Assignment 2

Prabhath Chellingi - CS20BTECH11038

Download all python codes from

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https://github.com/PRABHATH−cs20−11038/
Assignment_1/tree/main/Assignment_2
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and latex-tikz codes from

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Assignment_1/tree/main/Assignment_2
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1 Problem

($GATE-EC-66$) Consider two identical boxes B_1 and B_2, where the box $B_i(i = 1, 2)$ contains $i+2$ red and $5 - i - 1$ white balls. A fair die is cast. Let the number of dots shown on the top face of the die be N. If N is even or 5, then two balls are drawn with replacement from the box B_1, otherwise, two balls are drawn with replacement from the box B_2. The probability that the two drawn balls are of different colours is

2 Solution

Let $X \in \{1, 2, 3, 4, 5, 6\}$ be the random variables representing the outcome for a die. The PDF of die is

$$\Pr(X = N) = \begin{cases} \frac{1}{6} & 1 \leq N \leq 6 \\ 0 & otherwise \end{cases} \tag{2.0.1}$$

As we know

$$\Pr(X = m) . \Pr(X = n) = 0 \tag{2.0.2}$$

for all $m, n \in \{1, 2, 3, 4, 5, 6\}$ as a single die cannot show more than one outcome at a roll.

Let $Y \in \{0, 1\}$ represent the die where 1 denotes the die with outcome $N = \{2, 4, 5, 6\}$ and 0 denotes the remaining.

$$\Pr(Y = 1) = \Pr((X = 2) + (X = 4) + (X = 5) + (X = 6)) \tag{2.0.3}$$

by using *Boolean logic* and equation (2.0.2), we get,

$$\Pr(Y = 1) = \frac{2}{3} \tag{2.0.4}$$

$$\Pr(Y = 0) = 1 - \Pr(Y = 1) = \frac{1}{3} \tag{2.0.5}$$

we get,

(i) probability of selecting box B_1,

$$\Pr(B_1) = \Pr(Y = 1) = \frac{2}{3} \tag{2.0.6}$$

(ii) probability of selecting box B_2,

$$\Pr(B_2) = \Pr(Y = 0) = \frac{1}{3} \tag{2.0.7}$$

let $C \in \{0, 1\}$ where 0 denotes to red balls and 1 denotes to white balls.

TABLE 2
Table of number of balls

bag	no. of red balls $(i + 2)$	no. of white balls $(5 - i - 1)$	total balls
B_1	$n(C = 0\|B_1) = 3$	$n(C = 1\|B_1) = 3$	$n(C\|B_1) = 6$
B_2	$n(C = 0\|B_2) = 4$	$n(C = 1\|B_2) = 2$	$n(C\|B_2) = 6$

TABLE 2
Table of probability of taking balls from each bag

bag	Probability of taking red ball	Probability of taking white ball
B_1	$\Pr(C = 0\|B_1) = 1/2$	$\Pr(C = 1\|B_1) = 1/2$
B_2	$\Pr(C = 0\|B_2) = 2/3$	$\Pr(C = 1\|B_2) = 1/3$

The probability of picking second ball is not effected by picking first ball because the second ball is chose after replacement.

Selecting two balls with replacement is a Bernoulli distribution of 2 trails,

TABLE 2
Table of no. of ways of selecting two different coloured balls

Cases	Trail 1	Trail 2
$(B_1, C = 0, C = 1)$	$\Pr(C = 0\|B_1)$	$\Pr(C = 1\|B_1)$
$(B_1, C = 1, C = 0)$	$\Pr(C = 1\|B_1)$	$\Pr(C = 0\|B_1)$
$(B_2, C = 0, C = 1)$	$\Pr(C = 0\|B_2)$	$\Pr(C = 1\|B_2)$
$(B_2, C = 1, C = 0)$	$\Pr(C = 1\|B_2)$	$\Pr(C = 0\|B_2)$

TABLE 2
Table of variables description

Variables	Description
$\Pr((C = 0, C = 1)\|B_1)$	probability of selecting two different coloured balls from box B_1
$\Pr((C = 0, C = 1)\|B_2)$	probability of selecting two different coloured balls from box B_2
$\Pr(T)$	probability of selecting two different coloured balls

(i)

$$\Pr((C = 0, C = 1)|B_1) = \Pr(C = 0|B_1) . \Pr(C = 1|B_1) + \Pr(C = 1|B_1) . \Pr(C = 0|B_1) \quad (2.0.8)$$

$$\Pr((C = 0, C = 1)|B_1) = \frac{1}{2} \quad (2.0.9)$$

(ii)

$$\Pr((C = 0, C = 1)|B_2) = \Pr(C = 0|B_2) . \Pr(C = 1|B_2) + \Pr(C = 1|B_2) . \Pr(C = 0|B_2) \quad (2.0.10)$$

$$\Pr((C = 0, C = 1)|B_1) = \frac{4}{9} \quad (2.0.11)$$

Now,

$$\Pr(T) = \Pr(((C = 0, C = 1)|B_1)(B_1)) + pr((C = 0, C = 1)|B_2)(B_2) \quad (2.0.12)$$

$\rightarrow$ by using *conditional probability*,

$$\Pr(EF) = \Pr(E|F) . \Pr(F) \quad (2.0.13)$$

from equation (2.0.13)

$$\Pr(T) = \Pr((C = 0, C = 1)|B_1) . \Pr(B_1) + \Pr((C = 0, C = 1)|B_2) . \Pr(B_2) \quad (2.0.14)$$

$$\Pr(T) = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) + \left(\frac{4}{9}\right)\left(\frac{1}{3}\right) \quad (2.0.15)$$

Hence, the probability of selecting two different coloured balls from the bags is

$$\Pr(T) = \frac{13}{27} \quad (2.0.16)$$

Probability–
simulation: 0.48015,
actual: 0.48148148148148145