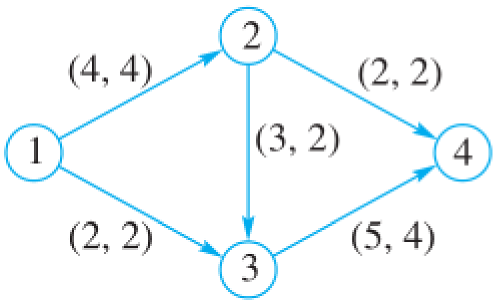
8.4

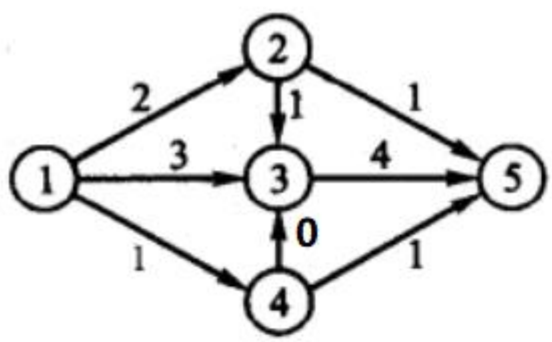
5

$value(F) = 6.$



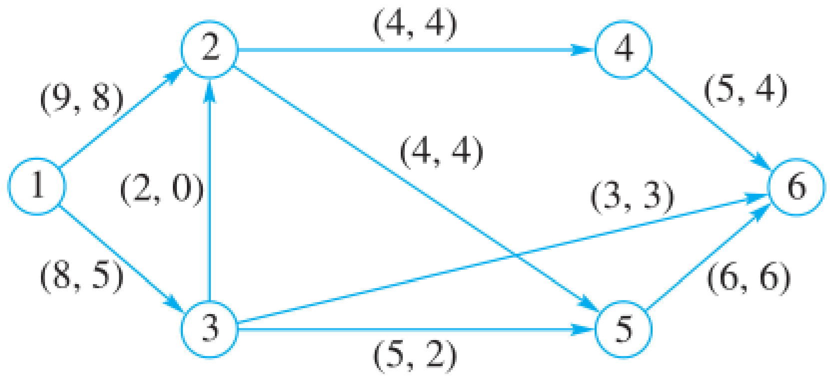
6

value(F) = 6



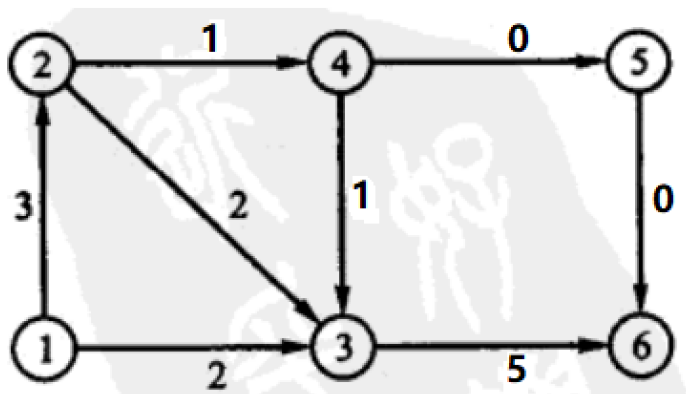
7

$value(F) = 13.$



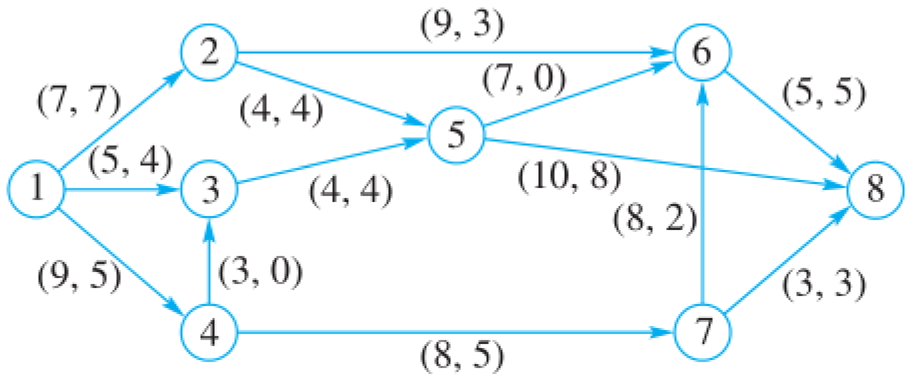
8

value(F) = 5



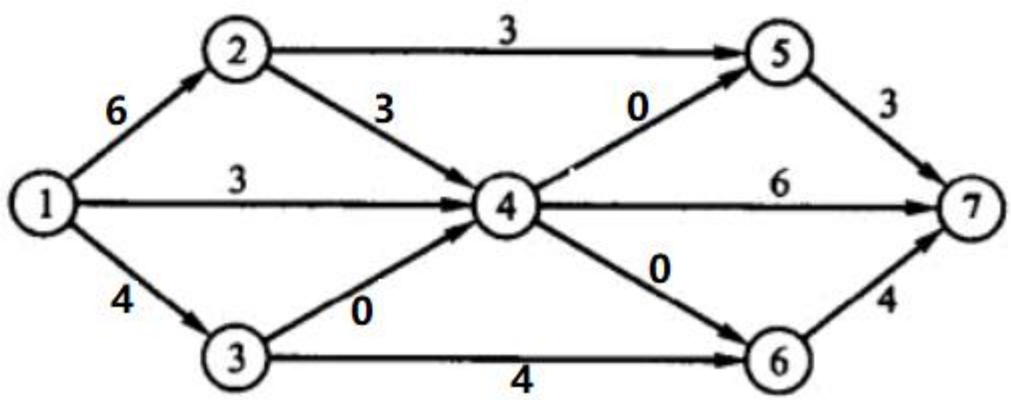
9

$value(F) = 16.$



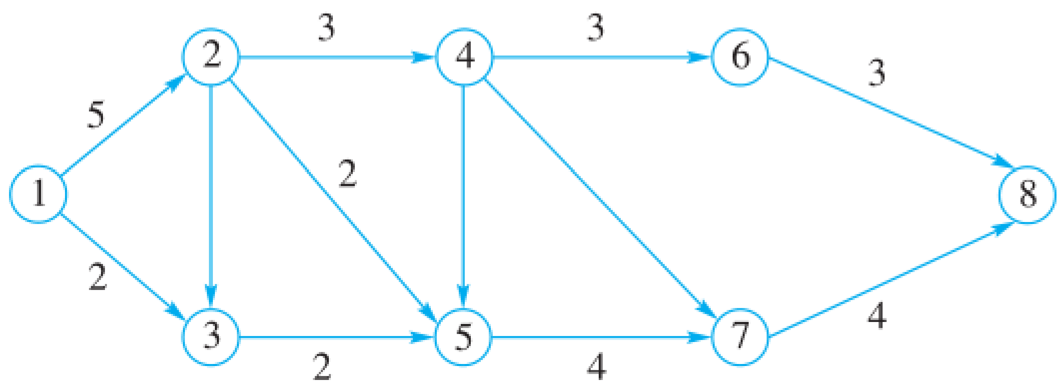
