Assignment of Session 4:

1. Consider a portfolio of options on a single asset. Suppose that the delta of the portfolio is 10, the gamma of the portfolio is -2, the value of the asset is $12, and the daily volatility of the asset is 1%. Derive a quadratic relationship between the change in the portfolio value and the percentage change in the underlying asset price in one day.

a) Calculate the first three moments of the change in the portfolio value.
b) Using the first two moments and assuming that the change in the portfolio is normally distributed, calculate the 1-day 95% VaR for the portfolio.
c) Use the third moment and the Cornish-Fisher expansion to revise your answer to b).

Solution:

The quadratic relationship is

$$\Delta P = 12 \times 10\Delta x + 0.5 \times 12^2 \times (-2)(\Delta x)^2$$

or

$$\Delta P = 120\Delta x - 144(\Delta x)^2$$

(a) The first three moments of ΔP are $-144\sigma^2$, $14{,}400\sigma^2+62{,}208\sigma^4$, and $-18{,}662{,}400\sigma^4-44{,}789{,}760\sigma^6$ where σ is the standard deviation of Δx. Substituting $\sigma = 0.01$, the first three moments are -0.0144, 1.44, and -0.1867.

(b) The first two moments imply that the mean and standard deviation of ΔP are -0.0144 and 1.20, respectively. The 5 percentile point of the distribution is $-0.0144-1.2\times1.65 = -1.9944$. The 1-day 95% VaR is therefore $1.9944.

(c) The skewness of the distribution is $1/1.2^3\times(-0.1867-3\times1.44\times(-0.0144)+2\times(-0.0144)^3)=-0.072$ We can use the Cornish-Fisher expansion. Setting $q = 0.05$ to obtain $w_q=-1.65+1/6\times(1.65^2-1)\times(-0.072)=-1.67$. So the 5 percentile point of the distribution is $-0.0144 - 1.2 \times 1.67 = -2.02$ 2.02 is the 1-day 95% VaR.

2. A company has a long position in a 2-year bond and a 3-year bond as well as a short position in a 5-year bond. Each bond has a principal of $100 and pays a 5% coupon annually. Calculate the company's exposure to the 1-year, 2-year, 3-year, 4-year and 5-year rate. Use the data on Page 12 of Handout 4 to calculate a 20-day 95% VaR on the assumption that rate changes are explained by (a) one factor, (b) two factors, and (c) three factors. Assume that the current zero-coupon yield curve is flat at 5% with continuous compounding, i.e., the zero rate now is 5% for all maturities. (Hint: First find the delta of the portfolio with respect to the 1-year, 2-year, 3-year, 4-year and 5-year rate, like the one on Page 13 of Handout 4)

Solution:

The cash flow of the portfolio is as follows:

Year	1	2	3	4	5
2-yr bond	5	105			
3-yr bond	5	5	105		
5-yr bond	-5	-5	-5	-5	-105
Total	5	105	100	-5	-105

The present value of the portfolio can be written as:

$$P = 5e^{-r_1\times1} + 105e^{-r_2\times2} + 100e^{-r_3\times3} - 5e^{-r_4\times4} - 105e^{-r_5\times5}$$

So $\frac{dP}{dr_1} = -5e^{-r_1\times1} = -5e^{-0.05\times1} = -4.756$ So when the one-year rate increases by one basis point, the value of the portfolio decreases by 0.0001 × 4.756 = 0.0005

$\frac{dP}{dr_2} = -2\times105e^{-r_2\times2} = -2\times105e^{-0.05\times2} = -190.02$ So when the two year rate increases by one basis point, the value of the portfolio decreases by 0.0001×190.02 = 0.0190; and so on.

