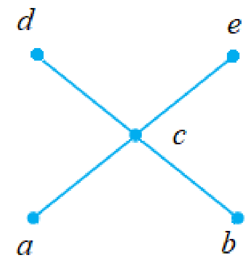
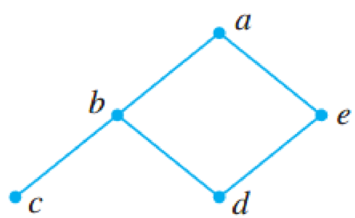
6.1

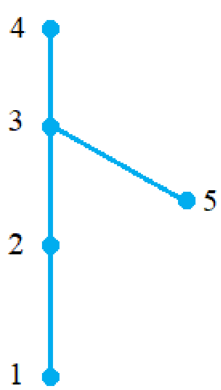
10.



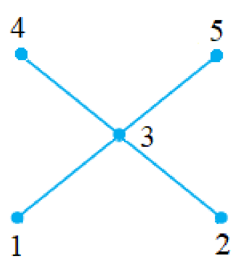
13.



14.



16.



18.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

26.

(a) eg. {{1,2,4},{3,12}}

(b) eg. {{a,d,f},{b,e},{c}}

(c) {{1,2,3,4}}

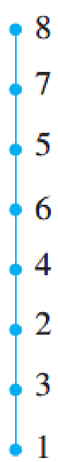
(d) eg. {{1,2,4,5,7,8},{3,6}}

(e) eg. {{1,2},{4,3},{5,6,7},{8,9}}

27. (a){2,3}   (b) {b,c,d}   (c){3}   (d) {2,3}   (e) {2,3,7,8}

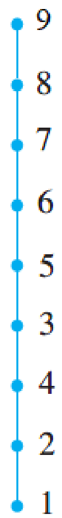
28. 相等，证明略。

29.



1 2 3 4 5 6 7 8, 1 2 3 4 6 5 7 8, 1 3 2 4 5 6 7 8, 1 3 2 4 6 5 7 8

30.



共 320 种结果，不一一写出。

34. 略. (证明非自反和传递)

35.

Suppose $a\ R^{-1}\ b$ and $b\ R^{-1}\ c$. Then $c\ R\ b$, $b\ R\ a$, and $c\ R\ a$. Hence $a\ R^{-1}\ c$ and R^{-1} is transitive. Suppose that $x\ R^{-1}\ x$. Then $x\ R\ x$, but this is a contradiction. Hence R^{-1} is irreflexive and a quasiorder.

36. aRb if and only if $a \mid b$ and $a \neq b$

38. 略. (证明自反，反对称，传递)

40. 略. (构造函数 $f: A \to A'$, 证明其是同构的)

6.2

6.

Maximal: none; minimal: 0.

8.

Maximal: 48; minimal: 2, 3.

12.

Greatest: 5; least: none.

14.

Greatest: 1; least: 0.

17. No, a may be maximal and there exists an element of A, b, such that a and b are incomparable.

18. No, a may be minimal and there exists an element of A, b, such that a and b are incomparable.

19.

(a) True. There cannot be $a_1 < a_2 < \cdots$ since A is finite.
(b) False. Not all elements have to be comparable.
(c) True. There cannot be $\cdots < a_2 < a_1$ since A is finite.
(d) False. Not all elements have to be comparable.

20. Suppose a and b are greatest elements of $(A, \leq)$. Then $a \leq b$ and $b \leq a$. Since $\leq$ is antisymmetric, $a = b$.

22.

Let $(A, \leq)$ be a poset, B is a subset of A.

Suppose $a, b \in A$, a and b are the LUB of B.

Since a is the LUB of B, then $a \leq b$. Similarily, $b \leq a$.

Since $\leq$ is antisymmetric, $a=b$. Hence B has at most one LUB.

Similarily, B has at most one GUB.

23. (a) f, g, h. **(b)** a, b, c. **(c)** f. **(d)** c.

24. (a) none. **(b)** none. **(c)** none. **(d)** none.

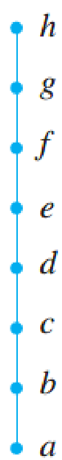
25. (a) d, e, f. **(b)** b, a. **(c)** d. **(d)** b.

26. (a) 5. **(b)** 1, 2, 3. **(c)** 5. **(d)** 3.

32.

(a) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. (b) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. (c) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. (d) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

33.



a b c d e f g h

a b c d e f h g

a b c e d f g h

a b c e d f h g

b a c d e f g h

b a c d e f h g

b a c e d f g h

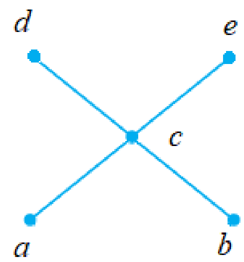
b a c e d f h g

35.

