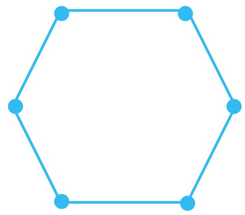
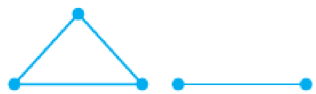
8.1

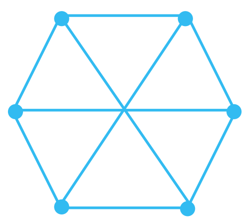
16.

One possible solution is 

17.

One possible solution is 

18.

One possible solution is 

27.

Two graphs, G_1 and G_2, are isomorphic if there is a one-to-one correspondence f between the vertices of G_1 and G_2 and (v_i, v_j) is an edge in G_1 if and only if $(f(v_i), f(v_j))$ is an edge in G_2.

28. Disproof. The graphs in Figures 8.24(a) and 8.24(b) are not isomorphic, because one has two vertices of degree 1, while the other has only one vertex of degree 1.

29. Disproof. The graphs in Figures 8.24(a) and 8.24(c) are not isomorphic, because one has a vertex of degree four and the other does not.

31.

In a digraph there are no multiple edges between vertices. In a graph, the edges are not directed.

32.

If a graph G has no loops or multiple edges, then each edge contributes 1 degree to a vertex degree to which it is joined, and each edge is joined by two vertices, so that the sum of all vertex degrees is twice the number of edges.

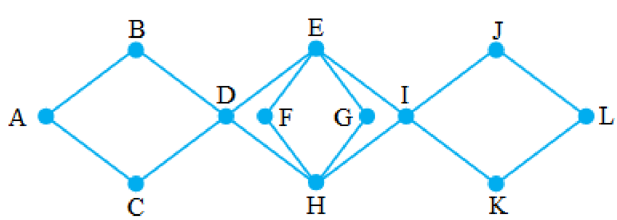
8.2

4. Euler path only, since exactly two vertices (4, 7) have odd degree.

6. Euler path only, since exactly two vertices have odd degree.

9. Yes, all vertices have even degree.

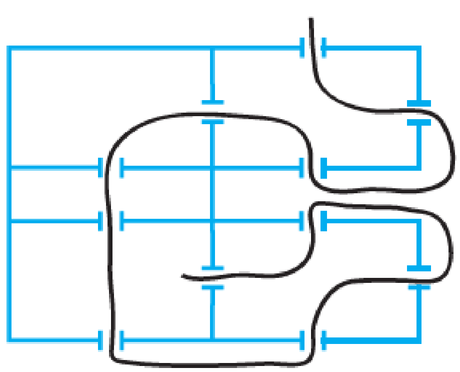
12.



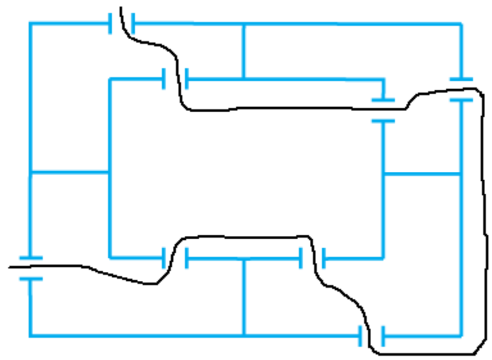
Euler circuit: $A, B, D, E, I, J, L, K, I, H, G, E, F, H, D, C, A$.
（过程略）

13.

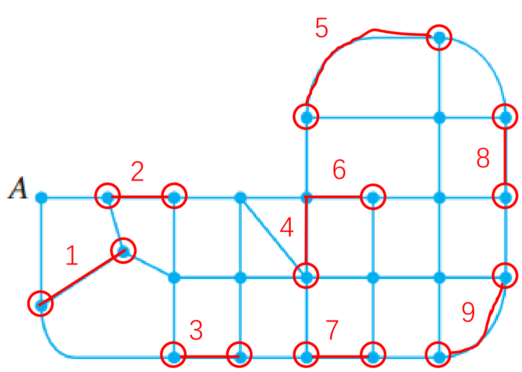
Yes. Note that if a circuit is required, it is not possible.



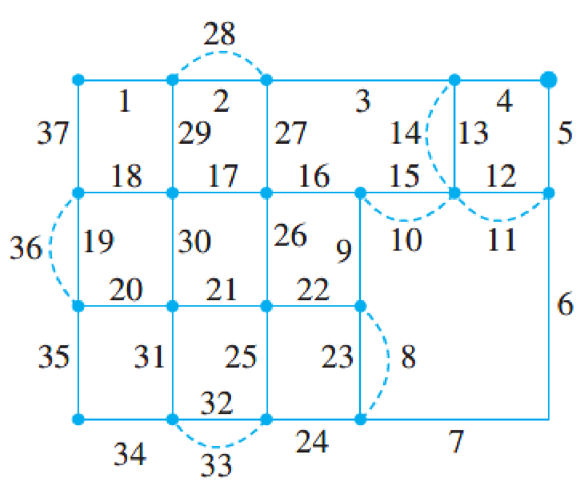
14. Yes.



