MAHATMA GANDHI COLLEGE, IRITTY

PROJECT REPORT

OPTICAL SYSTEM USING MATRIX METHOD

SUBMITTED BY

ARJUN P R

MG20CPHR01

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ABSTRACT

To find the focal length and cardinal points of system of lenses using Matrix method.

Introduction

Optical systems are in general made up of a large number of refracting surfaces. In order to obtain the position of the final image due to an optical system, one has to calculate step by step the position of the image due to each surface and consider this image as an object for the next surface. Such a step by step analysis becomes lengthy and tedious. In order to solve such problems easily and more efficiently K.Hallback has introduced in 1964 matrix methods in the study of geometrical optics. The matrix method is less cumbersome and above that, it is more amenable to computer use. That is why we are coding a python program to construct system matrix of a single lens and multiple lens systems.

Aim

- To construct the system matrix and to find cardinal points of a single lens using a python code.
- To construct the system matrix and to find cardinal points of multiple lens systems using a python code.
- To define Ramsden’s eyepiece of given lens separations and to find its focal length and cardinal points.
- To define Huygen’s eyepiece of given lens separations and to find its focal length and cardinal points.
- To define basic telescope and to find focal length of the telescope.
- To define compound microscope and to find focal length of the microscope.

Theory

• Refraction and Translation

A ray of light propagating through a cylindrically symmetric optical system undergoes two operations. At eachsurface boundary the direction of the ray changes due to refraction. In between the surfaces, the height of the ray changes while the direction remains the same. This is known as translation. Therefore, to fully describe the ray propagation through an optical system, we make use of two operators, one for the refraction process and the other for the translation process. These two operators are known as refraction matrix and the translation matrix respectively.

• Translation matrix

Let us consider a paraxial ray traveling in a homogeneous medium of refractive index μ_1 which is at a distance of x, from the z-axis (axis of symmetry). Let A be a point on the ray. Let it be at a distance x_1 from the z-axis and be inclined at an angle α_1 with the z-axis. Thus, (x_1, α_1) are the coordinates of the ray at point A. Let (x_2, α_2) be the coordinates of the ray at B through which the ray passes at a later instant. In traveling from A to B the ray undergoes translation.

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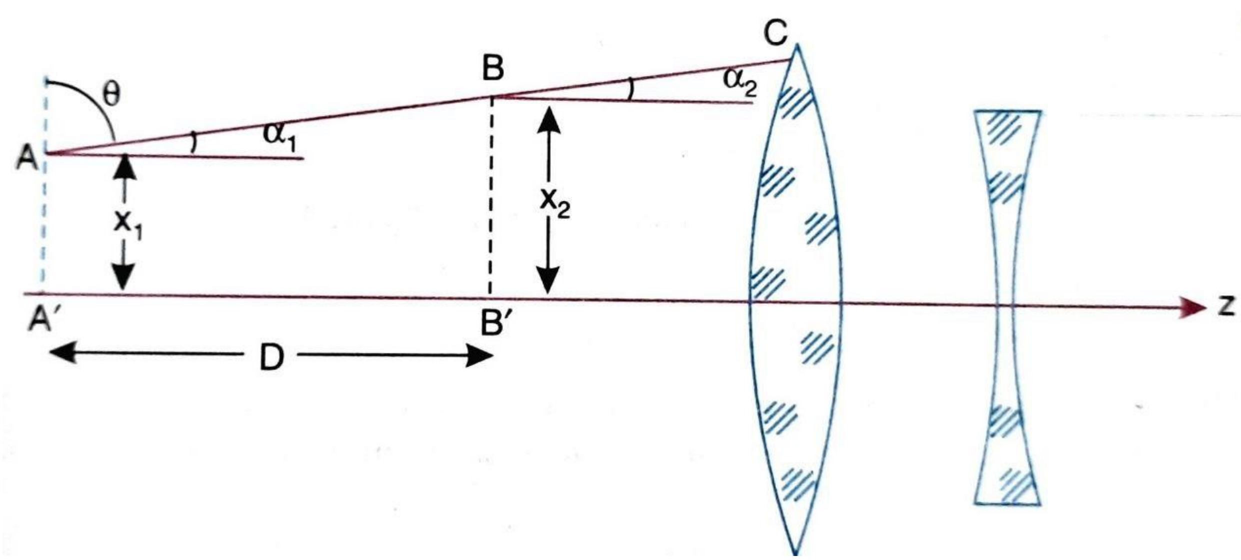


