2.1

12.

(a) $p \wedge \sim r$

(b) $q \vee p$

(c) $\sim p \wedge \sim q$

(d) $r \wedge q$

15.

(a) For all x there exists a y such that $x + y$ is even.

(b) There exists an x such that, for all y, $x + y$ is even.

16.

(a) For all integer x, x is not a prime number.

(b) There exists an integer y, y is not even.

18.

(a) $\forall x \ \sim \mathrm{P}(x)$

(b) $\forall x \forall y \ R(x, y)$

(c) $\forall x \ \sim (P(x) \wedge Q(x))$ / $\sim (\exists x \ (P(x) \wedge Q(x)))$

(d) $\forall x \ (P(x) \vee Q(x))$

28.

p	q	$p \vee q$	$\sim q$	$(p \vee q) \vee \sim q$
T	T	T	F	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	T

30.

p	q	r	$\sim p$	$\sim p \vee q$	$\sim r$	$(\sim p \vee q) \wedge \sim r$
T	T	T	F	T	F	F
T	T	F	F	T	T	T
T	F	T	F	F	F	F
T	F	F	F	F	T	F
F	T	T	T	T	F	F
F	T	F	T	T	T	T
F	F	T	T	T	F	F
F	F	F	T	T	T	T

32.

p	q	r	$p \downarrow q$	$p \downarrow r$	$(p \downarrow q) \wedge (p \downarrow r)$
T	T	T	F	F	F
T	T	F	F	F	F
T	F	T	F	F	F
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	T	F
F	F	T	T	F	F
F	F	F	T	T	T

35.

p	q	$p \wedge q$	$(p \wedge q)\, \Delta\, p$
T	T	T	F
T	F	F	T
F	T	F	F
F	F	F	F

2.2

6.

(a) $\sim r \Rightarrow q$
(b) $\sim q \wedge p$
(c) $q \Rightarrow \sim p$
(d) $\sim p \Rightarrow \sim r$

7.

(a) If I do not study discrete structures and I go to a movie, then I am in a good mood.

(b) If I am in a good mood, then I will study discrete structures or I will go to a movie.

(c) If I am not in a good mood, then I will not go to a movie or I will study discrete structures.

(d) I will go to a movie and I will not study discrete structures if and only if I am in a good mood.

12.

(a) contingency

p	q	$\sim p$	$q \wedge p$	$q \wedge \sim p$	$(q \wedge p) \vee (q \wedge \sim p)$
T	T	F	T	F	T
T	F	F	F	F	F
F	T	T	F	T	T
F	F	T	F	F	F

(b) tautology

p	q	$p \wedge q$	$p \wedge q \Rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

(c) contingency

p	q	$p \wedge q$	$p \Rightarrow p \wedge q$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

13.

Yes. If $p \Rightarrow q$ is false, then p is true and q is false. Hence $p \wedge q$ is false, $\sim(p \wedge q)$ is true, and $(\sim(p \wedge q)) \Rightarrow q$ is false.

14.

Yes. If $p \Rightarrow q$ is false, then p is true and q is false. Hence $\sim p$ is false, $p \Leftrightarrow q$ is false, and $(\sim p) \vee (p \Leftrightarrow q)$ is false.

15.

No, because if $p \Rightarrow q$ is true, it may be that both p and q are true, so $(p \wedge q) \Rightarrow \sim q$ is false. But it could also be that p and q are both false, and then $(p \wedge q) \Rightarrow \sim q$ is true.

16.

Yes. If $p \Rightarrow q$ is true, then $\sim(p \Rightarrow q)$ is false. Hence $\sim(p \Rightarrow q) \wedge \sim p$ is false.

24.

(a) The weather is not bad or I will go to work.

(b) Carol is not sick, and she goes to the picnic, she will not have a good time.

(c) I will win the game and I will enter the contest.

27.

p	q	r	$p \wedge$	$(q \vee r)$	$(p \wedge q)$	$\vee$	$(p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	F
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	F	T	F	F	F
F	T	F	F	T	F	F	F
F	F	T	F	T	F	F	F
F	F	F	F	F	F	F	F
			(A)			(B)	

Because (A) and (B) are the same, the statements are equivalent.

28.

The truth table is as follows,

p	q	$p \wedge q$	$\sim p$	$\sim q$	$\sim (p \vee q)$	$(\sim p) \vee (\sim q)$
T	T	T	F	F	F	F
T	F	F	F	T	T	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T

$\therefore \ \sim (p \vee q) \equiv (\sim p) \vee (\sim q)$.

