University of Caloocan City

College of Liberal Arts & Sciences

Mathematics Department

COLLEGE ALGEBRA
Learning Module #1

Topic	**BASIC CONCEPTS IN ALGEBRA & POLYNOMIALS**
Duration	**3 hours**
Lesson Proper	**I. The Set of Real Numbers**

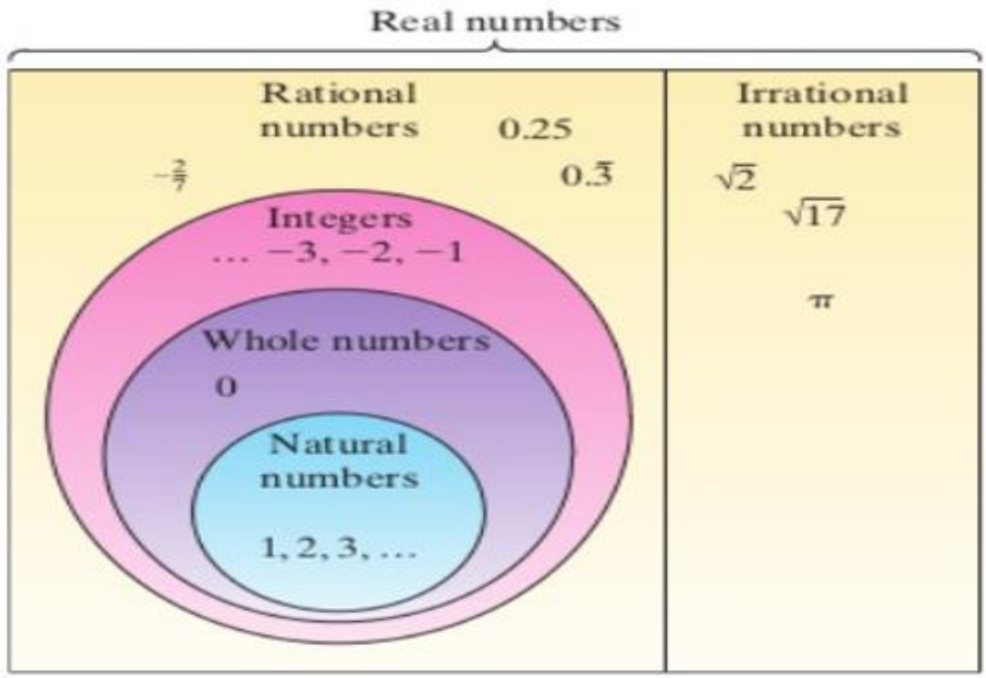

Fig.1 –The Real Number System

The real number system consists of a set of elements called real numbers. A real number may be positive, negative, or zero and can be classified as either rational or irrational.

2.5 = 5/2 and 0.2 = 1/5 are **terminating decimals**

0.666… = 2/3 and 0.1414… = 14/99 are **non-terminating repeating decimals**

Irrational Numbers are called **nonterminating nonrepeating decimals**

Ex. $\sqrt{2}$, $\sqrt[3]{5}$, π, **e = 2.71828…**

$\sqrt{2}$ **= 1.4142135623…** $\pi = 3.14159265358979323\ldots$

Prove that 0.999… = 1

Let x = 0.999…

10x = 9.999…

10x − x = 9

X = 1 **Thus, 0.999… = 1**

Exercises: Give the equivalent value in fraction of the following**:**

a. 0.777… Ans: 7/9
b. 0.41444… Ans: 14,003/9
c. 0.2156262…

Performing Operations on Series of Numbers

$$\mathbf{4^2 * 3 \div 6 + 2 - \sqrt{9}} = \_\_\_\_\_\_$$

- **Operations Involved: addition, subtraction, multiplication, division, involution and evolution.**
- **In a series of numbers involving the basic operations in Arithmetic, the following give the order of performing the operations**
  - From left to right, perform first involution/evolution
  - Second, perform the operations within the presence of parentheses or grouping symbols
  - Perform multiplication/division whichever comes first
  - Perform addition/subtraction whichever comes first

Examples:

a. 55 – 3 * 8
b. (6 – 3) + (4 * 9)
c. $16 \div 2 * 4 + 2^3$
d. $3^3 + 9 \div 3 * 6 * \sqrt[3]{8}$
e. $(-4^2 \div 4) \div 2$
g. $-4^2 \div (4 \div 2)$
h. $(-4)^2 \div 4 - 3(-2)$
i. $(-5^2 \div 5) * 3^2 - \sqrt{49}$
j. $[-(-6)^2 - \sqrt[3]{-64} - 2] \div [-4^2 - 1^2]$

Properties of Equality

1. Reflexive **: a = a**
2. Symmetric **: If a =b, then b = a**
3. Transitive **: If a = b and b = c, then a = c**
4. Addition Property of Equality (APE):
   If a = b, then a + c = b + c
5. Multiplication Property of Equality
   If a = b, then ac = bc
6. Substitution**: If a = b,** then a can be replaced by b in any mathematical statement without changing the value of the statement

