№1.

$\dot{\varphi} = a\sin\varphi + b\sin^2(\varphi)$

1) С.Р. $f = \sin\varphi(a + b\sin\varphi) = 0 \Rightarrow \sin\varphi = 0 \Rightarrow \varphi_{1,2} = \pi k,\ k \in \mathbb{Z}$

$\sin\varphi = -\frac{a}{b} \Rightarrow \varphi_4 = \arcsin\left(-\frac{a}{b}\right)$

$\varphi_3 = \pi k + \arcsin\left(-\frac{a}{b}\right)$

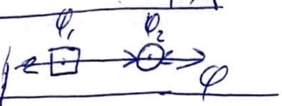
Т.к. $-1 \le \sin\varphi \le 1 \Rightarrow$

$-1 \le \frac{a}{b} \le 1$

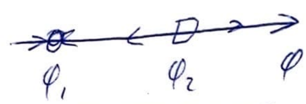
I $|a| > |b| \Rightarrow \frac{a}{b} > 1$ $\varphi_2 \varphi_3 - \nexists$ $\varphi_{1,2} = \pi k \Rightarrow \varphi_1 = 0, \pi$.

$$\boxed{\frac{\partial f}{\partial \varphi} = \cos\varphi\,(a + 2b\sin\varphi)}$$

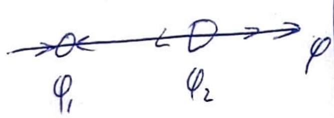
$a > 0;\ b > 0$



$a < 0;\ b > 0$



$a < 0;\ b < 0$



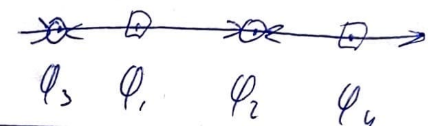
$a > 0;\ b < 0$

II) $|a| < |b|$ $\varphi_1 = 0$ $\varphi_2 = \pi$;

$a > 0;\ b > 0$

$\varphi_3 = -\arcsin\varphi$

$\varphi_4 = \pi + \arcsin\varphi$



$a < 0;\ b > 0$

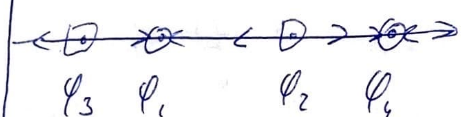
$\varphi_3 = a\sin\varphi$

$\varphi_4 = \pi - a\sin\varphi$

$a < 0,\ b < 0$

$\varphi_3 = -\arcsin\varphi$

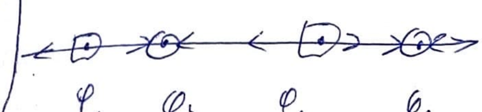
$\varphi_4 = \pi + \arcsin\varphi$



$a > 0;\ b < 0$

$\varphi_3 = \arcsin\varphi$

$\varphi_4 = \pi - \arcsin\varphi$



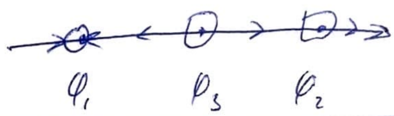
III) $|a| = |b|$ $\varphi_3 = \varphi_4$; $\varphi_1 = 0, \varphi_2 = \pi$.

$a > 0;\ b > 0$

$\varphi_3 = -\frac{\pi}{2}$

$a < 0;\ b > 0$

$\varphi_3 = \frac{\pi}{2}$

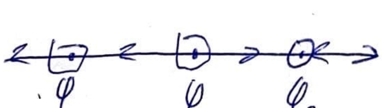


$a < 0;\ b < 0$

$\varphi_3 = -\frac{\pi}{2}$

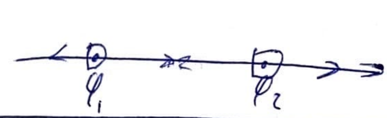
$a > 0;\ b < 0$

$\varphi = \frac{\pi}{2}$

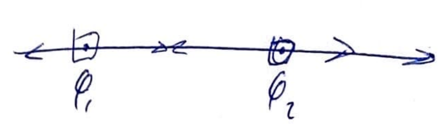


IV) $a = 0$; $\varphi_1 = \varphi_3 = 0$; $\varphi_4 = \varphi_2 = \pi$.

