Assignment 2

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Download all python codes from

https://github.com/PRABHATH−cs20−11038/AI1103/tree/main/Assignment_2

and latex-tikz codes from

https://github.com/PRABHATH−cs20−11038/AI1103/tree/main/Assignment_2

1 Problem

$(GATE-EC-66)$ Consider two identical boxes B_1 and B_2, where the box $B_i(i = 1, 2)$ contains $i+2$ red and $5 - i - 1$ white balls. A fair die is cast. Let the number of dots shown on the top face of the die be N. If N is even or 5, then two balls are drawn with replacement from the box B_1, otherwise, two balls are drawn with replacement from the box B_2. The probability that the two drawn balls are of different colours is

2 Solution

Let $X \in \{1, 2, 3, 4, 5, 6\}$ be the random variables of a die,

$$\Pr(X = N) = \begin{cases} \frac{1}{6} & 1 \le N \le 6 \\ 0 & otherwise \end{cases} \tag{2.0.1}$$

$$\Pr(X = m) . \Pr(X = n) = 0 \tag{2.0.2}$$

$\forall\ m, n \in \{1, 2, 3, 4, 5, 6\}$ as a single die cannot show more than one outcome at a roll.
Let $Y \in \{0, 1\}$ represent the die where,
$1 \implies$ the die with outcome $N = \{2, 4, 5, 6\}$,
$0 \implies N = \{1, 3\}$.

$$\Pr(Y = 1) =$$
$$\Pr((X = 2) + (X = 4) + (X = 5) + (X = 6)) \tag{2.0.3}$$

by using *Boolean logic* and (2.0.2),

$$\Pr(Y = 1) = \frac{2}{3} \tag{2.0.4}$$

$$\Pr(Y = 0) = 1 - \Pr(Y = 1) = \frac{1}{3} \tag{2.0.5}$$

$\implies$ probability of selecting box B_1,

$$\Pr(B_1) = \Pr(Y = 1) = \frac{2}{3} \tag{2.0.6}$$

$\implies$ probability of selecting box B_2,

$$\Pr(B_2) = \Pr(Y = 0) = \frac{1}{3} \tag{2.0.7}$$

Let $C \in \{0, 1\}$ where,
$0 \implies$ red balls,
$1 \implies$ white balls.

TABLE 0
Table of number of balls

Box	No. of red balls $(i + 2)$	No. of white balls $(5 - i - 1)$	Total balls
B_1	$n(C = 0\|B_1) = 3$	$n(C = 1\|B_1) = 3$	$n(C\|B_1) = 6$
B_2	$n(C = 0\|B_2) = 4$	$n(C = 1\|B_2) = 2$	$n(C\|B_2) = 6$

TABLE 0
Table of probability of taking balls from each box

Box	Probability of taking red ball	Probability of taking white ball
B_1	$\Pr(C = 0\|B_1) = 1/2$	$\Pr(C = 1\|B_1) = 1/2$
B_2	$\Pr(C = 0\|B_2) = 2/3$	$\Pr(C = 1\|B_2) = 1/3$

The probability of picking 2^{nd} ball is not effected by picking 1^{st} ball because the 2^{nd} ball is chose after **replacement**.

Selecting two balls with replacement is a *Bernoulli distribution* of 2 trails,

TABLE 0
Table of no. of ways of selecting two different coloured balls

Cases	Trail 1	Trail 2
$(B_1, C = 0, C = 1)$	$\Pr(C = 0\|B_1)$	$\Pr(C = 1\|B_1)$
$(B_1, C = 1, C = 0)$	$\Pr(C = 1\|B_1)$	$\Pr(C = 0\|B_1)$
$(B_2, C = 0, C = 1)$	$\Pr(C = 0\|B_2)$	$\Pr(C = 1\|B_2)$
$(B_2, C = 1, C = 0)$	$\Pr(C = 1\|B_2)$	$\Pr(C = 0\|B_2)$

TABLE 0
Table of variables description

Variables	Description
$\Pr\left((C=0, C=1) \mid B_1\right)$	Probability of selecting two different coloured balls from box B_1
$\Pr\left((C=0, C=1) \mid B_2\right)$	Probability of selecting two different coloured balls from box B_2
$\Pr\left(T\right)$	Total probability of selecting two different coloured balls

$$\implies \Pr\left((C=0, C=1) \mid B_1\right) = \Pr\left(C=0 \mid B_1\right) . \Pr\left(C=1 \mid B_1\right) + \Pr\left(C=1 \mid B_1\right) . \Pr\left(C=0 \mid B_1\right) \tag{2.0.8}$$

$$\Pr\left((C=0, C=1) \mid B_1\right) = \frac{1}{2} \tag{2.0.9}$$

$$\implies \Pr\left((C=0, C=1) \mid B_2\right) = \Pr\left(C=0 \mid B_2\right) . \Pr\left(C=1 \mid B_2\right) + \Pr\left(C=1 \mid B_2\right) . \Pr\left(C=0 \mid B_2\right) \tag{2.0.10}$$

$$\Pr\left((C=0, C=1) \mid B_1\right) = \frac{4}{9} \tag{2.0.11}$$

by using *Bayes theorem*,

$$\Pr\left(T\right) = \Pr\left((C=0, C=1) \mid B_1\right) . \Pr\left(B_1\right) + \Pr\left((C=0, C=1) \mid B_2\right) . \Pr\left(B_2\right) \tag{2.0.12}$$

$$\Pr\left(T\right) = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) + \left(\frac{4}{9}\right)\left(\frac{1}{3}\right) \tag{2.0.13}$$

Hence, the probability of selecting two different coloured balls from the boxes is

$$\Pr\left(T\right) = \frac{13}{27} \tag{2.0.14}$$

Probability−
simulation: 0.48015,
actual: 0.48148148148148145