Translation matrix can be expressed as

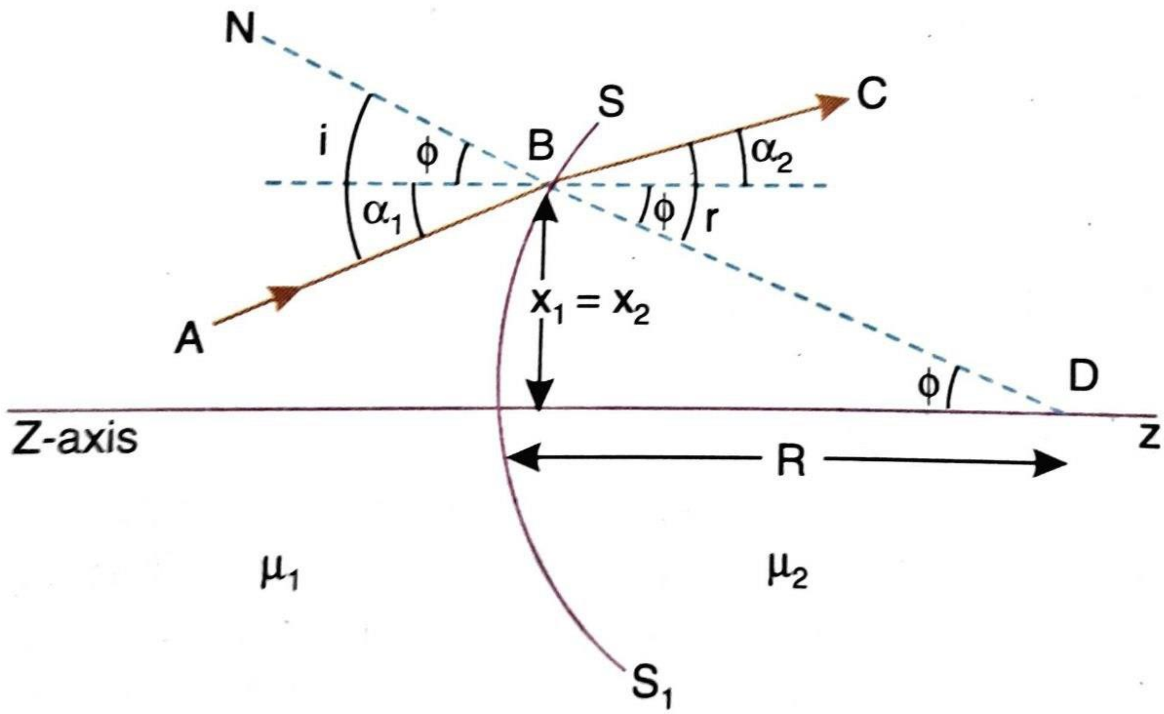
$$T = \begin{pmatrix} 1 & 0 \\ D/\mu_1 & 1 \end{pmatrix}$$

Where

D is the translation distance and μ_1 is the refractive index of material of lens.

• Refraction matrix

Let us consider a convex spherical surface of radius of curvature R separating two media of refracting index μ_1 and μ_2. Let a ray AB be incident on the surface SS_1 at a point B and be refracted along BC as shown in figure. If i and r be the angles of incidence and refraction with the normal to the surface.



Refraction matrix can be expressed as;

$$R = \begin{pmatrix} 1 & -P \\ 0 & 1 \end{pmatrix}$$

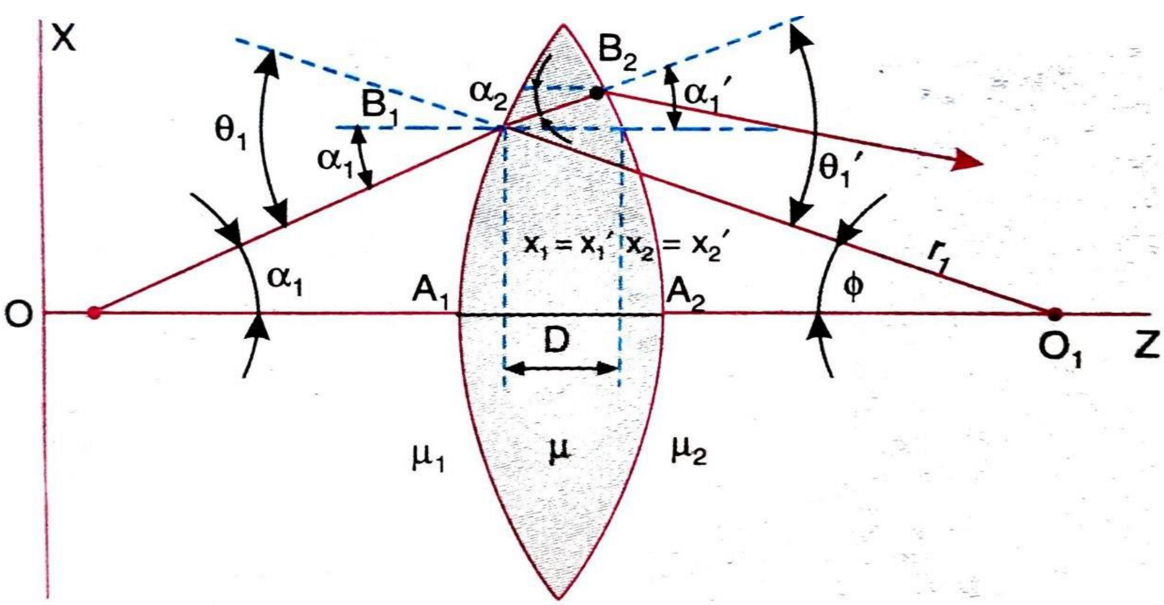
Where,

P= $(\mu_2-\mu_1)/R$, is known as power of the refractive surface.

