Introduction to Dynamic Programming

Part III: FE for identifying the optimal policy

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[1]material @ https://github.com/PPEphile

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Stokey, N.L., Lucas, R.E. and Prescott, E.C. (1989) *Recursive Methods in Economic Dynamics*. Cambridge, Harvard University Press.

Intro

- We want to solve:

$$\max_{0 \leq x_{t+1} \leq f(x_t)} \sum_{t=0}^{\infty} \beta^t F(x_t, x_{t+1}) \qquad \text{(SP)}$$

- Last time we identified $v^*(x)$, the solution to the (SP)
- *But is that really what we were after?*

A necessary condition

Theorem 4

If the path $\underset{\sim}{x}^*$ is optimal, then

$$v^*(x_t*) = F(x_t^*, x_{t+1}^*) + \beta v^*(x_{t+1}^*) = \max_{y \in \Gamma(x)} \{F(x, y) + \beta v^*(y)\}$$

for all t.

A sufficient condition

Theorem 5

If the candidate path $\hat{\underset{\sim}{x}}$ is feasible, satisfies Theorem 4 **and**

$$\lim_{t\to\infty} \sup \beta^t v^*(\hat{x}_t) \leq 0$$

then $\hat{\underset{\sim}{x}} = \underset{\sim}{x}^*$.

Least upper bound

Least upper bound

Example