29.

p	q	$\sim$	$(p \Leftrightarrow q)$	$(p \wedge \sim q)$	$\vee$	$(q \wedge \sim p)$
T	T	F	T	F	F	F
T	F	T	F	T	T	F
F	T	T	F	F	T	T
F	F	F	T	F	F	F
		(A)			(B)	

Because columns (A) and (B) are the same, the statements are equivalent.

30.

$\exists x\ (\mathrm{P}(x) \vee \mathrm{Q}(x))$ is true if and only if $\exists x$, either $\mathrm{P}(x)$ or $\mathrm{Q}(x)$ is true, and this means $\exists x\ \mathrm{P}(x)$ is true or $\exists x\ \mathrm{Q}(x)$ is true. This holds if and only if $\exists x\ (\mathrm{P}(x) \vee \mathrm{Q}(x))$ is true.

31.

The statement $\forall x(\mathrm{P}(x) \wedge \mathrm{Q}(x))$ is true if and only if $\forall x$ both $\mathrm{P}(x)$ and $\mathrm{Q}(x)$ are true, but this means $\forall x\mathrm{P}(x)$ is true and $\forall x\mathrm{Q}(x)$ is true. This holds if and only if $\forall x\mathrm{P}(x) \wedge \forall x\mathrm{Q}(x)$ is true.

32.

The truth table is as follows,

p	q	$p \wedge q$	$p \wedge q \Rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$\therefore\ p \wedge q \Rightarrow p$ is a tautology.

33.

p	q	$q \quad \Rightarrow$	$(p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	T	F
		$\uparrow$	

34.

The truth table is as follows,

p	q	$p \Rightarrow q$	$p \wedge (p \Rightarrow q)$	$(p \wedge (p \Rightarrow q)) \Rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$\therefore\ (p \wedge (p \Rightarrow q)) \Rightarrow q$ is a tautology.

35.

p	q	r	$((p \Rightarrow q)$	$\wedge$	$(q \Rightarrow r))$	$\Rightarrow$	$(p \Rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	F
T	F	T	F	F	T	T	T
T	F	F	F	F	T	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T
			(1)	(3)	(2)	↑	(4)

2.3

8.

Valid.

Let p: I graduate this semester, q: I have passed the physics course, r: I study physics for 10 hours a week, s: I play volleyball.

$$\Leftrightarrow \begin{array}{l} p \Rightarrow q \\ \sim r \Rightarrow \sim q \\ r \Rightarrow \sim s \\ \hline \therefore\ s \Rightarrow \sim p \end{array} \qquad \begin{array}{l} \because \sim r \Rightarrow \sim q \Leftrightarrow q \Rightarrow r \\ \therefore\ p \Rightarrow \sim s \\ \therefore\ s \Rightarrow \sim p \end{array}$$

12.

(a) Not valid.
(b) Valid.

18.

Let p: n^2 is even, q: n is even. We need to prove $p \Leftrightarrow q$ is true.

$\because p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$

$\therefore$ we can prove $p \Rightarrow q$ is true and $q \Rightarrow p$ is true.

1. Prove $p \Rightarrow q$ is true.

$p \Rightarrow q \equiv \sim q \Rightarrow \sim p$, hence, we can prove $\sim q \Rightarrow \sim p$.

Thus suppose n is odd, then $n = 2k+1$ $(k \in \mathbb{Z})$,

and then $n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$ $(k \in \mathbb{Z})$.

Thus n^2 is odd, $\sim q \Rightarrow \sim p$ is true.

$p \Rightarrow q$ is true.

2. Prove $q \Rightarrow p$ is true.

Suppose n is even, then $n = 2k$ $(k \in \mathbb{Z})$,

and then $n^2 = 4k^2 = 2(2k^2)$ $(k \in \mathbb{Z})$.

Thus n^2 is even, $q \Rightarrow p$ is true.

Thus, $p \Leftrightarrow q$, that is, n^2 is even if and only if n is even.

20.

Let p: $A \subseteq B$, q: $\bar{B} \subseteq \bar{A}$.

1. Prove $p \Rightarrow q$ is true.

Let $x \in U$, then

$A \subseteq B$ means $\forall x \in A,\ x \in B$.

