Chapter 4: Association Rules

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Mining Association Rules in Large Databases

- Association rule mining
- Algorithms for scalable mining of (single-dimensional Boolean) association rules in transactional databases
- Mining various kinds of association/correlation rules
- Constraint-based association mining
- Sequential pattern mining
- Applications/extensions of frequent pattern mining
- Summary

What Is Association Mining?

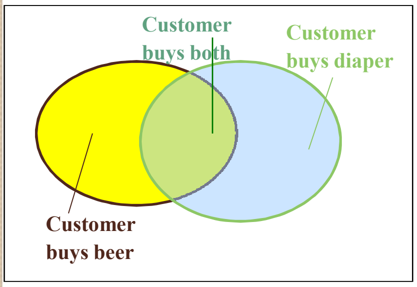
- Association rule mining:
  - Finding frequent patterns, associations, correlations, or causal structures among sets of items or objects in transaction databases, relational databases, and other information repositories.
  - Frequent pattern: pattern (set of items, sequence, etc.) that occurs frequently in a database [AIS93]
- Motivation: finding regularities in data
  - What products were often purchased together? — Beer and diapers?!
  - What are the subsequent purchases after buying a PC?
  - What kinds of DNA are sensitive to this new drug?
  - Can we automatically classify web documents?

Why Is Association Mining Important?

- Foundation for many essential data mining tasks
  - Association, correlation, causality
  - Sequential patterns, temporal or cyclic association, partial periodicity, spatial and multimedia association
  - Associative classification, cluster analysis, iceberg cube, fascicles (semantic data compression)
- Broad applications
  - Basket data analysis, cross-marketing, catalog design, sale campaign analysis
  - Web log (click stream) analysis, DNA sequence analysis, etc.

Basic Concepts: Association Rules

Transaction-id	Items bought
10	A, B, C
20	A, C
30	A, D
40	B, E, F



- Itemset X={x_1, …, x_k}
- Find all the rules $X \rightarrow Y$ with min confidence and support
  - support, s, probability that a transaction contains X∪Y
  - confidence, c, conditional probability that a transaction having X also contains Y.

Let min_support = 50%, min_conf = 50%:

$A \rightarrow C$ (50%, 66.7%)

$C \rightarrow A$ (50%, 100%)

Mining Association Rules: Example

Transaction-id	Items bought
10	A, B, C
20	A, C
30	A, D
40	B, E, F

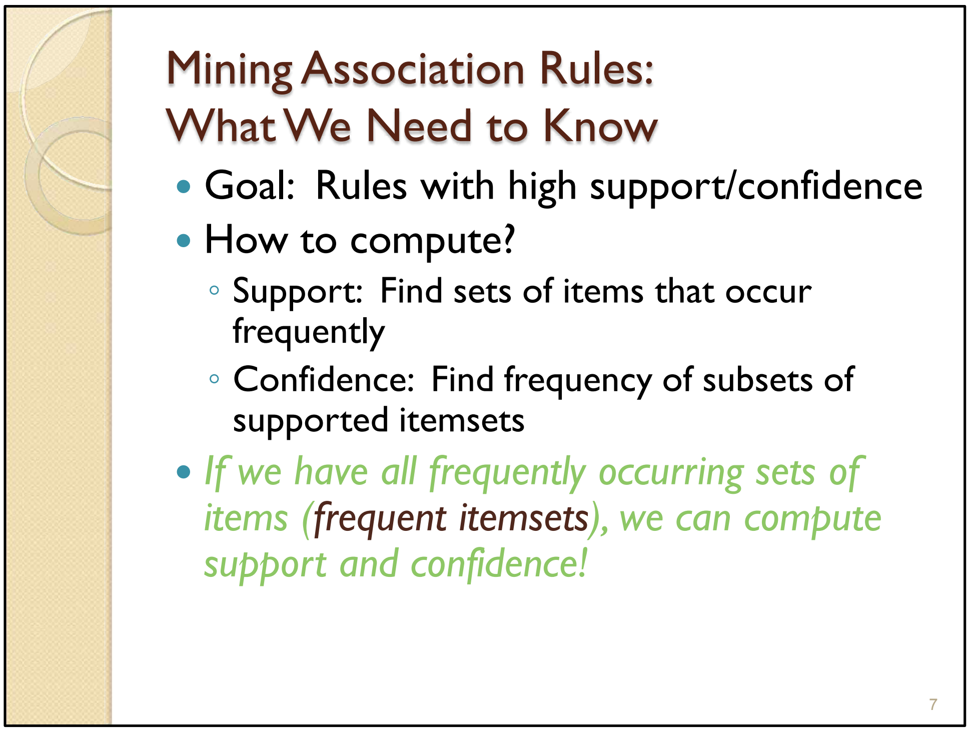
Min. support 50%
Min. confidence 50%

Frequent pattern	Support
{A}	75%
{B}	50%
{C}	50%
{A, C}	50%

For rule $A \Rightarrow C$:

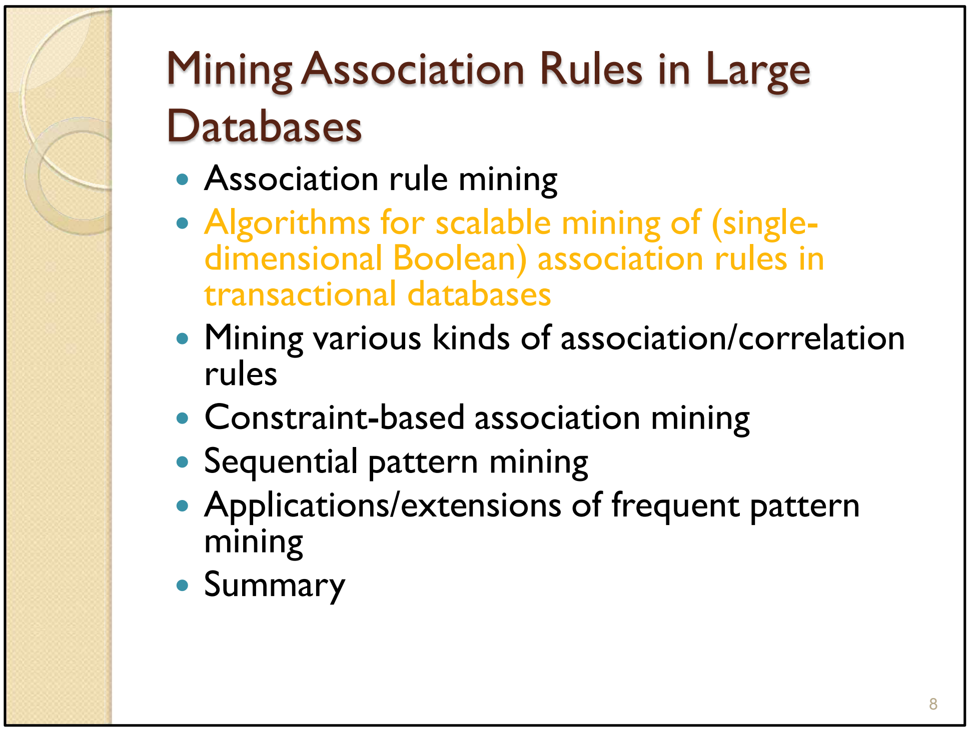
support = support({A}∪{C}) = 50%

confidence = support({A}∪{C})/support({A}) = 66.6%



Mining Association Rules: What We Need to Know

- Goal: Rules with high support/confidence
- How to compute?
  - Support: Find sets of items that occur frequently
  - Confidence: Find frequency of subsets of supported itemsets
- *If we have all frequently occurring sets of items (frequent itemsets), we can compute support and confidence!*



Mining Association Rules in Large Databases

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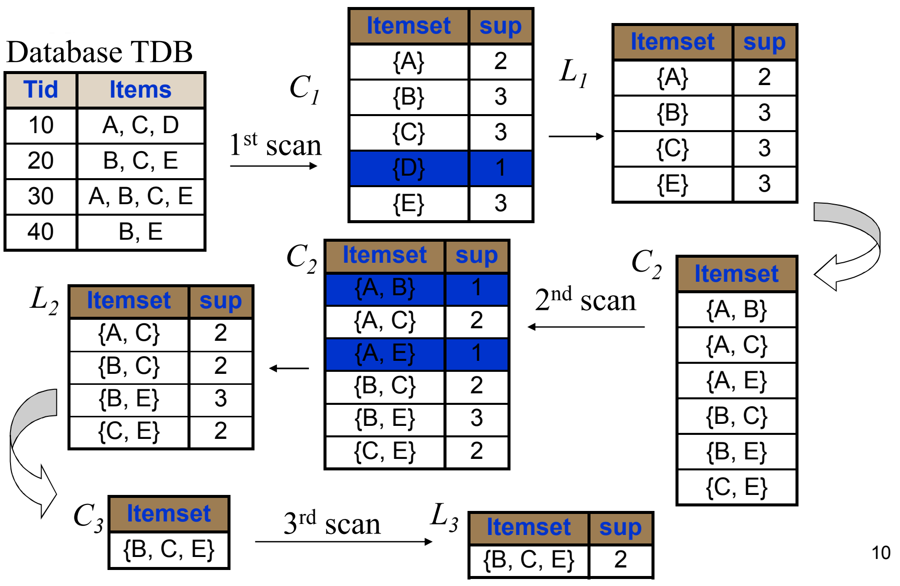
Apriori: A Candidate Generation-and-test Approach

- Any subset of a frequent itemset must be frequent
  - if **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
  - Every transaction having {beer, diaper, nuts} also contains {beer, diaper}
- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not be generated/tested!
- Method:
  - generate length (k+1) candidate itemsets from length k frequent itemsets, and
  - test the candidates against DB
- Performance studies show its efficiency and scalability
- Agrawal & Srikant 1994, Mannila, et al. 1994

The Apriori Algorithm—An Example



Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

L_2

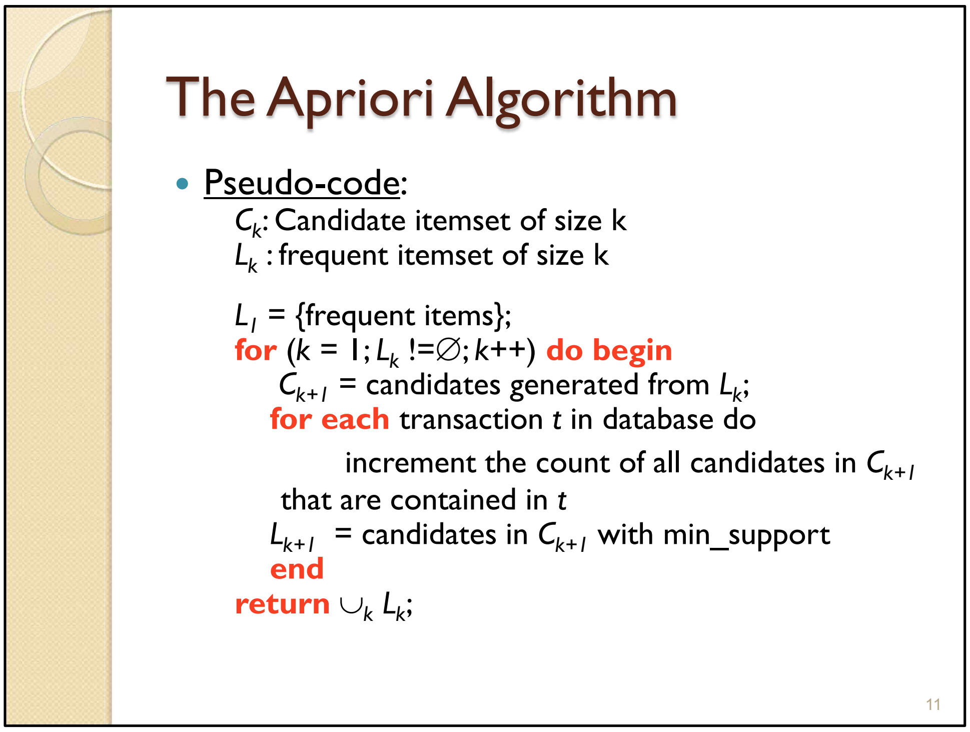
Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

L_3

Itemset	sup
{B, C, E}	2



The Apriori Algorithm

- Pseudo-code:

```
$C_k$: Candidate itemset of size k
$L_k$ : frequent itemset of size k

$L_1$ = {frequent items};
for ($k$ = 1; $L_k$ !=∅; $k$++) do begin
    $C_{k+1}$ = candidates generated from $L_k$;
    for each transaction $t$ in database do
            increment the count of all candidates in $C_{k+1}$
     that are contained in $t$
    $L_{k+1}$ = candidates in $C_{k+1}$ with min_support
    end
return $\cup_k L_k$;
```

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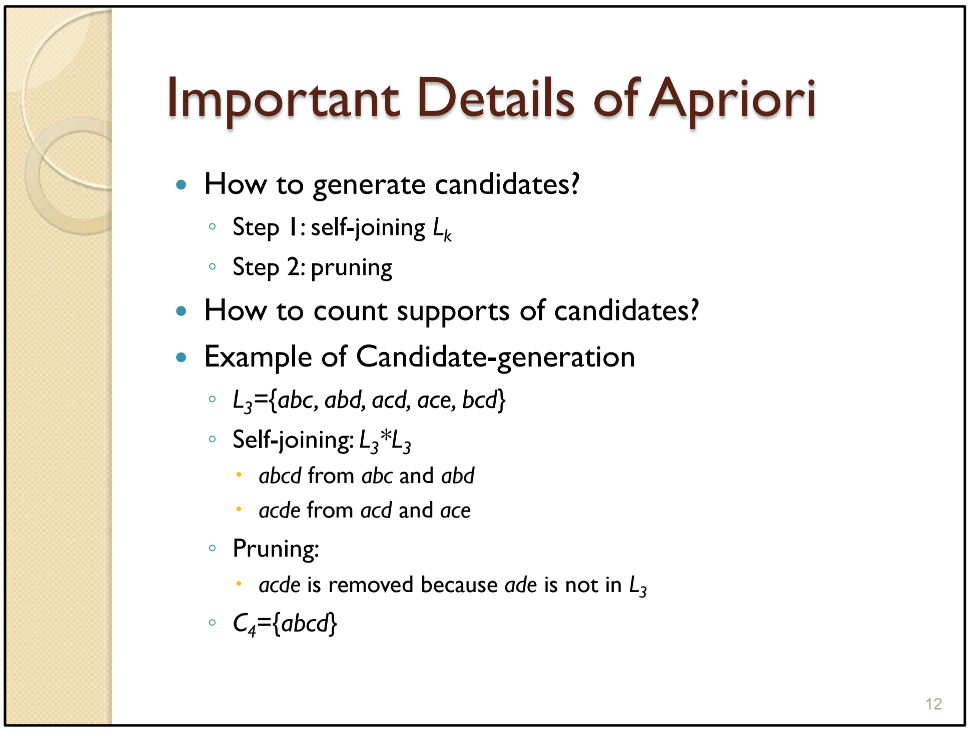

Important Details of Apriori

- How to generate candidates?
  - Step 1: self-joining L_k
  - Step 2: pruning
- How to count supports of candidates?
- Example of Candidate-generation
  - L_3={*abc, abd, acd, ace, bcd*}
  - Self-joining: L_3*L_3
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
  - Pruning:
    - *acde* is removed because *ade* is not in L_3
  - C_4={*abcd*}

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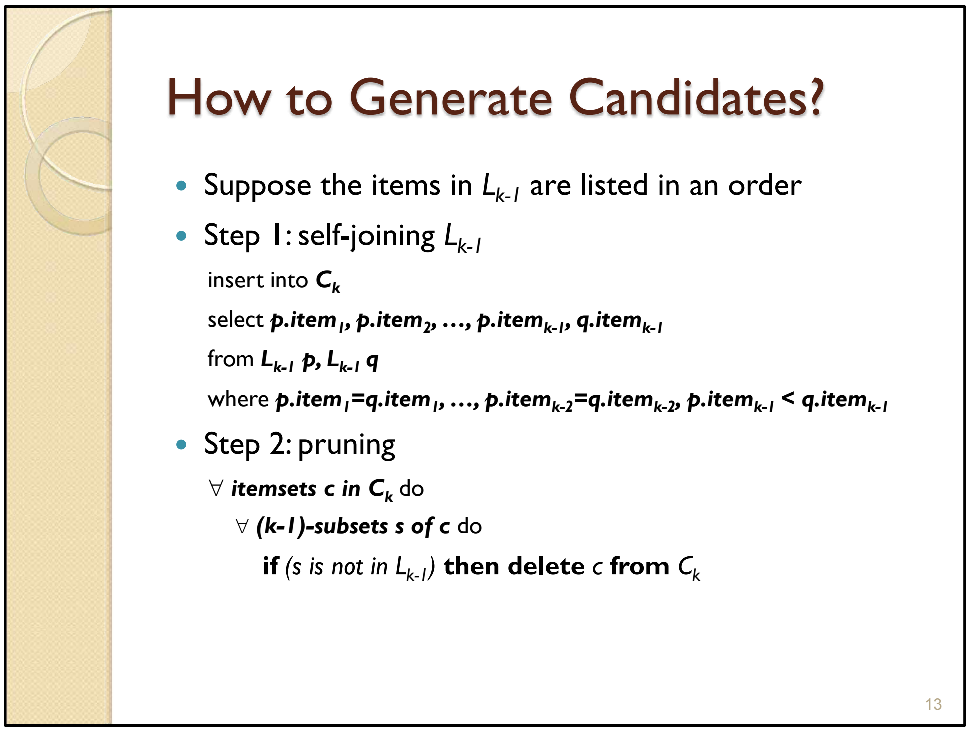

How to Generate Candidates?

- Suppose the items in L_{k-1} are listed in an order
- Step 1: self-joining L_{k-1}

```
insert into $C_k$
select $p.item_1, p.item_2, \ldots, p.item_{k-1}, q.item_{k-1}$
from $L_{k-1}$ $p$, $L_{k-1}$ $q$
where $p.item_1=q.item_1, \ldots, p.item_{k-2}=q.item_{k-2}, p.item_{k-1} < q.item_{k-1}$
```

- Step 2: pruning

```
$\forall$ itemsets $c$ in $C_k$ do
    $\forall$ $(k-1)$-subsets $s$ of $c$ do
        if ($s$ is not in $L_{k-1}$) then delete $c$ from $C_k$
```

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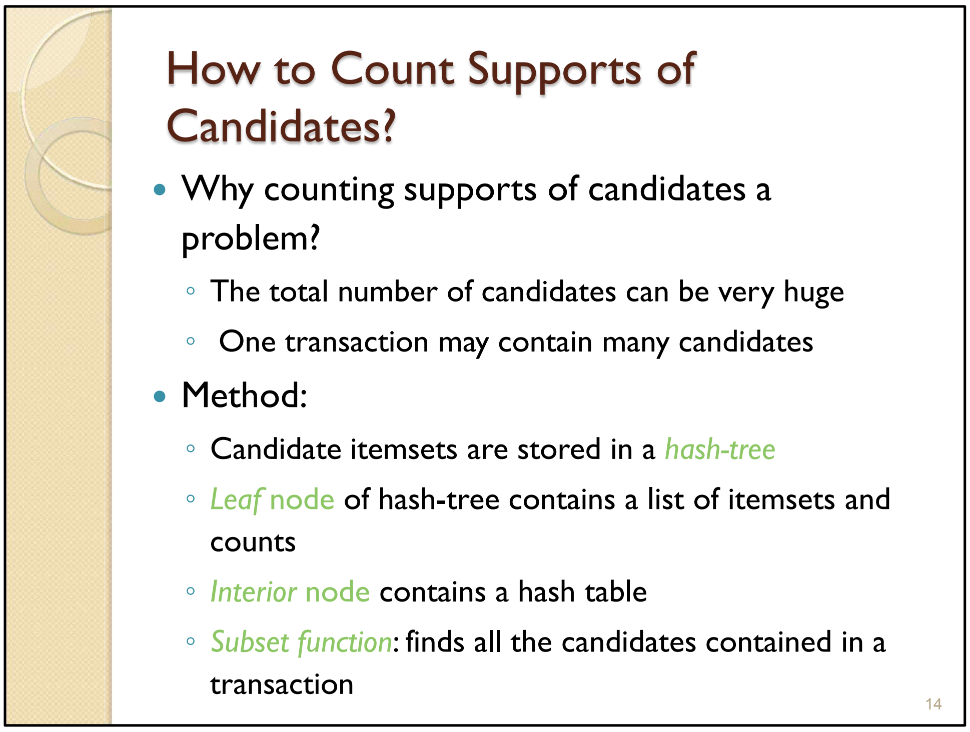

How to Count Supports of Candidates?

- Why counting supports of candidates a problem?
  - The total number of candidates can be very huge
  - One transaction may contain many candidates
- Method:
  - Candidate itemsets are stored in a *hash-tree*
  - *Leaf* node of hash-tree contains a list of itemsets and counts
  - *Interior* node contains a hash table
  - *Subset function*: finds all the candidates contained in a transaction

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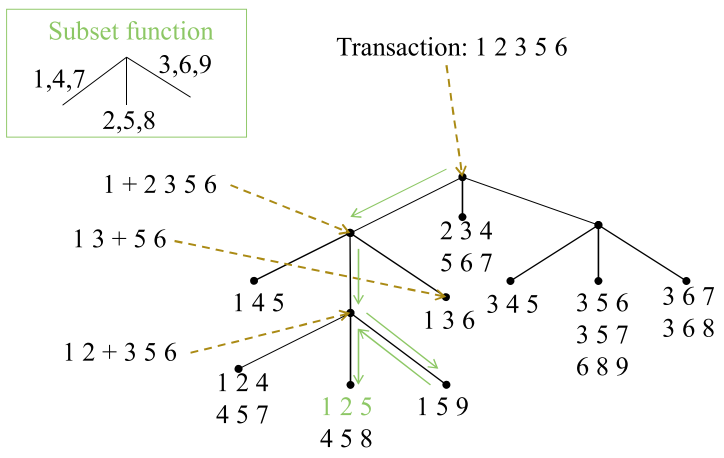
Example: Counting Supports of Candidates



Efficient Implementation of Apriori in SQL

- Hard to get good performance out of pure SQL (SQL-92) based approaches alone
- Make use of object-relational extensions like UDFs, BLOBs, Table functions etc.
  - Get orders of magnitude improvement
- S. Sarawagi, S. Thomas, and R. Agrawal. Integrating association rule mining with relational database systems: Alternatives and implications. In SIGMOD'98

Challenges of Frequent Pattern Mining

- Challenges
  - Multiple scans of transaction database
  - Huge number of candidates
  - Tedious workload of support counting for candidates
- Improving Apriori: general ideas
  - Reduce passes of transaction database scans
  - Shrink number of candidates
  - Facilitate support counting of candidates

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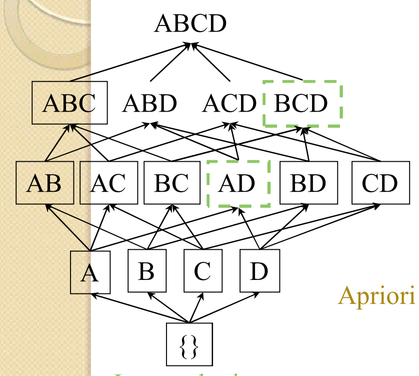
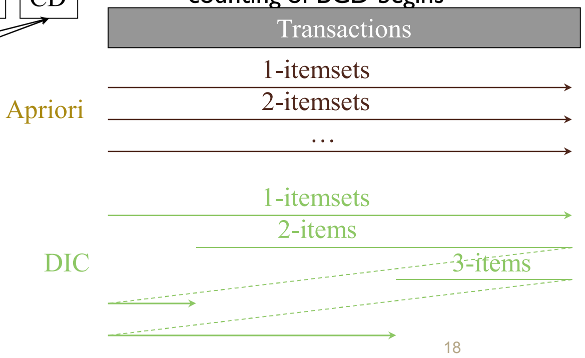
DIC: Reduce Number of Scans



Itemset lattice

- Once both A and D are determined frequent, the counting of AD begins
- Once all length-2 subsets of BCD are determined frequent, the counting of BCD begins



S. Brin R. Motwani, J. Ullman, and S. Tsur. Dynamic itemset counting and implication rules for market basket data. In *SIGMOD'97*

