Introduction to Dynamic Programming

Part III: FE for identifying the optimal policy

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[1]material @ https://github.com/PPEphile

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Stokey, N.L., Lucas, R.E. and Prescott, E.C. (1989) *Recursive Methods in Economic Dynamics*. Cambridge, Harvard University Press.

Intro

- We want to solve:

$$\max_{0 \leq x_{t+1} \leq f(x_t)} \sum_{t=0}^{\infty} \beta^t F(x_t, x_{t+1}) \qquad \text{(SP)}$$

- Last time we identified $v^*(x)$, the solution to the (SP)
- *But is that really what we were after?*

Corn-growing with linear utility

Consider the classical corn growing example with utility $U(c) = c$, $f(k) = 2k$ and $\beta = \frac{1}{3}$ and some $k_0 \geq 0$.

A necessary condition

This gives us an intuitive necessary condition:

Theorem 4

If the path $\underset{\sim}{x}^*$ is optimal, then

$$v^*(x_t*) = F(x_t^*, x_{t+1}^*) + \beta v^*(x_{t+1}^*) = \max_{y \in \Gamma(x)} \{F(x, y) + \beta v^*(y)\}$$

for all t.

A sufficient condition

Theorem 5

If the candidate path $\underset{\sim}{\hat{x}}$ is feasible and satisfies

$$v^*(\hat{x}_t) = F(\hat{x}_t, \hat{x}_{t+1}) + \beta v^*(\hat{x}_{t+1})$$

for all t **and**

$$\limsup_{t \to \infty} \beta^t v^*(\hat{x}_t) \leq 0$$

then $\underset{\sim}{\hat{x}}$ is optimal.

Proof

Example

Corn-growing with linear utility

Consider the classical corn growing example with utility $U(c) = \ln(c)$, $f(k) = 2k$ and $\beta = \frac{1}{3}$ and some $k_0 \geq 0$.