Assignment 2

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Download all python codes from

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https://github.com/PRABHATH−cs20−11038/
    Assignment_1/tree/main/Assignment_2
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and latex-tikz codes from

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    Assignment_1/tree/main/Assignment_2
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1 Problem

$(GATE-EC-66)$ Consider two identical boxes B_1 and B_2, where the box $B_i(i = 1, 2)$ contains $i+2$ red and $5-i-1$ white balls. A fair die is cast. Let the number of dots shown on the top face of the die be N. If N is even or 5, then two balls are drawn with replacement from the box B_1, otherwise, two balls are drawn with replacement from the box B_2. The probability that the two drawn balls are of different colours is

(A) $\frac{7}{25}$

(B) $\frac{9}{25}$

(C) $\frac{12}{25}$

(D) $\frac{16}{25}$

2 Solution

Let $X \in \{1, 2, 3, 4, 5, 6\}$ be the random variables representing the outcome for a die. The *PDF* of die is

$$\Pr(X = N) = \begin{cases} \frac{1}{6} & 1 \le N \le 6 \\ 0 & otherwise \end{cases} \tag{2.0.1}$$

As we know

$$\Pr(X = m) . \Pr(X = n) = 0 \tag{2.0.2}$$

for all $m, n \in \{1, 2, 3, 4, 5, 6\}$ as a single die cannot show more than one outcome at a roll.

Let $Y \in \{0, 1\}$ represent the die where 1 denotes the die with outcome $N = \{2, 4, 5, 6\}$ and 0 denotes the remaining.

$$\Pr(Y = 1) = \Pr((X = 2) + (X = 4) + (X = 5) + (X = 6)) \tag{2.0.3}$$

by using *Boolean logic* and equation (2.0.2), we get,

$$\Pr(Y = 1) = \Pr(X = 2) + \Pr(X = 4) + \Pr(X = 5) + \Pr(X = 6) \tag{2.0.4}$$

$$\Pr(Y = 1) = \frac{2}{3} \tag{2.0.5}$$

$$\begin{aligned} \Pr(Y = 0) &= 1 - \Pr(Y = 1) \\ &= \frac{1}{3} \end{aligned} \tag{2.0.6}$$

let $C \in \{0, 1\}$ where 0 denotes to red balls and 1 denotes to white balls.

TABLE 4
Table of number of balls

bag	no. of red balls $(i+2)$	no. of white balls $(5-i-1)$	total balls
B_1	$n(C = 0\|B_1) = 3$	$n(C = 1\|B_1) = 3$	$n(C\|B_1) = 6$
B_2	$n(C = 0\|B_2) = 4$	$n(C = 1\|B_2) = 2$	$n(C\|B_2) = 6$

TABLE 4
Table of probability of taking balls from each bag

bag	Probability of taking red ball	Probability of taking white ball
B_1	$\Pr(C = 0\|B_1) = 1/2$	$\Pr(C = 1\|B_1) = 1/2$
B_2	$\Pr(C = 0\|B_2) = 2/3$	$\Pr(C = 1\|B_2) = 1/3$

Given the probability of selecting boxes B_1, B_2 is same as probability of $Y = \{1, 0\}$ respectively,
from equations (2.0.5), (2.0.6),

$$\Pr(B_1) = \frac{2}{3} \tag{2.0.7}$$

$$\Pr(B_2) = \frac{1}{3} \quad (2.0.8)$$

The probability of second ball is not effected because the second ball is chose after replacement.

No. of ways of selecting two different coloured balls is
(1) $(B_1, C = 0, C = 1)$
(2) $(B_1, C = 1, C = 0)$
(3) $(B_2, C = 0, C = 1)$
(4) $(B_2, C = 1, C = 0)$

Let,

$\Pr((C = 0, C = 1)|B_1)$ = probability of selecting two different coloured balls from Bag B_1.

$\Pr((C = 0, C = 1)|B_2)$ = probability of selecting two different coloured balls from Bag B_2.

by using Boolean logic,

(i)

$$\Pr((C = 0, C = 1)|B_1) = \Pr(C = 0|B_1) . \Pr(C = 1|B_1) + \Pr(C = 1|B_1) . \Pr(C = 0|B_1) \quad (2.0.9)$$

$$\Pr((C = 0, C = 1)|B_1) = \frac{1}{2} \quad (2.0.10)$$

(ii)

$$\Pr((C = 0, C = 1)|B_2) = \Pr(C = 0|B_2) . \Pr(C = 1|B_2) + \Pr(C = 1|B_2) . \Pr(C = 0|B_2) \quad (2.0.11)$$

$$\Pr((C = 0, C = 1)|B_2) = \frac{4}{9} \quad (2.0.12)$$

Let,

$\Pr(T)$ = Probability of selecting two different coloured balls

$$\Pr(T) = \Pr(((C = 0, C = 1)|B_1)(B_1)) + pr((C = 0, C = 1)|B_2)(B_2) \quad (2.0.13)$$

→ by using *conditional probability*,

$$\Pr(EF) = \Pr(E|F) . \Pr(F) \quad (2.0.14)$$

from equation (2.0.14)

$$\Pr(T) = \Pr((C = 0, C = 1)|B_1) . \Pr(B_1) + \Pr((C = 0, C = 1)|B_2) . \Pr(B_2) \quad (2.0.15)$$

from (2.0.7), (2.0.8), (2.0.10), (2.0.12),

$$\Pr(T) = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) + \left(\frac{4}{9}\right)\left(\frac{1}{3}\right) \quad (2.0.16)$$

Hence, the probability of selecting two different coloured balls from the bags is

$$\Pr(T) = \frac{13}{27} \quad (2.0.17)$$

Probability−
simulation: 0.48015,
actual: 0.48148148148148145