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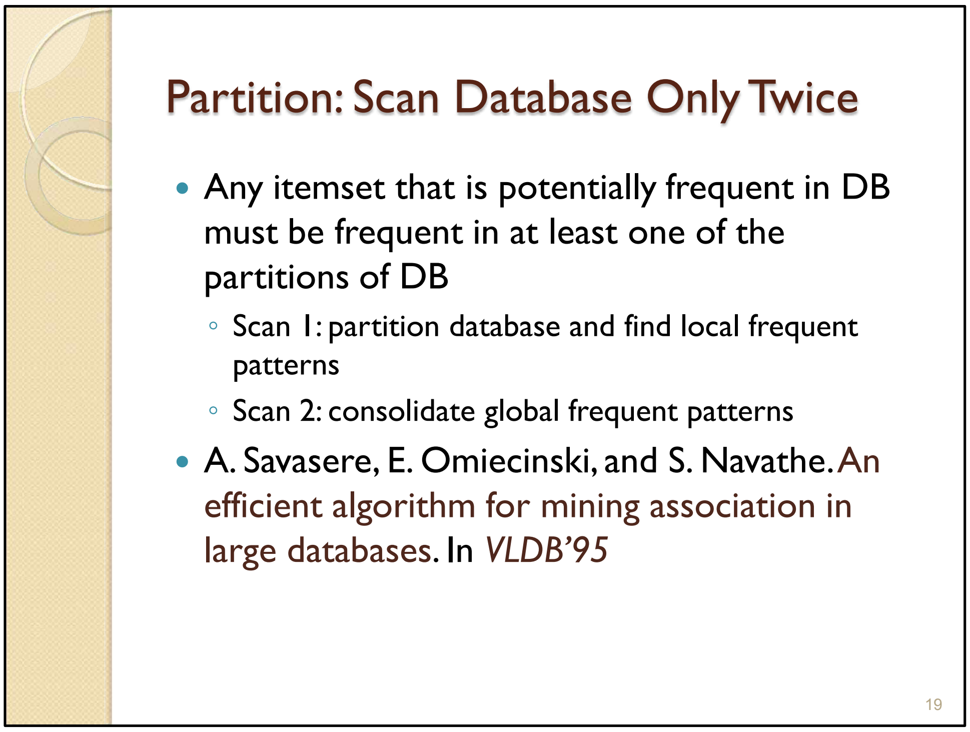
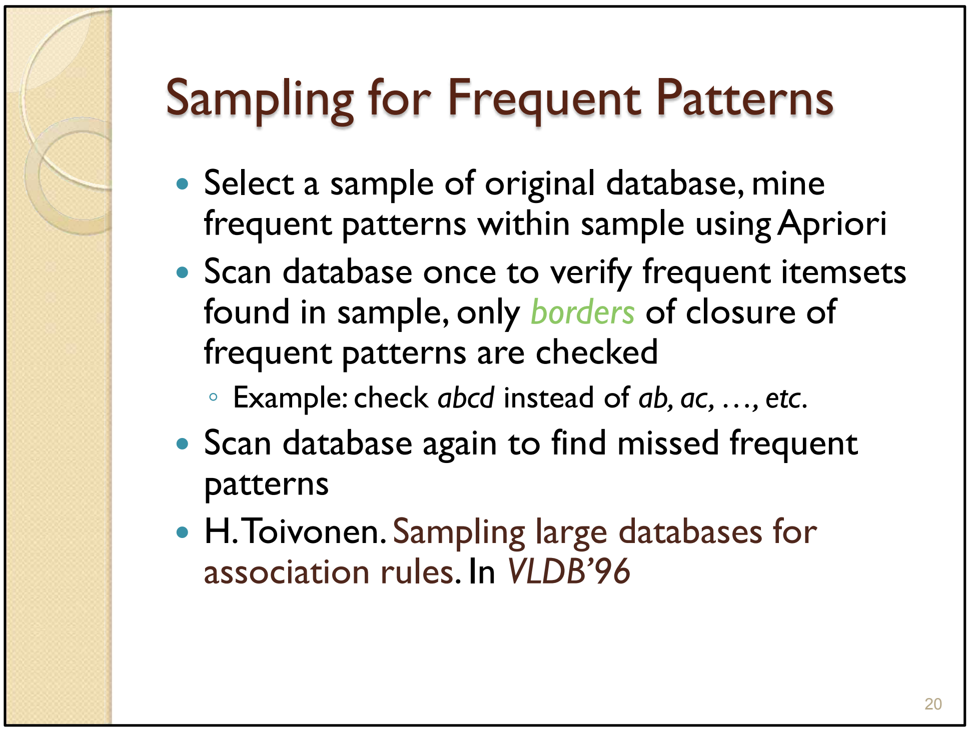


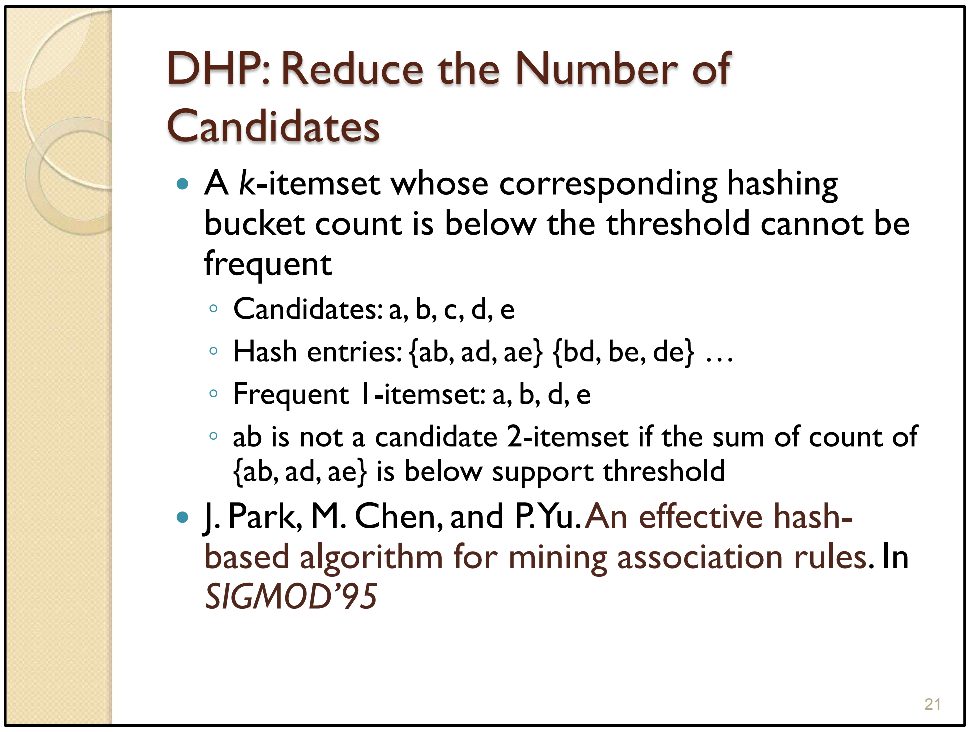
Partition: Scan Database Only Twice

- Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB
  - Scan 1: partition database and find local frequent patterns
  - Scan 2: consolidate global frequent patterns
- A. Savasere, E. Omiecinski, and S. Navathe. An efficient algorithm for mining association in large databases. In *VLDB'95*



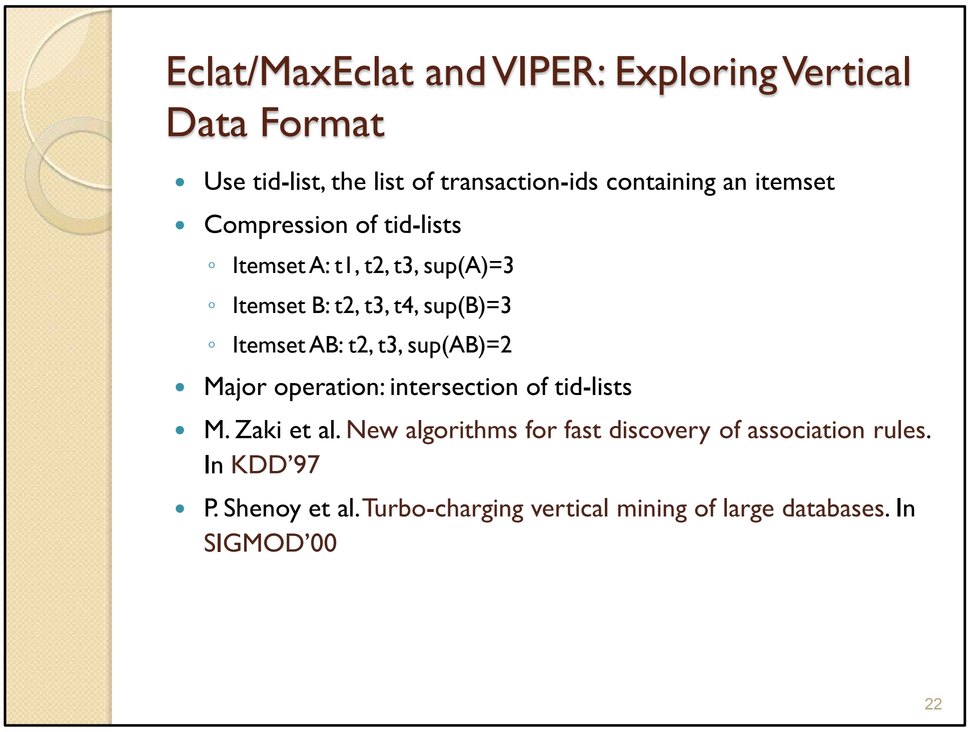
Sampling for Frequent Patterns

- Select a sample of original database, mine frequent patterns within sample using Apriori
- Scan database once to verify frequent itemsets found in sample, only *borders* of closure of frequent patterns are checked
  - Example: check *abcd* instead of *ab, ac, …, etc.*
- Scan database again to find missed frequent patterns
- H. Toivonen. Sampling large databases for association rules. In *VLDB'96*



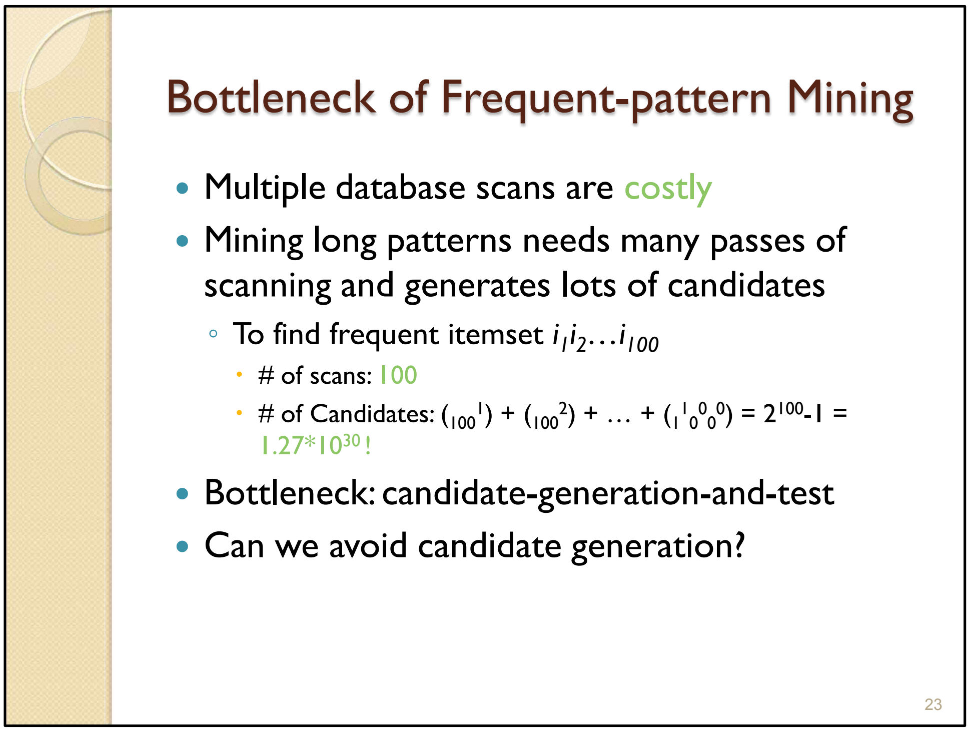
DHP: Reduce the Number of Candidates

- A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
  - Candidates: a, b, c, d, e
  - Hash entries: {ab, ad, ae} {bd, be, de} …
  - Frequent 1-itemset: a, b, d, e
  - ab is not a candidate 2-itemset if the sum of count of {ab, ad, ae} is below support threshold
- J. Park, M. Chen, and P. Yu. An effective hash-based algorithm for mining association rules. In *SIGMOD'95*



Eclat/MaxEclat and VIPER: Exploring Vertical Data Format

- Use tid-list, the list of transaction-ids containing an itemset
- Compression of tid-lists
  - Itemset A: t1, t2, t3, sup(A)=3
  - Itemset B: t2, t3, t4, sup(B)=3
  - Itemset AB: t2, t3, sup(AB)=2
- Major operation: intersection of tid-lists
- M. Zaki et al. New algorithms for fast discovery of association rules. In KDD'97
- P. Shenoy et al. Turbo-charging vertical mining of large databases. In SIGMOD'00



Bottleneck of Frequent-pattern Mining

- Multiple database scans are costly
- Mining long patterns needs many passes of scanning and generates lots of candidates
  - To find frequent itemset $i_1 i_2 \ldots i_{100}$
    - # of scans: 100
    - # of Candidates: $\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100}-1 =$ $1.27*10^{30}$!
- Bottleneck: candidate-generation-and-test
- Can we avoid candidate generation?

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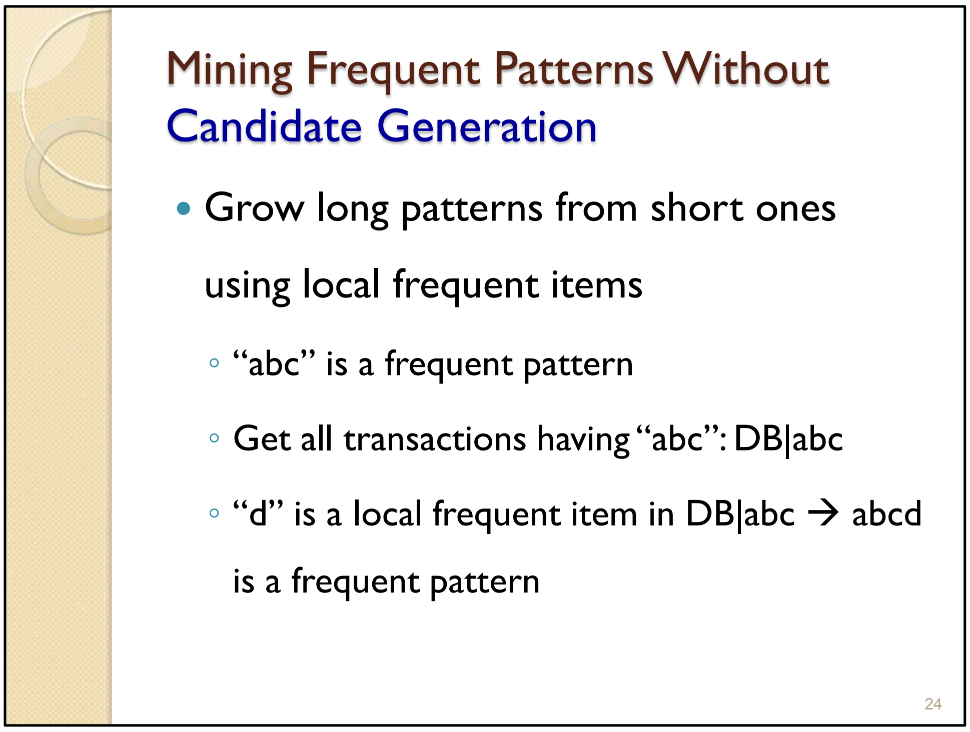

Mining Frequent Patterns Without Candidate Generation

- Grow long patterns from short ones using local frequent items
  - "abc" is a frequent pattern
  - Get all transactions having "abc": DB|abc
  - "d" is a local frequent item in DB|abc → abcd is a frequent pattern

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Construct FP-tree from a Transaction Database

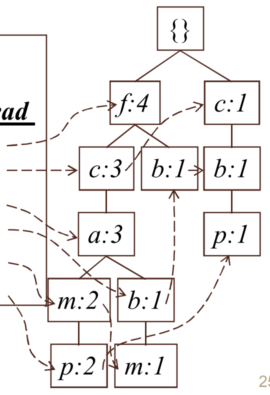
TID	*Items bought*	*(ordered) frequent items*
100	*{f, a, c, d, g, i, m, p}*	*{f, c, a, m, p}*
200	*{a, b, c, f, l, m, o}*	*{f, c, a, b, m}*
300	*{b, f, h, j, o, w}*	*{f, b}*
400	*{b, c, k, s, p}*	*{c, b, p}*
500	*{a, f, c, e, l, p, m, n}*	*{f, c, a, m, p}*

min_support = 3

1. Scan DB once, find frequent 1-itemset (single item pattern)
2. Sort frequent items in frequency descending order, f-list
3. Scan DB again, construct FP-tree

Header Table

Item	*frequency*	*head*
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	



F-list=f-c-a-b-m-p

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Benefits of the FP-tree Structure

- Completeness
  - Preserve complete information for frequent pattern mining
  - Never break a long pattern of any transaction
- Compactness
  - Reduce irrelevant info—infrequent items are gone
  - Items in frequency descending order: the more frequently occurring, the more likely to be shared
  - Never be larger than the original database (not count node-links and the *count* field)
  - For Connect-4 DB, compression ratio could be over 100

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Partition Patterns and Databases

- Frequent patterns can be partitioned into subsets according to f-list
  - F-list=f-c-a-b-m-p
  - Patterns containing p
  - Patterns having m but no p
  - …
  - Patterns having c but no a nor b, m, p
  - Pattern f
- Completeness and non-redundency

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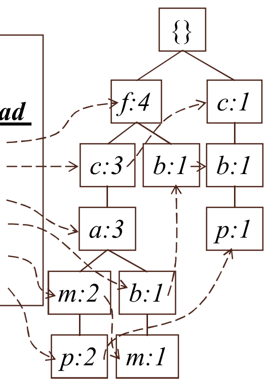
Find Patterns Having P From P-conditional Database

- Starting at the frequent item header table in the FP-tree
- Traverse the FP-tree by following the link of each frequent item *p*
- Accumulate all of *transformed prefix paths* of item *p* to form *p*'s conditional pattern base

Header Table

Item	*frequency*	*head*
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



***Conditional* pattern bases**

item	*cond. pattern base*
c	*f:3*
a	*fc:3*
b	*fca:1, f:1, c:1*
m	*fca:2, fcab:1*
p	*fcam:2, cb:1*

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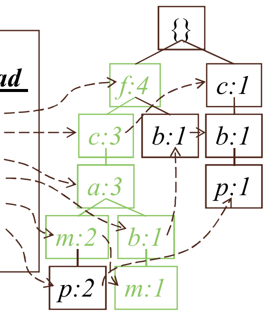
From Conditional Pattern-bases to Conditional FP-trees

- For each pattern-base
  - Accumulate the count for each item in the base
  - Construct the FP-tree for the frequent items of the pattern base

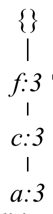
Header Table

Item	***frequency***	***head***
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	



m-conditional **pattern base:**
fca:2, fcab:1

→

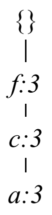


m-conditional **FP-tree**

→

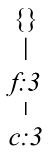
All frequent patterns relate to *m*
m,
fm, cm, am,
fcm, fam, cam,
fcam

Recursion: Mining Each Conditional FP-tree



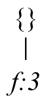
m-conditional **FP-tree**

Cond. pattern base of "am": (fc:3)



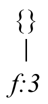
am-conditional **FP-tree**

Cond. pattern base of "cm": (f:3)



cm-conditional **FP-tree**

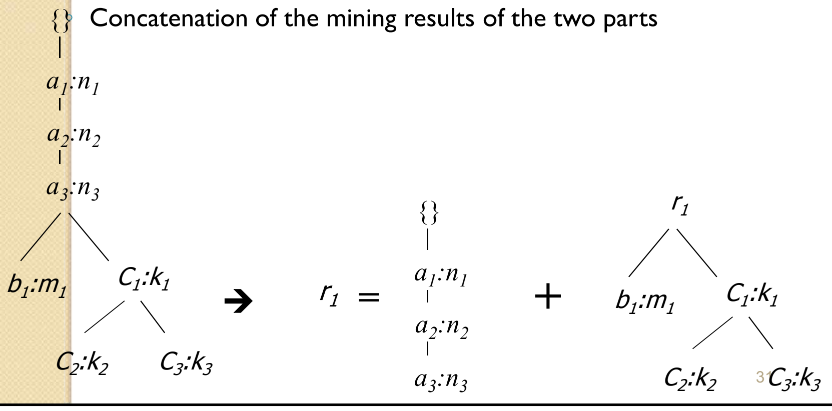
Cond. pattern base of "cam": (f:3)



cam-conditional **FP-tree**

A Special Case: Single Prefix Path in FP-tree

- Suppose a (conditional) FP-tree T has a shared single prefix-path P
- Mining can be decomposed into two parts
  - Reduction of the single prefix path into one node
  - Concatenation of the mining results of the two parts



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Mining Frequent Patterns With FP-trees

- Idea: Frequent pattern growth
  - Recursively grow frequent patterns by pattern and database partition
- Method
  - For each frequent item, construct its conditional pattern-base, and then its conditional FP-tree
  - Repeat the process on each newly created conditional FP-tree
  - Until the resulting FP-tree is empty, or it contains only one path—single path will generate all the combinations of its sub-paths, each of which is a frequent pattern

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Scaling FP-growth by DB Projection

- FP-tree cannot fit in memory?—DB projection
- First partition a database into a set of projected DBs
- Then construct and mine FP-tree for each projected DB
- Parallel projection vs. Partition projection techniques
    - Parallel projection is space costly

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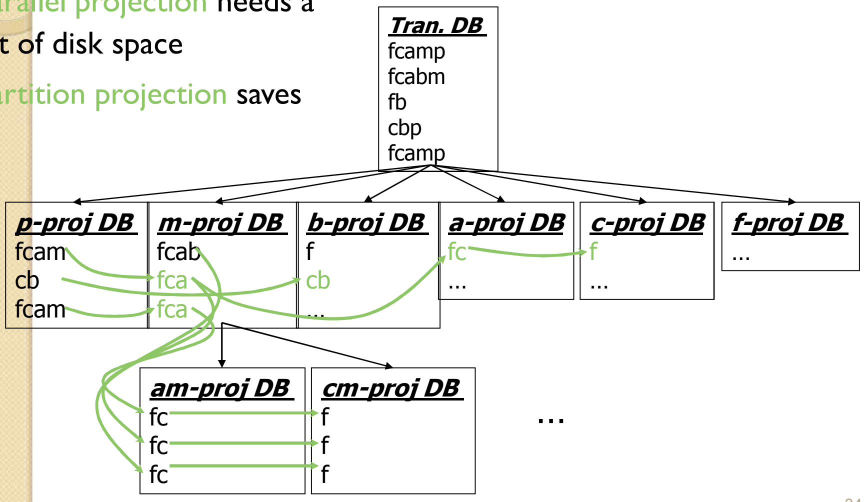
Partition-based Projection