R is the radius of curvature

- **System matrix**

Let us consider the example of a double convex lens as in figure. When a ray passes through a lens, refraction occurs twice, at each curved surface, and translation occurs once, between the two curved surfaces. Hence, we require two refraction matrices [R_1] and [R_2] and one translation matrix,[T_{21}]. Multiplication of these matrices, written from right to left, leads to the system matrix, [S].The system matrix indicates how a ray of light is affected in passing through the optical system



System matrix can be repressed as;

$$[S] = [R_2]\,[T_{21}]\,[R_1] = \begin{pmatrix} 1 & -P_2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ D/\mu_1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -P_1 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} b & -a \\ -d & c \end{pmatrix}$$

where

$$a = P_1 + P_2 - P_1 P_2 D / \mu_1$$
$$b = 1 - P_2 D / \mu_1$$
$$c = 1 - P_1 D / \mu_1$$
$$d = -D / \mu_1$$

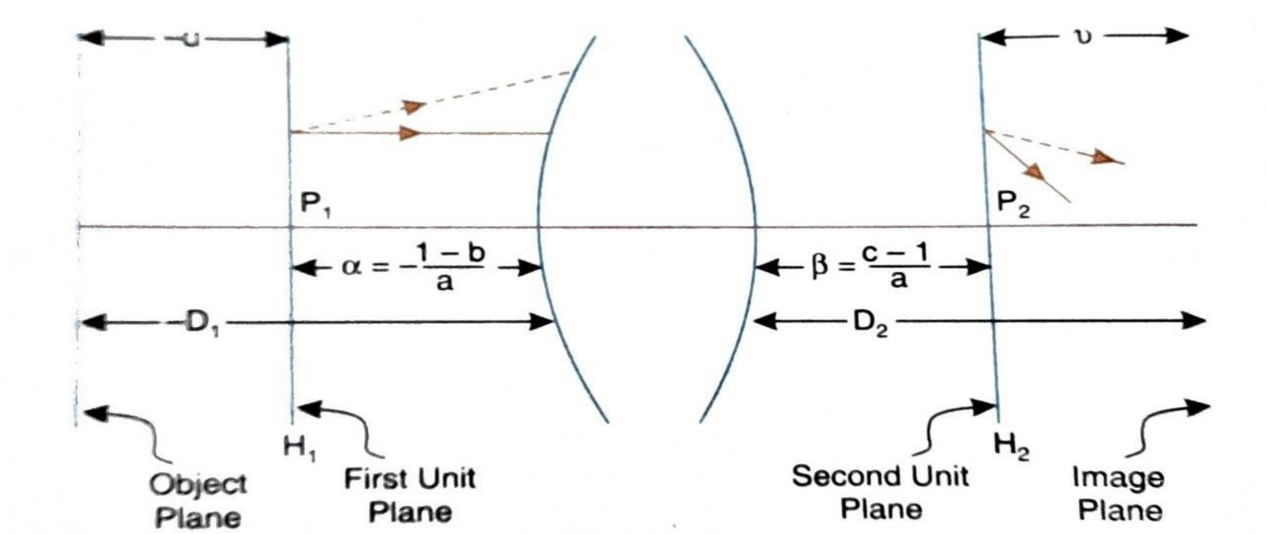
These four constants a, b, c and d are called Guass constants.

And the effective focal length of the system matrix is given by;

$$F=1/a$$

- **Cardinal points**
  i. **Two principal or unit points,**
  ii. **Two focal points, and**
  iii. **Two nodal points**

The position and size of the image of an image of an object placed in front of a coaxial system of lenses can be determined with the help of the cardinal points. The cardinal points and planes of an optical system are as shown in figure.



