# Assignment 1

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## 1 PROBLEM

(Prob 3.6) Find the probability distribution of
(i) number of heads in two tosses of a coin.
(ii) number of tails in the simultaneous tosses of three coins.
(iii) number of heads in four tosses of a coin.

## 2 SOLUTION

Let $X_i \in \{0, 1\}$ represent the $i^{Th}$ coin where 1 denotes the coin giving outcome as head. Then, $X_i$ has a Bernoulli distribution with parameter

$$p = \frac{1}{2} \tag{2.0.1}$$

Let

$$X = \sum_{i=1}^{n} X_i \tag{2.0.2}$$

where n is the total number of coins tossed. Then $X$ has a Binomial Distribution. Then for

$$\Pr(X_i = n) \overset{Z}{\rightleftharpoons} \Pr(X_i = z) \tag{2.0.3}$$

yielding

$$\Pr(X_i = z) = 1 - p + pz^{-1} \tag{2.0.4}$$

with using the fact that $X_i$ are i.i.d.,

$$\Pr(X = z) = (1 - p + pz^{-1})^n$$
$$= \sum_{k=0}^{n} {}^nC_k p^k (1 - p)^{n-k} z^{-k} \tag{2.0.5}$$

$$\Pr(X = k) = \begin{cases} {}^nC_k p^k (1 - p)^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \tag{2.0.6}$$

(i) Here we are seeing outcome of two tosses of a coin. We can say it as simultaneous tossing two coins. So, we get,

$$n = 2 \tag{2.0.7}$$

This is the **special case** with equation (2.0.6) with $n = 2$.
Now we obtain probability distribution of number of heads of two coins from (2.0.6),

$$\Pr(X = k) = \begin{cases} {}^2C_k \left(\frac{1}{2}\right)^k \left(1 - \frac{1}{2}\right)^{2-k} & 0 \le k \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr(X = k) = \begin{cases} {}^2C_k \left(\frac{1}{2}\right)^2 & \text{if } 0 \le k \le 2 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.8}$$

(a) probability of getting 0 heads,

$$\Pr(X = 0) = {}^2C_0 \left(\frac{1}{2}\right)^2$$
$$= 1.\frac{1}{4} \tag{2.0.9}$$
$$= \frac{1}{4}$$

(b) probability of getting 1 heads,

$$\Pr(X = 1) = {}^2C_1 \left(\frac{1}{2}\right)^2$$
$$= 2.\frac{1}{4} \tag{2.0.10}$$
$$= \frac{1}{2}$$

(c) probability of getting 2 heads,

$$\Pr(X = 2) = {}^2C_2 \left(\frac{1}{2}\right)^2$$
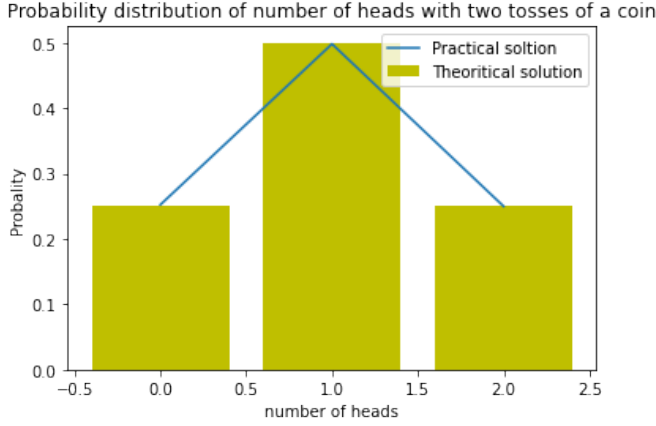
$$= 1.\frac{1}{4} \qquad (2.0.11)$$

$$= \frac{1}{4}$$



Fig. i. Plot of probability distribution of two tossed coins

(ii) Let $X_i' = 1 - X_i$, where in $X_i' \in 0, 1$ here 1 denotes outcome as tail.
This is the **special case** with equation (2.0.6) with $n = 3$ and complement of $X_i$.
With respect to tails the parameter would change to

$$q = 1 - p = \frac{1}{2} \qquad (2.0.12)$$

and there will be a small change in binomial distribution (2.0.6) and we get binomial distribution for tails as,

$$\Pr(X' = k) = \begin{cases} {}^nC_k q^k (1-q)^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \qquad (2.0.13)$$

Here we are seeing outcome of simultaneous tosses of three coin. So, we get,

$$n = 3 \qquad (2.0.14)$$

Now we obtain probability distribution of number of tails of three coins from (2.0.13),

$$\Pr(X' = k) = \begin{cases} {}^3C_k \left(\frac{1}{2}\right)^k \left(1 - \frac{1}{2}\right)^{3-k} & 0 \le k \le 3 \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr(X' = k) = \begin{cases} {}^3C_k \left(\frac{1}{2}\right)^3 & \text{if } 0 \le k \le 3 \\ 0 & \text{otherwise} \end{cases} \qquad (2.0.15)$$

