Assignment 3

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1 Problem

$(GATE(MA)2011 − 49Q))$ Let X and Y be two continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} 2, & 0 < x + y < 1, x > 0, y > 0, \\ 0, & elsewhere. \end{cases} \quad (1.0.1)$$

$E\left(X|Y = \frac{1}{2}\right)$ is
(A) 1/4
(B) 1/2
(C) 1
(D) 2

2 Solution

The PDF of X and Y is,

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy \quad (2.0.1)$$

$$f_X(x) = \int_{0}^{1-x} 2dy \quad (2.0.2)$$

$$f_X(x) = 2 - 2x \quad (2.0.3)$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx \quad (2.0.4)$$

$$f_Y(y) = \int_{0}^{1-y} 2dx \quad (2.0.5)$$

$$f_Y(y) = 2 - 2y \quad (2.0.6)$$

$$f_Y\left(\frac{1}{2}\right) = 1 \quad (2.0.7)$$

by using Bayes theorem,

$$f\left(X|Y = \frac{1}{2}\right) = \frac{f\left(x, \frac{1}{2}\right)}{f_Y\left(\frac{1}{2}\right)} \quad (2.0.8)$$

$$f\left(X|Y = \frac{1}{2}\right) = \begin{cases} 2, & 0 < x < \frac{1}{2}, \\ 0, & elsewhere. \end{cases} \quad (2.0.9)$$
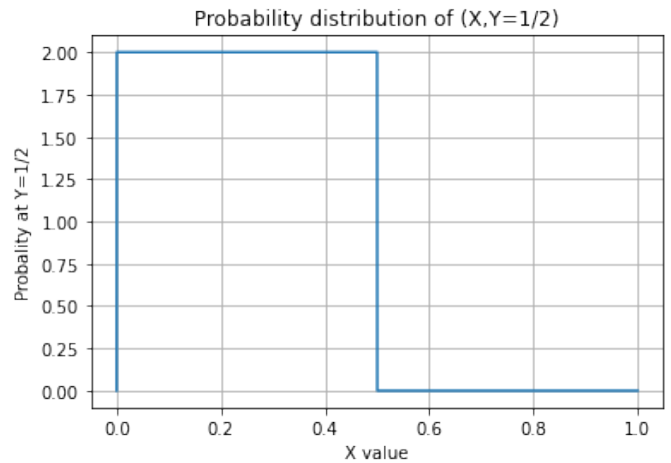

Fig. 4. Plot of of probability function

It is in the form of Bernoulli distribution, the expectation value is given by,

$$E\left(X|Y = \frac{1}{2}\right) = \sum_{-\infty}^{\infty} kf\left(X = k|Y = \frac{1}{2}\right) \quad (2.0.10)$$

$$E\left(X|Y = \frac{1}{2}\right) = \int_{-\infty}^{0} x(0)dx + \int_{0}^{\frac{1}{2}} x(2)dx + \int_{0}^{-\infty} x(0)dx \quad (2.0.11)$$

$$E\left(X|Y=\frac{1}{2}\right) = \int_0^{\frac{1}{2}} 2x\,dx \qquad (2.0.12)$$

$$E\left(X|Y=\frac{1}{2}\right) = \frac{1}{4} \qquad (2.0.13)$$

Option (A) is correct.

Expected Value−
   simulated: 0.25008392175344957,
   actual: 0.25