

Assignment 1

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Download all python codes from

https://github.com/PRABHATH-cs20-11038/Assignment_1/tree/main/Assignment_1/Codes

and latex-tikz codes from

https://github.com/PRABHATH-cs20-11038/Assignment_1/blob/main/Assignment_1/Assignment_1.tex

1 PROBLEM

- (Prob 3.6) Find the probability distribution of
- (i) number of heads in two tosses of a coin.
 - (ii) number of tails in the simultaneous tosses of three coins.
 - (iii) number of heads in four tosses of a coin.

2 SOLUTION

Let $X_i \in \{0, 1\}$ represent the i^{th} coin where 1 denotes the coin giving outcome as head. Then, X_i has a Bernoulli distribution with parameter

$$p = \frac{1}{2} \quad (2.0.1)$$

Let

$$X = \sum_{i=1}^n X_i \quad (2.0.2)$$

where n is the total number of coins tossed. Then X has a Binomial Distribution. Then for

$$\Pr(X_i = n) \stackrel{Z}{\rightleftharpoons} \Pr(X_i = z) \quad (2.0.3)$$

yielding

$$\Pr(X_i = z) = 1 - p + pz^{-1} \quad (2.0.4)$$

with using the fact that X_i are i.i.d.,

$$\begin{aligned} \Pr(X = z) &= (1 - p + pz^{-1})^n \\ &= \sum_{k=0}^n {}^nC_k p^k (1 - p)^{n-k} z^{-k} \end{aligned} \quad (2.0.5)$$

$$\Pr(X = k) = \begin{cases} {}^nC_k p^k (1 - p)^{n-k} & 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases} \quad (2.0.6)$$

- (i) Here we are seeing outcome of two tosses of a coin. We can say it as simultaneous tossing two coins. So, we get,

$$n = 2 \quad (2.0.7)$$

This is the **special case** with equation (2.0.6) with $n = 2$.

Now we obtain probability distribution of number of heads of two coins from (2.0.6),

$$\Pr(X = k) = \begin{cases} {}^2C_k \left(\frac{1}{2}\right)^k \left(1 - \frac{1}{2}\right)^{2-k} & 0 \leq k \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr(X = k) = \begin{cases} {}^2C_k \left(\frac{1}{2}\right)^2 & \text{if } 0 \leq k \leq 2 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.8)$$

- (a) probability of getting 0 heads,

$$\begin{aligned} \Pr(X = 0) &= {}^2C_0 \left(\frac{1}{2}\right)^2 \\ &= 1 \cdot \frac{1}{4} \\ &= \frac{1}{4} \end{aligned} \quad (2.0.9)$$

- (b) probability of getting 1 heads,

$$\begin{aligned} \Pr(X = 1) &= {}^2C_1 \left(\frac{1}{2}\right)^2 \\ &= 2 \cdot \frac{1}{4} \\ &= \frac{1}{2} \end{aligned} \quad (2.0.10)$$

(c) probability of getting 2 heads,

$$\begin{aligned}\Pr(X = 2) &= {}^2C_2 \left(\frac{1}{2}\right)^2 \\ &= 1 \cdot \frac{1}{4} \\ &= \frac{1}{4}\end{aligned}\quad (2.0.11)$$

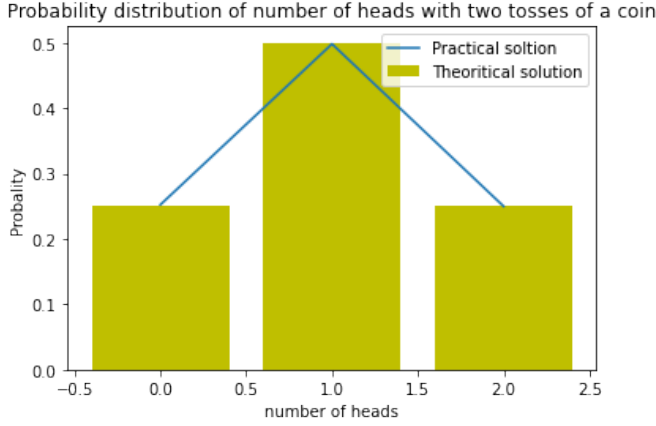


Fig. i. Plot of probability distribution of two tossed coins

(ii) Let $X'_i = 1 - X_i$, where in $X'_i \in 0, 1$ here 1 denotes outcome as tail. This is the **special case** with equation (2.0.6) with $n = 3$ and complement of X_i . With respect to tails the parameter would change to

$$q = 1 - p = \frac{1}{2} \quad (2.0.12)$$

and there will be a small change in binomial distribution (2.0.6) and we get binomial distribution for tails as,

$$\Pr(X' = k) = \begin{cases} {}^nC_k q^k (1 - q)^{n-k} & 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases} \quad (2.0.13)$$

Here we are seeing outcome of simultaneous tosses of three coin. So, we get,

$$n = 3 \quad (2.0.14)$$

Now we obtain probability distribution of number of tails of three coins from (2.0.13),

$$\Pr(X' = k) = \begin{cases} {}^3C_k \left(\frac{1}{2}\right)^k \left(1 - \frac{1}{2}\right)^{3-k} & 0 \leq k \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr(X' = k) = \begin{cases} {}^3C_k \left(\frac{1}{2}\right)^3 & \text{if } 0 \leq k \leq 3 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.15)$$

(a) probability of getting 0 tails,

$$\begin{aligned}\Pr(X' = 0) &= {}^3C_0 \left(\frac{1}{2}\right)^3 \\ &= 1 \cdot \frac{1}{8} \\ &= \frac{1}{8}\end{aligned}\quad (2.0.16)$$

