Logic Programming Languages

COEN 171
Design and Implementation of Programming Languages

Topics

- Introduction
- A Brief Introduction to Predicate Calculus
- Predicate Calculus and Proving Theorems
- An Overview of Logic Programming
- The Origins of Prolog
- The Basic Elements of Prolog
- Deficiencies of Prolog
- Applications of Logic Programming

Introduction

- Programs in logic languages are expressed in a form of symbolic logic
- Use a logical inferencing process to produce results
- Declarative rather that procedural
  - Only specification of results are stated (not detailed procedures for producing them)

Predicate Calculus

- Proposition – A logical statement that may or may not be true
  - Consists of objects and relationships of objects to each other
- Symbolic logic can be used for the basic needs of formal logic
  - Express propositions
  - Express relationships between propositions
  - Describe how new propositions can be inferred from other propositions
- Particular form of symbolic logic used for logic programming called predicate calculus

Predicate Calculus
Propositions

- Objects in propositions are represented by simple terms
  - Constant – a symbol that represents an object
  - Variable – a symbol that can represent different objects at different times
- Atomic (simplest) propositions consist of compound terms
  - Compound term – one element of a mathematical relation and written like a mathematical function
    - Functor – function symbol that names the relationship
    - Ordered list of parameters (tuple)
  - Example: student(john), like(seth, linux), like(nick, windows)

Predicate Calculus
Propositions

- Propositions can be stated in two forms
  - Fact – proposition is assumed to be true
  - Query – truth of proposition is to be determined
- Compound proposition
  - Two or more atomic propositions connected by operators

Name	Symbol	Example	Meaning
negation	$\neg$	$\neg$ a	not a
conjunction	$\cap$	a $\cap$ b	a and b
disjunction	$\cup$	a $\cup$ b	a or b
equivalence	$\equiv$	a $\equiv$ b	a is equivalent to b
implication	$\supset$	a $\supset$ b	a implies b
	$\subset$	a $\subset$ b	b implies a

Predicate Calculus
Propositions

- Quantifier enables variables to be part of a proposition

Name	Example	Meaning
universal	$\forall X.P$	For all X, P is true
existential	$\exists X.P$	There exists a value of X such that P is true

- Example
  - $\forall x.(\text{Student}(x) \cap \text{Class}(\text{CS171}, x) \supset \text{Major}(\text{COEN}, x))$
  - $\exists y.(\text{Student}(y) \cap \text{Name}(\text{Mike}, y) \cap \text{Major}(\text{COEN}, y))$

Predicate Calculus
Clausal Form

- Too many ways to state the same thing
- Use a standard form for propositions
  - $B_1 \cup B_2 \cup \ldots \cup B_n \subset A_1 \cap A_2 \cap \ldots \cap A_m$
    - consequent $\subset$ antecedent
- Example
  - likes(jack,cheddar) $\subset$ likes(jack, cheese) $\cap$ cheese(cheddar)

Predicate Calculus
Resolution

- A use of propositions is to discover new theorems that can be inferred from known axioms and theorems
- Resolution – an inference principle that allows inferred propositions to be computed from given propositions
  - Example

$A \subset X \cap Y$
$B \subset A \cap Z$

$B \subset X \cap Y \cap Z$
$\cancel{A} \cup B \subset X \cap Y \cap \cancel{A} \cap Z$

$A \subset X \cap Y$
$B \subset M$
$C \cup D \subset A \cap N$

$B \cup C \cup D \subset X \cap Y \cap M \cap N$

Predicate Calculus
Resolution

- Unification – finding values for variables in propositions that allows matching process to succeed
- Instantiation – assigning temporary values to variables to allow unification to succeed
- After instantiating a variable with a value, if matching fails, may need to backtrack and instantiate with a different value
- Theorem is proved by finding an inconsistency
  - Hypotheses: a set of pertinent propositions
  - Goal: negation of theorem stated as a proposition

Predicate Calculus
Theorem Proving

- Basis for logic programming
- When propositions used for resolution, only restricted form can be used
- Horn clause - can have only two forms
  - Headed: single atomic proposition on left side
  - Headless: empty left side (used to state facts)
- Most propositions can be stated as Horn clauses

Overview of Logic Programming

- Declarative semantics
  - There is a simple way to determine the meaning of each statement
  - Simpler than the semantics of imperative languages
- Programming is nonprocedural
  - Programs do not state now a result is to be computed, but rather the form of the result
  - Example – describe the characteristics of a sorted list, not the process of rearranging a list

sort(A, B) $\subset$ permute(A, B) $\cap$ sorted(B)
sorted(list) $\subset$ $\forall$j such that $1 \le j < n$, list(j) $\le$ list(j+1)

The Origins of Prolog

- University of Aix-Marseille (Calmerauer & Roussel)
  - Natural language processing
- University of Edinburgh (Kowalski)
  - Automated theorem proving

Edinburgh syntax

- Term – a constant, variable or structure
- Constant – an atom or an integer
  - Atom – symbolic value of Prolog
    - A string of letters, digits and underscores beginning with a lowercase letter
    - A string of printable ASCII characters delimited by apostrophes
- Variable – any string of letters, digits and underscores beginning with an uppercase letter or an underscore
  - Instantiation – binding of a variable to a value
    - Lasts only as long as it takes to satisfy one complete goal
- Structure – represents atomic proposition
  - functor(parameter list)

Fact Statements

- Used for the hypotheses
- Headless Horn clauses

```
female(shelley).
male(bill).
father(bill, jake).
```


