Assignment 6

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Download all python codes from

https://github.com/PRABHATH−cs20−11038/AI1103/tree/main/Assignment_6/Codes

and latex-tikz codes from

https://github.com/PRABHATH−cs20−11038/AI1103/tree/main/Assignment_6

1 Problem

$(GATE(ME)2005-2Q)$ A lot has 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly 2 of the chosen items are defective is

2 Solution

Let $X_i \in \{0,1\}$ represent the i^{Th} item where 1 denotes the item is defective. Then, X_i has a Bernoulli distribution with parameter

$$p = \frac{10}{100} = \frac{1}{10} \tag{2.0.1}$$

Let

$$X = \sum_{i=1}^{n} X_i \tag{2.0.2}$$

Where n is the total no. of items chosen. Then X has a Binomial Distribution. Then for

$$\Pr(X_i = n) \overset{Z}{\rightleftharpoons} \Pr(X_i = z) \tag{2.0.3}$$

Yielding

$$\Pr(X_i = z) = 1 - p + pz^{-1} \tag{2.0.4}$$

with using the fact that X_i are $i.i.d.$,

$$\begin{aligned} \Pr(X = z) &= (1 - p + pz^{-1})^n \\ &= \sum_{k=0}^{n} {}^nC_k p^k (1-p)^{n-k} z^{-k} \end{aligned} \tag{2.0.5}$$

The probability of getting k defective items is,

$$\Pr(X = k) = \begin{cases} {}^nC_k p^k (1-p)^{n-k} & 0 \le k \le n \\ 0 & otherwise \end{cases} \tag{2.0.6}$$

Total no. of items chosen,

$$n = 10 \tag{2.0.7}$$

Probability of getting exactly 2 items defective,

$$\Pr(X = 2) = {}^{10}C_2 \left(\frac{1}{10}\right)^2 \left(1 - \frac{1}{10}\right)^{10-2} \tag{2.0.8}$$

$$\Pr(X = 2) = {}^{10}C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^8 \tag{2.0.9}$$

$$\Pr(X = 2) = 0.1937102445 \tag{2.0.10}$$

Probability−
simulated: 0.19205
actual:0.1937102445