• Principal or Unit points

The principal or unit points (P_1 and P_2 in figure) are a pair of conjugate points on the principal axis for which linear transverse magnification is unity and positive. The planes passing through these points and perpendicular to the principal axis are called principal planes or unit planes. The unit planes are two conjugate planes, one of which lies in object plane and the other in image plane. If a ray strikes the first unit plane (in object side) at a certain height, it emerges out from the second unit plane (on the image side) at the same height. Let α and ß be the respective distances of the first and second unit planes from the refracting surfaces of a lens system as shown in figure. For these planes magnification is unity, i.e. m = 1. We have

$$(b + aD_1) = \frac{1}{c - aD_2} = \frac{1}{m}$$

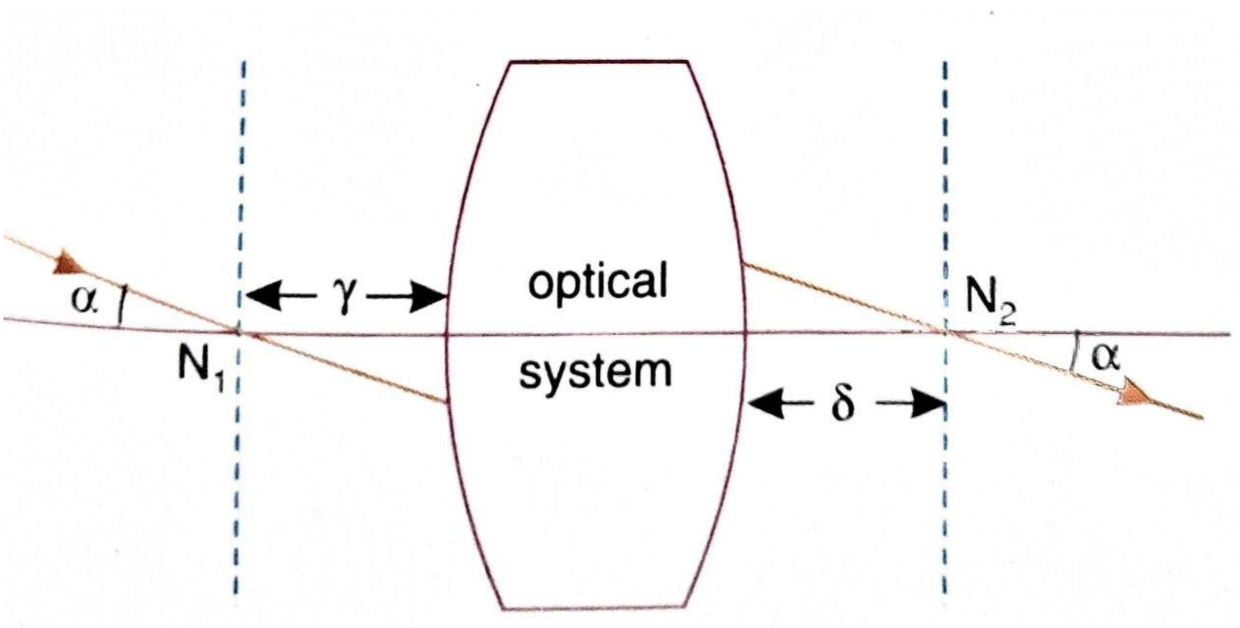
Using $D_1 = \alpha$ and $D_2 = \beta$ in the above equation, we get

$$(b + a\alpha) = 1 \qquad \text{or} \qquad \alpha = \frac{1-b}{a}$$

$$c - a\beta = 1 \qquad \text{or} \qquad \beta = \frac{c-1}{a}$$

- **Nodal planes**

Nodal points are a pair of conjugate points on the axis, which have a relative angular magnification of unity. According to the property of nodal points, a ray striking the first nodal point at an angle α emerges from the second nodal point at the same angle. The planes passing through these points and normal to the axis are known as nodal planes. In most of the cases nodal planes are situated at the focal point of the lens so we didn't include a code to find nodal point of the lens.



• System matrix for a two lens system

Let us now consider two thin lenses of focal lengths f_1 and f_2 placed coaxially and separated by a distance 't'.The system matrix can be written by combining the system matrices of individual lenses and the translation matrix for the distance of separation between them.

The system matrix for first lens,

$$L_1 = \begin{pmatrix} 1 & \frac{-1}{f_1} \\ 0 & 1 \end{pmatrix}$$

The system matrix for second lens,

$$L_2 = \begin{pmatrix} 1 & -\frac{1}{f_2} \\ 0 & 1 \end{pmatrix}$$

The translation matrix for distance 't' between the lenses,

$$T = \begin{pmatrix} 1 & 0 \\ t & 1 \end{pmatrix} \quad (\text{as } \mu = 1)$$

Now the total system matrix is given by,

$$\boldsymbol{S} = \begin{pmatrix} 1 & -\frac{1}{f_2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ t & 1 \end{pmatrix} \begin{pmatrix} 1 & -\frac{1}{f_1} \\ 0 & 1 \end{pmatrix}$$

