4.6

2.

$\text{EDGE}(I, J)$

$\text{RESULT} \leftarrow F$

$Y \leftarrow VERT[I]$

$WHILE(Y \neq 0)$

$\quad IF(HEAD(X) == J) \;\; THEN$

$\quad\quad \text{RESULT} \leftarrow T$

$\quad \text{ELSE}$

$\quad\quad \text{X} \leftarrow \text{NEXT(X)}$

3. On average, EDGE must look at the average number of edges from any vertex. If R has P edges and N vertices, then EDGE examines $\dfrac{\sum P_{ij}}{N} = \dfrac{P}{N}$ edges on average.

4.

$LOOK(NUM, NEXT, START, N, K)$

$\text{X} \leftarrow \text{START}$

$\text{WHILE(X} \neq 0)$

$\quad \text{IF(K==NUM(X)) THEN}$

$\quad\quad RETUEN \;\; \text{X}$

$\quad \text{ELSE}$

$\quad\quad \text{X} \leftarrow \text{NEXT(X)}$

PRINT("NOT FOUND")

6.

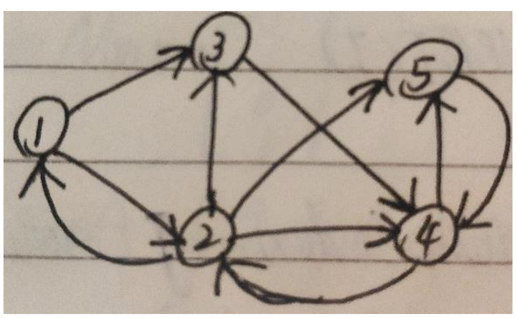
一种解法:

VERT	TAIL	HEAD	NEXT
2	4	1	9

4	1	1	3
7	1	2	5
1	2	1	6
	1	3	0
	2	4	8
	3	4	10
	2	3	0
	4	3	0
	3	3	0

8.

$$M_R=\begin{pmatrix}0&1&1&0&0\\1&0&1&1&1\\0&0&0&1&0\\0&1&0&0&1\\0&0&0&1&0\end{pmatrix}$$



12.

一种解法:

VERT	TAIL	HEAD	NEXT
1	F	F	2
4	F	M	3

7	F	W	0
11	M	M	5
	M	F	6
	M	W	0
	R	F	8
	R	M	9
	R	R	10
	R	W	0
	W	F	12
	W	M	13
	W	W	0

4.7

7. (a) $\{(2,1), (3,1), (3,2), (3,3), (4,2), (4,3), (4,4), (1,4)\}$.

(b) $\{(1,1), (1,2), (2,2), (2,3), (2,4), (4,1), (3,4)\}$.

(c) $\{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1), (4,4)\}$.

(d) $\{(1,1), (2,1), (2,2), (1,4), (4,1), (2,3), (3,2), (1,3), (4,2), (3,4), (4,4)\}$.

8.

(a) {(a,c),(a,e) , (b,a),(b,c) , (b,d),(c,a) , (c,c),(c,d) , (c,e),(d,a) , (d,b),(d,e) , (e,c),(e,d) , (e,e)}

(b) {(a,a), (a,d),(c,b) , (e,a),(e,b)}

(c){(a,a),(a,b),(a,d) ,(a,e),(b,a),(b,b),(b,e),(c,b),(d,c),(d,d),(e,a),(e,b),(e,d),(e,e)}

(d) {(a,a), (e,a),(d,a) , (a,b),(b,c),(d,c),(e,e),(a,e),(b,e),(d,e)}

12.

$M_{R\cap S}=\begin{pmatrix}0&0&0\\0&0&1\\0&0&0\\0&1&0\end{pmatrix}$ $M_{R\cup S}=\begin{pmatrix}1&1&1\\1&1&1\\0&1&1\\1&1&1\end{pmatrix}$ $M_{R^{-1}}=\begin{pmatrix}0&0&0&1\\1&1&0&1\\0&1&1&1\end{pmatrix}$ $M_{\bar{S}}=\begin{pmatrix}0&1&0\\0&1&0\\1&0&1\\1&0&1\end{pmatrix}$

14.

