Assignment 5

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Download all python codes from

https://github.com/PRABHATH−cs20−11038/AI1103/tree/main/Assignment_5/Codes

and latex-tikz codes from

https://github.com/PRABHATH−cs20−11038/AI1103/tree/main/Assignment_5

1 Problem

$(GATE(EC)2013-26Q)$ Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $\Pr(3V \geq 2U)$ is

(A) 4/9
(B) 1/2
(C) 2/3
(D) 5/9

2 Solution

U and V are independent random variables,

$$V \sim N\left(0, \frac{1}{9}\right) \tag{2.0.1}$$

$$U \sim N\left(0, \frac{1}{4}\right) \tag{2.0.2}$$

Let,

$$Z = 3V - 2U \tag{2.0.3}$$

$$Z \sim N\left(0, 9 \times \frac{1}{9} + 4 \times \frac{1}{4}\right) \tag{2.0.4}$$

$$Z \sim N(0, 2) \tag{2.0.5}$$

For Z, $\mu = 0$, and $\sigma^2 = 2$.
By Gaussian Distribution,
PDF of Z,

$$f_Z(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \tag{2.0.6}$$

$$f_Z(z) = \frac{1}{2\sqrt{\pi}} e^{-\frac{z^2}{4}} \tag{2.0.7}$$

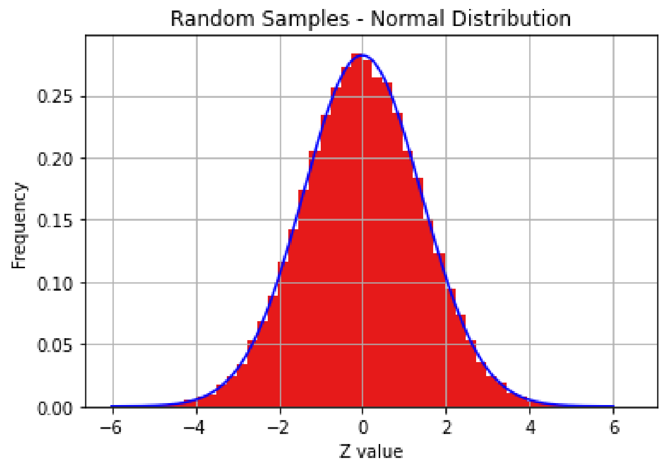

Fig. 4. Plot of of distribution function

$$\Pr(Z \geq 0) = \int_0^\infty f_Z(z)dz \tag{2.0.8}$$

$$\Pr(Z \geq 0) = \frac{1}{2}\int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi}} e^{-\frac{z^2}{4}} dz \tag{2.0.9}$$

$$\Pr(Z \geq 0) = \frac{1}{2}\frac{1}{2\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-\frac{z^2}{4}} dz \tag{2.0.10}$$

$$\Pr(Z \geq 0) = \frac{1}{2}\frac{1}{2\sqrt{\pi}}\left(\sqrt{4\pi}\right) \tag{2.0.11}$$

$$\Pr(Z \geq 0) = \frac{1}{2} \tag{2.0.12}$$

$$\Pr(3V - 2U \geq 0) = \frac{1}{2} \tag{2.0.13}$$

$$\Pr(3V \geq 2U) = \frac{1}{2} \tag{2.0.14}$$

Option (B) is correct.

Probability−
simulated: 0.49873
actual: 0.5