Assignment 1

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Download all python codes from

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https://github.com/PRABHATH-cs20-11038/
    Assignment_1/tree/main/Assignment_1/Codes
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and latex-tikz codes from

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https://github.com/PRABHATH-cs20-11038/
    Assignment_1/blob/main/Assignment_1/
    Assignment_1.tex
```

1 Problem

(Prob 3.6) Find the probability distribution of
(i) number of heads in two tosses of a coin.
(ii) number of tails in the simultaneous tosses of three coins.
(iii) number of heads in four tosses of a coin.

2 Solution

Let $X_i \in \{0, 1\}$ represent the i^{Th} coin where 1 denotes the coin giving outcome as head. Then, X_i has a Bernoulli distribution with parameter

$$p = \frac{1}{2} \tag{2.0.1}$$

Let

$$X = \sum_{i=1}^{n} X_i \tag{2.0.2}$$

where n is the total number of coins tossed. Then X has a Binomial Distribution. Then for

$$\Pr(X_i = n) \overset{Z}{\rightleftharpoons} \Pr(X_i = z) \tag{2.0.3}$$

yielding

$$\Pr(X_i = z) = 1 - p + pz^{-1} \tag{2.0.4}$$

with using the fact that X_i are i.i.d.,

$$\begin{aligned} \Pr(X = z) &= (1 - p + pz^{-1})^n \\ &= \sum_{k=0}^{n} {}^nC_k p^k (1-p)^{n-k} z^{-k} \end{aligned} \tag{2.0.5}$$

$$\Pr(X = k) = \begin{cases} {}^nC_k p^k (1-p)^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \tag{2.0.6}$$

(i) Here we are seeing outcome of two tosses of a coin. We can say it as simultaneous tossing two coins. So, we get,

$$n = 2 \tag{2.0.7}$$

Now we obtain probability distribution of number of heads of two coins from (2.0.6),

$$\Pr(X = k) = \begin{cases} {}^2C_k (\frac{1}{2})^k (1 - \frac{1}{2})^{2-k} & 0 \le k \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr(X = k) = \begin{cases} {}^2C_k (\frac{1}{2})^2 & \text{if } 0 \le k \le 2 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.8}$$

(a) probability of getting 0 heads,

$$\begin{aligned} \Pr(X = 0) &= {}^2C_0 (\frac{1}{2})^2 \\ &= 1.\frac{1}{4} \\ &= \frac{1}{4} \end{aligned} \tag{2.0.9}$$

(b) probability of getting 1 heads,

$$\begin{aligned} \Pr(X = 1) &= {}^2C_1 (\frac{1}{2})^2 \\ &= 2.\frac{1}{4} \\ &= \frac{1}{2} \end{aligned} \tag{2.0.10}$$

(c) probability of getting 2 heads,

$$\Pr(X = 2) = {}^{2}C_2(\frac{1}{2})^2$$
$$= 1.\frac{1}{4} \tag{2.0.11}$$
$$= \frac{1}{4}$$

k	0	1	2
Pr(k)	1/4	1/2	1/4

Fig. i: Table of probability distribution number of heads with two tossed coins

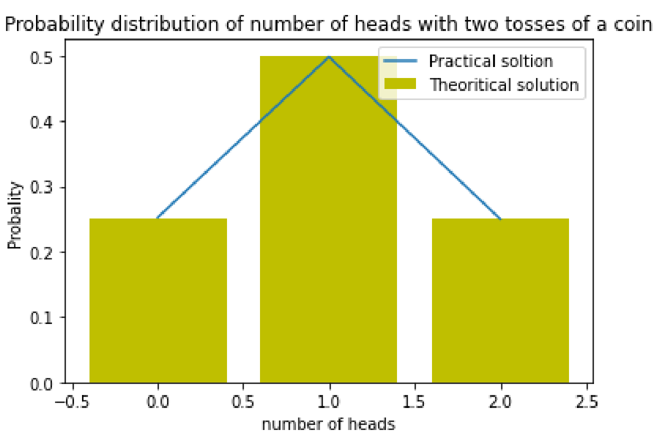

Fig. i: Plot of probability distribution of two tossed coins

(ii) Let $X_i' = 1 - X_i$, where in $X_i' \in 0, 1$ here 1 denotes outcome as tail.
With respect to tails the parameter would change to

$$q = 1 - p = \frac{1}{2} \tag{2.0.12}$$

and there will be a small change in binomial distribution (2.0.6) and we get binomial distribution for tails as,

