Assignment 2

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Download all python codes from

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https://github.com/PRABHATH−cs20−11038/
    Assignment_1/tree/main/Assignment_2
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and latex-tikz codes from

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https://github.com/PRABHATH−cs20−11038/
    Assignment_1/tree/main/Assignment_2
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1 Problem

($GATE-EC-66$) Consider two identical boxes B_1 and B_2, where the box $B_i(i=1,2)$ contains $i+2$ red and $5-i-1$ white balls. A fair die is cast. Let the number of dots shown on the top face of the die be N. If N is even or 5, then two balls are drawn with replacement from the box B_1, otherwise, two balls are drawn with replacement from the box B_2. The probability that the two drawn balls are of different colours is

2 Solution

Let $X \in \{1,2,3,4,5,6\}$ be the random variables representing the outcome for a die. The *PDF* of die is

$$\Pr(X=N) = \begin{cases} \frac{1}{6} & 1 \le N \le 6 \\ 0 & otherwise \end{cases} \quad (2.0.1)$$

As we know

$$\Pr(X=m).\Pr(X=n) = 0 \quad (2.0.2)$$

for all $m,n \in \{1,2,3,4,5,6\}$ as a single die cannot show more than one outcome at a roll.

Let $Y \in \{0,1\}$ represent the die where 1 denotes the die with outcome $N=\{2,4,5,6\}$ and 0 denotes the remaining.
Y forms a *Bernoulli distribution*.

$$\Pr(Y=1) = \Pr((X=2)+(X=4)+(X=5)+(X=6)) \quad (2.0.3)$$

by using *Boolean logic* and equation (2.0.2), we get,

TABLE 0
Table of probability of selecting boxes

$\Pr(Y=1)$	$\Pr(B_1)=\Pr(Y=1)$	$\Pr(B_1)=\frac{2}{3}$
$\Pr(Y=0) = 1 - \Pr(Y=1)$	$\Pr(B_2)=\Pr(Y=0)$	$\Pr(B_2)=\frac{1}{3}$

let $C \in \{0,1\}$ where 0 denotes to red balls and 1 denotes to white balls.

TABLE 0
Table of number of balls

bag	no. of red balls $(i+2)$	no. of white balls $(5-i-1)$	total balls
B_1	$n(C=0\|B_1)=3$	$n(C=1\|B_1)=3$	$n(C\|B_1)=6$
B_2	$n(C=0\|B_2)=4$	$n(C=1\|B_2)=2$	$n(C\|B_2)=6$

TABLE 0
Table of probability of taking balls from each bag

bag	Probability of taking red ball	Probability of taking white ball
B_1	$\Pr(C=0\|B_1)=1/2$	$\Pr(C=1\|B_1)=1/2$
B_2	$\Pr(C=0\|B_2)=2/3$	$\Pr(C=1\|B_2)=1/3$

No. of ways of selecting two different coloured balls is

1) $(B_1, C=0, C=1)$
2) $(B_1, C=1, C=0)$
3) $(B_2, C=0, C=1)$
4) $(B_2, C=1, C=0)$

The probability of picking second ball is not effected by picking first ball because the second ball is chose after replacement.

TABLE 4
TABLE OF VARIABLES DESCRIPTION

variables	description
$\Pr((C=0, C=1)\vert B_1)$	probability of selecting two different coloured balls from box B_1
$\Pr((C=0, C=1)\vert B_2)$	probability of selecting two different coloured balls from box B_2
$\Pr(T)$	probability of selecting two different coloured balls

TABLE 4
TABLE OF PROBABILITY OF SELECTING TWO DIFFERENT COLOURED BALLS FROM EACH BAG

by using *Boolean logic*,

Equation	Value
$\Pr((C=0, C=1)\vert B_1) = \Pr(C=0\vert B_1).\Pr(C=1\vert B_1) + \Pr(C=1\vert B_1).\Pr(C=0\vert B_1)$	$\frac{1}{2}$
$\Pr((C=0, C=1)\vert B_2) = \Pr(C=0\vert B_2).\Pr(C=1\vert B_2) + \Pr(C=1\vert B_2).\Pr(C=0\vert B_2)$	$\frac{4}{9}$

Now,

$$\Pr(T) = \Pr(((C=0, C=1)|B_1)(B_1)) + pr((C=0, C=1)|B_2)(B_2) \tag{2.0.4}$$

$\rightarrow$ by using *conditional probability*,

$$\Pr(EF) = \Pr(E|F).\Pr(F) \tag{2.0.5}$$

from equation (2.0.5)

$$\Pr(T) = \Pr((C=0, C=1)|B_1).\Pr(B_1) + \Pr((C=0, C=1)|B_2).\Pr(B_2) \tag{2.0.6}$$

$$\Pr(T) = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) + \left(\frac{4}{9}\right)\left(\frac{1}{3}\right) \tag{2.0.7}$$

Hence, the probability of selecting two different coloured balls from the bags is

$$\Pr(T) = \frac{13}{27} \tag{2.0.8}$$

Probability–
simulation: 0.48015,
actual: 0.48148148148148145