

# Assignment 1

Prabhath Chellingi - CS20BTECH11038

Download all python codes from

[https://github.com/PRABHATH-cs20-11038/Assignment\\_1/tree/main/Assignment\\_1/Codes](https://github.com/PRABHATH-cs20-11038/Assignment_1/tree/main/Assignment_1/Codes)

and latex-tikz codes from

[https://github.com/PRABHATH-cs20-11038/Assignment\\_1/blob/main/Assignment\\_1/Assignment\\_1.tex](https://github.com/PRABHATH-cs20-11038/Assignment_1/blob/main/Assignment_1/Assignment_1.tex)

$$\Pr(X = k) = \begin{cases} {}^nC_k p^k (1-p)^{n-k} & 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases} \quad (2.0.6)$$

TABLE 0  
TABLE

Case	n (no. of coins)	k (no. of required outcomes)	0	1	2	3	4
(i)	2	$\Pr(X = k) (\text{heads})$	1/2	1/4	1/2	0	0
(ii)	3	$\Pr(X = k) (\text{tails})$	1/8	3/8	3/8	1/8	0
(iii)	4	$\Pr(X = k) (\text{heads})$	1/16	1/4	3/8	1/4	1/16

## 1 PROBLEM

- (Prob 3.6) Find the probability distribution of
- (i) number of heads in two tosses of a coin.
  - (ii) number of tails in the simultaneous tosses of three coins.
  - (iii) number of heads in four tosses of a coin.

## 2 SOLUTION

Let  $X_i \in \{0, 1\}$  represent the  $i^{\text{th}}$  coin where 1 denotes the coin giving outcome as head. Then,  $X_i$  has a Bernoulli distribution with parameter

$$p = \frac{1}{2} \quad (2.0.1)$$

Let

$$X = \sum_{i=1}^n X_i \quad (2.0.2)$$

where n is the total number of coins tossed. Then X has a Binomial Distribution. Then for

$$\Pr(X_i = n) \stackrel{Z}{\Leftrightarrow} \Pr(X_i = z) \quad (2.0.3)$$

yielding

$$\Pr(X_i = z) = 1 - p + pz^{-1} \quad (2.0.4)$$

with using the fact that  $X_i$  are i.i.d.,

$$\begin{aligned} \Pr(X = z) &= (1 - p + pz^{-1})^n \\ &= \sum_{k=0}^n {}^nC_k p^k (1-p)^{n-k} z^{-k} \end{aligned} \quad (2.0.5)$$

- (i) Two coins. So,

$$n = 2 \quad (2.0.7)$$

from (2.0.6),

$$\Pr(X = k) = \begin{cases} {}^2C_k \left(\frac{1}{2}\right)^2 & \text{if } 0 \leq k \leq 2 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.8)$$

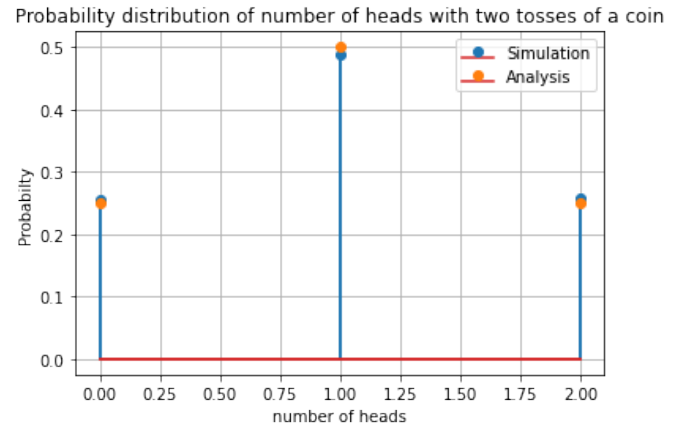


Fig. i. Plot of probability distribution of two tossed coins

- (ii) To get probability of tails,

$$\Pr(X = n - k) = \begin{cases} {}^nC_k (p)^k (1-p)^{n-k} & 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases} \quad (2.0.9)$$

$$\Pr(X = k) = \begin{cases} {}^nC_k(p)^{n-k}(1-p)^k & 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases} \quad (2.0.10)$$

$$\Pr(X = k) = \begin{cases} {}^4C_k\left(\frac{1}{2}\right)^4 & \text{if } 0 \leq k \leq 4 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.14)$$

Three coins. So,

$$n = 3 \quad (2.0.11)$$

from (2.0.10),

$$\Pr(X = k) = \begin{cases} {}^3C_k\left(\frac{1}{2}\right)^3 & \text{if } 0 \leq k \leq 3 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.12)$$

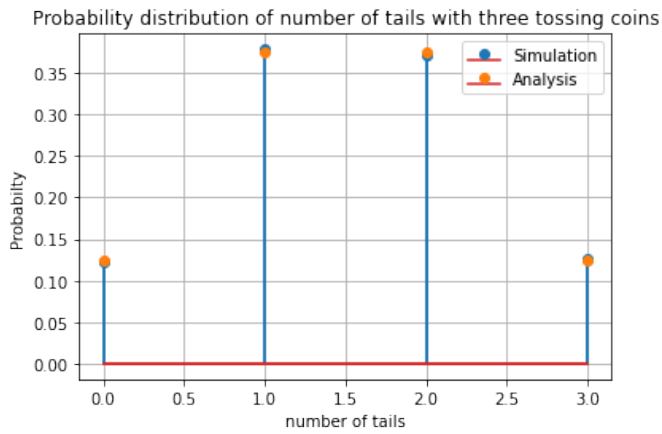


Fig. ii. Plot of probability distribution of no of tails with three tossed coins

(iii) Four coins. So,

$$n = 4 \quad (2.0.13)$$

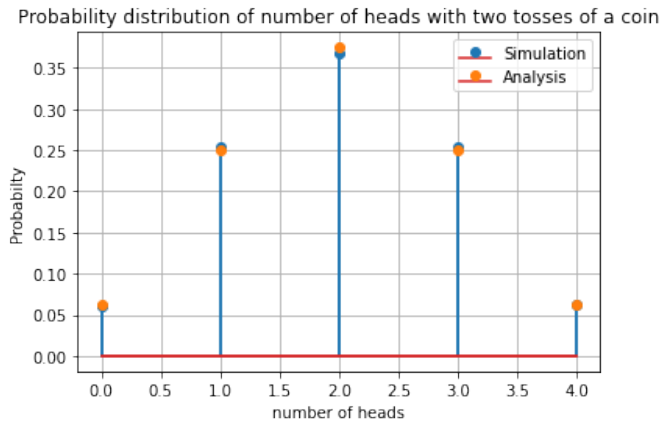


Fig. iii. Plot of probability distribution of four tossed coins

from (2.0.6),