The least element of A is the label on the row that is all ones. The greatest element of A is the label on the column that is all ones.

36.

偏序的哈塞图:



37. (a) 50. (b) {2, 4, 8, 16, 32, 64}

38. 25.

6.3

1~6 是，否，否，是，是，是

13

For each $T_1, T_2 \subseteq T$, $T_1 \cap T_2$, and $T_1 \cup T_2$ are subsets of T so $P(T)$ is a sublattice of $P(S)$.

14 对于区间[a,b]中元素 x 均有，a<=x<=b,设区间[a,b]内两元素 x1,x2。则有 a<=x1<=b,a<=x2<=b。又因为 x1,x2∈L,所以 a<=x1Vx2<=b，a<=x1Λx2<=b，。所以 x1Vx2∈S 和 x1Λx2∈S 均成立。所以 S 是 L 的子格。

15

For any elements x, y of a linearly ordered poset, $x \leq y$ or $y \leq x$. Say $x \leq y$. Then $x = x \wedge y$ and $y = x \vee y$. Hence any subset of a linearly ordered poset is a sublattice.

18 若 L 是有界格，则 L 必有最大元 I 和最小元 0。
如果 I=0 则 L 必只有一个元素，因为所有 x∈L 都有 0<=x<=I
所以如果某个有界格有两个或更多的元素，那么 0≠I。

19

Suppose $a \wedge b = a$. $a \leq a \vee b = (a \wedge b) \vee b = (a \vee b) \wedge b \leq b$. Thus $a \leq b$.

Suppose $a \leq b$. $a \wedge b \leq a$ and $a \leq a$, $a \leq b$ gives $a \leq a \wedge b$. Hence $a \wedge b = a$.

20 Dn 中 a∧b 表示 a 与 b 的最大公约数，aVb 表示 a 和 b 的最小公倍数.

证明其是否为分配格，可根据其是否有图 6.44(a)(b)的结构

若有(a)结构

有质数 k1,k2,k3,p1,p2 使得；

b = k10 ,a = k2b,I = k3a => I = k1k2k30

c = p10, I = p2c => I = p1p20

得出 k1k2k3 = p1p2, 此等式必不成立，故假设不成立。

若有(b)结构

根据上边思路，则有质数 m1,m2,m3,n1,n2,n3

I = m1n1a = m2n2b = m3n3c

得出 m1n2 = m2n2 = m3n3,此等式必不成立，故假设不成立。

所以 Dn 对于任意 n 是分配的。

22 若格 L 是分配格，则对于 L 中任意 a,b,c 有

a∧(bVc) = (a∧b)V(a∧c)

aV(b∧c) = (aVb) ∧(aVc)

若 a,b,c 属于 L 的子格 S

则 bVc∈S，b∧c∈S

进一步 a∧(bVc)∈S，aV(b∧c)∈S

分配格的子格是分配格。

24 (1) aV(a1∧b) = (aVa1) ∧(aVb) = I∧(aVb) = (aVb)

(2)a∧(a1Vb) = (a∧a1)V(a∧b) = 0V(a∧b) = (aVb)

25

Suppose $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$. Then

$$
\begin{aligned}
y \leq y \vee (y \wedge a) &= (y \wedge y) \vee (y \wedge a) \\
&= y \wedge (y \vee a) \\
&= y \wedge (a \vee x) \\
&= (y \wedge a) \vee (y \wedge x) \\
&= (a \wedge x) \vee (y \wedge x) \\
&= x \wedge (a \vee y) \leq x.
\end{aligned}
$$

Hence $y \leq x$. A similar argument shows $x \leq y$. Thus $x = y$.

26

(a)设格 L 是分配格，对于 L 上所有 a,b,c,a<=c

有 aV(b∧c) = (aVb)∧(aVc) = (aVb)∧c

所以一个分配格是模格。

(b)图中 aV(b∧c) = aV0 = a

(aVb)∧(aVc) = IVI = I

所以图中所示的格是一个非分配格。

图中有如下关系：0<=a<=I , 0<=b<=I, 0<=c<=I

所以图中格想要成为模格，必须满足 a 为 0 或 c 为 I,(在此描述的 a,c 均为模格定义中的 a,c a<=c)

1)a 为 0

0V(m∧n) = m∧n = (0Vm)∧n (m,n∈{0,a,b,I},满足 0<=n)

2)c 为 I

mV(n∧I) = mVn = (mVn)∧I (m,n∈{0,a,b,I}满足 m<=I)

以上情况模格定义均成立，所以如图所示的格是一个非分配的模格。

27

$1' = 42$, $42' = 1$, $2' = 21$, $21' = 2$, $3' = 14$, $14' = 3$, $7' = 6$, $6' = 7$.