Properties of Real Numbers

Property Name	Algebraic Representation	Example	Description/Notes
Commutative property of addition	$a + b = b + a$	$5 + 3 = 3 + 5$	The order in which two real numbers are added or multiplied does not affect the result.
Commutative property of multiplication	$a \cdot b = b \cdot a$	$(5)(3) = (3)(5)$	
Associative property of addition	$(a + b) + c = a + (b + c)$	$(2 + 3) + 7 = 2 + (3 + 7)$	The manner in which two real numbers are grouped under addition or multiplication does not affect the result.
Associative property of multiplication	$(a \cdot b)c = a(b \cdot c)$	$(2 \cdot 3)7 = 2(3 \cdot 7)$	
Distributive property of multiplication over addition	$a(b + c) = ab + ac$	$3(5 + 2) = 3 \cdot 5 + 3 \cdot 2$	A factor outside the parentheses is multiplied by each term inside the parentheses.
Identity property of addition	0 is the identity element for addition because $a + 0 = 0 + a = a$	$5 + 0 = 0 + 5 = 5$	Any number added to the identity element 0 will remain unchanged.
Identity property of multiplication	1 is the identity element for multiplication because $a \cdot 1 = 1 \cdot a = a$	$5 \cdot 1 = 1 \cdot 5 = 5$	Any number multiplied by the identity element 1 will remain unchanged.
Inverse property of addition	a and $(-a)$ are additive inverses because $a + (-a) = 0$ and $(-a) + a = 0$	$3 + (-3) = 0$	The sum of a number and its additive inverse (opposite) is the identity element 0.
Inverse property of multiplication	a and $\frac{1}{a}$ are multiplicative inverses because $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$ (provided $a \neq 0$)	$5 \cdot \frac{1}{5} = 1$	The product of a number and its multiplicative inverse (reciprocal) is the identity element 1.

Mathematics Department

COLLEGE ALGEBRA
Learning Module #2

Topic	**POLYNOMIALS**
Duration	**3 hours**
Lesson Proper	**II. POLYNOMIALS** ➢ Expressions combining numbers and letters involving at least one of the basic operations. **Definitions of Basic Terms in Polynomials** ➢ A ***constant*** is a symbol that assumes one specific value. ➢ A ***variable*** is a symbol that assumes many values. ➢ An ***algebraic expression, or simply expression***, is a collection of constants and variables involving at least one of the basic operations in mathematics ➢ A ***term*** is an expression preceded by plus or minus sign **one term:** r^2; $\frac{2}{x}$; $\frac{a+b}{a-b}$; $\sqrt{y}$; $(x-y)^4$; $\frac{(a+b+c)}{3}$; $x \div y$ **two terms:** $ab-3$; $\frac{x-abc}{3}-1$; $\frac{1}{x}-\frac{1}{y}$; $x-\frac{x}{x+1}$; $a+x \div y$ ➢ ***A monomial*** is a term involving only the product of a real number and variables with nonnegative integral exponents **Monomial:** 6; $3b$; $15xyz^2$; x^2y^4; $\frac{xy}{3}$; abc; $\sqrt{2}$; abx **Not monomial**: $\frac{1}{x}$; $\frac{a}{b}-5$; $\sqrt{x}$; $x+2$; $\frac{1}{a+b}$; $x+y$ ➢ A ***polynomial*** is a sum of finite number of monomials. The general polynomial in one variable of degree n is of the form $a_nx^n+\cdots+a_1x+a_0$ A **binomial** is a polynomial consisting of exactly two terms A **trinomial** is a polynomial consisting of exactly three terms If a monomial is expressed as a product of two or more symbols, each of the symbol is called the coefficient of the rest of the product. Ex: ***2xy*** - **2** is called the numerical coefficient and ***Xy*** is called the literal coefficient ➢ Two monomials (or two terms) are similar if they have the same literal coefficient

Addition of Expressions or Polynomials

Rule1: To add two or more monomials with the same literal coefficients, add only their numerical coefficients, and affix the literal coefficients:

Ex. 1: -8x + 15x = (-8 +15)x = 7x

Rule 2: To add two or more polynomials, add similar or like terms together. **Ex. 2: $3x^2 - 4x - 4y$; $7x^2 - 2y - 2$ and $-4x^2 + x - y - 7$**

Ex. 3: Subtract 4x –y – 3 from 2x –y -4

Rule 3: To remove a grouping symbol preceded by a:

- minus sign, change the sign of each of the terms;
- plus sign, no further change is done;
- Factor, use the distributive law

Ex. 4: 4x – 2y – 5 – 2(8x – 7y) – (3x – 4y -1)

Rule 4: When one symbol of grouping is within another symbol of grouping, the innermost symbol must be removed first.

Ex. 4: – {– 2x – y – [3x – (4x + y – 3) – y] – 7}

Exercises: Simplify the following:

a. – {12x – 3y -8 – [6x – (x + 7y -3)]} **(ans. – 7x – 4y +11)**
b. – [–3 x –y – (4y – z)] – [8y + 7x – (5x – 9z)] **(ans. x – 3y – 10z)**

Powers with Positive Integral Exponents

Laws of Exponents:

1. The Product of Powers: $a^m * a^n = a^{m+n}$ **Ex. 5:** $x^5 * x^4 = x^9$
2. The Quotient of Powers: $\frac{a^m}{a^n} = a^{m-n}$ **Ex. 6:** $\frac{x^7}{x^3} = x^4$; $\frac{x^4}{x^6} = \frac{1}{x^2}$
3. The Power of a Power: $(a^m)^n = a^{mn}$ **Ex. 7:** $(x^2)^4 = x^8$
4. The Power of a Product: $(ab)^m = a^m b^m$ **Ex. 8:** $(2a^2b^4)^3 = 8a^6b^{12}$
5. The Power of a Quotient: $[\frac{a}{b}]^m = \frac{a^m}{b^m}$ **Ex.9:** $[\frac{\overset{312}{-2x}}{3}]^4 = \frac{16x}{81}$

Exercises: Simplify the following:

a. -4^2 b. $-[\frac{-3}{2}]^4$ c. $\frac{(a-b)^3}{a-b}$ d. $\frac{8^{3x}}{4^{4x}}$

e. $\frac{25^{100}}{125^{50}}$ f. $\frac{(x^4y^8z)^3}{(x^5y^8zw)^2}$ g. $[\frac{12x^3y^4z}{18x^2y^6}]^2$

Product of Polynomials

Rule 1: To multiply two monomials, use commutative, associative and the laws of exponents in multiplication. **Ex. 10:** $(3x^2y^4z^2w^9)\ (-4xyz^4wv)$

Rule 2: To multiply two polynomials, use the distributive law and apply Rule 1. **Ex. 11:** $(2x - 3y)\ (4x + 5y)$

