Queens College, CUNY, Department of Computer Science
Computational Finance
CSCI 365 / 765
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6 Homework: Options 2

- In all questions below, time is measured in years and the time today is $t_0 = 0$.
- Continuous interest rate compounding is used in all questions.
- In all questions, the following notation is employed for American/European put/call options:
  1. **European call: c**
  2. **European put: p**
  3. **American call: C**
  4. **American put: P**
- Some of the questions below require the cumulative normal function, given by

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2}\, du\,. \tag{6.0.1}$$

Fortunately C++ provides a function `erf(x)` which can be used to compute $N(x)$:

$$N(x) = \frac{1+\mathrm{erf}(x/\sqrt{2})}{2}\,. \tag{6.0.2}$$

You may use the C++ function below to compute the value of $N(x)$.

```
double cum_norm(double x)
{
  const double root = sqrt(0.5);
  return 0.5*(1.0 + erf(x*root));
}
```

6.1 Put-call parity

Question: For each case below, either calculate the value of the missing parameter, or else prove that there is not enough information to solve the problem.

6.1.1

1. The market price of a stock is 100 today. The stock does not pay dividends.
2. The interest rate is 10%.
3. A European call option has a strike of 101 and expiration time of 0.5 years.
4. The price of the above European call option is 8.
5. **Calculate the price of a European put option with the same strike and expiration as the call.**

6.1.2

1. The market price of a stock is 100 today.
2. The stock pays continuous dividends at a rate of 3%.
3. The interest rate is 10%.
4. A European put option has a strike of 101 and expiration time of 0.75 years.
5. The price of the above European put option is 4.
6. **Calculate the price of a European call option with the same strike and expiration as the put.**

6.1.3

1. The market price of a stock is 100 today. The stock does not pay dividends.
2. The interest rate is 5%.
3. The price of a European call option (with expiration time of 1 year) is 6.
4. The price of a European put option (with expiration time of 1 year) is 7.
5. Both options have the same strike price.
6. **Calculate the value of the strike price of the options.**

6.2 Put-call parity & option pricing bounds

- **To answer the question below, you need to consult the inequalities in Lecture 7.**
- **Consult Lecture 7, eqs. (7.2.1), (7.2.4) and (7.2.6).**

1. The market price of a stock is 20 today. The stock does not pay dividends.
2. The interest rate is 8%.
3. A European call option has a strike of 21 and an expiration time of 1 year.
4. A European put option has a strike of 21 and an expiration time of 1 year.
5. **Calculate the present value of the strike price of the options.**
6. **Calculate the minimum fair value the call option must have, to be consistent with the above data and the inequalities in Lecture 7.**
7. **Calculate the maximum fair value of the put option, to be consistent with the above data and the inequalities in Lecture 7. Calculate the corresponding fair value of the call option, when the put option has its maximum value.**
8. **Calculate the maximum fair value of the call option, to be consistent with the above data and the inequalities in Lecture 7. Calculate the corresponding fair value of the put option, when the call option has its maximum value.**

6.3 Delta

1. The market price of a stock is 10 today. The stock does not pay dividends.
2. The volatility of the stock is $\sigma = 50\%$ ($\sigma = 0.5$ in decimal).
3. The interest rate is 6%.
4. A European call option has a strike of 12 and expiration time of 0.8 years.
5. A European put option has a strike of 11 and expiration time of 0.8 years.
6. There is a futures contract on the same stock, with the same expiration time as the options.
7. The Delta of a European call option is given by

$$\Delta_c = N(d_1)\,. \tag{6.3.1}$$

8. The Delta of a European put option is given by

$$\Delta_p = -N(-d_1)\,. \tag{6.3.2}$$

9. The definition of d_1 is

$$d_1 = \frac{\ln(S/K) + r(T-t_0)}{\sigma\sqrt{T-t_0}} + \tfrac{1}{2}\sigma\sqrt{T-t_0}\,. \tag{6.3.3}$$

10. **Calculate the Delta of the call option.**
11. **Calculate the Delta of the put option.**
12. **Calculate the Delta of the futures contract.**
13. **Calculate the number N_{fp} of futures contracts, so that the total Delta of a portfolio of long one put option and long N_{fp} futures contracts equals zero.**
14. **Calculate the number N_{fc} of futures contracts, so that the total Delta of a portfolio of short one call option and long N_{fc} futures contracts equals zero.**