

Assignment 3

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Download all python codes from

https://github.com/PRABHATH-cs20-11038/AI1103/tree/main/Assignment_3/codes

and latex-tikz codes from

https://github.com/PRABHATH-cs20-11038/AI1103/tree/main/Assignment_3

1 PROBLEM

(GATE(MA)2011-49Q)) Let X and Y be two continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} 2, & 0 < x + y < 1, x > 0, y > 0, \\ 0, & \text{elsewhere.} \end{cases} \quad (1.0.1)$$

$E(X | Y = \frac{1}{2})$ is

- (A) $1/4$
- (B) $1/2$
- (C) 1
- (D) 2

2 SOLUTION

The PDF of X and Y is,

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy \quad (2.0.1)$$

$$f_X(x) = \int_0^{1-x} 2 dy \quad (2.0.2)$$

$$f_X(x) = 2 - 2x \quad (2.0.3)$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx \quad (2.0.4)$$

$$f_Y(y) = \int_0^{1-y} 2 dx \quad (2.0.5)$$

$$f_Y(y) = 2 - 2y \quad (2.0.6)$$

$$f_Y\left(\frac{1}{2}\right) = 1 \quad (2.0.7)$$

by using Bayes theorem,

$$f_{X|Y}\left(x \mid \frac{1}{2}\right) = \frac{f_{X,Y}\left(x, \frac{1}{2}\right)}{f_Y\left(\frac{1}{2}\right)} \quad (2.0.8)$$

$$f_{X|Y}\left(x \mid \frac{1}{2}\right) = \begin{cases} 2, & 0 < x < \frac{1}{2}, \\ 0, & \text{elsewhere.} \end{cases} \quad (2.0.9)$$

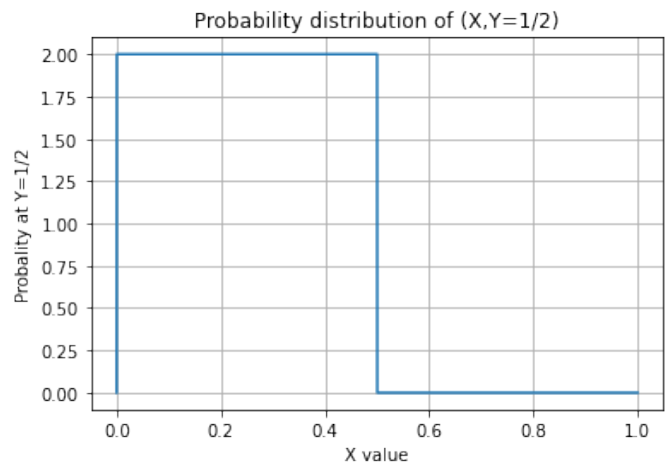


Fig. 4. Plot of probability function

It is in the form of Bernoulli distribution, the expectation value is given by,

$$E\left(X \mid Y = \frac{1}{2}\right) = \sum_{-\infty}^{\infty} x f_{X|Y}\left(x \mid \frac{1}{2}\right) \quad (2.0.10)$$

$$E\left(X \mid Y = \frac{1}{2}\right) = \int_{-\infty}^0 x(0) dx + \int_0^{\frac{1}{2}} x(2) dx + \int_{\frac{1}{2}}^{\infty} x(0) dx \quad (2.0.11)$$

$$E\left(X \mid Y = \frac{1}{2}\right) = \int_0^{\frac{1}{2}} 2x dx \quad (2.0.12)$$

$$E\left(X \mid Y = \frac{1}{2}\right) = \frac{1}{4} \quad (2.0.13)$$

Option (A) is correct.

Expected Value–

simulated: 0.25008392175344957,

actual: 0.25