Exercises: Find the products of the following:

a. $(5x^4 - 8y^3)\ (5x^4 + 8y^3)$

b. $(5x^4 - 8y^3)\ (5x^4 - 8y^3)$

c. $(a^2 - b)(a^2 + a^2b + b^2)$

d. $(x^7 - x^6 + x^4 - x^3 - 5x + 3)\ (x^5 - x^2 + 2)$

Simplify the following:

1. $-(2x - 5y) - \{4 - [5y - (2y - 9x) - 3] - 6x\}$
2. $-\{2x - [4y - 3x - (4 - 3y + 5x)] - 7\}$
3. $-[8x - 2y - (5 + 7y)] - \{6x - (2 - 8y - 2x)\}$
4. Subtract the sum of the second and third expressions from the first
   $3x - 2y + 8z;\ 12x - y - 10z;\ 3x - y + 7z$
5. Subtract the first from the sum of the second and the third expressions
   $2x - y - 3;\ 4x + y - 5;\ x + 11y - 8$
6. $\dfrac{3x^n - 2x^n}{(x^n)^2}$ 7. $\dfrac{2x^4 + 3x^4}{2x^4 * 3x^4}$ 8. $\dfrac{64^a}{128^a}$ 9. $\dfrac{(2x^4y^8)^3}{(4x^8y^4)^2}$
10. $(3x^4y^2z^2)^2\ (4x^2y^4z^3w)^3$
11. $(7x - 8y)(7x - 8y)$
12. $(21x^4 - 13y^3)\ (21x^4 + 13y^3)$
13. $(x^5 - 3x^4 - x^3 + x - 3)(x^3 - x + 5)$

Mathematics Department

COLLEGE ALGEBRA
Learning Module #3

Topic	**SPECIAL PRODUCTS AND FACTORING**
Duration	**3 hours**

Lesson Proper

Types of Special Products

1. Product of Two Binomials

$$(ax+by)(cx+dy) = acx^2 + (ad+bc)xy + bdy^2$$

Example #1: Find the product: $(2x-3y)(4x+5y)$

$$(2x-3y)(4x+5y) = (2)(4)x^2 + (2\cdot 5 - 3\cdot 4)xy - (3)(5)y^2 = 8x^2 - 2xy - 5y^2$$

2. Square of Binomials

$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x-y)^2 = x^2 - 2xy + y^2$$

- ***The result of the square of a binomial is a perfect square trinomial***

It is important to emphasize that in $(x+y)^2$: x refers to first term, and y refers to second term

Example#2: $(3x-7)^2 = (3x)^2 - 2(3x)(7) + 7^2 = 9x^2 - 42x + 49$

Example#3: $(5x^3-9y^4)^2 = (5x^3)^2 - 2(5x^3)(9y^4) + (9y^4)^2 = 25x^2 - 90x^3y^3 + 81y^8$

3. Product of the Sum and the Difference of the Same Two Terms:

$$(x+y)(x-y) = x^2 - y^2$$

Example#4: $(4x+5y)(4x-5y) = (4x)^2 - (5y)^2 = 16x^2 - 25y^2$

4. Cube of a Binomial

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$
$$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

Example#5: $(a^2-4)^3 = (a^2)^3 - 3(a^2)^2(4) + 3(a^2)(4)^2 - (4)^3$

5. Special Case Of Product of Binomial and Trinomial

$$(x+y)\left(x^2 - xy + y^2\right) = x^3 + y^3$$
$$(x-y)\left(x^2 + xy + y^2\right) = x^3 - y^3$$

Example#6: $(7a-4b)\left(49a^2 + 28ab + 16b^2\right) = (7a)^3 - (4b)^3 = 343a^3 - 64b^3$

Example#7: $(3c^2-5d^4)\left(9c^4 + 15c^2d^4 + 25d^8\right) = \left(3c^2\right)^3 - (5d^4)^3 = 27c^6 - 125d^{12}$

	6. Square of Trinomial $$(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})^2 = \boldsymbol{a}^2+\boldsymbol{b}^2+\boldsymbol{c}^2+2\boldsymbol{ab}+2\boldsymbol{ac}+2\boldsymbol{bc}$$ **Examples:** $(3x+y-5)^2 = (3x)^2+(y)^2+(-5)^2+2(3x)(y)+2(3x)(-5)+2(y)(-5)$ $= 9x^2+y^2+25+6xy-30x-10y$
Exercises	**Determine the product by identifying the type of special product to be used in the following:** 1. $(4x^3-9y^5)(4x^3+9y^5)$ 2. $(2a^4-y^5)(2a^4-y^5) =$ 3. $(2x-5y)^3$ 4. $(2x^3-3y^5)(x^3+4y^5)$ 5. $(5c-2d^3)(25c^2+10cd^3+4d^6)$ 6. $(6x-y-2z)^2$

Mathematics Department

COLLEGE ALGEBRA
Learning Module #4

Topic	**FACTORING**
Duration	**3 hours**
Lesson Proper	Factoring of Polynomials is simply the reverse process of the special product formulas. Thus, we shall be adapting the reverse process of the special product formulas to factor polynomials. A polynomial with integral exponents is no longer factorable if: 1. the coefficients have no common factor, and 2. it cannot be expressed as the product of two polynomials of lower degree. __Types of Factoring__: 1. **Common Monomial Factor:** $ax + ay = a(x + y)$ **Example #1:** $24x^2 - 18x^3 = 6x^2(4 - 3x)$ 2. **Difference of Two Squares:** $x^2 - y^2 = (x + y)(x - y)$ **Example #2:** $9a^2 - 25b^2 = (3a)^2 - (5b)^2$ $= (3a + 5b)(3a - 5b)$ **Example #3:** $49b^4 - 16a^8 = (7b^2)^2 - (4a^4)^2$ $= (7b^2 + 4a^4)(7b^2 - 4a^4)$ 3. **Sum and Difference of Two Cubes:** $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ **Example #4:** $8x^3 - 125y^3 = (2x)^3 - (5y)^3$ $= (2x - 5y)(4x^2 + 10xy + 25y^2)$ **Example #5:** $27a^6 + 64b^9 = (3a^2)^3 + (4b^3)^3$ $= (3a^2 + 4b^3)(9a^4 - 12a^2b^3 + 16b^2)$ 4. **Perfect Square Trinomial:** $x^2 + 2xy + y^2 = (x + y)^2$ $x^2 - 2xy + y^2 = (x - y)^2$ **Example #6:** $4x^2 - 12xy + 9y^2 = (2x - 3y)^2$ **Example #7:** $25x^4 + 80x^2y^4 + 64y^8 = (5x^2 + 8y^4)^2$