(b) probability of getting 1 tails,

$$\begin{aligned}\Pr(X' = 1) &= {}^3C_1 \left(\frac{1}{2}\right)^3 \\ &= 3 \cdot \frac{1}{8} \\ &= \frac{3}{8}\end{aligned}\quad (2.0.17)$$

(c) probability of getting 2 tails,

$$\begin{aligned}\Pr(X' = 2) &= {}^3C_2 \left(\frac{1}{2}\right)^3 \\ &= 3 \cdot \frac{1}{8} \\ &= \frac{3}{8}\end{aligned}\quad (2.0.18)$$

(d) probability of getting 3 tails,

$$\begin{aligned}\Pr(X' = 3) &= {}^3C_3 \left(\frac{1}{2}\right)^3 \\ &= 1 \cdot \frac{1}{8} \\ &= \frac{1}{8}\end{aligned}\quad (2.0.19)$$

(iii) Here we are seeing outcome of four tosses of a coin. We can say it as simultaneous tossing four coins. So, we get,

$$n = 4 \quad (2.0.20)$$

This is the **special case** with equation (2.0.6) with $n = 4$.

Now we obtain probability distribution of number of heads of two coins from (2.0.6),

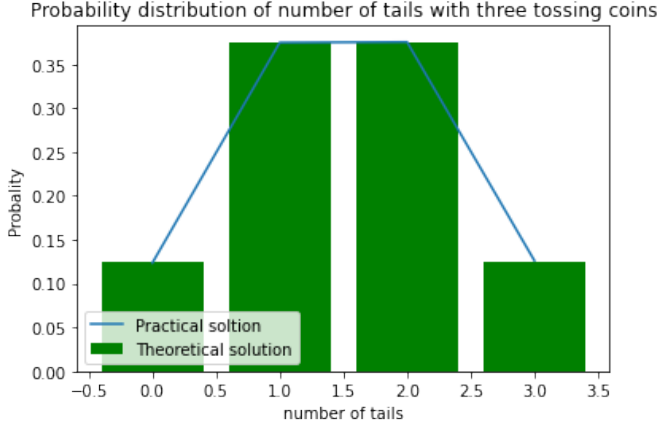


Fig. ii. Plot of probability distribution of no of tails with three tossed coins

$$\Pr(X = k) = \begin{cases} {}^4C_k \left(\frac{1}{2}\right)^k \left(1 - \frac{1}{2}\right)^{4-k} & 0 \leq k \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr(X = k) = \begin{cases} {}^4C_k \left(\frac{1}{2}\right)^4 & \text{if } 0 \leq k \leq 4 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.21)$$

(a) probability of getting 0 heads,

$$\begin{aligned} \Pr(X = 0) &= {}^4C_0 \left(\frac{1}{2}\right)^4 \\ &= 1 \cdot \frac{1}{16} \\ &= \frac{1}{16} \end{aligned} \quad (2.0.22)$$

(b) probability of getting 1 heads,

$$\begin{aligned} \Pr(X = 1) &= {}^4C_1 \left(\frac{1}{2}\right)^4 \\ &= 4 \cdot \frac{1}{16} \\ &= \frac{1}{4} \end{aligned} \quad (2.0.23)$$

(c) probability of getting 2 heads,

$$\begin{aligned} \Pr(X = 2) &= {}^4C_2 \left(\frac{1}{2}\right)^4 \\ &= 6 \cdot \frac{1}{16} \\ &= \frac{3}{8} \end{aligned} \quad (2.0.24)$$

(d) probability of getting 3 heads,

$$\begin{aligned} \Pr(X = 3) &= {}^4C_3 \left(\frac{1}{2}\right)^4 \\ &= 4 \cdot \frac{1}{16} \\ &= \frac{1}{4} \end{aligned} \quad (2.0.25)$$

(e) probability of getting 4 heads,

$$\begin{aligned} \Pr(X = 4) &= {}^4C_4 \left(\frac{1}{2}\right)^4 \\ &= 1 \cdot \frac{1}{16} \\ &= \frac{1}{16} \end{aligned} \quad (2.0.26)$$

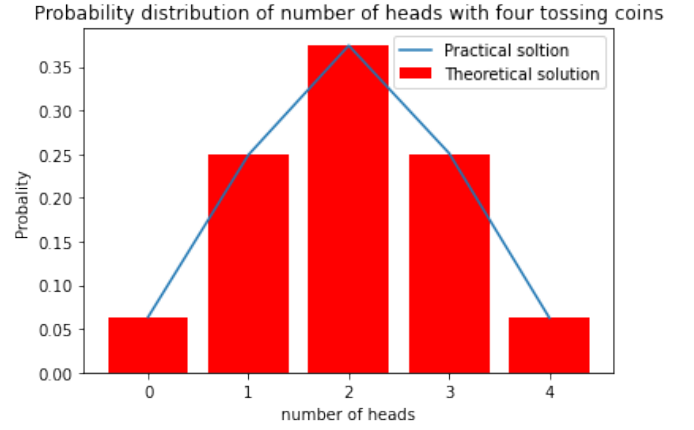


Fig. iii. Plot of probability distribution of four tossed coins

TABLE 3
TABLE

Case	n (no. of coins)	k (no. of required outcomes)	0	1	2	3	4
(i)	2	$\Pr(X = k) (\text{heads})$	1/2	1/4	1/2	0	0
(ii)	3	$\Pr(X' = k) (\text{tails})$	1/8	3/8	3/8	1/8	0
(iii)	4	$\Pr(X = k) (\text{heads})$	1/16	1/4	3/8	1/4	1/16