Assignment 1

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Download all python codes from

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https://github.com/PRABHATH-cs20-11038/
    Assignment_1/tree/main/Assignment_1/Codes
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and latex-tikz codes from

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https://github.com/PRABHATH-cs20-11038/
    Assignment_1/blob/main/Assignment_1/
    Assignment_1.tex
```

1 Problem

(Prob 3.6) Find the probability distribution of
(i) number of heads in two tosses of a coin.
(ii) number of tails in the simultaneous tosses of three coins.
(iii) number of heads in four tosses of a coin.

2 Solution

Let $X_i \in \{0, 1\}$ represent the i^{Th} coin where 1 denotes the coin giving outcome as head. Then, X_i has a Bernoulli distribution with parameter

$$p = \frac{1}{2} \tag{2.0.1}$$

Let

$$X = \sum_{i=1}^{n} X_i \tag{2.0.2}$$

where n is the total number of coins tossed. Then X has a Binomial Distribution. Then for

$$p_{X_i}(n) \overset{Z}{\rightleftharpoons} P_{X_i}(z) \tag{2.0.3}$$

yielding

$$P_{X_i}(z) = 1 - p + pz^{-1} \tag{2.0.4}$$

with using the fact that X_i are i.i.d.,

$$\begin{aligned} P_X(z) &= (1 - p + pz^{-1})^n \\ &= \sum_{k=0}^{n} {}^nC_k p^k (1-p)^{n-k} z^{-k} \end{aligned} \tag{2.0.5}$$

$$p_X(k) = \begin{cases} {}^nC_k p^k (1-p)^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \tag{2.0.6}$$

2.1 (i)

Here we are seeing outcome of two tosses of a coin. We can say it as simultaneous tossing two coins. So, we get,

$$n = 2 \tag{2.1.1}$$

Now we obtain probability distribution of number of heads of two coins from (2.0.6),

$$p_X(k) = \begin{cases} {}^2C_k (\frac{1}{2})^k (1 - \frac{1}{2})^{2-k} & 0 \le k \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$p_X(k) = \begin{cases} {}^2C_k (\frac{1}{2})^2 & \text{if } 0 \le k \le 2 \\ 0 & \text{otherwise} \end{cases} \tag{2.1.2}$$

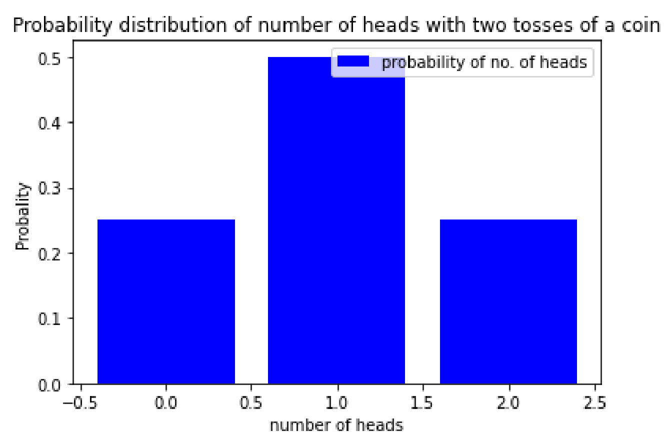

Fig. 0: Plot of probability distribution of two tossed coins

2.2 (ii)

With respect to tails the parameter would change to

$$q = 1 - p = \frac{1}{2} \tag{2.2.1}$$

and there will be a small change in binomial distribution (2.0.6) and we get binomial distribution for tails as,

$$p_{1-X}(k) = \begin{cases} {}^nC_k q^k (1-q)^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \tag{2.2.2}$$

Here we are seeing outcome of simultaneous tosses of three coin. So, we get,

$$n = 3 \tag{2.2.3}$$

Now we obtain probability distribution of number of tails of three coins from (2.2.2),

$$p_{1-X}(k) = \begin{cases} {}^3C_k (\frac{1}{2})^k (1-\frac{1}{2})^{3-k} & 0 \le k \le 3 \\ 0 & \text{otherwise} \end{cases}$$

$$p_{1-X}(k) = \begin{cases} {}^3C_k (\frac{1}{2})^3 & \text{if } 0 \le k \le 3 \\ 0 & \text{otherwise} \end{cases} \tag{2.2.4}$$

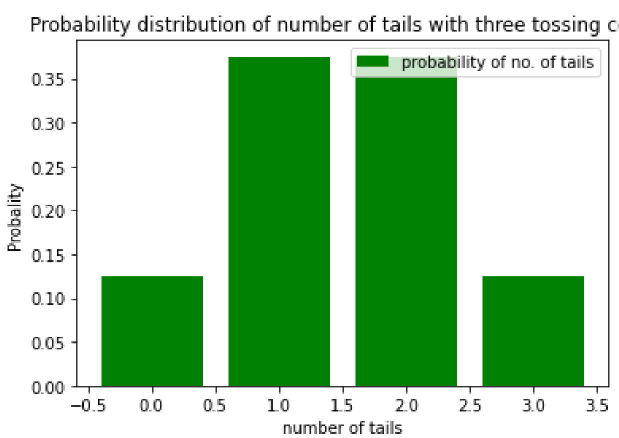

Fig. 0: Plot of probability distribution of no of tails with three tossed coins

2.3 (iii)

Here we are seeing outcome of four tosses of a coin. We can say it as simultaneous tossing four coins. So, we get,

$$n = 4 \tag{2.3.1}$$

Now we obtain probability distribution of number of heads of two coins from (2.0.6),

$$p_X(k) = \begin{cases} {}^4C_k (\frac{1}{2})^k (1-\frac{1}{2})^{4-k} & 0 \le k \le 4 \\ 0 & \text{otherwise} \end{cases}$$

$$p_X(k) = \begin{cases} {}^4C_k (\frac{1}{2})^4 & \text{if } 0 \le k \le 4 \\ 0 & \text{otherwise} \end{cases} \tag{2.3.2}$$

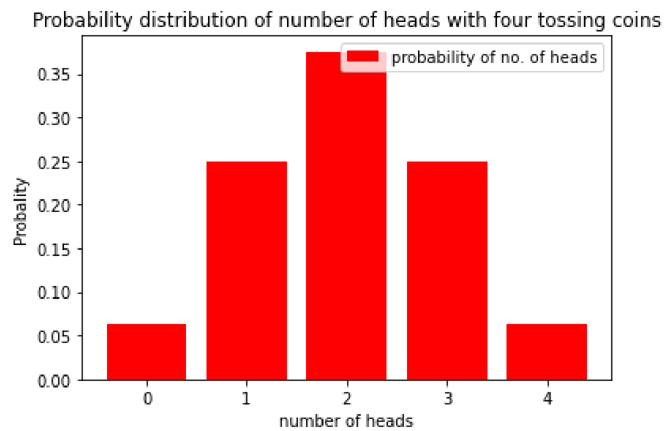

Fig. 0: Plot of probability distribution of four tossed coins