Example Exercises: Factor the following completely:

a. $x^2 - 6x + 9$

b. $2x^3 + 4x^2 + 2x$

c. $4x^2 - 12xy + 9y^2$

d. $81x^6 - 18x^4 + x^2$

Other Types of Factoring

5. Other Trinomials : $\boldsymbol{x^2 + (a+b)x + ab = (x+a)(x+b)}$

Example #8: $\boldsymbol{x^2 - x - 6}$

Solution: The set of factors of x^2 are x and x. While that of -6 are ± 2 and ∓ 3, or ± 1 and ∓ 6. The correct choice depends on the sum of products Of the means and the extremes.

Thus, $\boldsymbol{x^2 - x - 6 = (x+2)(x-3)}$

6. Factoring by Grouping

Sometimes proper grouping of terms is necessary to make the given polynomial factorable. This type of factoring is usually applied to algebraic expressions consisting of at least four terms.

Example #9: $\boldsymbol{ax + ay - bx - by = a(x+y) - b(x+y)}$

$$\boldsymbol{= (x+y)(a-b)}$$

Example #10: $\boldsymbol{x^2 - y^2 + 2y - 1 = x^2 - (y^2 - 2y + 1)}$

$$\boldsymbol{= x^2 - (y-1)^2}$$
$$\boldsymbol{= [x + (y-1)][x - (y-1)]}$$
$$\boldsymbol{= (x + y - 1)(x - y + 1)}$$

7. Addition and Subtraction of Suitable Terms

This type of factoring is usually applied to polynomials of degree 4 with two terms being perfect squares and both preceded by positive sign. Through the addition and subtraction of suitable terms, the given will always lead to the difference of two squares.

Example #11: $\boldsymbol{x^4 + 4}$

Solution: Add and subtract $\boldsymbol{4x^2}$ **based on** $\boldsymbol{(x^2+2)^2}$

$$\boldsymbol{x^4 + 4 = x^4 + 4x^2 + 4 - 4x^2}$$
$$\boldsymbol{= (x^2+2)^2 - (2x)^2}$$
$$\boldsymbol{= (x^2 + 2x + 1)(x^2 - 2x + 2)}$$

8. Sum and Difference of Two Odd Primes

$$\boldsymbol{x^n + y^n = (x+y)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 - \cdots + y^{n-1})}$$
$$\boldsymbol{x^n - y^n = (x-y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 - \cdots + y^{n-1})}$$

Example #12: $\boldsymbol{x^5 + y^5 = (x+y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)}$

Exercises

Factor the following completely. Tell what type of factoring is be used.

1. $x^5 + 32y^5$
2. $x^4 - 14x^2 + 25$
3. $x^3 - x$
4. $a^2 - 121$
5. $x(m+n) - (m+n)$
6. $xy^4 + x^4y$
7. $x^2 - 6x + 9$
8. $25x^2 - 10x + 1$
9. $6x^2 - 9x - 15$
10. $x^3 + x^2 - x - 1$

University of Caloocan City

College of Liberal Arts & Sciences

Mathematics Department

COLLEGE ALGEBRA
Learning Module #5

Topic	**Simplifying, Addition, Multiplication & Division of Fractions**
Duration	**3 hours**
Lesson Proper	

I. Simplifying Fractions

A. Lecture:

- ✓ A fraction is said to be in simplified form if the numerator and denominator have no common factor except 1.
- ✓ If a common factor appears in the numerator and denominator such can be renowned by division using the property of real numbers.

$$\frac{ac}{bc} = \frac{a}{b} \qquad b \neq 0\,, \qquad c \neq 0$$

$$\frac{(\boldsymbol{x}-\boldsymbol{y})}{(\boldsymbol{x}-\boldsymbol{y})} = \mathbf{1} \qquad \boldsymbol{and} \qquad \frac{(\boldsymbol{x}-\boldsymbol{y})}{\boldsymbol{y}-\boldsymbol{x}} = \frac{(\boldsymbol{x}-\boldsymbol{y})}{-(\boldsymbol{x}-\boldsymbol{y})} = -\mathbf{1}$$

B. Illustrations:

Problem #1: $\frac{x^2-4}{2x-4}$

Solution:

Factoring the polynomial in the Numerator and Denominator

$$\frac{(x+2)(x-2)}{2(x-2)} \qquad \frac{\rightarrow \text{ Difference of Two Squares}}{\rightarrow \text{ Common Monomial Factor}}$$

Take note: $(\boldsymbol{x}-\mathbf{2})$ resulted to be the Common Factor

Making Use of Cancellation Method: $\frac{(x+2)\cancel{(x-2)}}{2\cancel{(x-2)}} = \frac{\boldsymbol{x}+\mathbf{2}}{\mathbf{2}}$

Problem #2: $\frac{\mathbf{2}\boldsymbol{x}^{\mathbf{2}}-\mathbf{3}\boldsymbol{x}+\mathbf{1}}{\mathbf{2}\boldsymbol{x}^{\mathbf{2}}+\boldsymbol{x}-\mathbf{1}}$

Solution: $\frac{\cancel{(2x-1)}(x-1)}{\cancel{(2x-1)}(x+1)} = \frac{(\boldsymbol{x}-\mathbf{1})}{(\boldsymbol{x}+\mathbf{1})}$

Take note: Numerator and Denominator are both factorable by other Types of factoring.

