Assignment 2

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1 Problem

$(GATE(MA)2011-49Q)$) Let X and Y be two continuous random variables with the joint probability density function

$$f(x,y) = \begin{cases} 2, & 0 < x+y < 1, x > 0, y > 0, \\ 0, & elsewhere. \end{cases} \tag{1.0.1}$$

$E\left(X|Y=\frac{1}{2}\right)$ is

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 1

(D) 2

2 Solution

We are checking at $y = \frac{1}{2}$,

$$\Pr\left(Y = \frac{1}{2}\right) = 1 \tag{2.0.1}$$

when $y = \frac{1}{2}$,

$$\Pr\left((X) \cap \left(Y = \frac{1}{2}\right)\right) = f\left(x, \frac{1}{2}\right) = \begin{cases} 2, & 0 < x < \frac{1}{2}, \\ 0, & elsewhere. \end{cases} \tag{2.0.2}$$

by using Bayes theorem,

$$\Pr\left(X|Y = \frac{1}{2}\right) = \frac{\Pr\left((X) \cap \left(Y = \frac{1}{2}\right)\right)}{\Pr\left(Y = \frac{1}{2}\right)} \tag{2.0.3}$$

$$\Pr\left(X|Y = \frac{1}{2}\right) = \begin{cases} 2, & 0 < x < \frac{1}{2}, \\ 0, & elsewhere. \end{cases} \tag{2.0.4}$$

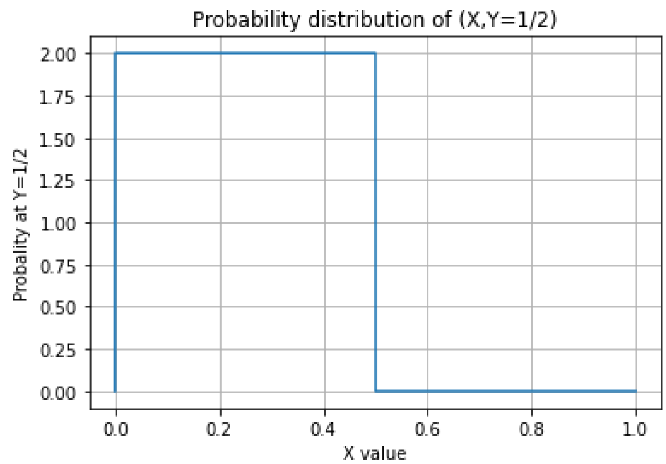

Fig. 4. Plot of of probability function

It is in the form of Bernoulli distribution, the expectation value is given by,

$$E\left(X|Y = \frac{1}{2}\right) = \sum_{-\infty}^{\infty} k \Pr\left(X = k|Y = \frac{1}{2}\right) \tag{2.0.5}$$

$$E\left(X|Y = \frac{1}{2}\right) = \int_{0}^{\frac{1}{2}} x(2)dx + \int_{-\infty}^{0} x(0)dx + \int_{0}^{-\infty} x(0)dx \tag{2.0.6}$$

$$E\left(X|Y = \frac{1}{2}\right) = \int_{0}^{\frac{1}{2}} 2xdx \tag{2.0.7}$$

$$E\left(X|Y = \frac{1}{2}\right) = \frac{1}{4} \tag{2.0.8}$$

Option (A) is correct.