Year	1	2	3	4	5
Impact of 1bp change	-0.0005	-0.0190	-0.0258	0.0016	0.0409

The sensitivity to the first factor is
$-0.0005 \times 0.32 - 0.0190 \times 0.35 - 0.0258 \times 0.36 + 0.0016 \times 0.36 + 0.0409 \times 0.36 = 0.000798$.
The sensitivity to the second factor is
$-0.0005\times(-0.32)-0.0190\times(-0.10)-0.0258\times0.02+0.0016\times0.14+0.0409\times0.17= 0.0087$
The sensitivity to the third factor is
$-0.0005 \times (-0.37) - 0.0190 \times (-0.38) - 0.0258 \times (-0.30) + 0.0016 \times (-0.12) +0.0409 \times (-0.04) = 0.0133$.
Assuming one factor, the standard deviation of the one-day change in the portfolio value is 0.000798 × 17.49 = 0.0140. The 20-day 95% VaR is therefore 0.0140 × 1.645×sqrt(20) = 0.103.
Assuming two factors, the standard deviation of the one-day change in the portfolio value is $\text{Sqrt}(0.000798^2 \times17.49^2+0.0087^2 \times6.05^2)=0.0545$
The 20-day 95% VaR is therefore 0.0545 × 1.645×sqrt(20) = 0.401.
Assuming three factors, the standard deviation of the one-day change in the portfolio value is $\text{Sqrt}(0.000798^2 \times17.49^2+0.0087^2 \times6.05^2+0.0133^2 \times3.10^2)=0.0683$
The 20-day 95% VaR is therefore 0.0683 × 1.645×sqrt(20) = 0.502.

In this case the second and third factors have an important impact on VaR.

3. The calculations in Section II of Handout 4 assume that the investments in the DJIA, FTSE 100, CAC 40, and Nikkei 225 are $4 million, $3 million, $1 million, and $2 million, respectively. How do the VaR and ES change if the investment are $3million, $3 million, $1 million, and $3 million, respectively? Carry out calculations when

(a) volatilities and correlations are estimated using the equally weighted model and

(b) when they are estimated using the EWMA model.

What is the effect of changing λ from 0.94 to 0.90 in the EWMA calculations? Please explain the effect with one sentence.

Use the spreadsheets named “HW4Q3_VaRExampleRMFI4eModelBuilding.xls”.

Solution:

(a) When the equally weighted model is used the worksheet shows that one-day 99% VaR is $215,007 and the 1-day ES is $246,326

(b) When the EWMA model is used the worksheet shows that one-day 99% VaR is $447,404 and the 1-day ES is $512,575.

Changing λ to 0.90 leads to a VaR of $500,403 and an ES of $573,293. This is higher because more recent (high) returns are given more weight.

Assignment of Session 5:

1. A bank has the following transaction with an AA-rated corporation:

a) A two-year interest rate swap with a principal of $200 million that is worth $5 million
b) A nine-month foreign exchange forward contract with a principal of $300 million that is worth -$10 million
c) A long position in a six-month option on gold, with a principal of $100 million that is worth $10 million

What is the capital requirement under Basel I if there is no netting? What difference does it make if the netting amendment applies? What is the capital required under Basel II when the standardized approach is used?

Solution:

Using Table on Page 7 the credit equivalent amount under Basel I (in millions of dollars) for the three transactions are

(a) $5 + 0.005 \times 200 = 6$

(b) $0.01 \times 300 = 3$

(c) $10 + 0.01 \times 100 = 11$

The total credit equivalent amount is $6 + 3 + 11 = 20$. Because the corporation has a risk weight of 50% the risk weighted amount is 10. The capital required is 0.08×10 or \$0.8 million.

If netting applies, the current exposure after netting is in millions of dollars 5-10+10 =5. The NRR is therefore 5/150 = 1/3. The credit equivalent amount is in millions of dollars

$$5 + (0.4 + 0.6 \times 1/3)\times(0.005 \times 200 + 0.01 \times 300 + 0.01 \times 100) = 8$$

The risk weighted amount is 4 and the capital required is $0.08 \times 4 = 0.32$. In this case the netting amendment reduces the capital by 60%.

Under Basel II when the standardized approach is used the corporation has a risk weight of 20% and the capital required is therefore 40% of that required under Basel I or 0.4×0.32 or \$0.128 million.

2. Suppose that the assets of a bank consist of \$500 million of loans to A-rated corporations. The PD for the corporations is estimated as 0.2%. The average maturity is three years and the LGD is 50%. What are the risk-weighted assets for credit risk under the Basel II IRB approach? What are the Tier 1 and Tier 2 capital requirement? How does this compare with the capital required under the Basel II standardized approach and under Basel I?

