Assignment 1

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Download all python codes from

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https://github.com/PRABHATH-cs20-11038/
    Assignment_1/tree/main/Assignment_1/Codes
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and latex-tikz codes from

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https://github.com/PRABHATH-cs20-11038/
    Assignment_1/blob/main/Assignment_1/
    Assignment_1.tex
```

1 Problem

(Prob 3.6) Find the probability distribution of
(i) number of heads in two tosses of a coin.
(ii) number of tails in the simultaneous tosses of three coins.
(iii) number of heads in four tosses of a coin.

2 Solution

Let $X_i \in \{0, 1\}$ represent the i^{Th} coin where 1 denotes the coin giving outcome as head. Then, X_i has a Bernoulli distribution with parameter

$$p = \frac{1}{2} \tag{2.0.1}$$

Let

$$X = \sum_{i=1}^{n} X_i \tag{2.0.2}$$

where n is the total number of coins tossed. Then X has a Binomial Distribution. Then for

$$\Pr(X_i = n) \overset{Z}{\rightleftharpoons} \Pr(X_i = z) \tag{2.0.3}$$

yielding

$$\Pr(X_i = z) = 1 - p + pz^{-1} \tag{2.0.4}$$

with using the fact that X_i are i.i.d.,

$$\begin{aligned}\Pr(X = z) &= (1 - p + pz^{-1})^n \\ &= \sum_{k=0}^{n} {}^nC_k p^k (1-p)^{n-k} z^{-k}\end{aligned} \tag{2.0.5}$$

$$\Pr(X = k) = \begin{cases} {}^nC_k p^k (1-p)^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \tag{2.0.6}$$

TABLE 0
Table of Probability distribution of different cases

Case	n (no. of coins)	k (no. of required outcomes)	0	1	2	3	4
(i)	2	$\Pr(X = k)\,(heads)$	1/2	1/4	1/2	0	0
(ii)	3	$\Pr(X = k)\,(tails)$	1/8	3/8	3/8	1/8	0
(iii)	4	$\Pr(X = k)\,(heads)$	1/16	1/4	3/8	1/4	1/16

The above table represents the probability distribution of following of following cases at different k values.

(i) Two coins. So,

$$n = 2 \tag{2.0.7}$$

from (2.0.6),

$$\Pr(X = k) = \begin{cases} {}^2C_k \left(\frac{1}{2}\right)^2 & \text{if } 0 \le k \le 2 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.8}$$

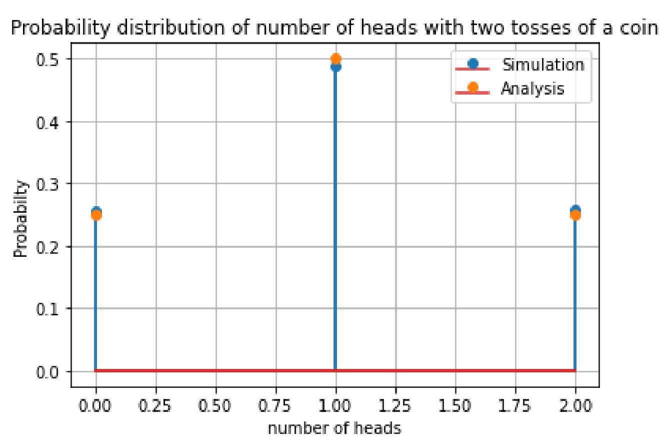

Fig. i. Plot of probability distribution of two tossed coins

(ii) To get probability of tails,

$$\Pr(X = n-k) = \begin{cases} {}^nC_k (p)^k (1-p)^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \quad (2.0.9)$$

$$\Pr(X = k) = \begin{cases} {}^nC_k (p)^{n-k} (1-p)^{k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \quad (2.0.10)$$

Three coins. So,

$$n = 3 \quad (2.0.11)$$

from (2.0.10),

$$\Pr(X = k) = \begin{cases} {}^3C_k \left(\frac{1}{2}\right)^3 & \text{if } 0 \le k \le 3 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.12)$$

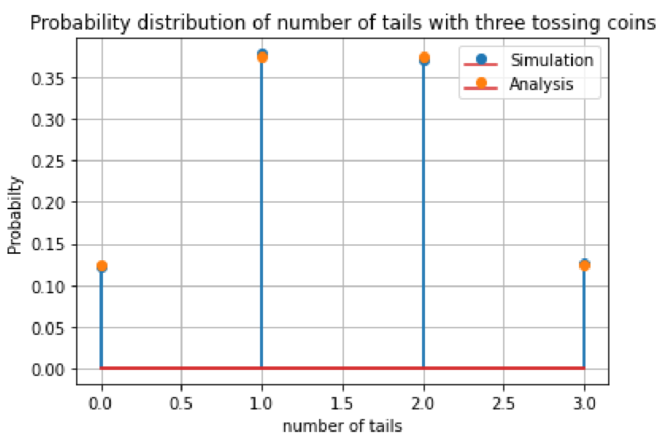

Fig. ii. Plot of probability distribution of no of tails with three tossed coins

(iii) Four coins. So,

$$n = 4 \quad (2.0.13)$$

from (2.0.6),

$$\Pr(X = k) = \begin{cases} {}^4C_k \left(\frac{1}{2}\right)^4 & \text{if } 0 \le k \le 4 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.14)$$

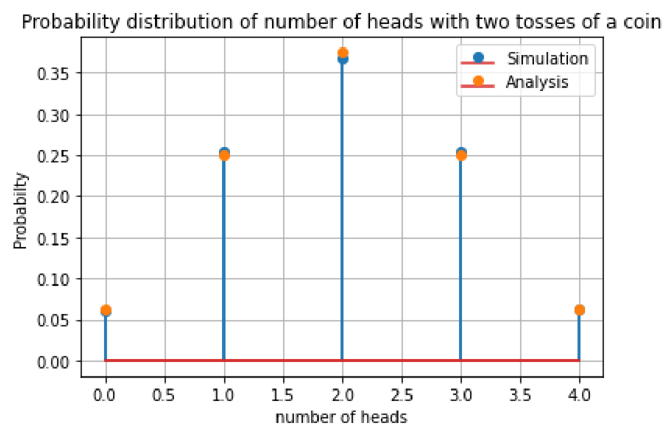

Fig. iii. Plot of probability distribution of four tossed coins