Assignment 3

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Download all python codes from

```
https://github.com/PRABHATH−cs20−11038/
    AI1103/tree/main/Assignment_3/codes
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and latex-tikz codes from

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https://github.com/PRABHATH−cs20−11038/
    AI1103/tree/main/Assignment_3
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1 Problem

$(GATE(MA)2011-49Q)$) Let X and Y be two continuous random variables with the joint probability density function

$$f(x,y) = \begin{cases} 2, & 0 < x+y < 1, x > 0, y > 0, \\ 0, & elsewhere. \end{cases} \tag{1.0.1}$$

$E\left(X|Y = \frac{1}{2}\right)$ is
(A) 1/4
(B) 1/2
(C) 1
(D) 2

2 Solution

The PDF of X and Y is,

$$f(X = x) = \int_{-\infty}^{\infty} f(x,y)dy \tag{2.0.1}$$

$$f(X = x) = \int_{0}^{1-x} 2dy \tag{2.0.2}$$

$$f(X = x) = 2 - 2x \tag{2.0.3}$$

$$f(Y = y) = \int_{-\infty}^{\infty} f(x,y)dx \tag{2.0.4}$$

$$f(Y = y) = \int_{0}^{1-y} 2dx \tag{2.0.5}$$

$$f(Y = y) = 2 - 2y \tag{2.0.6}$$

$$f\left(Y = \frac{1}{2}\right) = 1 \tag{2.0.7}$$

by using Bayes theorem,

$$f\left(X|Y = \frac{1}{2}\right) = \frac{f\left(x, \frac{1}{2}\right)}{f\left(Y = \frac{1}{2}\right)} \tag{2.0.8}$$

$$f\left(X|Y = \frac{1}{2}\right) = \begin{cases} 2, & 0 < x < \frac{1}{2}, \\ 0, & elsewhere. \end{cases} \tag{2.0.9}$$

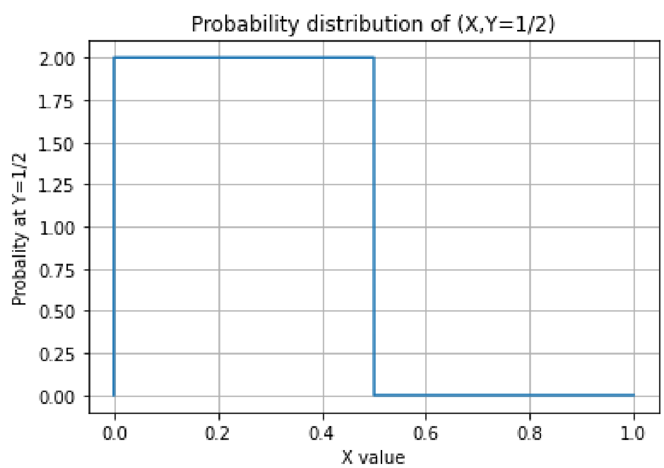

Fig. 4. Plot of of probability function

It is in the form of Bernoulli distribution, the expectation value is given by,

$$E\left(X|Y = \frac{1}{2}\right) = \sum_{-\infty}^{\infty} kf\left(X = k|Y = \frac{1}{2}\right) \tag{2.0.10}$$

$$E\left(X|Y = \frac{1}{2}\right) = \int_{-\infty}^{0} x(0)dx + \int_{0}^{\frac{1}{2}} x(2)dx + \int_{0}^{-\infty} x(0)dx \tag{2.0.11}$$

$$E\left(X|Y = \frac{1}{2}\right) = \int_{0}^{\frac{1}{2}} 2x dx \tag{2.0.12}$$

$$E\left(X|Y = \frac{1}{2}\right) = \frac{1}{4} \tag{2.0.13}$$

Option (A) is correct.

Expected Value−
simulated: 0.25008392175344957,
actual: 0.25