \sin(u)$$

Undo substitution $u = \arctan\left(\frac{x}{4}\right)$, use:

$$\tan\left(\arctan\left(\frac{x}{4}\right)\right) = \frac{x}{4}$$

$$\sin\left(\arctan\left(\frac{x}{4}\right)\right) = \frac{x}{4\sqrt{\frac{x^2}{16}+1}}$$

$$\sec\left(\arctan\left(\frac{x}{4}\right)\right) = \sqrt{\frac{x^2}{16}+1}$$

$$= \ln\left(\sqrt{\frac{x^2}{16}+1}+\frac{x}{4}\right) - \frac{x}{4\sqrt{\frac{x^2}{16}+1}}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{x^2}{(x^2+16)^{\frac{3}{2}}}\,dx$$

$$= \ln\left(\left|\sqrt{\frac{x^2}{16}+1}+\frac{x}{4}\right|\right) - \frac{x}{4\sqrt{\frac{x^2}{16}+1}} + C$$

Rewrite/simplify:

$$= \ln\left(\left|\sqrt{x^2+16}+x\right|\right) - \frac{x}{\sqrt{x^2+16}} + C$$

Problem:

$$\int \frac{x^2}{(9-x^2)^{\frac{3}{2}}}\,dx$$

Perform trigonometric substitution:

Substitute $x = 3\sin(u) \longrightarrow u = \arcsin\left(\frac{x}{3}\right)$, $dx = 3\cos(u)\,du$ (steps):

$$= \int \frac{27\cos(u)\sin^2(u)}{\left(9-9\sin^2(u)\right)^{\frac{3}{2}}}\,du$$

Simplify using $9 - 9\sin^2(u) = 9\cos^2(u)$:

$$= \int \frac{\sin^2(u)}{\cos^2(u)}\,du$$

... or choose an alternative:

Perform hyperbolic substitution

Rewrite/simplify using trigonometric/hyperbolic identities:

$$= \int \tan^2(u)\,du$$

Rewrite/simplify using trigonometric/hyperbolic identities:

$$\tan^2(u) = \sec^2(u) - 1$$

$$= \int \left(\sec^2(u)-1\right)du$$

Apply linearity:

$$= \int \sec^2(u)\,du - \int 1\,du$$

Now solving:

$$\int \sec^2(u)\,du$$

This is a standard integral:

$$= \tan(u)$$

Now solving:

$$\int 1\,du$$

Apply constant rule:

$$= u$$

Plug in solved integrals:

$$\int \sec^2(u)\,du - \int 1\,du$$

$$= \tan(u) - u$$

Undo substitution $u = \arcsin\left(\frac{x}{3}\right)$, use:

$$\tan\left(\arcsin\left(\frac{x}{3}\right)\right) = \frac{x}{3\sqrt{1-\frac{x^2}{9}}}$$

$$= \frac{x}{3\sqrt{1-\frac{x^2}{9}}} - \arcsin\left(\frac{x}{3}\right)$$

The problem is solved:

$$\int \frac{x^2}{(9-x^2)^{\frac{3}{2}}}\,dx$$

$$= \frac{x}{3\sqrt{1-\frac{x^2}{9}}} - \arcsin\left(\frac{x}{3}\right) + C$$

Problem:

$$\int x^2\sqrt{9-4x^2}\,dx$$

Perform trigonometric substitution:

Substitute $x = \frac{3\sin(u)}{2} \longrightarrow u = \arcsin\left(\frac{2x}{3}\right), dx = \frac{3\cos(u)}{2}\,du$ (steps):

$$= \int \frac{27\cos(u)\sin^2(u)\sqrt{9-9\sin^2(u)}}{8}\,du$$

Simplify using $9 - 9\sin^2(u) = 9\cos^2(u)$:

$$= \frac{81}{8}\int \cos^2(u)\sin^2(u)\,du$$

... or choose an alternative:

Perform hyperbolic substitution

Now solving:

$$\int \cos^2(u)\sin^2(u)\,du$$

Rewrite/simplify using trigonometric/hyperbolic identities:

$$\cos^2(u) = 1 - \sin^2(u)$$

$$= \int \sin^2(u)\left(1-\sin^2(u)\right)\,du$$

... or choose an alternative:

Apply product-to-sum formulas

Expand:

$$= \int \left(\sin^2(u) - \sin^4(u)\right)\,du$$

Apply linearity:

$$= \int \sin^2(u)\,du - \int \sin^4(u)\,du$$

Now solving:

$$\int \sin^2(u)\,du$$

Apply reduction formula:

$$\int \sin^n(u)\,du = \frac{n-1}{n}\int \sin^{n-2}(u)\,du - \frac{\cos(u)\sin^{n-1}(u)}{n}$$

with $n = 2$:

$$= -\frac{\cos(u)\sin(u)}{2} + \frac{1}{2}\int 1\,du$$

... or choose an alternative:

Apply product-to-sum formulas

Now solving:

$$\int 1\,du$$

Apply constant rule:

$$= u$$

Plug in solved integrals:

$$-\frac{\cos(u)\sin(u)}{2} + \frac{1}{2}\int 1\,du$$

$$= \frac{u}{2} - \frac{\cos(u)\sin(u)}{2}$$

Now solving:

$$\int \sin^4(u)\,du$$

Apply the last reduction formula again with $n = 4$:

$$= -\frac{\cos(u)\sin^3(u)}{4} + \frac{3}{4}\int \sin^2(u)\,du$$

... or choose an alternative:

Apply product-to-sum formulas

Now solving:

$$\int \sin^2(u)\,du$$

Use previous result:

$$= \frac{u}{2} - \frac{\cos(u)\sin(u)}{2}$$

Plug in solved integrals:

$$-\frac{\cos(u)\sin^3(u)}{4} + \frac{3}{4}\int \sin^2(u)\,du$$

$$= -\frac{\cos(u)\sin^3(u)}{4} - \frac{3\cos(u)\sin(u)}{8} + \frac{3u}{8}$$

Plug in solved integrals:

$$\int \sin^2(u)\,du - \int \sin^4(u)\,du$$

$$= \frac{\cos(u)\sin^3(u)}{4} - \frac{\cos(u)\sin(u)}{8} + \frac{u}{8}$$

Plug in solved integrals:

$$\frac{81}{8}\int \cos^2(u)\sin^2(u)\,du$$

$$= \frac{81\cos(u)\sin^3(u)}{32} - \frac{81\cos(u)\sin(u)}{64} + \frac{81u}{64}$$

Undo substitution $u = \arcsin\left(\frac{2x}{3}\right)$, use:

$$\sin\left(\arcsin\left(\frac{2x}{3}\right)\right) = \frac{2x}{3}$$

$$\cos\left(\arcsin\left(\frac{2x}{3}\right)\right) = \sqrt{1-\frac{4x^2}{9}}$$

$$= \frac{3x^3\sqrt{1-\frac{4x^2}{9}}}{4} - \frac{27x\sqrt{1-\frac{4x^2}{9}}}{32} + \frac{81\arcsin\left(\frac{2x}{3}\right)}{64}$$

The problem is solved:

$$\int x^2\sqrt{9-4x^2}\,dx$$

$$= \frac{3x^3\sqrt{1-\frac{4x^2}{9}}}{4} - \frac{27x\sqrt{1-\frac{4x^2}{9}}}{32} + \frac{81\arcsin\left(\frac{2x}{3}\right)}{64} + C$$

Rewrite/simplify:

$$= \frac{\sqrt{9-4x^2}\left(16x^3-18x\right) + 81\arcsin\left(\frac{2x}{3}\right)}{64} + C$$

Problem:

$$\int \frac{\sqrt{9-4x^2}}{x^2}\,dx$$

Integrate by parts: $\int fg' = fg - \int f'g$

$$f = \sqrt{9-4x^2}, \quad g' = \frac{1}{x^2}$$

$\downarrow$ steps $\quad$ $\downarrow$ steps

$$f' = -\frac{4x}{\sqrt{9-4x^2}}, \ g = -\frac{1}{x}:$$

$$= -\frac{\sqrt{9-4x^2}}{x} - \int \frac{4}{\sqrt{9-4x^2}}\,dx$$

Now solving:

$$\int \frac{4}{\sqrt{9-4x^2}}\,dx$$

Substitute $u = \frac{2x}{3} \longrightarrow du = \frac{2}{3}\,dx$ (steps):

$$= \int \frac{6}{\sqrt{9-9u^2}}\,du$$

Simplify:

$$= 2\int \frac{1}{\sqrt{1-u^2}}\,du$$

Now solving:

$$\int \frac{1}{\sqrt{1-u^2}}\,du$$

This is a standard integral:

$$= \arcsin(u)$$

Plug in solved integrals:

$$2\int \frac{1}{\sqrt{1-u^2}}\,du$$

$$= 2\arcsin(u)$$

Undo substitution $u = \frac{2x}{3}$:

$$= 2\arcsin\left(\frac{2x}{3}\right)$$

Plug in solved integrals:

$$-\frac{\sqrt{9-4x^2}}{x} - \int \frac{4}{\sqrt{9-4x^2}}\,dx$$

$$= -\frac{\sqrt{9-4x^2}}{x} - 2\arcsin\left(\frac{2x}{3}\right)$$

The problem is solved:

$$\int \frac{\sqrt{9-4x^2}}{x^2}\,dx$$

$$= -\frac{\sqrt{9-4x^2}}{x} - 2\arcsin\left(\frac{2x}{3}\right) + C$$

Problem:

$$\int \frac{1}{x^2\sqrt{x^2+1}}\,dx$$

Perform trigonometric substitution:

Substitute $x = \tan(u) \longrightarrow u = \arctan(x), dx = \sec^2(u)\,du$ (steps):

$$= \int \frac{\sec^2(u)}{\tan^2(u)\sqrt{\tan^2(u)+1}}\,du$$

Simplify using $\tan^2(u) + 1 = \sec^2(u)$:

$$= \int \frac{\sec(u)}{\tan^2(u)}\,du$$

... or choose an alternative:

Perform hyperbolic substitution

Rewrite/simplify using trigonometric/hyperbolic identities:

$$= \int \frac{\cos(u)}{\sin^2(u)}\,du$$

Substitute $v = \sin(u) \longrightarrow dv = \cos(u)\,du$ (steps):

$$= \int \frac{1}{v^2}\,dv$$

Apply power rule:

$$\int v^n\,dv = \frac{v^{n+1}}{n+1} \text{ with } n = -2:$$

$$= -\frac{1}{v}$$

Undo substitution $v = \sin(u)$:

$$= -\frac{1}{\sin(u)}$$

Undo substitution $u = \arctan(x)$, use:

$$\sin(\arctan(x)) = \frac{x}{\sqrt{x^2+1}}$$

$$= -\frac{\sqrt{x^2+1}}{x}$$

The problem is solved:

$$\int \frac{1}{x^2\sqrt{x^2+1}}\,dx$$

$$= -\frac{\sqrt{x^2+1}}{x} + C$$

Problem:

$$\int \frac{1}{\left(x^2+4\right)^2}\,\mathrm{d}x$$

Apply reduction formula:

$$\int \frac{1}{\left(\mathrm{a}x^2+\mathrm{b}\right)^{\mathrm{n}}}\,\mathrm{d}x = \frac{2\mathrm{n}-3}{2\mathrm{b}\left(\mathrm{n}-1\right)}\int \frac{1}{\left(\mathrm{a}x^2+\mathrm{b}\right)^{\mathrm{n}-1}}\,\mathrm{d}x + \frac{x}{2\mathrm{b}\left(\mathrm{n}-1\right)\left(\mathrm{a}x^2+\mathrm{b}\right)^{\mathrm{n}-1}}$$

with $\mathrm{a}=1, \mathrm{b}=4, \mathrm{n}=2$:

$$= \frac{x}{8\left(x^2+4\right)} + \frac{1}{8}\int \frac{1}{x^2+4}\,\mathrm{d}x$$

Now solving:

$$\int \frac{1}{x^2+4}\,\mathrm{d}x$$

Substitute $u = \frac{x}{2} \longrightarrow \mathrm{d}u = \frac{1}{2}\,\mathrm{d}x$ (steps):

$$= \int \frac{2}{4u^2+4}\,\mathrm{d}u$$

Simplify:

$$= \frac{1}{2}\int \frac{1}{u^2+1}\,\mathrm{d}u$$

Now solving:

$$\int \frac{1}{u^2+1}\,\mathrm{d}u$$

This is a standard integral:

$$= \arctan(u)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{u^2+1}\,\mathrm{d}u$$

$$= \frac{\arctan(u)}{2}$$

Undo substitution $u = \frac{x}{2}$:

$$= \frac{\arctan\left(\frac{x}{2}\right)}{2}$$

Plug in solved integrals:

$$\frac{x}{8\left(x^2+4\right)} + \frac{1}{8}\int \frac{1}{x^2+4}\,\mathrm{d}x$$

$$= \frac{x}{8\left(x^2+4\right)} + \frac{\arctan\left(\frac{x}{2}\right)}{16}$$

The problem is solved:

$$\int \frac{1}{\left(x^2+4\right)^2}\,\mathrm{d}x$$

$$= \frac{x}{8\left(x^2+4\right)} + \frac{\arctan\left(\frac{x}{2}\right)}{16} + C$$