Solution:

Under the Basel II IRB approach

$$\rho = 0.12[1 + e^{-50\times0.002}] = 0.2285$$

$$b = [0.11852 - 0.05478 \times ln(0.002)]^2 = 0.2106$$

$$MA = \frac{1+(3-2.5)\times0.2106}{1-1.5\times0.2106} = 1.62$$

and $WCDR = N\left[\frac{N^{-1}[\,0.002] + \sqrt{0.2285}N^{-1}(99.9\%)}{\sqrt{1-0.2285}}\right] = 0.0554$

The RWA is

$$500 \times 0.5 \times (0.0554 - 0.002) \times 1.62 \times 12.5 = 270.3375$$

The total capital is 8% of this or \$21.63 million. Half of this must be Tier I.

Under the Basel II standardized approach the risk weight for A- corporation is 50% and the total capital required is 8% of $500 *50% or $20 million. Under the Basel I the risk weight is 100% and the total capital required is 8% of $500 or $40 million.

3. *A bank has the following balance sheet*

Cash	2	Retail Deposits (stable)	35
Treasury Bonds (>1 year)	2	Retail Deposits (less stable)	25
Corporate Bonds Rated AA	2	Wholesale Deposits	24
Residential Mortgages	4	Preferred Stock (> 1 yr)	4
Small Business Loans (<1 yr)	10	Tier 2 capital	3
Fixed Assets	80	Tier 1 Capiral	9
	100		100

(a) What is the Net Stable Funding Ratio?
(b) The bank decides to satisfy Basel III by raising more stable retail deposits and keeping the proceeds in Treasury bonds. How much extra retail deposits need to be raised?

The amount of stable funding is

$$35 \times 0.9 + 25 \times 0.8 + 24 \times 0.5 + 16 \times 1.0 = 79.5$$

The required stable funding is

$$2 \times 0 + 2 \times 0.05 + 2 \times 0.2 + 4 \times 0.65 + 10 \times 0.85 + 80 \times 1.0 = 91.6$$

The net stable funding ratio is

$$\frac{79.5}{91.6} = 0.8679$$

If X is the amount of retail deposits we require

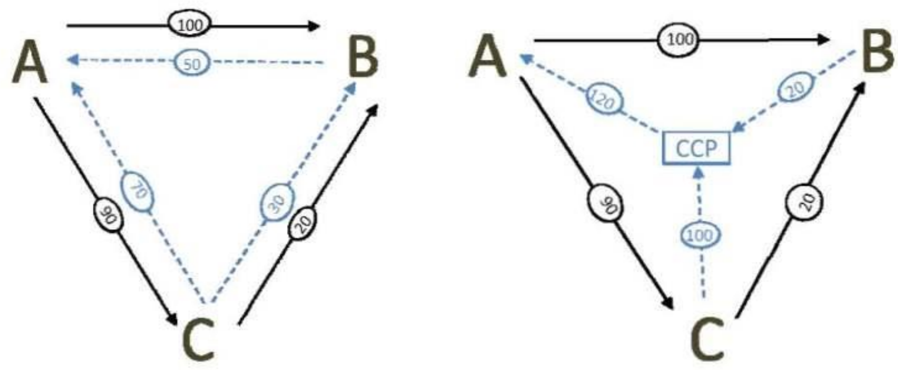
$$79.5+0.9X=91.6+0.05X$$

so that

$$0.85X=12.1 \text{ or } X=14.23$$

Assignment of Session 6:

1. The following example illustrates that CCP can sometimes make matters worse than better. Suppose there are three market participants and one CCP. The exposures represented by the dotted lines are standard transactions that can be cleared centrally. Those represented by solid line are non-standard transactions that cannot be cleared centrally. For example, in party B’s dealing with A, the nonstandard OTC transactions are worth 100 to B and -100 to A; the standard OTC transactons are worth +50 to A and -50 to B. Note that netting is done with EACH counterparty.



Dealer	Exposure after bilateral netting
A	0
B	100
C	20
Ave	40

Dealer	Exposure after netting incl. CCP	Exposure after netting excl. CCP
A	120	0
B	120	120
C	90	90
Ave	110	70

In the figure above, suppose that an extra standard transaction between A and C which is worth 160 to A is cleared *through the CCP*. What effect does this have on the tables in the figure?

