

These problem sets, with included solutions, form the core of an Advanced Course offered at the 2024 Ross Summer Mathematics Program. They chart a guided path through the initially bewildering but quickly exhilarating world of p -adic numbers.

There are two primary goals. First, to build up the foundational theory in a gradual and organic fashion. Second, to show off some striking applications of p -adic numbers to elementary number theory. Here highlights include (a) classical congruences for the Bernoulli numbers, (b) Thue's theorem on the finiteness of solutions to cubic Pell-like equations, and (c) Skolem's theorem on the structure of zeros of linear recurrences.

Designed for advanced undergraduates and beginning graduate students, these problems require no prior knowledge of algebraic number theory or topology. A strong foundation in calculus, elementary number theory, abstract algebra, and matrix algebra is sufficient for fully engaging with the text.

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UNREAL ANALYSIS

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