Rewrite/simplify:

$$= \frac{x}{8x^2+32} + \frac{\arctan\left(\frac{x}{2}\right)}{16} + C$$

Problem:

$$\int x^3\sqrt{4x^2-1}\,\mathrm{d}x$$

Substitute $u = 4x^2-1 \longrightarrow \mathrm{d}u = 8x\,\mathrm{d}x$ (steps):

$$= \frac{1}{32}\int \left(u^{\frac{3}{2}}+\sqrt{u}\right)\mathrm{d}u$$

… or choose an alternative:

Substitute x^2

Don't substitute

Now solving:

$$\int \left(u^{\frac{3}{2}}+\sqrt{u}\right)\mathrm{d}u$$

Apply linearity:

$$= \int u^{\frac{3}{2}}\,\mathrm{d}u + \int \sqrt{u}\,\mathrm{d}u$$

Now solving:

$$\int u^{\frac{3}{2}}\,\mathrm{d}u$$

Apply power rule:

$$\int u^{\mathrm{n}}\,\mathrm{d}u = \frac{u^{\mathrm{n}+1}}{\mathrm{n}+1} \text{ with } \mathrm{n} = \frac{3}{2}:$$

$$= \frac{2u^{\frac{5}{2}}}{5}$$

Now solving:

$$\int \sqrt{u}\,\mathrm{d}u$$

Apply power rule with $\mathrm{n} = \frac{1}{2}$:

$$= \frac{2u^{\frac{3}{2}}}{3}$$

Plug in solved integrals:

$$\int u^{\frac{3}{2}}\,\mathrm{d}u + \int \sqrt{u}\,\mathrm{d}u$$

$$= \frac{2u^{\frac{5}{2}}}{5} + \frac{2u^{\frac{3}{2}}}{3}$$

Plug in solved integrals:

$$\frac{1}{32}\int \left(u^{\frac{3}{2}}+\sqrt{u}\right)\mathrm{d}u$$

$$= \frac{u^{\frac{5}{2}}}{80} + \frac{u^{\frac{3}{2}}}{48}$$

Undo substitution $u = 4x^2-1$:

$$= \frac{\left(4x^2-1\right)^{\frac{5}{2}}}{80} + \frac{\left(4x^2-1\right)^{\frac{3}{2}}}{48}$$

The problem is solved:

$$\int x^3\sqrt{4x^2-1}\,\mathrm{d}x$$

$$= \frac{\left(4x^2-1\right)^{\frac{5}{2}}}{80} + \frac{\left(4x^2-1\right)^{\frac{3}{2}}}{48} + C$$

Rewrite/simplify:

$$= \frac{\left(4x^2-1\right)^{\frac{3}{2}}\left(6x^2+1\right)}{120} + C$$

Problem:

$$\int \frac{1}{x\sqrt{x^2-9}}\,dx$$

Substitute $u = \sqrt{x^2-9} \longrightarrow du = \frac{x}{\sqrt{x^2-9}}\,dx$ (steps):

$$= \int \frac{1}{u^2+9}\,du$$

... or choose an alternative:

Substitute x^2

Substitute x^2-9

Don't substitute

Substitute $v = \frac{u}{3} \longrightarrow dv = \frac{1}{3}\,du$ (steps):

$$= \int \frac{3}{9v^2+9}\,dv$$

Simplify:

$$= \frac{1}{3}\int \frac{1}{v^2+1}\,dv$$

Now solving:

$$\int \frac{1}{v^2+1}\,dv$$

This is a standard integral:

$$= \arctan(v)$$

Plug in solved integrals:

$$\frac{1}{3}\int \frac{1}{v^2+1}\,dv$$

$$= \frac{\arctan(v)}{3}$$

Undo substitution $v = \frac{u}{3}$:

$$= \frac{\arctan\left(\frac{u}{3}\right)}{3}$$

Undo substitution $u = \sqrt{x^2-9}$:

$$= \frac{\arctan\left(\frac{\sqrt{x^2-9}}{3}\right)}{3}$$

The problem is solved:

$$\int \frac{1}{x\sqrt{x^2-9}}\,dx$$

$$= \frac{\arctan\left(\frac{\sqrt{x^2-9}}{3}\right)}{3} + C$$

Problem:

$$\int \frac{1}{x^3\sqrt{x^2-4}}\,dx$$

Substitute $u = \sqrt{x^2-4} \longrightarrow du = \frac{x}{\sqrt{x^2-4}}\,dx$ (steps):

$$= \int \frac{1}{(u^2+4)^2}\,du$$

... or choose an alternative:

Substitute x^2

Substitute x^2-4

Don't substitute

Apply reduction formula:

$$\int \frac{1}{(au^2+b)^n}\,du = \frac{2n-3}{2b(n-1)}\int \frac{1}{(au^2+b)^{n-1}}\,du + \frac{u}{2b(n-1)(au^2+b)^{n-1}}$$

with $a = 1, b = 4, n = 2$:

$$= \frac{u}{8(u^2+4)} + \frac{1}{8}\int \frac{1}{u^2+4}\,du$$

Now solving:

$$\int \frac{1}{u^2+4}\,du$$

Substitute $v = \frac{u}{2} \longrightarrow dv = \frac{1}{2}\,du$ (steps):

$$= \int \frac{2}{4v^2+4}\,dv$$

Simplify:

$$= \frac{1}{2}\int \frac{1}{v^2+1}\,dv$$

Now solving:

$$\int \frac{1}{v^2+1}\,dv$$

This is a standard integral:

$$= \arctan(v)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{v^2+1}\,dv$$

$$= \frac{\arctan(v)}{2}$$

Undo substitution $v = \frac{u}{2}$:

$$= \frac{\arctan\left(\frac{u}{2}\right)}{2}$$

Plug in solved integrals:

$$\frac{u}{8(u^2+4)} + \frac{1}{8}\int \frac{1}{u^2+4}\,du$$

$$= \frac{u}{8(u^2+4)} + \frac{\arctan\left(\frac{u}{2}\right)}{16}$$

Undo substitution $u = \sqrt{x^2-4}$:

$$= \frac{\arctan\left(\frac{\sqrt{x^2-4}}{2}\right)}{16} + \frac{\sqrt{x^2-4}}{8x^2}$$

The problem is solved:

$$\int \frac{1}{x^3\sqrt{x^2-4}}\,dx$$

$$= \frac{\arctan\left(\frac{\sqrt{x^2-4}}{2}\right)}{16} + \frac{\sqrt{x^2-4}}{8x^2} + C$$

Rewrite/simplify:

$$= \frac{\arctan\left(\frac{\sqrt{x^2-4}}{2}\right) + \frac{2\sqrt{x^2-4}}{x^2}}{16} + C$$

Problem:

$$\int \frac{(x^2-16)^{\frac{3}{2}}}{x^3}\,dx$$

Substitute $u=\sqrt{x^2-16} \longrightarrow du=\frac{x}{\sqrt{x^2-16}}\,dx$ (steps):

$$=\int \frac{u^4}{(u^2+16)^2}\,du$$

... or choose an alternative:

Substitute x^2

Substitute x^2-16

Don't substitute

Perform polynomial long division:

$$=\int\left(\frac{-32u^2-256}{(u^2+16)^2}+1\right)du$$

Apply linearity:

$$=\int 1\,du-32\int\frac{u^2+8}{(u^2+16)^2}\,du$$

Now solving:

$$\int 1\,du$$

Apply constant rule:

$$=u$$

Now solving:

$$\int\frac{u^2+8}{(u^2+16)^2}\,du$$

Write u^2+8 as u^2+16-8 and split:

$$=\int\left(\frac{u^2+16}{(u^2+16)^2}-\frac{8}{(u^2+16)^2}\right)du$$

$$=\int\left(\frac{1}{u^2+16}-\frac{8}{(u^2+16)^2}\right)du$$

Apply linearity:

$$=\int\frac{1}{u^2+16}\,du-8\int\frac{1}{(u^2+16)^2}\,du$$

Now solving:

$$\int\frac{1}{u^2+16}\,du$$

Substitute $v=\frac{u}{4}\longrightarrow dv=\frac{1}{4}\,du$ (steps):

$$=\int\frac{4}{16v^2+16}\,dv$$

Simplify:

$$=\frac{1}{4}\int\frac{1}{v^2+1}\,dv$$

Now solving:

$$\int\frac{1}{v^2+1}\,dv$$

This is a standard integral:

$$=\arctan(v)$$

Plug in solved integrals:

$$\frac{1}{4}\int\frac{1}{v^2+1}\,dv$$

$$=\frac{\arctan(v)}{4}$$

Undo substitution $v=\frac{u}{4}$:

$$=\frac{\arctan\left(\frac{u}{4}\right)}{4}$$

Now solving:

$$\int\frac{1}{(u^2+16)^2}\,du$$

Apply reduction formula:

$$\int\frac{1}{(au^2+b)^n}\,du=\frac{2n-3}{2b(n-1)}\int\frac{1}{(au^2+b)^{n-1}}\,du+\frac{u}{2b(n-1)(au^2+b)^{n-1}}$$

with $a=1, b=16, n=2$:

$$=\frac{u}{32(u^2+16)}+\frac{1}{32}\int\frac{1}{u^2+16}\,du$$

Now solving:

$$\int\frac{1}{u^2+16}\,du$$

Use previous result:

$$=\frac{\arctan\left(\frac{u}{4}\right)}{4}$$

Plug in solved integrals:

$$\frac{u}{32(u^2+16)}+\frac{1}{32}\int\frac{1}{u^2+16}\,du$$

$$=\frac{u}{32(u^2+16)}+\frac{\arctan\left(\frac{u}{4}\right)}{128}$$

Plug in solved integrals:

$$\int\frac{1}{u^2+16}\,du-8\int\frac{1}{(u^2+16)^2}\,du$$

$$=\frac{3\arctan\left(\frac{u}{4}\right)}{16}-\frac{u}{4(u^2+16)}$$

Plug in solved integrals:

$$\int 1\,du-32\int\frac{u^2+8}{(u^2+16)^2}\,du$$

$$=\frac{8u}{u^2+16}+u-6\arctan\left(\frac{u}{4}\right)$$

Undo substitution $u=\sqrt{x^2-16}$:

$$=-6\arctan\left(\frac{\sqrt{x^2-16}}{4}\right)+\frac{8\sqrt{x^2-16}}{x^2}+\sqrt{x^2-16}$$

The problem is solved:

$$\int\frac{(x^2-16)^{\frac{3}{2}}}{x^3}\,dx$$

$$=-6\arctan\left(\frac{\sqrt{x^2-16}}{4}\right)+\frac{8\sqrt{x^2-16}}{x^2}+\sqrt{x^2-16}+C$$

Rewrite/simplify:

$$=\frac{\sqrt{x^2-16}\,(x^2+8)}{x^2}-6\arctan\left(\frac{\sqrt{x^2-16}}{4}\right)+C$$

Problem:

$$\int\frac{x^2}{(1-x^2)^{\frac{5}{2}}}\,dx$$

Perform trigonometric substitution:

Substitute $x=\sin(u)\longrightarrow u=\arcsin(x), dx=\cos(u)\,du$ (steps):

$$=\int\frac{\cos(u)\sin^2(u)}{(1-\sin^2(u))^{\frac{5}{2}}}\,du$$

Simplify using $1-\sin^2(u)=\cos^2(u)$:

$$=\int\frac{\sin^2(u)}{\cos^4(u)}\,du$$

... or choose an alternative:

Perform hyperbolic substitution

Rewrite/simplify using trigonometric/hyperbolic identities:

$$=\int\sec^2(u)\tan^2(u)\,du$$

Substitute $v=\tan(u)\longrightarrow dv=\sec^2(u)\,du$ (steps):

$$=\int v^2\,dv$$

Apply power rule:

$$\int v^n\,dv=\frac{v^{n+1}}{n+1}\text{ with } n=2:$$

$$=\frac{v^3}{3}$$

Undo substitution $v=\tan(u)$:

$$=\frac{\tan^3(u)}{3}$$

Undo substitution $u=\arcsin(x)$, use:

$$\tan(\arcsin(x))=\frac{x}{\sqrt{1-x^2}}$$

$$=\frac{x^3}{3(1-x^2)^{\frac{3}{2}}}$$

The problem is solved:

$$\int\frac{x^2}{(1-x^2)^{\frac{5}{2}}}\,dx$$

$$=\frac{x^3}{3(1-x^2)^{\frac{3}{2}}}+C$$

Problem:

$$\int \frac{1}{(2x-3)\sqrt{4x^2-12x+5}}\,dx$$

Substitute $u = \sqrt{4x^2-12x+5} \longrightarrow du = \frac{8x-12}{2\sqrt{4x^2-12x+5}}\,dx$ (steps):

$$= \frac{1}{2}\int \frac{1}{u^2+4}\,du$$

... or choose an alternative:

Substitute $4x^2-12x+5$

Substitute $2x-3$

Don't substitute

Now solving:

$$\int \frac{1}{u^2+4}\,du$$

Substitute $v = \frac{u}{2} \longrightarrow dv = \frac{1}{2}\,du$ (steps):

$$= \int \frac{2}{4v^2+4}\,dv$$

Simplify:

$$= \frac{1}{2}\int \frac{1}{v^2+1}\,dv$$

Now solving:

$$\int \frac{1}{v^2+1}\,dv$$

This is a standard integral:

$$= \arctan(v)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{v^2+1}\,dv$$

$$= \frac{\arctan(v)}{2}$$

Undo substitution $v = \frac{u}{2}$:

$$= \frac{\arctan\left(\frac{u}{2}\right)}{2}$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{u^2+4}\,du$$

$$= \frac{\arctan\left(\frac{u}{2}\right)}{4}$$

Undo substitution $u = \sqrt{4x^2-12x+5}$:

$$= \frac{\arctan\left(\frac{\sqrt{4x^2-12x+5}}{2}\right)}{4}$$

The problem is solved:

$$\int \frac{1}{(2x-3)\sqrt{4x^2-12x+5}}\,dx$$

$$= \frac{\arctan\left(\frac{\sqrt{4x^2-12x+5}}{2}\right)}{4} + C$$

Integration of Rational Functions

1. $\displaystyle\int \frac{12x+18}{(x+2)(x+4)(x-1)}\,dx$

2. $\displaystyle\int \frac{x-5}{x^2-x-2}\,dx$

3. $\displaystyle\int \frac{dx}{(x-1)(x-4)}$

4. $\displaystyle\int \frac{3x-11}{(2x+1)(x-3)}\,dx$

5. $\displaystyle\int \frac{6x^2+23x-9}{x^3+2x^2-3x}\,dx$

6. $\displaystyle\int \frac{-2x^2+6x+8}{x^2(x+2)}\,dx$

7. $\displaystyle\int \frac{x^3+5x^2+9x+7}{x^2+5x+4}\,dx$

8. $\displaystyle\int \frac{2x^2+x+6}{x^2+x-2}\,dx$

9. $\displaystyle\int \frac{(2x+1)}{(x-2)(x-3)^2}\,dx$

10. $\displaystyle\int \frac{4x^2+3x+2}{x^2(x+1)}\,dx$

11. $\displaystyle\int \frac{2x-5}{x(x-1)^3}\,dx$

12. $\displaystyle\int \frac{3x^2-6x+7}{x^3-x^2-x+1}\,dx$

13. $\displaystyle\int \frac{3x^2+17x+32}{x^3+8x^2+16x}\,dx$

14. $\displaystyle\int \frac{13x^2+17}{(2x+1)(x^2+4)}\,dx$

15. $\displaystyle\int \frac{2x+1}{(3x-1)(x^2+2x+2)}\,dx$

16. $\displaystyle\int \frac{2x^2-3x-7}{(x+3)(x^2+1)}\,dx$

17. $\displaystyle\int \frac{5x^2-x+17}{(x+2)(x^2+9)}\,dx$

18. $\displaystyle\int \frac{3x^3+6x^2+17x-5}{x^2+2x+5}\,dx$

19. $\displaystyle\int \frac{4x^2+21x+54}{x^2+6x+13}\,dx$

20. $\displaystyle\int \frac{x^3}{x^2+x+1}\,dx$

21 $\displaystyle\int \frac{x^3+7x^2+25x+35}{x^2+5x+6}\,dx$

22. $\displaystyle\int \frac{x^2+3x+5}{x^3+8}\,dx$

23. $\displaystyle\int \frac{x^2-x-8}{(2x-3)(x^2+2x+2)}\,dx$

24. $\displaystyle\int \frac{dx}{x^4+16x^2}$

25. $\displaystyle\int \frac{x^5+2x^3-3x}{(x^2+1)^3}\,dx$

26. $\displaystyle\int \frac{dx}{x^2(x^2+1)^2}$

27. $\displaystyle\int \frac{x^4+2x^3+11x^2+8x+16}{x(x^2+4)^2}\,dx$

28. $\displaystyle\int \frac{x+2}{(x^2-1)(x^2+1)^2}\,dx$

Problem:

$$\int \frac{12x+18}{(x-1)(x+2)(x+4)}\,dx$$

Apply linearity:

$$=6\int \frac{2x+3}{(x-1)(x+2)(x+4)}\,dx$$

Now solving:

$$\int \frac{2x+3}{(x-1)(x+2)(x+4)}\,dx$$

Perform partial fraction decomposition:

$$=\int\left(-\frac{1}{2(x+4)}+\frac{1}{6(x+2)}+\frac{1}{3(x-1)}\right)dx$$

Apply linearity:

$$=-\frac{1}{2}\int\frac{1}{x+4}\,dx+\frac{1}{6}\int\frac{1}{x+2}\,dx+\frac{1}{3}\int\frac{1}{x-1}\,dx$$

Now solving:

$$\int\frac{1}{x+4}\,dx$$

Substitute $u=x+4 \longrightarrow du=dx$ (steps):

$$=\int\frac{1}{u}\,du$$

This is a standard integral:

$$=\ln(u)$$

Undo substitution $u=x+4$:

$$=\ln(x+4)$$

Now solving:

$$\int\frac{1}{x+2}\,dx$$

Substitute $u=x+2 \longrightarrow du=dx$ (steps):

$$=\int\frac{1}{u}\,du$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x+2$:

$$=\ln(x+2)$$

Now solving:

$$\int\frac{1}{x-1}\,dx$$

Substitute $u=x-1 \longrightarrow du=dx$ (steps):

$$=\int\frac{1}{u}\,du$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x-1$:

$$=\ln(x-1)$$

Plug in solved integrals:

$$-\frac{1}{2}\int\frac{1}{x+4}\,dx+\frac{1}{6}\int\frac{1}{x+2}\,dx+\frac{1}{3}\int\frac{1}{x-1}\,dx$$
$$=-\frac{\ln(x+4)}{2}+\frac{\ln(x+2)}{6}+\frac{\ln(x-1)}{3}$$

Plug in solved integrals:

$$6\int\frac{2x+3}{(x-1)(x+2)(x+4)}\,dx$$
$$=-3\ln(x+4)+\ln(x+2)+2\ln(x-1)$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int\frac{12x+18}{(x-1)(x+2)(x+4)}\,dx$$
$$=-3\ln(|x+4|)+\ln(|x+2|)+2\ln(|x-1|)+C$$

Problem:

$$\int \frac{x-5}{x^2-x-2}\,\mathrm{d}x$$

Write $x-5$ as $\frac{1}{2}(2x-1)-\frac{9}{2}$ and split:

$$=\int\left(\frac{2x-1}{2(x^2-x-2)}-\frac{9}{2(x^2-x-2)}\right)\mathrm{d}x$$

Apply linearity:

$$=\frac{1}{2}\int\frac{2x-1}{x^2-x-2}\,\mathrm{d}x-\frac{9}{2}\int\frac{1}{x^2-x-2}\,\mathrm{d}x$$

Now solving:

$$\int\frac{2x-1}{x^2-x-2}\,\mathrm{d}x$$

Substitute $u=x^2-x-2 \longrightarrow \mathrm{d}u=(2x-1)\,\mathrm{d}x$ (steps):

$$=\int\frac{1}{u}\,\mathrm{d}u$$

This is a standard integral:

$$=\ln(u)$$

Undo substitution $u=x^2-x-2$:

$$=\ln(x^2-x-2)$$

Now solving:

$$\int\frac{1}{x^2-x-2}\,\mathrm{d}x$$

Factor the denominator:

$$=\int\frac{1}{(x-2)(x+1)}\,\mathrm{d}x$$

Perform partial fraction decomposition:

$$=\int\left(\frac{1}{3(x-2)}-\frac{1}{3(x+1)}\right)\mathrm{d}x$$

Apply linearity:

$$=\frac{1}{3}\int\frac{1}{x-2}\,\mathrm{d}x-\frac{1}{3}\int\frac{1}{x+1}\,\mathrm{d}x$$

Now solving:

$$\int\frac{1}{x-2}\,\mathrm{d}x$$

Substitute $u=x-2 \longrightarrow \mathrm{d}u=\mathrm{d}x$ (steps):

$$=\int\frac{1}{u}\,\mathrm{d}u$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x-2$:

$$=\ln(x-2)$$

Now solving:

$$\int\frac{1}{x+1}\,\mathrm{d}x$$

Substitute $u=x+1 \longrightarrow \mathrm{d}u=\mathrm{d}x$ (steps):

$$=\int\frac{1}{u}\,\mathrm{d}u$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x+1$:

$$=\ln(x+1)$$

Plug in solved integrals:

$$\frac{1}{3}\int\frac{1}{x-2}\,\mathrm{d}x-\frac{1}{3}\int\frac{1}{x+1}\,\mathrm{d}x$$

$$=\frac{\ln(x-2)}{3}-\frac{\ln(x+1)}{3}$$

Plug in solved integrals:

$$\frac{1}{2}\int\frac{2x-1}{x^2-x-2}\,\mathrm{d}x-\frac{9}{2}\int\frac{1}{x^2-x-2}\,\mathrm{d}x$$

$$=\frac{\ln(x^2-x-2)}{2}+\frac{3\ln(x+1)}{2}-\frac{3\ln(x-2)}{2}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int\frac{x-5}{x^2-x-2}\,\mathrm{d}x$$

$$=\frac{\ln(|x^2-x-2|)}{2}+\frac{3\ln(|x+1|)}{2}-\frac{3\ln(|x-2|)}{2}+C$$

Rewrite/simplify:

$$=\frac{\ln(|x-2|\,|x+1|)+3\ln(|x+1|)-3\ln(|x-2|)}{2}+C$$

Problem:

$$\int\frac{1}{(x-4)(x-1)}\,\mathrm{d}x$$

Substitute $u=x-1 \longrightarrow \mathrm{d}u=\mathrm{d}x$ (steps):

$$=\int\frac{1}{(u-3)\,u}\,\mathrm{d}u$$

... or choose an alternative:

Substitute $x-4$

Don't substitute

This integral could be solved using partial fraction decomposition, but there is a simpler way.

Expand fraction by $\frac{1}{u^2}$:

$$=\int\frac{1}{\left(1-\frac{3}{u}\right)u^2}\,\mathrm{d}u$$

... or choose an alternative:

Don't apply this trick

Substitute $v=1-\frac{3}{u} \longrightarrow \mathrm{d}v=\frac{3}{u^2}\,\mathrm{d}u$ (steps):

$$=\frac{1}{3}\int\frac{1}{v}\,\mathrm{d}v$$

Now solving:

$$\int\frac{1}{v}\,\mathrm{d}v$$

This is a standard integral:

$$=\ln(v)$$

Plug in solved integrals:

$$\frac{1}{3}\int\frac{1}{v}\,\mathrm{d}v$$

$$=\frac{\ln(v)}{3}$$

Undo substitution $v=1-\frac{3}{u}$:

$$=\frac{\ln\left(1-\frac{3}{u}\right)}{3}$$

Undo substitution $u=x-1$:

$$=\frac{\ln\left(1-\frac{3}{x-1}\right)}{3}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int\frac{1}{(x-4)(x-1)}\,\mathrm{d}x$$

$$=\frac{\ln\left(\left|\frac{3}{x-1}-1\right|\right)}{3}+C$$

Problem:

$$\int \frac{3x-11}{(x-3)(2x+1)}\,dx$$

Perform partial fraction decomposition:

$$= \int \left(\frac{25}{7(2x+1)} - \frac{2}{7(x-3)}\right) dx$$

Apply linearity:

$$= \frac{25}{7}\int \frac{1}{2x+1}\,dx - \frac{2}{7}\int \frac{1}{x-3}\,dx$$

Now solving:

$$\int \frac{1}{2x+1}\,dx$$

Substitute $u = 2x+1 \longrightarrow du = 2\,dx$ (steps):

