Assignment 2

Prabhath Chellingi - CS20BTECH11038

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1 Problem

($GATE - EC - 66$) Consider two identical boxes B_1 and B_2, where the box $B_i(i = 1, 2)$ contains $i+2$ red and $5 - i - 1$ white balls. A fair die is cast. Let the number of dots shown on the top face of the die be N. If N is even or 5, then two balls are drawn with replacement from the box B_1, otherwise, two balls are drawn with replacement from the box B_2. The probability that the two drawn balls are of different colours is

(A) $\frac{7}{25}$

(B) $\frac{9}{25}$

(C) $\frac{12}{25}$

(D) $\frac{16}{25}$

2 Solution

Let $X \in \{1, 2, 3, 4, 5, 6\}$ be the random variables representing the outcome for a die. The *PDF* of die is

$$\Pr(X = N) = \begin{cases} \frac{1}{6} & 1 \leq N \leq 6 \\ 0 & otherwise \end{cases} \tag{2.0.1}$$

where $N =$ the number on top face of the die.

As we know

$$\Pr(X = m) . \Pr(X = n) = 0 \tag{2.0.2}$$

for all $m, n \in \{1, 2, 3, 4, 5, 6\}$ as a single die cannot show more than one outcome at a roll.

Let $Y \in \{0, 1\}$ represent the die where 1 denotes the die with outcome $N = \{2, 4, 5, 6\}$ and 0 denotes the remaining.

$$\Pr(Y = 1) = \\ \Pr((X = 2) + (X = 4) + (X = 5) + (X = 6)) \tag{2.0.3}$$

by using *Boolean logic* and equation (2.0.2), we get,

$$\Pr(Y = 1) = \\ \Pr(X = 2) + \Pr(X = 4) + \Pr(X = 5) + \Pr(X = 6) \tag{2.0.4}$$

$$\begin{aligned} \Pr(Y = 1) &= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \\ &= \frac{2}{3} \end{aligned} \tag{2.0.5}$$

$$\begin{aligned} \Pr(Y = 0) &= 1 - \Pr(Y = 1) \\ &= \frac{1}{3} \end{aligned} \tag{2.0.6}$$

let $C \in \{0, 1\}$ where 0 denotes to red balls and 1 denotes to white balls.

There are two boxes $B_i(i = 1, 2)$ containing $i+2$ red and $5 - i - 1$ white balls.

(a) number of red balls($C = 0$) in B_1,

$$n(C = 0|B_1) = (1) + 2 \tag{2.0.7}$$
$$n(C = 0|B_1) = 3 \tag{2.0.8}$$

(b) number of white balls($C = 1$) in B_1,

$$n(C = 1|B_1) = 5 - (1) - 1 \tag{2.0.9}$$
$$n(C = 1|B_1) = 3 \tag{2.0.10}$$

number of balls in box B_1,

$$n(C|B_1) = n(C = 0|B_1) + n(C = 1|B_1) \tag{2.0.11}$$
$$n(C|B_1) = 6 \tag{2.0.12}$$

(a) probability of selecting red ball from box B_1 is,

$$\Pr(C=0|B_1) = \frac{n(C=0|B_1)}{n(C=0|B_1)+n(C=1|B_1)} = \frac{1}{2} \tag{2.0.13}$$

(b) probability of selecting white ball from box B_1 is,

$$\Pr(C=1|B_1) = \frac{n(C=1|B_1)}{n(C=0|B_1)+n(C=1|B_1)} = \frac{1}{2} \tag{2.0.14}$$

(a) number of red balls($C = 0$) in B_2,

$$n(C=0|B_2) = (2)+2 \tag{2.0.15}$$
$$n(C=0|B_2) = 4 \tag{2.0.16}$$

(b) number of white balls($C = 1$) in B_2,

$$n(C=1|B_1) = 5-(2)-1 \tag{2.0.17}$$
$$n(C=1|B_1) = 2 \tag{2.0.18}$$

number of balls in box B_2,

$$n(C|B_2) = n(C=0|B_2)+n(C=1|B_2) \tag{2.0.19}$$
$$n(C|B_1) = 6 \tag{2.0.20}$$

(a) probability of selecting red ball from box B_2 is,

$$\Pr(C=0|B_2) = \frac{n(C=0|B_2)}{n(C=0|B_2)+n(C=1|B_2)} = \frac{2}{3} \tag{2.0.21}$$