R∩S={(1,1),(1,2),(2,1),(2,2),(3,3),(4,4),(5,5),(6,6)}

{{1,2},{3},{4},{5},{6}}

19. The definitions of irreflexive, asymmetric, and antisymmetric each require that a certain pair does not belong to R. We cannot “fix” this by including more pairs in R.

20.(a)成立，(b)不成立

23. **(a)** Reflexive. $a\ R\ a \wedge a\ S\ a \Rightarrow a\ S \circ R\ a$.
Irreflexive. No. $1\ R\ 2 \wedge 2\ S\ 1 \Rightarrow 1\ S \circ R\ 1$.
Symmetric. No. $1\ R\ 3, 3\ R\ 1, 3\ S\ 2, 2\ S\ 3 \Rightarrow$ $1\ S \circ R\ 2$, but $2 \not{S \circ R}\ 1$.
Asymmetric. No. $R = \{(1,2),(3,4)\}$ and $S = \{(2,3),(4,1)\}$ provide a counterexample.
Antisymmetric. No. $R = \{(a,b),(c,d)\}$ and $S = \{(b,c),(d,a)\}$ provide a counterexample.
Transitive. No. $R = \{(a,d),(b,e)\}$ and $S = \{(d,b),(e,c)\}$ provide a counterexample.

(b) No, symmetric and transitive properties are not preserved.

24.

(a) $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{pmatrix}$

26.

If a(R∩S)b, then a R b, since R is asymmetric， there is no b R a. Then there is no b(R∩S)a, so R∩S is asymmetric.

Let R={(1,2)}, S={(2,1)} ， so R∪S={(1,2),(2,1)} ， as we can see, R∪S is not asymmetric.

27. $R \cap S$ is antisymmetric. If $a(R \cap S)b$ and $b(R \cap S)a$, then $a\ R\ b$ and $b\ R\ a$. Hence $a = b$ because R is antisymmetric. $R \cup S$ may not be antisymmetric. Let $R = \{(1, 2)\}$, $S = \{(2, 1)\}$.

28.

Let a(S ∪ T) ◦R c, then there is a(S ∪ T)b and b R c. Then a S b or a T b, so we have a S◦R c or a T◦R c. Then a (S◦R) ∪ (T◦R) c. Hence, *(S ∪ T) ◦ R = (S ◦ R) ∪ (T ◦ R)*

30.

Let a T ◦R c , so a R b and b T c. As R *⊆S, we have a S b, then* a T ◦S c. So if R ⊆ S, then T ◦ R ⊆ T ◦S.

31. (a) Let $\mathbf{M}_{R\cap S} = \left[\, m_{ij} \,\right]$, $\mathbf{M}_R = \left[\, r_{ij} \,\right]$, $\mathbf{M}_S = \left[\, s_{ij} \,\right]$. $m_{ij} = 1$ if and only if $(i, j) \in R \cap S$. $(i, j) \in R$ if and only if $r_{ij} = 1$ and $(i, j) \in S$ if and only if $s_{ij} = 1$. But this happens if and only if the i, jth entry of $\mathbf{M}_R \wedge \mathbf{M}_S$ is 1.

(b) Let $\mathbf{M}_{R\cup S} = \left[\, m_{ij} \,\right]$, $\mathbf{M}_R = \left[\, r_{ij} \,\right]$, $\mathbf{M}_S = \left[\, s_{ij} \,\right]$. $m_{ij} = 1$ if and only if $(i, j) \in R \cup S$. $(i, j) \in R$ if and only if $r_{ij} = 1$ or $(i, j) \in S$ if and only if $s_{ij} = 1$. But this happens if and only if the i, jth entry of $\mathbf{M}_R \vee \mathbf{M}_S$ is 1.

(c) The i, jth entry of $\mathbf{M}_{R^{-1}}$ is 1 if and only if $(i, j) \in R^{-1}$ if and only if $(j, i) \in R$ if and only if the j, ith entry of $\mathbf{M}_R$ is 1 if and only if the i, jth entry of $\mathbf{M}_R^T$ is 1.

(d) The i, jth entry of $\mathbf{M}_{\overline{R}}$ is 1 if and only if $(i, j) \in \overline{R}$ if and only if $(i, j) \notin R$ if and only if the i, jth entry of $\mathbf{M}_R$ is 0 if and only if the i, jth entry of $\overline{\mathbf{M}_R}$ is 1.

