**考试科目名称离散数学 （A卷）**

考试方式：开卷 考试日期2020年8 月15日 教师 赵建华，姚远

系（专业）软件学院（软件工程） 年级 2019级 班级

学号 姓名 成绩

注意：所有作答请写在答题纸上。

1. (8 points) Let *p* be the statement “It’s raining”, *q* be the statement “The field is wet”, *r* be the statement “The flowers need watering”. Please represent the following statements as logical formulas.

a) It’s raining, the filed is wet, and the flowers need watering.

b) It is not raining, the field is wet, and the flowers need watering.

c) If it is raining and the filed is wet, the flowers need watering.

d) If the flowers don’t need watering, then it is not raining or the field is not wet.

1. (8 points)Supposethere are 10 persons and each of them flips a coin. We know that the probability of the ‘HEAD’ outcome of the *i*-th person is 1/(2i+1). What is probability that the number of ‘HEAD’ outcomes is even?
2. (10 points) Let relation R be a reflexive and transitive relation on the set A. Define relation R' asx R' y if and only if x R y and y R x.

a) Prove that R' is an equivalence relation.

b) Let RP be a relation on the quotient set A/R' defined as:

[x] RP [y] if and only if x R y

Prove that RP is a partial order relation on A/R'.

1. (8 points) Define relation on the set of all integers. Give and prove the transitive closure of R.
2. (10 points) Prove the following properties by mathematical induction.a) For any two elements a, b in a communitive group (S,\*), and any positive integer n, abn=bna.b) Using the above property to prove that ambn=bnam holds for any two elements a, b in S, and any two positive integers m, n.
3. (8 points) Given a sequence of m numbers, prove that there must be a continuous non-empty subsequence such that the sum of this subsequence can be divided by m.
4. (8 points) Prove that a connected graph G has a unique minimal span tree if the weights of the edges of G are mutually different with each other.
5. (10 points) A subset of set A = {1, 2, 3,... , n} is called *alternating*, if the first number is odd and odd numbers and even numbers alternatingly appear after we sort all its elements in ascending order. For example, {1} and {1,2,3,4} are alternating; {2}, {1,3,4} and {1,4,6} are not alternating. Define that the empty set is alternating. Find the number of alternating subsets of A.
6. (10 points) Given the following network with source S and sink T:

7

3

3

3

7

4

5

2

2

2

6

S

T

a) Calculate the maximal flow of this network.

b) Give the minimal cut of this network.

1. (10 points) Calculate the number of different ways to color the following graph with 5 different colors such that any two adjacent vertexes have different colors. The calculation process is required.
2. (10 points) Prove that: on a 4\*n Chinese chessboard, it is impossible for "horse" to traverse each grid exactly once and return to the origin.

中文参考

1. (8分) 假设p表示“天正在下雨”，q表示“地上是湿的”，r表示“花需要浇水”，请用逻辑公式表示下列命题：
2. 天正在下雨，地上不是湿的，并且花需要浇水
3. 天不在下雨，地上是湿的，花需要浇水
4. 如果天在下雨并且地上是湿的，那么花不需要浇水
5. 如果花需要浇水，那么天不在下雨或者地上不是湿的
6. (8分) 假设第i个人抛硬币正面向上的概率是1/(2i+1)。10个人抛硬币，正面向上的个数是偶数的概率是多少？
7. (10分) 假设集合A上的关系R是自反的和传递的。定义A上的关系R'如下：                     x R' y当且仅当x R y 且 y R x。a)  请证明R'是一个等价关系。b)  A的商集A/R'上的关系RP定义如下：                            [x] RP [y] 当且仅当 x R y  请证明RP是A/R'上的偏序关系。
8. (8分)定义整数集上的关系，请给出R的传递闭包并证明之。
9. (10分) 使用数学归纳法证明下列性质：a) 可交换群(S,\*)的元素a和b，对于任意的正整数n，都有abn=bna。b)利用这个性质证明对于S中的任意元素a,b和任意正整数m,n，ambn=bnam。
10. (8分)给定m个数组成的序列，请证明一定能够从该序列中选出一个连续子序列，使得这个子序列的和能够被m整除。
11. (8分)请证明如果图G中各条边的权重各不相同，那么G的最小生成树是唯一的。
12. (10分)集合的某个子集称为是交替的，如果其元素按照升序排列时是奇数、偶数交替出现的，且第一个数是奇数。例如是交替的，与不是交替的。规定空集是交替的。求的交替子集的个数。
13. (10分)已知网络如下，其中源点是S，汇点是T：

7

3

3

3

7

4

5

2

2

2

6

S

T

a）请计算出这个图的最大流。b）给出这个图的最小割。

1. (10分)要使用5种颜色对下图中顶点进行染色并使得相邻顶点的颜色不同。请问总共有多少种染色方法。请给出演算过程。
2. (10分)试证明：在4\*n的中国象棋棋盘上，“马”不可能不重复的遍历每一个格子并回到原点。