Assignment 3

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1 PROBLEM

$(GATE(MA)2011 - 49Q))$ Let X and Y be two continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} 2, & 0 < x + y < 1, x > 0, y > 0, \\ 0, & elsewhere. \end{cases} \quad (1.0.1)$$

$E\left(X|Y = \frac{1}{2}\right)$ is

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 1

(D) 2

2 SOLUTION

We are checking at $y = \frac{1}{2}$,

$$\Pr\left(Y = \frac{1}{2}\right) = 1 \quad (2.0.1)$$

when $y = \frac{1}{2}$,

$$\Pr\left((X) \cap \left(Y = \frac{1}{2}\right)\right) = f\left(x, \frac{1}{2}\right) = \begin{cases} 2, & 0 < x < \frac{1}{2}, \\ 0, & elsewhere. \end{cases} \quad (2.0.2)$$

by using Bayes theorem,

$$\Pr\left(X|Y = \frac{1}{2}\right) = \frac{\Pr\left((X) \cap \left(Y = \frac{1}{2}\right)\right)}{\Pr\left(Y = \frac{1}{2}\right)} \quad (2.0.3)$$

$$\Pr\left(X|Y = \frac{1}{2}\right) = \begin{cases} 2, & 0 < x < \frac{1}{2}, \\ 0, & elsewhere. \end{cases} \quad (2.0.4)$$
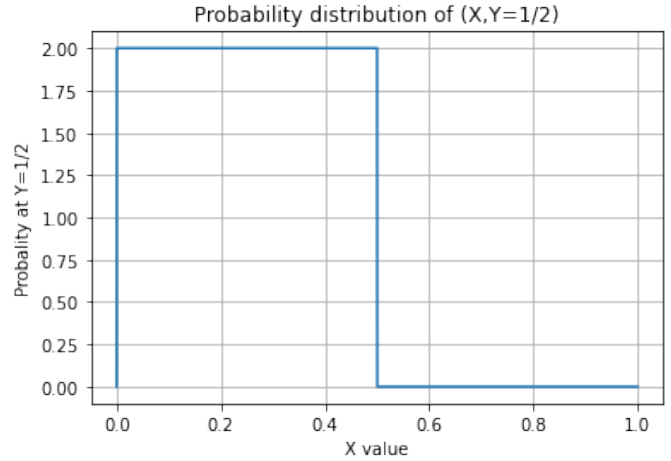

Fig. 4. Plot of of probability function

It is in the form of Bernoulli distribution, the expectation value is given by,

$$E\left(X|Y = \frac{1}{2}\right) = \sum_{-\infty}^{\infty} k \Pr\left(X = k|Y = \frac{1}{2}\right) \quad (2.0.5)$$

$$E\left(X|Y = \frac{1}{2}\right) = \int_0^{\frac{1}{2}} x(2)dx + \int_{-\infty}^0 x(0)dx + \int_0^{-\infty} x(0)dx \quad (2.0.6)$$

$$E\left(X|Y = \frac{1}{2}\right) = \int_0^{\frac{1}{2}} 2xdx \quad (2.0.7)$$

$$E\left(X|Y = \frac{1}{2}\right) = \frac{1}{4} \quad (2.0.8)$$

Option (A) is correct.