

Queens College, CUNY, Department of Computer Science

Computational Finance

CSCI 365 / 765

Fall 2017

Instructor: Dr. Sateesh Mane

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due Friday October 13, 2017 at 11.59 pm

4 Homework: Forwards & Futures

Note: Continuous interest rate compounding is used in all questions.

4.1 Forward price

- Suppose the price of a stock is $S_0 = 100.5$ at time $t_0 = 0$. The stock pays no dividends. Suppose also the interest rate is $r = 5.5\%$.

Question: Calculate the fair value forward price at the time $T = 0.75$ years.

$$F = S_0 e^{r(T-t_0)} . \quad (4.1.1)$$

- Suppose the price of a stock is $S_0 = 95.5$ at time $t_0 = 0$ and it pays a continuous dividend yield $q = 1.1\%$. Suppose also the interest rate is $r = 5.1\%$.

Question: Calculate the fair value forward price at the time $T = 0.65$ years.

$$F = S_0 e^{(r-q)(T-t_0)} . \quad (4.1.2)$$

4.2 Forwards: arbitrage

- The price of a stock is $S_0 = 100.0$ at time $t_0 = 0$.
- The stock pays no dividends.
- The interest rate is $r = 5.0\%$.
- The expiration time of the forward contract is $T = 1.0$ years.
- **Question: For each case below, formulate an arbitrage strategy to take advantage of the forward price.**
 1. The forward price is $F = 105.0$.
 2. The forward price is $F = 106.0$.
- **Question: For each case above, state how much profit your arbitrage strategy yields at the expiration time T .**

4.3 Futures: mark to market

- Suppose the stock price today is $S_0 = 100.0$. We have a futures contract which will expire in 5 days. The futures price today is $F_0 = 105.5$. Today is $t_0 = 0$ and the expiration date is $t_5 = 5$ (measured in days, not years).
- Every day for $t_i = i$, $i = 1, 2, 3, 4, 5$, the stock price is S_1, S_2, S_3, S_4, S_5 . The futures price every day is F_1, F_2, F_3, F_4, F_5 . On the expiration day, $F_5 = S_5$ (the futures price converges to the stock price).
- Every day the futures contract is **marked to market**. This means that if the futures price on day i is F_i , (for $i = 1, 2, 3, 4, 5$), the following happens.
 1. If $F_i > F_{i-1}$, the investor receives a cash amount $F_i - F_{i-1}$ in the mark to market account.
 2. If $F_i < F_{i-1}$, the investor pays a cash amount $F_{i-1} - F_i$ into the mark to market account.
 3. If $F_i = F_{i-1}$, nothing happens.
- **Note: the money in the mark to market account does not earn interest.**
- **See next page.**

4.3.1 Random walk 1

- Here is a list of the stock and futures prices, for $i = 1, 2, 3, 4, 5$.

i	S_i	F_i	money received	money paid
1	99.5	103.3		
2	101.3	104.1		
3	101.3	102.1		
4	100.2	101.3		
5	99.3	99.3		

- Question:** For each day, calculate the amount of money paid or received in the mark to market account.
- On the final day (expiration), the futures price converges to the stock price so $F_5 = S_5$. The investor pays the stock price S_5 and takes delivery of the underlying asset.
- Question:** Add up the money the investor pays on the final day (to take delivery) plus the money in the mark to market account. What is the total amount of money the investor pays?

4.3.2 Random walk 2

- Here is a **different** list of the stock and futures prices, for $i = 1, 2, 3, 4, 5$. (Remember that $S_0 = 100.0$ and $F_0 = 105.5$.)

i	S_i	F_i	money received	money paid
1	100.9	106.3		
2	103.8	108.7		
3	106.1	109.2		
4	107.5	108.3		
5	108.3	108.3		

- Question:** For each day, calculate the amount of money paid or received in the mark to market account.
- On the final day (expiration), the futures price converges to the stock price so $F_5 = S_5$. The investor pays the stock price S_5 and takes delivery of the underlying asset.
- Question:** Add up the money the investor pays on the final day (to take delivery) plus the money in the mark to market account. What is the total amount of money the investor pays?

4.3.3 Summary

- Question:** Does the random walk of the stock prices affect the total amount paid by the investor?