Introduction to Dynamic Programming

Part II: Functional Equation

PPE Phil(e)[1]

[1]material @ https://github.com/PPEphile

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Stokey, N.L., Lucas, R.E. and Prescott, E.C. (1989) *Recursive Methods in Economic Dynamics*. Cambridge, Harvard University Press.

The Functional Equation (Recap)

- We eliminated infinitely many constraints at the cost of now having to find a function
- **Theorem 2**: For any candidate function v to be the true value function v^*, it needs to satisfy the (FE).

Functional Equation

$$v(x) = \max_{y \in \Gamma(x)} \{F(x, y) + \beta v(y)\} \qquad \text{(FE)}$$

where $\Gamma(x)$ is the set of admissible values of y given the current state x.

Sufficient condition

Theorem 3

If v is real-valued, solves the (FE) **and**

$$\lim_{t\to\infty} \beta^t v(x_t) = 0 \quad \text{for all feasible paths of } (x_t)_{t=0}^{\infty}$$

then $v = v^*$

Least upper bound

- (FE) only necessary but not sufficient
  - e.g. $v(x) = \pm\infty$ is an universal solution
- Supremum: least upper bound

Example

Consider the classical corn growing example with utility $U(c) = c$, $f(k) = k^{\alpha}$ a