1 作业 (必做部分)

题目 1 (TJ 9-11)

解答:
$\mathbb{Z}_8, \mathbb{Z}_4 \times \mathbb{Z}_2, \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2, \mathbb{D}_4, \mathbb{D}_8$

题目 2 (TJ 9-16)

解答:
According to Theorem 9.17 and Corollary 9.18
(a)12(b)30(c)5(d)30

题目 3 (TJ 9-23)

解答:
If K is infinite, let G be $\mathbb{Z}$ and H be $\mathbb{Z}\times\mathbb{Z}$, K be $\mathbb{Z}\times\mathbb{Z}\times\cdots = \prod_{i\in\mathbb{N}}\mathbb{Z}$, $G\times K \cong H\times K$ but $G \ncong H$.
If K is finite, assuming that $G \ncong H$, let K be {e}, then $G \times K \ncong H \times K$, so $G \cong H$.

题目 4 (TJ 10-1(a,c))

解答：
(a)S_4：All permutation groups of order 4. A_4：All even permutation groups of order 4.
(b)S_4：All permutation groups of order 4. D_4：{(1),(1234),(13)(24),(1432),(24),(12)(34),(13),(14)(23)}

题目 5 (TJ 10-11)

解答：
Assuming H is not normal, so $\exists g_0 \in G, \exists h_0 \in H, g_0 h_0 g_0^{-1} \notin H$, $K = \{g_0 h g_0^{-1} | h \in H\}$ will be a subgroup of G. As $e = g_0 e g_0^{-1} \in K$, $K \neq \emptyset$. $\forall k_1, k_2 \in K, \exists h_1, h_2 \in H$, s.t.$k_1 = g_0 h_1 g_0^{-1}, k_2 = g_0 h_2 g_0^{-1}$, we have $k_1 k_2^{-1} = g_0(h_1 h_2^{-1}) g_0^{-1}$, so $k_1 k_2^{-1} \in K$, K is a subgroup of G. Let $f : H \to K$, $f(h) = g_0 h g_0^{-1}$, if it has h and h' that $f(h) = f(h')$, we have $g_0 h g_0^{-1} = g_0 h' g_0^{-1}$, so $h = h'$, as f is onto, we can get that f is one to one. So $|H| = |K|$. As $g_0 h_0 g_0^{-1} \notin H$, $H \neq K$, there exists a contradiction, so H is normal.

题目 6 (TJ 10-12)

解答：
(1)As $eg = ge = g$, $e \in C(g)$. Let $a, b \in C(g)$, then $abg = agb = gab$, so $ab \in C(g)$. Let $a \in C(g)$, then $ag = ga \Rightarrow g = a^{-1} g a \Rightarrow g a^{-1} = a^{-1} g$, so $a^{-1} \in C(g)$. So $C(g)$ is a subgroup of G.
(2)Let $a \in G, c \in C\,(g)$, as $\langle g \rangle$ is normal, $ag^k = g^k a$. Let $x \in G$, as $\langle g \rangle$ is normal, $x\langle g \rangle = \langle g \rangle x$, $\exists k, k' (x = g^k x g^{-1}, x = g^{-1} x g^{k'})$. So

$$\begin{aligned} aca^{-1}g &= aca^{-1}g \\ &= (g^{k_1} a^{-1} g^{-1})^{-1} c (g^{k_1} a^{-1} g^{-1}) g \\ &= g a g^{-k_1} c g^{k_1} u^{-1} \\ &= g a g^{-k_1} g^{k_1} c a^{-1} \\ &= g a c a^{-1} \end{aligned}$$

So, $aca^{-1} \in C\,(g)$, $C(g)$ is nomal.

题目 7 (TJ 11-5)

解答：
Define $\phi_k(x) \equiv kx(mod 18)$, let P be $\{\phi_k : 3|k\}$, P is the set of all homomorphisms from $\mathbb{Z}_{24}$ to $\mathbb{Z}_{18}$.

题目 8 (TJ 11-2(b,d,e))

解答：
All of the following maps are homomorphisms.

(b)Kernel is $\{0\}$.(d)Kernel is $\{M \in GL_2(\mathbb{R}) | \det M = 1\}$.(e)Kernel is $\{\begin{pmatrix} a & 0 \\ c & d \end{pmatrix} \in \mathbb{M}_2(\mathbb{R})\}$.

2 作业 (选做部分)

题目 1 (SageMath 学习)
学习 TJ 第 9、10/11 章关于 SageMath 的内容

解答:

题目 2 (TJ 11-17)

解答:

题目 3 (6、8 阶群)
请给出同构意义下的所有 6 阶、8 阶群。

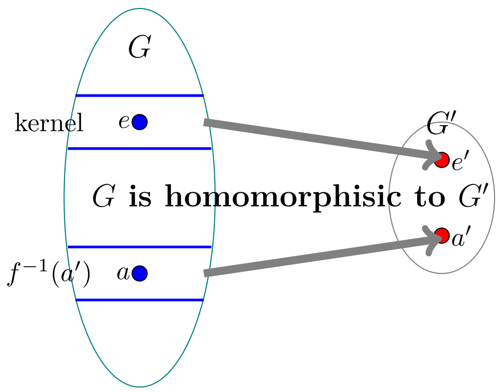
解答:

---

3 Open Topics

Open Topics 1 (群同态第二定理)
请证明群同态第二定理。

Open Topics 2 (同态猜想)



请证明或证否下列猜想

- Kernel 和任意的 G' 中非单位元元素的逆像不相交
- Kernel 和任意的 G' 中非单位元元素的逆像同势
- 任意的 G' 中元素的逆像不相交且同势
- 任意的 G' 中元素的逆像必定是 kernel 的某个陪集

4 反馈