36.

If a R – S b, then a R b and there is no a S b. As R *and* S *are symmetric relations, so* b R a and there is no b S a.Then b R – S a . Hence R - S *is also a symmetric relation on* A.

37. **(a)** R is symmetric if and only if $x\ R\ y \Rightarrow y\ R\ x$ if and only if $R \subseteq R^{-1} \subseteq R$.

(b) Suppose R is antisymmetric. Let $(x, y) \in R \cap R^{-1}$, then $x = y$ and $(x, y) \in \Delta$. Suppose $R \cap R^{-1} \subseteq \Delta$. If $x\ R\ y$ and $y\ R\ x$, then $x\ R\ y$ and $x\ R^{-1}\ y$. Thus $(x, y) \in \Delta$, so $x = y$.

(c) Suppose R is asymmetric. Let $(x, y) \in R \cap R^{-1}$. This contradicts the fact that R is asymmetric. Hence $R \cap R^{-1} = \varnothing$. Let $x\ R\ y$ and $y\ R\ x$. Then $(x, y) \in R \cap R^{-1} = \varnothing$. Hence R is asymmetric.

4.8

8.

Since $S^{\infty} = S \cup S^2 \cup \cdots \cup S^n$, $S = R^2$, hence, $S^{\infty} = R^2 \cup R^4 \cup \cdots \cup R^{2n}$

Therefore, $aS^{\infty}b \Leftrightarrow a(R^2 \cup R^4 \cup \cdots \cup R^{2n})b \Leftrightarrow aR^2b \vee aR^4b \vee \cdots \vee aR^{2n}b \Leftrightarrow$ there is a path in R from a to b having an even number of edges.

10.

$n = 4$

Let $W_0 = M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, then $W_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $W_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$,

$$W_3 = W_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \text{ therefore, } M_{R^\infty} = W_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

12.

$n = 4$

Let $W_0 = M_R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, then $W_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $W_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$,

$$W_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \text{ therefore, } M_{R^\infty} = W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

14.

Disproof.

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$M_{R_t} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \quad M_{s(R_t)} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$$M_{R_s} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad M_{t(R_s)} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Therefore, $M_{s(R_t)} \neq M_{t(R_s)}$, the symmetric closure of R_t is not the same relation

as the transitive closure of R_s.

18.

$$M_{(R\cup S)^\infty} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$A/R = \{\{1\},\{2,3\},\{4,5\}\}$, $A/S = \{\{1,2\},\{3\},\{4\},\{5\}\}$, $A/(R\cup S)^\infty = \{\{1,2,3\},\{4,5\}\}$.

20.

19 题中的过程仅适用于R和S均为等价关系，且计算$(R\cup S)^\infty$过程中需要用到 Warshall 算法，所以无法替代 Warshall 算法。

23.

We first show R^∞ is transitive. Then we show it is the smallest relation that contains R. It is a direct proof.

24.

首先证明若从a到b之间存在环，则存在更短的路径；再证明a到b之间，若a, b不同，则最长路径长度为n-1，若a, b相同，则最长路径长度为n；因此，$\forall k \in Z$, $1 \le k \le n$，$aR^k b$，因此$R^\infty = R \cup R^2 \cup R^n$。间接证明。

25.

$\mathrm{R} \times \mathrm{R}$.

5.1

5. Each integer has a unique square that is also an integer.

6. For each $a \in \mathrm{R}$, $e^a \in \mathrm{R}$, and for each $a \in \mathrm{R}$, there is a unique e^a.

7. Each $r \in \mathrm{R}$ is either an integer or it is not.

8. For each $a \in \mathbb{R}$, there is a unique integer which is the greatest integer less than or equal to a.

11. (a) Both. (b) Neither.

12. (a) One to one. (b) Onto.

13. (a) Both. (b) Onto.

14. (a) Onto. (b)Neither.

15. (a) Both. (b) Onto.

29. n^m**.**

30.

If $f: A \to B$ and $g: B \to C$ are one-to-one functions, then $g \circ f$ is one to one.

