

1. $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$ 是 $\lim_{x \rightarrow x_0} f(x)$ 存在的 (充要条件).

$f(x)$ 在点 x_0 连续是 $f(x)$ 在点 x_0 处可导的 (必要) 条件.

2. $\lim_{n \rightarrow \infty} \frac{n^2 + 1}{2n^2 + 2n} = \frac{1}{2}$. $= \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n^2}}{2 + \frac{2}{n}} = \frac{1}{2}$

3. 求极限 $\lim_{x \rightarrow \infty} \frac{(x+2021)^2(2x+1)^3}{x^5+3}$. $= \lim_{x \rightarrow \infty} \frac{(1+\frac{2021}{x})^2 \cdot (2+\frac{1}{x})^3}{1+\frac{3}{x^5}} = 8$

4. $\lim_{x \rightarrow +\infty} x(\sqrt{1+x^2} - x) = \lim_{x \rightarrow +\infty} x \frac{1}{\sqrt{1+x^2} + x} = \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{\frac{1}{x^2} + 1} + 1} = \frac{1}{2}$

5. 计算 $\lim_{x \rightarrow 0} (1-2\sin x)^{\cot x} = \lim_{x \rightarrow 0} (1-2\sin x)^{\frac{1}{\tan x}} = \lim_{x \rightarrow 0} (1-2\sin x)^{\frac{1}{\sin x} \cdot \frac{\sin x}{\tan x}} = \lim_{x \rightarrow 0} (1-2\sin x)^{\frac{1}{\sin x} \cdot \frac{1}{2}} = e^{-2} = e^{-2}$

6. 求极限: $\lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{\sin x} - \frac{1}{\tan x} \right) = \frac{1}{2}$. $= \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right) = \lim_{x \rightarrow 0} \frac{1}{x} \frac{1 - \cos x}{\sin x} = \lim_{x \rightarrow 0} \frac{\frac{1}{2}x^2}{x^2} = \frac{1}{2}$

$\lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{\sin x} - \frac{1}{\tan x} \right) = \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{\tan x - \sin x}{\sin x \tan x} \right) = \lim_{x \rightarrow 0} \frac{x \cdot \frac{1}{2}x^2}{x^3} = \frac{1}{2}$

7. 计算 $\lim_{x \rightarrow 0} \frac{(1+x^2)^{\frac{1}{3}} - 1}{\cos x - 1} = \lim_{x \rightarrow 0} \frac{\frac{1}{3}x^2}{-\frac{1}{2}x^2} = -\frac{2}{3}$. $(1+x)^2 - 1 = 2x$

8. $\lim_{x \rightarrow 0} \left(\frac{1+x}{1-e^{-x}} - \frac{1}{x} \right) = \lim_{x \rightarrow 0} \frac{x+x^2-1+e^{-x}}{(1-e^{-x})x} = \lim_{x \rightarrow 0} \frac{x+x^2-1+e^{-x}}{x^2} = \lim_{x \rightarrow 0} \frac{1+2x-e^{-x}}{2x} = \lim_{x \rightarrow 0} \frac{2+e^{-x}}{2} = \frac{3}{2}$

9. 求极限 $\lim_{x \rightarrow 0} (\sec x - \tan x)$.

10. $\lim_{x \rightarrow 0} \frac{e^x - \sin x - 1}{(\arcsin x)^2} = \lim_{x \rightarrow 0} \frac{e^x - \sin x - 1}{x^2} = \lim_{x \rightarrow 0} \frac{e^x - \cos x}{2x} = \lim_{x \rightarrow 0} \frac{e^x + \sin x}{2} = \frac{1}{2}$

解: 原式 $= \lim_{x \rightarrow 0} \frac{e^x - \sin x - 1}{x^2} = \lim_{x \rightarrow 0} \frac{e^x - \cos x}{2x} = \lim_{x \rightarrow 0} \frac{e^x + \sin x}{2} = \frac{1}{2}$

11. $\lim_{x \rightarrow 0} \frac{2x(e^x - 1)}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{2x^2}{x^2} = 2$

多项式不能直接化简, 此时可上下同求导求值.