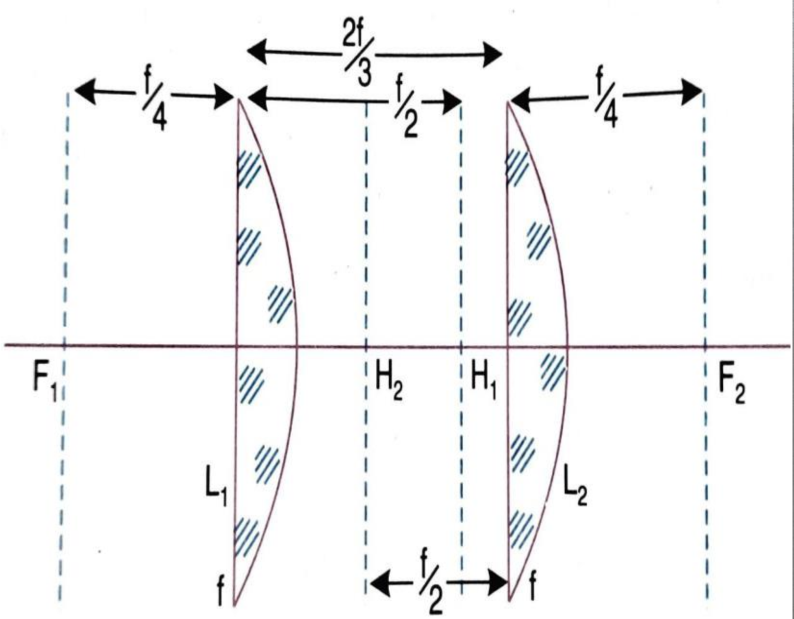
From this system matrix of combination of two lenses the effective focal length is found just as we did before.

F=1/a

Also the unit points is found just as we did earlier.

• Ramsden's Eyepiece

Ramsden's eyepiece is an eyepiece consisting of two plano-convex crown-glass lenses of equal focal length, placed with the convex sides facing each other and with a separation between the lenses of about two-thirds of the focal length of each.



The above figures represent Ramsden's eyepiece.

$$\text{The system matrix for the lens } L_1 = \begin{pmatrix} 1 & \frac{-1}{f} \\ 0 & 1 \end{pmatrix}$$

$$\text{The translation matrix from } L_1 \text{ to } L_2 = \begin{pmatrix} 1 & 0 \\ \frac{2f}{3} & 1 \end{pmatrix}$$

$$\text{The system matrix for the second lens} = \begin{pmatrix} 1 & \frac{-1}{f} \\ 0 & 1 \end{pmatrix}$$

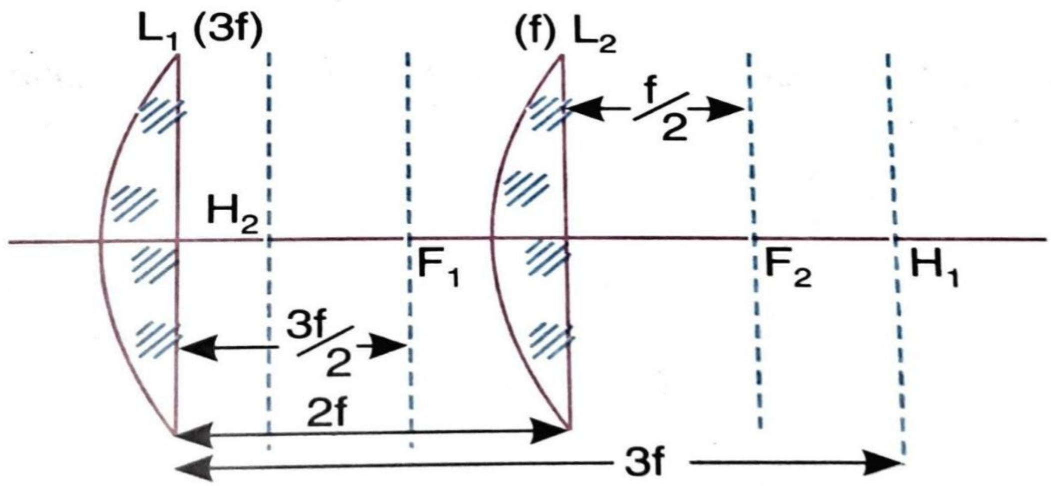
The system matrix of Ramsden's eye piece

$$S = \begin{pmatrix} 1 & \frac{-1}{f} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{2f}{3} & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{-1}{f} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{-4}{3f} \\ \frac{2f}{3} & \frac{1}{3} \end{pmatrix}$$

The focal length and Unit points of the combination of lenses can be found from the system matrix we obtained.

• Huygen's Eyepiece

Huygen's eyepiece is a telescope eyepiece consisting of two planoconvex lenses separated by a distance equal to half the sum of their focal lengths, which are in the ratio of three to one, and oriented so that their curved surfaces face the incident light.



