

Assignment 1

Prabhath Chellingi - CS20BTECH11038

Download all python codes from

https://github.com/PRABHATH-cs20-11038/Assignment_1/tree/main/Assignment_1/Codes

and latex-tikz codes from

https://github.com/PRABHATH-cs20-11038/Assignment_1/blob/main/Assignment_1/Assignment_1.tex

$$\Pr(X = k) = \begin{cases} {}^nC_k p^k (1-p)^{n-k} & 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases} \quad (2.0.6)$$

TABLE 0
TABLE OF PROBABILITY DISTRIBUTION OF DIFFERENT CASES

Case	n (no. of coins)	k (no. of re- quired out- comes)	0	1	2	3	4
(i)	2	$\Pr(X = k) = {}^2C_k \left(\frac{1}{2}\right)^2$	1/2	1/4	1/2	0	0
(ii)	3	$\Pr(X = k) = {}^3C_k \left(\frac{1}{2}\right)^3$	1/8	3/8	3/8	1/8	0
(iii)	4	$\Pr(X = k) = {}^4C_k \left(\frac{1}{2}\right)^4$	1/16	1/4	3/8	1/4	1/16

1 PROBLEM

- (Prob 3.6) Find the probability distribution of
(i) number of heads in two tosses of a coin.
(ii) number of tails in the simultaneous tosses of three coins.
(iii) number of heads in four tosses of a coin.

2 SOLUTION

Let $X_i \in \{0, 1\}$ represent the i^{th} coin where 1 denotes the coin giving outcome as head. Then, X_i has a Bernoulli distribution with parameter

$$p = \frac{1}{2} \quad (2.0.1)$$

Let

$$X = \sum_{i=1}^n X_i \quad (2.0.2)$$

where n is the total number of coins tossed. Then X has a Binomial Distribution. Then for

$$\Pr(X_i = n) \stackrel{Z}{\Leftrightarrow} \Pr(X_i = z) \quad (2.0.3)$$

yielding

$$\Pr(X_i = z) = 1 - p + pz^{-1} \quad (2.0.4)$$

with using the fact that X_i are i.i.d.,

$$\begin{aligned} \Pr(X = z) &= (1 - p + pz^{-1})^n \\ &= \sum_{k=0}^n {}^nC_k p^k (1-p)^{n-k} z^{-k} \end{aligned} \quad (2.0.5)$$

The above table represents the probability distribution of following cases at different k values.

(i) from (2.0.6)

$$\Pr(X = k) = \begin{cases} {}^2C_k \left(\frac{1}{2}\right)^2 & \text{if } 0 \leq k \leq 2 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.7)$$

Probability distribution of number of heads with two tosses of a coin

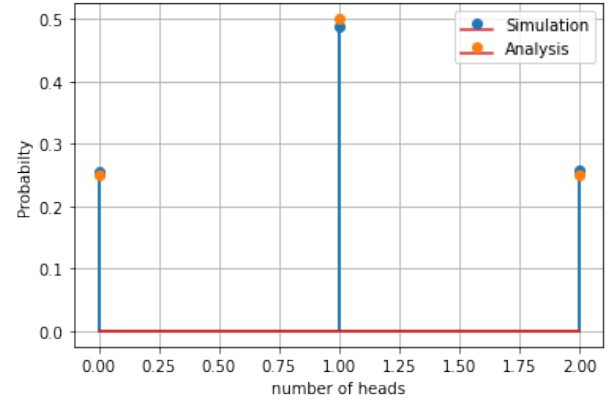


Fig. i. Plot of probability distribution of two tossed coins

(ii) probability of tails,
from (2.0.6)

$$\Pr(X = k) = \begin{cases} {}^nC_k(p)^{n-k}(1-p)^k & 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases} \quad (2.0.8)$$

from (2.0.8),

$$\Pr(X = k) = \begin{cases} {}^3C_k\left(\frac{1}{2}\right)^3 & \text{if } 0 \leq k \leq 3 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.9)$$

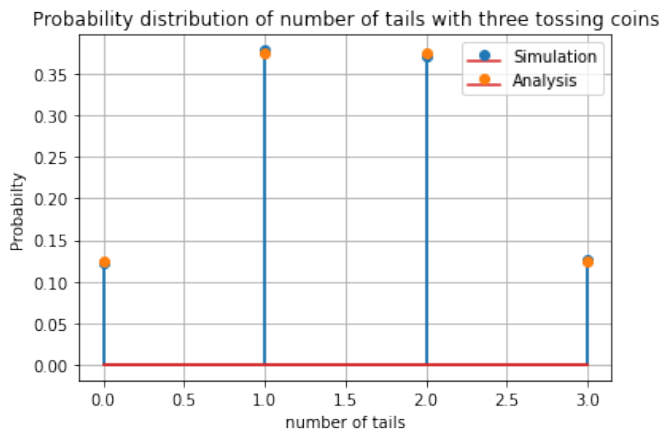


Fig. ii. Plot of probability distribution of no of tails with three tossed coins

(iii) from (2.0.6),

$$\Pr(X = k) = \begin{cases} {}^4C_k\left(\frac{1}{2}\right)^4 & \text{if } 0 \leq k \leq 4 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.10)$$

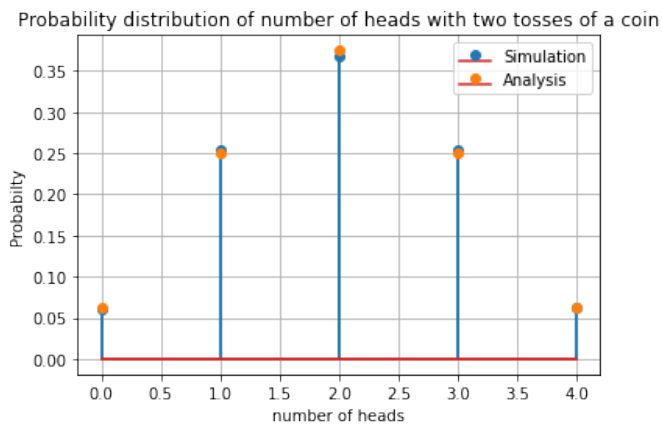


Fig. iii. Plot of probability distribution of four tossed coins