$\forall x \in A,\ x \in B \ \Leftrightarrow\ \forall x \in \bar{B},\ x \in \bar{A}$

$\therefore\ \bar{B} \subseteq \bar{A}$.

$\therefore\ p \Rightarrow q$ is true.

2. Prove $q \Rightarrow p$ is true.

Let $x \in U$, then

$\bar{B} \subseteq \bar{A}$ means $\forall x \in \bar{B},\ x \in \bar{A}$.

$\forall x \in \bar{B},\ x \in \bar{A} \ \Leftrightarrow\ \forall x \in A,\ x \in B$

$\therefore\ A \subseteq B$.

$\therefore\ q \Rightarrow p$ is true.

Thus, $p \Leftrightarrow q$, that is, $A \subseteq B$ if and only if $\bar{B} \subseteq \bar{A}$.

22.

Let p: k is odd, $q: k^3$ is odd.

1. Prove $p \Rightarrow q$ is true.

Suppose k is odd, then $k = 2n+1\ (n \in \mathbb{Z})$,

and then $k^3 = 2(4n^3 + 6n^2 + 3n) + 1\ (n \in \mathbb{Z})$.

Thus k^3 is odd, $p \Rightarrow q$ is true.

2. Prove $q \Rightarrow p$ is true.

$q \Rightarrow p \equiv\ \sim p \Rightarrow \sim q$, hence, we can prove $\sim p \Rightarrow \sim q$.

Suppose k is even, then $k = 2n\ (n \in \mathbb{Z})$,

and then $k^3 = 2(4n^3)\ (n \in \mathbb{Z})$.

Thus k^3 is even, $\sim p \Rightarrow \sim q$ is true, $q \Rightarrow p$ is true.

Thus, $p \Leftrightarrow q$, that is, k is odd is a necessary and sufficient condition for k^3 to be odd.

23.

For $n = 41$, we have a counterexample. $41^2 + 41 \cdot 41 + 41$ is $41(41 + 41 + 1)$ or $41 \cdot 83$.

24.

Prove.

Let these 5 numbers are: $k-2,\ k-1,\ k,\ k+1,\ k+2\ (k \in \mathbb{Z})$.

Thus, the sum of them is $5k$, it is divisible by 5.

30.

Valid. The proof technique is an example of an indirect method of proof, follows from the tautology (p ⇒ q) ⇔ ((~q) ⇒ (~p)). An implication is equivalent to its contrapositive. (~q) ⇒ (~p) proof by contradiction.

2.4

20.

(a)
Suppose P(k) is true, that is, $2\,|\,(2k-1)$.
Let $2t = 2k-1\ (t \in \mathbb{Z})$, then $2(k+1)-1 = 2k-1+2 = 2t+2 = 2(t+1)$,
and then $2\,|\,(2(k+1)-1)$, thus P($k+1$) is true.
(b)
$\forall n \in \mathbb{Z}$, $2n-1$ is odd, P(n) is not true.
(c)
No, in (a), we don't have the "Basic Step" which have to be done in the mathematical induction. And the principle of mathematical induction doesn't consider whether P(k) is true or not, we suppose P(k) is true.

22.

In the *Induction Step*, we didn't suppose that $z^{k-1} = 1$.

26.34.36.

Using mathematical induction to prove.

28.

The proposition is False.
It can be changed like this:

Prove that every integer greater than 27 can be written as
$5a + 8b$, where $a, b \in \mathbb{Z}^{+}$. remove '+'

Then, you can use mathematical induction to prove it.

30.

Proof by contradiction.

Assume there exists an $x \neq 1$, such that $\mathrm{GCD}(a^n, b^n) = x$.

Then $x \mid a^n$, $x \mid b^n$,Obtained from 29., $x \mid a$, $x \mid b$,

that is, $x \mid \mathrm{GCD}(a,b)$.

This is a contradiction to GCD(a, b) =1.

2.5

20.

A reasonable conjecture would be $g_2 + g_4 + \cdots + g_{2n} = g_{2n+1} - 1\ (n \in \mathbb{Z},\ n \geq 1)$.

21.

A reasonable conjecture would be $g_1 + g_2 + \cdots + g_n = g_{n+2} - 3\ (n \in \mathbb{Z},\ n \geq 1)$.

22.

A reasonable conjecture would be $g_1 + g_3 + \cdots + g_{2n-1} = g_{2n} - 2.\ (n \in \mathbb{Z},\ n \geq 1)$

23.24.25

Using mathematical induction to prove.