

Assignment 5

Prabhath Chellingi - CS20BTECH11038

Download all python codes from

https://github.com/PRABHATH-cs20-11038/AI1103/tree/main/Assignment_5/Codes

and latex-tikz codes from

https://github.com/PRABHATH-cs20-11038/AI1103/tree/main/Assignment_5

1 PROBLEM

(GATE(EC)2013 – 26Q) Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $\Pr(3V \geq 2U)$ is

- (A) $4/9$
- (B) $1/2$
- (C) $2/3$
- (D) $5/9$

2 SOLUTION

U and V are independent random variables,

$$V \sim N\left(0, \frac{1}{9}\right) \quad (2.0.1)$$

$$U \sim N\left(0, \frac{1}{4}\right) \quad (2.0.2)$$

Let,

$$Z = 3V - 2U \quad (2.0.3)$$

$$Z \sim N\left(0, 9 \times \frac{1}{9} + 4 \times \frac{1}{4}\right) \quad (2.0.4)$$

$$Z \sim N(0, 2) \quad (2.0.5)$$

For Z , $\mu = 0$, and $\sigma^2 = 2$.

By Gaussian Distribution,
PDF of Z ,

$$f_Z(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \quad (2.0.6)$$

$$f_Z(z) = \frac{1}{2\sqrt{\pi}} e^{-\frac{z^2}{4}} \quad (2.0.7)$$

The Q – function is,

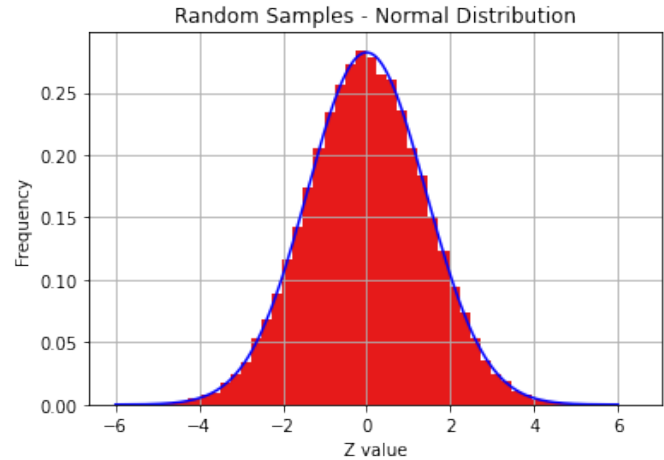


Fig. 4. Plot of of distribution function

$$Q(x) = \frac{1}{2} \operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right) \quad (2.0.8)$$

$$\Pr(Z \geq 0) = Q(x) \quad (2.0.9)$$

where x is standard normal variable,

$$x = \frac{z - \mu}{\sigma} \quad (2.0.10)$$

$$x = \frac{0 - 0}{\sqrt{2}} = 0 \quad (2.0.11)$$

$$\Pr(Z \geq 0) = Q(0) \quad (2.0.12)$$

$$\Pr(Z \geq 0) = \frac{1}{2} \quad (2.0.13)$$

$$\Pr(3V - 2U \geq 0) = \frac{1}{2} \quad (2.0.14)$$

$$\Pr(3V \geq 2U) = \frac{1}{2} \quad (2.0.15)$$

Option (B) is correct.

Probability–

simulated: 0.49873

actual: 0.5