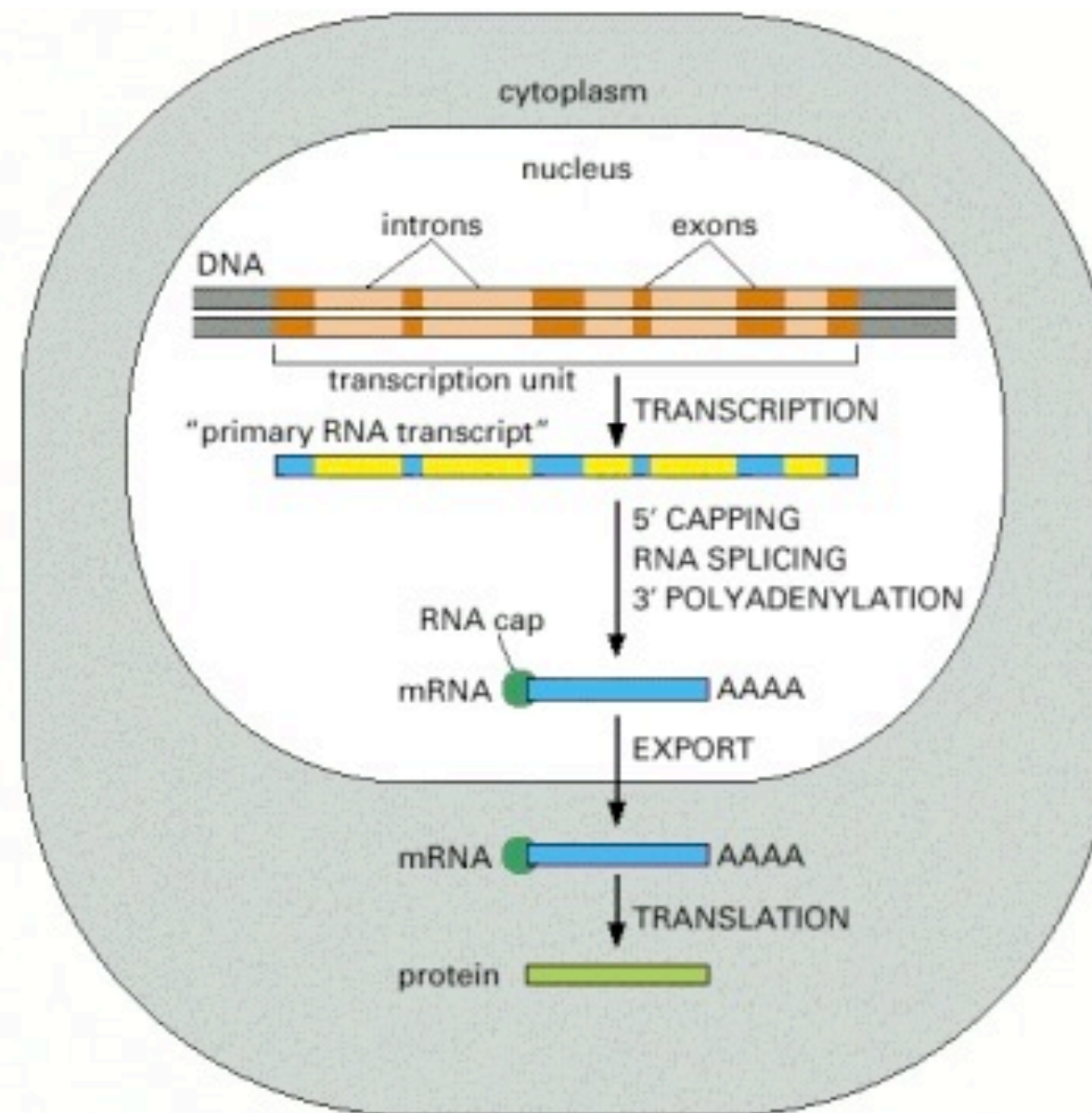
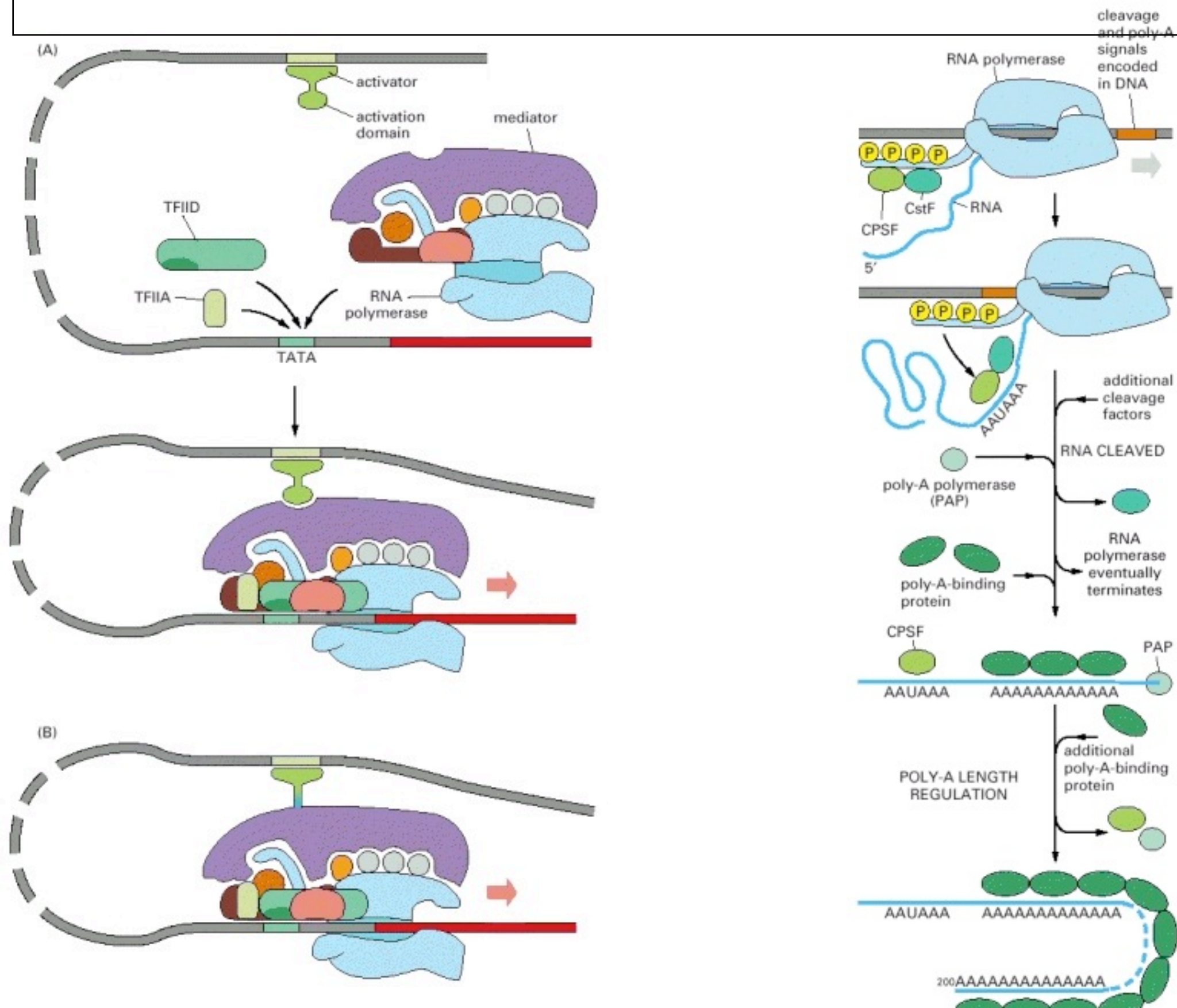