29 两者都不成立。

34

a’= e , e’= a , b’= c , d’= c , c′=b 和 d

37

The sublattice $\{a, b, d\}$ of Figure 6.57 is not complemented.

38 格({x|x∈R 且 0<=x<=1},<=) 是有界格

但其子格({x|x∈R 且 0<x<1},<=) 不是有界格

39

For any a, b, c in the sublattice with $a \leq c$, $a \vee (b \wedge c) = (a \vee b) \wedge c$, because this is true in the full lattice.

40

设 L 是一个全序集，则对于 L 中每两个元素 a,b 均有 a<=b 或 b<=a，则集合{a,b}均有最小上界和最大下界。所以 L 是一个格。对于 L 中任意三个元素 a,b,c 有以下六种情况：

1)a<=b<=c

2)a<=c<=b

3)b<=a<=c

4)b<=c<=a

5)c<=a<=b

6)c<=b<=a

可验证以上六种情况均满足 aV(b∧c) = (aVb)∧(aVc)和 a∧(bVc) = (a∧b)V(a∧c)

所以 L 是分配格。

所以任何一个全序是一个分配格。

6.4

6 否，图中顶点数为 8 只可能和 B3 同构，第 5 题图给出了与 B3 同构的哈塞图，对比可知此图与 B3 不同构。

8 是，此图与 B2 同构

10
B7 对应哈塞图元素个数为 $7^2 = 49$
B8 对应哈塞图元素个数为 $8^2 = 64$
$49<60<64$,所以不存在 Bn 与之同构。

16
1)若(a)成立，则 $a \vee b = b$,所以 $a <= b$,所以 $a \wedge b = a$
对应于集合关系 $A \subseteq B$,又因为 $B \cap B' = \emptyset$,$A \cup A' = S$ (用 B'表示 B 的补，A'表示 A 的补，S 表示全集)，即有 $A \cap B' = \emptyset$,$A' \cup B = S$,所以 $a \wedge b' = 0$,$a' \vee b$
2),3),4),5),6)后边可采用相似思想证明。

17
$$(a \wedge b) \vee (a \wedge b') = a \wedge (b \vee b') = a \wedge I = a.$$

18
$$b \wedge (a \vee (a' \wedge (b \vee b'))) = b \wedge (a \vee (a' \wedge I)) = b \wedge (a \vee a') = b \wedge I = b$$

19
$$(a \wedge b \wedge c) \vee (b \wedge c) = (a \vee I) \wedge (b \wedge c) = I \wedge (b \wedge c) = b \wedge c.$$

20
$$((a \vee c) \wedge (b' \vee c))' = ((a \wedge b') \vee c))' = (a' \vee b) \wedge c'$$

21

Suppose $a \leq b$. Then $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) = b \wedge (a \vee c)$.

27

(A, R) is not a Boolean algebra; complements are not unique.

29

(a) $\{a\}, \{b\}, \{c\}$. **(b)** 2, 3, 5.

32

(a) 原子为 001，010，100

110 = 100 V 010,

101 = 100 V 001,

011 = 010 V 001,

111 = 100 V 010 V 001

(b) 原子为 2，3，7

6 = 2 V 3

14 = 2 V 7

21 = 3 V 7

42 = 2 V 3 V 7

(c) 原子 $\begin{matrix}1 & 0\\ 0 & 0\end{matrix}$ = a1, $\begin{matrix}0 & 1\\ 0 & 0\end{matrix}$ = a2, $\begin{matrix}0 & 0\\ 1 & 0\end{matrix}$ = a3, $\begin{matrix}0 & 0\\ 0 & 1\end{matrix}$ = a4,

$\begin{matrix}1 & 1\\ 0 & 0\end{matrix}$=a1Va2

$\begin{matrix}1 & 0\\ 1 & 0\end{matrix}$=a1Va3

$\begin{matrix}1 & 0\\ 0 & 1\end{matrix}$=a1Va4

$\begin{matrix}0 & 1\\ 1 & 0\end{matrix}$=a2Va3

$\begin{matrix}0 & 1\\ 0 & 1\end{matrix}$=a2Va4

$\begin{matrix}0 & 0\\ 1 & 1\end{matrix}$ =a3Va4

$\begin{matrix}1 & 1\\ 1 & 0\end{matrix}$ =a1Va2Va3

$\begin{matrix}1 & 1\\ 0 & 1\end{matrix}$ =a1Va2Va4

$\begin{matrix}1 & 0\\ 1 & 1\end{matrix}$ =a1Va3Va4

$\begin{matrix}0 & 1\\ 1 & 1\end{matrix}$ =a2Va3Va4

$\begin{matrix}1 & 1\\ 1 & 1\end{matrix}$ =a1Va2Va3Va4

6.5

11. $x \wedge z$.