Proof: Let $a_1, a_2 \in A$. Suppose $(g \circ f)(a_1) = (g \circ f)(a_2)$. Then $g(f(a_1)) = g(f(a_2))$ and $f(a_1) = f(a_2)$, because $(g \circ f)(a_1) = g(f(a_1))$, $(g \circ f)(a_2) = g(f(a_2))$ and g is one-to-one functions. Thus a1 = a2, because f is one-to-one functions. Hence $g \circ f$ is one to one.

31.

If $f: A \to B$ and $g: B \to C$ are onto functions, then $g \circ f$ is onto.

Proof: Choose $x \in C$. Then there exists $y \in B$ such that $g(y) = x$. (Why? because g is onto) Then there exists $z \in A$ such that $f(z) = y$ (why? because f is onto) and $(g \circ f)(z) = x$. Hence, $g \circ f$ is onto.

33.

Suppose $g \circ f$ is onto. Let $c \in C$. Then $\exists a \in A$ such that $(g \circ f)(a) = c$. But $(g \circ f)(a) = g(f(a))$, $f(a) \in B$, so g is onto.

34.

Use mathematical induction to prove, $\forall k \in Z$, f^k is bijection.

Therefore, if $a_1, a_2 \in A$,

$$O(a_1, f) = \{ f^0(a_1), f^1(a_1), f^{-1}(a_1), f^2(a_1), f^{-2}(a_1), \cdots \},$$

$$O(a_2, f) = \{ f^0(a_2), f^1(a_2), f^{-1}(a_2), f^2(a_2), f^{-2}(a_2), \cdots \}$$

If $a_1 \neq a_2$, $\forall k \in Z$, $f^k(a_1) \neq f^k(a_2)$, then $O(a_1, f) \cap O(a_2, f) = \varnothing$.

Therefore, if $O(a_1, f) \cap O(a_2, f) \neq \varnothing$, then $a_1 = a_2$, then $O(a_1, f) = O(a_2, f)$.

40.

(a) Disproof.

$$f(a_1 + a_2) = (a_1 + a_2)^2, \ f(a_1) + f(a_2) = a_1^2 + a_2^2$$

If $a_1, a_2 \neq 0$, then $(a_1 + a_2)^2 \neq a_1^2 + a_2^2$, then $f(a_1 + a_2) \neq f(a_1) + f(a_2)$.

(b) Proof.

The length of $s_1 \cdot s_2$ is equal to the length of s_1 plus the length of s_2.

Therefore, $f(s_1 \cdot s_2) = f(s_1) + f(s_2)$.

41. Disproof.

(a) and **(b)**. Consider the table for $\diamond$.

$\diamond$	0	1
0	0	1
1	1	0

Since $f(0) = \mathit{true}$ and $f(1) = \mathit{false}$, we see this is not the table for either $\vee$ or $\wedge$.

5.2

7.

$n = ak + r, 0 \le r < n$. Since $k < 2k < 3k < \cdots < ak \le n$ the number of multiples of k between 1 and n is a. But $\frac{n}{k} = a + \frac{r}{n}$ with $0 \le \frac{r}{n} < 1$ so $\lfloor \frac{n}{k} \rfloor = a$.

8.

设 n=2k+1,(k∈Z) $\frac{n^2}{4} = \frac{4k^2+4k+1}{4} = k^2$+k+$\frac{1}{4}$,向上取整为$k^2$+k+1。

$\frac{n^2+3}{4} = \frac{4k^2+4k+4}{4} = k^2$+k+1

所以$\lceil \frac{n^2}{4} \rceil = \frac{n^2+3}{4}$

18. (例 7.(a)中 l : A^* -> Z 为 l(w) , 它是字符串 w 的长度)

(a) 由于A^*中元素均为字符串，字符串必有唯一长度，所以 l 是处处有定义的。

(b) ‘ab’∈A^*，‘ba’∈A^*，l(‘ab’) =2 ,l(‘ba’) =2,所以 l 不是单射。

(c) l(w)为字符串 w 的长度，l(w)>=0，但 l(w)∈Z，所以 l 不是满射.

20.

存在 2 个不同的关于 p 的布尔函数,f(p) = p,g(p) = ！ p

存在 4 个有两个布尔变量的布尔函数 ,设这两个布尔变量为 p,q

f1(p，q) = p&q ，f2(p，q) = p&!q，f3(p，q) = !p&q，f1(p，q) = !p&!q

28.