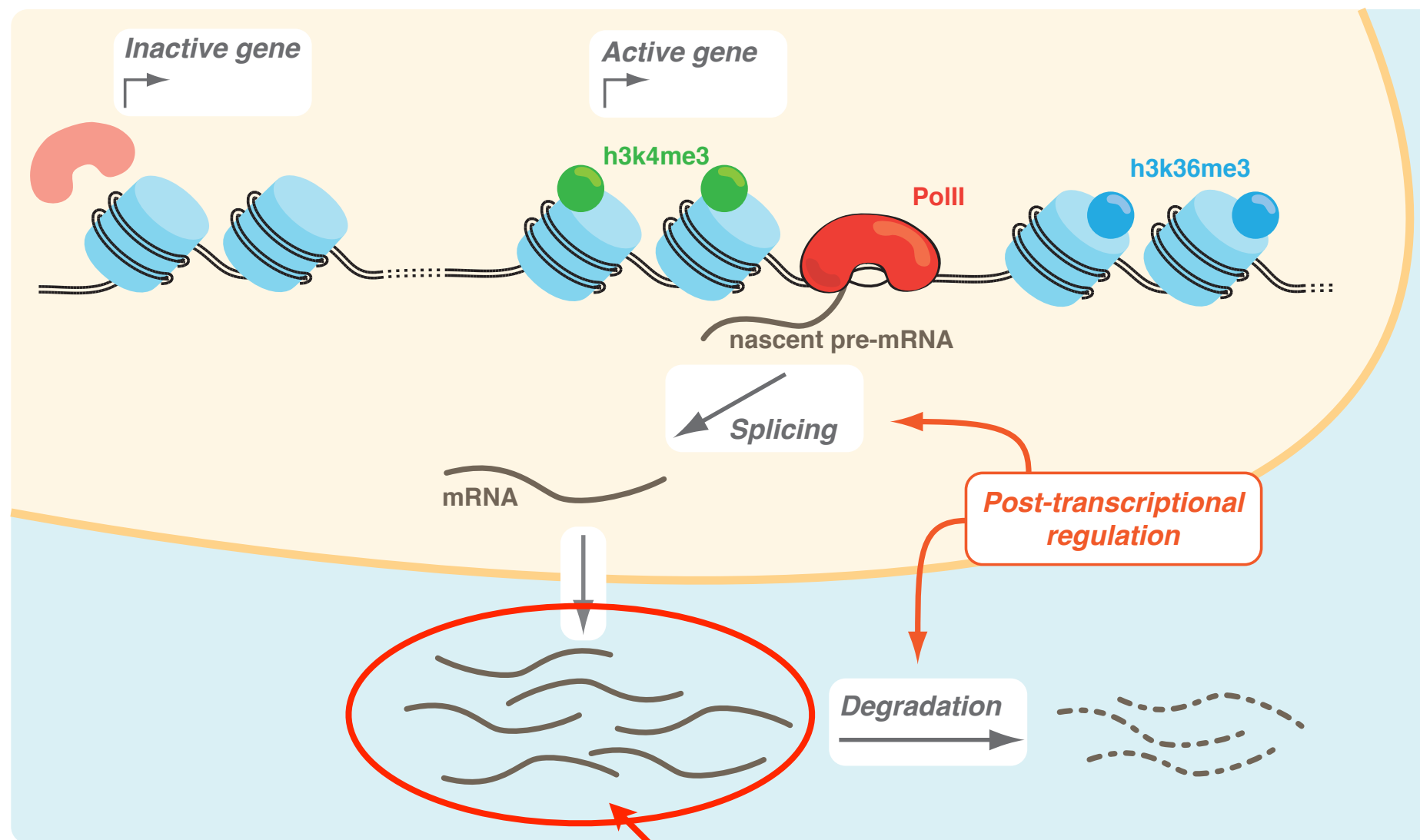
Transcription



Transcription



Transcription



Equilibrium pool of
mRNA for each gene

Simplest model

Transcription rate: $P(t)$ [#mRNA/time]

mRNA pool: $m(t)$ [#mRNA]

Degradation rate: γ [1/time]

$$\dot{m}(t) = P(t) - \gamma m(t) .$$

Case 1: $P(t) = P_0$

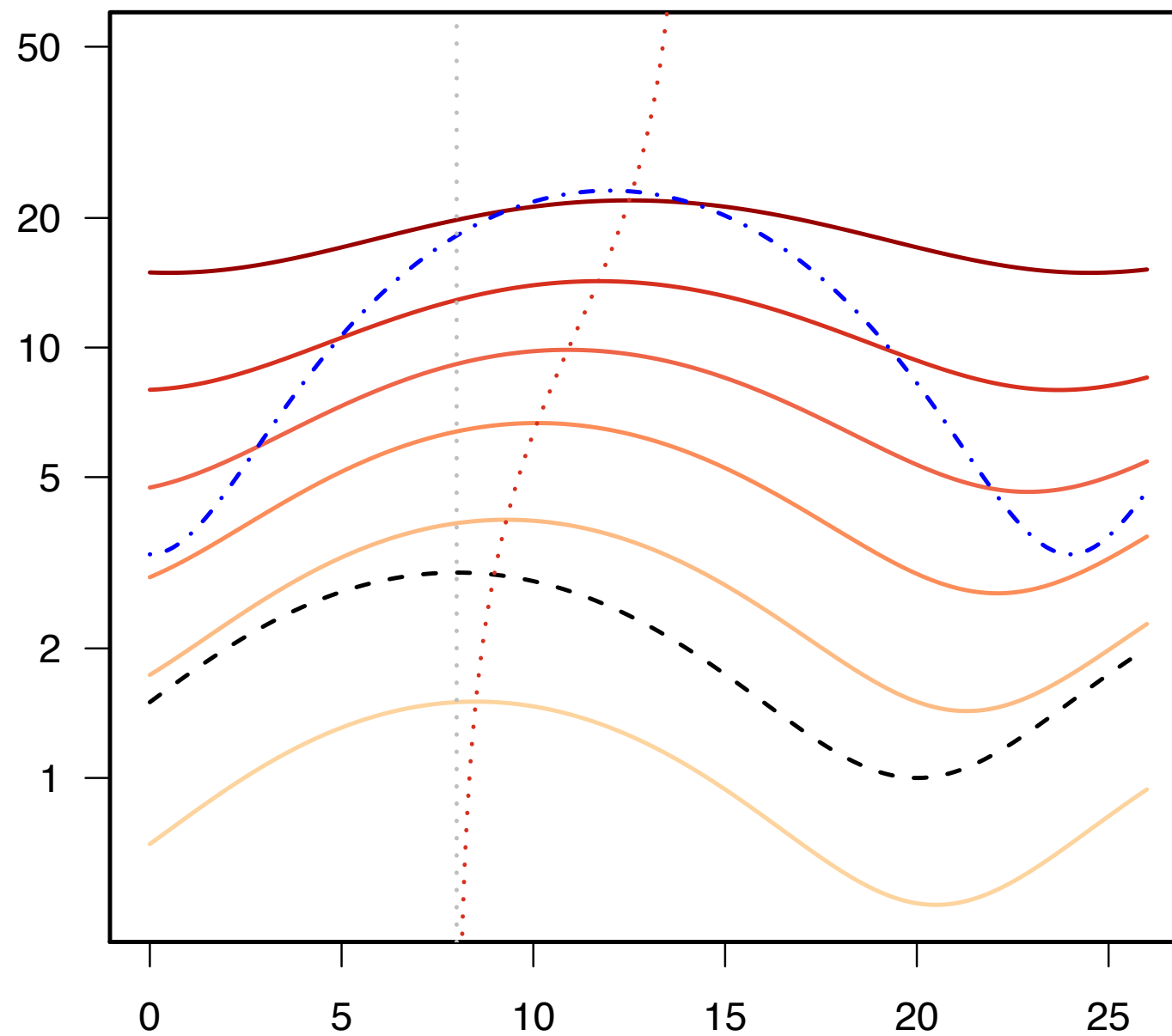
$$m(t) = \frac{P_0}{\gamma} (1 - e^{-\gamma t}) + e^{-\gamma t} m(0) \rightarrow \frac{P_0}{\gamma} .$$