Problem #3: $\dfrac{-x^2-1}{1-x^4}$

Solution:
Arrange polynomial in the Denominator in Standard Form $ax^2 + bx + c$

$$\frac{-x^2-1}{-x^4+1} \quad = \quad \text{Factoring} - 1$$

$$\frac{-(x^2+1)}{-(x^4-1)} = \frac{-\cancel{(x^2+1)}}{-\cancel{(x^2+1)}(x^2-1)} = \frac{1}{x^2-1} = \frac{\mathbf{1}}{(\boldsymbol{x}+\mathbf{1})(\boldsymbol{x}-\mathbf{1})}$$

C. Self-Evaluation:

Solve the following problems:

1. $\frac{5x-6}{10x-12}$
2. $\frac{4cd-xcd}{8-2x}$
3. $\frac{x^2-16}{x^2+4x}$
4. $\frac{3+2x-x^2}{1+5x+4x^2}$
5. . $\frac{6abc-18ab}{3a^2bc-9a^2b}$
6. $\frac{2x+4y}{x^2+2xy}$
7. $\frac{x^2-1}{x^2-x}$
8. $\frac{x^2-x-6}{x^2-5x+6}$

II. Addition and Subtraction of Fractions

A. Lecture:

- ✓ Addition of Fractions depends on the kinds of denominator the fraction have.

1. To add two fractions with the same denominator, use

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \quad , \quad c \neq 0$$

Illustrations:

a. $\frac{x-3}{x-2} + \frac{1}{x-2} = \frac{x-3+1}{x-2} = \frac{x-2}{x-2} = \mathbf{1}$

b. $\frac{2x}{x-1} + \frac{2}{1-x} = \frac{2x}{x-1} + \frac{2}{-(x-1)} = \frac{2x}{x-1} - \frac{2}{x-1} = \frac{2x-2}{x-1} = \frac{2(x-1)}{(x-1)} = \mathbf{2}$

2. To add two fractions with different denominators

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \quad , \quad b \neq 0 \quad , \quad d \neq 0$$

Illustrations:

a. $\frac{2}{3} + \frac{5}{4}$

Find the LCD which is 12. Then find the equivalent fraction

$\frac{2}{3} = \frac{8}{12}$ ÷

$\frac{5}{4} = \frac{15}{12}$ 3

$\frac{2}{3} + \frac{5}{4}$ **is equivalent to** $\frac{8+15}{12} = \frac{23}{12}$

b. $\frac{2x}{x-1} + \frac{2}{1-x} = \frac{2x}{x-1} + \frac{2}{-(x-1)} = \frac{2x-2}{x-1} = 2$

How to find the LCM (Least Common Multiple) of the following sets of expressions:

C. Examples:

Find the least common multiple of the following sets of expressions:

a. x^2 and x^3

b. $uw^4x^3y^9$ and $w^6x^7y^2z$

c. $(x^3-1); (x^2-1)\,; (x^2-2x-1)$

d. $(x+1)(x^2-2x+1);\ (x^2-x-2)^2\,; (x^2-1)(x-2)^3$

e. $(x^2-1)(x-1)\,;\ x^3-1\,; (x^2+x+1)^2$

Ans. $(x+1)(x-1)^2(x^2+x+1)^2$

Solutions:

a. The greatest power serves as the LCM. Thus, the LCM of x^2 and x^3 is x^3.

b. Selecting the greatest power for each kind of variable gives the LCM of $uw^4x^3y^9$ and $w^6x^7y^2z$ as $u6x^7y^9z$

c. $x^3-1 = (x-1)(x^2+x+1)$; $x^2-1 = (x+1)(x-1)$; $x^2-2x-1 = (x-1)^2$.

Selecting the greatest power for each kind of factor, we get:

LCM $=(x-1)^2(x^2+x+1)(x+1)$

d. Express the given into its prime factors. For each kind of factor, choose the greatest power to comprise the LCM.

$$\begin{array}{rcl|l|l}
(x+1)(x^2-2x+1) & = & (x+1) & (x-1)^2 & \\
(x^2-x-2)^2 & = & (x+1)^2 & & (x-2)^2 \\
(x^2-1)(x-2)^3 & = & (x+1) & (x-1) & (x-2)^3
\end{array}$$

Selecting the greatest power for each kind of factor, we get:

LCM $=(x+1)^2(x-1)^2(x+2)^3$

D. Self-Evaluation:

Solve the following problems:

1. $\frac{7}{36} - \frac{2}{45} + \frac{11}{60}$

2. $\frac{11}{42} - \frac{2}{63} + \frac{5}{72}$

3. $\frac{2}{x-y} - \frac{3}{x+y}$

4. $\frac{2x-1}{x^2(x-1)^2} - \frac{2}{(x^2-1)(x-1)}$

III. Multiplication and Division of Fractions

A. Lecture:

✓ To multiply and divide two fractions, make use of the following theories:

$$\frac{a}{b},\frac{c}{d} = \frac{ac}{bd} \qquad where \quad b \neq 0\,, \quad d \neq 0$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc} \qquad where \quad b \neq 0\,, \quad d \neq 0$$

B. Illustrations:

a. $\frac{3}{5}\cdot\frac{10}{21} = \frac{2}{7}$ $\Rightarrow$ Dividing 3 & 21 by a 3 CF · Dividing 5 & 10 by 5

b. $\frac{8x^3}{9y^2} \div \frac{4x^2}{3y} = \frac{8x^3}{9y^2}\cdot\frac{3y}{4x^2} = \frac{2x}{3y}$

c. $\frac{x^2-1}{x-1} \div (x+1) \Rightarrow \frac{(x+1)(x-1)}{(x-1)}\cdot\frac{1}{(x+1)} = 1$

