

# Assignment 2

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Download all python codes from

[https://github.com/PRABHATH-cs20-11038/AI1103/tree/main/Assignment\\_2](https://github.com/PRABHATH-cs20-11038/AI1103/tree/main/Assignment_2)

and latex-tikz codes from

[https://github.com/PRABHATH-cs20-11038/AI1103/tree/main/Assignment\\_2](https://github.com/PRABHATH-cs20-11038/AI1103/tree/main/Assignment_2)

## 1 PROBLEM

(GATE – EC – 66) Consider two identical boxes  $B_1$  and  $B_2$ , where the box  $B_i (i = 1, 2)$  contains  $i + 2$  red and  $5 - i - 1$  white balls. A fair die is cast. Let the number of dots shown on the top face of the die be  $N$ . If  $N$  is even or 5, then two balls are drawn with replacement from the box  $B_1$ , otherwise, two balls are drawn with replacement from the box  $B_2$ . The probability that the two drawn balls are of different colours is

## 2 SOLUTION

Let  $X \in \{1, 2, 3, 4, 5, 6\}$  be the random variables of a die,

$$\Pr(X = N) = \begin{cases} \frac{1}{6} & 1 \leq N \leq 6 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.1)$$

$$\Pr(X = m) \cdot \Pr(X = n) = 0 \quad (2.0.2)$$

$\forall m, n \in \{1, 2, 3, 4, 5, 6\}$  as a single die cannot show more than one outcome at a roll.

Let  $Y \in \{0, 1\}$  represent the die where,

$1 \Rightarrow$  the die with outcome  $N = \{2, 4, 5, 6\}$ ,

$0 \Rightarrow N = \{1, 3\}$ .

$$\Pr(Y = 1) =$$

$$\Pr((X = 2) + (X = 4) + (X = 5) + (X = 6)) \quad (2.0.3)$$

by using *Boolean logic* and (2.0.2),

$$\Pr(Y = 1) = \frac{2}{3} \quad (2.0.4)$$

$$\Pr(Y = 0) = 1 - \Pr(Y = 1) = \frac{1}{3} \quad (2.0.5)$$

$\Rightarrow$  probability of selecting box  $B_1$ ,

$$\Pr(B_1) = \Pr(Y = 1) = \frac{2}{3} \quad (2.0.6)$$

$\Rightarrow$  probability of selecting box  $B_2$ ,

$$\Pr(B_2) = \Pr(Y = 0) = \frac{1}{3} \quad (2.0.7)$$

Let  $C \in \{0, 1\}$  where,

$0 \Rightarrow$  red balls,

$1 \Rightarrow$  white balls.

TABLE 0

TABLE OF NUMBER OF BALLS

Box	No. of red balls ( $i + 2$ )	No. of white balls ( $5 - i - 1$ )	Total balls
$B_1$	$n(C = 0 B_1) = 3$	$n(C = 1 B_1) = 3$	$n(C B_1) = 6$
$B_2$	$n(C = 0 B_2) = 4$	$n(C = 1 B_2) = 2$	$n(C B_2) = 6$

TABLE 0

TABLE OF PROBABILITY OF TAKING BALLS FROM EACH BOX

Box	Probability of taking red ball	Probability of taking white ball
$B_1$	$\Pr(C = 0 B_1) = 1/2$	$\Pr(C = 1 B_1) = 1/2$
$B_2$	$\Pr(C = 0 B_2) = 2/3$	$\Pr(C = 1 B_2) = 1/3$

The probability of picking  $2^{nd}$  ball is not effected by picking  $1^{st}$  ball because the  $2^{nd}$  ball is chose after **replacement**.

Selecting two balls with replacement is a *Bernoulli distribution* of 2 trails,

TABLE 0

TABLE OF NO. OF WAYS OF SELECTING TWO DIFFERENT COLOURED BALLS

Cases	Trail 1	Trail 2
$(B_1, C = 0, C = 1)$	$\Pr(C = 0 B_1)$	$\Pr(C = 1 B_1)$
$(B_1, C = 1, C = 0)$	$\Pr(C = 1 B_1)$	$\Pr(C = 0 B_1)$
$(B_2, C = 0, C = 1)$	$\Pr(C = 0 B_2)$	$\Pr(C = 1 B_2)$
$(B_2, C = 1, C = 0)$	$\Pr(C = 1 B_2)$	$\Pr(C = 0 B_2)$

TABLE 0  
TABLE OF VARIABLES DESCRIPTION

Variables	Description
$\Pr((C = 0, C = 1) B_1)$	Probability of selecting two different coloured balls from box $B_1$
$\Pr((C = 0, C = 1) B_2)$	Probability of selecting two different coloured balls from box $B_2$
$\Pr(T)$	Total probability of selecting two different coloured balls

$\Rightarrow$

$$\begin{aligned}\Pr((C = 0, C = 1)|B_1) = \\ \Pr(C = 0|B_1) \cdot \Pr(C = 1|B_1) \\ + \Pr(C = 1|B_1) \cdot \Pr(C = 0|B_1) \quad (2.0.8)\end{aligned}$$

$$\Pr((C = 0, C = 1)|B_1) = \frac{1}{2} \quad (2.0.9)$$

$\Rightarrow$

$$\begin{aligned}\Pr((C = 0, C = 1)|B_2) = \\ \Pr(C = 0|B_2) \cdot \Pr(C = 1|B_2) \\ + \Pr(C = 1|B_2) \cdot \Pr(C = 0|B_2) \quad (2.0.10)\end{aligned}$$

$$\Pr((C = 0, C = 1)|B_1) = \frac{4}{9} \quad (2.0.11)$$

by using *Bayes theorem*,

$$\begin{aligned}\Pr(T) = \\ \Pr((C = 0, C = 1)|B_1) \cdot \Pr(B_1) + \\ \Pr((C = 0, C = 1)|B_2) \cdot \Pr(B_2) \quad (2.0.12)\end{aligned}$$

$$\Pr(T) = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) + \left(\frac{4}{9}\right)\left(\frac{1}{3}\right) \quad (2.0.13)$$

Hence, the probability of selecting two different coloured balls from the boxes is

$$\Pr(T) = \frac{13}{27} \quad (2.0.14)$$

Probability– simulation: 0.48015, actual: 0.48148148148148145
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