(b) probability of selecting white ball from box B_2 is,

$$\Pr(C=1|B_2) = \frac{n(C=1|B_2)}{n(C=0|B_2)+n(C=1|B_2)} = \frac{1}{3} \tag{2.0.22}$$

Given the probability of selecting box B_1 is same as $\Pr(Y=1)$, from equation (2.0.5),

$$\Pr(B_1) = \Pr(Y=1) \tag{2.0.23}$$

$$\Pr(B_1) = \frac{2}{3} \tag{2.0.24}$$

Given the probability of selecting box B_2 is same as $\Pr(Y=0)$, from equation (2.0.6),

$$\Pr(B_2) = \Pr(Y=0) \tag{2.0.25}$$

$$\Pr(B_2) = \frac{1}{3} \tag{2.0.26}$$

No of ways of selecting two different coloured balls is

(1) $(B_1, C=0, C=1)$
(2) $(B_1, C=1, C=0)$
(3) $(B_2, C=0, C=1)$
(4) $(B_2, C=1, C=0)$

The probability of second ball is not effected because the second ball is chose after replacement.

Let $\Pr((C=0,C=1)|B_1)$ be probability of selecting two different coloured balls from Bag B_1. Probability of selecting two different colored balls from bag B_1 is, (by using Boolean logic)

$$\Pr((C=0,C=1)|B_1) = \Pr(C=0|B_1).\Pr(C=1|B_1) + \Pr(C=1|B_1).\Pr(C=0|B_1) \tag{2.0.27}$$

$$\Pr((C=0,C=1)|B_1) = \frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2} \tag{2.0.28}$$

$$\Pr((C=0,C=1)|B_1) = \frac{1}{2} \tag{2.0.29}$$

Let $\Pr((C=0,C=1)|B_2)$ be probability of selecting two different coloured balls from Bag B_2. Probability of selecting two different colored balls from bag B_2 is, (by using Boolean logic)

$$\Pr((C=0,C=1)|B_2) = \Pr(C=0|B_2).\Pr(C=1|B_2) + \Pr(C=1|B_2).\Pr(C=0|B_2) \tag{2.0.30}$$

$$\Pr((C=0,C=1)|B_2) = \frac{2}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3} \tag{2.0.31}$$

$$\Pr\left((C = 0, C = 1)|B_2\right) = \frac{4}{9} \tag{2.0.32}$$

let probability of selecting two different coloured balls be $\Pr(T)$

Now we can obtain $\Pr(T)$ by using *conditional probability*

$$\Pr(EF) = \Pr(E|F) \, . \Pr(F) \tag{2.0.33}$$

$$\begin{aligned}\Pr(T) = \\ &\Pr\left(((C = 0, C = 1)|B_1)(B_1)\right) + \\ &\quad pr((C = 0, C = 1)|B_2)(B_2)\end{aligned} \tag{2.0.34}$$

from equation (2.0.33)

$$\begin{aligned}\Pr(T) = \\ &\Pr\left((C = 0, C = 1)|B_1\right) . \Pr(B_1) + \\ &\quad \Pr\left((C = 0, C = 1)|B_2\right) . \Pr(B_2)\end{aligned} \tag{2.0.35}$$

from (2.0.24), (2.0.26), (2.0.29), (2.0.32),

$$\Pr(T) = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) + \left(\frac{4}{9}\right)\left(\frac{1}{3}\right) \tag{2.0.36}$$

$$\Pr(T) = \frac{13}{27} \tag{2.0.37}$$

Hence, the probability of selecting two different coloured balls from the bags is

$$\frac{13}{27}$$

Probability−
simulation: 0.48015,
actual: 0.48148148148148145