1. Consider an $\mathsf{ARMA}(1,1)$

$$y_t = \varphi_0 + \varphi_1 y_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1}.$$

   (a) Precisely describe the two types of stationarity.

   (b) Why is stationarity a useful property?

   (c) What is a minimal set of assumptions sufficient to ensure $\{y_t\}$ is covariance stationary if $\{\varepsilon_t\}$ is a white noise sequence?

   (d) What are the values of the following quantities? You may assume that y_t is weakly stationary (for this question only). $\mathcal{I}_t$ denotes the information set available at time t.

      i. $\mathsf{E}[y_{t+1}]$

      ii. $\mathsf{E}_t[y_{t+1}] = \mathsf{E}[y_{t+1}|\mathcal{I}_t]$

      iii. $\mathsf{V}[y_{t+1}]$

      iv. $\mathsf{V}_t[y_{t+1}] = \mathsf{Var}[y_{t+1}|\mathcal{I}_t]$

   (e) Suppose you were trying to differentiate between an $\mathsf{AR}(1)$ and an $\mathsf{MA}(1)$ but could not estimate any regressions. What information would you try to obtain?

   (f) Now suppose that $\varphi_1 = 1$. What can you say about $\mathsf{E}_t[y_{t+h}]$ and $\mathsf{V}_t[y_{t+h}]$ for $h > 2$?

2. **The timing of expectations in forward looking models with an emphasis in monetary policy** as presented in Clarida, Galí and Gertler (Journal of Economic Literature, 1999).
The model of interest is the so-called new Keynesian Phillips curve

$$\pi_t = \beta \pi^e_{t+1} + x_t \tag{1}$$

where π_t denote inflation at time t, x_t is some measure of activity (say the output gap, unemployment or the share of labor in aggregate output) and π^e_{t+1} denotes a representative agent's expectation of future inflation at time $t+1$. The question of interest here relates to the timing of this expectation. We consider two possibilities, and denote them by $\pi^e_{t+1|t}$ and $\pi^e_{t+1|t-1}$ respectively. These are defined as

$$\text{model } \mathsf{CI}\text{: } \pi^e_{t+1|t} = E(\pi_{t+1}|\mathcal{I}_t), \text{ and}$$
$$\text{model } \mathsf{LI}\text{: } \pi^e_{t+1|t-1} = E(\pi_{t+1}|\mathcal{I}_{t-1})$$

where $E(\cdot|\mathcal{I})$ denotes the expectation conditional on information $\mathcal{I}$. Hence $\pi^e_{t+1|t}$ denotes the expectation conditional on contemporanous information (CI) available to the agent at time t and $\pi^e_{t+1|t}$ denotes the expectation conditional on lagged information (LI) available at time $t-1$.We consider the forward solutions to expression (1) both under model CI and LI. We assume first $\beta \in (-1,1)$.
Expression (1) can be solved as

$$\pi_t = E\left[\left.\sum_{j=0}^{\infty} \beta^j x_{t+j}\right| \mathcal{I}_{t-\delta}\right] \tag{2}$$

where $\delta = 0$ for model CI and $\delta = 1$ for model LI.

(a) What assumptions do you need to make on the long run conditional expectations of π_{t+k} as $k \to \infty$ to obtain (2)?

(b) We make in turn the following assumptions on the dynamics of x_t :

$$\begin{aligned} &(i) \quad x_t = \mu + \varepsilon_t + \theta \varepsilon_{t-1} \\ &(ii) \quad x_t = \rho x_{t-1} + \varepsilon_t \end{aligned}$$

where $\mu \neq 0$, $\theta \in (-1, 1)$, $\rho \in \mathbb{R}$, and $\varepsilon_t \sim \mathsf{NID}\left(0, \sigma_\varepsilon^2\right)$. We assume that $\mathcal{I}_t$ consists of all the information available at time t regarding the processes $\pi_{t'}, x_{t'}$ and $\varepsilon_{t'}$ for $t' \leq t$; $\mathcal{I}_{t-1}$ is similarly defined.

i. For each of (i) and (ii) above, compute the conditional expectations of future $x_{t'}$ for $t' \geq t$ and solve expression (2) for π_t under models CI and LI.

ii. What conditions do β and ρ need to satisfy for (2) to provide a valid solution to expression (1)?

iii. Under what conditions are π_t and x_t integrated and cointegrated?