# Introduction to Dynamic Programming

Part III: FE for identifying the optimal policy

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## Identifying the optimal policy

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Stokey, N.L., Lucas, R.E. and Prescott, E.C. (1989) *Recursive Methods in Economic Dynamics*. Cambridge, Harvard University Press.

## Intro

• We want to solve:

$$\max_{0 \le x_{t+1} \le f(x_t)} \sum_{t=0}^{\infty} \beta^t F(x_t, x_{t+1})$$
 (SP)

- Last time we identified  $v^*(x)$ , the solution to the (SP)
- But is that really what we were after?

# Corn-growing with linear utility

Consider the classical corn growing example with utility U(c)=c, f(k)=2k and  $\beta=\frac{1}{3}$  and some  $k_0\geq 0$ .

# A necessary condition

This gives us an intuitive necessary condition:

#### Theorem 4

If the path  $\underline{x}^*$  is optimal, then

$$v^*(x_t^*) = F(x_t^*, x_{t+1}^*) + \beta v^*(x_{t+1}^*) = \max_{y \in \Gamma(x)} \{ F(x, y) + \beta v^*(y) \}$$

for all t.

## A sufficient condition

#### Theorem 5

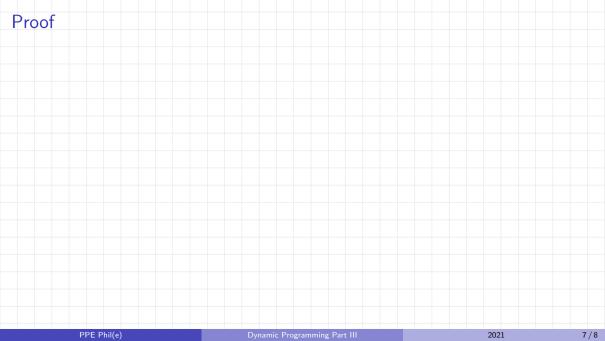
If the candidate path  $\hat{x}$  is feasible and satisfies

$$v^*(\hat{x}_t) = F(\hat{x}_t, \hat{x}_{t+1}) + \beta v^*(\hat{x}_{t+1})$$

for all t and

$$\limsup_{t\to\infty}\beta^t v^*(\hat{x}_t)\leq 0$$

then  $\hat{x}$  is optimal.



# Example

# Corn-growing with linear utility

Consider the classical corn growing example with utility  $U(c) = \ln(c)$ , f(k) = 2k and  $\beta = \frac{1}{3}$  and some  $k_0 \ge 0$ .