- Parallel projection needs a lot of disk space
- Partition projection saves it



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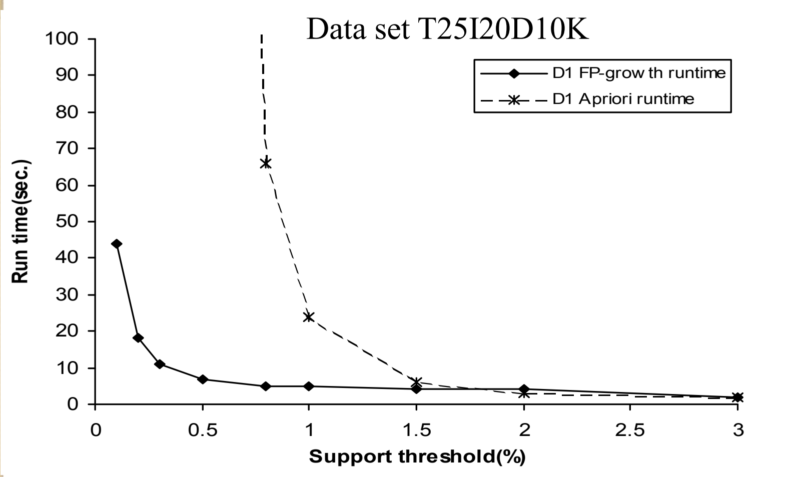
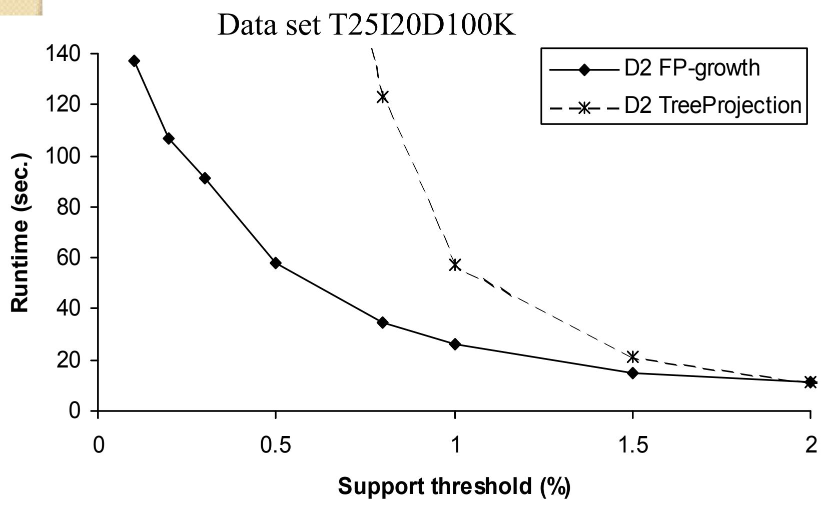
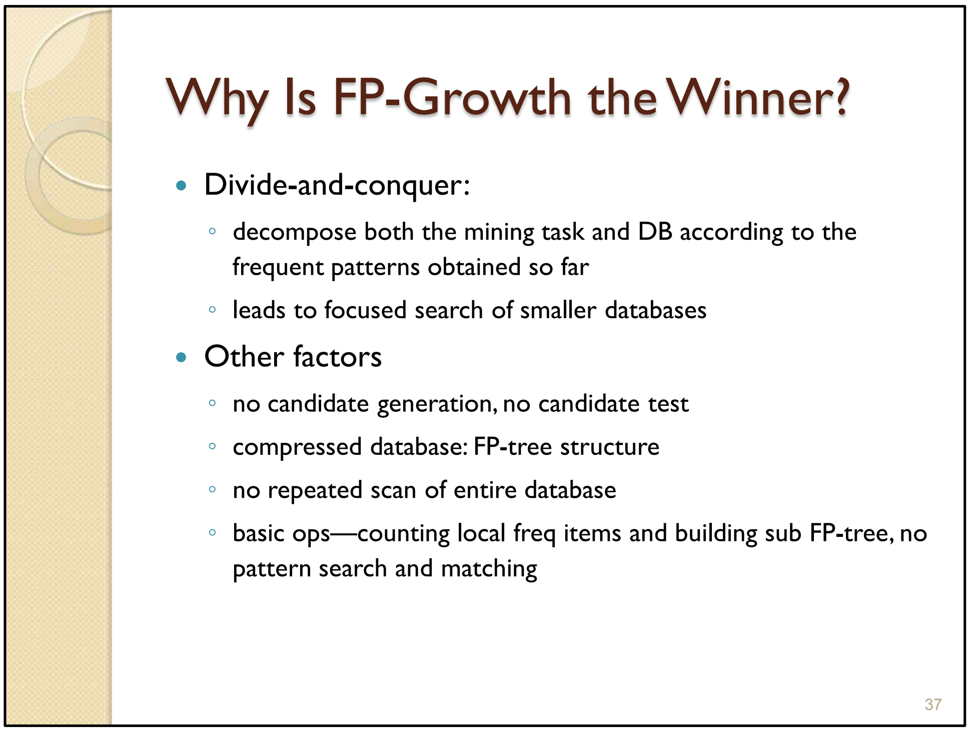
FP-Growth vs. Apriori: Scalability With the Support Threshold



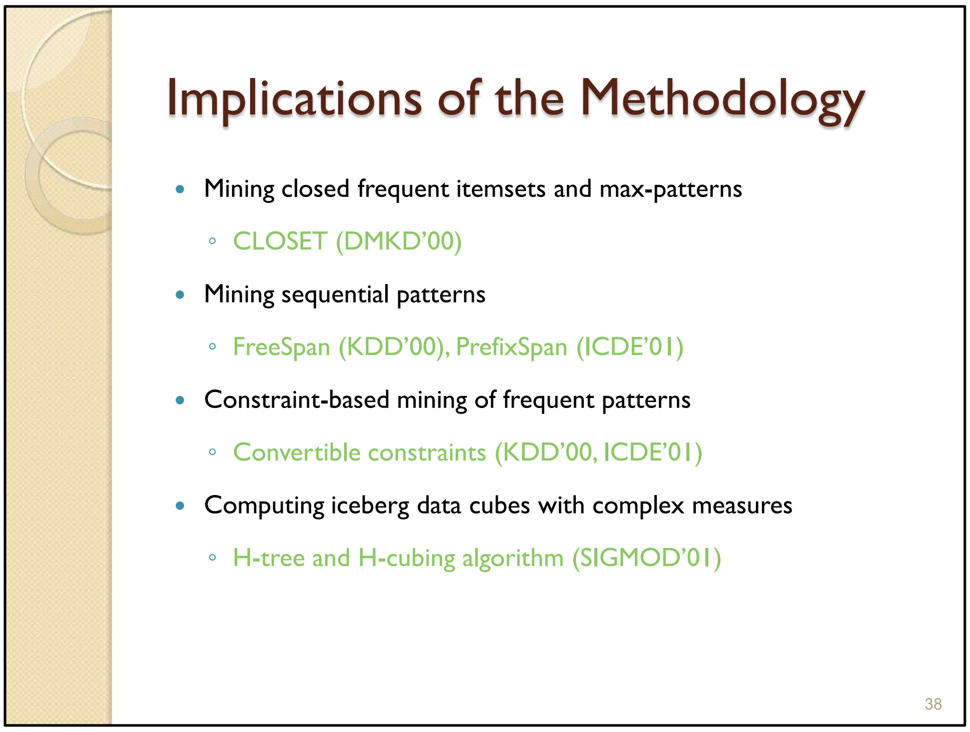
FP-Growth vs. Tree-Projection: Scalability with the Support Threshold





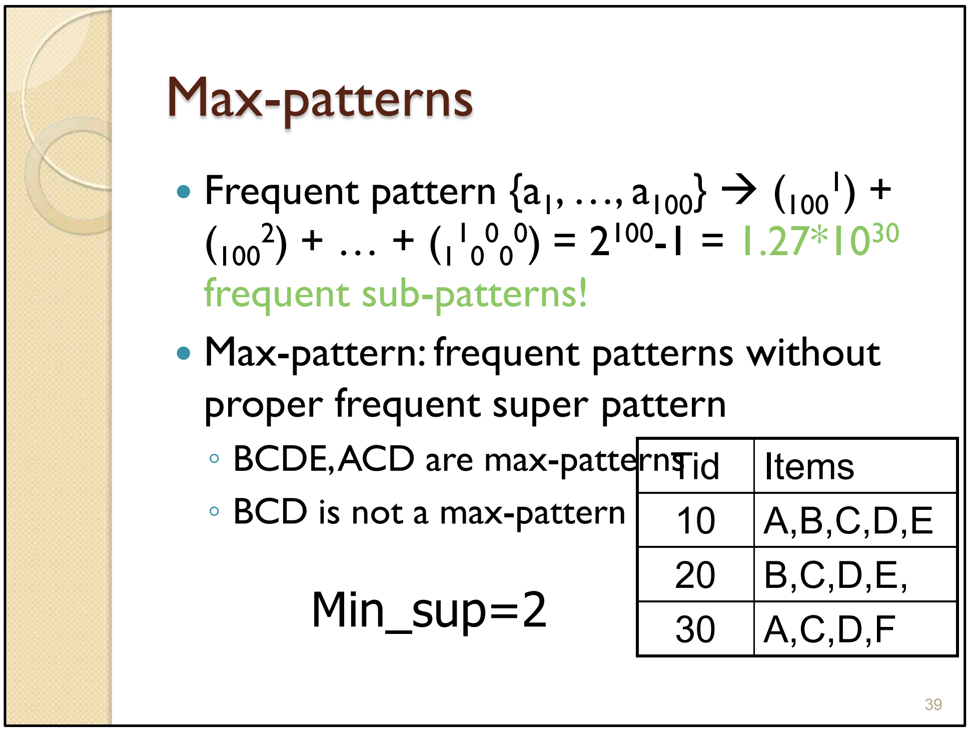
Why Is FP-Growth the Winner?

- Divide-and-conquer:
  - decompose both the mining task and DB according to the frequent patterns obtained so far
  - leads to focused search of smaller databases
- Other factors
  - no candidate generation, no candidate test
  - compressed database: FP-tree structure
  - no repeated scan of entire database
  - basic ops—counting local freq items and building sub FP-tree, no pattern search and matching



Implications of the Methodology

- Mining closed frequent itemsets and max-patterns
  - CLOSET (DMKD'00)
- Mining sequential patterns
  - FreeSpan (KDD'00), PrefixSpan (ICDE'01)
- Constraint-based mining of frequent patterns
  - Convertible constraints (KDD'00, ICDE'01)
- Computing iceberg data cubes with complex measures
  - H-tree and H-cubing algorithm (SIGMOD'01)



Max-patterns

- Frequent pattern $\{a_1, \ldots, a_{100}\} \rightarrow ({}_{100}{}^{1}) + ({}_{100}{}^{2}) + \ldots + ({}_{1}{}^{1}{}_{0}{}^{0}{}_{0}{}^{0}) = 2^{100}-1 = 1.27*10^{30}$ frequent sub-patterns!
- Max-pattern: frequent patterns without proper frequent super pattern
  - BCDE, ACD are max-patterns
  - BCD is not a max-pattern

Min_sup=2

Tid	Items
10	A,B,C,D,E
20	B,C,D,E,
30	A,C,D,F

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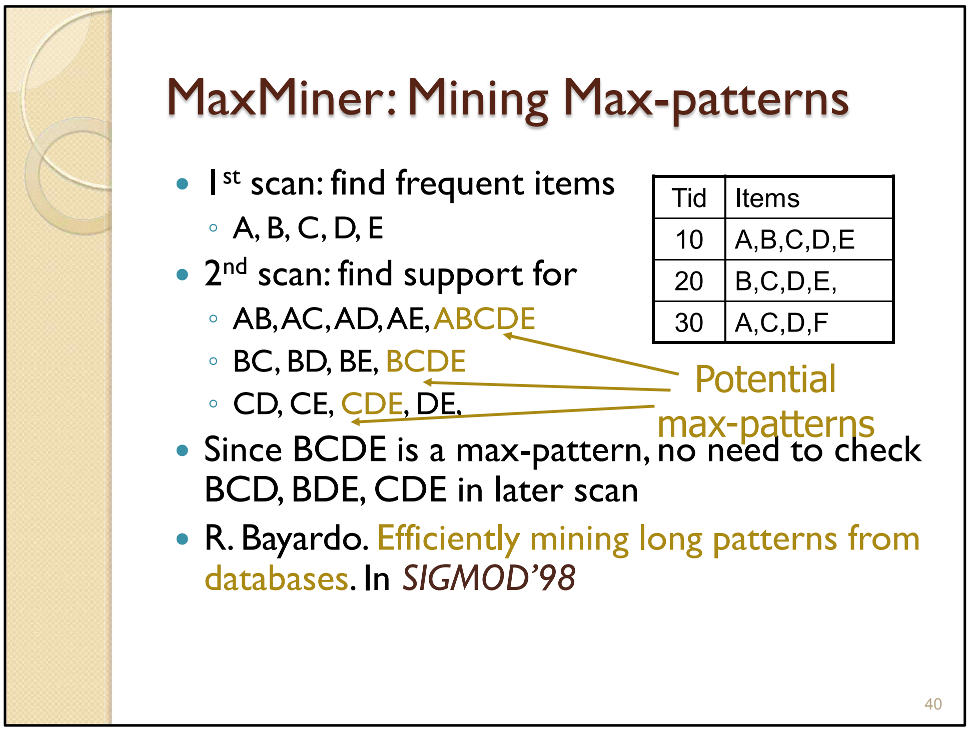

MaxMiner: Mining Max-patterns

- 1st scan: find frequent items
  - A, B, C, D, E
- 2nd scan: find support for
  - AB, AC, AD, AE, ABCDE
  - BC, BD, BE, BCDE
  - CD, CE, CDE, DE,

Tid	Items
10	A,B,C,D,E
20	B,C,D,E,
30	A,C,D,F

Potential max-patterns

- Since BCDE is a max-pattern, no need to check BCD, BDE, CDE in later scan
- R. Bayardo. Efficiently mining long patterns from databases. In *SIGMOD'98*

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Frequent Closed Patterns

- Conf(ac→d)=100% ➔ record acd only
- For frequent itemset X, if there exists no item y s.t. every transaction containing X also contains y, then X is a frequent closed pattern
  - "acd" is a frequent closed pattern
- Concise rep. of freq pats
- Reduce # of patterns and rules
- N. Pasquier et al. In ICDT'99

Min_sup=2

TID	Items
10	a, c, d, e, f
20	a, b, e
30	c, e, f
40	a, c, d, f
50	c, e, f

Mining Frequent Closed Patterns: CLOSET

- Flist: list of all frequent items in support ascending order
  - Flist: d-a-f-e-c
- Divide search space
  - Patterns having d
  - Patterns having d but no a, etc.
- Find frequent closed pattern recursively
  - Every transaction having d also has cfa → cfad is a frequent closed pattern
- J. Pei, J. Han & R. Mao. CLOSET: An Efficient Algorithm for Mining Frequent Closed Itemsets", DMKD'00.

Min_sup=2

TID	Items
10	a, c, d, e, f
20	a, b, e
30	c, e, f
40	a, c, d, f
50	c, e, f

Mining Frequent Closed Patterns: CHARM

- Use vertical data format: $t(AB)=\{T_1, T_{12}, \ldots\}$
- Derive closed pattern based on vertical intersections
  - $t(X)=t(Y)$: X and Y always happen together
  - $t(X)\subset t(Y)$: transaction having X always has Y
- Use diffset to accelerate mining
  - Only keep track of difference of tids
  - $t(X)=\{T_1, T_2, T_3\}$, $t(Xy)=\{T_1, T_3\}$
  - $\text{Diffset}(Xy, X)=\{T_2\}$
- M. Zaki. CHARM: An Efficient Algorithm for Closed Association Rule Mining, CS-TR99-10, Rensselaer Polytechnic Institute
- M. Zaki, Fast Vertical Mining Using Diffsets, TR01-1, Department of Computer Science, Rensselaer Polytechnic Institute

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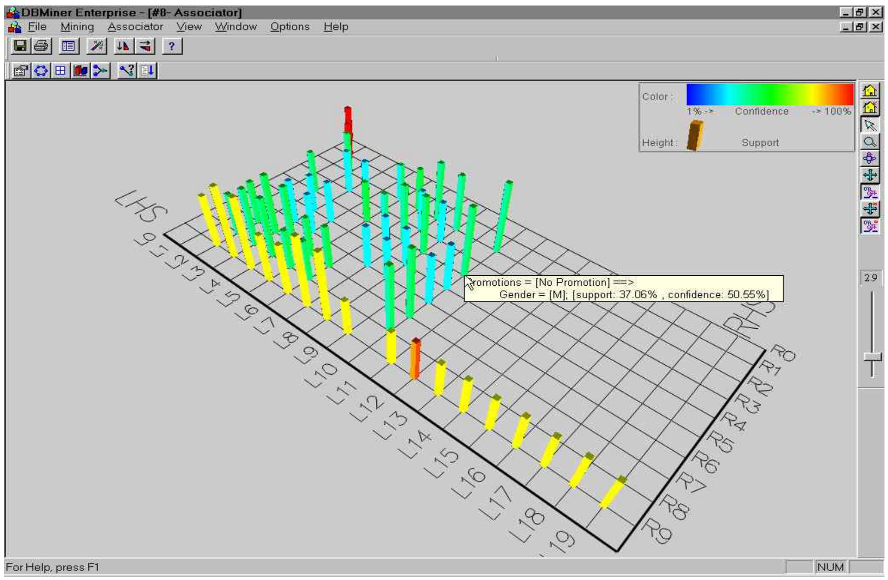
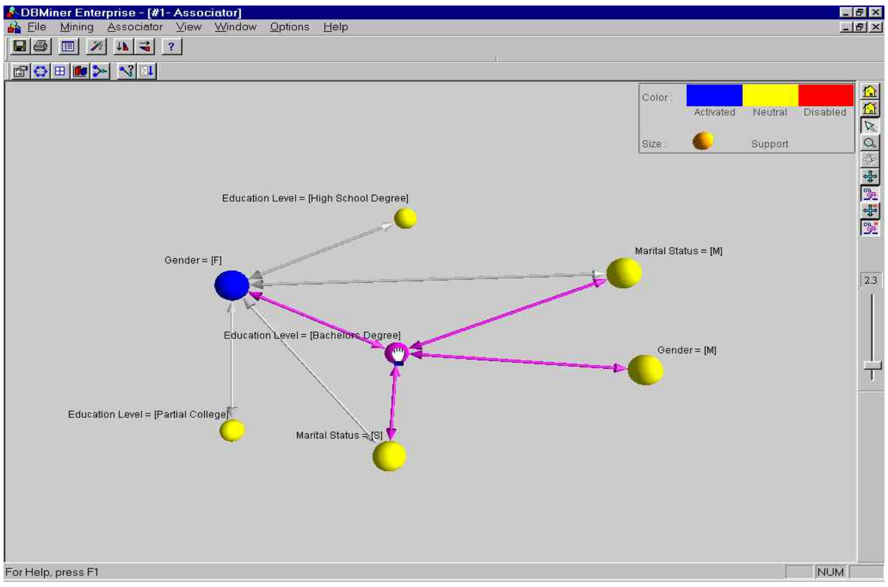
Visualization of Association Rules: Pane Graph



Promotions = [No Promotion] ==>
Gender = [M]; [support: 37.06% , confidence: 50.55%]

Visualization of Association Rules: Rule Graph


DBMiner Enterprise - [#1- Associator]
File Mining Associator View Window Options Help
Color : Activated Neutral Disabled
Size : Support
Education Level = [High School Degree]
Gender = [F]
Marital Status = [M]
Education Level = [Bachelors Degree]
Gender = [M]
Education Level = [Partial College]
Marital Status = [S]
For Help, press F1
NUM

Mining Association Rules in Large Databases

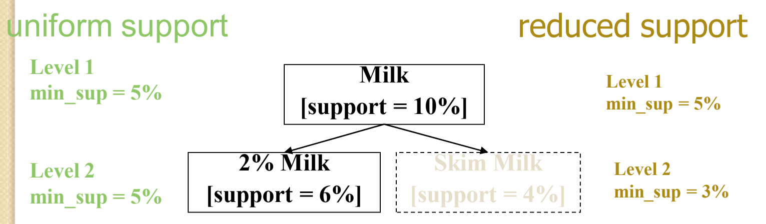
- Association rule mining
- Algorithms for scalable mining of (single-dimensional Boolean) association rules in transactional databases
- Mining various kinds of association/correlation rules
- Constraint-based association mining
- Sequential pattern mining
- Applications/extensions of frequent pattern mining
- Summary

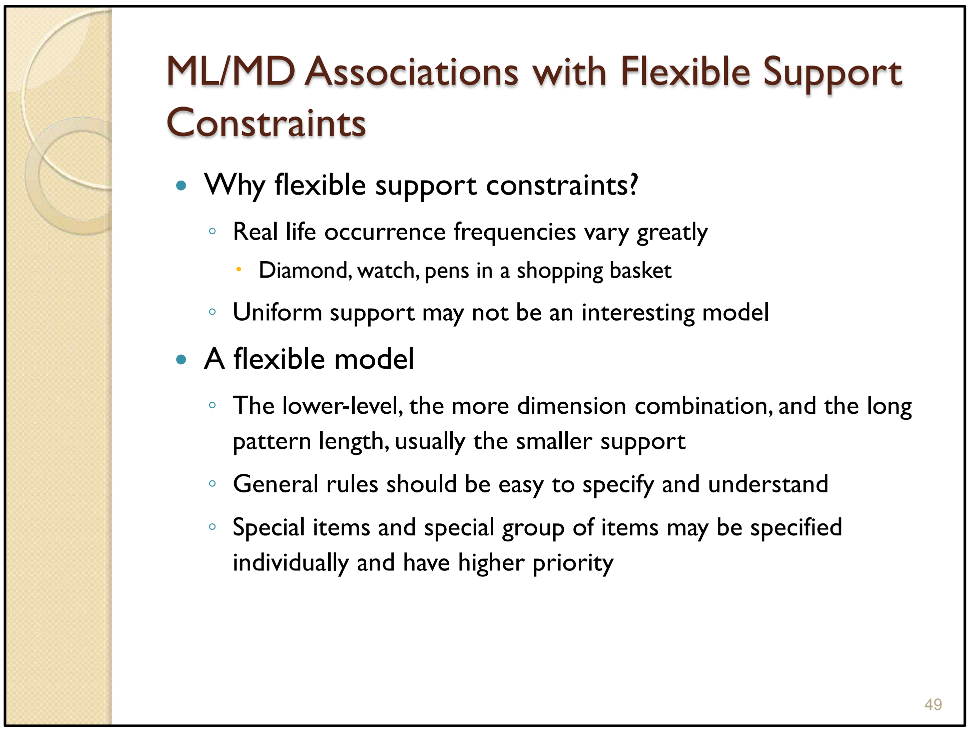
Mining Various Kinds of Rules or Regularities

- Multi-level, quantitative association rules, correlation and causality, ratio rules, sequential patterns, emerging patterns, temporal associations, partial periodicity
- Classification, clustering, iceberg cubes, etc.

Multiple-level Association Rules

- Items often form hierarchy
- Flexible support settings: Items at the lower level are expected to have lower support.
- Transaction database can be encoded based on dimensions and levels
- explore shared multi-level mining





ML/MD Associations with Flexible Support Constraints

- Why flexible support constraints?
  - Real life occurrence frequencies vary greatly
    - Diamond, watch, pens in a shopping basket
  - Uniform support may not be an interesting model
- A flexible model
  - The lower-level, the more dimension combination, and the long pattern length, usually the smaller support
  - General rules should be easy to specify and understand
  - Special items and special group of items may be specified individually and have higher priority

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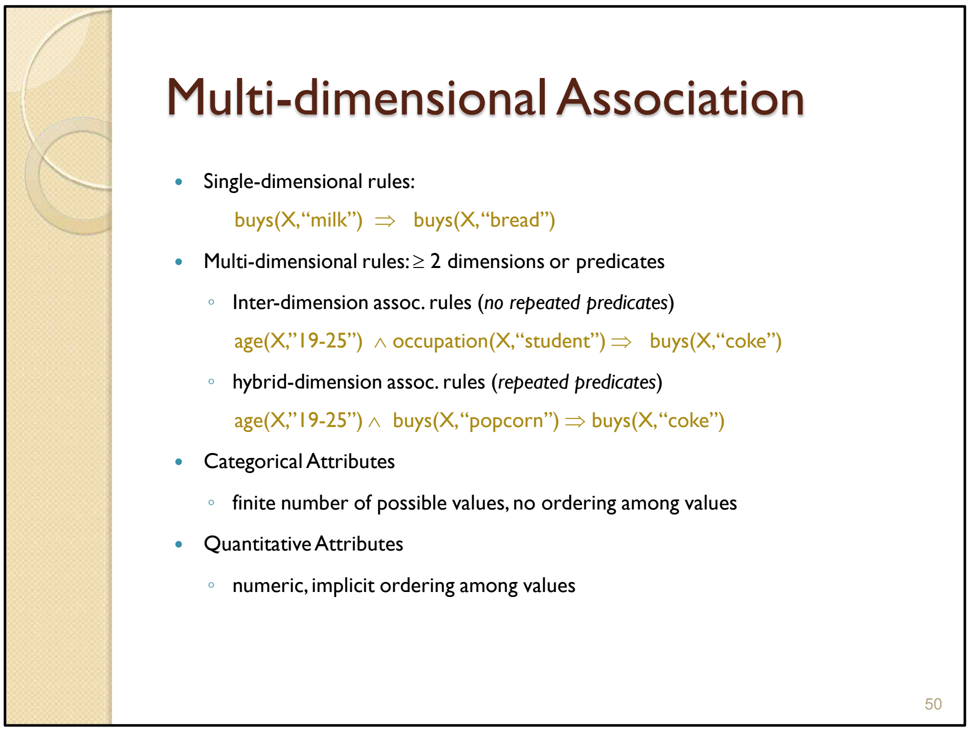

Multi-dimensional Association

- Single-dimensional rules:

  buys(X, "milk") ⇒ buys(X, "bread")
- Multi-dimensional rules: ≥ 2 dimensions or predicates
  - Inter-dimension assoc. rules (*no repeated predicates*)

    age(X,"19-25") ∧ occupation(X,"student") ⇒ buys(X, "coke")
  - hybrid-dimension assoc. rules (*repeated predicates*)

    age(X,"19-25") ∧ buys(X, "popcorn") ⇒ buys(X, "coke")
- Categorical Attributes
  - finite number of possible values, no ordering among values
- Quantitative Attributes
  - numeric, implicit ordering among values

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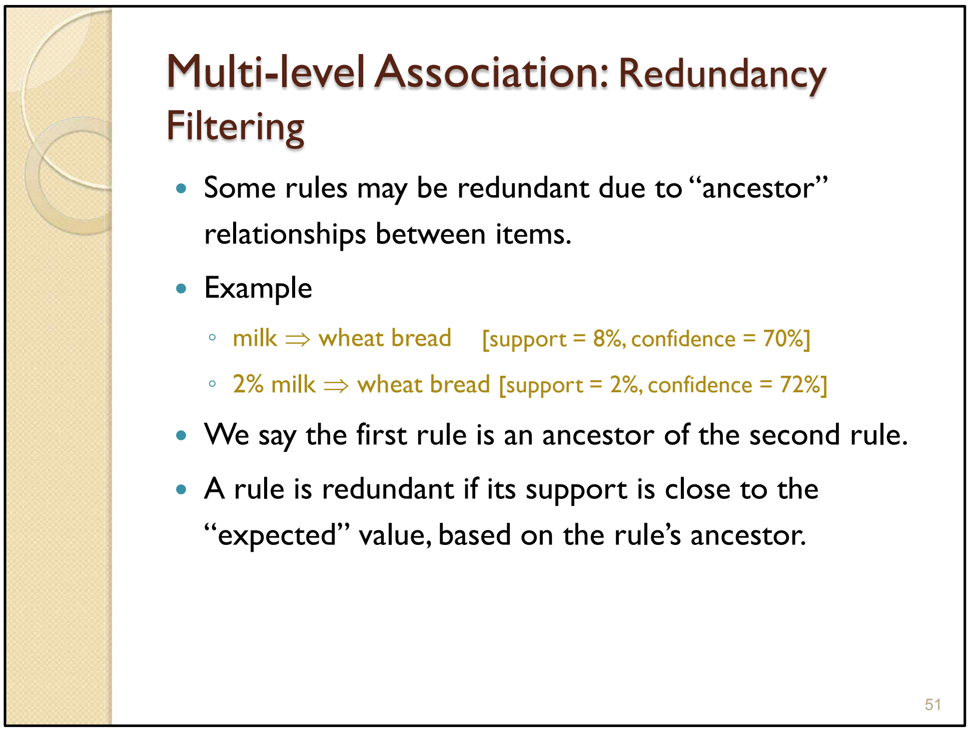

Multi-level Association: Redundancy Filtering

- Some rules may be redundant due to "ancestor" relationships between items.
- Example
  - milk ⇒ wheat bread [support = 8%, confidence = 70%]
  - 2% milk ⇒ wheat bread [support = 2%, confidence = 72%]
- We say the first rule is an ancestor of the second rule.
- A rule is redundant if its support is close to the "expected" value, based on the rule's ancestor.

