

# Assignment 5

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Download all python codes from

[https://github.com/PRABHATH-cs20-11038/AI1103/tree/main/Assignment\\_5/Codes](https://github.com/PRABHATH-cs20-11038/AI1103/tree/main/Assignment_5/Codes)

and latex-tikz codes from

[https://github.com/PRABHATH-cs20-11038/AI1103/tree/main/Assignment\\_5](https://github.com/PRABHATH-cs20-11038/AI1103/tree/main/Assignment_5)

By Gaussian Distribution,

Let  $X$  is standard normal variable,

$$X = \frac{Z - \mu}{\sigma} \quad (2.0.7)$$

$$\Pr(Z \geq 0) = \Pr(X\sigma + \mu \geq 0) \quad (2.0.8)$$

$$\Pr(Z \geq 0) = \Pr\left(X\left(\sqrt{2}\right) + 0 \geq 0\right) \quad (2.0.9)$$

$$\Pr(Z \geq 0) = \Pr(X \geq 0) \quad (2.0.10)$$

$$\Pr(Z \geq 0) = Q(0) \quad (2.0.11)$$

where  $Q(x)$  is the  $Q$ -function,

$$Q(0) = \frac{1}{2} \quad (2.0.12)$$

$$\Pr(Z \geq 0) = \frac{1}{2} \quad (2.0.13)$$

$$\Pr(3V - 2U \geq 0) = \frac{1}{2} \quad (2.0.14)$$

$$\Pr(3V \geq 2U) = \frac{1}{2} \quad (2.0.15)$$

Option (B) is correct.

Probability–  
simulated: 0.49873  
actual: 0.5

## 1 PROBLEM

(GATE(EC)2013 – 26Q) Let  $U$  and  $V$  be two independent zero mean Gaussian random variables of variances  $\frac{1}{4}$  and  $\frac{1}{9}$  respectively. The probability  $\Pr(3V \geq 2U)$  is

- (A) 4/9
- (B) 1/2
- (C) 2/3
- (D) 5/9

## 2 SOLUTION

$U$  and  $V$  are independent random variables,

For  $V$ ,  $\mu_V = 0$ , and  $\sigma_V^2 = \frac{1}{9}$

$$V \sim N\left(0, \frac{1}{9}\right) \quad (2.0.1)$$

For  $U$ ,  $\mu_U = 0$ , and  $\sigma_U^2 = \frac{1}{4}$

$$U \sim N\left(0, \frac{1}{4}\right) \quad (2.0.2)$$

Let,

$$Z = 3V - 2U \quad (2.0.3)$$

$$Z \sim N\left((3\mu_V - 2\mu_U), ((3)^2\sigma_V^2 + (2)^2\sigma_U^2)\right) \quad (2.0.4)$$

$$Z \sim N\left(0, 9 \times \frac{1}{9} + 4 \times \frac{1}{4}\right) \quad (2.0.5)$$

$$Z \sim N(0, 2) \quad (2.0.6)$$

For  $Z$ ,  $\mu = 0$ , and  $\sigma^2 = 2$ .