Case 2: $P(t) = P_0 + \cos(\omega t)$

$$m(t) \rightarrow \frac{P_0}{\gamma} + \frac{1}{\sqrt{\gamma^2 + \omega^2}} \cos(\omega(t - \tau)) .$$

$$\text{with } \sin \omega \tau = \frac{\omega}{\sqrt{\gamma^2 + \omega^2}}$$

Simplest model

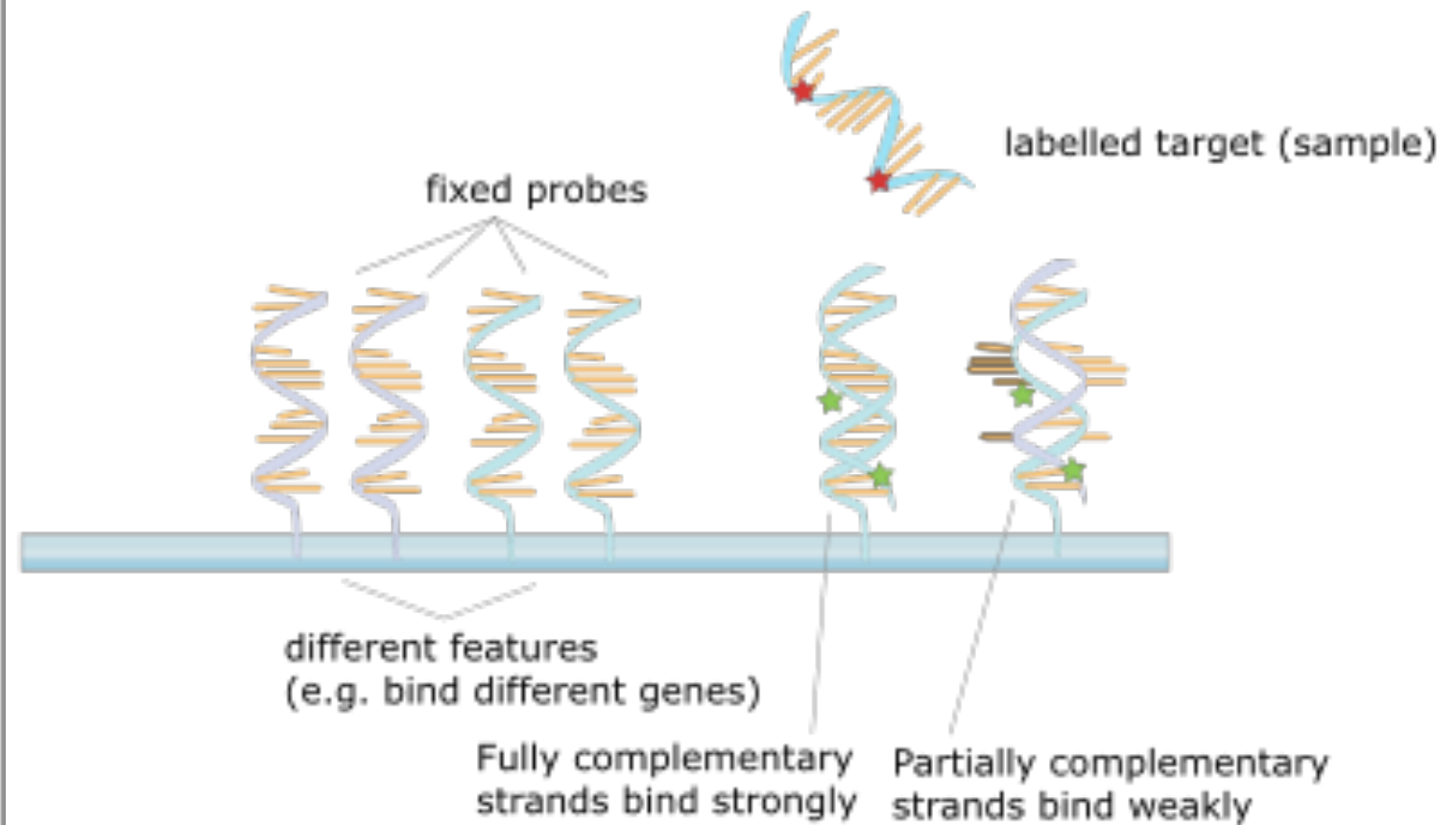
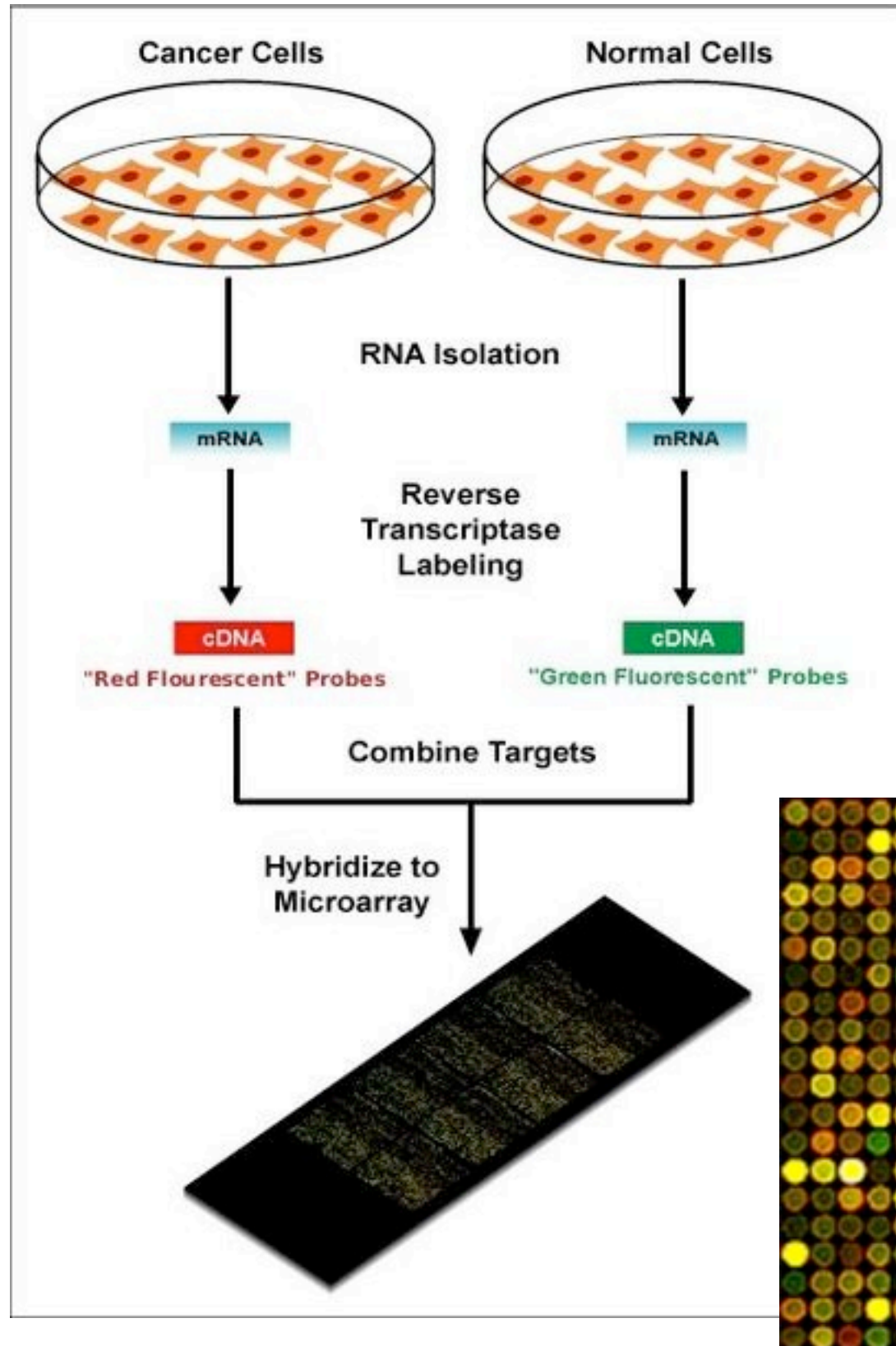


