Tutorial 1
Robotics

Direct and inverse kinematics

1 Introduction

We propose to study the geometric and kinematic modeling of a manipulator arm developed by the *Interactive Robotics Laboratory* of the *CEA List* (Fig. 1). This robot, which kinematic chain is of serial type, has 6 revolute joints (j_i with $i = 1, \ldots, 6$).

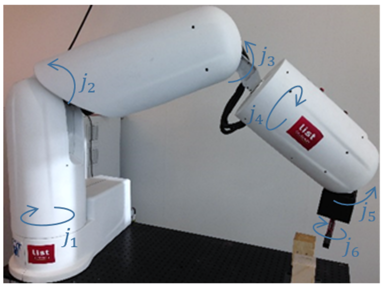

FIGURE 1 – Prototype of the robotic manipulator of *CEA-LIST*.

The numerical values of the robot parameters, required for the completion of this tutorial, are specified in the table 1. The use of *Matlab*^{TM} sofware is required to perform the tutorial.

2 Direct geometric model

Q1. Fill in the figure 2 giving the frames attached to the successive links of the robot according to the MDH convention (axis names and geometric distances should be reported on the completed figure).

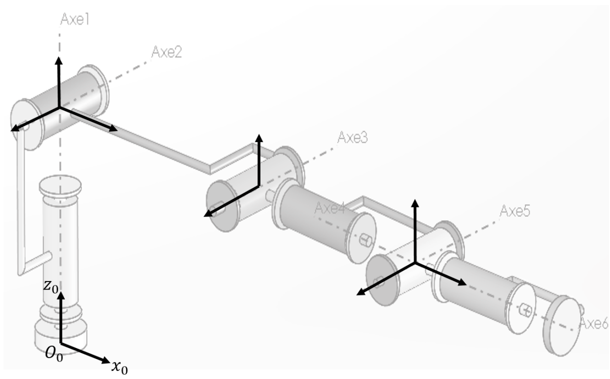

FIGURE 2 – Description of the robot's geometry.

Q2. Fill in the table with the geometric parameters of the robot.

i	α_i	d_i	θ_i	r_i
1	?	?	?	?
2	?	?	?	?
3	?	?	?	?
4	?	?	?	?
5	?	?	?	?
6	?	?	?	?

Q3. Programming of the Matlab function to compute the direct geometric model of the robot :

- Q3a) Write a generic function `TransformMatElem`$(\alpha_i, d_i, \theta_i, r_i)$ which output argument is the homogeneous transform matrix $\overline{g}$ between two successive frames ;
- Q3b) Write a function `ComputeDGM`(α, d, θ, r) which computes the direct geometric model of any robot with series open kinematic chain, taking as input arguments the robot's geometric parameters vectors (α, d, θ, r) ;
- Q3c) Using the results of question $Q2$, compute the homogeneous transform matrix $\overline{g}_{0E}$ which gives the position and the orientation of the frame $\mathcal{R}_E$ attached to the end-effector of the robot, expressed in the base frame $\mathcal{R}_0$ ($\mathcal{R}_E$ is defined by a translation of the frame $\mathcal{R}_6$ by a distance r_E along the z_6 axis).

Q4. What are the values of positions P_x, P_y, P_z and the parameters related to the orientation $R_{n,q}$ (n being the direction vector and $q \in [0, \pi]$ the rotation angle such that $R_{n,q} = R_{0E}$) of the end-effector frame for the two joint configurations $q_i = \left[-\frac{\pi}{2}, 0, -\frac{\pi}{2}, -\frac{\pi}{2}, -\frac{\pi}{2}, -\frac{\pi}{2}\right]^t$ and $q_f = \left[0, \frac{\pi}{4}, 0, \frac{\pi}{2}, \frac{\pi}{2}, 0\right]^t$ ($q = [q_1, \ldots, q_6]^t$) ?

Q5. Write a function `PlotFrame`(q) which provides a visualization of the position and the orientation of the end-effector frame $\mathcal{R}_E$ with respect to the base frame $\mathcal{R}_0$ for the joint configurations q_i and q_f.

3 Direct kinematic model

Q6. Write a function `ComputeJac`(α, d, θ, r) which output is the Jacobian matrix ${}^0J$ (computed by the method of velocities composition).
Reminder : the Jacobian matrix relates the velocities in the task coordinates of the end-effector frame in $\mathcal{R}_0$, for a given joint configuration q, to the joint velocities :

$$ {}^0\mathcal{V}_{0,E} = \begin{bmatrix} {}^0V_{0,E}\left(O_E\right) \\ {}^0\omega_{0,E} \end{bmatrix} = \begin{bmatrix} {}^0J_v\left(q\right) \\ {}^0J_\omega\left(q\right) \end{bmatrix} \dot{q} = {}^0J\left(q\right)\dot{q} $$

What are the values of the twists at O_E evaluated with $q = q_i$ and $q = q_f$ with the joint velocities $\dot{q} = [0.5, 1.0, -0.5, 0.5, 1.0, -0.5]^t$?

