EXERCISES

March, 2017

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1. Stationarity and Autocovariance.

   (a) Is the following MA(2) covariance stationary?

   $$y_t = \epsilon_t + 2.4\epsilon_{t-1} + 0.8\epsilon_{t-2}$$
   $$\epsilon_t \overset{iid}{\sim} \mathsf{N}(0,1).$$

   If so, calculate the autocovariance function and compute the forecasts $y_{t+k|t} = E_t y_{t+k}$.

   (b) Is the following AR(2) covariance stationary?

   $$y_t = 1.1y_{t-1} - 0.18y_{t-2} + \epsilon_t$$
   $$\epsilon_t \overset{iid}{\sim} \mathsf{N}(0,1).$$

   If so, compute the forecasts $y_{t+k|t}$ for $k \geq 1$.

2. Consider the VAR(1)

   $$x_t = x_{t-1} + \varepsilon_t$$
   $$y_t = \beta x_t + \eta_t$$

   where $(\varepsilon_t, \eta_t)' \sim \mathsf{iid}(\mathbf{0}, \Omega)$.

   (a) Are x_t and y_t cointegrated?

   (b) Write the model in Vector Error (or Equilibrium) form (VEC).

   (c) Suggest a method for computing $\partial x_{t+j}/\partial \eta_t$ for $j = 0, 1, ...$ (THIS IS A BIT MORE COMPLICATED)

3. Consider the following AR(1) data generating process (DGP)

   $$y_t = \tau + \delta \times 1_{\{t>t_0\}} + \rho y_{t-1} + \varepsilon_t$$

   where $1_{\{\cdot\}}$ denotes the indicator function that takes value 1 if $\{\cdot\}$ is true and zero otherwise. We assume $|\rho| < 1$.

   (a) Is y_t defined above stationary?

   (b) We now want to compare several forecasting techniques. We write $y_{T+1|T}$ the forecast of y_{T+1} made at time T and $e_{T+1|T}$ the forecast error $y_{T+1} - y_{T+1|T}$. For each of the models below, compute the forecast errors $e_{t_0|t_0-1}$, $e_{t_0+1|t_0}$ and $e_{t_0+2|t_0+1}$, their expectations and variances.
   (i) forecasting model $y^i_{t+1|t} = \tau + \rho y_t$
   (ii) forecasting model $y^{ii}_{t+1|t} = \tau + \rho y_t + e^i_{t|t-1}$
   (iii) forecasting model $y^{iii}_{t+1|t} = y_t$.

   What forecasting model seems best to you?