Queens College, CUNY, Department of Computer Science

Computational Finance

CSCI 365 / 765

Fall 2017

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due Friday October 20, 2017 at 11.59 pm

5 Homework: Options 1

Note: Continuous interest rate compounding is used in all questions.

5.1 Option price: arbitrage 1

- Suppose the market price of a stock is S_0 at time t_0.
- The stock does not pay dividends.
- The interest rate is $r > 0$ (a constant).
- The following options all have strike $K > 0$ and expiration time $T > t_0$.
  1. European call c_{Eur}.
  2. American call C_{Am}.
  3. European put p_{Eur}.
  4. American put P_{Am}.
- **Question: For each case below, either formulate an arbitrage strategy to take advantage of the option price, else explain why an arbitrage strategy is not possible for that situation.**
  1. Market price $c_{\text{Eur}} = S_0 + 1.5$.
  2. Market price $C_{\text{Am}} = S_0 + 2.5$.
  3. Market price $p_{\text{Eur}} = e^{-r(T-t_0)}K + 0.75$.
  4. Market price $P_{\text{Am}} = K + 1.75$.

5.2 Option price: arbitrage 2

- Suppose the market price of a stock is S_0 at time t.
- The interest rate is $r > 0$ (a constant).
- The following options all have strike $K > 0$ and expiration time $T > t_0$.
- The stock pays **one dividend**, of amount D_1, at a time t_1, where $t_0 < t_1 < T$.

  1. European call c_{Eur}.
  2. American call C_{Am}.
  3. European put p_{Eur}.
  4. American put P_{Am}.

- **Question: For each case below, either formulate an arbitrage strategy to take advantage of the option price, else explain why an arbitrage strategy is not possible for that situation.**

  1. Market price $c_{\mathrm{Eur}} = S_0 + 1.5$.
  2. Market price $C_{\mathrm{Am}} = S_0 + 2.5$.
  3. Market price $p_{\mathrm{Eur}} = e^{-r(T-t_0)}K + 0.75$.
  4. Market price $P_{\mathrm{Am}} = K + 1.75$.

- Suppose the stock pays two dividends in the time interval from t_0 to T.
  The dividend amounts are D_1 and D_2, paid at times t_1 and t_2, where $t_0 < t_1 < t_2 < T$.

  Question: Describe your arbitrage strategy (if it exists) in this scenario, for each case above.

5.3 Call option spreads: arbitrage

- Suppose the market price of a stock is S at time t.
- The stock does not pay dividends.
- The interest rate is $r > 0$ (a constant).
- All the options below have the same expiration time T (where $T > t$).

5.3.1 European options

- We are given two European call options on the stock:
  c_1 with strike K_1 and c_2 with strike K_2 (where $K_2 > K_1 > 0$).
- Recall that European options can be exercised only at the the expiration time T.
- For simplicity, denote the prices of the options by c_1 and c_2, respectively.
  Create a bull call spread with price $c_1 - c_2$.
- **Question: Show that at expiration, the option prices must satisfy the following inequality:**
$$c_1(T) - c_2(T) \le K_2 - K_1 \qquad \text{(at expiration)}\,. \tag{5.3.1}$$
- **Question: Formulate an arbitrage strategy to show that before expiration (time $t < T$), the option prices must satisfy the following inequality:**
$$c_1(t) - c_2(t) \le \mathrm{PV}(K_2 - K_1) = e^{-r(T-t)}(K_2 - K_1) \qquad (t < T)\,. \tag{5.3.2}$$

5.3.2 American options

- Next we are given two American call options on the stock:
  C_1 with strike K_1 and C_2 with strike K_2 (where $K_2 > K_1 > 0$).
- Recall that American options can be exercised at any time $t \le T$.
- For simplicity, denote the prices of the options by C_1 and C_2, respectively.
  Create a bull call spread with price $C_2 - C_1$.
- **Question: Show that at expiration, the option prices must satisfy the following inequality:**
$$C_1(T) - C_2(T) \le K_2 - K_1 \qquad \text{(at expiration)}\,. \tag{5.3.3}$$
- **Question: Formulate an arbitrage strategy to show that before expiration (time $t < T$), the American option prices must satisfy the following inequality:**
$$C_1(t) - C_2(t) \le K_2 - K_1 \qquad (t < T)\,. \tag{5.3.4}$$

5.3.3 Stock dividends

- **(Optional question)**
  Suppose the stock pays n dividends D_i at times t_i, $i = 1, 2, \ldots, n$ during the lifetime of the options. Explain how the inequalities in eqs. (5.3.2) and (5.3.4) would be modified.

5.4 Put option spreads: arbitrage

- Suppose the market price of a stock is S at time t.
- The stock does not pay dividends.
- The interest rate is $r > 0$ (a constant).
- All the options below have the same expiration time T (where $T > t$).

5.4.1 European options

- We are given two European put options on the stock:
  p_1 with strike K_1 and p_2 with strike K_2 (where $K_2 > K_1 > 0$).
- Recall that European options can be exercised only at the the expiration time T.
- For simplicity, denote the prices of the options by p_1 and p_2, respectively.
  Create a bear put spread with price $p_2 - p_1$.
- **Question: Show that at expiration, the option prices must satisfy the following inequality:**
$$p_2(T) - p_1(T) \le K_2 - K_1 \qquad \text{(at expiration)}\,. \tag{5.4.1}$$
- **Question: Formulate an arbitrage strategy to show that before expiration (time $t < T$), the option prices must satisfy the following inequality:**
$$p_2(t) - p_1(t) \le \mathrm{PV}(K_2 - K_1) = e^{-r(T-t)}(K_2 - K_1) \qquad (t < T)\,. \tag{5.4.2}$$

5.4.2 American options

- Next we are given two American put options on the stock:
  P_1 with strike K_1 and P_2 with strike K_2 (where $K_2 > K_1 > 0$).
- Recall that American options can be exercised at any time $t \le T$.
- For simplicity, denote the prices of the options by P_1 and P_2, respectively.
  Create a bull put spread with price $P_2 - P_1$.
- **Question: Show that at expiration, the option prices must satisfy the following inequality:**
$$P_2(T) - P_1(T) \le K_2 - K_1 \qquad \text{(at expiration)}\,. \tag{5.4.3}$$
- **Question: Formulate an arbitrage strategy to show that before expiration (time $t < T$), the American option prices must satisfy the following inequality:**
$$P_2(t) - P_1(t) \le K_2 - K_1 \qquad (t < T)\,. \tag{5.4.4}$$

5.4.3 Stock dividends

- **(Optional question)**
  Suppose the stock pays n dividends D_i at times t_i, $i = 1, 2, \dots, n$ during the lifetime of the options. Explain how the inequalities in eqs. (5.4.2) and (5.4.4) would be modified.