C. Examples:

Problem #1:

$$\frac{x^2y-xy}{y^2-1}\cdot\frac{y^3+y^2}{x^3-x^2} \div \frac{y^2}{y-1}$$

$$= \frac{\cancel{x}y\cancel{(x-1)}}{\cancel{(y+1)}\cancel{(y-1)}}\cdot\frac{\cancel{y^2}\cancel{(y+1)}}{\cancel{x^2}\cancel{(x-1)}}\cdot\frac{\cancel{y-1}}{\cancel{y^2}} = \frac{y}{x}$$

Problem #2:

$$\frac{x^4-16}{x^3-64}\cdot\frac{x^2+4x+16}{x^4-5x^2+4} \div \frac{x^3+4x}{x^3-3x^2-4x}$$

$$= \frac{\cancel{(x^2+4)}\cancel{(x+2)}\cancel{(x-2)}}{\cancel{(x-4)}\cancel{(x^2+4x+16)}}\cdot\frac{\cancel{x^2+4x+16}}{\cancel{(x-2)}\cancel{(x+2)}\cancel{(x+1)}(x-1)}\cdot\frac{x\cancel{(x+1)}\cancel{(x-4)}}{x\cancel{(x^2+4)}}$$

$$= \frac{1}{x-1}$$

Problem #3:

$$\frac{y^3-27}{9y-y^3} \div \frac{y^4+3y^3+9y^2}{y^4+3y^3}$$

$$= \frac{(y-3)\cancel{(y^2+3y+9)}}{y(3-y)\cancel{(3+y)}}\cdot\frac{y^3\cancel{(y+3)}}{y^2\cancel{(y^2+3y+9)}} = -1$$

D. Self-Evaluation:

Solve the following problems:

1. $\frac{16a^2b^4c^3}{27a^4c^2} \div \frac{8b^2c^3}{9a^2c^3}$

2. $\frac{xy-x^2}{x^2} \div \frac{xy-y^2}{y^2}$

3. $\frac{4x^3-4x^2+x}{8x^2-4x} \div \frac{4x^3-2x^2}{4x^3}$

4. $\frac{x^4-27x}{x^2-6x+9} \div \frac{x^3+3x^2+9x}{x^2-9}$

5. $\frac{x^2-y^2}{ax+bx-ay-by} \div \frac{ax-2ay+bx-2by}{x^2+3xy+2y^2}$

COLLEGE ALGEBRA
Learning Module #6

Topic	Complex Fractions and Exponents
Duration	3 hours
Lesson Proper	**I. Complex fraction** is a fraction whose numerator or denominator or both contains fractions. **Examples:** $\frac{2}{\frac{5}{7}} = \frac{14}{5}$; $\frac{\frac{2}{5}}{7} = \frac{2}{35}$; $\frac{\frac{3}{4}}{\frac{2}{5}}$; $\frac{\frac{x}{y}}{2}$; $\frac{\frac{a+b}{c}}{\frac{a-b}{d}}$; $\frac{\frac{x-1}{y+1}}{\frac{a}{b}}$ ➢ Complex fractions can be reduced to lowest form by simplifying the numerator and denominator individually and then apply the division operation. $\frac{a/b}{c/d} = \frac{a}{b} * \frac{d}{c} = \frac{ad}{bc}$ **Example #1:** $\frac{2x/3}{4/9x} = \frac{2x}{3} * \frac{9x}{4} = \frac{3x^2}{2}$ **2.** $\frac{\frac{1}{x}+\frac{1}{y}}{\frac{x}{y}+\frac{y}{x}} = \frac{\frac{y+x}{xy}}{\frac{x^2-y^2}{xy}} = \frac{x+y}{xy} * \frac{xy}{(x+y)(x-y)} = \frac{1}{x-y}$ **3.** $\frac{25-\frac{x^2}{y^2}}{5-\frac{x}{y}} = \frac{\frac{25y^2-x^2}{y^2}}{\frac{5y-x}{y}} = \frac{(5y+x)(5y-x)}{y^2} * \frac{y}{5y-x} = \frac{5y+x}{y}$ **Self-evaluation:** a. $\frac{\frac{5}{12}+\frac{3}{8}}{\frac{7}{18}-\frac{11}{12}}$ b. $\frac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}}$ c. $\frac{\frac{5-2y}{4-y}-1}{\frac{y^2-y+3}{y-4}+1}$

Example #4: Simplify the following:

$$\frac{4}{1-\frac{1}{1-\frac{1}{7}}} = \frac{4}{1-\frac{1}{\frac{7-1}{7}}} = \frac{4}{1-\frac{1}{\frac{6}{7}}} = \frac{4}{1-\frac{7}{6}} = \frac{4}{-\frac{1}{6}} = -24$$

II. Zero and Negative Exponents

The power a^n is defined as: $a^n = \underbrace{a * a \ldots a}_{\text{n factors}}$

$$a^0 = 1, \ a \neq 0$$

$$a^{-n} = \frac{1}{a^n}, \ n \text{ is any positive integer}, a \neq 0$$

Example #5: $(4a)^{-2} = \frac{1}{(4a)^2} = \frac{1}{16a^2}$

- **The rules on positive integral powers applies for negative and rational exponents.**

$$\left[\frac{a}{b}\right]^{-n} = \frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n}$$

Example #6: $\left[\frac{2a}{b}\right]^{-3} = \left[\frac{b}{2a}\right]^3 = \frac{b^3}{8a^3}$

- This means that: 1) a fraction raised to a negative integral exponent can be expressed to its reciprocal raised to a positive integral exponent.
2) Moreover, a factor of the numerator may be moved in the denominator, or vice-versa, if the sign of the exponent is changed.

Exercises: Simplify the following without negative exponents.

