Assignment of Session 1:

1. Suppose that each of two investments has 3% chance of a loss of \$20 million, a 2% chance of a loss of \$2 million, and a 95% chance of a profit of \$2 million during a one-year period. They are independent of each other.

a) What is the VaR for one of the investments when the confidence level is 96%?
b) What is the expected shortfall for one of the investments when the confidence level is 96%?
c) What is the VaR for a portfolio consisting of the two investments when the confidence level is 96%?
d) What is the expected shortfall for a portfolio consisting of the two investments when the confidence level is 96%?
e) Show that, in this example, VaR does not satisfy the subadditivity condition whereas expected shortfall does.

Solution:

(a) A loss of \$2 million extends from the 95 percentile point of the loss distribution to the 97 percentile point. The 96% VaR is therefore \$2 million.

(b) The expected shortfall for one of the investments is the expected loss conditional that the loss is in the 4 percent tail. Given that we are in the tail there is a 25% chance that the loss is \$2 million and an 75% chance that the loss is \$20 million. The expected loss is therefore \$15.5 million. This is the expected shortfall.

(c) For a portfolio consisting of the two investments there is a $0.03 \times 0.03 = 0.0009$ chance that the loss is \$40 million; there is a $2 \times 0.03 \times 0.02 = 0.0012$ chance that the loss is \$22 million; there is a $2 \times 0.03 \times 0.95 = 0.057$ chance that the loss is \$18 million; there is a $0.02 \times 0.02 = 0.0004$ chance that the loss is \$4million; there is a $2 \times 0.02 \times 0.95 = 0.038$ chance that the loss is zero; there is a $0.95 \times 0.95 = 0.9025$ chance that the profit is \$4 million. It follows that the 96% VaR is \$18 million.

(d) The expected shortfall for the portfolio consisting of the two investments is the expected loss conditional that the loss is in the 4% tail. Given that we are in the tail, there is a $0.0379/0.04 = 0.9475$ chance of a loss of \$18 million, a $0.0012/0.04 = 0.03$ chance of a loss of \$22 million; and a $0.0009/0.04=0.0225$ chance of a loss of \$40 million. The expected loss is therefore \$18.615.

(e) VaR does not satisfy the subadditivity condition because $18 > 2 + 2$. However, expected shortfall does because $18.615 < 15.5+15.5$.

2. Suppose that we back-test a VaR model using 1,000 days of data. The VaR confidence level is 99% and we observe 15 exceptions. Should be reject the model at the 95% confidence level (i.e., 5% significance level)? Use the one-tailed test in the lecture note.

Solution:

p=1-99%=1%

m/n=15/1000=1.5%>1%, so use the right-tail test

Ho: the probability of an exception on any given day = p

H1: the probability of an exception on any given day > p

prob (number of exception >=15|Ho)=1-BINOMDIST(14,1000,0.01,TRUE)=8.24%>5%

So cannot reject the hypothesis.

3. The change in the value of a portfolio in three months is normally distributed with a mean of \$500,000 and a standard deviation of \$3 million. Calculate the VaR and ES for a confidence level of 99.5% and a time horizon of three months.

Solution:
The loss has a mean of −500 and a standard deviation of 3000. Also, $N^{-1}(0.995)$ =2.576. The 99.5% VaR in \$'000s is −500+3000×2.576) =7,227. We are 99.5% certain that the loss will not be greater than \$7.227 million.
The ES is

$$-500 + 3000\frac{e^{-\frac{2.576^2}{2}}}{\sqrt{2\pi} \times 0.005} = 8{,}172$$

The expected loss conditional that it is in the 0.5% tail of the distribution is \$8.172 million.

Assignment of Session 2:

1. Suppose that the parameters in a GARCH (1,1) model are $\alpha = .05, \beta = .92$ and $\omega = .000003$

a) What is the long-run average volatility?
b) If the current volatility is 2% per day, what is your estimate of the volatility in 20, 40, and 60 days?
c) Suppose that there is an event that increases the current volatility by 0.5 percentage points to 2.5% per day. Estimate the effect on your forecasted volatility in 20, 40, and 60 days.

Solution:

(a) The long-run average variance, *VL*, is $\frac{\omega}{1-\alpha-\beta} = \frac{0.000003}{0.03} = 0.0001$
The long run average volatility is 0.01 or 1% per day.

(b) The expected variance in 20 days is $0.0001 + 0.97^{20}(0.02^2 - 0.0001) = 0.000263$
The expected volatility per day is therefore $\sqrt{0.000263}$ = 0.0162 or 1.62%. Similarly the expected volatilities in 40 and 60 days are 1.37% and 1.22%, respectively.

(c) The expected variance in 20 days is

$$0.0001 + 0.97^{20}(0.025^2 - 0.0001) = 0.00039$$

The expected volatility per day is therefore $\sqrt{.00039}$= 0.0196 or 1.96%. Similarly the expected volatilities in 40 and 60 days are 1.60% and 1.36% per day, respectively.

2. *(Spreadsheets Provided)*

In the file “HW2Q2Example_GARCHCALCSS&P500.xls”, the maximum likelihood estimation of *parameters in the EWMA and GARCH(1,1) model is illustrated using S&P500 data between July 15, 2005 and August 13, 2010.*

Download the file “HW2Q2_EURUSDExchangerates.xls”. Estimate parameters for the EWMA and GARCH(1,1) model on the euro-USD exchange rate data between July 27, 2005, and July 27, 2010.