The system matrix for the lens $L_1 = \begin{pmatrix} 1 & \frac{-1}{3f} \\ 0 & 1 \end{pmatrix}$

The translation matrix from L_1 to $L_2 = \begin{pmatrix} 1 & 0 \\ 2f & 1 \end{pmatrix}$

The system matrix for the second lens $= \begin{pmatrix} 1 & \frac{-1}{f} \\ 0 & 1 \end{pmatrix}$

The system matrix of Ramsden's eye piece

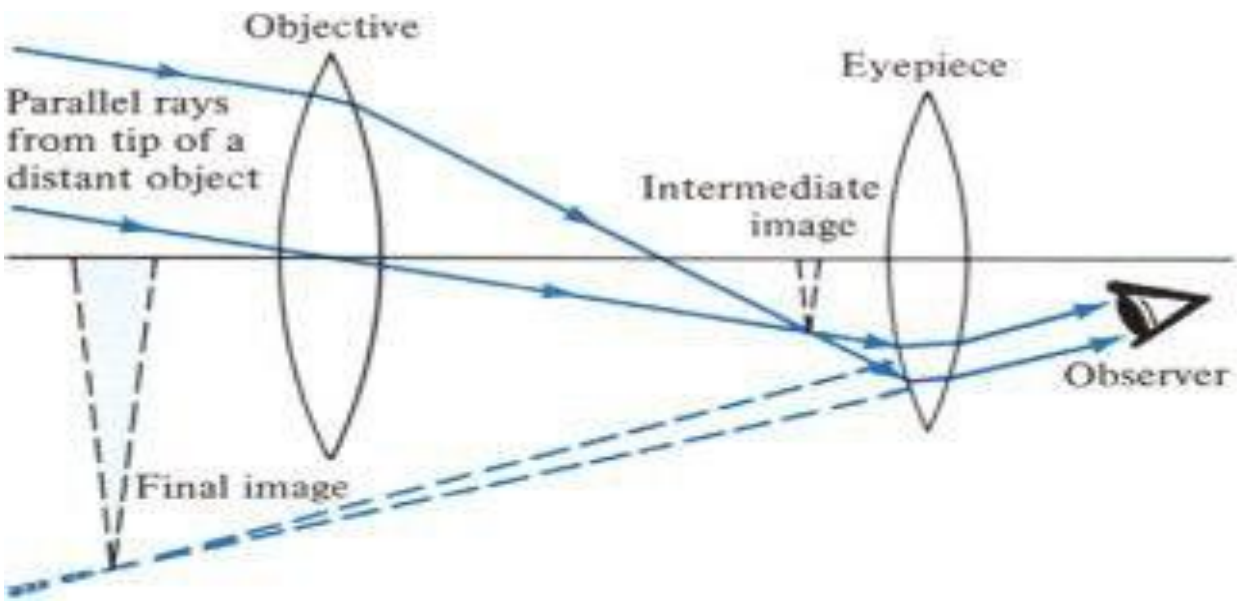
$$S = \begin{pmatrix} 1 & \frac{-1}{f} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2f & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{-1}{3f} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{-2}{3f} \\ 2f & \frac{1}{3} \end{pmatrix}$$

The focal length and Unit points of the combination of lenses can be found from the system matrix we obtained.

• Simple telescope

A simple telescope can be described as a pair of convex lenses mounted inside a hollow tube. The first lens through which we see is the objective and it helps to focus on an object or the image. The lens behind the first lens is the eyepiece and it magnifies the image.

Telescopes are an indispensable tool in the study of astronomy. It allows the user to magnify and observe distant celestial objects in the sky. The first-ever telescopes are believed to be invented in the Netherlands with a simple glass lens. Galileo is often credited for the discovery of the telescope. He created several designs of the telescope using different lenses.



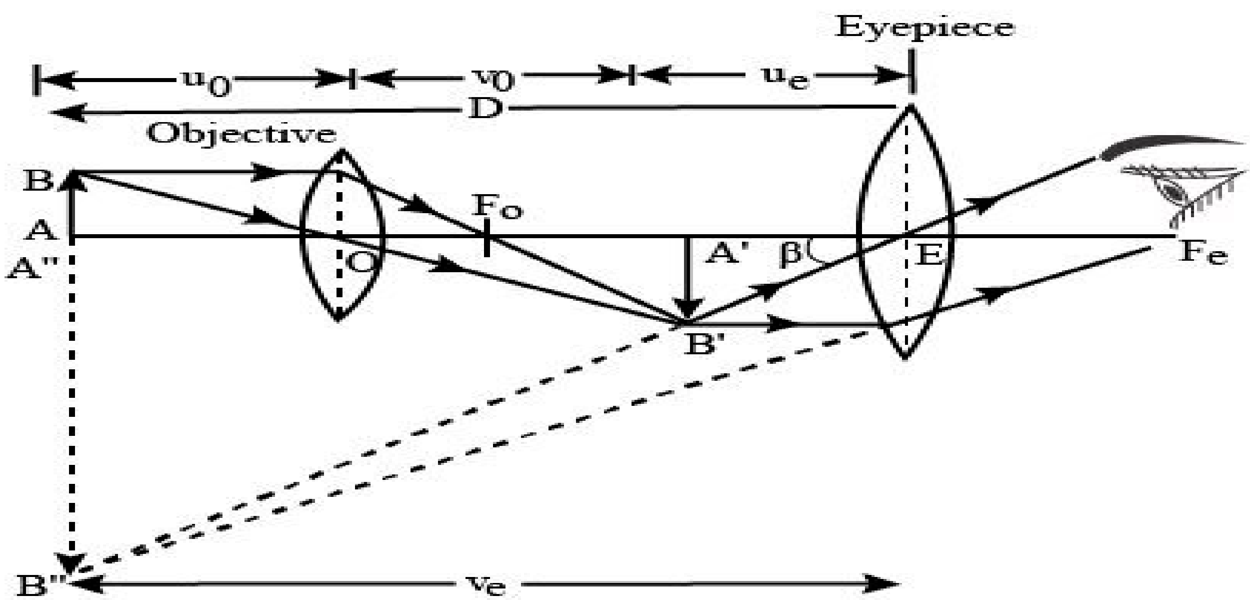
A simple ray diagram representing a telescope is given above.