1. $-2^0 - 2 + 2^{-1} - 2^{-2} + 2^{-3}$
2. $-2^{-1} - (-2^2)^{-1} - (2^{-2})^0$
3. $\frac{2^{-1} - 3^{-1}}{2^{-1} + 3^{-1}}$
4. $\left[\frac{b}{a}\right]^{-9} * \left[\frac{a^2b^{-2}}{a^{-1}b^4}\right]^{-3}$

Self-evaluation

a. $-4 - 4^{-1} - 4^0 - 4^{-2}$

b. $(x-y)(x^{-1} - y^{-1})$

c. $\frac{5^{-1} - 5^{-2}}{5^{-1} + 5^{-2}}$

d. $\frac{x^{-1} - y^{-1}}{x - y}$

e. $\left[\frac{x^{-1} + y^{-1}}{x^{-1}y^2 + x^2y^{-1}}\right]^{-1}$

Mathematics Department

COLLEGE ALGEBRA
Learning Module #6b

Topic	**Fractional Exponents/Radicals**
Duration	**3 hours**

Lesson Proper

We shall extend the definition of a^n to include fractions or rational numbers for n. If m/n is a rational number with positive integer a, then

$$\boldsymbol{a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m}$$

The form $\boldsymbol{\sqrt[n]{a^m} = a^{m/n}}$ is called the **principal root** of $\boldsymbol{a^m}$. The numerator **m** indicates a power and the denominator **n** is called the **root or order**.

Specifically, $\boldsymbol{a^{1/n}}$ means the **principal nth root of a**. The symbol $\boldsymbol{\sqrt[n]{a}}$ is called a **radica**l, where **a** is called the **radicand**, and **n** is called the **index or order**.

To evaluate radicals, it is sometimes convenient to express the radical with fractional exponent or apply the rule $\boldsymbol{\sqrt[n]{a^n} = a}$. The following are examples on radicals:

1. $(-64)^{\frac{2}{3}} = (\sqrt[3]{-64})^2 = (-4)^2 = 16$
2. $(-32)^{-\frac{2}{3}} = \frac{1}{(-32)^{2/5}} = \frac{1}{(-2)^2} = \frac{1}{4}$
3. $\sqrt[6]{x^{18}} = x^{18/6} = x^3$
4. $\sqrt[3]{8x^6} = \sqrt[3]{(2x^2)^3} = [(2x^2)^3)]^{\frac{1}{3}} = 2x^2$
5. $\sqrt[6]{16x^2} = (16x^2)^{1/6} = (2^4x^2)^{1/6} = (2^4x)^{1/3} = \sqrt[3]{4x}$
6. $\sqrt[3]{-64x^6} = -4x^2$
7. $\sqrt[5]{-32x^{10}} = -2x^2$

Notice that we impose the condition a >0 in the definition of square root of the number a because it will not always hold if a<0. For example:

$$(\sqrt{-7})^2 \neq -7$$

So to avoid conflict we give a stronger definition for the square of a number, i.e.

$$\sqrt{a^2} = |a|, \quad \text{for any number a.}$$

In general, for an even n and any real number a.

$$\sqrt[n]{a^n} = |a| \ .$$

Examples: a. $\sqrt[3]{x^3} = x$ b. $\sqrt[5]{32a^5} = \sqrt[5]{2^5a^5} = 2a$

Laws on Radicals

1. $(\sqrt[n]{a})^n = a, \quad a > 0.$ Example: $(\sqrt[5]{6})^5 = 6$
2. $\sqrt[n]{ab} = \sqrt[n]{a}\ \sqrt{b}\,,\ a, b > 0$ Example: $\sqrt{18} = \sqrt{9 * 2} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$
3. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}\,, \quad a, b > 0$ Example: $\sqrt[5]{\frac{5}{32}} = \frac{\sqrt[5]{5}}{\sqrt[5]{32}} = \frac{\sqrt[5]{5}}{2}$
4. $\sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a} = \sqrt[m]{\sqrt[n]{a}}$ Example: $\sqrt[6]{4} = \sqrt[3]{\sqrt{4}} = \sqrt[3]{2}$

Simplified Radical Form

1. All exponents in the radicand must be less than the index.
2. Any exponents in the radicand can have no factors in common with the index.
3. No fractions appear under a radical.
4. No radicals appear in the denominator of a fraction.

EXERCISES:

Simplify the following radicals:

a. $\sqrt[4]{x^7}$ **b.** $\sqrt[3]{-16}$ **c.** $\sqrt[5]{-64}$ **d.** $\sqrt[6]{x^{11}}$

e. $\sqrt[3]{\frac{9}{x^{12}}}$ **f.** $\sqrt[8]{x^2}$ **g.** $\sqrt[6]{x^4y^8}$ **h.** $\sqrt[3]{\frac{8}{x^{12}}}$

i. $\sqrt[3]{-64x^6z^{24}}$

COLLEGE ALGEBRA
Learning Module #6c

Topic	**Radicals and its Operations**
Duration	**3 hours**
Lesson Proper	**Three Ways to Simplify Radicals:** 1. Removal of Perfect nth powers ➢ Break down the radicand into perfect and nonperfect nth powers and apply the $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$. 2. Reducing the index to the lowest possible order ➢ Reducing the index is done by applying the property $\sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a} = \sqrt[m]{\sqrt[n]{a}}$ or expressing the radical into its exponential form and simplifying the fractional exponent. 3. Rationalizing the denominator of the radicand ➢ It is the process to remove the radical sign in the denominator, Which is making the radicand nonfrational. Example#1: $\sqrt[3]{\frac{2}{5x^2}} = \sqrt[3]{\frac{2}{5x^2} * \frac{5^2x}{5^2x}} = \frac{\sqrt[3]{50x}}{5x}$ Example#2: $\sqrt[3]{\frac{1}{75}} = \sqrt[3]{\frac{1}{3 \cdot 5^2} * \frac{3^2 \cdot 5}{3^2 \cdot 5}} = \frac{\sqrt[3]{45}}{15}$ Exercises: Simplify the following: 1. $\sqrt[4]{\frac{1}{192\,x^{11}y^5}}$ 2. $\sqrt[5]{\frac{1}{3xy^3}}$ 3. $\sqrt[3]{\frac{1}{25x^8y^{10}}}$ 4. $\sqrt[5]{\frac{1}{3xy^3}}$ 5. $\sqrt[6]{\frac{1}{72x^2y}}$

Addition & Subtraction of Radicals

Similar Radicals are radicals of the same order and the same radicand. Similar radicals can be combined into a single radical by the use of the distributive law. Radicals of different indices and radicands are called dissimilar radicals.

