Assignment 1

Prabhath Chellingi - CS20BTECH11038

Download all python codes from

```
https://github.com/PRABHATH−cs20−11038/
    Assignment_1/tree/main/Assignment_1/Codes
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and latex-tikz codes from

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https://github.com/PRABHATH−cs20−11038/
    Assignment_1/blob/main/Assignment_1/
    Assignment_1.tex
```

1 Problem

(Prob 3.6) Find the probability distribution of
(i) number of heads in two tosses of a coin.
(ii) number of tails in the simultaneous tosses of three coins.
(iii) number of heads in four tosses of a coin.

2 Solution

Let $X_i \in \{0, 1\}$ represent the i^{Th} coin where 1 denotes the coin giving outcome as head. Then, X_i has a Bernoulli distribution with parameter

$$p = \frac{1}{2} \tag{2.0.1}$$

Let

$$X = \sum_{i=1}^{n} X_i \tag{2.0.2}$$

where n is the total number of coins tossed. Then X has a Binomial Distribution. Then for

$$p_{X_i}(n) \overset{Z}{\rightleftharpoons} P_{X_i}(z) \tag{2.0.3}$$

yielding

$$P_{X_i}(z) = 1 - p + pz^{-1} \tag{2.0.4}$$

with using the fact that X_i are i.i.d.,

$$\begin{aligned} P_X(z) &= (1 - p + pz^{-1})^n \\ &= \sum_{k=0}^{n} {}^nC_k p^k (1 - p)^{n-k} z^{-k} \end{aligned} \tag{2.0.5}$$

$$p_X(k) = \begin{cases} {}^nC_k p^k (1 - p)^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \tag{2.0.6}$$

2.1 (i)

Here we are seeing outcome of two tosses of a coin. We can say it as simultaneous tossing two coins. So, we get,

$$n = 2 \tag{2.1.1}$$

Now we obtain probability distribution of number of heads of two coins from (2.0.6),

$$p_X(k) = \begin{cases} {}^2C_k(\frac{1}{2})^k(1 - \frac{1}{2})^{2-k} & 0 \le k \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$p_X(k) = \begin{cases} {}^2C_k(\frac{1}{2})^2 & \text{if } 0 \le k \le 2 \\ 0 & \text{otherwise} \end{cases} \tag{2.1.2}$$

probability of getting 0 heads,

$$\begin{aligned} P_X(0) &= {}^2C_0(\frac{1}{2})^2 \\ &= 1.\frac{1}{4} \\ &= \frac{1}{4} \end{aligned} \tag{2.1.3}$$

probability of getting 1 heads,

$$\begin{aligned} P_X(1) &= {}^2C_1(\frac{1}{2})^2 \\ &= 2.\frac{1}{4} \\ &= \frac{1}{2} \end{aligned} \tag{2.1.4}$$

probability of getting 2 heads,

$$\begin{aligned} P_X(2) &= {}^2C_2(\frac{1}{2})^2 \\ &= 1.\frac{1}{4} \\ &= \frac{1}{4} \end{aligned} \tag{2.1.5}$$

k	0	1	2
Pr(k)	1/4	1/2	1/4

Fig. 0: Table of probability distribution number of heads with two tossed coins

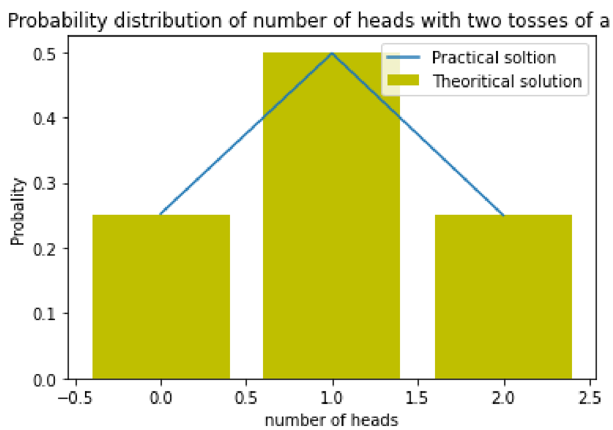

Fig. 0: Plot of probability distribution of two tossed coins

2.2 (ii)

With respect to tails the parameter would change to

$$q = 1 - p = \frac{1}{2} \tag{2.2.1}$$

and there will be a small change in binomial distribution (2.0.6) and we get binomial distribution for tails as,

$$p_{1-X}(k) = \begin{cases} {}^{n}C_k q^k (1-q)^{n-k} & 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases} \tag{2.2.2}$$

