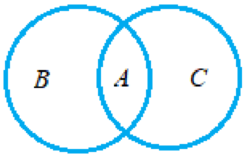
1.1

5. (a) False. (b) True. (c) False. (d) True. (e) False. (f) False.

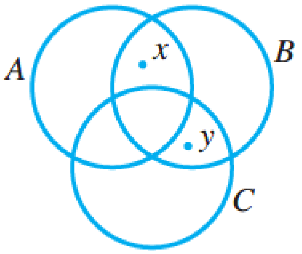
10. (a), (e)=A

16. A={0,±1,±2,±3} $\therefore a, b, c, d, e$ are True.

30.



31.



34.

$\because$ A $\subseteq$ B

$\therefore \forall\ x \in A,\ x \in B$

as well , $\because$ B $\subseteq C$

$\therefore \forall\ x \in B,\ x \in C$

$\therefore \forall\ x \in A,\ x \in C$

$\therefore A \in C$

36.

The number of subsets of A $= 2^{|A|} = n$

$\therefore$ The number of subsets of B $= 2^{|B|} = 2^{|A|+1} = 2n$

1.2

1.

(a) $\{a, b, c, d, e, f, g\}$. **(b)** $\{a, c, d, e, f, g\}$.

(c) $\{a, c\}$. **(d)** $\{f\}$.

(e) $\{b, g, d, e\}$. **(f)** $\{a, b, c\}$.

(g) $\{d, e, f, h, k\}$. **(h)** $\{a, b, c, d, e, f\}$.

(i) $\{b, g, f\}$. **(j)** $\{g\}$.

6.

$C = \{1, 2, 3, 4\}$

(a)$\{1, 6, 8\}$

(b){5,9}

(c){1,2,3,4}

(d){5,6,7,8,9}

(e){3,5,7,9}

(f){1,5,6,8,9}

(g){1,2,3,4,7,8}

(h){1,3,5,9}

10.

(a){b,d,h}

(b){a,b,c,d,e,f,g,h}

(c){b,d,e,h}

(d){a,c,d,e,f,g}

(e){c,e,f,g}

(f){a,b,e,h}

23.

$|A| = 6$, $|B| = 5$, $|C| = 6$, $|A \cap B| = 2$, $|A \cap C| = 3$, $|B \cap C| = 3$, $|A \cap B \cap C| = 2$, $|A \cup B \cup C| = 11$. Hence $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

24.

A={1,2,3,4,5,6,7},|A|=7

B={2,3,4},|B|=3

C=$\{0, \pm 1, \pm 2, \pm 3\}$,|C|=7

$|A \cap B| = 3, |A \cap C| = 3, |B \cap C| = 2, |A \cap B \cap C| = 2, |A \cup B \cup C| = 11.$

Hence $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$.

34.

True: None;

False: None;

Not possible to identify: (a),(b),(c),(d),(e),(f)

36.

True: (a),(e),(f)

False: (d),

Not possible to identify: (b),(c)

39.

A and to B

1.3

19.

$e_n = e_{n-1} + 3,\ e_1 = 1,$ recursive

$e_n = 3n - 2,\ 1 \le n \le 6,$ explicit

20.

$a_n = \frac{1}{2} a_{n-1},\ a_1 = 1,$ recursive

$a_n = \frac{1}{2^{n-1}},\ 1 \le n,$ explicit

22.

$a_1 = 2,\ a_2 = 5,\ a_n = a_{n-1} + a_{n-2},\ 3 \le n$

25.

(a) Yes. **(b)** No. **(c)** Yes.

(d) Yes. **(e)** No. **(f)** No.

29.

$$\begin{aligned} f_{(A\oplus B)\oplus C} &= f_{A\oplus B} + f_C - 2 f_{A\oplus B} f_C \quad \text{by Theorem 4} \\ &= (f_A + f_B - 2 f_A f_B) \\ &\quad + f_C - 2(f_A + f_B - 2 f_A f_B) f_C \\ &= f_A + (f_B + f_C - 2 f_B f_C) \\ &\quad - 2 f_A (f_B + f_C - 2 f_B f_C) \\ &= f_A + f_{B\oplus C} - 2 f_A f_{B\oplus C} \\ &= f_{A\oplus(B\oplus C)} \end{aligned}$$

Since the characteristic functions are the same, the sets must be the same.