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

Now solving:

$$\int \frac{1}{u}\,du$$

This is a standard integral:

$$= \ln(u)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{\ln(u)}{2}$$

Undo substitution $u = 2x+1$:

$$= \frac{\ln(2x+1)}{2}$$

Now solving:

$$\int \frac{1}{x-3}\,dx$$

Substitute $u = x-3 \longrightarrow du = dx$ (steps):

$$= \int \frac{1}{u}\,du$$

Use previous result:

$$= \ln(u)$$

Undo substitution $u = x-3$:

$$= \ln(x-3)$$

Plug in solved integrals:

$$\frac{25}{7}\int \frac{1}{2x+1}\,dx - \frac{2}{7}\int \frac{1}{x-3}\,dx$$

$$= \frac{25\ln(2x+1)}{14} - \frac{2\ln(x-3)}{7}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{3x-11}{(x-3)(2x+1)}\,dx$$

$$= \frac{25\ln(|2x+1|)}{14} - \frac{2\ln(|x-3|)}{7} + C$$

Problem:

$$\int \frac{6x^2+23x-9}{x^3+2x^2-3x}\,dx$$

Factor the denominator:

$$= \int \frac{6x^2+23x-9}{(x-1)\,x\cdot(x+3)}\,dx$$

Perform partial fraction decomposition:

$$= \int \left(-\frac{2}{x+3} + \frac{3}{x} + \frac{5}{x-1}\right) dx$$

Apply linearity:

$$= -2\int \frac{1}{x+3}\,dx + 3\int \frac{1}{x}\,dx + 5\int \frac{1}{x-1}\,dx$$

Now solving:

$$\int \frac{1}{x+3}\,dx$$

Substitute $u = x+3 \longrightarrow du = dx$ (steps):

$$= \int \frac{1}{u}\,du$$

This is a standard integral:

$$= \ln(u)$$

Undo substitution $u = x+3$:

$$= \ln(x+3)$$

Now solving:

$$\int \frac{1}{x}\,dx$$

Use previous result:

$$= \ln(x)$$

Now solving:

$$\int \frac{1}{x-1}\,dx$$

Substitute $u = x-1 \longrightarrow du = dx$ (steps):

$$= \int \frac{1}{u}\,du$$

Use previous result:

$$= \ln(u)$$

Undo substitution $u = x-1$:

$$= \ln(x-1)$$

Plug in solved integrals:

$$-2\int \frac{1}{x+3}\,dx + 3\int \frac{1}{x}\,dx + 5\int \frac{1}{x-1}\,dx$$

$$= -2\ln(x+3) + 3\ln(x) + 5\ln(x-1)$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{6x^2+23x-9}{x^3+2x^2-3x}\,dx$$

$$= -2\ln(|x+3|) + 3\ln(|x|) + 5\ln(|x-1|) + C$$

Problem:

$$\int \frac{-2x^2+6x+8}{x^2\cdot(x+2)}\,dx$$

Apply linearity:

$$=-2\int \frac{x^2-3x-4}{x^2\cdot(x+2)}\,dx$$

Now solving:

$$\int \frac{x^2-3x-4}{x^2\cdot(x+2)}\,dx$$

Perform partial fraction decomposition:

$$=\int\left(\frac{3}{2(x+2)}-\frac{1}{2x}-\frac{2}{x^2}\right)dx$$

Apply linearity:

$$=\frac{3}{2}\int\frac{1}{x+2}\,dx-\frac{1}{2}\int\frac{1}{x}\,dx-2\int\frac{1}{x^2}\,dx$$

Now solving:

$$\int\frac{1}{x+2}\,dx$$

Substitute $u=x+2 \longrightarrow du=dx$ (steps):

$$=\int\frac{1}{u}\,du$$

This is a standard integral:

$$=\ln(u)$$

Undo substitution $u=x+2$:

$$=\ln(x+2)$$

Now solving:

$$\int\frac{1}{x}\,dx$$

Use previous result:

$$=\ln(x)$$

Now solving:

$$\int\frac{1}{x^2}\,dx$$

Apply power rule:

$$\int x^n\,dx=\frac{x^{n+1}}{n+1} \text{ with } n=-2:$$

$$=-\frac{1}{x}$$

Plug in solved integrals:

$$\frac{3}{2}\int\frac{1}{x+2}\,dx-\frac{1}{2}\int\frac{1}{x}\,dx-2\int\frac{1}{x^2}\,dx$$

$$=\frac{3\ln(x+2)}{2}-\frac{\ln(x)}{2}+\frac{2}{x}$$

Plug in solved integrals:

$$-2\int\frac{x^2-3x-4}{x^2\cdot(x+2)}\,dx$$

$$=-3\ln(x+2)+\ln(x)-\frac{4}{x}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{-2x^2+6x+8}{x^2\cdot(x+2)}\,dx$$

$$=-3\ln(|x+2|)+\ln(|x|)-\frac{4}{x}+C$$

Problem:

$$\int\frac{x^3+5x^2+9x+7}{x^2+5x+4}\,dx$$

Perform polynomial long division:

$$=\int\left(\frac{5x+7}{x^2+5x+4}+x\right)dx$$

Apply linearity:

$$=\int\frac{5x+7}{x^2+5x+4}\,dx+\int x\,dx$$

Now solving:

$$\int\frac{5x+7}{x^2+5x+4}\,dx$$

Write $5x+7$ as $\frac{5}{2}(2x+5)-\frac{11}{2}$ and split:

$$=\int\left(\frac{5(2x+5)}{2(x^2+5x+4)}-\frac{11}{2(x^2+5x+4)}\right)dx$$

Apply linearity:

$$=\frac{5}{2}\int\frac{2x+5}{x^2+5x+4}\,dx-\frac{11}{2}\int\frac{1}{x^2+5x+4}\,dx$$

Now solving:

$$\int\frac{2x+5}{x^2+5x+4}\,dx$$

Substitute $u=x^2+5x+4 \longrightarrow du=(2x+5)\,dx$ (steps):

$$=\int\frac{1}{u}\,du$$

This is a standard integral:

$$=\ln(u)$$

Undo substitution $u=x^2+5x+4$:

$$=\ln(x^2+5x+4)$$

Now solving:

$$\int\frac{1}{x^2+5x+4}\,dx$$

Factor the denominator:

$$=\int\frac{1}{(x+1)(x+4)}\,dx$$

Perform partial fraction decomposition:

$$=\int\left(\frac{1}{3(x+1)}-\frac{1}{3(x+4)}\right)dx$$

Apply linearity:

$$=\frac{1}{3}\int\frac{1}{x+1}\,dx-\frac{1}{3}\int\frac{1}{x+4}\,dx$$

Now solving:

$$\int\frac{1}{x+1}\,dx$$

Substitute $u=x+1 \longrightarrow du=dx$ (steps):

$$=\int\frac{1}{u}\,du$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x+1$:

$$=\ln(x+1)$$

Now solving:

$$\int\frac{1}{x+4}\,dx$$

Substitute $u=x+4 \longrightarrow du=dx$ (steps):

$$=\int\frac{1}{u}\,du$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x+4$:

$$=\ln(x+4)$$

Plug in solved integrals:

$$\frac{1}{3}\int\frac{1}{x+1}\,dx-\frac{1}{3}\int\frac{1}{x+4}\,dx$$

$$=\frac{\ln(x+1)}{3}-\frac{\ln(x+4)}{3}$$

Plug in solved integrals:

$$\frac{5}{2}\int\frac{2x+5}{x^2+5x+4}\,dx-\frac{11}{2}\int\frac{1}{x^2+5x+4}\,dx$$

$$=\frac{5\ln(x^2+5x+4)}{2}+\frac{11\ln(x+4)}{6}-\frac{11\ln(x+1)}{6}$$

Now solving:

$$\int x\,dx$$

Apply power rule:

$$\int x^n\,dx=\frac{x^{n+1}}{n+1} \text{ with } n=1:$$

$$=\frac{x^2}{2}$$

Plug in solved integrals:

$$\int\frac{5x+7}{x^2+5x+4}\,dx+\int x\,dx$$

$$=\frac{5\ln(x^2+5x+4)}{2}+\frac{11\ln(x+4)}{6}-\frac{11\ln(x+1)}{6}+\frac{x^2}{2}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int\frac{x^3+5x^2+9x+7}{x^2+5x+4}\,dx$$

$$=\frac{5\ln(|x^2+5x+4|)}{2}+\frac{11\ln(|x+4|)}{6}-\frac{11\ln(|x+1|)}{6}+\frac{x^2}{2}+C$$

Rewrite/simplify:

$$=\frac{15\ln(|x+1|\,|x+4|)+11\ln(|x+4|)-11\ln(|x+1|)+3x^2}{6}+C$$

Problem:

$$\int \frac{2x^2+x+6}{x^2+x-2}\,dx$$

Perform polynomial long division:

$$=\int \left(\frac{10-x}{x^2+x-2}+2\right)dx$$

Apply linearity:

$$=2\int 1\,dx-\int \frac{x-10}{x^2+x-2}\,dx$$

Now solving:

$$\int 1\,dx$$

Apply constant rule:

$$=x$$

Now solving:

$$\int \frac{x-10}{x^2+x-2}\,dx$$

Write $x-10$ as $\frac{1}{2}(2x+1)-\frac{21}{2}$ and split:

$$=\int \left(\frac{2x+1}{2(x^2+x-2)}-\frac{21}{2(x^2+x-2)}\right)dx$$

Apply linearity:

$$=\frac{1}{2}\int \frac{2x+1}{x^2+x-2}\,dx-\frac{21}{2}\int \frac{1}{x^2+x-2}\,dx$$

Now solving:

$$\int \frac{2x+1}{x^2+x-2}\,dx$$

Substitute $u=x^2+x-2 \longrightarrow du=(2x+1)\,dx$ (steps):

$$=\int \frac{1}{u}\,du$$

This is a standard integral:

$$=\ln(u)$$

Undo substitution $u=x^2+x-2$:

$$=\ln(x^2+x-2)$$

Now solving:

$$\int \frac{1}{x^2+x-2}\,dx$$

Factor the denominator:

$$=\int \frac{1}{(x-1)(x+2)}\,dx$$

Perform partial fraction decomposition:

$$=\int \left(\frac{1}{3(x-1)}-\frac{1}{3(x+2)}\right)dx$$

Apply linearity:

$$=\frac{1}{3}\int \frac{1}{x-1}\,dx-\frac{1}{3}\int \frac{1}{x+2}\,dx$$

Now solving:

$$\int \frac{1}{x-1}\,dx$$

Substitute $u=x-1 \longrightarrow du=dx$ (steps):

$$=\int \frac{1}{u}\,du$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x-1$:

$$=\ln(x-1)$$

Now solving:

$$\int \frac{1}{x+2}\,dx$$

Substitute $u=x+2 \longrightarrow du=dx$ (steps):

$$=\int \frac{1}{u}\,du$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x+2$:

$$=\ln(x+2)$$

Plug in solved integrals:

$$\frac{1}{3}\int \frac{1}{x-1}\,dx-\frac{1}{3}\int \frac{1}{x+2}\,dx$$

$$=\frac{\ln(x-1)}{3}-\frac{\ln(x+2)}{3}$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{2x+1}{x^2+x-2}\,dx-\frac{21}{2}\int \frac{1}{x^2+x-2}\,dx$$

$$=\frac{\ln(x^2+x-2)}{2}+\frac{7\ln(x+2)}{2}-\frac{7\ln(x-1)}{2}$$

Plug in solved integrals:

$$2\int 1\,dx-\int \frac{x-10}{x^2+x-2}\,dx$$

$$=-\frac{\ln(x^2+x-2)}{2}-\frac{7\ln(x+2)}{2}+2x+\frac{7\ln(x-1)}{2}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{2x^2+x+6}{x^2+x-2}\,dx$$

$$=-\frac{\ln(|x^2+x-2|)}{2}-\frac{7\ln(|x+2|)}{2}+2x+\frac{7\ln(|x-1|)}{2}+C$$

Rewrite/simplify:

$$=-\frac{\ln(|x-1|\,|x+2|)+7\ln(|x+2|)-4x-7\ln(|x-1|)}{2}+C$$

Problem:

$$\int \frac{2x+1}{(x-3)^2(x-2)}\,dx$$

Perform partial fraction decomposition:

$$=\int \left(\frac{5}{x-2}-\frac{5}{x-3}+\frac{7}{(x-3)^2}\right)dx$$

Apply linearity:

$$=5\int \frac{1}{x-2}\,dx-5\int \frac{1}{x-3}\,dx+7\int \frac{1}{(x-3)^2}\,dx$$

Now solving:

$$\int \frac{1}{x-2}\,dx$$

Substitute $u=x-2 \longrightarrow du=dx$ (steps):

$$=\int \frac{1}{u}\,du$$

This is a standard integral:

$$=\ln(u)$$

Undo substitution $u=x-2$:

$$=\ln(x-2)$$

Now solving:

$$\int \frac{1}{x-3}\,dx$$

Substitute $u=x-3 \longrightarrow du=dx$ (steps):

$$=\int \frac{1}{u}\,du$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x-3$:

$$=\ln(x-3)$$

Now solving:

$$\int \frac{1}{(x-3)^2}\,dx$$

Substitute $u=x-3 \longrightarrow du=dx$ (steps):

$$=\int \frac{1}{u^2}\,du$$

Apply power rule:

$\int u^n\,du=\frac{u^{n+1}}{n+1}$ with $n=-2$:

$$=-\frac{1}{u}$$

Undo substitution $u=x-3$:

$$=-\frac{1}{x-3}$$

Plug in solved integrals:

$$5\int \frac{1}{x-2}\,dx-5\int \frac{1}{x-3}\,dx+7\int \frac{1}{(x-3)^2}\,dx$$

$$=-\frac{7}{x-3}+5\ln(x-2)-5\ln(x-3)$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{2x+1}{(x-3)^2(x-2)}\,dx$$

$$=-\frac{7}{x-3}+5\ln(|x-2|)-5\ln(|x-3|)+C$$

Problem:

$$\int \frac{4x^2+3x+2}{x^2 \cdot (x+1)}\,dx$$

Perform partial fraction decomposition:

$$= \int \left(\frac{3}{x+1} + \frac{1}{x} + \frac{2}{x^2}\right) dx$$

Apply linearity:

$$= 3\int \frac{1}{x+1}\,dx + \int \frac{1}{x}\,dx + 2\int \frac{1}{x^2}\,dx$$

Now solving:

$$\int \frac{1}{x+1}\,dx$$

Substitute $u = x+1 \longrightarrow du = dx$ (steps):

$$= \int \frac{1}{u}\,du$$

This is a standard integral:

$$= \ln(u)$$

Undo substitution $u = x+1$:

$$= \ln(x+1)$$

Now solving:

$$\int \frac{1}{x}\,dx$$

Use previous result:

$$= \ln(x)$$

Now solving:

$$\int \frac{1}{x^2}\,dx$$

Apply power rule:

$\int x^n\,dx = \frac{x^{n+1}}{n+1}$ with $n = -2$:

$$= -\frac{1}{x}$$

Plug in solved integrals:

$$3\int \frac{1}{x+1}\,dx + \int \frac{1}{x}\,dx + 2\int \frac{1}{x^2}\,dx$$

$$= 3\ln(x+1) + \ln(x) - \frac{2}{x}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{4x^2+3x+2}{x^2 \cdot (x+1)}\,dx$$

$$= 3\ln(|x+1|) + \ln(|x|) - \frac{2}{x} + C$$

Problem:

$$\int \frac{2x-5}{(x-1)^3 x}\,dx$$

Perform partial fraction decomposition:

$$= \int \left(\frac{5}{x} - \frac{5}{x-1} + \frac{5}{(x-1)^2} - \frac{3}{(x-1)^3}\right) dx$$

Apply linearity:

$$= 5\int \frac{1}{x}\,dx - 5\int \frac{1}{x-1}\,dx + 5\int \frac{1}{(x-1)^2}\,dx - 3\int \frac{1}{(x-1)^3}\,dx$$

Now solving:

$$\int \frac{1}{x}\,dx$$

This is a standard integral:

$$= \ln(x)$$

Now solving:

$$\int \frac{1}{x-1}\,dx$$

Substitute $u = x-1 \longrightarrow du = dx$ (steps):

$$= \int \frac{1}{u}\,du$$

Use previous result:

$$= \ln(u)$$

Undo substitution $u = x-1$:

$$= \ln(x-1)$$

Now solving:

$$\int \frac{1}{(x-1)^2}\,dx$$

Substitute $u = x-1 \longrightarrow du = dx$ (steps):

$$= \int \frac{1}{u^2}\,du$$

Apply power rule:

$\int u^n\,du = \frac{u^{n+1}}{n+1}$ with $n = -2$:

$$= -\frac{1}{u}$$

Undo substitution $u = x-1$:

$$= -\frac{1}{x-1}$$

Now solving:

$$\int \frac{1}{(x-1)^3}\,dx$$

Substitute $u = x-1 \longrightarrow du = dx$ (steps):

$$= \int \frac{1}{u^3}\,du$$

Apply power rule with $n = -3$:

$$= -\frac{1}{2u^2}$$

Undo substitution $u = x-1$:

$$= -\frac{1}{2(x-1)^2}$$

Plug in solved integrals:

$$5\int \frac{1}{x}\,dx - 5\int \frac{1}{x-1}\,dx + 5\int \frac{1}{(x-1)^2}\,dx - 3\int \frac{1}{(x-1)^3}\,dx$$

$$= 5\ln(x) - \frac{5}{x-1} + \frac{3}{2(x-1)^2} - 5\ln(x-1)$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{2x-5}{(x-1)^3 x}\,dx$$

$$= 5\ln(|x|) - \frac{5}{x-1} + \frac{3}{2(x-1)^2} - 5\ln(|x-1|) + C$$

Problem:

$$\int \frac{3x^2-6x+7}{x^3-x^2-x+1}\,\mathrm{d}x$$

Factor the denominator:

$$=\int \frac{3x^2-6x+7}{(x-1)^2(x+1)}\,\mathrm{d}x$$

Perform partial fraction decomposition:

$$=\int\left(\frac{4}{x+1}-\frac{1}{x-1}+\frac{2}{(x-1)^2}\right)\mathrm{d}x$$

Apply linearity:

$$=4\int\frac{1}{x+1}\,\mathrm{d}x-\int\frac{1}{x-1}\,\mathrm{d}x+2\int\frac{1}{(x-1)^2}\,\mathrm{d}x$$

Now solving:

$$\int\frac{1}{x+1}\,\mathrm{d}x$$

Substitute $u=x+1 \longrightarrow \mathrm{d}u=\mathrm{d}x$ (steps):

$$=\int\frac{1}{u}\,\mathrm{d}u$$

This is a standard integral:

$$=\ln(u)$$

Undo substitution $u=x+1$:

$$=\ln(x+1)$$

Now solving:

$$\int\frac{1}{x-1}\,\mathrm{d}x$$

Substitute $u=x-1 \longrightarrow \mathrm{d}u=\mathrm{d}x$ (steps):

$$=\int\frac{1}{u}\,\mathrm{d}u$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x-1$:

$$=\ln(x-1)$$

Now solving:

$$\int\frac{1}{(x-1)^2}\,\mathrm{d}x$$

Substitute $u=x-1 \longrightarrow \mathrm{d}u=\mathrm{d}x$ (steps):

$$=\int\frac{1}{u^2}\,\mathrm{d}u$$

Apply power rule:

$$\int u^{\mathrm{n}}\,\mathrm{d}u=\frac{u^{\mathrm{n}+1}}{\mathrm{n}+1} \text{ with } \mathrm{n}=-2:$$

$$=-\frac{1}{u}$$

Undo substitution $u=x-1$:

$$=-\frac{1}{x-1}$$

Plug in solved integrals:

$$4\int\frac{1}{x+1}\,\mathrm{d}x-\int\frac{1}{x-1}\,\mathrm{d}x+2\int\frac{1}{(x-1)^2}\,\mathrm{d}x$$

$$=4\ln(x+1)-\frac{2}{x-1}-\ln(x-1)$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{3x^2-6x+7}{x^3-x^2-x+1}\,\mathrm{d}x$$

$$=4\ln(|x+1|)-\frac{2}{x-1}-\ln(|x-1|)+C$$

Problem:

$$\int \frac{3x^2+17x+32}{x^3+8x^2+16x}\,\mathrm{d}x$$

Factor the denominator:

$$=\int \frac{3x^2+17x+32}{x\cdot(x+4)^2}\,\mathrm{d}x$$

Perform partial fraction decomposition:

$$=\int\left(\frac{1}{x+4}-\frac{3}{(x+4)^2}+\frac{2}{x}\right)\mathrm{d}x$$

Apply linearity:

$$=\int\frac{1}{x+4}\,\mathrm{d}x-3\int\frac{1}{(x+4)^2}\,\mathrm{d}x+2\int\frac{1}{x}\,\mathrm{d}x$$

Now solving:

$$\int\frac{1}{x+4}\,\mathrm{d}x$$

Substitute $u=x+4 \longrightarrow \mathrm{d}u=\mathrm{d}x$ (steps):

$$=\int\frac{1}{u}\,\mathrm{d}u$$

This is a standard integral:

$$=\ln(u)$$

Undo substitution $u=x+4$:

$$=\ln(x+4)$$

Now solving:

$$\int\frac{1}{(x+4)^2}\,\mathrm{d}x$$

Substitute $u=x+4 \longrightarrow \mathrm{d}u=\mathrm{d}x$ (steps):

$$=\int\frac{1}{u^2}\,\mathrm{d}u$$

Apply power rule:

$$\int u^{\mathrm{n}}\,\mathrm{d}u=\frac{u^{\mathrm{n}+1}}{\mathrm{n}+1} \text{ with } \mathrm{n}=-2:$$

$$=-\frac{1}{u}$$

Undo substitution $u=x+4$:

$$=-\frac{1}{x+4}$$

Now solving:

$$\int\frac{1}{x}\,\mathrm{d}x$$

Use previous result:

$$=\ln(x)$$

Plug in solved integrals:

$$\int\frac{1}{x+4}\,\mathrm{d}x-3\int\frac{1}{(x+4)^2}\,\mathrm{d}x+2\int\frac{1}{x}\,\mathrm{d}x$$

$$=\ln(x+4)+2\ln(x)+\frac{3}{x+4}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{3x^2+17x+32}{x^3+8x^2+16x}\,\mathrm{d}x$$

$$=\ln(|x+4|)+2\ln(|x|)+\frac{3}{x+4}+C$$

Problem:

$$\int \frac{13x^2+17}{(2x+1)(x^2+4)}\,dx$$

Perform partial fraction decomposition:

$$=\int\left(\frac{70x-35}{17(x^2+4)}+\frac{81}{17(2x+1)}\right)dx$$

Apply linearity:

$$=\frac{35}{17}\int\frac{2x-1}{x^2+4}\,dx+\frac{81}{17}\int\frac{1}{2x+1}\,dx$$

Now solving:

$$\int\frac{2x-1}{x^2+4}\,dx$$

Expand:

$$=\int\left(\frac{2x}{x^2+4}-\frac{1}{x^2+4}\right)dx$$

Apply linearity:

$$=2\int\frac{x}{x^2+4}\,dx-\int\frac{1}{x^2+4}\,dx$$

Now solving:

$$\int\frac{x}{x^2+4}\,dx$$

Substitute $u=x^2+4 \longrightarrow du=2x\,dx$ (steps):

$$=\frac{1}{2}\int\frac{1}{u}\,du$$

Now solving:

$$\int\frac{1}{u}\,du$$

This is a standard integral:

$$=\ln(u)$$

Plug in solved integrals:

$$\frac{1}{2}\int\frac{1}{u}\,du$$

$$=\frac{\ln(u)}{2}$$

Undo substitution $u=x^2+4$:

$$=\frac{\ln(x^2+4)}{2}$$

Now solving:

$$\int\frac{1}{x^2+4}\,dx$$

Substitute $u=\frac{x}{2}\longrightarrow du=\frac{1}{2}\,dx$ (steps):

$$=\int\frac{2}{4u^2+4}\,du$$

Simplify:

$$=\frac{1}{2}\int\frac{1}{u^2+1}\,du$$

Now solving:

$$\int\frac{1}{u^2+1}\,du$$

This is a standard integral:

$$=\arctan(u)$$

Plug in solved integrals:

$$\frac{1}{2}\int\frac{1}{u^2+1}\,du$$

$$=\frac{\arctan(u)}{2}$$

Undo substitution $u=\frac{x}{2}$:

$$=\frac{\arctan(\frac{x}{2})}{2}$$

Plug in solved integrals:

$$2\int\frac{x}{x^2+4}\,dx-\int\frac{1}{x^2+4}\,dx$$

$$=\ln(x^2+4)-\frac{\arctan(\frac{x}{2})}{2}$$

Now solving:

$$\int\frac{1}{2x+1}\,dx$$

Substitute $u=2x+1\longrightarrow du=2\,dx$ (steps):

$$=\frac{1}{2}\int\frac{1}{u}\,du$$

Use previous result:

$$=\frac{\ln(u)}{2}$$

Undo substitution $u=2x+1$:

$$=\frac{\ln(2x+1)}{2}$$

Plug in solved integrals:

$$\frac{35}{17}\int\frac{2x-1}{x^2+4}\,dx+\frac{81}{17}\int\frac{1}{2x+1}\,dx$$

$$=\frac{35\ln(x^2+4)}{17}+\frac{81\ln(2x+1)}{34}-\frac{35\arctan(\frac{x}{2})}{34}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int\frac{13x^2+17}{(2x+1)(x^2+4)}\,dx$$

$$=\frac{81\ln(|2x+1|)}{34}+\frac{35\ln(x^2+4)}{17}-\frac{35\arctan(\frac{x}{2})}{34}+C$$

Rewrite/simplify:

$$=\frac{81\ln(|2x+1|)+70\ln(x^2+4)-35\arctan(\frac{x}{2})}{34}+C$$

Problem:

$$\int\frac{2x+1}{(3x-1)(x^2+2x+2)}\,dx$$

Perform partial fraction decomposition:

$$=\int\left(\frac{3}{5(3x-1)}-\frac{x-1}{5(x^2+2x+2)}\right)dx$$

Apply linearity:

$$=\frac{3}{5}\int\frac{1}{3x-1}\,dx-\frac{1}{5}\int\frac{x-1}{x^2+2x+2}\,dx$$

Now solving:

$$\int\frac{1}{3x-1}\,dx$$

Substitute $u=3x-1\longrightarrow du=3\,dx$ (steps):

$$=\frac{1}{3}\int\frac{1}{u}\,du$$

Now solving:

$$\int\frac{1}{u}\,du$$

This is a standard integral:

$$=\ln(u)$$

Plug in solved integrals:

$$\frac{1}{3}\int\frac{1}{u}\,du$$

$$=\frac{\ln(u)}{3}$$

Undo substitution $u=3x-1$:

$$=\frac{\ln(3x-1)}{3}$$

Now solving:

$$\int\frac{x-1}{x^2+2x+2}\,dx$$

Write $x-1$ as $\frac{1}{2}(2x+2)-2$ and split:

$$=\int\left(\frac{2x+2}{2(x^2+2x+2)}-\frac{2}{x^2+2x+2}\right)dx$$

Apply linearity:

$$=\int\frac{x+1}{x^2+2x+2}\,dx-2\int\frac{1}{x^2+2x+2}\,dx$$

Now solving:

$$\int\frac{x+1}{x^2+2x+2}\,dx$$

Substitute $u=x^2+2x+2\longrightarrow du=(2x+2)\,dx$ (steps):

$$=\frac{1}{2}\int\frac{1}{u}\,du$$

Now solving:

$$\int\frac{1}{u}\,du$$

Use previous result:

$$=\ln(u)$$

Plug in solved integrals:

$$\frac{1}{2}\int\frac{1}{u}\,du$$

$$=\frac{\ln(u)}{2}$$

Undo substitution $u=x^2+2x+2$:

$$=\frac{\ln(x^2+2x+2)}{2}$$

Now solving:

$$\int\frac{1}{x^2+2x+2}\,dx$$

Complete the square:

$$=\int\frac{1}{(x+1)^2+1}\,dx$$

Substitute $u=x+1\longrightarrow du=dx$ (steps):

$$=\int\frac{1}{u^2+1}\,du$$

This is a standard integral:

$$=\arctan(u)$$

Undo substitution $u=x+1$:

$$=\arctan(x+1)$$

Plug in solved integrals:

$$\int\frac{x+1}{x^2+2x+2}\,dx-2\int\frac{1}{x^2+2x+2}\,dx$$

$$=\frac{\ln(x^2+2x+2)}{2}-2\arctan(x+1)$$

Plug in solved integrals:

$$\frac{3}{5}\int\frac{1}{3x-1}\,dx-\frac{1}{5}\int\frac{x-1}{x^2+2x+2}\,dx$$

$$=-\frac{\ln(x^2+2x+2)}{10}+\frac{\ln(3x-1)}{5}+\frac{2\arctan(x+1)}{5}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int\frac{2x+1}{(3x-1)(x^2+2x+2)}\,dx$$

$$=\frac{\ln(|3x-1|)}{5}-\frac{\ln(x^2+2x+2)}{10}+\frac{2\arctan(x+1)}{5}+C$$

Rewrite/simplify:

$$=-\frac{-2\ln(|3x-1|)+\ln(x^2+2x+2)-4\arctan(x+1)}{10}+C$$

Problem:

$$\int \frac{2x^2-3x-7}{(x+3)(x^2+1)}\,dx$$

Perform partial fraction decomposition:

$$=\int\left(\frac{2}{x+3}-\frac{3}{x^2+1}\right)dx$$

Apply linearity:

$$=2\int\frac{1}{x+3}\,dx-3\int\frac{1}{x^2+1}\,dx$$

Now solving:

$$\int\frac{1}{x+3}\,dx$$

Substitute $u=x+3 \longrightarrow du=dx$ (steps):

$$=\int\frac{1}{u}\,du$$

This is a standard integral:

$$=\ln(u)$$

Undo substitution $u=x+3$:

$$=\ln(x+3)$$

Now solving:

$$\int\frac{1}{x^2+1}\,dx$$

This is a standard integral:

$$=\arctan(x)$$

Plug in solved integrals:

$$2\int\frac{1}{x+3}\,dx-3\int\frac{1}{x^2+1}\,dx$$

$$=2\ln(x+3)-3\arctan(x)$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{2x^2-3x-7}{(x+3)(x^2+1)}\,dx$$

$$=2\ln(|x+3|)-3\arctan(x)+C$$

Problem:

$$\int \frac{5x^2-x+17}{(x+2)(x^2+9)}\,dx$$

Perform partial fraction decomposition:

$$=\int\left(\frac{2x-5}{x^2+9}+\frac{3}{x+2}\right)dx$$

Apply linearity:

$$=\int\frac{2x-5}{x^2+9}\,dx+3\int\frac{1}{x+2}\,dx$$

Now solving:

$$\int\frac{2x-5}{x^2+9}\,dx$$

Expand:

$$=\int\left(\frac{2x}{x^2+9}-\frac{5}{x^2+9}\right)dx$$

Apply linearity:

$$=2\int\frac{x}{x^2+9}\,dx-5\int\frac{1}{x^2+9}\,dx$$

Now solving:

$$\int\frac{x}{x^2+9}\,dx$$

Substitute $u=x^2+9 \longrightarrow du=2x\,dx$ (steps):

$$=\frac{1}{2}\int\frac{1}{u}\,du$$

Now solving:

$$\int\frac{1}{u}\,du$$

This is a standard integral:

$$=\ln(u)$$

Plug in solved integrals:

$$\frac{1}{2}\int\frac{1}{u}\,du$$

$$=\frac{\ln(u)}{2}$$

Undo substitution $u=x^2+9$:

$$=\frac{\ln(x^2+9)}{2}$$

Now solving:

$$\int\frac{1}{x^2+9}\,dx$$

Substitute $u=\frac{x}{3} \longrightarrow du=\frac{1}{3}\,dx$ (steps):

$$=\int\frac{3}{9u^2+9}\,du$$

Simplify:

$$=\frac{1}{3}\int\frac{1}{u^2+1}\,du$$

Now solving:

$$\int\frac{1}{u^2+1}\,du$$

This is a standard integral:

$$=\arctan(u)$$

Plug in solved integrals:

$$\frac{1}{3}\int\frac{1}{u^2+1}\,du$$

$$=\frac{\arctan(u)}{3}$$

Undo substitution $u=\frac{x}{3}$:

$$=\frac{\arctan\left(\frac{x}{3}\right)}{3}$$

Plug in solved integrals:

$$2\int\frac{x}{x^2+9}\,dx-5\int\frac{1}{x^2+9}\,dx$$

$$=\ln(x^2+9)-\frac{5\arctan\left(\frac{x}{3}\right)}{3}$$

Now solving:

$$\int\frac{1}{x+2}\,dx$$

Substitute $u=x+2 \longrightarrow du=dx$ (steps):

$$=\int\frac{1}{u}\,du$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x+2$:

$$=\ln(x+2)$$

Plug in solved integrals:

$$\int\frac{2x-5}{x^2+9}\,dx+3\int\frac{1}{x+2}\,dx$$

$$=\ln(x^2+9)+3\ln(x+2)-\frac{5\arctan\left(\frac{x}{3}\right)}{3}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{5x^2-x+17}{(x+2)(x^2+9)}\,dx$$

$$=3\ln(|x+2|)+\ln(x^2+9)-\frac{5\arctan\left(\frac{x}{3}\right)}{3}+C$$

Problem:

$$\int \frac{3x^3+6x^2+17x-5}{x^2+2x+5}\,dx$$

Perform polynomial long division:

$$= \int \left(\frac{2x-5}{x^2+2x+5}+3x\right) dx$$

Apply linearity:

$$= \int \frac{2x-5}{x^2+2x+5}\,dx + 3\int x\,dx$$

Now solving:

$$\int \frac{2x-5}{x^2+2x+5}\,dx$$

Write $2x-5$ as $2x+2-7$ and split:

$$= \int \left(\frac{2x+2}{x^2+2x+5} - \frac{7}{x^2+2x+5}\right) dx$$

Apply linearity:

$$= 2\int \frac{x+1}{x^2+2x+5}\,dx - 7\int \frac{1}{x^2+2x+5}\,dx$$

Now solving:

$$\int \frac{x+1}{x^2+2x+5}\,dx$$

Substitute $u = x^2+2x+5 \longrightarrow du = (2x+2)\,dx$ (steps):

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

Now solving:

$$\int \frac{1}{u}\,du$$

This is a standard integral:

$$= \ln(u)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{\ln(u)}{2}$$

Undo substitution $u = x^2+2x+5$:

$$= \frac{\ln(x^2+2x+5)}{2}$$

Now solving:

$$\int \frac{1}{x^2+2x+5}\,dx$$

Complete the square:

$$= \int \frac{1}{(x+1)^2+4}\,dx$$

Substitute $u = \frac{x+1}{2} \longrightarrow du = \frac{1}{2}\,dx$ (steps):

$$= \int \frac{2}{4u^2+4}\,du$$

Simplify:

$$= \frac{1}{2}\int \frac{1}{u^2+1}\,du$$

Now solving:

$$\int \frac{1}{u^2+1}\,du$$

This is a standard integral:

$$= \arctan(u)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{u^2+1}\,du$$

$$= \frac{\arctan(u)}{2}$$

Undo substitution $u = \frac{x+1}{2}$:

$$= \frac{\arctan\left(\frac{x+1}{2}\right)}{2}$$

Plug in solved integrals:

$$2\int \frac{x+1}{x^2+2x+5}\,dx - 7\int \frac{1}{x^2+2x+5}\,dx$$

$$= \ln(x^2+2x+5) - \frac{7\arctan\left(\frac{x+1}{2}\right)}{2}$$

Now solving:

$$\int x\,dx$$

Apply power rule:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \text{ with } n = 1:$$

$$= \frac{x^2}{2}$$

Plug in solved integrals:

$$\int \frac{2x-5}{x^2+2x+5}\,dx + 3\int x\,dx$$

$$= \ln(x^2+2x+5) - \frac{7\arctan\left(\frac{x+1}{2}\right)}{2} + \frac{3x^2}{2}$$

The problem is solved:

$$\int \frac{3x^3+6x^2+17x-5}{x^2+2x+5}\,dx$$

$$= \ln(x^2+2x+5) - \frac{7\arctan\left(\frac{x+1}{2}\right)}{2} + \frac{3x^2}{2} + C$$

Problem:

$$\int \frac{4x^2+21x+54}{x^2+6x+13}\,dx$$

Perform polynomial long division:

$$= \int \left(\frac{2-3x}{x^2+6x+13}+4\right) dx$$

Apply linearity:

$$= 4\int 1\,dx - \int \frac{3x-2}{x^2+6x+13}\,dx$$

Now solving:

$$\int 1\,dx$$

Apply constant rule:

$$= x$$

Now solving:

$$\int \frac{3x-2}{x^2+6x+13}\,dx$$

Write $3x-2$ as $\frac{3}{2}(2x+6)-11$ and split:

$$= \int \left(\frac{3(2x+6)}{2(x^2+6x+13)} - \frac{11}{x^2+6x+13}\right) dx$$

Apply linearity:

$$= 3\int \frac{x+3}{x^2+6x+13}\,dx - 11\int \frac{1}{x^2+6x+13}\,dx$$

Now solving:

$$\int \frac{x+3}{x^2+6x+13}\,dx$$

Substitute $u = x^2+6x+13 \longrightarrow du = (2x+6)\,dx$ (steps):

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

Now solving:

$$\int \frac{1}{u}\,du$$

This is a standard integral:

$$= \ln(u)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{\ln(u)}{2}$$

Undo substitution $u = x^2+6x+13$:

$$= \frac{\ln(x^2+6x+13)}{2}$$

Now solving:

$$\int \frac{1}{x^2+6x+13}\,dx$$

Complete the square:

$$= \int \frac{1}{(x+3)^2+4}\,dx$$

Substitute $u = \frac{x+3}{2} \longrightarrow du = \frac{1}{2}\,dx$ (steps):

$$= \int \frac{2}{4u^2+4}\,du$$

Simplify:

$$= \frac{1}{2}\int \frac{1}{u^2+1}\,du$$

Now solving:

$$\int \frac{1}{u^2+1}\,du$$

This is a standard integral:

$$= \arctan(u)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{u^2+1}\,du$$

$$= \frac{\arctan(u)}{2}$$

Undo substitution $u = \frac{x+3}{2}$:

$$= \frac{\arctan\left(\frac{x+3}{2}\right)}{2}$$

Plug in solved integrals:

$$3\int \frac{x+3}{x^2+6x+13}\,dx - 11\int \frac{1}{x^2+6x+13}\,dx$$

$$= \frac{3\ln(x^2+6x+13)}{2} - \frac{11\arctan\left(\frac{x+3}{2}\right)}{2}$$

Plug in solved integrals:

$$4\int 1\,dx - \int \frac{3x-2}{x^2+6x+13}\,dx$$

$$= -\frac{3\ln(x^2+6x+13)}{2} + \frac{11\arctan\left(\frac{x+3}{2}\right)}{2} + 4x$$

The problem is solved:

$$\int \frac{4x^2+21x+54}{x^2+6x+13}\,dx$$

$$= -\frac{3\ln(x^2+6x+13)}{2} + \frac{11\arctan\left(\frac{x+3}{2}\right)}{2} + 4x + C$$

Problem:

$$\int \frac{x^3}{x^2+x+1}\,dx$$

Perform polynomial long division:

$$=\int\left(\frac{1}{x^2+x+1}+x-1\right)dx$$

Apply linearity:

$$=\int\frac{1}{x^2+x+1}\,dx+\int x\,dx-\int 1\,dx$$

Now solving:

$$\int\frac{1}{x^2+x+1}\,dx$$

Complete the square:

$$=\int\frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}\,dx$$

Substitute $u=\dfrac{2x+1}{\sqrt{3}}\longrightarrow du=\dfrac{2}{\sqrt{3}}\,dx$ (steps):

$$=\int\frac{\sqrt{3}}{2\left(\frac{3u^2}{4}+\frac{3}{4}\right)}\,du$$

Simplify:

$$=\frac{2}{\sqrt{3}}\int\frac{1}{u^2+1}\,du$$

Now solving:

$$\int\frac{1}{u^2+1}\,du$$

This is a standard integral:

$$=\arctan(u)$$

Plug in solved integrals:

$$\frac{2}{\sqrt{3}}\int\frac{1}{u^2+1}\,du$$

$$=\frac{2\arctan(u)}{\sqrt{3}}$$

Undo substitution $u=\dfrac{2x+1}{\sqrt{3}}$:

$$=\frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Now solving:

$$\int x\,dx$$

Apply power rule:

$$\int x^n\,dx=\frac{x^{n+1}}{n+1}\text{ with } n=1:$$

$$=\frac{x^2}{2}$$

Now solving:

$$\int 1\,dx$$

Apply constant rule:

$$=x$$

Plug in solved integrals:

$$\int\frac{1}{x^2+x+1}\,dx+\int x\,dx-\int 1\,dx$$

$$=\frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^2}{2}-x$$

The problem is solved:

$$\int\frac{x^3}{x^2+x+1}\,dx$$

$$=\frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^2}{2}-x+C$$

Rewrite/simplify:

$$=\frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{(x-2)x}{2}+C$$

Problem:

$$\int\frac{x^3+7x^2+25x+35}{x^2+5x+6}\,dx$$

Perform polynomial long division:

$$=\int\left(\frac{9x+23}{x^2+5x+6}+x+2\right)dx$$

Apply linearity:

$$=\int\frac{9x+23}{x^2+5x+6}\,dx+\int x\,dx+2\int 1\,dx$$

Now solving:

$$\int\frac{9x+23}{x^2+5x+6}\,dx$$

Write $9x+23$ as $\frac{9}{2}(2x+5)+\frac{1}{2}$ and split:

$$=\int\left(\frac{9(2x+5)}{2(x^2+5x+6)}+\frac{1}{2(x^2+5x+6)}\right)dx$$

Apply linearity:

$$=\frac{9}{2}\int\frac{2x+5}{x^2+5x+6}\,dx+\frac{1}{2}\int\frac{1}{x^2+5x+6}\,dx$$