Here we are seeing outcome of simultaneous tosses of three coin. So, we get,

$$n = 3 \tag{2.2.3}$$

Now we obtain probability distribution of number of tails of three coins from (2.2.2),

$$p_{1-X}(k) = \begin{cases} {}^{3}C_k (\frac{1}{2})^k (1-\frac{1}{2})^{3-k} & 0 \leq k \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

$$p_{1-X}(k) = \begin{cases} {}^{3}C_k (\frac{1}{2})^3 & \text{if } 0 \leq k \leq 3 \\ 0 & \text{otherwise} \end{cases} \tag{2.2.4}$$

k	0	1	2	3
Pr(k)	1/8	3/8	3/8	1/8

Fig. 0: Table of probability distribution of no of tails with three tossed coins

probability of getting 0 tails,

$$\begin{aligned} P_{1-X}(0) &= {}^{3}C_0 (\frac{1}{2})^3 \\ &= 1.\frac{1}{8} \\ &= \frac{1}{8} \end{aligned} \tag{2.2.5}$$

probability of getting 1 tails,

$$\begin{aligned} P_{1-X}(1) &= {}^{3}C_1 (\frac{1}{2})^3 \\ &= 3.\frac{1}{8} \\ &= \frac{3}{8} \end{aligned} \tag{2.2.6}$$

probability of getting 2 tails,

$$\begin{aligned} P_{1-X}(2) &= {}^{3}C_2 (\frac{1}{2})^3 \\ &= 3.\frac{1}{8} \\ &= \frac{3}{8} \end{aligned} \tag{2.2.7}$$

probability of getting 3 tails,

$$
\begin{aligned}
P_{1-X}(3) &= {}^{3}C_{3}(\frac{1}{2})^{3} \\
&= 1.\frac{1}{8} \\
&= \frac{1}{8}
\end{aligned}
\tag{2.2.8}
$$

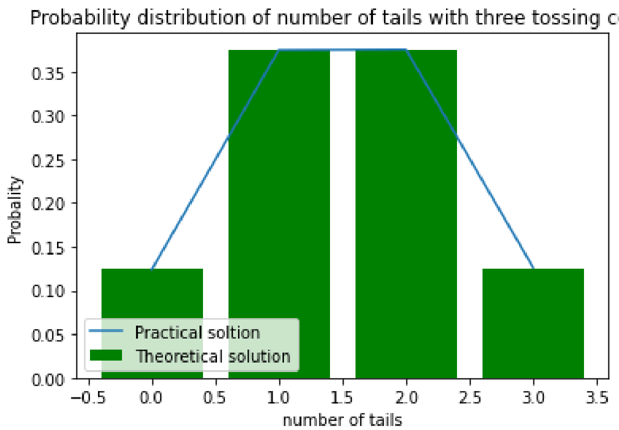

Fig. 0: Plot of probability distribution of no of tails with three tossed coins

2.3 (iii)

Here we are seeing outcome of four tosses of a coin. We can say it as simultaneous tossing four coins. So, we get,

$$n = 4 \tag{2.3.1}$$

Now we obtain probability distribution of number of heads of two coins from (2.0.6),

$$
p_X(k) = \begin{cases} {}^{4}C_k(\frac{1}{2})^k(1-\frac{1}{2})^{4-k} & 0 \le k \le 4 \\ 0 & \text{otherwise} \end{cases}
$$

$$
p_X(k) = \begin{cases} {}^{4}C_k(\frac{1}{2})^4 & \text{if } 0 \le k \le 4 \\ 0 & \text{otherwise} \end{cases}
\tag{2.3.2}
$$

probability of getting 0 tails,

$$
\begin{aligned}
P_X(0) &= {}^{4}C_0(\frac{1}{2})^4 \\
&= 1.\frac{1}{16} \\
&= \frac{1}{16}
\end{aligned}
\tag{2.3.3}
$$

k	0	1	2	3	4
Pr(k)	1/16	1/4	3/8	1/4	1/16

Fig. 0: Table of probability distribution of number of heads with four tossed coins

probability of getting 1 tails,

$$
\begin{aligned}
P_X(1) &= {}^{4}C_1(\frac{1}{2})^4 \\
&= 4.\frac{1}{16} \\
&= \frac{1}{4}
\end{aligned}
\tag{2.3.4}
$$

probability of getting 2 tails,

$$
\begin{aligned}
P_X(2) &= {}^{4}C_2(\frac{1}{2})^4 \\
&= 6.\frac{1}{16} \\
&= \frac{3}{8}
\end{aligned}
\tag{2.3.5}
$$

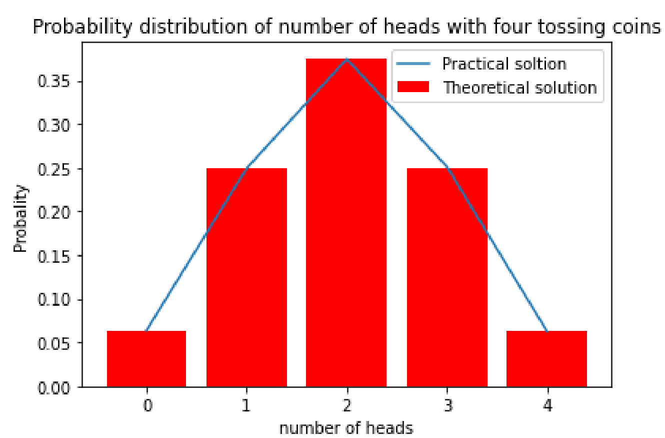

Fig. 0: Plot of probability distribution of four tossed coins

probability of getting 3 tails,

$$\begin{aligned} P_X(3) &= {}^4C_3(\frac{1}{2})^4 \\ &= 4.\frac{1}{16} \\ &= \frac{1}{4} \end{aligned} \tag{2.3.6}$$

probability of getting 4 tails,

$$\begin{aligned} P_X(4) &= {}^4C_4(\frac{1}{2})^4 \\ &= 1.\frac{1}{16} \\ &= \frac{1}{16} \end{aligned} \tag{2.3.7}$$