Solution:

In the case of the first table where all transactions are cleared bilaterally the exposures of A, B, and C become: 140, 100, and 0 for an average of 80. The second table becomes

Dealer	Exposure after netting incl CCP	Exposure after netting excl CCP
A	280	0
B	120	120
C	90	90
Ave	163.3	70

2. Suppose that the spread between the yield on a three-year riskless zero-coupon bond and a three-year zero-coupon bond issued by a bank is 210 basis points. Assume that the Black-Scholes–Merton model is a good model for option pricing. The Black-Scholes–Merton price of an option is $4.10. If you buy this option from the bank, will you pay a price higher, equal to, or lower than $4.10? Explain why. (You do not need to compute exactly how much you are going to pay)

Solution:

You should pay lower than $4.10 due to the probability that the bank may default.

To be more precise, you should be prepared to pay $4.10\times e^{-0.0210\times 3}$ = $3.85. This assumes that the option will rank equally with the bond in the event of a default, no collateral is posted by the bank, and the option is not netted with any other derivatives transactions.

3. Explain very briefly the difference between Vasicek's model, the Credit Risk Plus Model, and CreditMetrics as far as the following are concerned: (a) When a credit loss is recognized and (b) the way in which default correlation is modeled

Solution:

In Vasicke's model and Credit Risk Plus, a credit loss is recognized when a default occurs. In CreditMetrics, both downgrades and defaults lead to credit losses. In Vasicek's model, a Gaussian copula model of time to default is used. In Credit Risk Plus, a probability distribution is assumed for the default rate per year. In CreditMetrics, a Gaussian copula model is used to define rating transitions.

Assignment of Session 7:

1. What difference does it make to the VaR calculated in the example of Page 7 of Handout 7 if the exponentially weighted model is used to assign weights to scenarios as described on Page 9 of Handout3?

Solution:
The weights for the historical scenarios in the table on Page 10 of Handout 3 must be multiplied by 0.99. So we have the table below. The VaR with a 99% confidence is $345,435.

Scenario	**Loss ($000s)**	**Probability**	**Cumul. Probability**
s5	850.000	0.0005	0.0005
s4	750.000	0.0005	0.001
v494	477.814	0.00528279	0.00628279

s3	450.000	0.002	0.00828279
v339	345.435	0.002429074	0.010711864
s2	300.000	0.002	0.012711864
v349	282,204	0.002553936	0.0152658
v329	277.041	0.002310317	0.017576117
v487	253.385	0.005100642	0.02267676
s1	235.000	0.005	0.02767676
v227	217.974	0.001385562	0.029062321

2. The worksheet (OPRISKSIMULATION.xls) used to produce the figure on Page 11 of Handout 7 is attached. There the expected loss frequency is 3 per year and the mean and standard deviation of the log of the loss is 0 and $0.4 million respectively. What is the mean and standard deviation of the loss distribution? What is the mean and standard deviation of the loss distribution if the expected loss frequency is 4 per year?

Solution:
When the expected loss frequency is 3 per year, the mean total loss is about 3.3 and the standard deviation is about 2.0. When the expected loss frequency is increased to 4 per year, the mean loss is about 4.3 and the standard deviation is about 2.3

Assignment of Session 8:

A trader wishes to unwind a position of 100,000 units over **EIGHT** days. The dollar bid-offer spread, as a function of daily trading volume q, is $p(q) = a + be^{cq}$ with $a = 0.2$, $b = 0.15$, and $c = 0.1$, and q is measured in thousands. The standard deviation of the price change per day is $1.5. What is the optimal trading strategy for minimizing the 99% confidence level for the cost? What if the confidence level is 95%, 90% and 80%?

Solution:

Day	q_i ($,000)	q_i ($,000)	q_i ($,000)	q_i ($,000)
	99%	95%	90%	80%
1	30.3	28.0	26.4	23.8
2	25.6	23.9	22.7	20.7
3	20.0	19.2	18.6	17.4
4	13.6	14.0	14.0	13.8
5	7.1	8.6	9.4	10.2

6	2.6	4.1	5.2	6.9
7	0.7	1.6	2.5	4.3
8	0.2	0.6	1.2	2.9