

# Introduction to Dynamic Programming

## Part II: Functional Equation

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<sup>1</sup>material @ <https://github.com/PPEphile>

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  - ▶ Theorem 2 (necessary condition)
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Stokey, N.L., Lucas, R.E. and Prescott, E.C. (1989) *Recursive Methods in Economic Dynamics*. Cambridge, Harvard University Press.

# The Functional Equation (Recap)

## Functional Equation

$$v(x) = \max_{y \in \Gamma(x)} \{F(x, y) + \beta v(y)\} \quad (\text{FE})$$

where  $\Gamma(x)$  is the set of admissible values of  $y$  given the current state  $x$ .

- (FE) only necessary but not sufficient
  - ▶ e.g.  $v(x) = \pm\infty$  is an universal solution
- Supremum: least upper bound

# Upper bound