Assignment 5

Prabhath Chellingi - CS20BTECH11038

Download all python codes from

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https://github.com/PRABHATH−cs20−11038/
    AI1103/tree/main/Assignment_5/Codes
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and latex-tikz codes from

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https://github.com/PRABHATH−cs20−11038/
    AI1103/tree/main/Assignment_5
```

1 Problem

$(GATE(EC)2013 - 26Q)$ Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $\Pr(3V \geq 2U)$ is

(A) 4/9
(B) 1/2
(C) 2/3
(D) 5/9

2 Solution

U and V are independent random variables,
For V, $\mu_V = 0$, and $\sigma_V^2 = \frac{1}{9}$

$$V \sim N\left(0, \frac{1}{9}\right) \tag{2.0.1}$$

For U, $\mu_U = 0$, and $\sigma_U^2 = \frac{1}{4}$

$$U \sim N\left(0, \frac{1}{4}\right) \tag{2.0.2}$$

Let,

$$Z = 3V - 2U \tag{2.0.3}$$

$$Z \sim N\left((3\mu_V - 2\mu_U), \left((3)^2\sigma_U^2 + (2)^2\sigma_V^2\right)\right) \tag{2.0.4}$$

$$Z \sim N\left(0, 9 \times \frac{1}{9} + 4 \times \frac{1}{4}\right) \tag{2.0.5}$$

$$Z \sim N\left(0, 9 \times \frac{1}{9} + 4 \times \frac{1}{4}\right) \tag{2.0.6}$$

$$Z \sim N(0, 2) \tag{2.0.7}$$

For Z, $\mu = 0$, and $\sigma^2 = 2$.
By Gaussian Distribution,
Let X is standard normal variable,

$$X = \frac{Z - \mu}{\sigma} \tag{2.0.8}$$

$$\Pr(Z \geq 0) = \Pr(X\sigma + \mu \geq 0) \tag{2.0.9}$$

$$\Pr(Z \geq 0) = \Pr\left(X\left(\sqrt{2}\right) + 0 \geq 0\right) \tag{2.0.10}$$

$$\Pr(Z \geq 0) = \Pr(X \geq 0) \tag{2.0.11}$$

$$\Pr(Z \geq 0) = Q(0) \tag{2.0.12}$$

where $Q(x)$ is the $Q - function$,

$$Q(0) = \frac{1}{2} \tag{2.0.13}$$

$$\Pr(Z \geq 0) = \frac{1}{2} \tag{2.0.14}$$

$$\Pr(3V - 2U \geq 0) = \frac{1}{2} \tag{2.0.15}$$

$$\Pr(3V \geq 2U) = \frac{1}{2} \tag{2.0.16}$$

Option (B) is correct.

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Probability−
    simulated: 0.49873
    actual: 0.5
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