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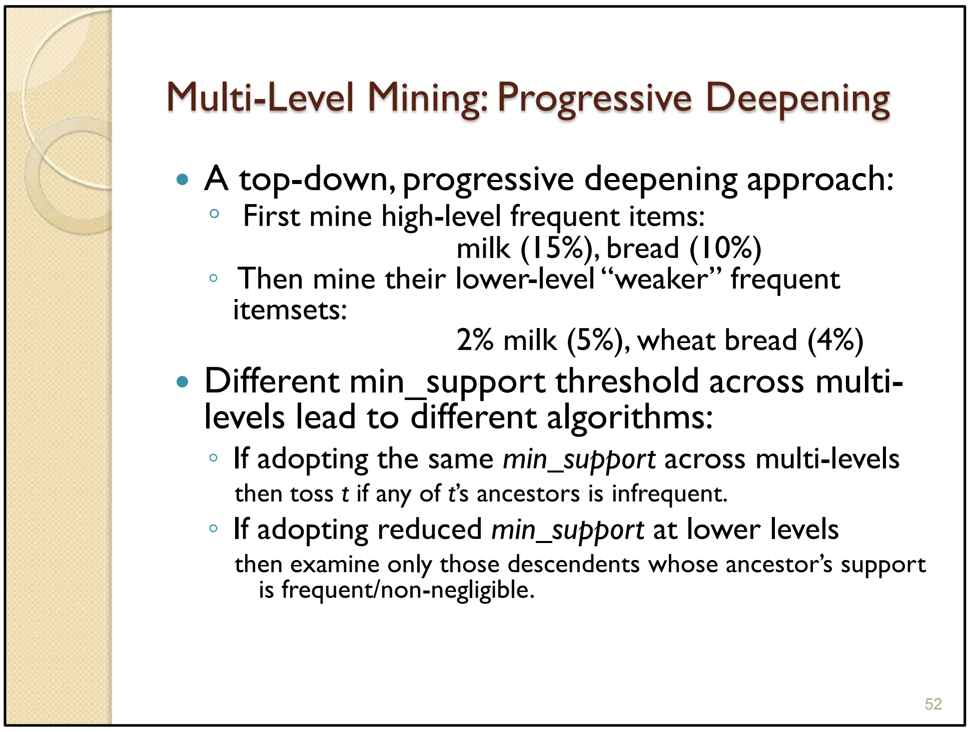

Multi-Level Mining: Progressive Deepening

- A top-down, progressive deepening approach:
  - First mine high-level frequent items:
    milk (15%), bread (10%)
  - Then mine their lower-level "weaker" frequent itemsets:
    2% milk (5%), wheat bread (4%)
- Different min_support threshold across multi-levels lead to different algorithms:
  - If adopting the same *min_support* across multi-levels
    then toss *t* if any of *t*'s ancestors is infrequent.
  - If adopting reduced *min_support* at lower levels
    then examine only those descendents whose ancestor's support is frequent/non-negligible.

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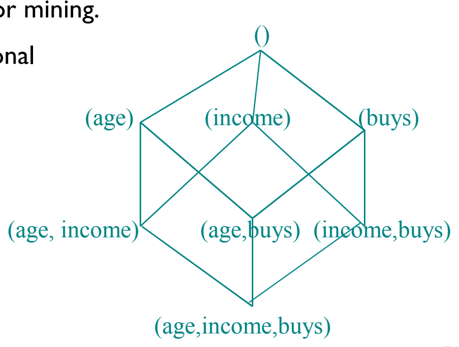
Techniques for Mining MD Associations

- Search for frequent *k*-predicate set:
  - Example: {age, occupation, buys} is a 3-predicate set
  - Techniques can be categorized by how age are treated

1. Using static discretization of quantitative attributes
   - Quantitative attributes are statically discretized by using predefined concept hierarchies
2. Quantitative association rules
   - Quantitative attributes are dynamically discretized into "bins"based on the distribution of the data
3. Distance-based association rules
   - This is a dynamic discretization process that considers the distance between data points

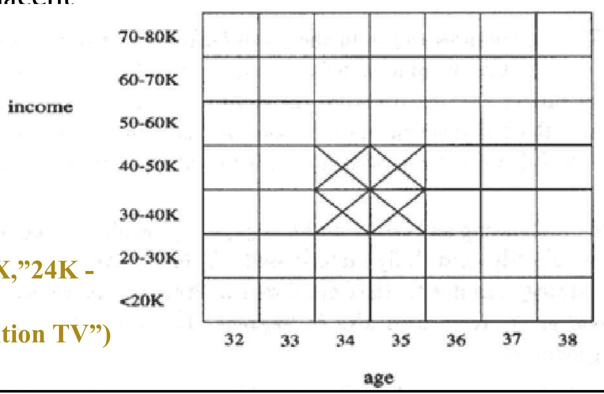
Static Discretization of Quantitative Attributes

- Discretized prior to mining using concept hierarchy.
- Numeric values are replaced by ranges.
- In relational database, finding all frequent k-predicate sets will require *k* or *k*+1 table scans.
- Data cube is well suited for mining.
- The cells of an n-dimensional cuboid correspond to the predicate sets.
- Mining from data cubes can be much faster.



Quantitative Association Rules

- Numeric attributes are *dynamically* discretized
  - Such that the confidence or compactness of the rules mined is maximized
- 2-D quantitative association rules: $A_{quan1} \wedge A_{quan2} \Rightarrow A_{cat}$
- Cluster "adjacent"
  association r
  to form gene
  rules using a
  grid
- Example



age(X,"30-34") ∧ income(X,"24K - 48K")
⇒ buys(X,"high resolution TV")

Mining Distance-based Association Rules

- Binning methods do not capture the semantics of interval data

Price($)	Equi-width (width $10)	Equi-depth (depth 2)	Distance-based
7	[0,10]	[7,20]	[7,7]
20	[11,20]	[22,50]	[20,22]
22	[21,30]	[51,53]	[50,53]
50	[31,40]		
51	[41,50]		
53	[51,60]		

- Distance-based partitioning, more meaningful discretization considering:
  - density/number of points in an interval
  - "closeness" of points in an interval

Interestingness Measure: Correlations (Lift)

- *play basketball* $\Rightarrow$ *eat cereal* [40%, 66.7%] is misleading
  - The overall percentage of students eating cereal is 75% which is higher than 66.7%.
- *play basketball* $\Rightarrow$ *not eat cereal* [20%, 33.3%] is more accurate, although with lower support and confidence
- Measure of dependent/correlated events: lift

$$corr_{A,B} = \frac{P(A \cup B)}{P(A)P(B)}$$

	Basketbal l	Not basketball	Sum (row)
Cereal	2000	1750	3750
Not cereal	1000	250	1250
Sum(col.)	3000	2000	5000

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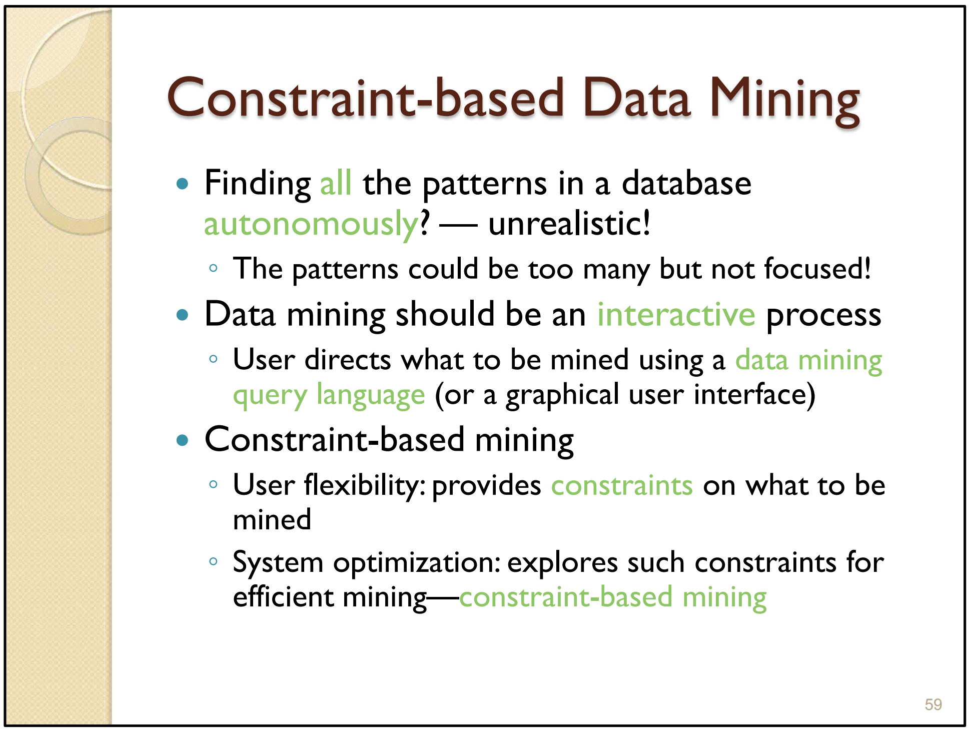
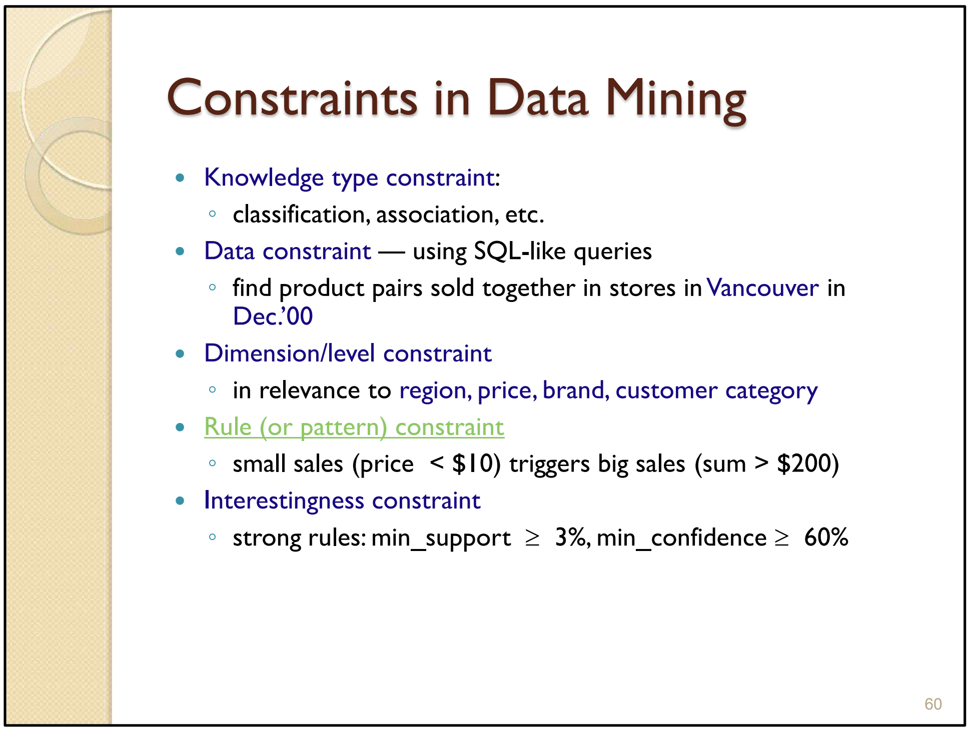


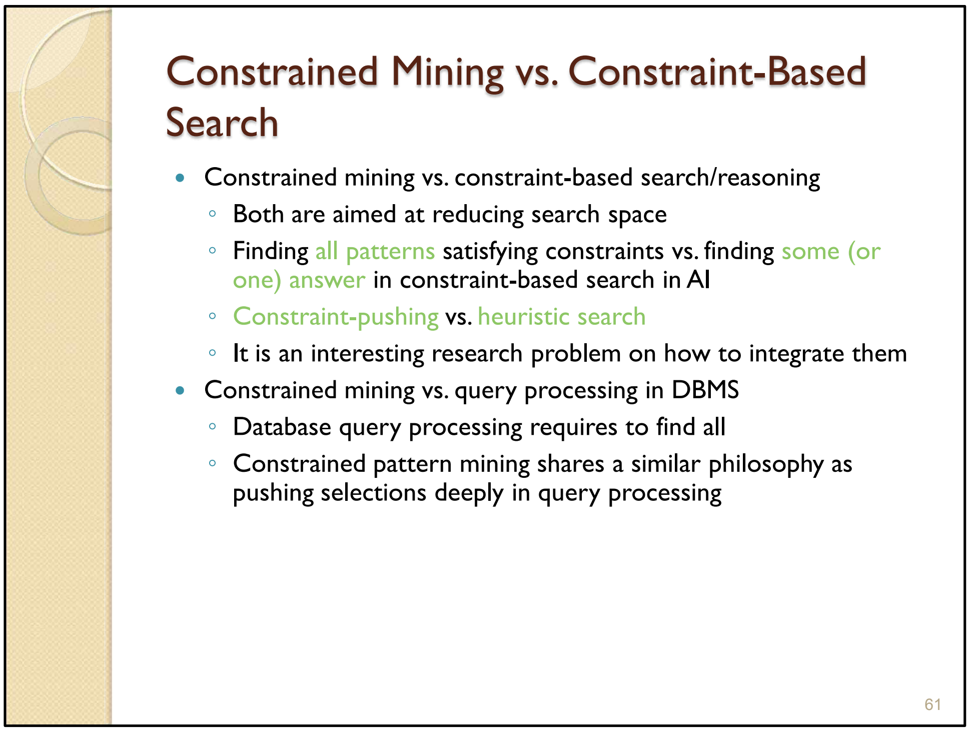
Constraint-based Data Mining

- Finding all the patterns in a database autonomously? — unrealistic!
  - The patterns could be too many but not focused!
- Data mining should be an interactive process
  - User directs what to be mined using a data mining query language (or a graphical user interface)
- Constraint-based mining
  - User flexibility: provides constraints on what to be mined
  - System optimization: explores such constraints for efficient mining—constraint-based mining



Constraints in Data Mining

- Knowledge type constraint:
  - classification, association, etc.
- Data constraint — using SQL-like queries
  - find product pairs sold together in stores in Vancouver in Dec.'00
- Dimension/level constraint
  - in relevance to region, price, brand, customer category
- Rule (or pattern) constraint
  - small sales (price $< \$10$) triggers big sales (sum $> \$200$)
- Interestingness constraint
  - strong rules: min_support $\geq$ 3%, min_confidence $\geq$ 60%



Constrained Mining vs. Constraint-Based Search

- Constrained mining vs. constraint-based search/reasoning
  - Both are aimed at reducing search space
  - Finding all patterns satisfying constraints vs. finding some (or one) answer in constraint-based search in AI
  - Constraint-pushing vs. heuristic search
  - It is an interesting research problem on how to integrate them
- Constrained mining vs. query processing in DBMS
  - Database query processing requires to find all
  - Constrained pattern mining shares a similar philosophy as pushing selections deeply in query processing

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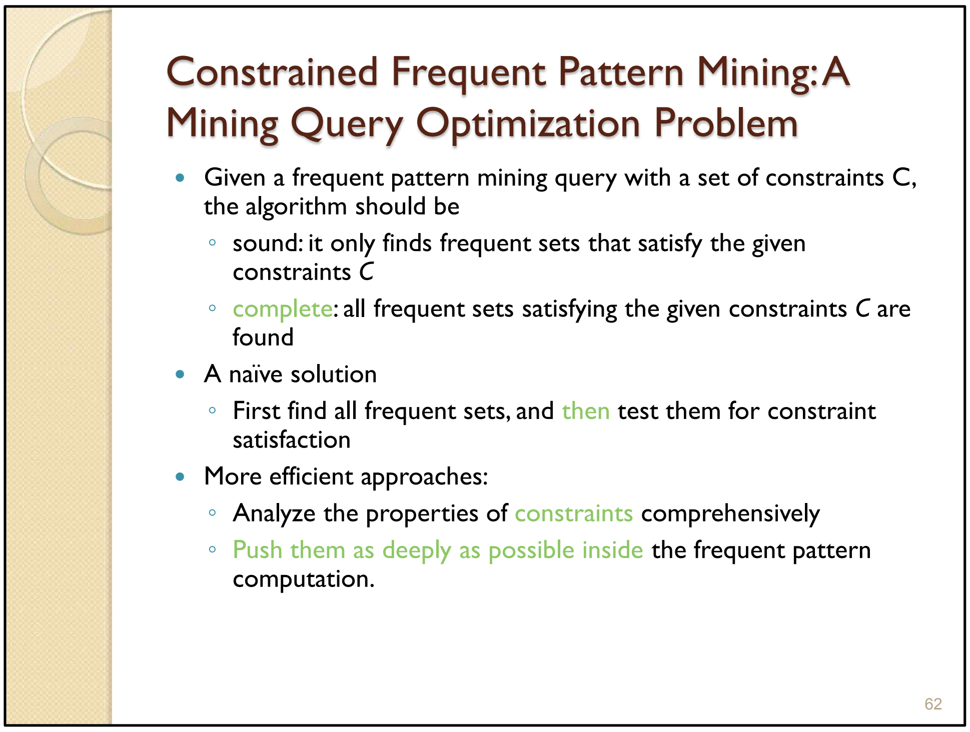

Constrained Frequent Pattern Mining: A Mining Query Optimization Problem

- Given a frequent pattern mining query with a set of constraints C, the algorithm should be
  - sound: it only finds frequent sets that satisfy the given constraints *C*
  - complete: all frequent sets satisfying the given constraints *C* are found
- A naïve solution
  - First find all frequent sets, and then test them for constraint satisfaction
- More efficient approaches:
  - Analyze the properties of constraints comprehensively
  - Push them as deeply as possible inside the frequent pattern computation.

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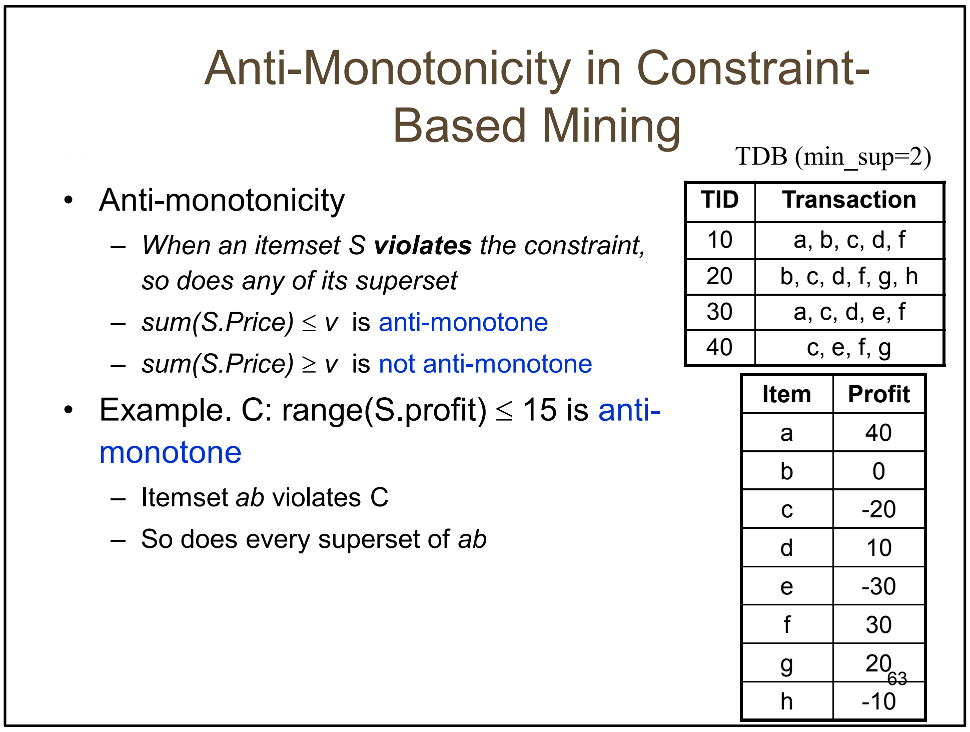

Anti-Monotonicity in Constraint-Based Mining

- Anti-monotonicity
  - *When an itemset S* ***violates*** *the constraint, so does any of its superset*
  - *sum(S.Price)* ≤ *v* is anti-monotone
  - *sum(S.Price)* ≥ *v* is not anti-monotone
- Example. C: range(S.profit) ≤ 15 is anti-monotone
  - Itemset *ab* violates C
  - So does every superset of *ab*

TDB (min_sup=2)

TID	Transaction
10	a, b, c, d, f
20	b, c, d, f, g, h
30	a, c, d, e, f
40	c, e, f, g

Item	Profit
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

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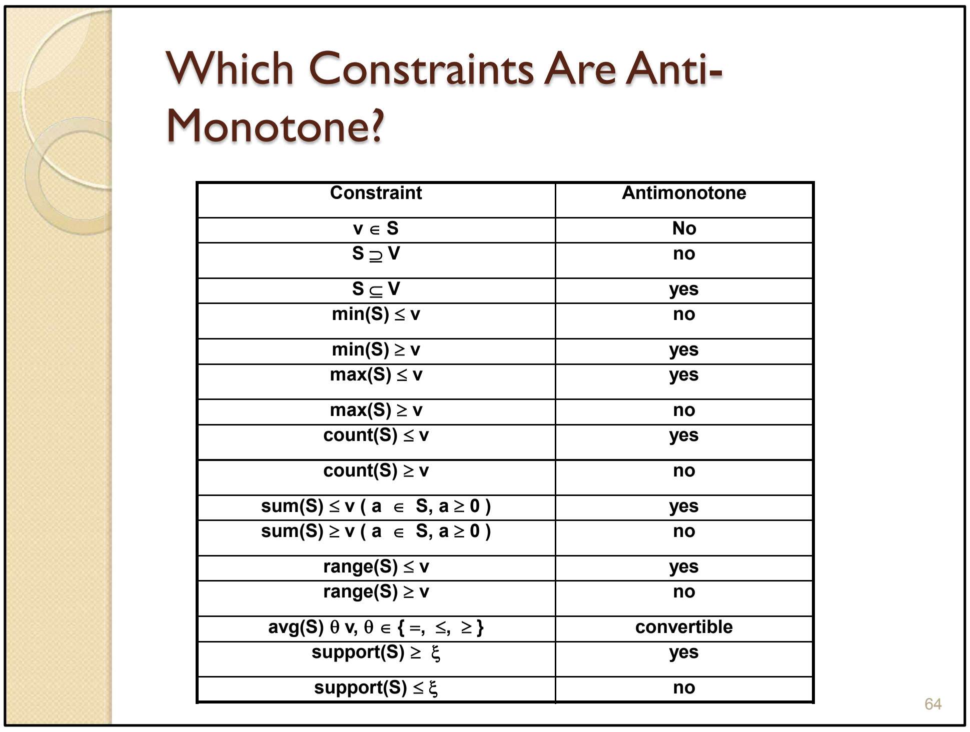

Which Constraints Are Anti-Monotone?