对任意 S 为 A 的子集, 令 f(x)=1, x∈S, 0, x∈A\S；这样对每个元素 x∈A, f(x)有两个取值, 0 或 1；因此, 根据乘法原理, 这样的 f 有 2^n 个, 每个 f 唯一的对应 A 的一个子集, 因此|pow(A)|=2^n

29.

$f^{-1}(1)$ is the set of elements of A.

5.3

11.

$\{f_5\}$, $\{f_6, f_{10}, f_{11}\}$, $\{f_7\}$, $\{f_4\}$, $\{f_8\}$, $\{f_1\}$, $\{f_2\}$, $\{f_3\}$, $\{f_9\}$, $\{f_{12}\}$.

12.

{f5},{f7},{f4},{f8},{f6,f10,f11},{f12},{f1},{f2},{f3},{f9}

13.

f_1, $\Theta(n \lg n)$; f_2, $\Theta(n^2)$; f_4, $\Theta(\lg n)$; f_5, $\Theta(1)$; f_6, $\Theta(n)$; f_{10}, $\Theta(n)$; f_{11}, $\Theta(n)$.

20.

Θ (n)

21.

$f(n, m, q) = 1 + nq + 3nmq + 1$. Let $N = \max(n, m, q)$, then f is $\Theta(N^3)$.

22.

f(n+2) = 2*f(n) + f(n+1)

f(0) = 0 ,f(1) = 1

得 f(n) = $2^n/6+(-1)^n/3$

f(n)∈Θ（2^n）

伪代码：

1. f0 <- 0

2. f1<-1

3. **FOR** I=0 **THRU** N

    a. f2 = 2*f0+f1

    b. f0 = f1

    c. f1 = f2

4. **RETURN** f2

计算 F(N)所需要的运行时间为 O(n)

23.

(a) $P_n = P_{n-1} + (n-2) + (n-3)$, $P_3 = 1$, $P_4 = 4$.

(b) $\theta(n^2)$.

24.

若Θ(n^a)比 Θ(n^b)低，则存在 n ≥ k 和常数 c, $|n^a|$<c$|n^b|$ = c$|n^a|.|n^{b-a}|$

即 c$|n^{b-a}|>1$，c 为常数,所以 b>a>0

若 b>a>0，则存在则存在 $\mathrm{n} \geq 2$ 和常数 $c = 1, |n^a| < |n^b|$ = $|n^a|.|n^{b-a}|$

即Θ(n^a)比 Θ(n^b)低

所以Θ(n^a)比 Θ(n^b)低，当且仅当 b>a>0。

25.

若Θ(a^n)比 Θ(b^n)低，则存在 $n \geq k$ 和常数 c,| a^n|<c| b^n| ,即 c> $(a/b)^n$

则 0<a<b。

若 0<a<b，则存在 $n \geq 2$ 和常数 $c = 1$, | a^n|<| b^n|，所以Θ(a^n)比 Θ(b^n)低。

综上Θ(a^n)比 Θ(b^n)低，当且仅当 0<a<b

26.

设 g(x)= r f(x)

若 r>0 存在 n>=1，和常数 c=2r，使得 |g(x)| = |r f(x)|<=2r|f(x)|成立

同时存在 n>=1，和常数 c=2/r，使得 |f(x)| <=2/r*|rf(x)| = 2|f(x)|成立

若 r<0 存在 n>=1，和常数 c=-2r，使得 |g(x)| = |r f(x)|<=-2r|f(x)|成立

同时存在 n>=1，和常数 c=-2/r，使得 |f(x)| <=-2/r*|rf(x)| = 2|f(x)|成立

所以如果 r!=0,那么对于任意函数 f,有Θ(rf) = Θ(f)

27.

