

Assignment 8

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Download all python codes from

https://github.com/PRABHATH-cs20-11038/AI1103/tree/main/Assignment_8/Codes

and latex-tikz codes from

https://github.com/PRABHATH-cs20-11038/AI1103/tree/main/Assignment_8

1 PROBLEM

(CSIR – UGC – NET – EXAM (June – 2013), Q.60)
Consider the quadratic equation $x^2 + 2Ux + V = 0$ where U and V are chosen independently and randomly from $\{1, 2, 3\}$ with equal probabilities. Then probability that the equation has both roots real

- 1) $\frac{2}{3}$ 2) $\frac{1}{2}$ 3) $\frac{7}{9}$ 4) $\frac{1}{3}$

2 SOLUTION

Let $U \in \{1, 2, 3\}$ and $V \in \{1, 2, 3\}$

TABLE 4
PROBABILITY OF SELECTING VALUES FOR U

k	1	2	3
$\Pr(U = k)$	$1/3$	$1/3$	$1/3$

TABLE 4
PROBABILITY OF SELECTING VALUES FOR V

k	1	2	3
$\Pr(V = k)$	$1/3$	$1/3$	$1/3$

For $x^2 + 2Ux + V = 0$ to have real roots,

$$b^2 - 4ac \geq 0 \quad (2.0.1)$$

$$(2U)^2 - 4(1)(V) \geq 0 \quad (2.0.2)$$

$$U^2 \geq V \quad (2.0.3)$$

$$\Pr(U^2 \geq V) = 1 - \Pr(U^2 < V) \quad (2.0.4)$$

The possible pairs (U, V) for $\Pr(U^2 < V)$,

TABLE 4
TABLE FOR $\Pr(U^2 < V)$

(U, V) for $U^2 < V$	Probability
$(1, 2)$	$\Pr(U = 1)\Pr(V = 2) = 1/9$
$(1, 3)$	$\Pr(U = 1)\Pr(V = 3) = \frac{1}{9}$
Total	$\Pr(U^2 < V) = \frac{2}{9}$

$$\Pr(U^2 \geq V) = 1 - \frac{2}{9} \quad (2.0.5)$$

$$\Pr(U^2 \geq V) = \frac{7}{9} \quad (2.0.6)$$

Hence, Option 3 is correct.

Probability –
actual: 0.7778
simulated: 0.7769