(a) probability of getting 0 tails,

$$\Pr(X' = 0) = {}^3C_0 \left(\frac{1}{2}\right)^3$$

$$= 1.\frac{1}{8} \qquad (2.0.16)$$

$$= \frac{1}{8}$$

(b) probability of getting 1 tails,

$$\Pr(X' = 1) = {}^3C_1 \left(\frac{1}{2}\right)^3$$

$$= 3.\frac{1}{8} \qquad (2.0.17)$$

$$= \frac{3}{8}$$

(c) probability of getting 2 tails,

$$\Pr(X' = 2) = {}^3C_2 \left(\frac{1}{2}\right)^3$$

$$= 3.\frac{1}{8} \qquad (2.0.18)$$

$$= \frac{3}{8}$$

(d) probability of getting 3 tails,

$$\Pr(X' = 3) = {}^3C_3 \left(\frac{1}{2}\right)^3$$

$$= 1.\frac{1}{8} \qquad (2.0.19)$$

$$= \frac{1}{8}$$

(iii) Here we are seeing outcome of four tosses of a coin. We can say it as simultaneous tossing four coins. So, we get,

$$n = 4 \qquad (2.0.20)$$

This is the **special case** with equation (2.0.6) with $n = 4$.
Now we obtain probability distribution of number of heads of two coins from (2.0.6),
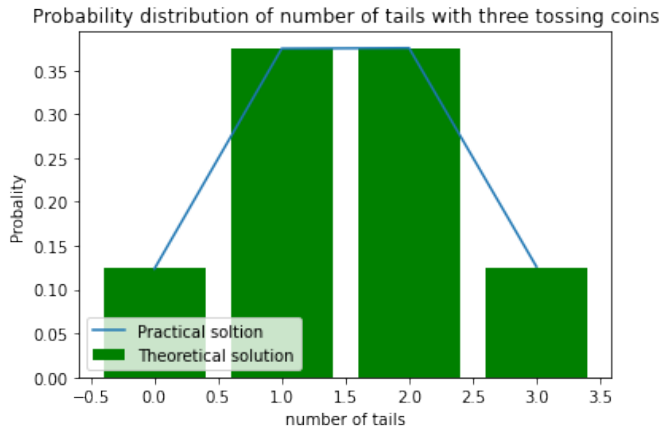
Fig. ii. Plot of probability distribution of no of tails with three tossed coins

$$\Pr(X = k) = \begin{cases} {}^4C_k \left(\frac{1}{2}\right)^k \left(1 - \frac{1}{2}\right)^{4-k} & 0 \leq k \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr(X = k) = \begin{cases} {}^4C_k \left(\frac{1}{2}\right)^4 & \text{if } 0 \leq k \leq 4 \\ 0 & \text{otherwise} \end{cases}$$ (2.0.21)

(a) probability of getting 0 heads,

$$\Pr(X = 0) = {}^4C_0 \left(\frac{1}{2}\right)^4$$

$$= 1.\frac{1}{16}$$ (2.0.22)

$$= \frac{1}{16}$$

(b) probability of getting 1 heads,

$$\Pr(X = 1) = {}^4C_1 \left(\frac{1}{2}\right)^4$$

$$= 4.\frac{1}{16}$$ (2.0.23)

$$= \frac{1}{4}$$

(c) probability of getting 2 heads,

$$\Pr(X = 2) = {}^4C_2 \left(\frac{1}{2}\right)^4$$

$$= 6.\frac{1}{16}$$ (2.0.24)

$$= \frac{3}{8}$$

(d) probability of getting 3 heads,

$$\Pr(X = 3) = {}^4C_3 \left(\frac{1}{2}\right)^4$$

$$= 4.\frac{1}{16}$$ (2.0.25)

$$= \frac{1}{4}$$

(e) probability of getting 4 heads,

$$\Pr(X = 4) = {}^4C_4 \left(\frac{1}{2}\right)^4$$

$$= 1.\frac{1}{16}$$ (2.0.26)

$$= \frac{1}{16}$$
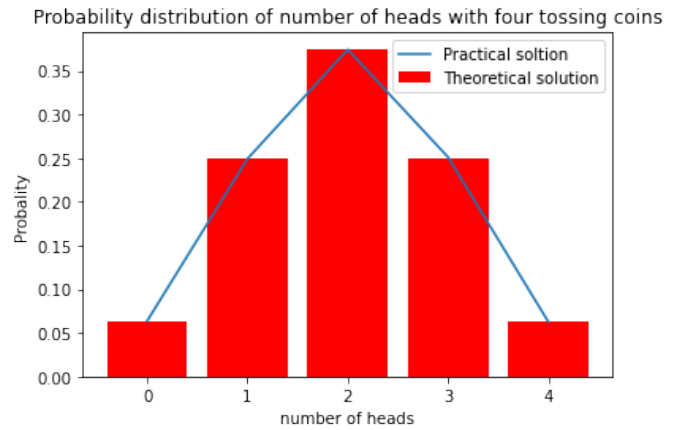


Fig. iii. Plot of probability distribution of four tossed coins

| n (no of coins) | k (no of required outcome) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| 2 | Pr(X=k) | 1/4 | 1/2 | 1/4 | 0 | 0 |
| 3 | Pr(X'=k) | 1/8 | 3/8 | 3/8 | 1/8 | 0 |
| 4 | Pr(X=k) | 1/16 | 1/4 | 3/8 | 1/4 | 1/16 |

Fig. 3. Table of probability distribution tossed coins