A8

ddl: 2023/12/17 23:59

1 Coin changing (CLRS 15-1)

Consider the problem of making change for n cents using the smallest number of coins. Assume that each coin's value is an integer.

1. Describe a greedy algorithm to make change consisting of quarters, dimes, nickels, and pennies [1]. Prove that your algorithm yields an optimal solution.
2. Suppose that the available coins are in denominations that are powers of c: the denominations are $c^0, c^1, \ldots, c^k$ for some integers $c > 1$ and $k \geq 1$. Show that the greedy algorithm always yields an optimal solution.
3. Give a set of coin denominations [2] for which the greedy algorithm does not yield an optimal solution. Your set should include a penny so that there is a solution for every value of n.
4. Give an $O(nk)$-time algorithm that makes change for any set of k different coin denominations using the smallest number of coins, assuming that one of the coins is a penny.

2 Yen's improvement to Bellman-Ford (CLRS 22-1)

The Bellman-Ford algorithm does not specify the order in which to relax edges in each pass. Consider the following method for deciding upon the order. Before the first pass, assign an arbitrary linear order $v_1, v_2, \ldots, v_{|V|}$ to the vertices of the input graph $G = (V, E)$. Then partition the edge set E into $E_f \cup E_b$, where $E_f = \{(v_i, v_j) \in E : i < j\}$ and $E_b = \{(v_i, v_j) \in E : i > j\}$ (Assume that G contains no self-loops, so that every edge belongs to either E_f or E_b. Define $G_f = (V, E_f)$ and $G_b = (V, E_b)$.

1. Prove that G_f is acyclic with topological sort $\langle v_1, v_2, \ldots, v_{|V|}\rangle$ and that G_b is acyclic with topological sort $\langle v_{|V|}, v_{|V|-1}, \ldots, v_1\rangle$.

Suppose that each pass of the Bellman-Ford algorithm relaxes edges in the following way. First, visit each vertex in the order $v_1, v_2, \ldots, v_{|V|}$, relaxing edges of E_f that leave the vertex. Then visit each vertex in the order $v_{|V|}, v_{|V|-1}, \ldots, v_1$, relaxing edges of E_b that leave the vertex.

2. Prove that with this scheme, if G contains no negative-weight cycles that are reachable from the source vertex s, then after only $\lceil |V|/2 \rceil$ passes over the edges, $v.d = \delta(s, v)$ for all vertices $v \in V$.
3. Does this scheme improve the asymptotic running time of the Bellman-Ford algorithm?

1. quarter = 25 cents, dime = 10 cents, nickel = 5 cents, and penny = 1 cent ↩

2. **N-COUNT**(钞票或硬币的)面额，面值
The **denomination** of a banknote or coin is its official value. ↩