Constraint	Antimonotone
v ∈ S	No
S ⊇ V	no
S ⊆ V	yes
min(S) ≤ v	no
min(S) ≥ v	yes
max(S) ≤ v	yes
max(S) ≥ v	no
count(S) ≤ v	yes
count(S) ≥ v	no
sum(S) ≤ v (a ∈ S, a ≥ 0)	yes
sum(S) ≥ v (a ∈ S, a ≥ 0)	no
range(S) ≤ v	yes
range(S) ≥ v	no
avg(S) θ v, θ ∈ { =, ≤, ≥ }	convertible
support(S) ≥ ξ	yes
support(S) ≤ ξ	no

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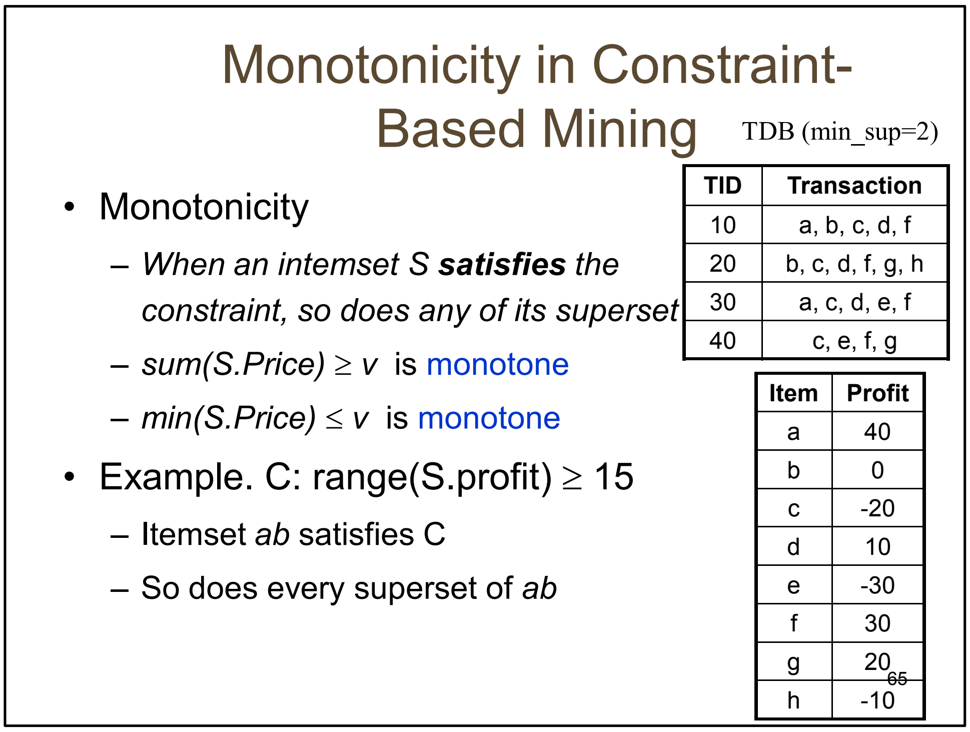


Monotonicity in Constraint-Based Mining

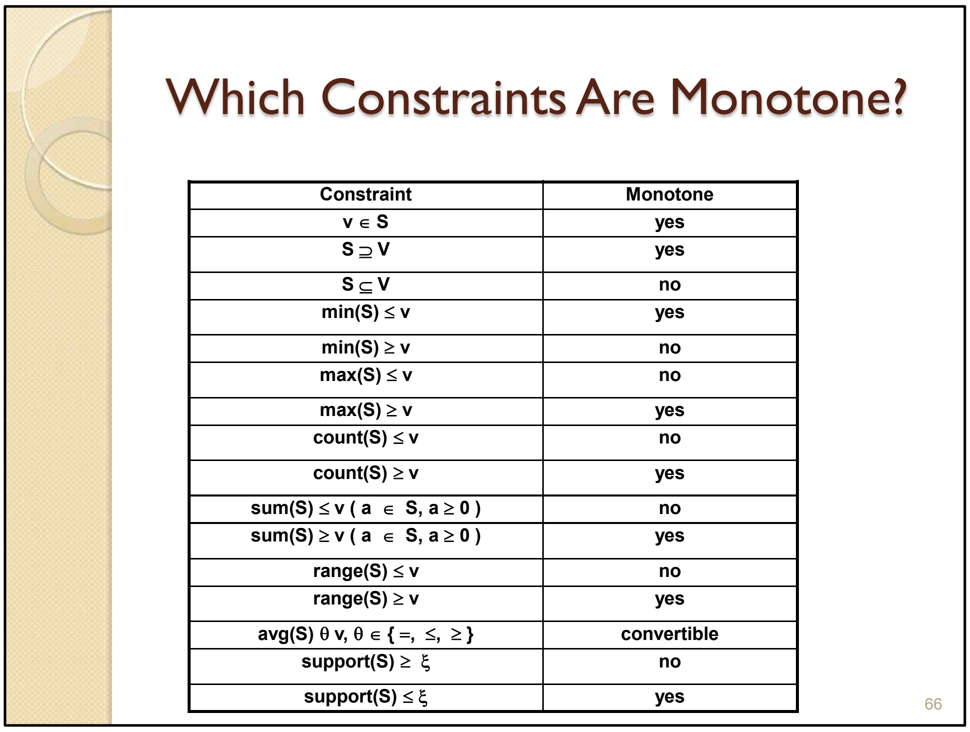
- Monotonicity
  - *When an intemset S **satisfies** the constraint, so does any of its superset*
  - *sum(S.Price)* $\geq$ *v* is monotone
  - *min(S.Price)* $\leq$ *v* is monotone
- Example. C: range(S.profit) $\geq$ 15
  - Itemset *ab* satisfies C
  - So does every superset of *ab*

TDB (min_sup=2)

TID	Transaction
10	a, b, c, d, f
20	b, c, d, f, g, h
30	a, c, d, e, f
40	c, e, f, g

Item	Profit
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

65



Which Constraints Are Monotone?

Constraint	Monotone
$v \in S$	yes
$S \supseteq V$	yes
$S \subseteq V$	no
$\min(S) \leq v$	yes
$\min(S) \geq v$	no
$\max(S) \leq v$	no
$\max(S) \geq v$	yes
count(S) $\leq$ v	no
count(S) $\geq$ v	yes
sum(S) $\leq$ v (a $\in$ S, a $\geq$ 0)	no
sum(S) $\geq$ v (a $\in$ S, a $\geq$ 0)	yes
range(S) $\leq$ v	no
range(S) $\geq$ v	yes
avg(S) θ v, $\theta \in \{=, \leq, \geq\}$	convertible
support(S) $\geq \xi$	no
support(S) $\leq \xi$	yes

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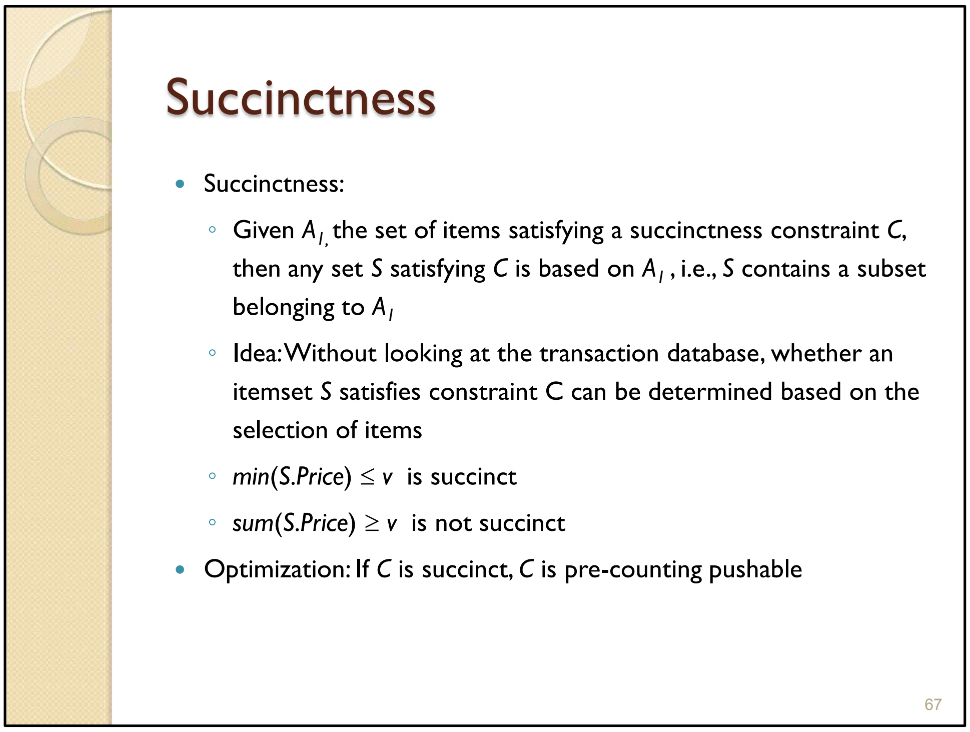

Succinctness

- Succinctness:
  - Given A_I, the set of items satisfying a succinctness constraint *C*, then any set *S* satisfying *C* is based on A_I , i.e., *S* contains a subset belonging to A_I
  - Idea: Without looking at the transaction database, whether an itemset *S* satisfies constraint C can be determined based on the selection of items
  - *min*(*S.Price*) $\leq v$ is succinct
  - *sum*(*S.Price*) $\geq v$ is not succinct
- Optimization: If *C* is succinct, *C* is pre-counting pushable

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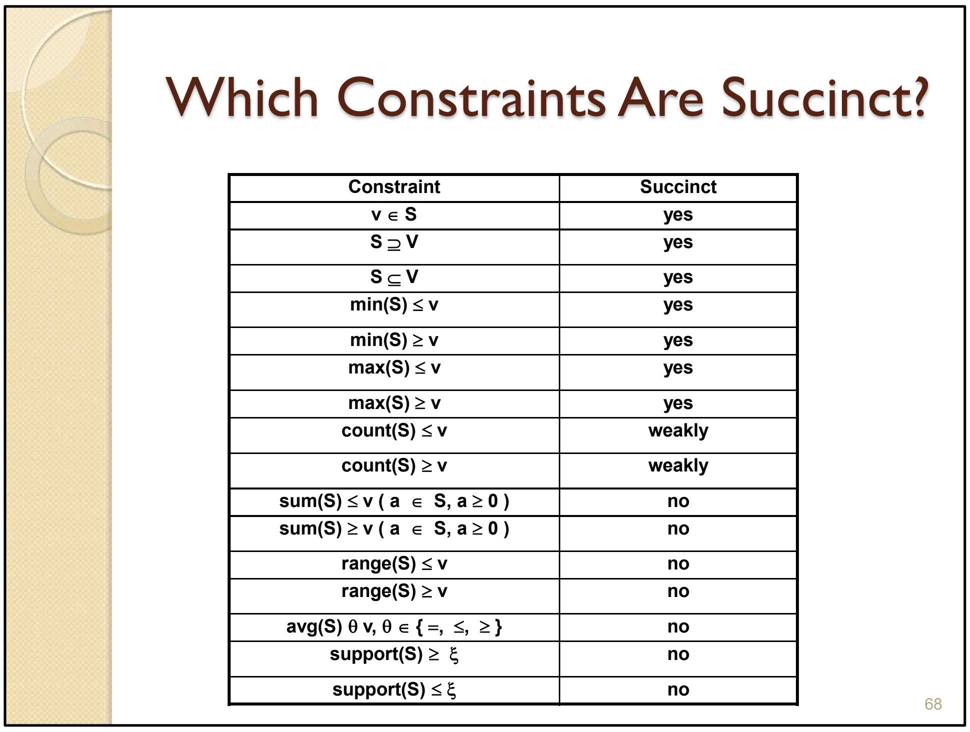

Which Constraints Are Succinct?

Constraint	Succinct
$v \in S$	yes
$S \supseteq V$	yes
$S \subseteq V$	yes
$\min(S) \leq v$	yes
$\min(S) \geq v$	yes
$\max(S) \leq v$	yes
$\max(S) \geq v$	yes
$\text{count}(S) \leq v$	weakly
$\text{count}(S) \geq v$	weakly
$\text{sum}(S) \leq v$ ($a \in S, a \geq 0$)	no
$\text{sum}(S) \geq v$ ($a \in S, a \geq 0$)	no
$\text{range}(S) \leq v$	no
$\text{range}(S) \geq v$	no
$\text{avg}(S)\ \theta\ v, \theta \in \{=, \leq, \geq\}$	no
$\text{support}(S) \geq \xi$	no
$\text{support}(S) \leq \xi$	no

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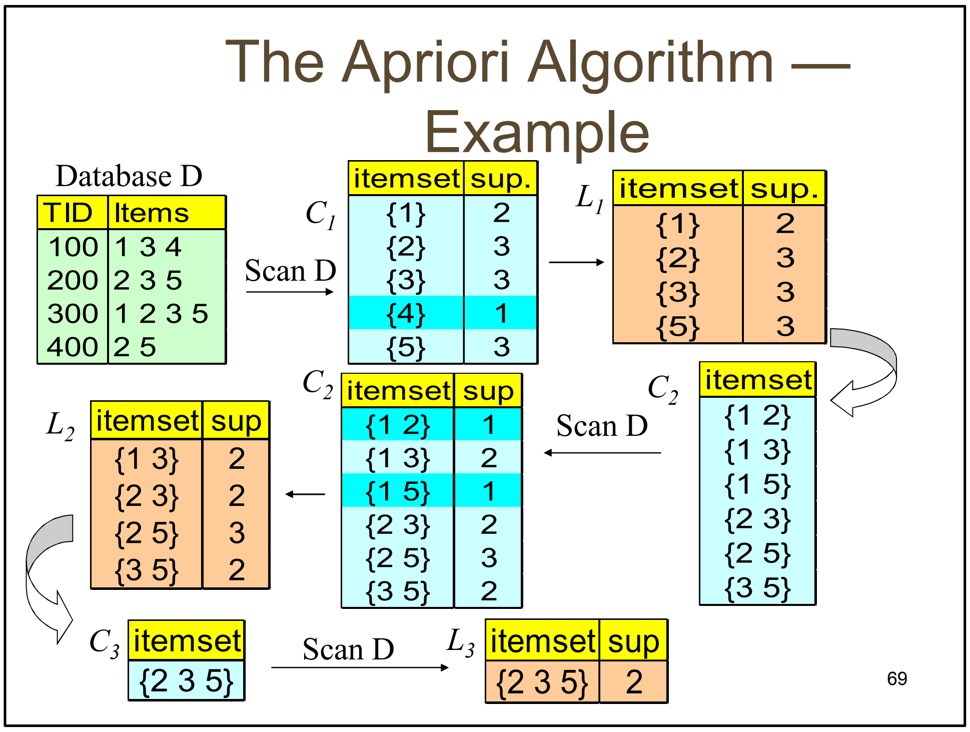

The Apriori Algorithm — Example

Database D

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Scan D

C_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

L_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{5}	3

C_2

itemset
{1 2}
{1 3}
{1 5}
{2 3}
{2 5}
{3 5}

Scan D

C_2

itemset	sup
{1 2}	1
{1 3}	2
{1 5}	1
{2 3}	2
{2 5}	3
{3 5}	2

L_2

itemset	sup
{1 3}	2
{2 3}	2
{2 5}	3
{3 5}	2

C_3

itemset
{2 3 5}

Scan D

L_3

itemset	sup
{2 3 5}	2

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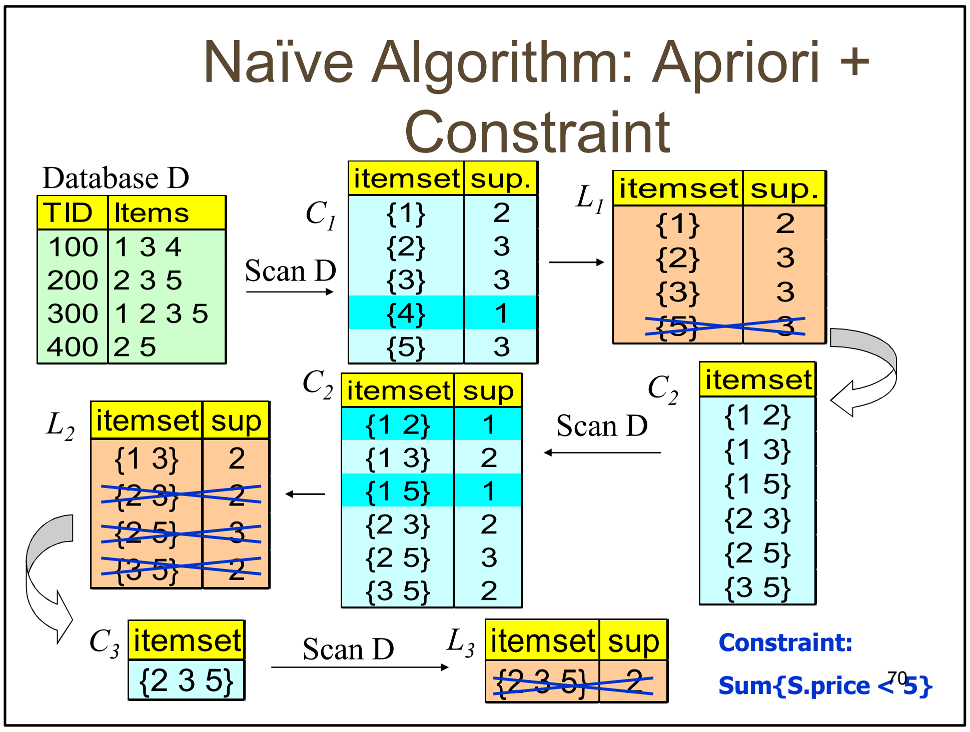

Naïve Algorithm: Apriori + Constraint

Database D

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Scan D

C_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

L_1

itemset	sup.
{1}	2
{2}	3
{3}	3
~~{5}~~	~~3~~

C_2

itemset
{1 2}
{1 3}
{1 5}
{2 3}
{2 5}
{3 5}

Scan D

C_2

itemset	sup
{1 2}	1
{1 3}	2
{1 5}	1
{2 3}	2
{2 5}	3
{3 5}	2

L_2

itemset	sup
{1 3}	2
~~{2 3}~~	~~2~~
~~{2 5}~~	~~3~~
~~{3 5}~~	~~2~~

C_3

itemset
{2 3 5}

Scan D

L_3

itemset	sup
~~{2 3 5}~~	~~2~~

Constraint:

Sum{S.price < 5}

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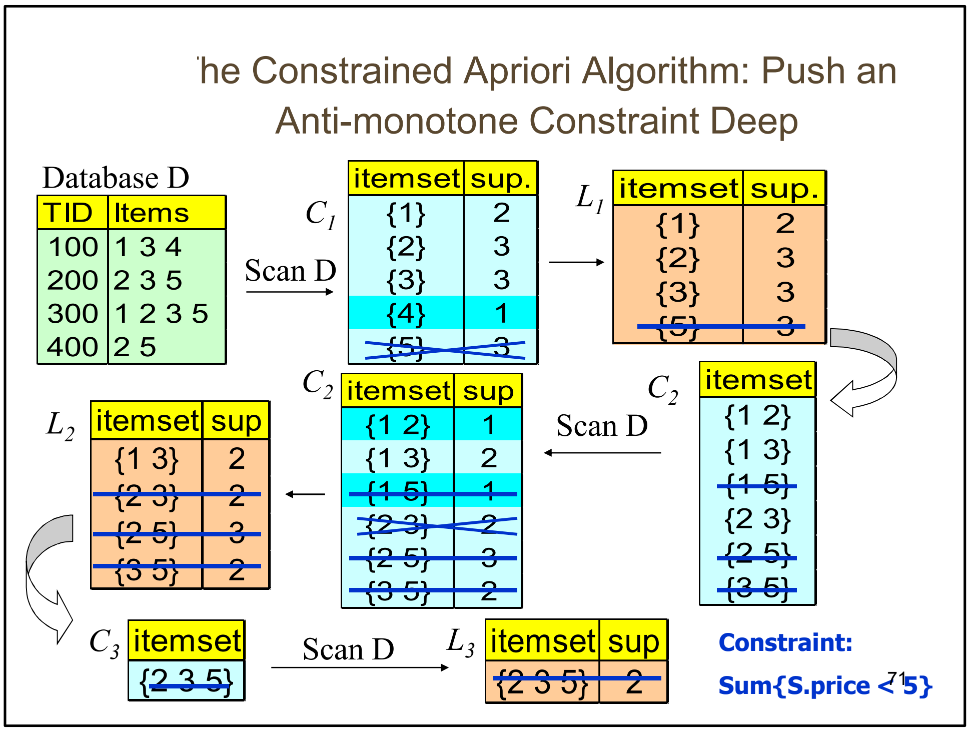


he Constrained Apriori Algorithm: Push an Anti-monotone Constraint Deep

Database D

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Scan D $\rightarrow$ C_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{4}	1
~~{5}~~	~~3~~

$\rightarrow$ L_1

itemset	sup.
{1}	2
{2}	3
{3}	3
~~{5}~~	~~3~~

C_2

itemset
{1 2}
{1 3}
~~{1 5}~~
{2 3}
~~{2 5}~~
~~{3 5}~~

Scan D $\leftarrow$ C_2

itemset	sup
{1 2}	1
{1 3}	2
~~{1 5}~~	~~1~~
~~{2 3}~~	~~2~~
~~{2 5}~~	~~3~~
~~{3 5}~~	~~2~~

$\leftarrow$ L_2

itemset	sup
{1 3}	2
~~{2 3}~~	~~2~~
~~{2 5}~~	~~3~~
~~{3 5}~~	~~2~~

C_3

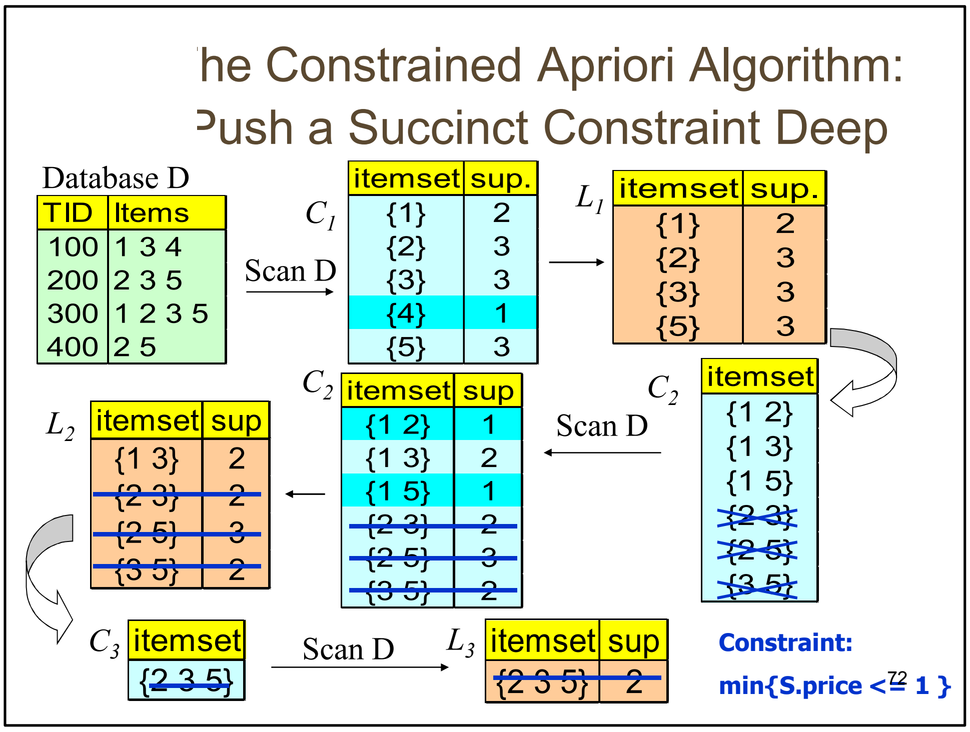
itemset
~~{2 3 5}~~

Scan D $\rightarrow$ L_3

itemset	sup
~~{2 3 5}~~	~~2~~

Constraint:

Sum{S.price < 5}



he Constrained Apriori Algorithm: Push a Succinct Constraint Deep

Database D

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Scan D $\rightarrow$ C_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

$\rightarrow$ L_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{5}	3

C_2

itemset
{1 2}
{1 3}
{1 5}
~~{2 3}~~
~~{2 5}~~
~~{3 5}~~

Scan D $\leftarrow$ C_2

itemset	sup
{1 2}	1
{1 3}	2
{1 5}	1
~~{2 3}~~	~~2~~
~~{2 5}~~	~~3~~
~~{3 5}~~	~~2~~

$\leftarrow$ L_2

itemset	sup
{1 3}	2
~~{2 3}~~	~~2~~
~~{2 5}~~	~~3~~
~~{3 5}~~	~~2~~

C_3

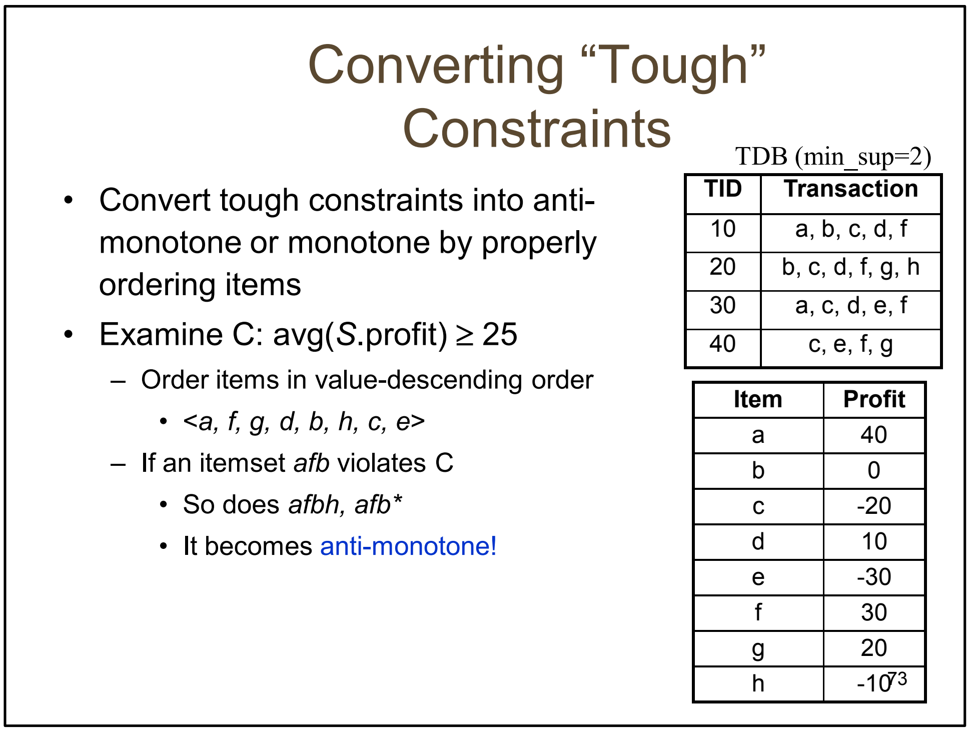
itemset
~~{2 3 5}~~

Scan D $\rightarrow$ L_3

itemset	sup
~~{2 3 5}~~	~~2~~

Constraint:

min{S.price <= 1 }



Converting “Tough” Constraints

- Convert tough constraints into anti-monotone or monotone by properly ordering items
- Examine C: avg(*S*.profit) ≥ 25
  - Order items in value-descending order
    - *<a, f, g, d, b, h, c, e>*
  - If an itemset *afb* violates C
    - So does *afbh, afb**
    - It becomes anti-monotone!