10

value(F) = 13



11

$value(F) = 7.$



14 符合题意即可

19

$\{(2, 5), (3, 5), (6, 8), (7, 8)\}.$

20

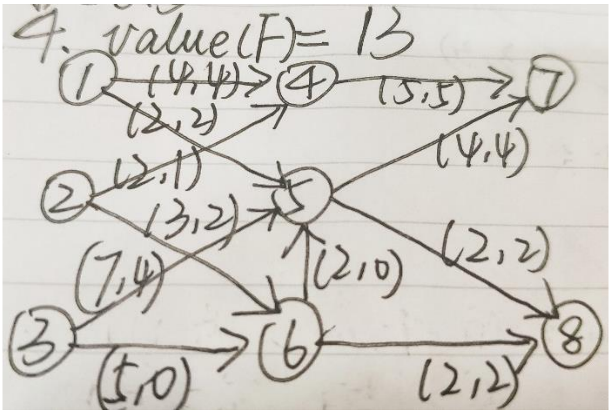
{(5 ,7), (4 ,7), (6, 7)}

21

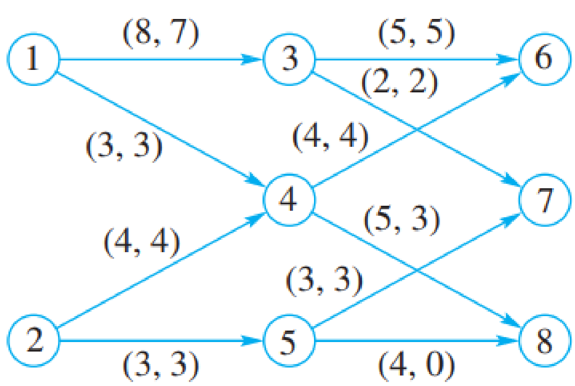
$\{(2, 4), (5, 7)\}$ with capacity 7.

8.5

4. 值为 13，参考答案如下：



5. *value*(*F*) = 17.



8.一个最大匹配是：

$M(a_1)=b_1$, $M(a_2)=b_2$, $M(a_3)=b_3$,$M(a_4)=b_4$.