32.

(a) No (b) No (c) Yes

34.

(a)$\{pr, qr, prq, qrq, prqq, qrqq, \ldots\}$

(b)$\{pr, pqqr, pqqqqr, pqqqqqqr, \ldots\}$

37.

By (1), 8 is an S-number. By (3), 1 is an S-number. By (2), all multiples of 1, that is, all integers are S-numbers.

1.4

24.

If $a \mid b,$ then $b = ka,$ for some $k \in \mathbb{Z}$.

Thus, $mb = m(ka) = (mk)a$ and mb is a multiple of a.

$\therefore a \mid mb$

Similarly, if $a \mid c,$ then $a \mid nc$

By Theorem 2, $a \mid mb + nc$, for any $m, n \in \mathbb{Z}$.

25.

The only divisors of p are $\pm p$ and ± 1, but p does not divide a. (Multiple both sides by b) $p \mid sa \cdot b \Leftrightarrow p \mid sab$ and $p \mid tb \cdot p \Leftrightarrow p \mid tpb$. If p divides the right side of the equation, then it must divide the left side also.

26.

If $\text{GCD}(a,c) = 1,$ there are intergers s, t suth that $1 = sa + tc.$

$\therefore b = sab + tcb$

$\because c \mid ab \therefore c \mid sab$

as well $c \mid tbc$

$\therefore c \mid sab + tbc \Rightarrow c \mid sab + tcb \Rightarrow c \mid b$

27.

$\because a \mid m$

$\therefore ac \mid mc$

$\because c \mid m$

$\therefore ac \mid am$

If $\text{GCD}(a,c) = 1,$ there are intergers s, t suth that $1 = sa + tc$

Thus $m = sam + tcm.$

$\because ac \mid am \therefore ac \mid sam$

$\because ac \mid mc \therefore ac \mid tcm$

$\therefore ac \mid sam + tcm \Rightarrow ac \mid m$

28.

If $d = \mathrm{GCD}(a,b)$

then there are integers s, t

such that $d = sa + tb \Rightarrow bd = sab + tbb$

$\because c \mid b$

$\therefore b = qc$, q is integer

$\therefore bd = sab + tbqc = sab + tqbc$

$\because a \mid b$

$\therefore ac \mid bc \Rightarrow ac \mid tqbc$

$\because c \mid b$

$\therefore ac \mid ab \Rightarrow ac \mid sab$

$\therefore ac \mid sab + tqbc$

$\therefore ac \mid bd$

29.

Let $d = \mathrm{GCD}(a,b)$

Then $cd \mid ca,\ cd \mid cb$

$\Rightarrow cd$ is a common divisor of ca and cb ——(1)

Let $e = \mathrm{GCD}(ca, cb)$ ——(2)

$(1)(2) \Rightarrow cd \mid e \Rightarrow e = kcd$ ——(3)

$(2) \Rightarrow e \mid ca,\ e \mid cb$ ——(4)

$(3)(4) \Rightarrow kcd \mid ca,\ kcd \mid cb \Rightarrow kd \mid a,\ kd \mid b$

$\because d = \mathrm{GCD}(a,b)$

$\therefore k = 1 \Rightarrow e = cd \Rightarrow \mathrm{GCD}(ca, cb) = c\mathrm{GCD}(a,b)$

1.5

6.