Suppose $h(n) > 0$, $\forall n$, $\Theta(f)$ lower than (or the same as) $\Theta(g)$. $|f(n)| \leq c \cdot |g(n)|$, $n \geq k$ (and $|g(n)| \leq d \cdot |f(n)|$, $n \geq l$). $h(n)|f(n)| \leq c \cdot h(n) \cdot |g(n)|$, $n \geq k$ (and $h(n)|g(n)| \leq d \cdot h(n) \cdot |f(n)|$, $n \geq l$). Hence $|f(n) \cdot h(n)| \leq c \cdot |g(n) \cdot h(n)|$, $n \geq k$ (and $|g(n) \cdot h(n)| \leq d \cdot |f(n) \cdot h(n)|$, $n \geq l$). Hence $\Theta(fh)$ is lower than (or the same as) $\Theta(gh)$. Note that if $\Theta(f)$ is strictly lower than $\Theta(g)$, then $\Theta(fh)$ must be strictly lower than $\Theta(gh)$.

28.

若Θ(f) = Θ(h)，则 f = O(h)，有 n>=k1,和常数 c1，使得|f(n)|<=c1|h(n)|

同样若Θ(g) = Θ(h)，则 g = O(h)，有 n>=k2,和常数 c2，使得|g(n)|<=c2|h(n)|

则存在 n>=max{k1,k2}和常数 c1+k2，使得|f(n)|+|g(n)|<=(c1+c2)|h(n)|

|f(n)+g(n)|<= |f(n)|+|g(n)|

则结论 如果Θ(f) = Θ(g) = Θ(h),那么 f+g 是 O(h)的 成立。

29.

26 题已证明定理 6，结合关系Θ是传递的, 结论得证。

5.4

12.

（a）(6,8)。(2,3)。(1,4,5)

（b）(1,2,3,4)。(5,7,8,6)

13.

（a）(1,6,3,7,2,5,4,8)

（b）(5,6,7,8)。(1,2,3)

14.

（a）(a,g,e,c,b,d)

（b）(a,d,b,e,g,c)

15.

(a) $(2,6)\circ(2,8)\circ(2,5)\circ(2,4)\circ(2,1)$.

(b) $(3,6)\circ(3,1)\circ(4,5)\circ(4,2)\circ(4,8)$.

16.

RAEEU YEO HRW

20.

(a)将置换写成不相交循环的积

(1,4,6,8,3) = (1,3)。(1,8)。(1,6)。(1,4) 为偶置换

(b)将置换写成不相交循环的积

(1,7,6,8,5)。(2,3,4) = (1,5)。(1,8)。(1,6)。(1,7)。(2,4)。(2,3) 为偶置换

26.

p 是集合 A 到自身的双射。

p2 = p。p ,也是集合 A 到自身的双射。

所以 p2 是置换。

28.

(a) (2,3,5)。(1,4)

(b)

1	2	3	4	5	6
4	5	2	1	3	6

(c)

1	2	3	4	5	6
1	5	2	4	3	6

(d) p 的周期为 2 x 3 = 6

29.

(a) Basis step: $n = 1$. If p is a permutation of a finite set A, then p^1 is a permutation of A is true.
Induction step: The argument in Exercise 26 also shows that if p^{n-1} is a permutation of A, then $p^{n-1} \circ p$ is a permutation of A. Hence p^n is a permutation of A.

(b) If $|A| = n$, then there are $n!$ permutations of A. Hence, the sequence $1_A, p, p^2, p^3, \ldots$ is finite and $p^i = p^j$ for some $i \neq j$. Suppose $i < j$. Then $p^{-i} \circ p^i = 1_A = p^{-i} \circ p^j$. So $p^{j-i} = 1_A$, $j - i \in \mathbb{Z}$.

30.

证自反性。对于 A 中任一元素 a,有$p^0(a) = a$，所以 a R a。自反性成立。

证对称性。若 a R b ,则$p^n(a) = b$，则$p^{-n}(b) = a$，b R a。对称性成立。

证传递性。若 a R b,b R c，则$p^{n1}(a) = b$，$p^{n2}(b) = c$，则$p^{n1} \circ p^{n2}(a) = c$，

即$p^{n1+n2}(a) = c$,即 a R c。传递性成立。

所以 R 是一个等价关系。

37.

(a) 3

(b) 6

38.

10

39.

. For each increasing sequence of length $\lceil \frac{n}{2} \rceil$, there is exactly one associated up-down permutation of A, because there is just one way to arrange the remaining elements of A in decreasing order and insert them to fill the even positions.