Assignment 1

Prabhath Chellingi - CS20BTECH11038

Download all python codes from

```
https://github.com/PRABHATH-cs20-11038/
    Assignment_1/tree/main/Assignment_1/Codes
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and latex-tikz codes from

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https://github.com/PRABHATH-cs20-11038/
    Assignment_1/blob/main/Assignment_1/
    Assignment_1.tex
```

1 Problem

(Prob 3.6) Find the probability distribution of
(i) number of heads in two tosses of a coin.
(ii) number of tails in the simultaneous tosses of three coins.
(iii) number of heads in four tosses of a coin.

2 Solution

Let $X_i \in \{0, 1\}$ represent the i^{Th} coin where 1 denotes the coin giving outcome as head. Then, X_i has a Bernoulli distribution with parameter

$$p = \frac{1}{2} \tag{2.0.1}$$

Let

$$X = \sum_{i=1}^{n} X_i \tag{2.0.2}$$

where n is the total number of coins tossed. Then X has a Binomial Distribution. Then for

$$\Pr(X_i = n) \overset{Z}{\rightleftharpoons} \Pr(X_i = z) \tag{2.0.3}$$

yielding

$$\Pr(X_i = z) = 1 - p + pz^{-1} \tag{2.0.4}$$

with using the fact that X_i are i.i.d.,

$$\begin{aligned} \Pr(X = z) &= (1 - p + pz^{-1})^n \\ &= \sum_{k=0}^{n} {}^nC_k p^k (1-p)^{n-k} z^{-k} \end{aligned} \tag{2.0.5}$$

$$\Pr(X = k) = \begin{cases} {}^nC_k p^k (1-p)^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \tag{2.0.6}$$

(i) Here we are seeing outcome of two tosses of a coin. We can say it as simultaneous tossing two coins. So, we get,

$$n = 2 \tag{2.0.7}$$

This is the **special case** with equation (2.0.6) with $n = 2$.
Now we obtain probability distribution of number of heads of two coins from (2.0.6),

$$\Pr(X = k) = \begin{cases} {}^2C_k \left(\frac{1}{2}\right)^k \left(1 - \frac{1}{2}\right)^{2-k} & 0 \le k \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr(X = k) = \begin{cases} {}^2C_k \left(\frac{1}{2}\right)^2 & \text{if } 0 \le k \le 2 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.8}$$

(a) probability of getting 0 heads,

$$\begin{aligned} \Pr(X = 0) &= {}^2C_0 \left(\frac{1}{2}\right)^2 \\ &= \frac{1}{4} \end{aligned} \tag{2.0.9}$$

(b) probability of getting 1 heads,

$$\begin{aligned} \Pr(X = 1) &= {}^2C_1 \left(\frac{1}{2}\right)^2 \\ &= \frac{1}{2} \end{aligned} \tag{2.0.10}$$

(c) probability of getting 2 heads,

$$\begin{aligned} \Pr(X = 2) &= {}^2C_2 \left(\frac{1}{2}\right)^2 \\ &= \frac{1}{4} \end{aligned} \tag{2.0.11}$$

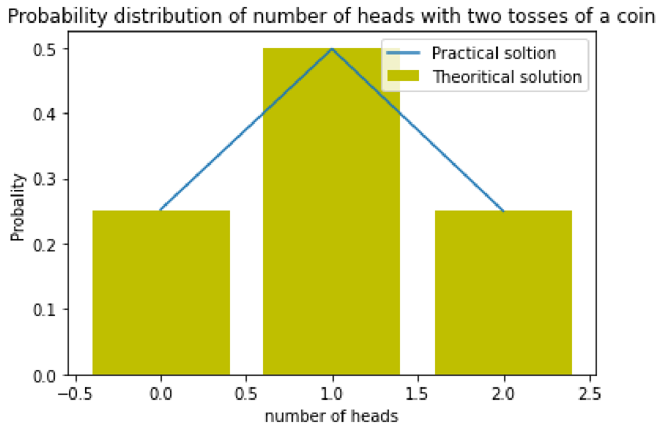

Fig. i. Plot of probability distribution of two tossed coins

(ii) This is the **special case** with equation (2.0.6) with $n = 3$ and complement of X_i.
For n coins if we want k tails then remaining should be heads,
Probability of getting k tails

$$= \Pr(X = n-k) \tag{2.0.12}$$

and there will be a small change in binomial distribution (2.0.6) and we get binomial distribution for tails as,

$$\Pr(X = n-k) = \begin{cases} {}^{n}C_k (p)^k (1-p)^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \tag{2.0.13}$$

$$\Pr(X = k) = \begin{cases} {}^{n}C_k (p)^{n-k} (1-p)^{k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \tag{2.0.14}$$

Here we are seeing outcome of simultaneous tosses of three coin. So, we get,

$$n = 3 \tag{2.0.15}$$

Now we obtain probability distribution of number of tails of three coins from (2.0.14),

$$\Pr(X = k) = \begin{cases} {}^{3}C_k \left(\frac{1}{2}\right)^{3-k} \left(1 - \frac{1}{2}\right)^{k} & 0 \le k \le 3 \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr(X = k) = \begin{cases} {}^{3}C_k \left(\frac{1}{2}\right)^{3} & \text{if } 0 \le k \le 3 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.16}$$

