

1. $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$ 是 $\lim_{x \rightarrow x_0} f(x)$ 存在的 ()

$f(x)$ 在点 x_0 连续是 $f(x)$ 在点 x_0 处可导的 () 条件.

2. $\lim_{n \rightarrow \infty} \frac{n^2 + 1}{2n^2 + 2n} = \frac{1}{2}.$

3、求极限 $\lim_{x \rightarrow \infty} \frac{(x+2021)^2(2x+1)^3}{x^5+3}.$

4、 $\lim_{x \rightarrow +\infty} x(\sqrt{1+x^2} - x) = \lim_{x \rightarrow +\infty} x \frac{1}{\sqrt{1+x^2} + x} = \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{\frac{1}{x^2} + 1} + 1} = \frac{1}{2}$

5、计算 $\lim_{x \rightarrow 0} (1 - 2 \sin x)^{\cot x} = \lim_{x \rightarrow 0} (1 - 2 \sin x)^{\cot x} = \lim_{x \rightarrow 0} (1 - 2 \sin x)^{\frac{1}{-2 \sin x} \frac{-2 \sin x \cos x}{\sin x}} = e^{-2}$

6、求极限： $\lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{\sin x} - \frac{1}{\tan x} \right) = \frac{1}{2}.$

$$\lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{\sin x} - \frac{1}{\tan x} \right) = \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{\tan x - \sin x}{\sin x \tan x} \right) = \lim_{x \rightarrow 0} \frac{x \cdot \frac{1}{2} x^2}{x^3} = \frac{1}{2}$$

7、计算 $\lim_{x \rightarrow 0} \frac{(1+x^2)^{\frac{1}{3}} - 1}{\cos x - 1} = \lim_{x \rightarrow 0} \frac{\frac{1}{3} x^2}{-\frac{1}{2} x^2} = -\frac{2}{3}.$

8. $\lim_{x \rightarrow 0} \left(\frac{1+x}{1-e^{-x}} - \frac{1}{x} \right)$

9. 求极限 $\lim_{x \rightarrow 0} (\sec x - \tan x).$

10. $\lim_{x \rightarrow 0} \frac{e^x - \sin x - 1}{(\arcsin x)^2}.$

解：原式 $= \lim_{x \rightarrow 0} \frac{e^x - \sin x - 1}{x^2} = \lim_{x \rightarrow 0} \frac{e^x - \cos x}{2x} = \lim_{x \rightarrow 0} \frac{e^x + \sin x}{2} = \frac{1}{2}.$

11. $\lim_{x \rightarrow 0} \frac{2x(e^x - 1)}{\sin^2 x}$