# Assignment 1

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Download all python codes from

```
https://github.com/PRABHATH-cs20-11038/
    Assignment_1/tree/main/Assignment_1/Codes
```

and latex-tikz codes from

```
https://github.com/PRABHATH-cs20-11038/
    Assignment_1/blob/main/Assignment_1/
    Assignment_1.tex
```

## 1 Problem

(Prob 3.6) Find the probability distribution of
(i) number of heads in two tosses of a coin.
(ii) number of tails in the simultaneous tosses of three coins.
(iii) number of heads in four tosses of a coin.

## 2 Solution

Let $X_i \in \{0, 1\}$ represent the $i^{Th}$ coin where 1 denotes the coin giving outcome as head. Then, $X_i$ has a Bernoulli distribution with parameter

$$p = \frac{1}{2} \tag{2.0.1}$$

Let

$$X = \sum_{i=1}^{n} X_i \tag{2.0.2}$$

where n is the total number of coins tossed. Then $X$ has a Binomial Distribution. Then for

$$\Pr(X_i = n) \overset{Z}{\rightleftharpoons} \Pr(X_i = z) \tag{2.0.3}$$

yielding

$$\Pr(X_i = z) = 1 - p + pz^{-1} \tag{2.0.4}$$

with using the fact that $X_i$ are i.i.d.,

$$\begin{aligned} \Pr(X = z) &= (1 - p + pz^{-1})^n \\ &= \sum_{k=0}^{n} {}^nC_k p^k (1-p)^{n-k} z^{-k} \end{aligned} \tag{2.0.5}$$

$$\Pr(X = k) = \begin{cases} {}^nC_k p^k (1-p)^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \tag{2.0.6}$$

TABLE 0
Table

| Case | n (no. of coins) | k (no. of required outcomes) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| (i) | 2 | $\Pr(X = k)\,(heads)$ | 1/2 | 1/4 | 1/2 | 0 | 0 |
| (ii) | 3 | $\Pr(X = k)\,(tails)$ | 1/8 | 3/8 | 3/8 | 1/8 | 0 |
| (iii) | 4 | $\Pr(X = k)\,(heads)$ | 1/16 | 1/4 | 3/8 | 1/4 | 1/16 |

(i) Here we are seeing outcome of two tosses of a coin. We can say it as simultaneous tossing two coins. So, we get,

$$n = 2 \tag{2.0.7}$$

This is the **special case** with equation (2.0.6) with $n = 2$.
Now we obtain probability distribution of number of heads of two coins from (2.0.6),

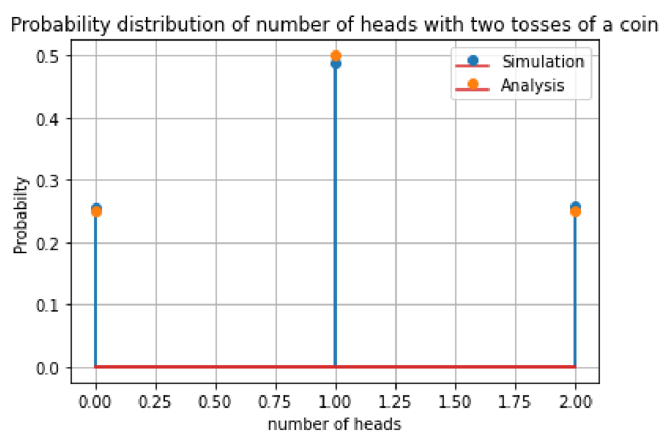


Fig. i. Plot of probability distribution of two tossed coins

$$\Pr(X = k) = \begin{cases} {}^2C_k \left(\frac{1}{2}\right)^2 & \text{if } 0 \le k \le 2 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.8}$$

(ii) This is the **special case** with equation (2.0.6) with $n = 3$ and complement of $X_i$.
For $n$ coins if we want $k$ tails then remaining should be heads,
Probability of getting k tails

$$= \Pr(X = n - k) \tag{2.0.9}$$

and there will be a small change in binomial distribution (2.0.6) and we get binomial distribution for tails as,

$$\Pr(X = n - k) = \begin{cases} {}^nC_k(p)^k(1-p)^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \tag{2.0.10}$$

$$\Pr(X = k) = \begin{cases} {}^nC_k(p)^{n-k}(1-p)^k & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \tag{2.0.11}$$

Here we are seeing outcome of simultaneous tosses of three coin. So, we get,

$$n = 3 \tag{2.0.12}$$

Now we obtain probability distribution of number of tails of three coins from (2.0.11),

$$\Pr(X = k) = \begin{cases} {}^3C_k\left(\frac{1}{2}\right)^3 & \text{if } 0 \le k \le 3 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.13}$$

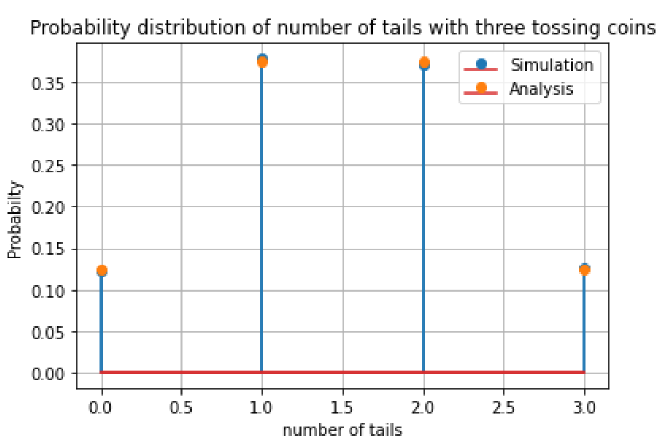


Fig. ii. Plot of probability distribution of no of tails with three tossed coins

(iii) Here we are seeing outcome of four tosses of a coin. We can say it as simultaneous tossing four coins. So, we get,

$$n = 4 \tag{2.0.14}$$

This is the **special case** with equation (2.0.6) with $n = 4$.
Now we obtain probability distribution of number of heads of two coins from (2.0.6),

$$\Pr(X = k) = \begin{cases} {}^4C_k\left(\frac{1}{2}\right)^4 & \text{if } 0 \le k \le 4 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.15}$$

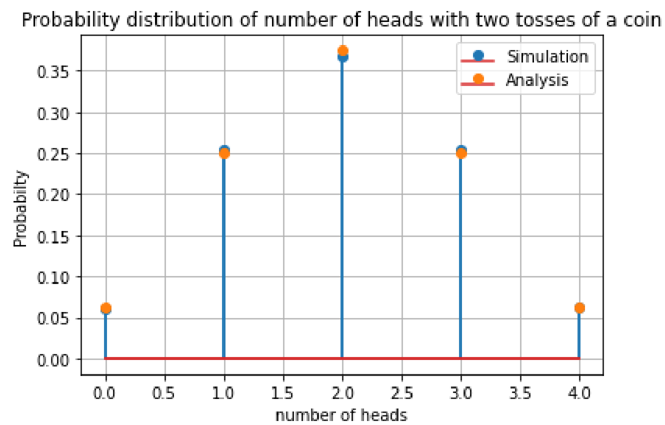


Fig. iii. Plot of probability distribution of four tossed coins