12.Z

13. $y \vee x'$.

(I)题目有误

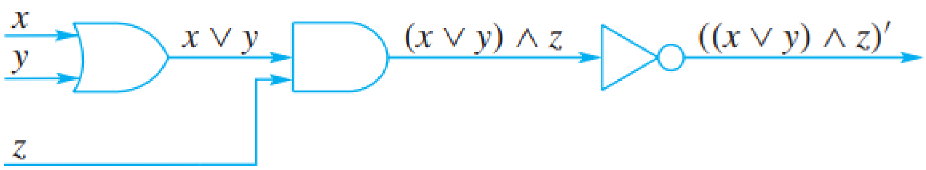
14. $X' \wedge Z'$

18. $(X \vee (Y \wedge Z))' \vee Z'$

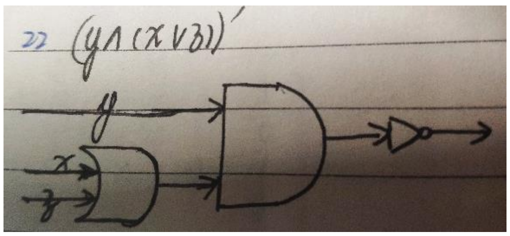
19. $((x \wedge y) \vee (y \wedge z))'$.

20. $(X' \wedge X)' \vee ((Y \wedge W') \vee ((Y \wedge W') \vee Z'))$

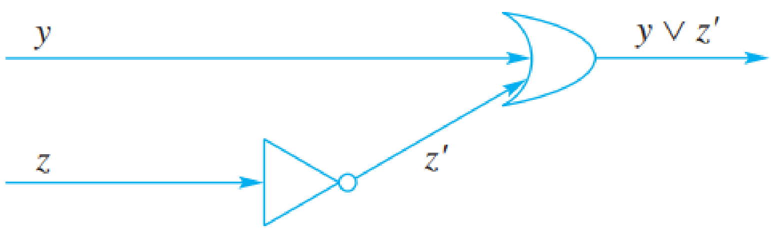
21. $((x \vee y) \wedge z)'$.



22.



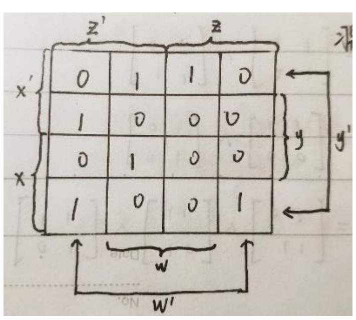
23. $y \vee z'$.



(I)题目有误

6.6

8.



12. $(Z'\wedge X')\vee(Z\wedge Y')\vee(X\wedge Y\wedge Z')$

14. $(X'\wedge Y')\vee(Z\wedge Y)\vee(X'\wedge Y)$

16. $(X'\wedge Z')\vee(W'\wedge X'\wedge Z)\vee(X\wedge Y'\wedge W')$

24. $(X'\wedge W\wedge Y')\vee(X\wedge Y'\wedge W')\vee(X'\wedge Z'\wedge W'\wedge Y)\vee(X\wedge Z'\wedge Y\wedge W)$

25. **(a)** $x'\wedge y', x'\wedge y, x\wedge y'$

(b) Since $\wedge$ is commutative and associative, we need only consider the case $(w_1\wedge w_2\wedge\cdots\wedge w_n\wedge y)\vee(w_1\wedge w_2\wedge\cdots\wedge w_n\wedge y')$. But this is equivalent to $w_1\wedge w_2\wedge\cdots\wedge w_n$.

26.（a）用 X’取代（0,0）和（0,1）,用 Y’取代（0,0）和（1,0）

(b)

$$
\begin{aligned}
&(X'\wedge Y')\vee(X'\wedge Y)\vee(X\wedge Y')\\
&=(X'\wedge(Y'\vee Y))\vee(X\wedge Y')\\
&=X'\vee(X\wedge Y')\\
&=(X'\vee X)\wedge(X'\vee Y')\\
&=X'\vee Y'
\end{aligned}
$$