Solution:

As the spreadsheets show the optimal value of λ in the EWMA model is 0.958 and the log likelihood objective function is 11,806.4767. In the GARCH (1,1) model, the optimal values of ω, α, and β are 0.0000001330, 0.04447, and0.95343, respectively. The long-run average daily volatility is 0.7954% and the log likelihood objective function is 11,811.1955.

3. Suppose that a bank has made a large number of loans of a certain type. The one-year probability of default on each loan is 2%. The bank uses a Gaussian copula for time to default. It is interested in estimating a “99.9% worst case” for the percentage of loans that default on the portfolio. Construct a table to show how this varies with the copula correlation.

Solution:
The WCDR with a 99.9% confidence level is from equation

$$WCDR(T,X) = N\left[\frac{N^{-1}[\,2\%]\;+\sqrt{\rho}N^{-1}(99.9\%)}{\sqrt{1-\rho}}\right]$$

The table below gives the variation of this with the copula correlation.

Copula Correlation	WCDR (%)
0	2
0.1	12.82
0.2	22.63
0.3	33.30
0.4	44.90
0.5	57.37
0.6	70.45
0.7	83.42

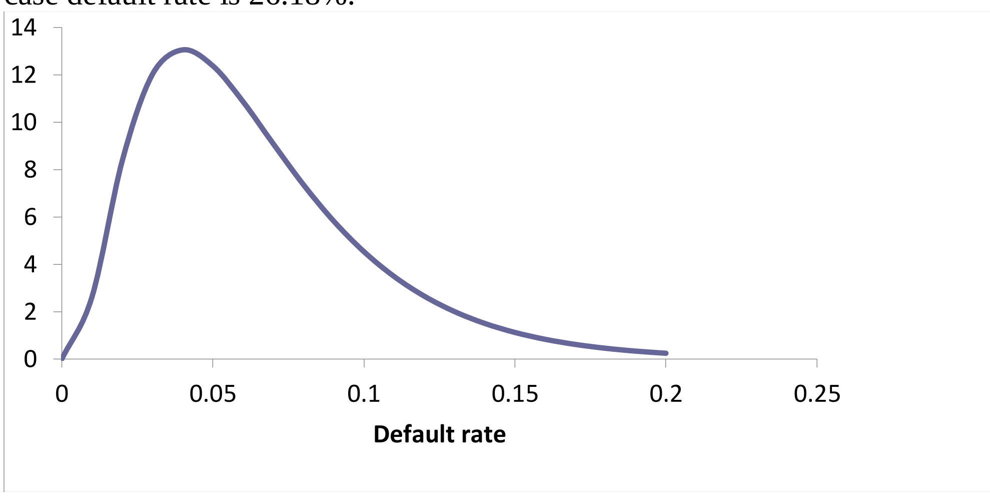
0.8	94.39
0.9	99.72

4. *(One Spreadsheet Provided)*
The file "Hw2Q4Example_DefaultRates4e.xls" shows the default rate for all rated companies between 1970 and 2013. The procedure for calculating maximum likelihood estimates for PD and ρ of the Vasicek model is also illustrated using this data.

Suppose the default rates in the last 15 years for a certain category of loans is 2%, 4%, 7%, 12%, 6%, 5%, 8%, 14%, 10%, 2%, 3%, 2%, 6%, 7%, 9%. Use the maximum likelihood method to calculate the best fit values of the parameters in Vasicek's model. What is the probability distribution of the default rate? What is the 99.9% worst case default rate?

Solution:

The maximum likelihood estimates of ρ and PD are 0.086 and 6.48%. The 99.9% worst case default rate is 26.18%.



Assignment of Session 3:

1. Suppose that a one-day 98% VaR is estimated as $12 million from 1,000 observations. The one-day changes are approximately normal with mean 0 and standard deviation $5 million. Estimate a 99% confidence interval for the VaR estimate.

Solution:
The standard error is

$$\frac{1}{f(x)}\sqrt{\frac{(1-q)q}{n}}$$

In this case the 0.98 point on the approximating normal distribution is NORMINV(0.98,0,5) = 10.27. $f(x)$ is estimated as NORMDIST(10.27,0,5,FALSE) = 0.0097. The standard error is therefore

$$\frac{1}{0.0097}\sqrt{\frac{(1-0.98)0.98}{1000}} = 0.457$$

A 99% confidence interval for the VaR is 12 − 2.576 × 0.457 to 12 + 2.576 × 0.457 or 10.823 to 13.177.

2. Suppose that the portfolio considered in Section I of Handout 3 has (in $000) 3,000 in DJIA, 3000 in FTSE, 1,000 in CAC40 and 3,000 in Nikkei 225. Use the spreadsheet named “VaRExampleRMFI4eHistoricalSimulation.xls” to calculate what difference this makes to

(a) The one-day 99% VaR and ES that are calculated in Section I
(b) The one-day 99% VaR and ES that are calculated using the weighting-of-observations procedure in Section III
(c) The one-day 99% VaR and ES that are calculated using the volatility-updating procedure in Section III
(d) The one-day 99% VaR and ES that are calculated using extreme value theory in Section IV

Solution:

(a) VaR is $230,785; ES is $324,857

(b) VaR is $262,456; ES is $413,774

(c) For the first procedure VaR is $629,943; ES is $699,460. For the second procedure VaR is $578,562 and ES is $687,700

(d) The values of β and ξ given by Solver are 44.94 and 0.306. The VaR with 99% confidence given by extreme value theory is $230,484. The expected shortfall is $326,336.