Assignment 7

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Download latex-tikz codes from

https://github.com/PRABHATH−cs20−11038/AI1103/tree/main/Assignment_7

1 Problem

$(CSIR-UGC-NET\_EXAM(Dec-2016), Q.107)$ Let X be a random variable with a certain non-degenerate distribution. Then identify the correct statements

1. If X has an exponential distribution then $median(X) < E(X)$
2. If X has a uniform distribution on an interval $[a, b]$, then $E(X) < median(X)$
3. If X has a Binomial distribution then $V(X) < E(X)$
4. If X has a normal distribution, then $E(X) < V(X)$

2 Solution

Expected value($E(X)$):
It is nothing but weighted average
Median($median(X)$):
It is the value separating the higher half from the lower half of a data sample
Variance($V(X)$):
It is the expectation of the squared deviation of a random variable from its mean

1) Let's consider X has an exponential distribution.

$$X \sim Exp(\lambda) \tag{2.0.1}$$

where λ is rate parameter.
Probability function of exponential distribution,

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{2.0.2}$$

The expected value of $X \sim Exp(\lambda)$,

$$E(X) = \frac{1}{\lambda} \tag{2.0.3}$$

The median of $X \sim Exp(\lambda)$,

$$median(X) = \frac{\ln 2}{\lambda} \tag{2.0.4}$$

$$\ln 2 < 1 \tag{2.0.5}$$

$$\frac{\ln 2}{\lambda} < \frac{1}{\lambda} \tag{2.0.6}$$

$$median(X) < E(X) \tag{2.0.7}$$

Hence, option 1 is correct.

2) Let's consider X has a uniform distribution in interval $[a, b]$,

$$X \sim U(a, b) \tag{2.0.8}$$

where, a = lower limit
b = upper limit
Probability function of uniform distribution,

$$f_X(k) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & x < a, x > b \end{cases} \tag{2.0.9}$$

The expected value of $X \sim U(a, b)$,

$$E(X) = \frac{1}{2}(a + b) \tag{2.0.10}$$

The median of $X \sim U(a, b)$,

$$median(X) = \frac{1}{2}(a + b) \tag{2.0.11}$$

$$E(X) = median(X) \tag{2.0.12}$$

Hence, option 2 is incorrect.

3) Let's consider X has a binomial distribution,

$$X \sim B(n, p) \tag{2.0.13}$$

where, n = no. of trails
p = success parameter
Probability function of binomial distribution,

$$f_X(k) = \begin{cases} {}^nC_k p^k (1-p)^{n-k} & 0 \leq k \leq n \\ 0 & otherwise \end{cases} \tag{2.0.14}$$

The expected value of $X \sim B(n, p)$,

$$E(X) = np \tag{2.0.15}$$

The variance of $X \sim B(n, p)$,

$$V(X) = \sigma^2 = np(1-p) \tag{2.0.16}$$

$$1 - p \leq 1 \tag{2.0.17}$$

$$np(1-p) \leq np \tag{2.0.18}$$

$$V(X) \leq E(X) \tag{2.0.19}$$

Hence, option 3 is incorrect.

4) Let's consider X has a normal distribution,

$$X \sim N\left(\mu, \sigma^2\right) \tag{2.0.20}$$

where, μ = mean of distribution
σ^2 = variance
Probability function of normal distribution,

$$f_X(k) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x-\mu}{2\sigma}\right)^2} \tag{2.0.21}$$

The expected value of $X \sim N\left(\mu, \sigma^2\right)$,

$$E(X) = \mu \tag{2.0.22}$$

The variance of $X \sim N\left(\mu, \sigma^2\right)$,

$$V(X) = \sigma^2 \tag{2.0.23}$$

$E(X)$ and $V(X)$ are user defined. So, they can take any value.
Hence, option 4 is incorrect.