$$\Pr(X' = k) = \begin{cases} {}^{n}C_k q^k (1-q)^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \tag{2.0.13}$$

Here we are seeing outcome of simultaneous tosses of three coin. So, we get,

$$n = 3 \tag{2.0.14}$$

Now we obtain probability distribution of number of tails of three coins from (2.2.2),

$$\Pr(X' = k) = \begin{cases} {}^{3}C_k (\frac{1}{2})^k (1-\frac{1}{2})^{3-k} & 0 \le k \le 3 \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr(X' = k) = \begin{cases} {}^{3}C_k (\frac{1}{2})^3 & \text{if } 0 \le k \le 3 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.15}$$

k	0	1	2	3
Pr(k)	1/8	3/8	3/8	1/8

Fig. ii: Table of probability distribution of no of tails with three tossed coins

(a) probability of getting 0 tails,

$$\Pr(X' = 0) = {}^{3}C_0(\frac{1}{2})^3$$
$$= 1.\frac{1}{8} \tag{2.0.16}$$
$$= \frac{1}{8}$$

(b) probability of getting 1 tails,

$$\begin{aligned}\Pr(X' = 1) &= {}^3C_1(\frac{1}{2})^3 \\ &= 3.\frac{1}{8} \\ &= \frac{3}{8}\end{aligned} \tag{2.0.17}$$

(c) probability of getting 2 tails,

$$\begin{aligned}\Pr(X' = 2) &= {}^3C_2(\frac{1}{2})^3 \\ &= 3.\frac{1}{8} \\ &= \frac{3}{8}\end{aligned} \tag{2.0.18}$$

(d) probability of getting 3 tails,

$$\begin{aligned}\Pr(X' = 3) &= {}^3C_3(\frac{1}{2})^3 \\ &= 1.\frac{1}{8} \\ &= \frac{1}{8}\end{aligned} \tag{2.0.19}$$

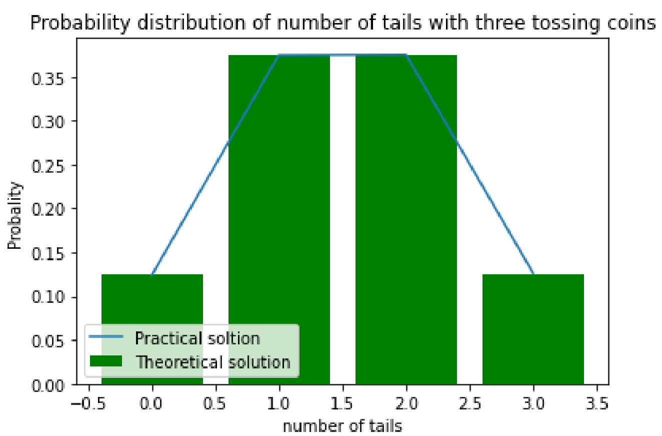

Fig. ii: Plot of probability distribution of no of tails with three tossed coins

(iii) Here we are seeing outcome of four tosses of a coin. We can say it as simultaneous tossing four coins. So, we get,

$$n = 4 \tag{2.0.20}$$

Now we obtain probability distribution of number of heads of two coins from (2.0.6),

$$\Pr(X = k) = \begin{cases} {}^4C_k(\frac{1}{2})^k(1-\frac{1}{2})^{4-k} & 0 \le k \le 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr(X = k) = \begin{cases} {}^4C_k(\frac{1}{2})^4 & \text{if } 0 \le k \le 4 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.21}$$

k	0	1	2	3	4
Pr(k)	1/16	1/4	3/8	1/4	1/16

Fig. iii: Table of probability distribution of number of heads with four tossed coins

(a) probability of getting 0 heads,

$$\begin{aligned}\Pr(X = 0) &= {}^4C_0(\frac{1}{2})^4 \\ &= 1.\frac{1}{16} \\ &= \frac{1}{16}\end{aligned} \tag{2.0.22}$$

(b) probability of getting 1 heads,

$$\begin{aligned}\Pr(X = 1) &= {}^4C_1(\frac{1}{2})^4 \\ &= 4.\frac{1}{16} \\ &= \frac{1}{4}\end{aligned} \tag{2.0.23}$$

(c) probability of getting 2 heads,

$$\begin{aligned}\Pr(X = 2) &= {}^4C_2(\frac{1}{2})^4 \\ &= 6.\frac{1}{16} \\ &= \frac{3}{8}\end{aligned} \tag{2.0.24}$$

(d) probability of getting 3 heads,

$$\begin{aligned}\Pr(X = 3) &= {}^4C_3(\frac{1}{2})^4 \\ &= 4.\frac{1}{16} \\ &= \frac{1}{4}\end{aligned} \tag{2.0.25}$$

(e) probability of getting 4 heads,

$$\begin{aligned}\Pr(X = 4) &= {}^4C_4(\frac{1}{2})^4 \\ &= 1.\frac{1}{16} \\ &= \frac{1}{16}\end{aligned} \tag{2.0.26}$$

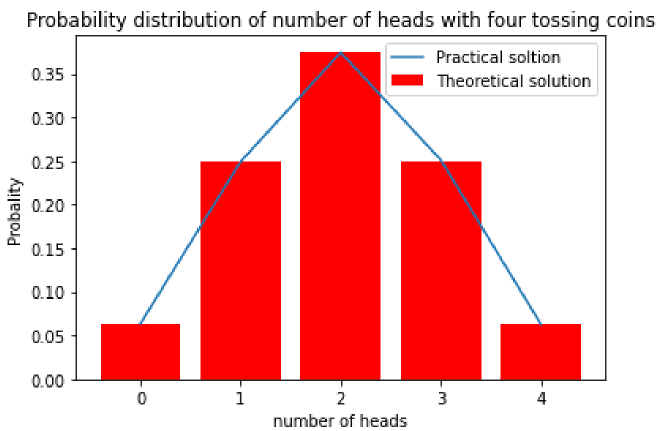

Fig. iii: Plot of probability distribution of four tossed coins