(a) probability of getting 0 tails,

$$\begin{aligned} \Pr(X = 0) &= {}^{3}C_0 \left(\frac{1}{2}\right)^3 \\ &= \frac{1}{8} \end{aligned} \tag{2.0.17}$$

(b) probability of getting 1 tails,

$$\begin{aligned} \Pr(X = 1) &= {}^{3}C_1 \left(\frac{1}{2}\right)^3 \\ &= \frac{3}{8} \end{aligned} \tag{2.0.18}$$

(c) probability of getting 2 tails,

$$\begin{aligned} \Pr(X = 2) &= {}^{3}C_2 \left(\frac{1}{2}\right)^3 \\ &= \frac{3}{8} \end{aligned} \tag{2.0.19}$$

(d) probability of getting 3 tails,

$$\begin{aligned} \Pr(X = 3) &= {}^{3}C_3 \left(\frac{1}{2}\right)^3 \\ &= \frac{1}{8} \end{aligned} \tag{2.0.20}$$

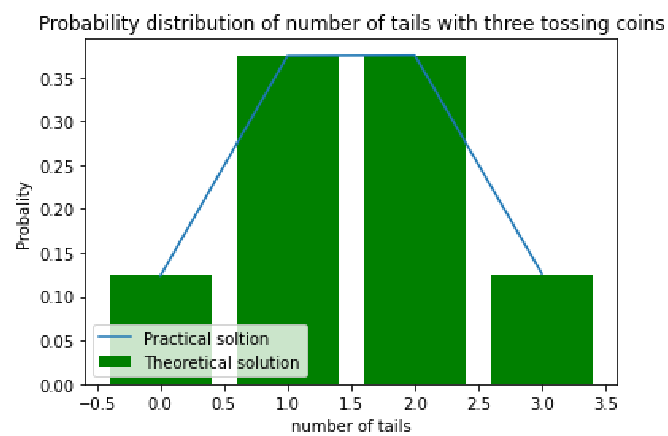

Fig. ii. Plot of probability distribution of no of tails with three tossed coins

(iii) Here we are seeing outcome of four tosses of a coin. We can say it as simultaneous tossing four coins. So, we get,

$$n = 4 \tag{2.0.21}$$

This is the **special case** with equation (2.0.6) with $n = 4$.
Now we obtain probability distribution of number of heads of two coins from (2.0.6),

$$\Pr(X = k) = \begin{cases} {}^4C_k \left(\frac{1}{2}\right)^k \left(1 - \frac{1}{2}\right)^{4-k} & 0 \le k \le 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr(X = k) = \begin{cases} {}^4C_k \left(\frac{1}{2}\right)^4 & \text{if } 0 \le k \le 4 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.22}$$

(a) probability of getting 0 heads,

$$\Pr(X = 0) = {}^4C_0 \left(\frac{1}{2}\right)^4 = \frac{1}{16} \tag{2.0.23}$$

(b) probability of getting 1 heads,

$$\Pr(X = 1) = {}^4C_1 \left(\frac{1}{2}\right)^4 = \frac{1}{4} \tag{2.0.24}$$

(c) probability of getting 2 heads,

$$\Pr(X = 2) = {}^4C_2 \left(\frac{1}{2}\right)^4 = \frac{3}{8} \tag{2.0.25}$$

(d) probability of getting 3 heads,

$$\Pr(X = 3) = {}^4C_3 \left(\frac{1}{2}\right)^4 = \frac{1}{4} \tag{2.0.26}$$

(e) probability of getting 4 heads,

$$\Pr(X = 4) = {}^4C_4 \left(\frac{1}{2}\right)^4 = \frac{1}{16} \tag{2.0.27}$$

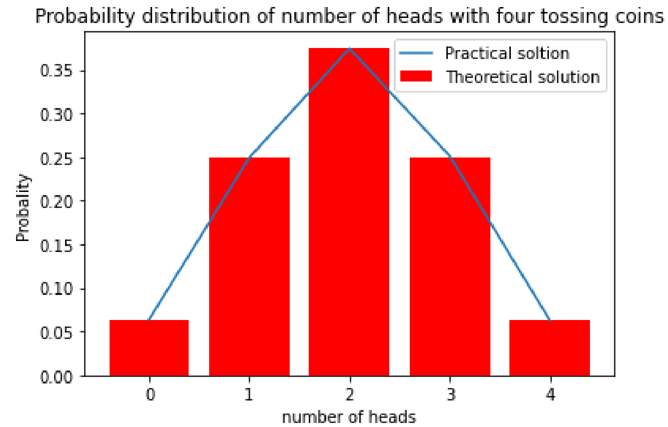

Fig. iii. Plot of probability distribution of four tossed coins

TABLE 3
Table

Case	n (no. of coins)	k (no. of required outcomes)	0	1	2	3	4
(i)	2	$\Pr(X = k)$ (*heads*)	1/2	1/4	1/2	0	0
(ii)	3	$\Pr(X = k)$ (*tails*)	1/8	3/8	3/8	1/8	0
(iii)	4	$\Pr(X = k)$ (*heads*)	1/16	1/4	3/8	1/4	1/16