$b > 0$



$b < 0$

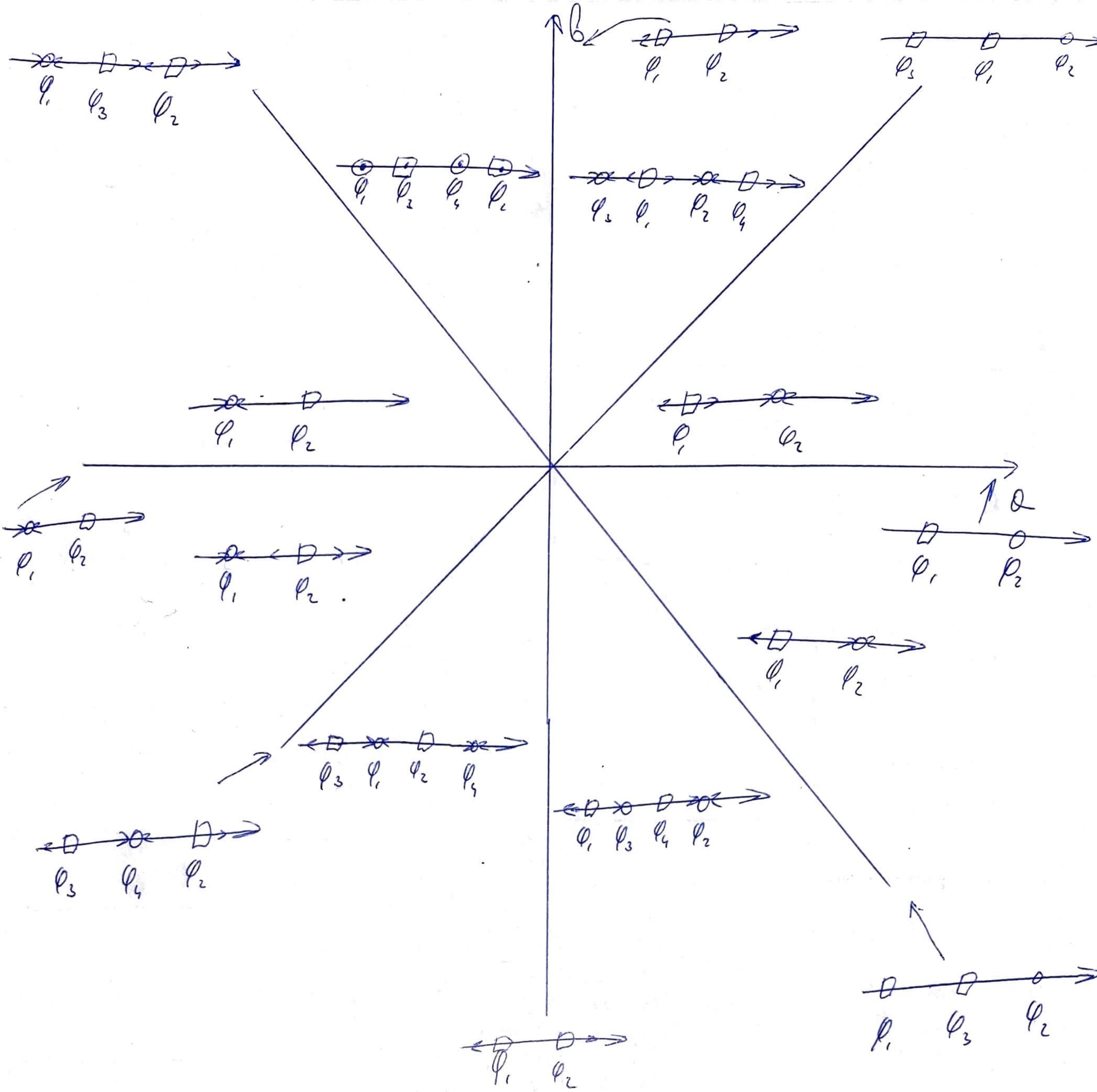


V) $b = 0$; $\varphi_3 \varphi_4 - \nexists$ $\varphi_1 = 0, \varphi_2 = \pi$.

$a > 0$

$a < 0$

VI) $a = 0$; $b = 0$ С.Р. нет.



b
a
φ_1 φ_2
φ_3 φ_1 φ_2
φ_1 φ_3 φ_2
φ_1 φ_3 φ_4 φ_2
φ_3 φ_1 φ_2 φ_4
φ_1 φ_2
φ_1 φ_2
φ_1 φ_2
φ_1 φ_2
φ_1 φ_2
φ_1 φ_2
φ_3 φ_1 φ_2 φ_4
φ_1 φ_3 φ_4 φ_2
φ_3 φ_4 φ_2
φ_1 φ_3 φ_2
φ_1 φ_2

№2.

$\dot{x} = x^8 + a\cdot x^4 + b$, $b<0$.

$y = x^4$

$\dot{x} = y^2 + ay + b$; $D = a^2 - 4b$, т.к $b<0$ $D>0$.

$y = \frac{-a \pm \sqrt{a^2-4b}}{2} \Rightarrow x = \pm\sqrt[4]{y} = \pm\sqrt[4]{\frac{-a\pm\sqrt{a^2-4b}}{2}}$

т.к $b<0$ $-a-\sqrt{a^2-4b} < 0 \Rightarrow x = \pm\sqrt[4]{\frac{-a-\sqrt{a^2-4b}}{2}}$ — комплексное число. $\Rightarrow$

$x_{1,2} = \pm\sqrt[4]{\frac{-a+\sqrt{a^2-4b}}{2}} = \pm\sqrt[4]{\frac{-a+a\sqrt{-4b}}{2}} = \sqrt[4]{\frac{2\sqrt{-b}}{2}} = \sqrt[4]{\sqrt{-b}} = \sqrt[8]{-b}$

Фазовые портреты не зависит от параметра a.

Бифуркаций нет.