Assignment 3

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Download all python codes from

https://github.com/PRABHATH−cs20−11038/AI1103/tree/main/Assignment_3/codes

and latex-tikz codes from

https://github.com/PRABHATH−cs20−11038/AI1103/tree/main/Assignment_3

1 Problem

$(GATE(MA)2011-49Q)$) Let X and Y be two continuous random variables with the joint probability density function

$$f(x,y)=\begin{cases}2, & 0<x+y<1, x>0, y>0,\\ 0, & elsewhere.\end{cases} \tag{1.0.1}$$

$E\left(X|Y=\frac{1}{2}\right)$ is
(A) 1/4
(B) 1/2
(C) 1
(D) 2

2 Solution

The PDF of Y is,

$$f_Y(y)=\int_{-\infty}^{\infty} f(x,y)dx \tag{2.0.1}$$

$$f_Y(y)=\int_{0}^{1-y} 2dx \tag{2.0.2}$$

$$f_Y(y)=2-2y \tag{2.0.3}$$

$$\Pr\left(Y=\frac{1}{2}\right)=f_Y\left(\frac{1}{2}\right)=1 \tag{2.0.4}$$

$$\Pr\left((X)\left(Y=\frac{1}{2}\right)\right)=f\left(x,\frac{1}{2}\right)=\begin{cases}2, & 0<x<\frac{1}{2},\\ 0, & elsewhere.\end{cases} \tag{2.0.5}$$

by using Bayes theorem,

$$\Pr\left(X|Y=\frac{1}{2}\right)=\frac{\Pr\left((X)\left(Y=\frac{1}{2}\right)\right)}{\Pr\left(Y=\frac{1}{2}\right)} \tag{2.0.6}$$

$$\Pr\left(X|Y=\frac{1}{2}\right)=\begin{cases}2, & 0<x<\frac{1}{2},\\ 0, & elsewhere.\end{cases} \tag{2.0.7}$$

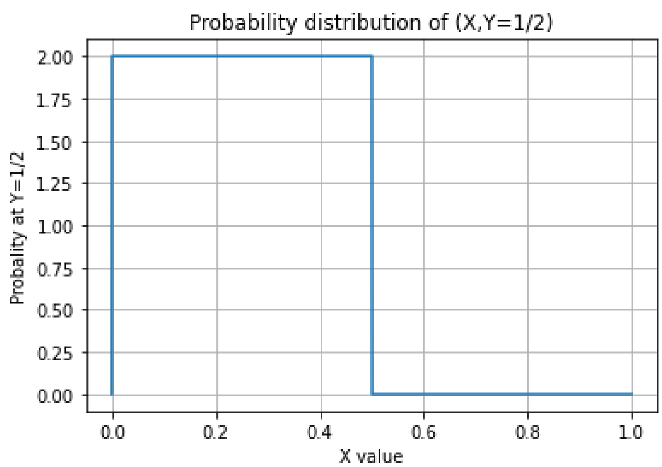

Fig. 4. Plot of of probability function

It is in the form of Bernoulli distribution, the expectation value is given by,

$$E\left(X|Y=\frac{1}{2}\right)=\sum_{-\infty}^{\infty} k\Pr\left(X=k|Y=\frac{1}{2}\right) \tag{2.0.8}$$

$$E\left(X|Y=\frac{1}{2}\right)=\int_{0}^{\frac{1}{2}} x(2)dx+\int_{-\infty}^{0} x(0)dx+\int_{0}^{-\infty} x(0)dx \tag{2.0.9}$$

$$E\left(X|Y=\frac{1}{2}\right)=\int_{0}^{\frac{1}{2}} 2xdx \tag{2.0.10}$$

$$E\left(X|Y=\frac{1}{2}\right)=\frac{1}{4} \tag{2.0.11}$$

Option (A) is correct.

Expected Value−
simulated: 0.25008392175344957,
actual: 0.25