10.一个最大匹配是：

$M(a_1)=b_1$, $M(a_2)=b_2$, $M(a_3)=b_3$,$M(a_4)=b_4$, $M(a_5)=b_5$, $M(a_6)=b_6$.

14. 设 S 为 A 的任意子集，E 为以 S 为开始节点的边的集合，则有, k|S|=|E|。每条边的结束节点必定在 R(S)中，最多有 k|R(S)|个这样的节点。k|S|=|E|≤k|R(S)|, 则有|S|≤|R|.由 Hall 婚姻定理，存在 A,B,R 的完全匹配。

15. Let S be any subset of A and E the set of edges that begin in S. Then $k|S| \le |E|$. Each edge in E must terminate in a node of $R(S)$. There are at most $j|R(S)|$ such nodes. Since $j \le k$, $j|S| \le k|S| \le j|R(S)|$ and $|S| \le |R(S)|$. By Hall's Marriage theorem, there is a complete matching for A, B, and R.

16.因为 A,B 和 R 的网络中，每一个节点的度数至少为 n/2,则图中存在一条哈密顿回路，因为每一条边的两点必定属于 A,B 两个不同的集合，所以 A 中的每个点都至少有一条边与 B 中的某点相连，所以存在一个完全匹配。

17. (a) No, vertices 1, 2, and 6 must be in different subsets, but there are only two sets in the partition.

(b) Yes, {{1, 3, 5, 7}, {2, 4, 6, 8}}.

（a）答案有误，可分为{{1,3,5}, {2,4,6}}

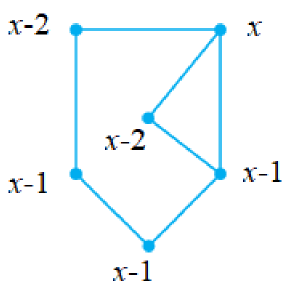
18.用 0,1 标记每个顶点，设一个顶点有 k 个 1，n-k 个 0，则它与只有一位不同的点相连，所以将有 0 个 1,2 个 1……n 个 1/n-1 个 1 的点的集合划为一部分，剩下的点划为一部分，两部分的集合大小相等，且集合内部无边，所以该图是哈塞图。

19. A triangle is formed by edges (u, v), (v, w), and (w, u). If u and v are placed in different subsets by a two-set partition, neither subset can contain w.

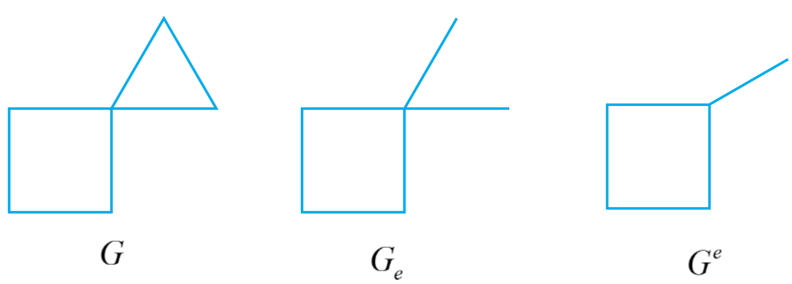
8.6

15. $P_G(x) = x(x-1)(x-2)(x-3)$; $\chi(G) = 4$.

16. $P_G(x) = x(x-1)^3(x-2)^2$; $\chi(G) = 3$



19. $P_G(x) = x(x-1)^3(x-2)^2; \chi(G) = 3$



$$P_G(x) = P_{G_e}(x) - P_{G^e}(x) = x(x-1)^4(x-2) - x(x-1)^3(x-2) = x(x-1)^3(x-2)^2$$

23. (Outline) Basis step: $n = 1$ P(1): $P_{L_1}(x) = x$ is true, because L_1 consists of a single vertex.
Induction step: We use P(k) to show P($k + 1$). Let $G = L_{k+1}$ and e be an edge $\{u, v\}$ with $\deg(v) = 1$. Then G_e has two components, L_k and v. Using Theorem 1 and P(k), we have $P_{G_e}(x) = x \cdot x(x-1)^{k-1}$. Merging v with u gives $G^e = L_k$. Thus $P_{G^e}(x) = x(x-1)^{k-1}$. By Theorem 2, $P_{L_{k+1}}(x) = x^2(x-1)^{k-1} - x(x-1)^{k-1} = x(x-1)^{k-1}(x-1)$ or $x(x-1)^k$.

26.

(a) 边连接的两门课程考试时间冲突（存在两门都要考的学生），不能安排在同一时间段
(b) 考试时间段
(c) 至少安排几个时间段考试使得所有学生能顺利参加全部的考试。

27. Label the vertices with the fish species. An edge connects two vertices if one species eats the other. The colors represent the tanks, so $\chi(G)$ will be the minimum number of tanks needed.