(a)

A(BD) = (AB)D

$$\mathbf{AB} = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 13 \\ -3 & 0 \end{bmatrix}$$

$$\mathbf{(AB)D} = \begin{bmatrix} 7 & 13 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 31 & 27 \\ 9 & -6 \end{bmatrix}$$

(b)

A(C + E) = AC + AE

$$\mathbf{C}+\mathbf{E}=\begin{bmatrix}4&0&2\\9&6&2\\3&2&4\end{bmatrix}\quad \mathbf{A}(\mathbf{C}+\mathbf{E})=\begin{bmatrix}2&1&3\\4&1&-2\end{bmatrix}\begin{bmatrix}4&0&2\\9&6&2\\3&2&4\end{bmatrix}=\begin{bmatrix}26&12&18\\19&2&2\end{bmatrix}$$

(c)

$$\mathbf{FD}=\begin{bmatrix}-2&3\\4&5\end{bmatrix}\begin{bmatrix}-3&2\\4&1\end{bmatrix}=\begin{bmatrix}18&-1\\8&13\end{bmatrix}\quad \mathbf{AB}=\begin{bmatrix}7&13\\-3&0\end{bmatrix}$$

$$\mathbf{FD}+\mathbf{AB}=\begin{bmatrix}25&12\\5&13\end{bmatrix}$$

9.

(a) $\begin{bmatrix}22&34\\3&11\\-31&3\end{bmatrix}$. **(b)** **BC** is not defined.

(c) $\begin{bmatrix}25&5&26\\20&-3&32\end{bmatrix}$.

(d) $\mathbf{D}^T+\mathbf{E}$ is not defined.

16.

(a)

Let $\mathbf{A}=\left[a_{ij}\right]$ is an $m\times p$ matrix,

A has a row of zeros: the k^{th} row

$\Rightarrow a_{kj}=0,\ k$ is an integer, $1\le k\le m,\ 1\le j\le p$

$\mathbf{B}=\left[b_{ij}\right]$ is an $p\times n$ matrix,

$\mathbf{C}=\mathbf{AB}=\left[c_{ij}\right]$ is an $m\times n$ matrix,

By the definition of **AB**,

$$c_{ij}=a_{i1}b_{1j}+a_{i2}b_{2j}+\cdots+a_{ip}b_{pj}\Rightarrow c_{kj}=a_{k1}b_{1j}+a_{k2}b_{2j}+\cdots+a_{kp}b_{pj}=0$$

$\therefore$ **AB** has a corresponding row of zeros.

(b)

Let $\mathbf{A}=\left[a_{ij}\right]$ is an $m\times p$ matrix,

$\mathbf{B}=\left[b_{ij}\right]$ is an $p\times n$ matrix,

B has a column of zeros: the k^{th} column

$\Rightarrow b_{ik}=0,\ k$ is an integer, $1\le k\le n,\ 1\le i\le p$

$\mathbf{C}=\mathbf{AB}=\left[c_{ij}\right]$ is an $m\times n$ matrix,

By the definition of **AB**,

$$c_{ij}=a_{i1}b_{1j}+a_{i2}b_{2j}+\cdots+a_{ip}b_{pj}\Rightarrow c_{ik}=a_{i1}b_{1k}+a_{i2}b_{2k}+\cdots+a_{ip}b_{pk}=0$$

$\therefore$ **AB** has a corresponding column of zeros.

17.

The jth column of $\mathbf{AB}$ has entries $c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$. Let $\mathbf{D} = \left[\, d_{ij} \,\right] = \mathbf{AB}_j$, where $\mathbf{B}_j$ is the jth column of $\mathbf{B}$. Then $d_{ij} = \sum_{m=1}^{n} a_{im}b_{mj} = c_{ij}$.

23.

(a) The i, jth element of $(\mathbf{A}^T)^T$ is the j, ith element of $\mathbf{A}^T$. But the j, ith element of $\mathbf{A}^T$ is the i, jth element of $\mathbf{A}$. Thus $(\mathbf{A}^T)^T = \mathbf{A}$.

(b) The i, jth element of $(\mathbf{A} + \mathbf{B})^T$ is the j, ith element of $\mathbf{A} + \mathbf{B}$, $a_{ji} + b_{ji}$. But this is the sum of the j, ith entry of $\mathbf{A}$ and the j, ith entry of $\mathbf{B}$. It is also the sum of the i, jth entry of $\mathbf{A}^T$ and the i, jth entry of $\mathbf{B}^T$. Thus $(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$.