18. Nine edges. There are 16 vertices have odd degree, and most of them are connected, except two of them, therefore the minimum number of edges that would need to be traveled twice is nine(14/2+2).



19. See the solution in the figure below， the consecutively numbered edges are one possible circuit.



20. See the solution for Exercise 18. (Euler circuit is omitted).

21.

If n is odd, each vertex in K_n has degree $n-1$, an even number. In this case, K_n has an Euler circuit. If n is even, then each vertex of K_n has odd degree; K_n does not have an Euler circuit.

22.

Proof.
If there is an Euler circuit in $G=(V_G,E,\gamma)$, then $\forall v_i \in V_G, d(v_i)=2k(k\in N_+)$, because G,H is a pair of isomorphic graphs, there exists a one-to-one correspondence f, apparently, $\forall f(v_i)\in V_H, d(f(v_i))=d(v_i)=2k(k\in N_+)$, therefore, there is an Euler circuit in H.

23.

Suppose the strings $a_1a_2\cdots a_n$ and $b_1b_2\cdots b_n$ differ only in the ith position. Then a_i or b_i is 1 (and the other is 0); say $a_i=1$. Let A be the subset represented by $a_1a_2\cdots a_n$ and B, the one represented by $b_1b_2\cdots b_n$. Then B is a subset of A, and there is no proper subset of A that contains B. Hence there is an edge in B_n between these vertices.

Suppose there is an edge between the vertices labeled $a_1a_2\cdots a_n$ and $b_1b_2\cdots b_n$. Then $a_i \leq b_i$, $1 \leq i \leq n$ (or vice versa). Hence there are at least as many 1's in $b_1b_2\cdots b_n$ as in $a_1a_2\cdots a_n$. If the strings differ in two or more positions, say $a_j=a_k=0$ and $b_j=b_k=1$, consider the label $c_1c_2\cdots c_n$ with $c_i=b_i$, $i\neq j$, and $c_j=0$. Then $c_1c_2\cdots c_n$ represents an subset between those represented by $a_1a_2\cdots a_n$ and $b_1b_2\cdots b_n$. But this is not possible if there is an edge between the vertices labeled $a_1a_2\cdots a_n$ and $b_1b_2\cdots b_n$.

24. (a), (c)

25.

If n is even, there is an Euler circuit. Each vertex is labeled with a string of even length. Hence it must have even degree as the value of each position could be changed in turn to create the label of a vertex connected to the original one. If n is odd, by the same reasoning every vertex has odd degree and there is no Euler circuit.

8.3

3. Hamiltonian path.

6. Hamiltonian circuit.

10. $A, B, C, D, E, F, J, G, H, I, A$

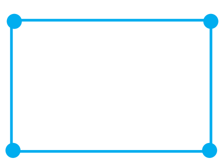
14. $A, B, C, E, D, F, J, G, H, I, A$

19.

Choose any vertex, v_1, in K_n, $n \geq 3$. Choose any one of the $n-1$ edges with v_1 as an endpoint. Follow this edge to v_2. Here we have $n-2$ edges from which to choose. Continuing in this way we see there are $(n-1)(n-2)\cdots 3\cdot 2\cdot 1$ Hamiltonian circuits we can choose.

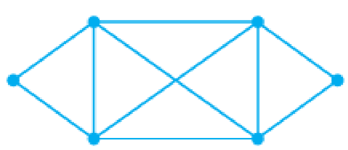
20.

One example is



21.

One example is



22.

Let $M_{R^\infty} = [m_{ij}]$, if $m_{ij} = 1$, then v_i, v_j is connected in the graph G.
If G is connected, there are no vertices of degree 0,which means $\forall i, j \in V_G$, $m_{ij} \neq 0$. If there are loop in the graph, then $\exists i \in V_G$, $m_{ii} = 1$, otherwise $m_{ii} = 0$.
Therefore, if $\forall m_{ij} \in M_{R^\infty}, i \neq j$, $m_{ij} = 1$, G is connected.

23. One solution is 000, 100, 110, 111, 101, 001, 011, 010, 000.

24. One solution is 0000, 0001, 0011,0010,0110,0111,0101,0100,1100,1101,1111,1110,1010,1011,1001,1000,0000.

25.

Construct a sequence of $2^n + 1$ strings of 0's and 1's of length n such that the first and the last terms are the same, and consecutive terms differ in exactly one position.