TDB (min_sup=2)

TID	Transaction
10	a, b, c, d, f
20	b, c, d, f, g, h
30	a, c, d, e, f
40	c, e, f, g

Item	Profit
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

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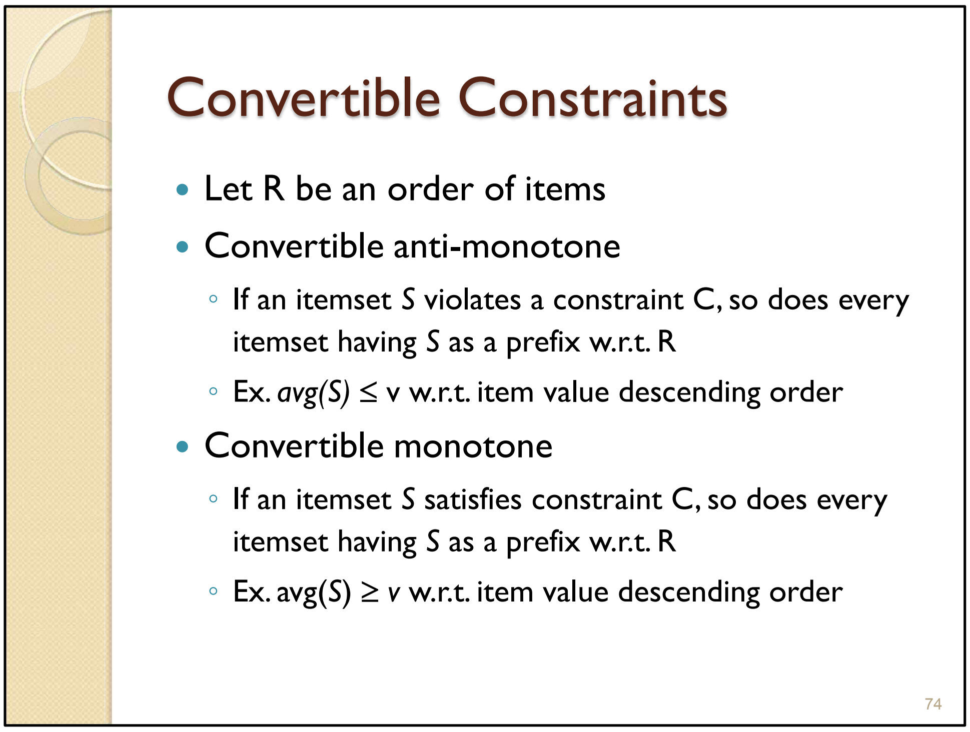

Convertible Constraints

- Let R be an order of items
- Convertible anti-monotone
  - If an itemset *S* violates a constraint C, so does every itemset having *S* as a prefix w.r.t. R
  - Ex. *avg(S)* ≤ v w.r.t. item value descending order
- Convertible monotone
  - If an itemset *S* satisfies constraint C, so does every itemset having *S* as a prefix w.r.t. R
  - Ex. avg(*S*) ≥ *v* w.r.t. item value descending order

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Strongly Convertible Constraints

- avg(X) ≥ 25 is convertible anti-monotone w.r.t. item value descending order R: <*a, f, g, d, b, h, c, e*>
  - If an itemset *af* violates a constraint C, so does every itemset with *af* as prefix, such as *afd*
- avg(X) ≥ 25 is convertible monotone w.r.t. item value ascending order R^{-1}: <*e, c, h, b, d, g, f, a*>
  - If an itemset *d* satisfies a constraint *C*, so does itemsets *df* and *dfa*, which having *d* as a prefix
- Thus, avg(X) ≥ 25 is strongly convertible

Item	Profit
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

What Constraints Are Convertible?

Constraint	Convertible anti-monotone	Convertible monotone	Strongly convertible
avg(S) ≤ , ≥ v	Yes	Yes	Yes
median(S) ≤ , ≥ v	Yes	Yes	Yes
sum(S) ≤ v (items could be of any value, v ≥ 0)	Yes	No	No
sum(S) ≤ v (items could be of any value, v ≤ 0)	No	Yes	No
sum(S) ≥ v (items could be of any value, v ≥ 0)	No	Yes	No
sum(S) ≥ v (items could be of any value, v ≤ 0)	Yes	No	No
……			

Combing Them Together—A General Picture

Constraint	Antimonotone	Monotone	Succinct
v ∈ S	no	yes	yes
S ⊇ V	no	yes	yes
S ⊆ V	yes	no	yes
min(S) ≤ v	no	yes	yes
min(S) ≥ v	yes	no	yes
max(S) ≤ v	yes	no	yes
max(S) ≥ v	no	yes	yes
count(S) ≤ v	yes	no	weakly
count(S) ≥ v	no	yes	weakly
sum(S) ≤ v (a ∈ S, a ≥ 0)	yes	no	no
sum(S) ≥ v (a ∈ S, a ≥ 0)	no	yes	no
range(S) ≤ v	yes	no	no
range(S) ≥ v	no	yes	no
avg(S) θ v, θ ∈ { =, ≤, ≥ }	convertible	convertible	no
support(S) ≥ ξ	yes	no	no
support(S) ≤ ξ	no	yes	no

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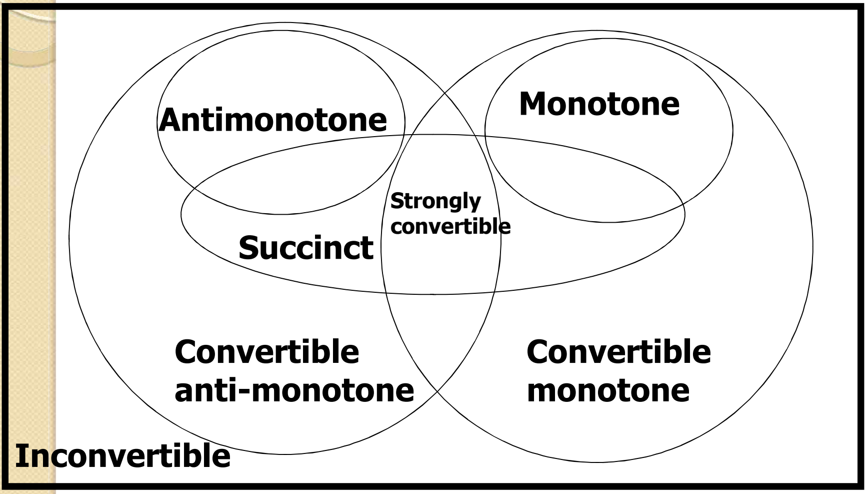
Classification of Constraints



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Mining With Convertible Constraints

- C: avg(S.profit) ≥ 25
- List of items in every transaction in value descending order R:

  <a, f, g, d, b, h, c, e>
  - C is convertible anti-monotone w.r.t. R
- Scan transaction DB once
  - remove infrequent items
    - Item *h* in transaction 40 is dropped
  - Itemsets *a* and *f* are good

TDB (min_sup=2)

TID	Transaction
10	a, f, d, b, c
20	f, g, d, b, c
30	a, f, d, c, e
40	f, g, h, c, e

Item	Profit
a	40
f	30
g	20
d	10
b	0
h	-10
c	-20
e	-30

Can Apriori Handle Convertible Constraint?

- A convertible, not monotone nor anti-monotone nor succinct constraint cannot be pushed deep into the an Apriori mining algorithm
  - Within the level wise framework, no direct pruning based on the constraint can be made
  - Itemset df violates constraint C: avg(X)>=25
  - Since adf satisfies C, Apriori needs df to assemble adf, df cannot be pruned
- But it can be pushed into frequent-pattern growth framework!

Item	Value
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

Mining With Convertible Constraints

- C: avg(X)>=25, min_sup=2
- List items in every transaction in value descending order R: <a, f, g, d, b, h, c, e>
  - C is convertible anti-monotone w.r.t. R
- Scan TDB once
  - remove infrequent items
    - Item h is dropped
  - Itemsets a and f are good, …
- Projection-based mining
  - Imposing an appropriate order on item projection
  - Many tough constraints can be converted into (anti)-monotone

Item	Value
a	40
f	30
g	20
d	10
b	0
h	-10
c	-20
e	-30

TDB (min_sup=2)

TID	Transaction
10	a, f, d, b, c
20	f, g, d, b, c
30	a, f, d, c, e
40	f, g, h, c, e

Handling Multiple Constraints

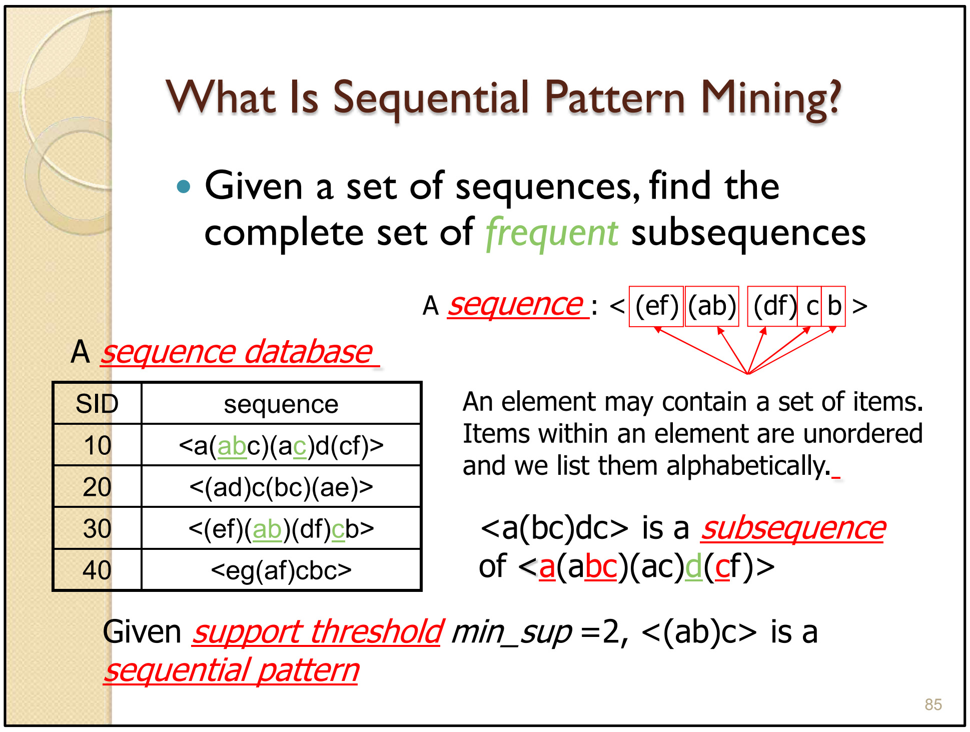
- Different constraints may require different or even conflicting item-ordering
- If there exists an order *R* s.t. both C_1 and C_2 are convertible w.r.t. *R*, then there is no conflict between the two convertible constraints
- If there exists conflict on order of items
  - Try to satisfy one constraint first
  - Then using the order for the other constraint to mine frequent itemsets in the corresponding projected database

Mining Association Rules in Large Databases

- Association rule mining
- Algorithms for scalable mining of (single-dimensional Boolean) association rules in transactional databases
- Mining various kinds of association/correlation rules
- Constraint-based association mining
- Sequential pattern mining
- Applications/extensions of frequent pattern mining
- Summary

Sequence Databases and Sequential Pattern Analysis

- Transaction databases, time-series databases vs. sequence databases
- Frequent patterns vs. (frequent) sequential patterns
- Applications of sequential pattern mining
  - Customer shopping sequences:
    - First buy computer, then CD-ROM, and then digital camera, within 3 months.
  - Medical treatment, natural disasters (e.g., earthquakes), science & engineering processes, stocks and markets, etc.
  - Telephone calling patterns, Weblog click streams
  - DNA sequences and gene structures



What Is Sequential Pattern Mining?

- Given a set of sequences, find the complete set of *frequent* subsequences

A *sequence* : < (ef) (ab) (df) c b >

A *sequence database*

SID	sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

An element may contain a set of items. Items within an element are unordered and we list them alphabetically.

<a(bc)dc> is a *subsequence* of <a(abc)(ac)d(cf)>

Given *support threshold* *min_sup* =2, <(ab)c> is a *sequential pattern*

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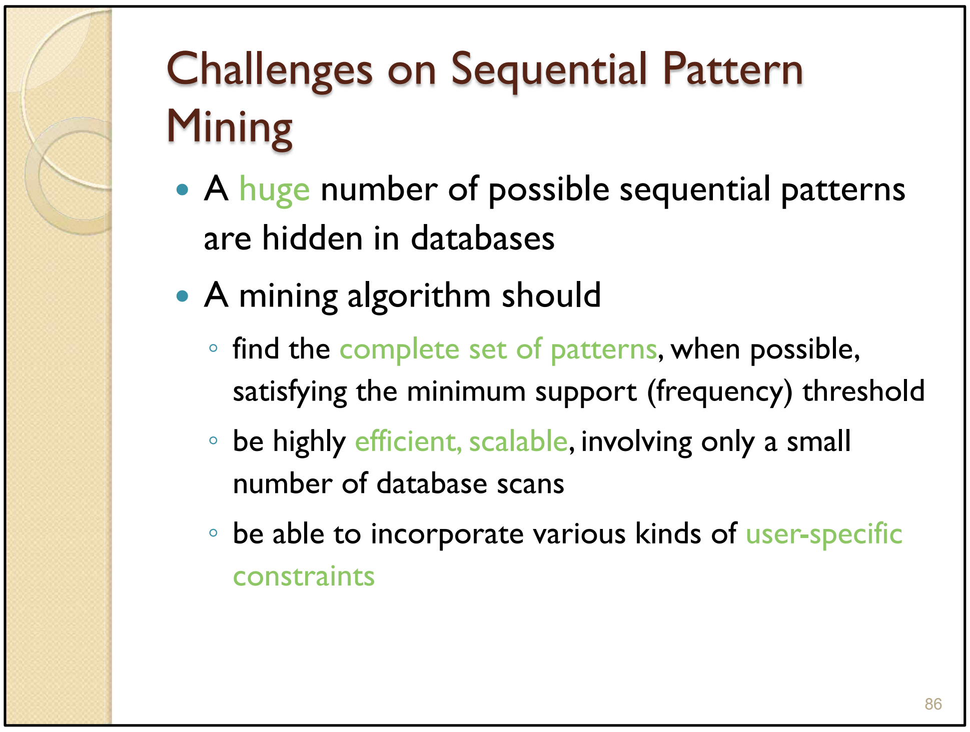

Challenges on Sequential Pattern Mining

- A huge number of possible sequential patterns are hidden in databases
- A mining algorithm should
  - find the complete set of patterns, when possible, satisfying the minimum support (frequency) threshold
  - be highly efficient, scalable, involving only a small number of database scans
  - be able to incorporate various kinds of user-specific constraints

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Studies on Sequential Pattern Mining

- Concept introduction and an initial Apriori-like algorithm
  - R. Agrawal & R. Srikant. "Mining sequential patterns," ICDE'95
- GSP—An Apriori-based, influential mining method (developed at IBM Almaden)
  - R. Srikant & R. Agrawal. "Mining sequential patterns: Generalizations and performance improvements," EDBT'96
- From sequential patterns to episodes (Apriori-like + constraints)
  - H. Mannila, H. Toivonen & A.I. Verkamo. "Discovery of frequent episodes in event sequences," Data Mining and Knowledge Discovery, 1997
- Mining sequential patterns with constraints
  - M.N. Garofalakis, R. Rastogi, K. Shim: SPIRIT: Sequential Pattern Mining with Regular Expression Constraints. VLDB 1999

A Basic Property of Sequential Patterns: Apriori

- A basic property: Apriori (Agrawal & Sirkant'94)
  - If a sequence S is not frequent
  - Then none of the super-sequences of S is frequent
  - E.g, <hb> is infrequent → so do <hab> and <(ah)b>

Seq. ID	Sequence
10	<(bd)cb(ac)>
20	<(bf)(ce)b(fg)>
30	<(ah)(bf)abf>
40	<(be)(ce)d>
50	<a(bd)bcb(ade)>

Given *support threshold*
min_sup =2

GSP—A Generalized Sequential Pattern Mining Algorithm

- GSP (Generalized Sequential Pattern) mining algorithm
  - proposed by Agrawal and Srikant, EDBT'96
- Outline of the method
  - Initially, every item in DB is a candidate of length-1
  - for each level (i.e., sequences of length-k) do
    - scan database to collect support count for each candidate sequence
    - generate candidate length-(k+1) sequences from length-k frequent sequences using Apriori
  - repeat until no frequent sequence or no candidate can be found
- Major strength: Candidate pruning by Apriori

Finding Length-1 Sequential Patterns

- Examine GSP using an example
- Initial candidates: all singleton sequences
  - <a>, <b>, <c>, <d>, <e>, <f>, <g>, <h>
- Scan database once, count support for candidates

min_sup =2

Seq. ID	Sequence
10	<(bd)cb(ac)>
20	<(bf)(ce)b(fg)>
30	<(ah)(bf)abf>
40	<(be)(ce)d>
50	<a(bd)bcb(ade)>

Cand	Sup
<a>	**3**
<b>	**5**
<c>	**4**
<d>	**3**
<e>	**3**
<f>	**2**
~~<g>~~	1
~~<h>~~	1

Generating Length-2 Candidates

51 length-2 Candidates

	<a>	<b>	<c>	<d>	<e>	<f>
<a>	<aa>	<ab>	<ac>	<ad>	<ae>	<af>
<b>	<ba>	<bb>	<bc>	<bd>	<be>	<bf>
<c>	<ca>	<cb>	<cc>	<cd>	<ce>	<cf>
<d>	<da>	<db>	<dc>	<dd>	<de>	<df>
<e>	<ea>	<eb>	<ec>	<ed>	<ee>	<ef>
<f>	<fa>	<fb>	<fc>	<fd>	<fe>	<ff>

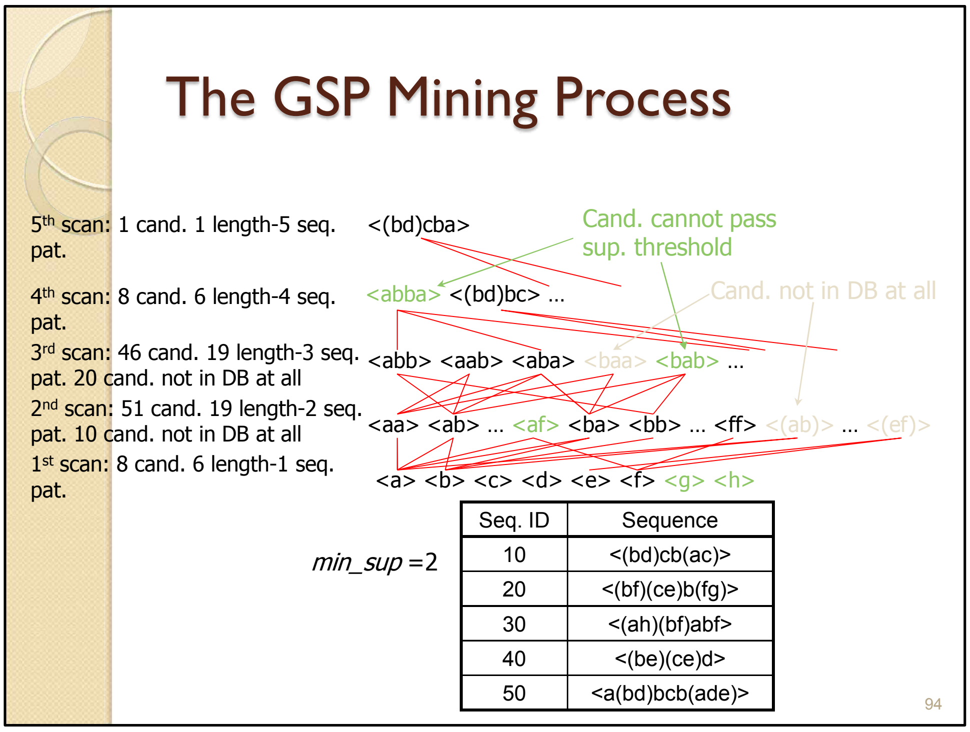
	<a>	<b>	<c>	<d>	<e>	<f>
<a>		<(ab)>	<(ac)>	<(ad)>	<(ae)>	<(af)>
<b>			<(bc)>	<(bd)>	<(be)>	<(bf)>
<c>				<(cd)>	<(ce)>	<(cf)>
<d>					<(de)>	<(df)>
<e>						<(ef)>
<f>						

Without Apriori property, 8*8+8*7/2=92 candidates

Apriori prunes 44.57% candidates

Generating Length-3 Candidates and Finding Length-3 Patterns

- Generate Length-3 Candidates
  - Self-join length-2 sequential patterns
    - Based on the Apriori property
    - <ab>, <aa> and <ba> are all length-2 sequential patterns → <aba> is a length-3 candidate
    - <(bd)>, <bb> and <db> are all length-2 sequential patterns → <(bd)b> is a length-3 candidate
  - 46 candidates are generated
- Find Length-3 Sequential Patterns
  - Scan database once more, collect support counts for candidates
  - 19 out of 46 candidates pass support threshold



The GSP Mining Process

5th scan: 1 cand. 1 length-5 seq. pat.

4th scan: 8 cand. 6 length-4 seq. pat.

3rd scan: 46 cand. 19 length-3 seq. pat. 20 cand. not in DB at all

2nd scan: 51 cand. 19 length-2 seq. pat. 10 cand. not in DB at all

1st scan: 8 cand. 6 length-1 seq. pat.