--- pre-mRNA
-.-.- post-trans. regulation

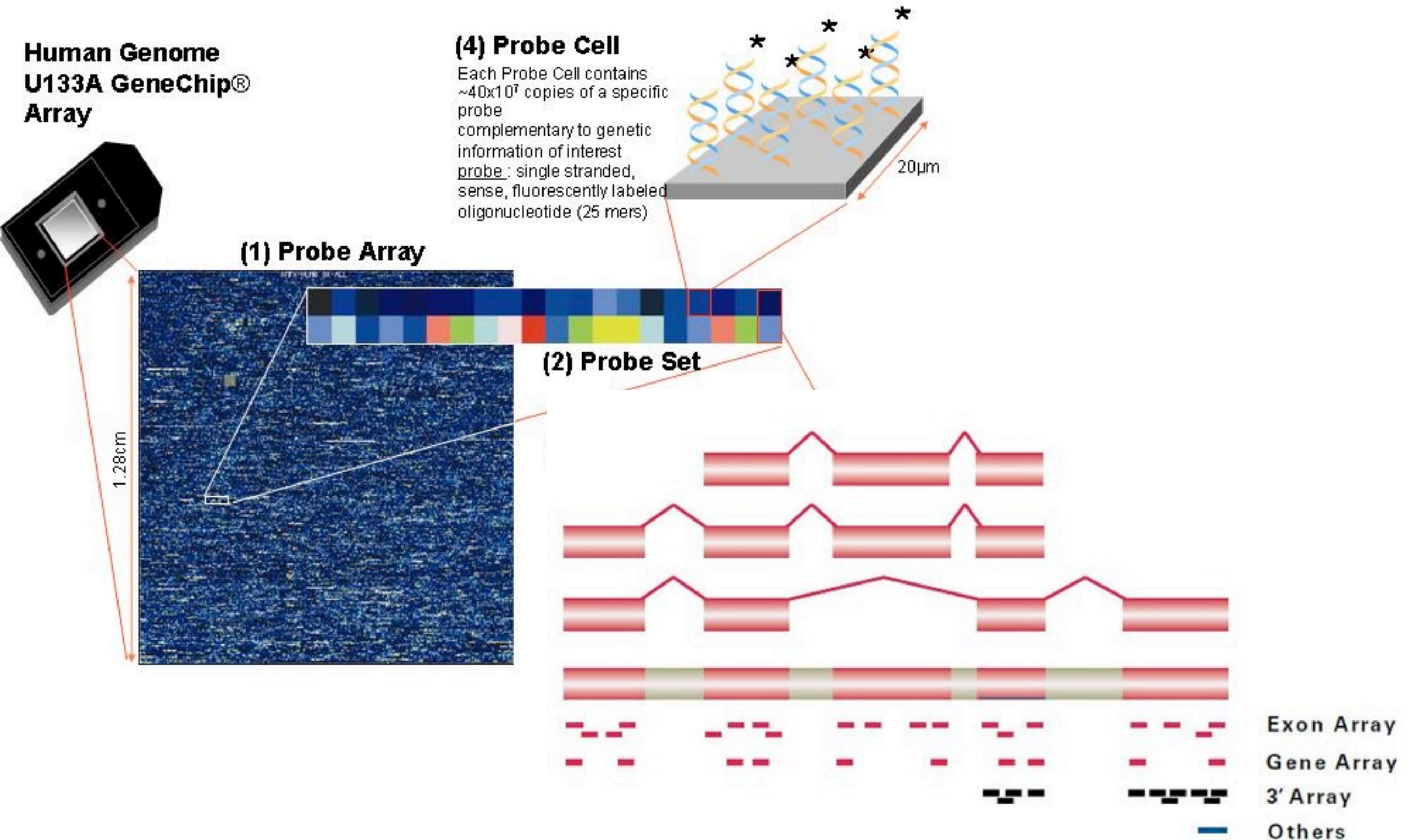
mRNA half-life [hr]

—	0.3
—	0.9
—	1.6
—	2.5
—	3.9
—	6.4

Microarrays



Microarrays



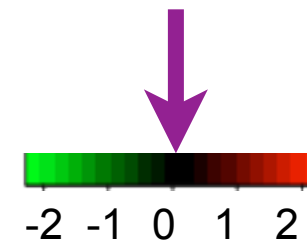
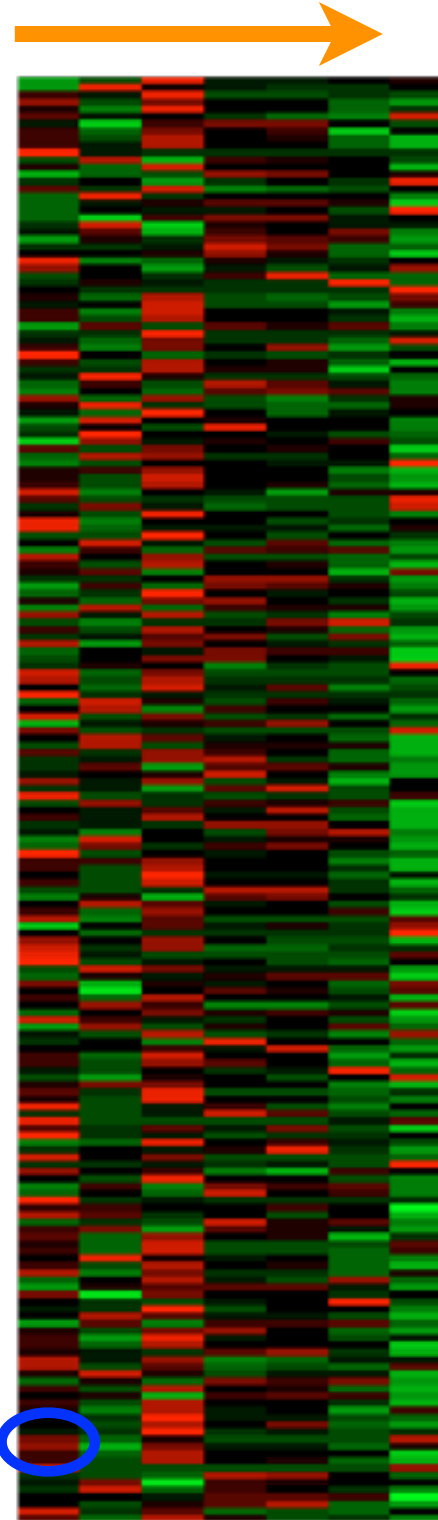
Output

Columns: conditions
(experiments)

Rows:
features
(genes)