By obtaining the system matrix of simple telescope system, its focal length is found.

- Compound telescope

A compound microscope is an optical instrument which uses two sets of lenses providing a high resolution and 2-dimensional image of the sample.

Typically, a compound microscope is used for viewing samples at high magnification (40 - 1000x), which is achieved by the combined effect of two sets of lenses: the ocular lens (in the eyepiece) and the objective lenses (close t o the sample).



Ray diagram of a two lens compound microscope is given above.

By obtaining the system matrix of compound microscope system, its focal length is found.

Python code for single lens and combination of lenses

```
importnumpy as np


def translation():
   u1 = float(input("Enter refractive index of first medium:"))
   D = float(input("Enter the distance:"))
   m2 = np.array([[1.0, 0.0], [D / u1, 1.0]])
print(f"Translation1:\n{m2}")


def assymmetric_system_function():
   u1 = float(input("Enter refractive index of first medium:"))
   u2 = float(input("Enter refractive index of second medium:"))
   R1 = float(input("Enter the radius of curvature of first
surface:"))
   R2 = float(input("Enter the radius of curvature second
surface:"))
   p1 = (u2 - u1) / R1
   p2 = (u2 - u1) / R2
   D = float(input("Enter the distance:"))
   m1 = np.array([[1.0, -p1], [0.0, 1.0]])
   m2 = np.array([[1.0, 0.0], [D / u1, 1.0]])
   m3 = np.array([[1.0, -p2], [0.0, 1.0]])
print("Matrices\n")
system = (m3).dot(m2).dot(m1)
```

```
print(f"Refraction1:\n{m1}")
print(f"Translation1:\n{m2}")
print(f"Refraction2:\n{m3}")
print(f"System matrix\n{system}")
    a = system[0][1]
    f = -1 / a
print("Focal length=", f)
    b = system[0][0]
    c = system[1][1]
    w = (1 - b) / a
    e = (c - 1) / a
print("Unit plane from first surface=", w)
print("Unit plane from second surface=", e)


def two_lens():
    system1=assymmetric_system_function()
    m2=translation()
    system2 = assymmetric_system_function()
lens = (system2).dot(m2)
two_lens = (lens).dot(system1)
print(f"System matrix for combination\n{two_lens}")
    a = two_lens[0][1]
    f = -1 / a
print("Focal length=", f)
    b = two_lens[0][0]
    c = two_lens[1][1]
    d = two_lens[1][0]
```

```
    w = (1 - b) / a
    e = (c - 1) / a
print("Unit plane from first surface=", w)
print("Unit plane from second surface=", e)

print("What would you like to do today?\n1. Define a system
matrix for a single lens.\n2. Define a system matrix for a
combination of two lens.")
q=int(input("Enter your choice:"))
if q == 1:
assymmetric_system_function()
elif q == 2:
two_lens()
else:
print("PLEASE ENTER THE RIGHT CHOICE!!!")
```