<(bd)cba>

<abba> <(bd)bc> …

<abb> <aab> <aba> <baa> <bab> …

<aa> <ab> … <af> <ba> <bb> … <ff> <(ab)> … <(ef)>

<a> <b> <c> <d> <e> <f> <g> <h>

Cand. cannot pass sup. threshold

Cand. not in DB at all

min_sup =2

Seq. ID	Sequence
10	<(bd)cb(ac)>
20	<(bf)(ce)b(fg)>
30	<(ah)(bf)abf>
40	<(be)(ce)d>
50	<a(bd)bcb(ade)>

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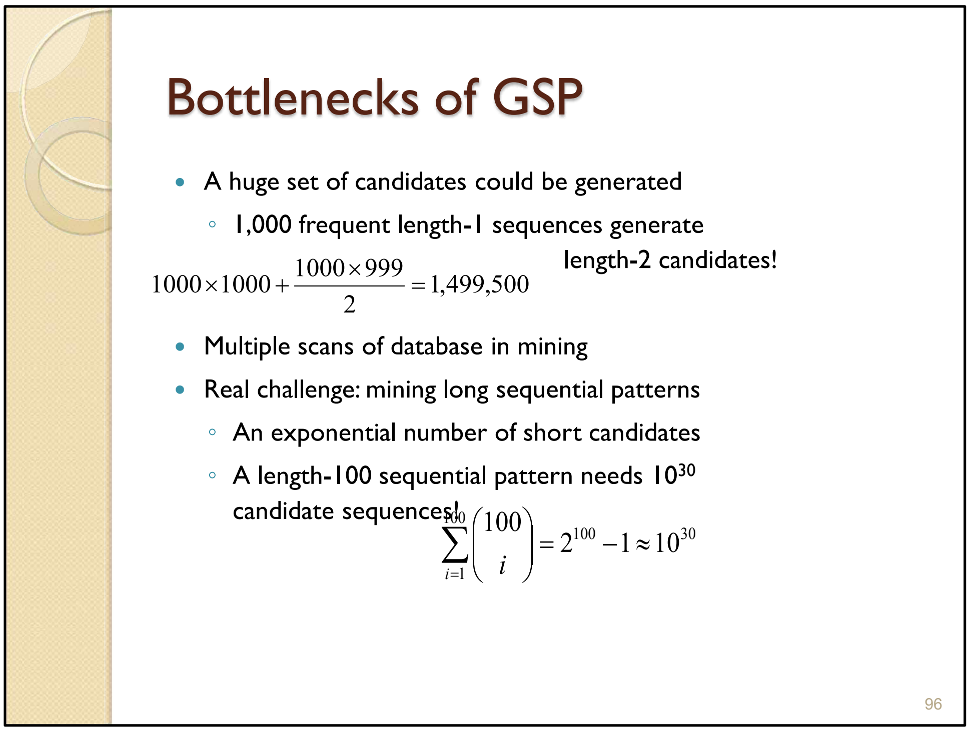

Bottlenecks of GSP

- A huge set of candidates could be generated
  - 1,000 frequent length-1 sequences generate length-2 candidates!

$$1000 \times 1000 + \frac{1000 \times 999}{2} = 1{,}499{,}500$$

- Multiple scans of database in mining
- Real challenge: mining long sequential patterns
  - An exponential number of short candidates
  - A length-100 sequential pattern needs 10^{30} candidate sequences!

$$\sum_{i=1}^{100} \binom{100}{i} = 2^{100} - 1 \approx 10^{30}$$

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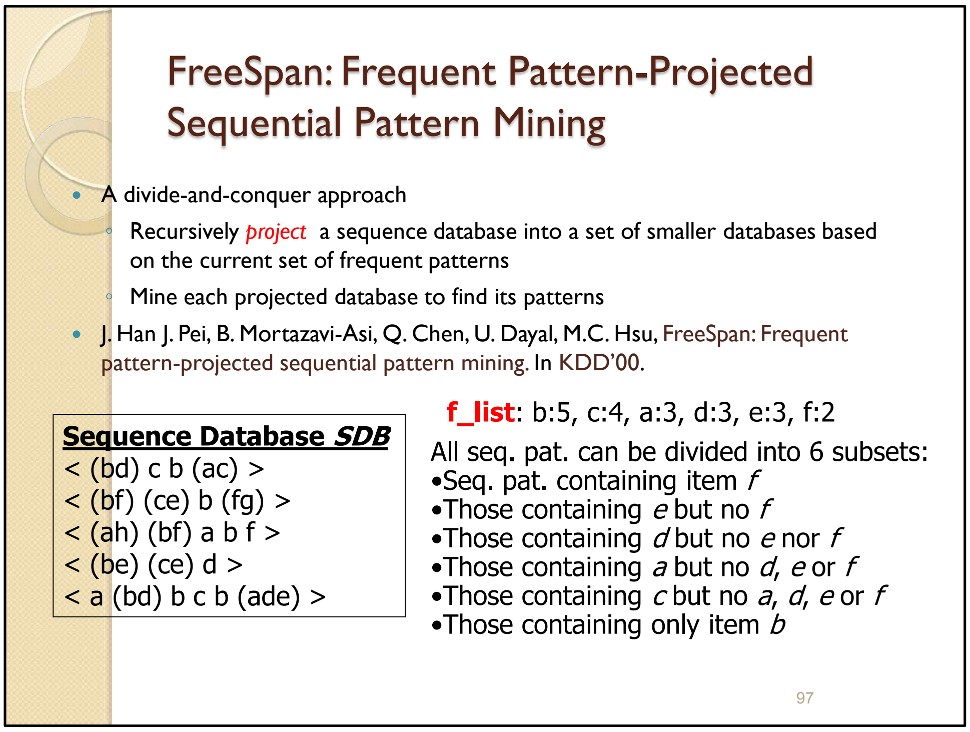

FreeSpan: Frequent Pattern-Projected Sequential Pattern Mining

- A divide-and-conquer approach
  - Recursively *project* a sequence database into a set of smaller databases based on the current set of frequent patterns
  - Mine each projected database to find its patterns
- J. Han J. Pei, B. Mortazavi-Asi, Q. Chen, U. Dayal, M.C. Hsu, FreeSpan: Frequent pattern-projected sequential pattern mining. In KDD'00.

Sequence Database *SDB*
< (bd) c b (ac) >
< (bf) (ce) b (fg) >
< (ah) (bf) a b f >
< (be) (ce) d >
< a (bd) b c b (ade) >

f_list: b:5, c:4, a:3, d:3, e:3, f:2

All seq. pat. can be divided into 6 subsets:
•Seq. pat. containing item *f*
•Those containing *e* but no *f*
•Those containing *d* but no *e* nor *f*
•Those containing *a* but no *d*, *e* or *f*
•Those containing *c* but no *a*, *d*, *e* or *f*
•Those containing only item *b*

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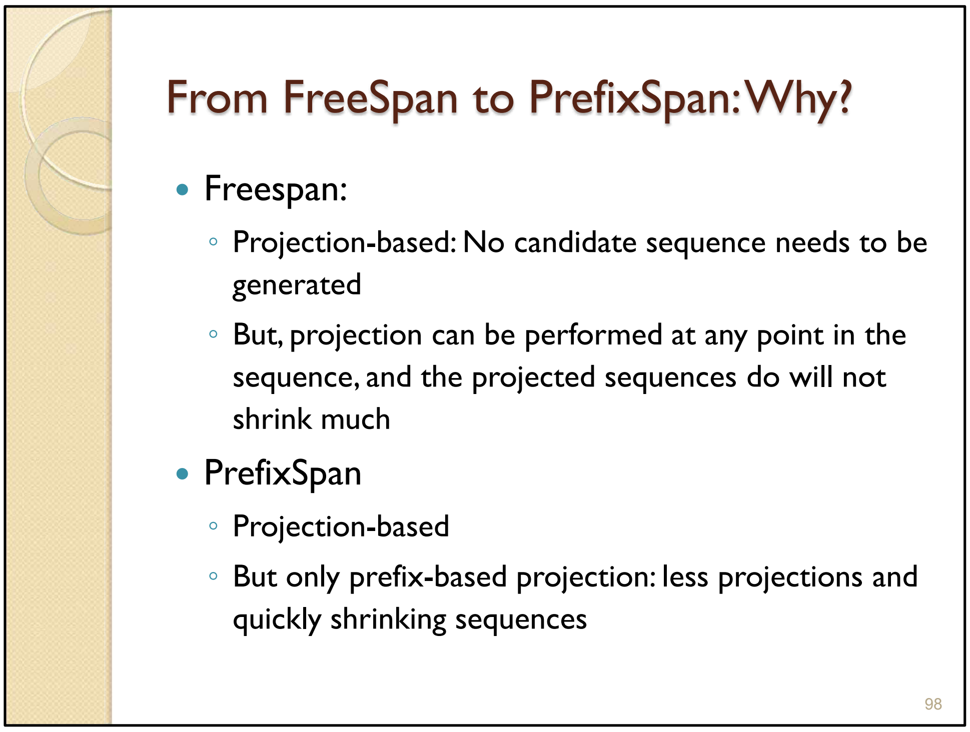

From FreeSpan to PrefixSpan: Why?

- Freespan:
  - Projection-based: No candidate sequence needs to be generated
  - But, projection can be performed at any point in the sequence, and the projected sequences do will not shrink much
- PrefixSpan
  - Projection-based
  - But only prefix-based projection: less projections and quickly shrinking sequences

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Prefix and Suffix (Projection)

- <a>, <aa>, <a(ab)> and <a(abc)> are *prefixes* of sequence <a(abc)(ac)d(cf)>
- Given sequence <a(abc)(ac)d(cf)>

Prefix	*Suffix* (Prefix-Based *Projection)*
<a>	<(abc)(ac)d(cf)>
<aa>	<(_bc)(ac)d(cf)>
<ab>	<(_c)(ac)d(cf)>

Mining Sequential Patterns by Prefix Projections

- Step 1: find length-1 sequential patterns
  - <a>, <b>, <c>, <d>, <e>, <f>
- Step 2: divide search space. The complete set of seq. pat. can be partitioned into 6 subsets:
  - The ones having prefix <a>;
  - The ones having prefix <b>;
  - …
  - The ones having prefix <f>

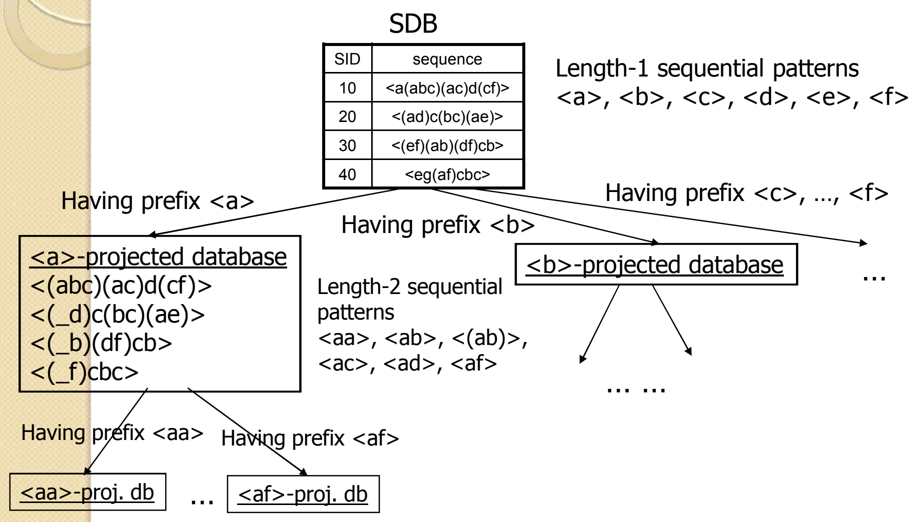
SID	sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

Finding Seq. Patterns with Prefix <a>

- Only need to consider projections w.r.t. <a>
  - <a>-projected database: <(abc)(ac)d(cf)>, <(_d)c(bc)(ae)>, <(_b)(df)cb>, <(_f)cbc>
- Find all the length-2 seq. pat. Having prefix <a>: <aa>, <ab>, <(ab)>, <ac>, <ad>, <af>
  - Further partition into 6 subsets
    - Having prefix <aa>;
    - …
    - Having prefix <af>

SID	sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

Completeness of PrefixSpan



Efficiency of PrefixSpan

- No candidate sequence needs to be generated
- Projected databases keep shrinking
- Major cost of PrefixSpan: constructing projected databases
  - Can be improved by bi-level projections

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Optimization Techniques in PrefixSpan

- Physical projection vs. pseudo-projection
  - Pseudo-projection may reduce the effort of projection when the projected database fits in main memory
- Parallel projection vs. partition projection
  - Partition projection may avoid the blowup of disk space

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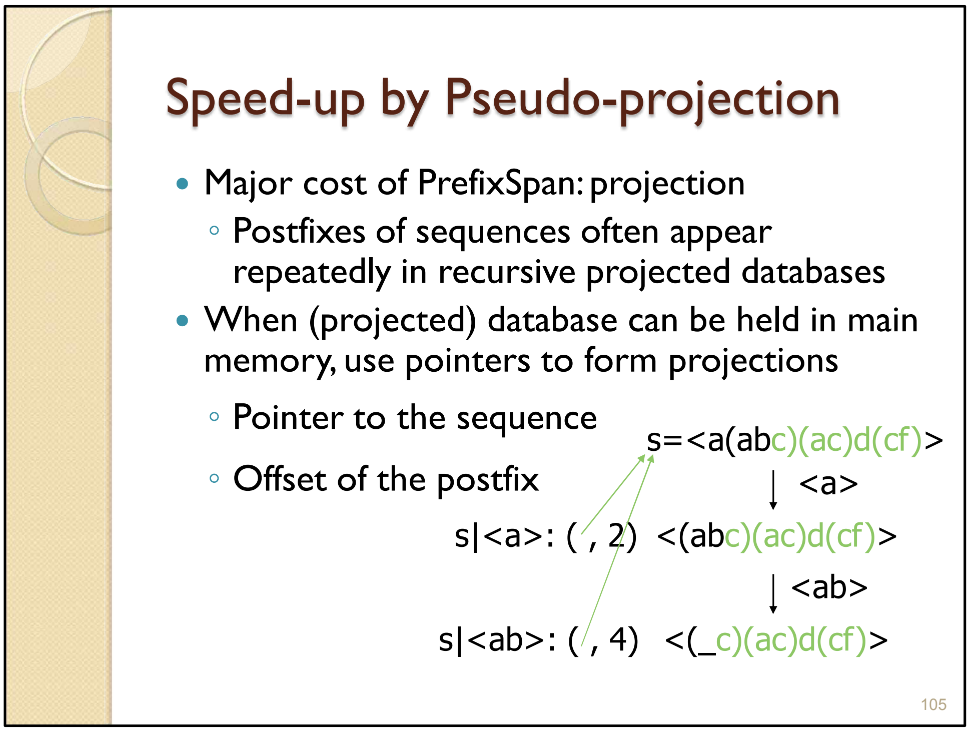

Speed-up by Pseudo-projection

- Major cost of PrefixSpan: projection
  - Postfixes of sequences often appear repeatedly in recursive projected databases
- When (projected) database can be held in main memory, use pointers to form projections
  - Pointer to the sequence
  - Offset of the postfix

```
                          s=<a(abc)(ac)d(cf)>
                                   ↓ <a>
s|<a>: ( , 2)  <(abc)(ac)d(cf)>
                                   ↓ <ab>
s|<ab>: ( , 4)  <(_c)(ac)d(cf)>
```

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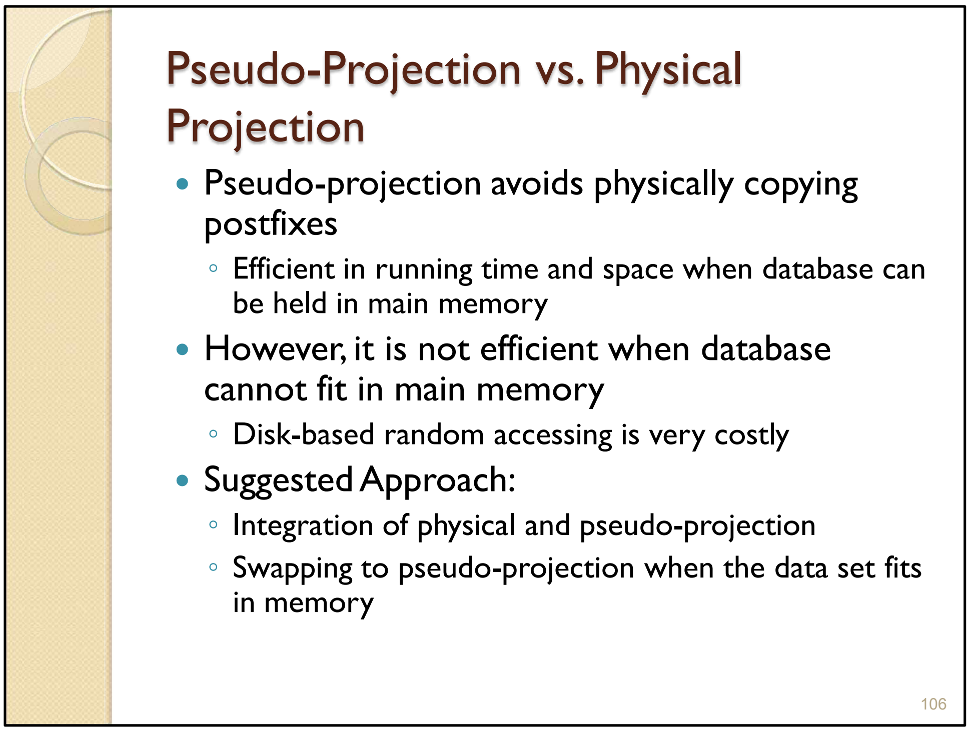

Pseudo-Projection vs. Physical Projection

- Pseudo-projection avoids physically copying postfixes
  - Efficient in running time and space when database can be held in main memory
- However, it is not efficient when database cannot fit in main memory
  - Disk-based random accessing is very costly
- Suggested Approach:
  - Integration of physical and pseudo-projection
  - Swapping to pseudo-projection when the data set fits in memory

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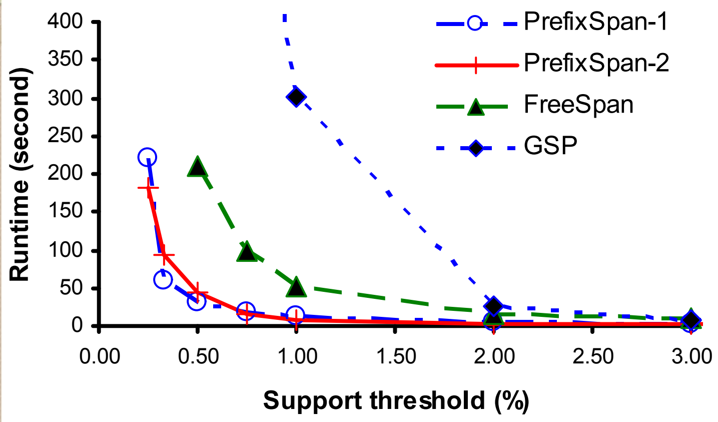
PrefixSpan Is Faster than GSP and FreeSpan



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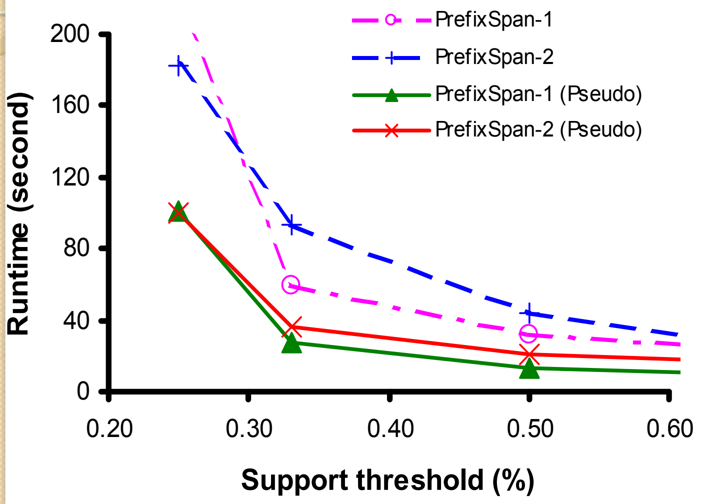
Effect of Pseudo-Projection



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Mining Association Rules in Large Databases

- Association rule mining
- Algorithms for scalable mining of (single-dimensional Boolean) association rules in transactional databases
- Mining various kinds of association/correlation rules
- Constraint-based association mining
- Sequential pattern mining
- Applications/extensions of frequent pattern mining
- Summary

Associative Classification

- Mine association possible rules (PR) in form of condset ➔ c
  - Condset: a set of attribute-value pairs
  - C: class label
- Build Classifier
  - Organize rules according to decreasing precedence based on confidence and support
- B. Liu, W. Hsu & Y. Ma. Integrating classification and association rule mining. In KDD'98

Spatial and Multi-Media Association: A Progressive Refinement Method

- Why progressive refinement?
  - Mining operator can be expensive or cheap, fine or rough
  - Trade speed with quality: step-by-step refinement.
- Superset coverage property:
  - Preserve all the positive answers—allow a positive false test but not a false negative test.
- Two- or multi-step mining:
  - First apply rough/cheap operator (superset coverage)
  - Then apply expensive algorithm on a substantially reduced candidate set (Koperski & Han, SSD'95).

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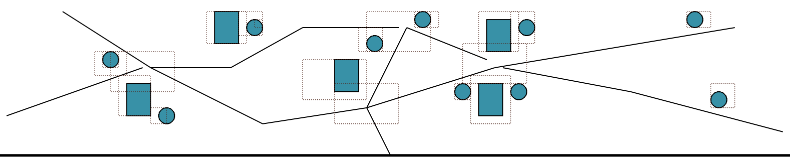
Progressive Refinement Mining of Spatial Associations

- Hierarchy of spatial relationship:
  - "g_close_to": near_by, touch, intersect, contain, etc.
  - First search for rough relationship and then refine it.
- Two-step mining of spatial association:
  - Step 1: rough spatial computation (as a filter)
    - Using MBR or R-tree for rough estimation.
  - Step2: Detailed spatial algorithm (as refinement)
    - Apply only to those objects which have passed the rough spatial association test (no less than *min_support*)



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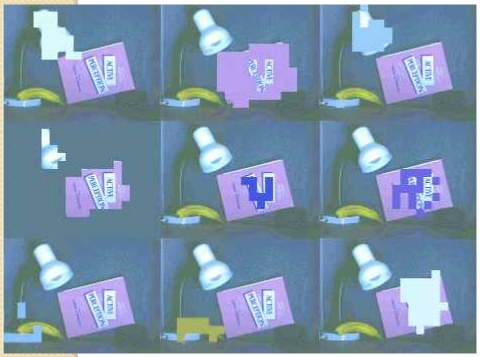
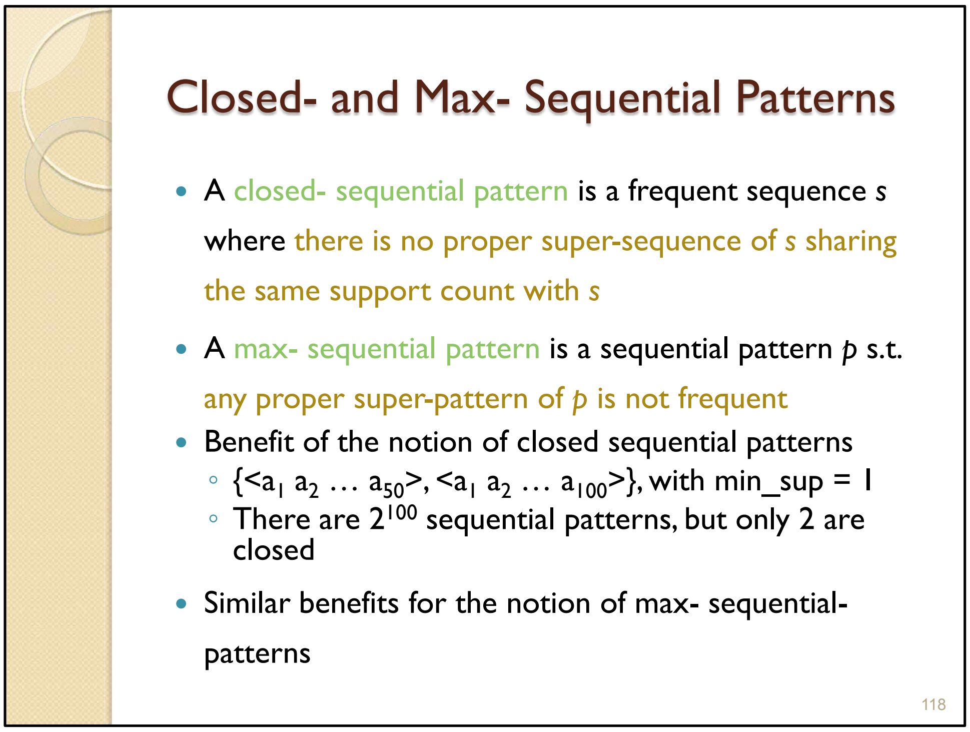
Mining Multimedia Associations

Correlations with color, spatial relationships, etc.
From coarse to Fine Resolution mining



Further Evolution of PrefixSpan

- Closed- and max- sequential patterns
  - Finding only the most meaningful (longest) sequential patterns
- Constraint-based sequential pattern growth
  - Adding user-specific constraints
- From sequential patterns to structured patterns
  - Beyond sequential patterns, mining structured patterns in XML documents



Closed- and Max- Sequential Patterns

- A closed- sequential pattern is a frequent sequence *s* where there is no proper super-sequence of *s* sharing the same support count with *s*
- A max- sequential pattern is a sequential pattern *p* s.t. any proper super-pattern of *p* is not frequent
- Benefit of the notion of closed sequential patterns
  - {<a_1 a_2 … a_{50}>, <a_1 a_2 … a_{100}>}, with min_sup = 1
  - There are 2^{100} sequential patterns, but only 2 are closed
- Similar benefits for the notion of max- sequential-patterns