(c) Let $\mathbf{C} = \left[\, c_{ij} \,\right] = (\mathbf{AB})^T$. Then $c_{ij} = \sum_{k=1}^{n} a_{jk}b_{ki}$, the j, ith entry of $\mathbf{AB}$. Let $\mathbf{D} = \left[\, d_{ij} \,\right] = \mathbf{B}^T\mathbf{A}^T$, then

$$d_{ij} = \sum_{k=1}^{n} b'_{ik}a'_{kj} = \sum_{k=1}^{n} b_{ki}a_{jk} = \sum_{k=1}^{n} a_{jk}b_{ki} = c_{ij}.$$

Hence $(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$.

29.

Since $(\mathbf{B}^{-1}\mathbf{A}^{-1})(\mathbf{AB}) = \mathbf{B}^{-1}(\mathbf{A}^{-1}\mathbf{A})\mathbf{B} = \mathbf{B}^{-1}\mathbf{I}_n\mathbf{B} = \mathbf{B}^{-1}\mathbf{B} = \mathbf{I}_n$, $(\mathbf{B}^{-1}\mathbf{A}^{-1}) = (\mathbf{AB})^{-1}$.

41.

Let $\left[\, d_{ij} \,\right] = \mathbf{B} \vee \mathbf{C}$, $\left[\, e_{ij} \,\right] = \mathbf{A} \vee (\mathbf{B} \vee \mathbf{C})$, $\left[\, f_{ij} \,\right] = \mathbf{A} \vee \mathbf{B}$, and $\left[\, g_{ij} \,\right] = (\mathbf{A} \vee \mathbf{B}) \vee \mathbf{C}$. Then

$$d_{ij} = \begin{cases} 1 & \text{if } b_{ij} = 1 \text{ or } c_{ij} = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$e_{ij} = \begin{cases} 1 & \text{if } a_{ij} = 1 \text{ or } d_{ij} = 1 \\ 0 & \text{otherwise.} \end{cases}$$

But this means $e_{ij} = 1$ if $a_{ij} = 1$ or $b_{ij} = 1$ or $c_{ij} = 1$ and $e_{ij} = 0$ otherwise.

$$f_{ij} = \begin{cases} 1 & \text{if } a_{ij} = 1 \text{ or } b_{ij} = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$g_{ij} = \begin{cases} 1 & \text{if } f_{ij} = 1 \text{ or } c_{ij} = 1 \\ 0 & \text{otherwise.} \end{cases}$$

But this means $g_{ij} = 1$ if $a_{ij} = 1$ or $b_{ij} = 1$ or $c_{ij} = 1$ and $g_{ij} = 0$ otherwise. Hence $\mathbf{A} \vee (\mathbf{B} \vee \mathbf{C}) = (\mathbf{A} \vee \mathbf{B}) \vee \mathbf{C}$.

1.6

24. Yes
25. Yes
26. No
27. No
28. No
29.

(a) Yes. **(b)** Yes. **(c)** Yes.

35. Yes

Let $\mathbf{C} = \left[\, c_{ij} \,\right] = \text{comp}(\mathbf{A} \vee \mathbf{B})$ and $\mathbf{D} = \left[\, d_{ij} \,\right] = \text{comp}(\mathbf{A}) \wedge \text{comp}(\mathbf{B})$. Then

$$c_{ij} = \begin{cases} 0 & \text{if } a_{ij} = 1 \text{ or } b_{ij} = 1 \\ 1 & \text{if } a_{ij} = 0 = b_{ij} \end{cases}$$

and

$$d_{ij} = \begin{cases} 0 & \text{if } a_{ij} = 1 \text{ or } b_{ij} = 1 \\ 1 & \text{if } a_{ij} = 0 = b_{ij}. \end{cases}$$

Hence, $\mathbf{C} = \mathbf{D}$. Similarly, we can show that $\text{comp}(\mathbf{A} \wedge \mathbf{B}) = \text{comp}(\mathbf{A}) \vee \text{comp}(\mathbf{B})$.