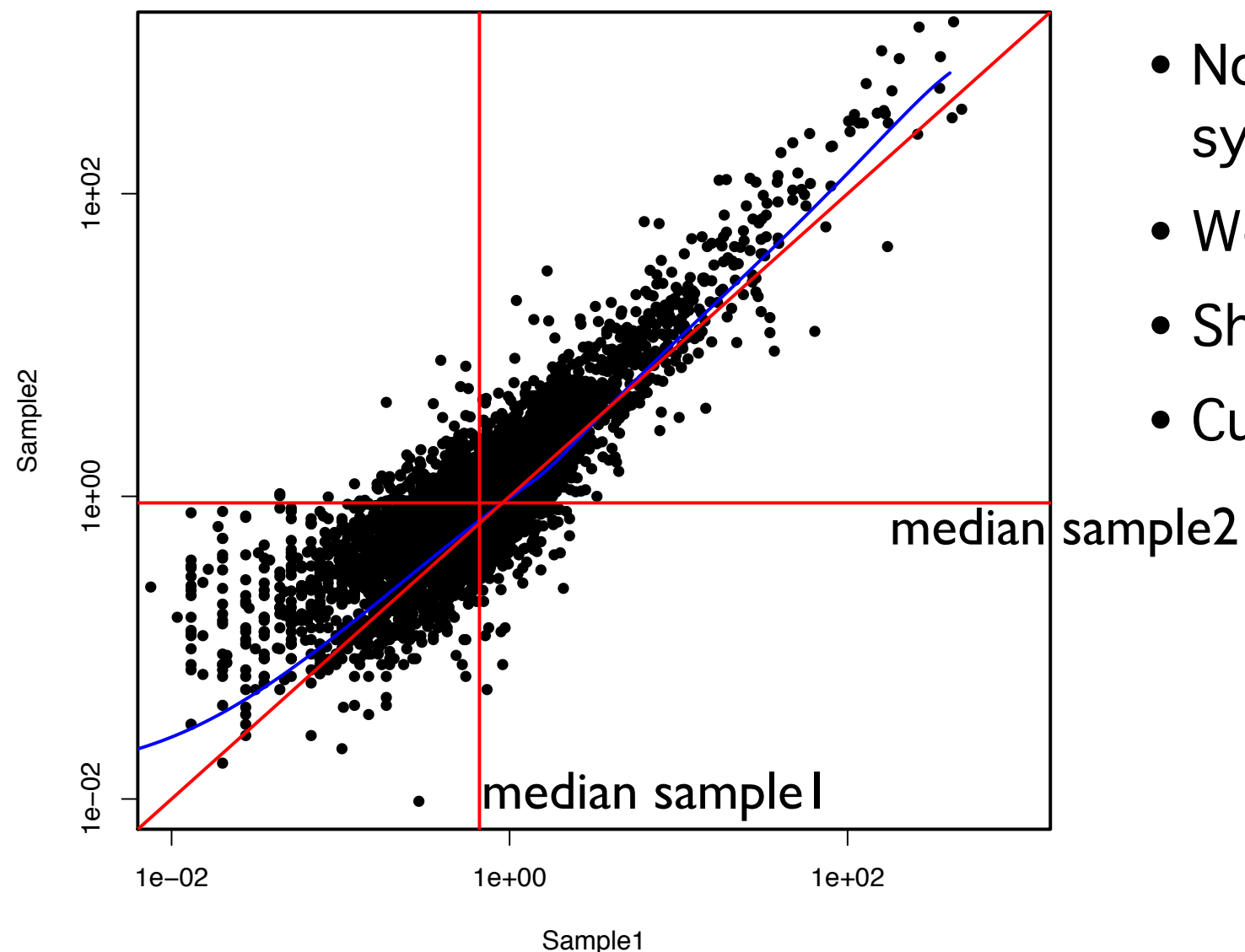
Signal is proportional
to concentration of
probe times
concentration of
mRNA

Cells: numbers
(measured signal)



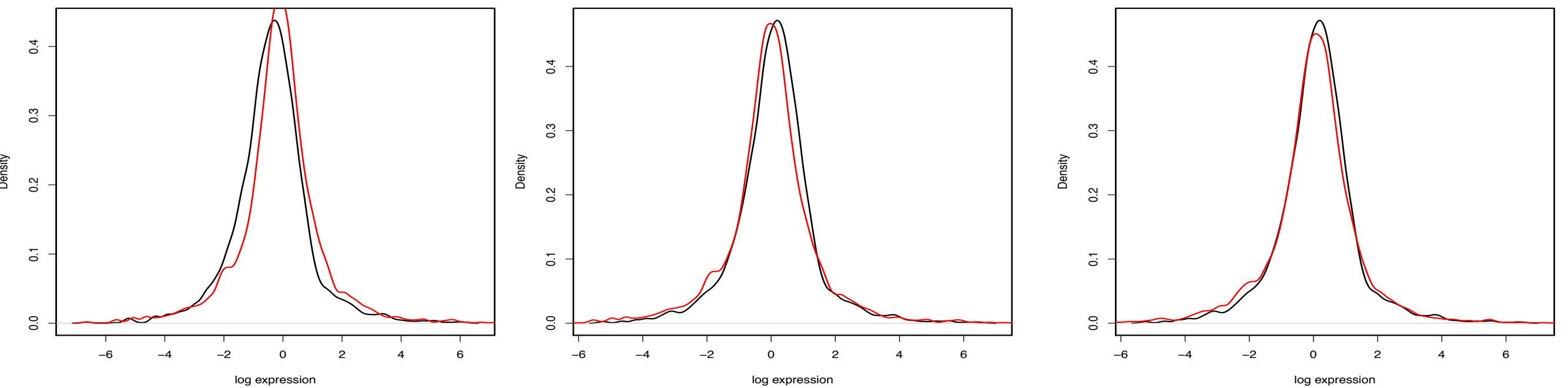
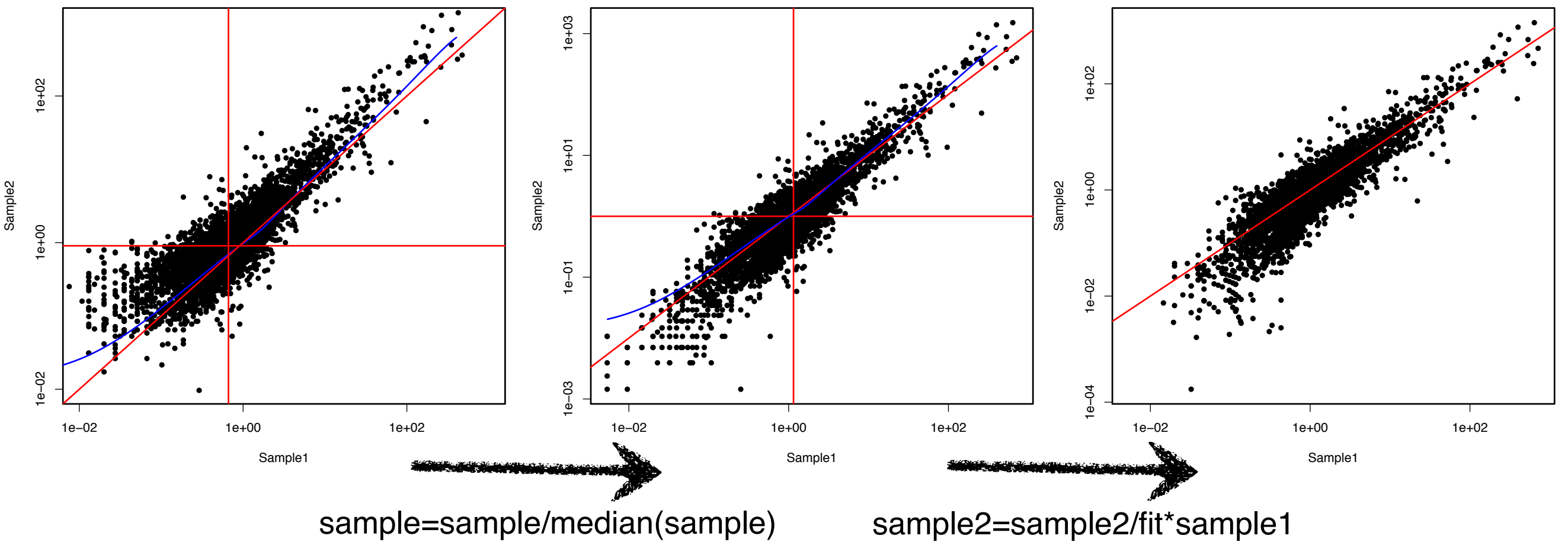
Normalization

Average variation between any 2 conditions is 0:
Systematic variation MUST BE technical artifact