Q7. In the rest of the study, we restrict the analysis of operational end-effector velocities to translational velocities via ${}^0J_v\left(q\right)$.
Qualify the transmission of velocities between the joint and task spaces for the corresponding q_i and q_f configurations : what is the preferred direction to transmit velocity in the task space when the manipulator configuration is q_i ? Same question for q_f ? What are the corresponding velocity manipulabilities ? On the figure obtained in the

question **Q**5 showing the frames in the task space, plot the velocities ellipsoids [1] corresponding to the q_i and q_f configurations.

4 Inverse geometric model

Q8. In this study, the resolution of the inverse geometric model is considered numerically by exploiting the inverse differential model. Moreover, the study is restricted to the position only of the robot's end-effector frame in the task space (no constraint on the orientation of the end-effector frame).
Using an iterative procedure exploiting the pseudo-inverse of the Jacobian matrix, program a function $q^* = \texttt{ComputeIGM}(X_d, q_0, k_{max}, \epsilon_x)$ having as input argument the desired task position X_d and the initial condition q_0. The maximum number of iterations k_{max} of the algorithm, as well as the norm of the tolerated Cartesian error $\|X_d - f(q_k)\| < \epsilon_x$, define the stopping criteria of the algorithm.
Compute q^* when the function is called with the following arguments :

1. $X_d = X_{d_i} = (-0.1, -0.7, 0.3)^t$, $q_0 = [-1.57, 0.00, -1.47, -1.47, -1.47, -1.47$ $, -1.47]$, $k_{max} = 100$, $\epsilon_x = 1\text{mm}$?
2. $X_d = X_{d_f} = (0.64, -0.10, 1.14)^t$, $q_0 = [0, 0.80, 0.00, 1.00, 2.00, 0.00]$, $k_{max} = 100$, $\epsilon_x = 1\text{mm}$?

Check the accuracy of the result using the function calculated in **Q**3.

5 Inverse kinematic model

Q9. In this question, the trajectory of the end effector to be followed in the task space must allow the desired final position X_{d_f} to be reached by following a straight line in the task space starting at the initial position X_{d_i}. This rectilinear motion is carried out at a constant speed $V = 1m.s^{-1}$ and is sampled at a period $T_e = 1\text{ms}$. The position of the end effector at the time instant kT_e is noted X_{d_k}. The initial configuration of the robot is given by q_i (found in question **Q**4).
Using the inverse differential kinematic model, write a function entitled $\texttt{ComputeIKM}(X_{d_i}, X_{d_f}, V, T_e, q_i)$ realizing the coordinate transform to provide the series of setpoint values q_{d_k} corresponding to the X_{d_k} to the joint drivers. To do this, after having programmed the time law corresponding to the required motion, you can use the function developed in question **Q**8 capable of calculating the iterative MGI from the pseudo-inverse of the Jacobian matrix.
On the figure obtained in question **Q**4, superimpose the reference trajectory made up of the sequence of positions X_{d_k} to be followed by the end effector. Using the function $\texttt{PlotFrame}(q_{d_k})$, display on the same figure the sequence of frames $\mathcal{R}_E$ for some configurations q_{d_k}. Comment on the trajectory actually followed by the end effector.

Q10. Plot the temporal evolution of the joint variables q_1 to q_6 calculated in the previous question. For each joint variable, graphically overlay the allowable extreme values corresponding to the joint limits :

$$q_{min} = \left[\ -\pi, -\tfrac{\pi}{2}, -\pi, -\pi, -\tfrac{\pi}{2}, -\pi\ \right]$$

1. You can use the functions `ellipsoid` and `rotate` provided by $Matlab^{TM}$.

and

$$q_{max} = \left[\begin{array}{c} 0, \frac{\pi}{2}, 0, \frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2} \end{array} \right]$$

Comment on the evolution of the joint variables obtained in the previous question.

Q11. In this question, we modify the algorithm developed in question ***Q***9. We wish to take into account the distance of the values taken by the articular variables from their limits in the computation of the inverse kinematic model. To do so, develop a new function `ComputeIKMlimits`$(X_{d_i}, X_{d_f}, V, T_e, q_i, q_{min}, q_{max})$ which considers a secondary task aiming at keeping some distance from the articular stops q_{min} and q_{max}. By the technique of the gradient projected into the null space of ${}^0J_v(q)$, you will consider minimizing the following potential function :

$$H_{lim}(q) = \sum_{i=1}^{n} \left(\frac{q_i - \overline{q}_i}{q_{max} - q_{min}} \right)^2 \quad \text{where} \quad \overline{q}_i = \frac{q_{max} - q_{min}}{2}$$

Plot the new temporal evolution of the joint variables q_1 to q_6 for the reference trajectory given in the question ***Q***9. Comment on the values taken by the joint variables.

Appendix

Parameters	Numerical values	Type of parameter
d_3	$0.7m$	Geometric parameter
r_1	$0.5m$	Geometric parameter
r_4	$0.2m$	Geometric parameter
r_E	$0.1m$	Geometric parameter

Table 1 – Numerical values of the robot parameters.