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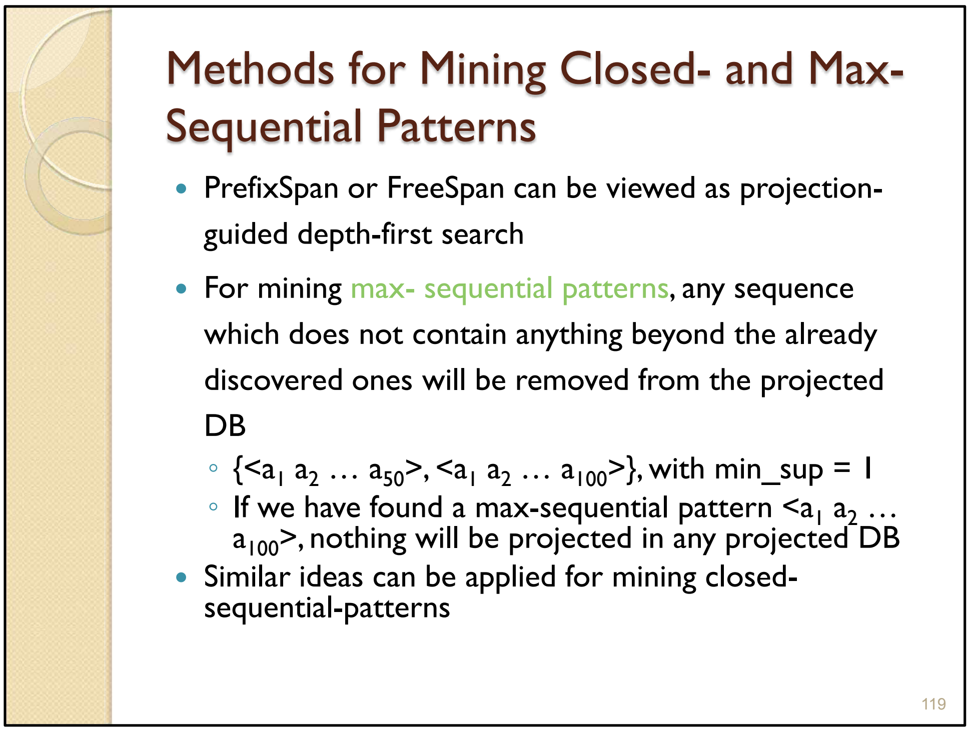

Methods for Mining Closed- and Max- Sequential Patterns

- PrefixSpan or FreeSpan can be viewed as projection-guided depth-first search
- For mining max- sequential patterns, any sequence which does not contain anything beyond the already discovered ones will be removed from the projected DB
  - {<a_1 a_2 … a_{50}>, <a_1 a_2 … a_{100}>}, with min_sup = 1
  - If we have found a max-sequential pattern <a_1 a_2 … a_{100}>, nothing will be projected in any projected DB
- Similar ideas can be applied for mining closed-sequential-patterns

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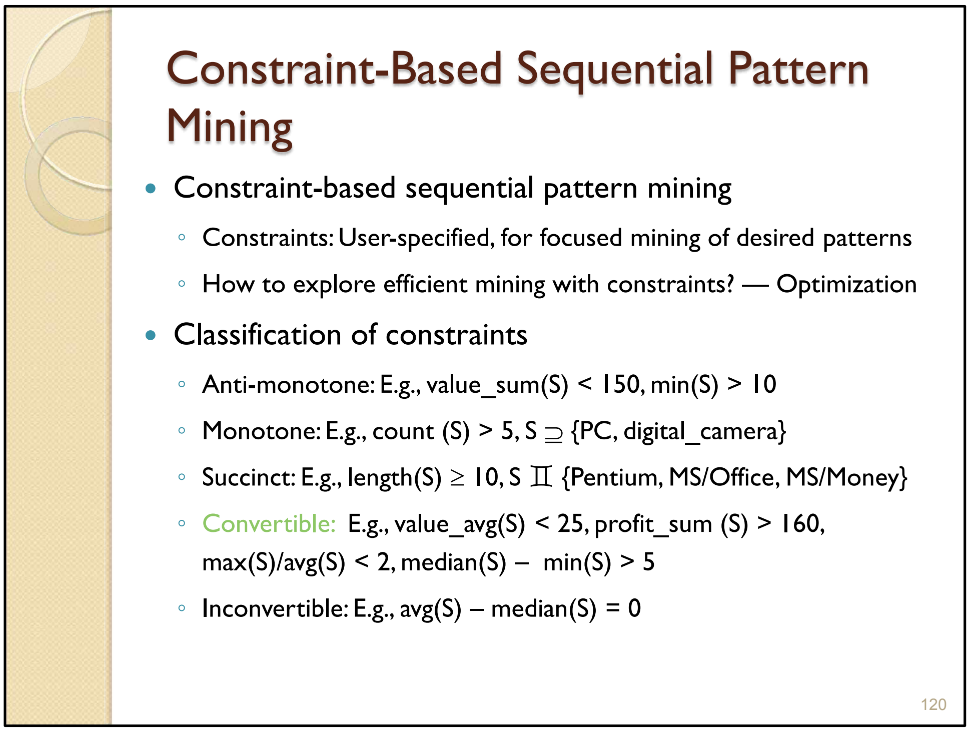

Constraint-Based Sequential Pattern Mining

- Constraint-based sequential pattern mining
  - Constraints: User-specified, for focused mining of desired patterns
  - How to explore efficient mining with constraints? — Optimization
- Classification of constraints
  - Anti-monotone: E.g., value_sum(S) < 150, min(S) > 10
  - Monotone: E.g., count (S) > 5, S ⊇ {PC, digital_camera}
  - Succinct: E.g., length(S) ≥ 10, S ∐ {Pentium, MS/Office, MS/Money}
  - Convertible: E.g., value_avg(S) < 25, profit_sum (S) > 160, max(S)/avg(S) < 2, median(S) – min(S) > 5
  - Inconvertible: E.g., avg(S) – median(S) = 0

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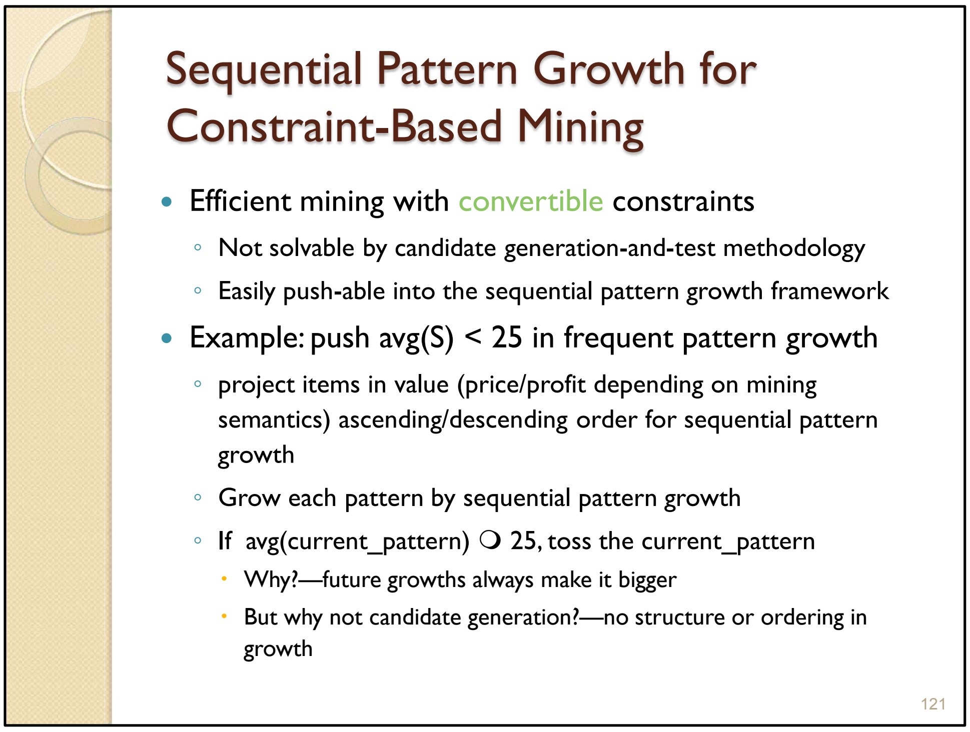

Sequential Pattern Growth for Constraint-Based Mining

- Efficient mining with convertible constraints
  - Not solvable by candidate generation-and-test methodology
  - Easily push-able into the sequential pattern growth framework
- Example: push avg(S) < 25 in frequent pattern growth
  - project items in value (price/profit depending on mining semantics) ascending/descending order for sequential pattern growth
  - Grow each pattern by sequential pattern growth
  - If avg(current_pattern) ❍ 25, toss the current_pattern
    - Why?—future growths always make it bigger
    - But why not candidate generation?—no structure or ordering in growth

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From Sequential Patterns to Structured Patterns

- Sets, sequences, trees and other structures
  - Transaction DB: Sets of items
    - $\{\{i_1, i_2, \ldots, i_m\}, \ldots\}$
  - Seq. DB: Sequences of sets:
    - $\{<\{i_1, i_2\}, \ldots, \{i_m, i_n, i_k\}>, \ldots\}$
  - Sets of Sequences:
    - $\{\{<i_1, i_2>, \ldots, <i_m, i_n, i_k>\}, \ldots\}$
  - Sets of trees (each element being a tree):
    - $\{t_1, t_2, \ldots, t_n\}$
- Applications: Mining structured patterns in XML documents

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Mining Association Rules in Large Databases

- Association rule mining
- Algorithms for scalable mining of (single-dimensional Boolean) association rules in transactional databases
- Mining various kinds of association/correlation rules
- Constraint-based association mining
- Sequential pattern mining
- Applications/extensions of frequent pattern mining
- Summary

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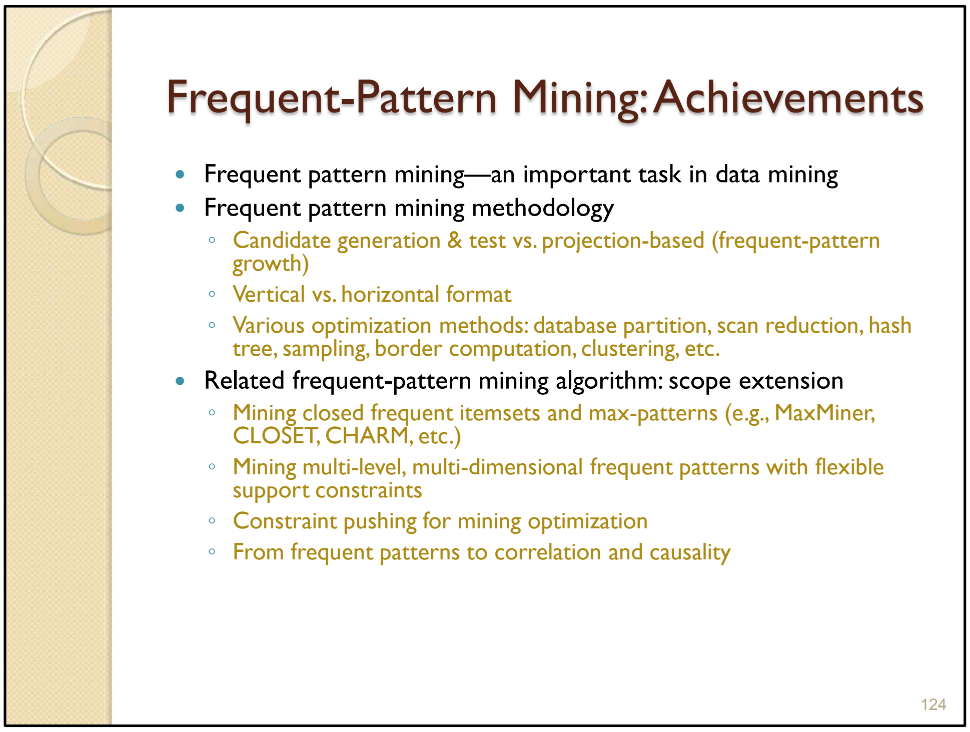

Frequent-Pattern Mining: Achievements

- Frequent pattern mining—an important task in data mining
- Frequent pattern mining methodology
  - Candidate generation & test vs. projection-based (frequent-pattern growth)
  - Vertical vs. horizontal format
  - Various optimization methods: database partition, scan reduction, hash tree, sampling, border computation, clustering, etc.
- Related frequent-pattern mining algorithm: scope extension
  - Mining closed frequent itemsets and max-patterns (e.g., MaxMiner, CLOSET, CHARM, etc.)
  - Mining multi-level, multi-dimensional frequent patterns with flexible support constraints
  - Constraint pushing for mining optimization
  - From frequent patterns to correlation and causality

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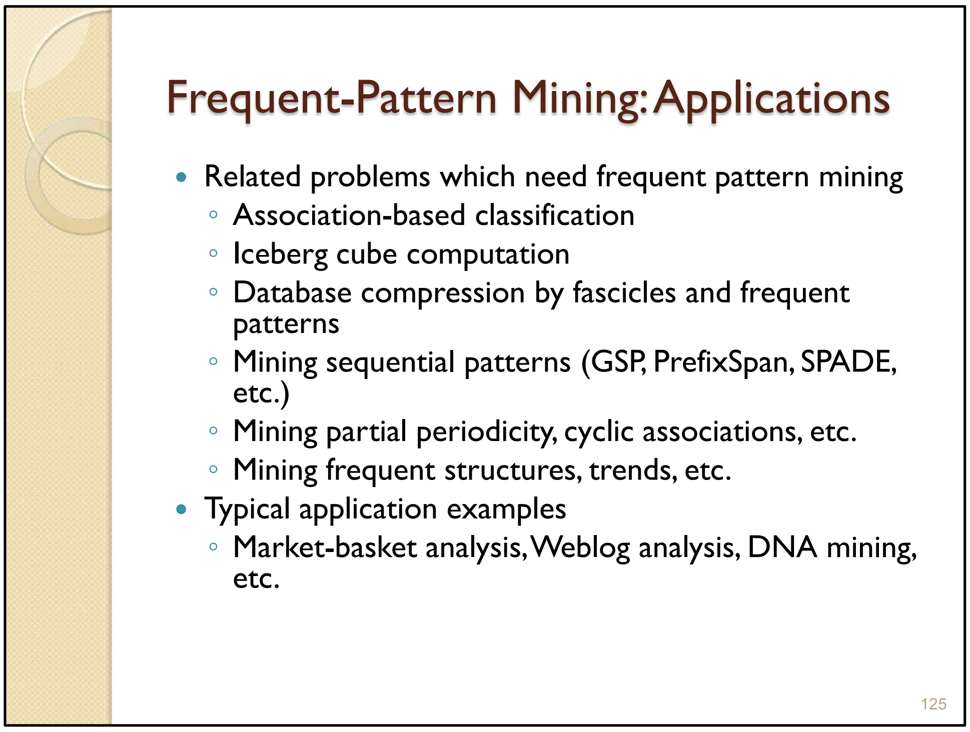

Frequent-Pattern Mining: Applications

- Related problems which need frequent pattern mining
  - Association-based classification
  - Iceberg cube computation
  - Database compression by fascicles and frequent patterns
  - Mining sequential patterns (GSP, PrefixSpan, SPADE, etc.)
  - Mining partial periodicity, cyclic associations, etc.
  - Mining frequent structures, trends, etc.
- Typical application examples
  - Market-basket analysis, Weblog analysis, DNA mining, etc.

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Frequent-Pattern Mining: Research Problems

- Multi-dimensional gradient analysis: patterns regarding changes and differences
    - Not just counts—other measures, e.g., avg(profit)
- Mining top-k frequent patterns without support constraint
- Mining fault-tolerant associations
    - "3 out of 4 courses excellent" leads to A in data mining
- Fascicles and database compression by frequent pattern mining
- Partial periodic patterns
- DNA sequence analysis and pattern classification

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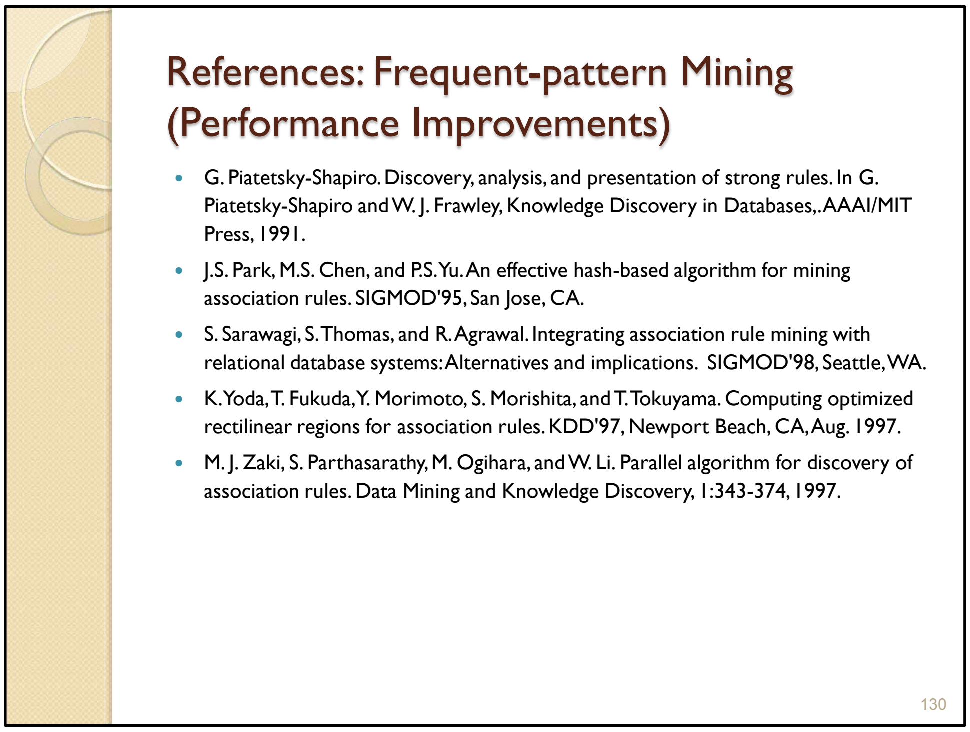

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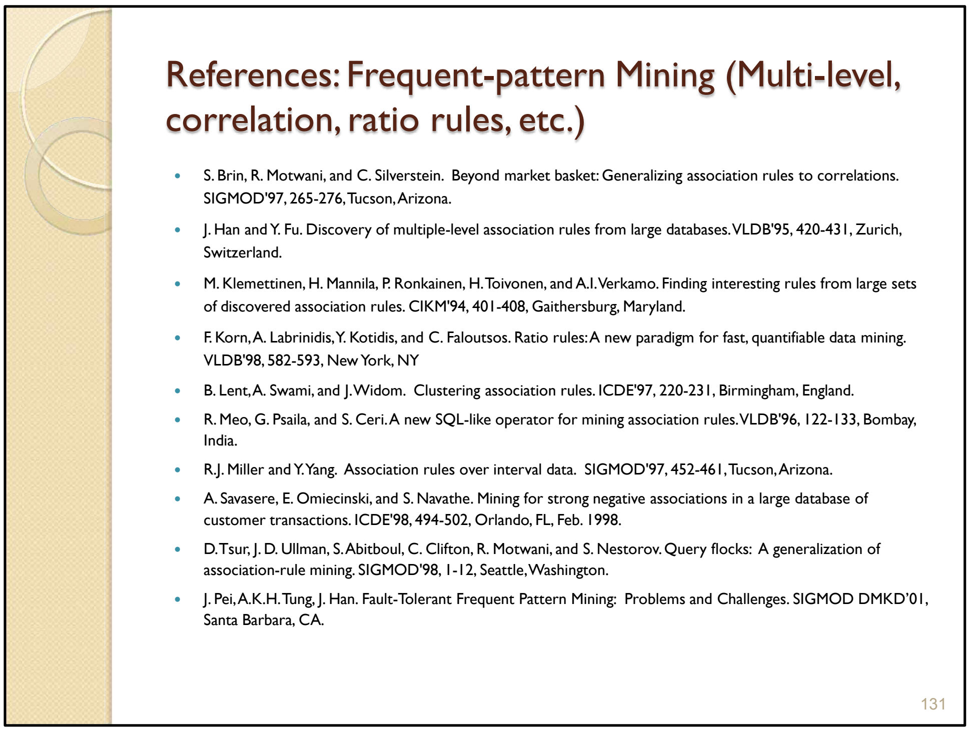

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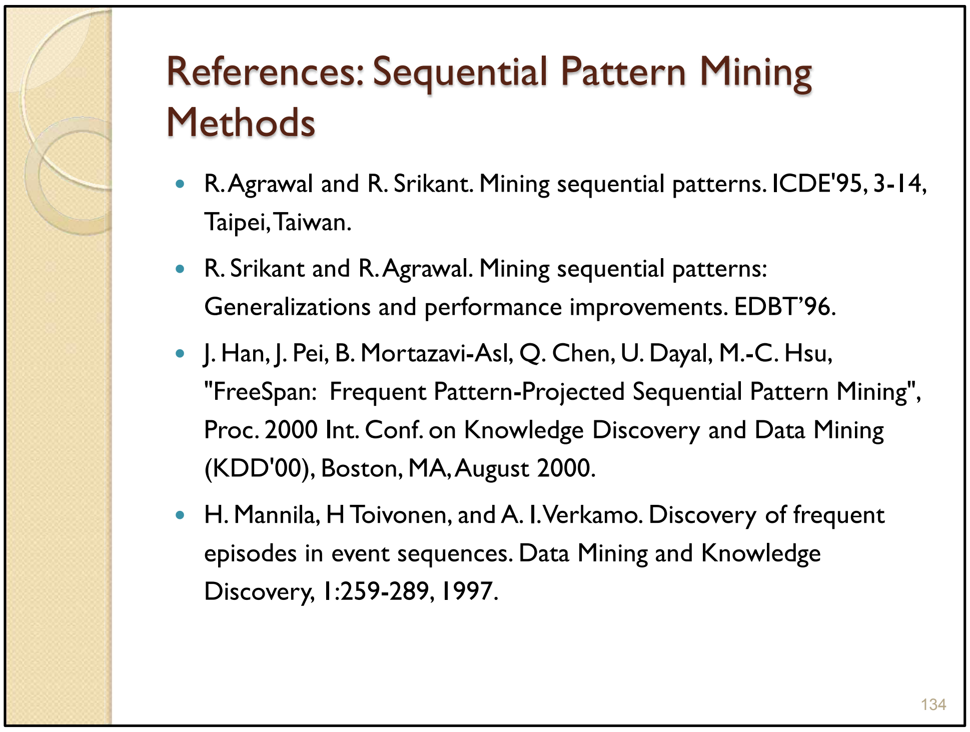

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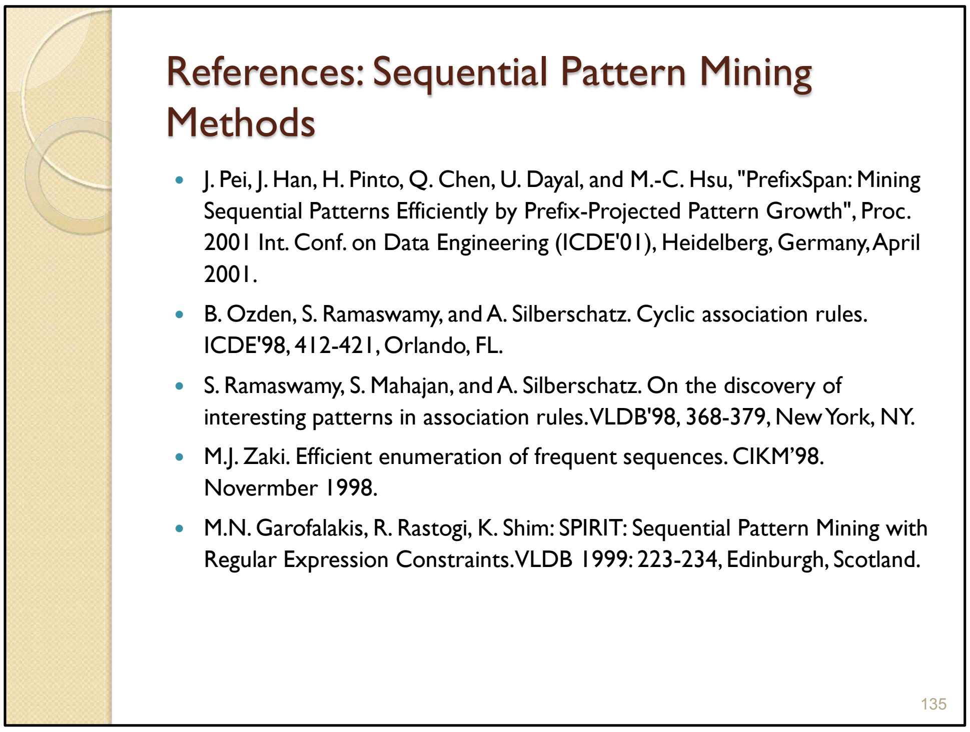

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