Rule Statements

- Used for the hypotheses
- Headed Horn clause
  - Right side: antecedent (if part)
    - May be single term or conjunction
  - Left side: consequent (then part)
    - Must be single term
- Conjunction: multiple terms separated by logical AND operations (implied)

```
ancestor(mary,shelley):- mother(mary,shelley).
```

```
parent(X,Y):- mother(X,Y).
parent(X,Y):- father(X,Y).
grandparent(X,Z):- parent(X,Y), parent(Y,Z).
```


Goal Statements

- For theorem proving, theorem is in form of proposition that we want system to prove or disprove – goal statement
- Same format as headless Horn

```
man(fred).
```

- Conjunctive propositions and propositions with variables also legal goals

```
father(X, mike).
```


Inferencing Process of Prolog

- Queries are called goals
- If a goal is a compound proposition, each of the facts is a subgoal
- To prove a goal is true, must find a chain of inference rules and/or facts. For goal Q:

```
P2 :- P1
P3 :- P2
...
Q :- Pn
```

- Process of proving a subgoal called matching, satisfying or resolution

Approaches

- Matching is the process of proving a proposition
- Bottom-up resolution, forward chaining
  - Begin with facts and rules of database and attempt to find sequence that leads to goal
  - Works well with a large set of possibly correct answers
- Top-down resolution, backward chaining
  - Begin with goal and attempt to find sequence that leads to set of facts in database
  - Works well with a small set of possibly correct answers
- Prolog implementations use backward chaining

Subgoal Strategies

- When goal has more than one subgoal, can use either
  - Depth-first search: find a complete proof for the first subgoal before working on others
  - Breadth-first search: work on all subgoals in parallel
- Prolog uses depth-first search
  - Can be done with fewer computer resources

Backtracking

- With a goal with multiple subgoals, if fail to show truth of one of subgoals, reconsider previous subgoal to find an alternative solution (backtracking)
- Begin search where previous search left off
- Can take lots of time and space because may find all possible proofs to every subgoal

Simple Arithmetic

- Prolog supports integer variables and integer arithmetic
- `is` operator: takes an arithmetic expression as right operand and variable as left operand

```
A is B / 17 + C
```

- Not the same as an assignment statement!

```
Sum is Sum + Number.
```


Example

```
speed(ford,100).
speed(chevy,105).
speed(dodge,95).
speed(volvo,80).
time(ford,20).
time(chevy,21).
time(dodge,24).
time(volvo,24).
distance(X,Y) :- speed(X,Speed),
                 time(X,Time),
                 Y is Speed * Time.

distance(chevy, Chevy_Distance).
```

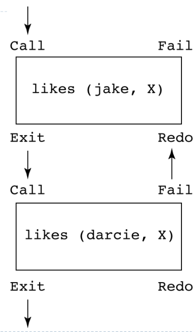
```
distance(chevy,?)
speed(chevy,?)
-> speed(chevy,105)
time(chevy,?)
-> time(chevy,21)
Chevy_Distance is 105*21
-> Chevy_Distance is 2205
```


Trace

- Built-in structure that displays instantiations at each step
- Tracing model of execution – four events:
  - Call (beginning of attempt to satisfy goal)
  - Exit (when a goal has been satisfied)
  - Redo (when backtrack occurs)
  - Fail (when goal fails)

Example

```
likes(jake,chocolate).
likes(jake, apricots).
likes(darcie, licorice).
likes(darcie, apricots).
trace.
likes(jake, X), likes(darcie, X).
(1) 1 Call: likes(jake, _0)?
(1) 1 Exit: likes(jake, chocolate)
(2) 1 Call: likes(darcie, chocolate)?
(2) 1 Fail: likes(darcie, chocolate)
(1) 1 Redo: likes(jake, _0)?
(1) 1 Exit: likes(jake, apricots)
(3) 1 Call: likes(darcie, apricots)?
(3) 1 Exit: likes(darcie, apricots)
X = apricots
```



List Structures

- List is a sequence of any number of elements
- Elements can be atoms, atomic propositions, or other terms (including other lists)
  - `[apple, prune, grape, kumquat]`
  - `[]` (empty list)
  - `[X | Y]` (head `X` and tail `Y`)

Example

- Append lists

```
append([], List, List).
append([Head | List_1], List_2, [Head | List_3]) :-
      append (List_1, List_2, List_3).
```

- Reverse list

```
reverse([], []).
reverse([Head | Tail], List) :-
      reverse (Tail, Result),
      append (Result, [Head], List).
```

- Member of a list

```
member(Element, [Element | _]).
member(Element, [_ | List]) :-
      member(Element, List).
```


Deficiencies of Prolog

- Resolution order control
  - In a pure logic programming environment, the order of attempted matches is nondeterministic and all matches would be attempted concurrently
- The closed-world assumption
  - The only knowledge is what is in the database
- The negation problem
  - Anything not stated in the database is assumed to be false
- Intrinsic limitations
  - It is easy to state a sort process in logic, but difficult to actually do—it doesn't know how to sort

Applications of Logic Programming

- Relational database management systems
- Expert systems
- Natural language processing

Summary

- Symbolic logic provides basis for logic programming
- Logic programs should be nonprocedural
- Prolog statements are facts, rules, or goals
- Resolution is the primary activity of a Prolog interpreter
- Although there are a number of drawbacks with the current state of logic programming it has been used in a number of areas

Assignments #10

- Reads chapter 16.1 – 16.8.
- Problem set: 16.5, 16.6 and 16.7.
- Programming exercises: 16.3, 16.4 and 16.5.
- Due date: December 2 at 23:59