Now solving:

$$\int\frac{2x+5}{x^2+5x+6}\,dx$$

Substitute $u=x^2+5x+6\longrightarrow du=(2x+5)\,dx$ (steps):

$$=\int\frac{1}{u}\,du$$

This is a standard integral:

$$=\ln(u)$$

Undo substitution $u=x^2+5x+6$:

$$=\ln(x^2+5x+6)$$

Now solving:

$$\int\frac{1}{x^2+5x+6}\,dx$$

Factor the denominator:

$$=\int\frac{1}{(x+2)(x+3)}\,dx$$

Perform partial fraction decomposition:

$$=\int\left(\frac{1}{x+2}-\frac{1}{x+3}\right)dx$$

Apply linearity:

$$=\int\frac{1}{x+2}\,dx-\int\frac{1}{x+3}\,dx$$

Now solving:

$$\int\frac{1}{x+2}\,dx$$

Substitute $u=x+2\longrightarrow du=dx$ (steps):

$$=\int\frac{1}{u}\,du$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x+2$:

$$=\ln(x+2)$$

Now solving:

$$\int\frac{1}{x+3}\,dx$$

Substitute $u=x+3\longrightarrow du=dx$ (steps):

$$=\int\frac{1}{u}\,du$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x+3$:

$$=\ln(x+3)$$

Plug in solved integrals:

$$\int\frac{1}{x+2}\,dx-\int\frac{1}{x+3}\,dx$$

$$=\ln(x+2)-\ln(x+3)$$

Plug in solved integrals:

$$\frac{9}{2}\int\frac{2x+5}{x^2+5x+6}\,dx+\frac{1}{2}\int\frac{1}{x^2+5x+6}\,dx$$

$$=\frac{9\ln(x^2+5x+6)}{2}-\frac{\ln(x+3)}{2}+\frac{\ln(x+2)}{2}$$

Now solving:

$$\int x\,dx$$

Apply power rule:

$$\int x^n\,dx=\frac{x^{n+1}}{n+1}\text{ with } n=1:$$

$$=\frac{x^2}{2}$$

Now solving:

$$\int 1\,dx$$

Apply constant rule:

$$=x$$

Plug in solved integrals:

$$\int\frac{9x+23}{x^2+5x+6}\,dx+\int x\,dx+2\int 1\,dx$$

$$=\frac{9\ln(x^2+5x+6)}{2}-\frac{\ln(x+3)}{2}+\frac{\ln(x+2)}{2}+\frac{x^2}{2}+2x$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int\frac{x^3+7x^2+25x+35}{x^2+5x+6}\,dx$$

$$=\frac{9\ln(|x^2+5x+6|)}{2}-\frac{\ln(|x+3|)}{2}+\frac{\ln(|x+2|)}{2}+\frac{x^2}{2}+2x+C$$

Rewrite/simplify:

$$=\frac{9\ln(|x+2|\,|x+3|)-\ln(|x+3|)+\ln(|x+2|)+x\cdot(x+4)}{2}+C$$

Problem:

$$\int \frac{x^2+3x+5}{x^3+8}\,dx$$

Factor the denominator:

$$=\int \frac{x^2+3x+5}{(x+2)(x^2-2x+4)}\,dx$$

Perform partial fraction decomposition:

$$=\int \left(\frac{3x+8}{4(x^2-2x+4)}+\frac{1}{4(x+2)}\right)dx$$

Apply linearity:

$$=\frac{1}{4}\int \frac{3x+8}{x^2-2x+4}\,dx+\frac{1}{4}\int \frac{1}{x+2}\,dx$$

Now solving:

$$\int \frac{3x+8}{x^2-2x+4}\,dx$$

Write $3x+8$ as $\frac{3}{2}(2x-2)+11$ and split:

$$=\int \left(\frac{3(2x-2)}{2(x^2-2x+4)}+\frac{11}{x^2-2x+4}\right)dx$$

Apply linearity:

$$=3\int \frac{x-1}{x^2-2x+4}\,dx+11\int \frac{1}{x^2-2x+4}\,dx$$

Now solving:

$$\int \frac{x-1}{x^2-2x+4}\,dx$$

Substitute $u=x^2-2x+4 \longrightarrow du=(2x-2)\,dx$ (steps):

$$=\frac{1}{2}\int \frac{1}{u}\,du$$

Now solving:

$$\int \frac{1}{u}\,du$$

This is a standard integral:

$$=\ln(u)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{u}\,du$$

$$=\frac{\ln(u)}{2}$$

Undo substitution $u=x^2-2x+4$:

$$=\frac{\ln(x^2-2x+4)}{2}$$

Now solving:

$$\int \frac{1}{x^2-2x+4}\,dx$$

Complete the square:

$$=\int \frac{1}{(x-1)^2+3}\,dx$$

Substitute $u=\frac{x-1}{\sqrt{3}} \longrightarrow du=\frac{1}{\sqrt{3}}\,dx$ (steps):

$$=\int \frac{\sqrt{3}}{3u^2+3}\,du$$

Simplify:

$$=\frac{1}{\sqrt{3}}\int \frac{1}{u^2+1}\,du$$

Now solving:

$$\int \frac{1}{u^2+1}\,du$$

This is a standard integral:

$$=\arctan(u)$$

Plug in solved integrals:

$$\frac{1}{\sqrt{3}}\int \frac{1}{u^2+1}\,du$$

$$=\frac{\arctan(u)}{\sqrt{3}}$$

Undo substitution $u=\frac{x-1}{\sqrt{3}}$:

$$=\frac{\arctan\left(\frac{x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Plug in solved integrals:

$$3\int \frac{x-1}{x^2-2x+4}\,dx+11\int \frac{1}{x^2-2x+4}\,dx$$

$$=\frac{3\ln(x^2-2x+4)}{2}+\frac{11\arctan\left(\frac{x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Now solving:

$$\int \frac{1}{x+2}\,dx$$

Substitute $u=x+2 \longrightarrow du=dx$ (steps):

$$=\int \frac{1}{u}\,du$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x+2$:

$$=\ln(x+2)$$

Plug in solved integrals:

$$\frac{1}{4}\int \frac{3x+8}{x^2-2x+4}\,dx+\frac{1}{4}\int \frac{1}{x+2}\,dx$$

$$=\frac{3\ln(x^2-2x+4)}{8}+\frac{\ln(x+2)}{4}+\frac{11\arctan\left(\frac{x-1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{x^2+3x+5}{x^3+8}\,dx$$

$$=\frac{\ln(|x+2|)}{4}+\frac{3\ln(x^2-2x+4)}{8}+\frac{11\arctan\left(\frac{x-1}{\sqrt{3}}\right)}{4\sqrt{3}}+C$$

Rewrite/simplify:

$$=\frac{6\ln(|x+2|)+9\ln(x^2-2x+4)+22\sqrt{3}\arctan\left(\frac{x-1}{\sqrt{3}}\right)}{24}+C$$

Problem:

$$\int \frac{x^2-x-8}{(2x-3)(x^2+2x+2)}\,dx$$

Perform partial fraction decomposition:

$$=\int \left(\frac{x+2}{x^2+2x+2}-\frac{1}{2x-3}\right)dx$$

Apply linearity:

$$=\int \frac{x+2}{x^2+2x+2}\,dx-\int \frac{1}{2x-3}\,dx$$

Now solving:

$$\int \frac{x+2}{x^2+2x+2}\,dx$$

Write $x+2$ as $\frac{1}{2}(2x+2)+1$ and split:

$$=\int \left(\frac{2x+2}{2(x^2+2x+2)}+\frac{1}{x^2+2x+2}\right)dx$$

Apply linearity:

$$=\int \frac{x+1}{x^2+2x+2}\,dx+\int \frac{1}{x^2+2x+2}\,dx$$

Now solving:

$$\int \frac{x+1}{x^2+2x+2}\,dx$$

Substitute $u=x^2+2x+2 \longrightarrow du=(2x+2)\,dx$ (steps):

$$=\frac{1}{2}\int \frac{1}{u}\,du$$

Now solving:

$$\int \frac{1}{u}\,du$$

This is a standard integral:

$$=\ln(u)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{u}\,du$$

$$=\frac{\ln(u)}{2}$$

Undo substitution $u=x^2+2x+2$:

$$=\frac{\ln(x^2+2x+2)}{2}$$

Now solving:

$$\int \frac{1}{x^2+2x+2}\,dx$$

Complete the square:

$$=\int \frac{1}{(x+1)^2+1}\,dx$$

Substitute $u=x+1 \longrightarrow du=dx$ (steps):

$$=\int \frac{1}{u^2+1}\,du$$

This is a standard integral:

$$=\arctan(u)$$

Undo substitution $u=x+1$:

$$=\arctan(x+1)$$

Plug in solved integrals:

$$\int \frac{x+1}{x^2+2x+2}\,dx+\int \frac{1}{x^2+2x+2}\,dx$$

$$=\frac{\ln(x^2+2x+2)}{2}+\arctan(x+1)$$

Now solving:

$$\int \frac{1}{2x-3}\,dx$$

Substitute $u=2x-3 \longrightarrow du=2\,dx$ (steps):

$$=\frac{1}{2}\int \frac{1}{u}\,du$$

Use previous result:

$$=\frac{\ln(u)}{2}$$

Undo substitution $u=2x-3$:

$$=\frac{\ln(2x-3)}{2}$$

Plug in solved integrals:

$$\int \frac{x+2}{x^2+2x+2}\,dx-\int \frac{1}{2x-3}\,dx$$

$$=\frac{\ln(x^2+2x+2)}{2}-\frac{\ln(2x-3)}{2}+\arctan(x+1)$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{x^2-x-8}{(2x-3)(x^2+2x+2)}\,dx$$

$$=-\frac{\ln(|2x-3|)}{2}+\frac{\ln(x^2+2x+2)}{2}+\arctan(x+1)+C$$

Rewrite/simplify:

$$=\frac{\ln(x^2+2x+2)-\ln(|2x-3|)}{2}+\arctan(x+1)+C$$

Problem:

$$\int \frac{1}{x^4+16x^2}\,dx$$

Factor the denominator:

$$=\int \frac{1}{x^2\cdot(x^2+16)}\,dx$$

Perform partial fraction decomposition:

$$=\int \left(\frac{1}{16x^2}-\frac{1}{16(x^2+16)}\right)dx$$

Apply linearity:

$$=\frac{1}{16}\int \frac{1}{x^2}\,dx-\frac{1}{16}\int \frac{1}{x^2+16}\,dx$$

Now solving:

$$\int \frac{1}{x^2}\,dx$$

Apply power rule:

$$\int x^n\,dx=\frac{x^{n+1}}{n+1} \text{ with } n=-2:$$

$$=-\frac{1}{x}$$

Now solving:

$$\int \frac{1}{x^2+16}\,dx$$

Substitute $u=\frac{x}{4} \longrightarrow du=\frac{1}{4}\,dx$ (steps):

$$=\int \frac{4}{16u^2+16}\,du$$

Simplify:

$$=\frac{1}{4}\int \frac{1}{u^2+1}\,du$$

Now solving:

$$\int \frac{1}{u^2+1}\,du$$

This is a standard integral:

$$=\arctan(u)$$

Plug in solved integrals:

$$\frac{1}{4}\int \frac{1}{u^2+1}\,du$$

$$=\frac{\arctan(u)}{4}$$

Undo substitution $u=\frac{x}{4}$:

$$=\frac{\arctan\left(\frac{x}{4}\right)}{4}$$

Plug in solved integrals:

$$\frac{1}{16}\int \frac{1}{x^2}\,dx-\frac{1}{16}\int \frac{1}{x^2+16}\,dx$$

$$=-\frac{1}{16x}-\frac{\arctan\left(\frac{x}{4}\right)}{64}$$

The problem is solved:

$$\int \frac{1}{x^4+16x^2}\,dx$$

$$=-\frac{1}{16x}-\frac{\arctan\left(\frac{x}{4}\right)}{64}+C$$

Rewrite/simplify:

$$=-\frac{\frac{4}{x}+\arctan\left(\frac{x}{4}\right)}{64}+C$$

Problem:

$$\int \frac{x^5+2x^3-3x}{\left(x^2+1\right)^3}\,dx$$

Substitute $u=x^2+1 \longrightarrow du=2x\,dx$ (steps), use:
$x^4=(u-1)^2$

$$=\frac{1}{2}\int \frac{u^2-4}{u^3}\,du$$

... or choose an alternative:

Substitute x^2

Don't substitute

Now solving:

$$\int \frac{u^2-4}{u^3}\,du$$

Expand:

$$=\int \left(\frac{1}{u}-\frac{4}{u^3}\right)du$$

Apply linearity:

$$=\int \frac{1}{u}\,du-4\int \frac{1}{u^3}\,du$$

Now solving:

$$\int \frac{1}{u}\,du$$

This is a standard integral:

$$=\ln(u)$$

Now solving:

$$\int \frac{1}{u^3}\,du$$

Apply power rule:

$$\int u^n\,du=\frac{u^{n+1}}{n+1} \text{ with } n=-3:$$

$$=-\frac{1}{2u^2}$$

Plug in solved integrals:

$$\int \frac{1}{u}\,du-4\int \frac{1}{u^3}\,du$$

$$=\ln(u)+\frac{2}{u^2}$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{u^2-4}{u^3}\,du$$

$$=\frac{\ln(u)}{2}+\frac{1}{u^2}$$

Undo substitution $u=x^2+1$:

$$=\frac{\ln\left(x^2+1\right)}{2}+\frac{1}{\left(x^2+1\right)^2}$$

The problem is solved:

$$\int \frac{x^5+2x^3-3x}{\left(x^2+1\right)^3}\,dx$$

$$=\frac{\ln\left(x^2+1\right)}{2}+\frac{1}{\left(x^2+1\right)^2}+C$$

Problem:

$$\int \frac{1}{x^2 \cdot (x^2+1)^2}\,\mathrm{d}x$$

Perform partial fraction decomposition:

$$= \int \left(-\frac{1}{x^2+1} - \frac{1}{(x^2+1)^2} + \frac{1}{x^2}\right)\mathrm{d}x$$

Apply linearity:

$$= -\int \frac{1}{x^2+1}\,\mathrm{d}x - \int \frac{1}{(x^2+1)^2}\,\mathrm{d}x + \int \frac{1}{x^2}\,\mathrm{d}x$$

Now solving:

$$\int \frac{1}{x^2+1}\,\mathrm{d}x$$

This is a standard integral:

$$= \arctan(x)$$

Now solving:

$$\int \frac{1}{(x^2+1)^2}\,\mathrm{d}x$$

Apply reduction formula:

$$\int \frac{1}{(ax^2+b)^n}\,\mathrm{d}x = \frac{2n-3}{2b(n-1)}\int \frac{1}{(ax^2+b)^{n-1}}\,\mathrm{d}x + \frac{x}{2b(n-1)(ax^2+b)^{n-1}}$$

with $a=1, b=1, n=2$:

$$= \frac{x}{2(x^2+1)} + \frac{1}{2}\int \frac{1}{x^2+1}\,\mathrm{d}x$$

Now solving:

$$\int \frac{1}{x^2+1}\,\mathrm{d}x$$

Use previous result:

$$= \arctan(x)$$

Plug in solved integrals:

$$\frac{x}{2(x^2+1)} + \frac{1}{2}\int \frac{1}{x^2+1}\,\mathrm{d}x$$

$$= \frac{\arctan(x)}{2} + \frac{x}{2(x^2+1)}$$

Now solving:

$$\int \frac{1}{x^2}\,\mathrm{d}x$$

Apply power rule:

$$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1} \text{ with } n=-2:$$

$$= -\frac{1}{x}$$

Plug in solved integrals:

$$-\int \frac{1}{x^2+1}\,\mathrm{d}x - \int \frac{1}{(x^2+1)^2}\,\mathrm{d}x + \int \frac{1}{x^2}\,\mathrm{d}x$$

$$= -\frac{3\arctan(x)}{2} - \frac{x}{2(x^2+1)} - \frac{1}{x}$$

The problem is solved:

$$\int \frac{1}{x^2 \cdot (x^2+1)^2}\,\mathrm{d}x$$

$$= -\frac{3\arctan(x)}{2} - \frac{x}{2(x^2+1)} - \frac{1}{x} + C$$

Problem:

$$\int \frac{x^4+2x^3+11x^2+8x+16}{x \cdot (x^2+4)^2}\,\mathrm{d}x$$

Perform partial fraction decomposition:

$$= \int \left(\frac{2}{x^2+4} + \frac{3x}{(x^2+4)^2} + \frac{1}{x}\right)\mathrm{d}x$$

Apply linearity:

$$= 2\int \frac{1}{x^2+4}\,\mathrm{d}x + 3\int \frac{x}{(x^2+4)^2}\,\mathrm{d}x + \int \frac{1}{x}\,\mathrm{d}x$$

Now solving:

$$\int \frac{1}{x^2+4}\,\mathrm{d}x$$

Substitute $u = \frac{x}{2} \longrightarrow \mathrm{d}u = \frac{1}{2}\,\mathrm{d}x$ (steps):

$$= \int \frac{2}{4u^2+4}\,\mathrm{d}u$$

Simplify:

$$= \frac{1}{2}\int \frac{1}{u^2+1}\,\mathrm{d}u$$

Now solving:

$$\int \frac{1}{u^2+1}\,\mathrm{d}u$$

This is a standard integral:

$$= \arctan(u)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{u^2+1}\,\mathrm{d}u$$

$$= \frac{\arctan(u)}{2}$$

Undo substitution $u = \frac{x}{2}$:

$$= \frac{\arctan\left(\frac{x}{2}\right)}{2}$$

Now solving:

$$\int \frac{x}{(x^2+4)^2}\,\mathrm{d}x$$

Substitute $u = x^2+4 \longrightarrow \mathrm{d}u = 2x\,\mathrm{d}x$ (steps):

$$= \frac{1}{2}\int \frac{1}{u^2}\,\mathrm{d}u$$

Now solving:

$$\int \frac{1}{u^2}\,\mathrm{d}u$$

Apply power rule:

$$\int u^n\,\mathrm{d}u = \frac{u^{n+1}}{n+1} \text{ with } n=-2:$$

$$= -\frac{1}{u}$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{u^2}\,\mathrm{d}u$$

$$= -\frac{1}{2u}$$

Undo substitution $u = x^2+4$:

$$= -\frac{1}{2(x^2+4)}$$

Now solving:

$$\int \frac{1}{x}\,\mathrm{d}x$$

This is a standard integral:

$$= \ln(x)$$

Plug in solved integrals:

$$2\int \frac{1}{x^2+4}\,\mathrm{d}x + 3\int \frac{x}{(x^2+4)^2}\,\mathrm{d}x + \int \frac{1}{x}\,\mathrm{d}x$$

$$= \ln(x) - \frac{3}{2(x^2+4)} + \arctan\left(\frac{x}{2}\right)$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{x^4+2x^3+11x^2+8x+16}{x \cdot (x^2+4)^2}\,\mathrm{d}x$$

$$= \ln(|x|) - \frac{3}{2(x^2+4)} + \arctan\left(\frac{x}{2}\right) + C$$

Problem:

$$\int \frac{x+2}{(x^2-1)(x^2+1)^2}\,dx$$

Factor the denominator:

$$=\int \frac{x+2}{(x-1)(x+1)(x^2+1)^2}\,dx$$

Perform partial fraction decomposition:

$$=\int\left(-\frac{x+2}{4(x^2+1)}-\frac{x+2}{2(x^2+1)^2}-\frac{1}{8(x+1)}+\frac{3}{8(x-1)}\right)dx$$

Apply linearity:

$$=-\frac{1}{4}\int\frac{x+2}{x^2+1}\,dx-\frac{1}{2}\int\frac{x+2}{(x^2+1)^2}\,dx-\frac{1}{8}\int\frac{1}{x+1}\,dx+\frac{3}{8}\int\frac{1}{x-1}\,dx$$

Now solving:

$$\int\frac{x+2}{x^2+1}\,dx$$

Expand:

$$=\int\left(\frac{x}{x^2+1}+\frac{2}{x^2+1}\right)dx$$

Apply linearity:

$$=\int\frac{x}{x^2+1}\,dx+2\int\frac{1}{x^2+1}\,dx$$

Now solving:

$$\int\frac{x}{x^2+1}\,dx$$

Substitute $u=x^2+1 \longrightarrow du=2x\,dx$ (steps):

$$=\frac{1}{2}\int\frac{1}{u}\,du$$

Now solving:

$$\int\frac{1}{u}\,du$$

This is a standard integral:

$$=\ln(u)$$

Plug in solved integrals:

$$\frac{1}{2}\int\frac{1}{u}\,du$$
$$=\frac{\ln(u)}{2}$$

Undo substitution $u=x^2+1$:

$$=\frac{\ln(x^2+1)}{2}$$

Now solving:

$$\int\frac{1}{x^2+1}\,dx$$

This is a standard integral:

$$=\arctan(x)$$

Plug in solved integrals:

$$\int\frac{x}{x^2+1}\,dx+2\int\frac{1}{x^2+1}\,dx$$
$$=\frac{\ln(x^2+1)}{2}+2\arctan(x)$$

Now solving:

$$\int\frac{x+2}{(x^2+1)^2}\,dx$$

Expand:

$$=\int\left(\frac{x}{(x^2+1)^2}+\frac{2}{(x^2+1)^2}\right)dx$$

Apply linearity:

$$=\int\frac{x}{(x^2+1)^2}\,dx+2\int\frac{1}{(x^2+1)^2}\,dx$$

Now solving:

$$\int\frac{x}{(x^2+1)^2}\,dx$$

Substitute $u=x^2+1 \longrightarrow du=2x\,dx$ (steps):

$$=\frac{1}{2}\int\frac{1}{u^2}\,du$$

Now solving:

$$\int\frac{1}{u^2}\,du$$

Apply power rule:

$$\int u^{n}\,du=\frac{u^{n+1}}{n+1}\text{ with } n=-2:$$
$$=-\frac{1}{u}$$

Plug in solved integrals:

$$\frac{1}{2}\int\frac{1}{u^2}\,du$$
$$=-\frac{1}{2u}$$

Undo substitution $u=x^2+1$:

$$=-\frac{1}{2(x^2+1)}$$

Now solving:

$$\int\frac{1}{(x^2+1)^2}\,dx$$

Apply reduction formula:

$$\int\frac{1}{(ax^2+b)^{n}}\,dx=\frac{2n-3}{2b(n-1)}\int\frac{1}{(ax^2+b)^{n-1}}\,dx+\frac{x}{2b(n-1)(ax^2+b)^{n-1}}$$

with $a=1, b=1, n=2$:

$$=\frac{x}{2(x^2+1)}+\frac{1}{2}\int\frac{1}{x^2+1}\,dx$$

Now solving:

$$\int\frac{1}{x^2+1}\,dx$$

Use previous result:

$$=\arctan(x)$$

Plug in solved integrals:

$$\frac{x}{2(x^2+1)}+\frac{1}{2}\int\frac{1}{x^2+1}\,dx$$
$$=\frac{\arctan(x)}{2}+\frac{x}{2(x^2+1)}$$

Plug in solved integrals:

$$\int\frac{x}{(x^2+1)^2}\,dx+2\int\frac{1}{(x^2+1)^2}\,dx$$
$$=\arctan(x)+\frac{x}{x^2+1}-\frac{1}{2(x^2+1)}$$

Now solving:

$$\int\frac{1}{x+1}\,dx$$

Substitute $u=x+1 \longrightarrow du=dx$ (steps):

$$=\int\frac{1}{u}\,du$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x+1$:

$$=\ln(x+1)$$

Now solving:

$$\int\frac{1}{x-1}\,dx$$

Substitute $u=x-1 \longrightarrow du=dx$ (steps):

$$=\int\frac{1}{u}\,du$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x-1$:

$$=\ln(x-1)$$

Plug in solved integrals:

$$-\frac{1}{4}\int\frac{x+2}{x^2+1}\,dx-\frac{1}{2}\int\frac{x+2}{(x^2+1)^2}\,dx-\frac{1}{8}\int\frac{1}{x+1}\,dx+\frac{3}{8}\int\frac{1}{x-1}\,dx$$
$$=-\frac{\ln(x^2+1)}{8}-\frac{\ln(x+1)}{8}-\arctan(x)-\frac{x}{2(x^2+1)}+\frac{1}{4(x^2+1)}+\frac{3\ln(x-1)}{8}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int\frac{x+2}{(x^2-1)(x^2+1)^2}\,dx$$
$$=-\frac{\ln(|x+1|)}{8}-\frac{\ln(x^2+1)}{8}-\arctan(x)-\frac{x}{2(x^2+1)}+\frac{1}{4(x^2+1)}+\frac{3\ln(|x-1|)}{8}+C$$

Rewrite/simplify:

$$=\frac{-\ln(|x+1|)-\ln(x^2+1)+3\ln(|x-1|)}{8}-\arctan(x)+\frac{1}{4x^2+4}-\frac{x}{2x^2+2}+C$$