Syllabus/Course Outline ECON0118: Term 1 Analysis and Term 2 Probability

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Texts

There are many books that cover the material of term 1; some are expensive. I have chosen *Real Analysis with Economic Applications* by E.A. Ok as a text for term 1, because it is kind of the standard text. You can download some of the chapters from Efe's website https://sites.google.com/a/nyu.edu/efeok/home/ There is a great deal in this book that we won't cover. (Some of the other books I mention below also have online versions.) A very good place to begin are the mathematical parts (Chapters 3,7,17) of *Recursive Methods in Economic Dynamics* by N. Stokey & R. Lucas. Another popular book with Economists is *Infinite Dimensional Analysis: A Hitchhiker's Guide* by C.D. Aliprantis & K.C. Border, this is too advanced for this course but is clear and comprehensive. A book that seems to cover much the same material is *An Introduction to Mathematical Analysis for Economic Theory and Econometrics* by D. Corbae & M. Stinchcombe. My favourite analysis books are *Real Analysis* by J.N McDonald & N.A. Weiss and *Real Analysis* by H.L. Royden. Both are crystal clear and try to make things simple – which I think is a very good thing. They obviously have no Economics applications. I have given references to Ok, Royden, and McDonald & Weiss below.

One text for term 2 is downloadable: *Measure and Probability Theory with Economic Applications* by E.A Ok. In the references I put a * to distinguish this text from the other by Ok. My favourite probability book is *Probability with Martingales* by David Williams, again because it tries to make things simple. I give references to this also. It can be obtained quite cheaply so I think this is a good substitute for Ok and I will stick to this book very closely in term 2.

Course Structure

The course will be delivered with the traditional lectures and problem sets that are done in the tutorials. There will be 2 take home exams at the end of each term

Course Outline

The material I would like to cover is listed below. As this is the first iteration of this course I am not sure how much of this we will do. I aim to do topics 1-5 in the first term and topics 6-10 in the second.

1. **Elementary Set Theory: (References: OK A1, M&W Chapter 1, Royden Chapter 1)**
   - *Definition of a set and examples: C,R,Q,Z,N*, intervals in *R*.
   - *Set Operations:* Complement, intersection, union, differences, de Morgan's Laws, set limsup & liminf, Cartesian products.
   - *Functions*: Definitions, inverses, bijective & surjective, sequences as functions, the Axiom of Choice, partial orderings and Zorn's Lemma.
   - *Countability:* Equivalence of sets, countability of the rationals, uncountability of the reals.
   - *Families/Classes of Sets*: Definitions of algebras, sigma algebras, topologies, and the Borel sets.

2. **Real Numbers, Sequences, and Functions: (References: Ok (A2-A4) M&W (Chapters 2&6) Royden (Chapter 2)**
   - *Reals:* The field axioms, the completeness axiom, supremum and infimum, density of the rationals & the irrationals.
   - *Sequences, Subsequences & Series:* Monotonicity, definition of convergence, Bolzano-Weierstrass, cluster points, limsup & liminf, Cauchy sequences, series convergence tests.
   - *Open and Closed Sets:* Definition of open and closed sets in *R* and relationship to sequences.
   - *Functions and Continuity:* Two definitions of continuity, Semi-continuity, Uniform continuity, Differentiation (Dini-derivatives & Lebesgue's Theorem on differentiability of monotone functions), intermediate value theorem, mean-value theorem.
   - *Sequences of functions*: Monotone sequences, pointwise convergence, and uniform convergence.
   - *Riemann Integrals*: Definition, properties, definition of sets of zero measure, characterization of Riemann-integrable functions.

3. **Metric Spaces 1: Ok (C1-7,D6), M&W (Chapter 7), R (Chapter 7)**
   - *A little topology:* Open and closed sets, bases, homeomorphic spaces.
   - *Metrics and Norms*: Definitions of both types of space; sequences, open & closed sets, continuity again; metric spaces as topological spaces.
   - *Connectedness & separability:* definitions, Hausdorff spaces, Tietze's Extension Th, Intermediate Value Theorem, open coverings?
   - *Completeness*: definitions, Cauchy sequences, Baire category.
   - *Compactness*: definition, Heine-Borel and its generalizations, totally boundedness.
   - *Weak topologies and non-metrizable* spaces (if there is time).

4. **Metric Spaces 2: Ok (D&E), M&W (Chapter 8), R (Chapter 9)**
   - *Continuity & Weierstrass*: definitions of continuity & semicontinuity again.
   - *Function Spaces*: Topology of uniform convergence.
   - *Contraction Mapping and Banach Fixed Point Theorem*:
   - *Compactness of Function spaces*: Topology of uniform convergence.
   - *Product Spaces:* Metrics for finite and countably infinite products.
   - *The Maximum Theorem:*
   - *Fixed Point Theorems*

5. **Normed Linear Spaces: Ok (F,G,J), M&W (Chapter 10), Royden (Chapter 10)**
   - Introductory concepts: Linear/vector space, linear operators & dual spaces.
   - Banach Spaces & Hilbert Spaces, L^p spaces.
   - Basis and Dimension,
   - Convex functions: Hahn Banach theorem & separating hyperplanes

6. **Probability and Measure: Ok* (Chapter B), M&W (Chapters 4&5), Williams (Chapters 1&2)**
   - Measurable Spaces: sigma algebras,
   - Borel sigma algebras,
   - regular probability measures,
   - examples,

- Caratheodory’s Extension Theorem,
- Lebesgue-Stieltjes probabilities,
- Lebesgue measure,
- First Borel-Cantelli.

7. **Random Variables: Ok* (Chapter C), Williams (Chapter 3)**
    - Measurability,
    - Monotone Class Theorem,
    - Sequences of Random Variables,
    - Egorov’s Theorem,
    - Lusin’s Theorem,
    - Distribution functions.

8. **Independence: Ok* (Chapter H&I), Williams (Chapter 4)**
    - Second Borel-Cantelli,
    - Stochastic Processes
    - Markov Chains,
    - Kolmogorov’s 0-1 Law.

9. **Integration and Expectation (Ok* Chapter D&E), Williams (Chapter 5&6)**
    - Definitions
    - Integrability & Fatou’s Lemma,
    - Monotone & Dominated Convergence,
    - Markov, Jensen, Cauchy-Schwartz and Holder’s inequalities.

10. **Conditional Expectation (Ok Chapter K), Williams (Chapter 9)**
    - Definition, Existence,
    - Regular conditional probabilities.