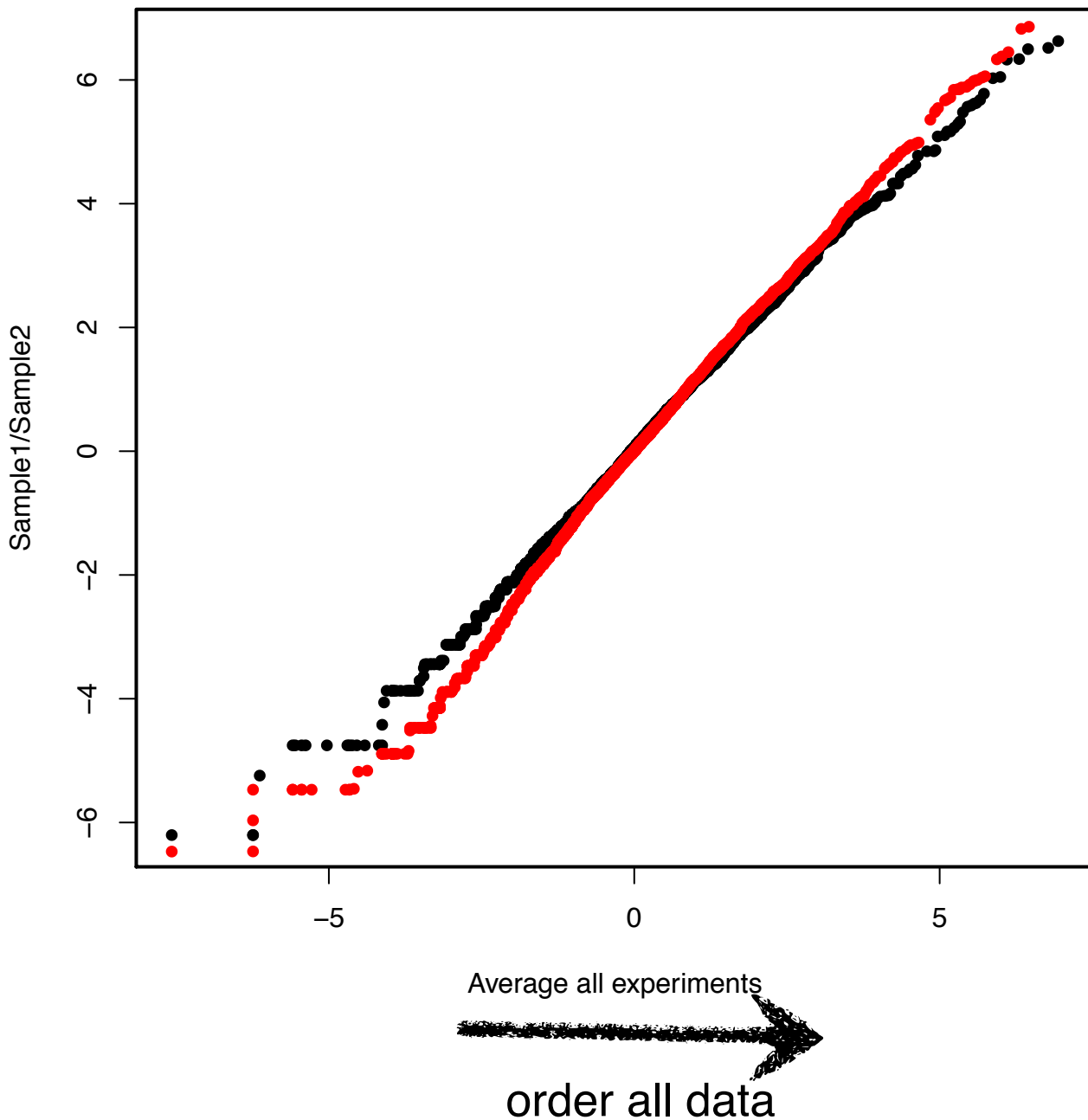


- Normalization consists in removing systematic variations
- Work in log-log coordinates
- Shift in medians
- Curved shaped

Normalization



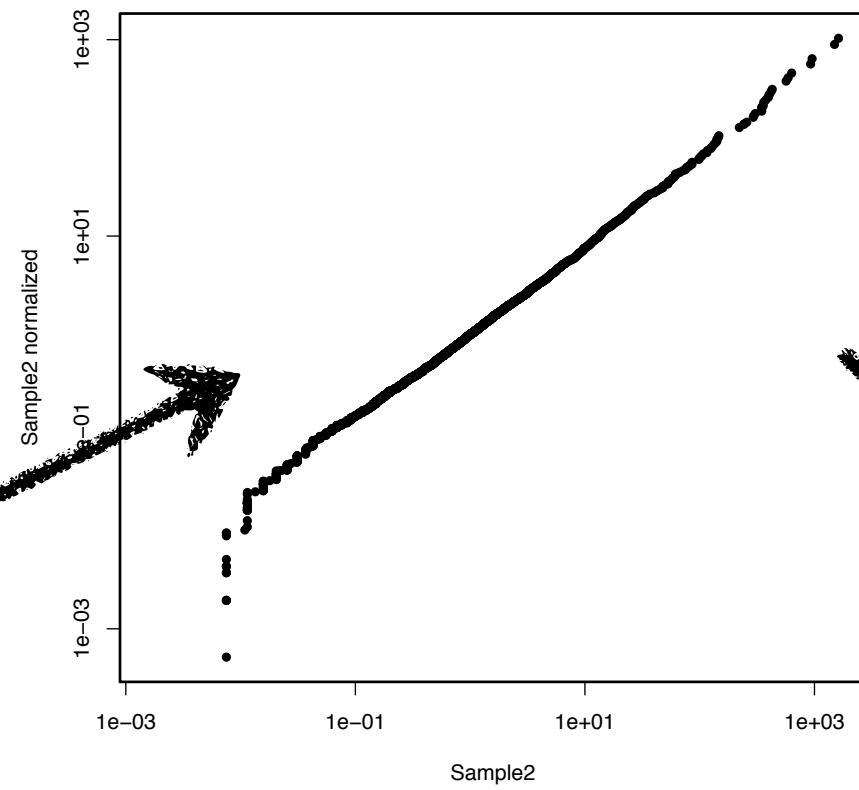
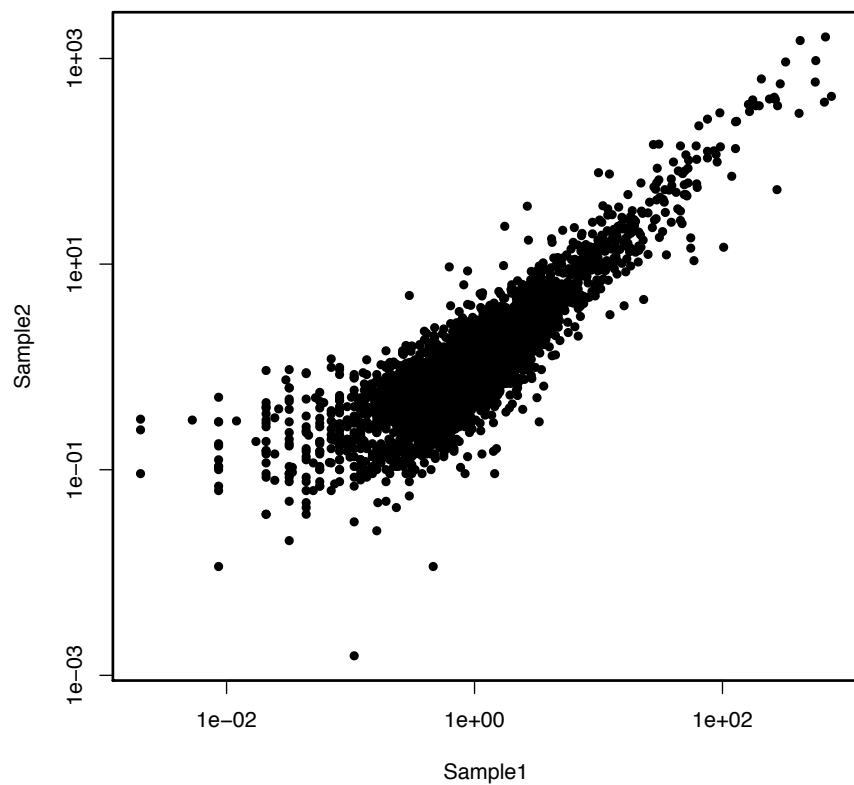
Quantile normalization