Example#1: $\sqrt{18} + \sqrt{8} - \sqrt{50}$

$= \sqrt{9 * 2} + \sqrt{4 * 2} - \sqrt{25 * 2}$

$= 3\sqrt{2} + 2\sqrt{2} - 5\sqrt{2} = (3 + 2 - 5)\sqrt{2} = \mathbf{0}$

Example#2: $\sqrt[3]{-8} - \sqrt[3]{-64} - \sqrt[4]{4} + \sqrt{8}$

$= -2 + 4 - \sqrt{2} + 2\sqrt{2}$

$= (-2 + 4) + \left(-\sqrt{2} + 2\sqrt{2}\right) = \mathbf{2} + \sqrt{\mathbf{2}}$

Multiplication of Radicals

- To multiply radicals of the same order, use the law of radicals

  $\sqrt[n]{a} * \sqrt[n]{b} = \sqrt[n]{ab}$

  Then simplify by the removal of the **n**th perfect powers from the radicand

Example#1: $\sqrt[3]{4x^3y^2} * \sqrt[3]{4x^5y^2}$

$= \sqrt[3]{16x^8y^4} = \sqrt[3]{8x^6y^3} * \sqrt[3]{2x^2y} = \mathbf{2x^2y\sqrt[3]{2x^2y}}$

Example#2: $\left(4\sqrt{6} - 5\sqrt{7}\right) * \left(2\sqrt{6} - 3\sqrt{7}\right)$

$= \left(4\sqrt{6} * 2\sqrt{6}\right) - \left(5\sqrt{7} * 2\sqrt{6}\right) - \left(4\sqrt{6} * 3\sqrt{7}\right) + \left(5\sqrt{7} * 3\sqrt{7}\right)$

$= 8\sqrt{36} - 10\sqrt{42} - 12\sqrt{42} + 15\sqrt{49}$

$= 8(6) - 22\sqrt{42} + 15(7) = (48 + 105) - 22\sqrt{42} = \mathbf{153} - \mathbf{22\sqrt{42}}$

- To multiply two radicals of different order, it is necessary to express them as radicals of the same order.

Example#3: $\sqrt[3]{\mathbf{2}} * \sqrt{\mathbf{3}} = \mathbf{2^{\frac{1}{3}}} * \mathbf{3^{\frac{1}{2}}} = \mathbf{2^{\frac{2}{6}}} * \mathbf{3^{\frac{3}{6}}}$

$= \sqrt[6]{\mathbf{2^2}} * \sqrt[6]{\mathbf{3^3}}$

$= \sqrt[6]{\mathbf{4 * 27}} = \sqrt[6]{\mathbf{108}}$

Example#4: $\sqrt[4]{4}\,\sqrt{2} = \sqrt{\sqrt{4}} * \sqrt{2} = \sqrt{2} * \sqrt{2} = \mathbf{2}$

Division of Radicals

- To divide radicals of the same order, use the law on radicals and rationalize the denominator of the radicand.

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Example#1: $\dfrac{\sqrt[4]{2x}}{\sqrt[4]{6x^3y^3}} = \sqrt[4]{\dfrac{1}{3x^2y^3}}$

$$= \sqrt[4]{\frac{1}{3x^2y^3} * \frac{3^3x^2y}{3^3x^2y}} = \frac{\sqrt[4]{27x^2y}}{3xy}$$

- To divide the radicals of different orders, it is necessary to express them as radicals with the same order.

Example#3: $\dfrac{\sqrt{3}}{\sqrt[3]{2}} = \dfrac{3^{\frac{1}{2}}}{2^{\frac{1}{3}}} = \dfrac{3^{\frac{3}{6}}}{2^{\frac{2}{6}}} = \sqrt[6]{\dfrac{3^3}{2^2} * \dfrac{2^4}{2^4}} = \dfrac{\sqrt[6]{432}}{2}$

- To divide the radicals with denominator of the radicand consisting of at least two terms, again rationalize the denominator.

Example#4: $\sqrt{\dfrac{a-b}{a+b}} = \sqrt{\dfrac{a-b}{a+b} * \dfrac{a+b}{a+b}} = \dfrac{\sqrt{a^2-b^2}}{a+b}$

Example#5: $\dfrac{4}{\sqrt{3}-2} = \dfrac{4}{\sqrt{3}-2} * \dfrac{\sqrt{3}+2}{\sqrt{3}+2} = \dfrac{4(\sqrt{3}+2)}{3-4} = -4(\sqrt{3}+2)$

Activities

Perform the indicated operations:

1. $\sqrt[3]{-16} - \sqrt[3]{-128} - \sqrt[3]{-2}$
2. $\sqrt[3]{\frac{1}{2}} - \sqrt[3]{\frac{1}{4}} - \sqrt[6]{16} - \sqrt[6]{4}$
3. $\sqrt[3]{18} * \sqrt[3]{4}$
4. $(2\sqrt{5}-1)^2$
5. $\sqrt[4]{27x^2y^3} * \sqrt[4]{15x^3y}$
6. $\dfrac{\sqrt[4]{3}}{\sqrt[4]{8y^2z}}$