Assignment 8

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Download all python codes from

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https://github.com/PRABHATH−cs20−11038/
    AI1103/tree/main/Assignment_8/Codes
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and latex-tikz codes from

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https://github.com/PRABHATH−cs20−11038/
    AI1103/tree/main/Assignment_8
```

1 Problem

$(CSIR-UGC-NET\_EXAM(June-2013), Q.60)$ Consider the quadratic equation $x^2+2Ux+V=0$ where U and V are chosen independently and randomly from $\{1,2,3\}$ with equal probabilities. Then probability that the equation has both roots real

1) $\frac{2}{3}$
2) $\frac{1}{2}$
3) $\frac{7}{9}$
4) $\frac{1}{3}$

2 Solution

Let $U \in \{1.2,3\}$ and $V \in \{1,2,3\}$

TABLE 4
Probability of selecting values for U

k	1	2	3
$\Pr(U=k)$	$1/3$	$1/3$	$1/3$

TABLE 4
Probability of selecting values for V

k	1	2	3
$\Pr(V=k)$	$1/3$	$1/3$	$1/3$

For $x^2+2Ux+V=0$ to have real roots,

$$b^2-4ac \geq 0 \tag{2.0.1}$$

$$(2U)^2-4(1)(V) \geq 0 \tag{2.0.2}$$

$$U^2 \geq V \tag{2.0.3}$$

The possible pairs of (U,V) for having real roots are

$$(U,V)=\{(1,1),(2,1),(2,2),(2,3),\\ (3,1),(3,2),(3,3)\} \tag{2.0.4}$$

Let $\Pr(T)$ be total probability,

$$\Pr(T)=\Pr((U=1)(V=1)) \\ +\Pr((U=2)(V=1))+\Pr((U=2)(V=2)) \\ +\Pr((U=2)(V=3))+\Pr((U=3)(V=1)) \\ +\Pr((U=3)(V=2))+\Pr((U=3)(V=3)) \tag{2.0.5}$$

as U and V are independent variables,

$$\Pr((U)(V))=\Pr(U).\Pr(V) \tag{2.0.6}$$

$$\Pr(T)=\Pr(U=1).\Pr(V=1) \\ +\Pr(U=2).\Pr(V=1)+\Pr(U=2).\Pr(V=2) \\ +\Pr(U=2).\Pr(V=3)+\Pr(U=3).\Pr(V=1) \\ +\Pr(U=3).\Pr(V=2)+\Pr(U=3).\Pr(V=3) \tag{2.0.7}$$

$$\Pr(T)=7\times\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) \tag{2.0.8}$$

$$\Pr(T)=\frac{7}{9} \tag{2.0.9}$$

Hence, Option 3 is correct.

Probability −
actual: 0.7778
simulated:0.7769