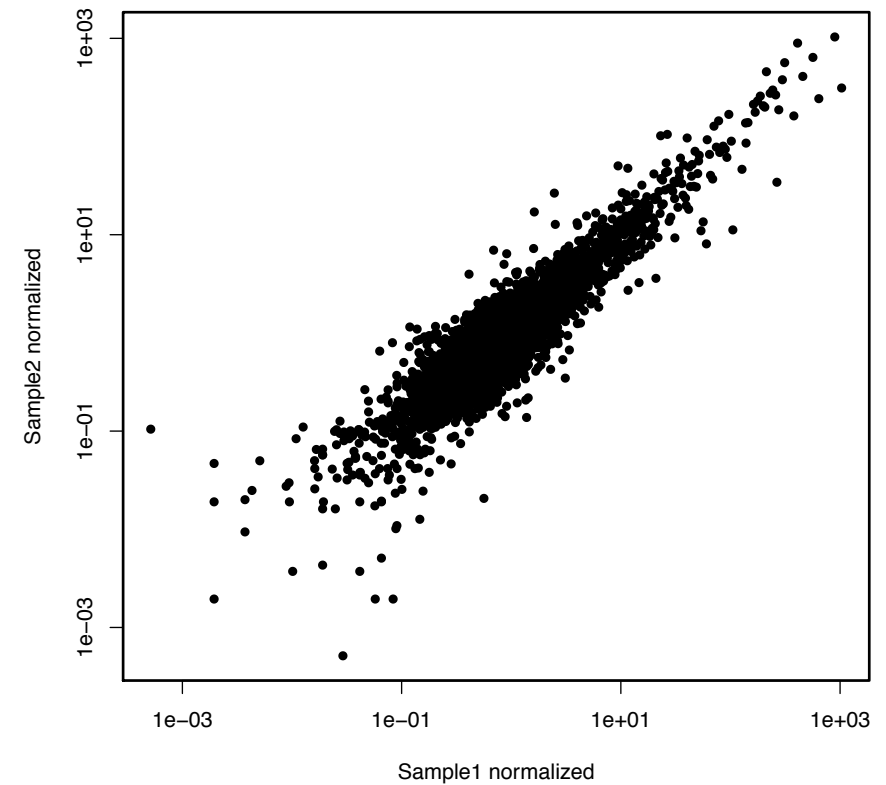
- Substitute ordered values from every sample with ordered values from average (or from specific distribution, e.g. gaussian)

Quantile normalization

before normalization

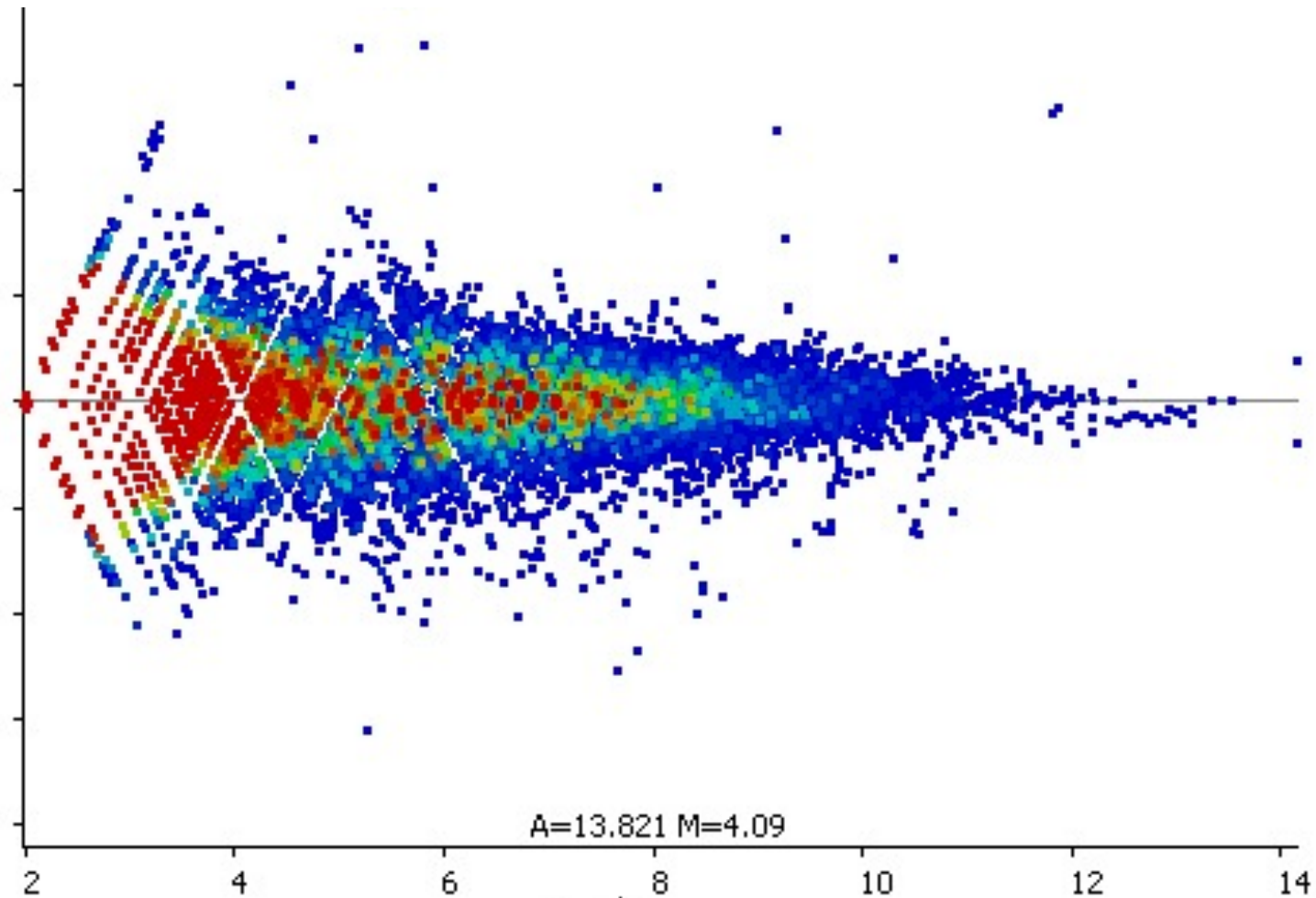


after normalization



MA plot: differential expression

$\text{Log}_2(\text{Sample1}/\text{Sample2})$



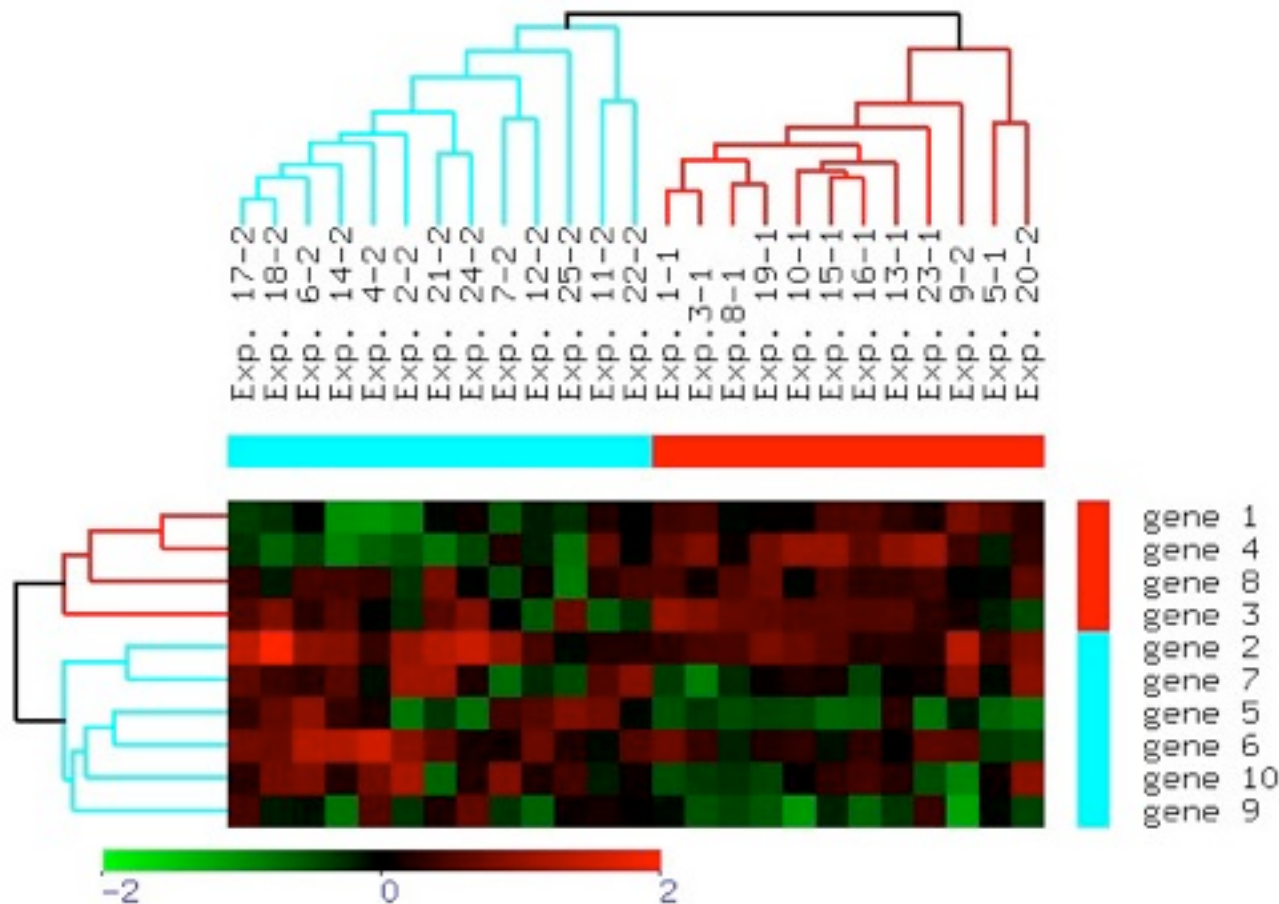
differential expression

$\text{Log}_{10}(\text{Sample1} * \text{Sample2}) / 2$

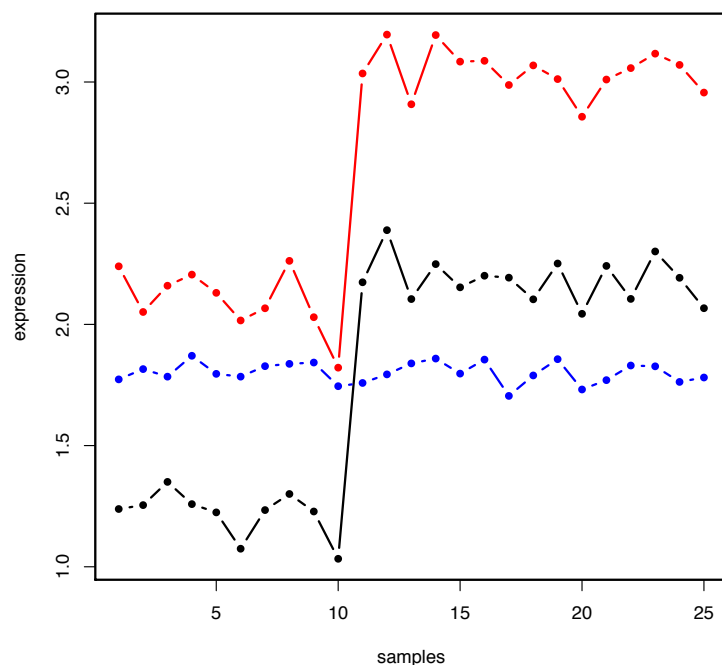
absolute expression



Clustering



- Same algorithm as UPGMA
- Distance matrix is $1 - \text{cor}(\text{row1}, \text{row2})$
- Update matrix with distance to average of two groups weighted by size
- Do the same for columns (rotate matrix)



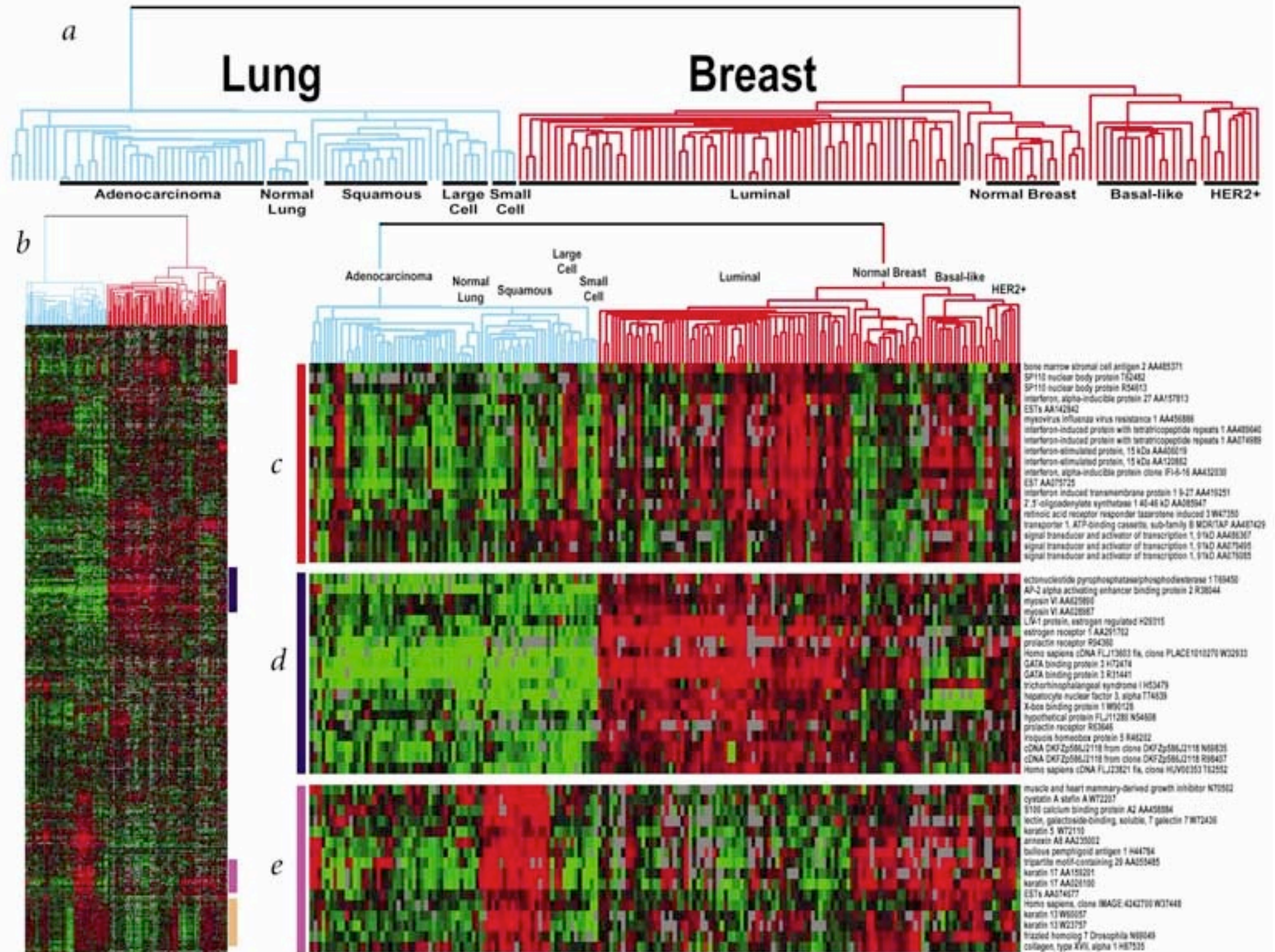
distance

	1	2
2	4.3	
3	2.4	4.9

correlation

	1	2
2	0.99	
3	-0.06	-0.03

Clustering



Clustering



- Similar expression patterns across many conditions probably imply a common set of regulators
- Looking for a shared set of functional annotations can help find the regulators

Linear models

	treat+WT	treat+KO	no treat+WT	no treat+KO
g	M11	M12	M13	M14

For each gene, make a linear relation between effect (expression) and factors (conditions)

$$\log(M_{cg}) = \alpha_g + \sum_n I_{cn} \beta_{ng} + \epsilon_g ,$$

$$\langle \log(M_{cg}) \rangle = \alpha_g + (I \cdot \beta)_{cg} ,$$

Design matrix:

	1	2	3	4
treat	1	1	0	0
KO	0	1	0	1