Python code for Ramsden's eyepiece and Huygen's eyepiece

```
importnumpy as np



def translation():
    u1 = float(input("Enter refractive index of first medium:"))
    D = float(input("Enter the distance:"))
    m2 = np.array([[1.0, 0.0], [D / u1, 1.0]])
print(f"Translation1:\n{m2}")



def assymmetric_system_function():
    u1 = float(input("Enter refractive index of first medium:"))
    u2 = float(input("Enter refractive index of second medium:"))
    R1 = float(input("Enter the radius of curvature of first surface:"))
    R2 = float(input("Enter the radius of curvature second surface:"))
    p1 = (u2 - u1) / R1
    p2 = (u2 - u1) / R2
    D = float(input("Enter the distance:"))
```

```
    m1 = np.array([[1.0, -p1], [0.0, 1.0]])
    m2 = np.array([[1.0, 0.0], [D / u1, 1.0]])
    m3 = np.array([[1.0, -p2], [0.0, 1.0]])
print("Matrices\n")
system = (m3).dot(m2).dot(m1)
print(f"Refraction1:\n{m1}")
print(f"Translation1:\n{m2}")
print(f"Refraction2:\n{m3}")
print(f"System matrix\n{system}")
    a = system[0][1]
    f = -1 / a
print("Focal length=", f)
    b = system[0][0]
    c = system[1][1]
    w = (1 - b) / a
    e = (c - 1) / a
print("Unit plane from first surface=", w)
print("Unit plane from second surface=", e)
```

```
def ramsden():
   o=float(input("Focal length of the lenses="))
   m4 = np.array([[1.0, -1/o], [0.0, 1.0]])
   m5 = np.array([[1.0, 0.0], [(2*o)/3, 1.0]])
   s=(m4).dot(m5).dot(m4)
print(f"System matrix of Ist lens:\n{m4}")
print(f"Translation:\n{m5}")
print(f"System matrix of IInd lens:\n{m4}")
print(f"System matrix of Ramsden's eyepiece\n{s}")
   a= s[0][1]
   f=-1/a
print("Focal length of combination=", f)
   b = s[0][0]
   c = s[1][1]
   w = (1 - b) / a
   e = (c - 1) / a
print("Unit plane from first surface=", w)
print("Unit plane from second surface=", e)


def huygens():
```

```
    o = float(input("Focal length of the lenses="))
    m4 = np.array([[1.0, -1/(3*o)], [0.0, 1.0]])
    m5 = np.array([[1.0, 0.0], [(2 * o), 1.0]])
    m6 = np.array([[1.0, -1 /o], [0.0, 1.0]])
    s = (m6).dot(m5).dot(m4)
print(f"System matrix of Ist lens:\n{m4}")
print(f"Translation:\n{m5}")
print(f"System matrix of IInd lens:\n{m6}")
print(f"System matrix of Huygen's eyepiece\n{s}")
    a = s[0][1]
    f = -1 / a
print("Focal length of combination=", f)
    b = s[0][0]
    c = s[1][1]
    w = (1 - b) / a
    e = (c - 1) / a
print("Unit plane from first surface=", w)
print("Unit plane from second surface=", e)
```

```
print("What would you like to do today?\n1.Ramsden's eyepiece.\n2.Huygen's eyepiece")

q=int(input("Enter your choice:"))

if q == 1:

ramsden()

elif q == 2:

huygens()

else:

print("PLEASE ENTER THE RIGHT CHOICE !!!")
```

Python code for telescope and microscope

```
importnumpy as np



def translation():
   u1 = float(input("Enter refractive index of first medium:"))
   D = float(input("Enter the distance:"))
   m2 = np.array([[1.0, 0.0], [D / u1, 1.0]])
print(f"Translation1:\n{m2}")



def assymmetric_system_function():
   u1 = float(input("Enter refractive index of first medium:"))
   u2 = float(input("Enter refractive index of second medium:"))
   R1 = float(input("Enter the radius of curvature of first surface:"))
   R2 = float(input("Enter the radius of curvature second surface:"))
   p1 = (u2 - u1) / R1
   p2 = (u2 - u1) / R2
   D = float(input("Enter the distance:"))
```

```
    m1 = np.array([[1.0, -p1], [0.0, 1.0]])
    m2 = np.array([[1.0, 0.0], [D / u1, 1.0]])
    m3 = np.array([[1.0, -p2], [0.0, 1.0]])
print("Matrices\n")
system = (m3).dot(m2).dot(m1)
print(f"Refraction1:\n{m1}")
print(f"Translation1:\n{m2}")
print(f"Refraction2:\n{m3}")
print(f"System matrix\n{system}")
    a = system[0][1]
    f = -1 / a
print("Focal length=", f)
    b = system[0][0]
    c = system[1][1]
    w = (1 - b) / a
    e = (c - 1) / a
print("Unit plane from first surface=", w)
print("Unit plane from second surface=", e)
```

```
def two_lens():
    system1=assymmetric_system_function()
    m2=translation()
    system2 = assymmetric_system_function()
lens = (system2).dot(m2)
two_lens = (lens).dot(system1)
print(f"System matrix for combination\n{two_lens}")
    a = two_lens[0][1]
    f = -1 / a
print("Focal length=", f)
    b = two_lens[0][0]
    c = two_lens[1][1]
    d = two_lens[1][0]
    w = (1 - b) / a
    e = (c - 1) / a
print("Unit plane from first surface=", w)
print("Unit plane from second surface=", e)



def telescope():
```

```
twolens=two_lens()
   m7=translation()
telescope=(m7).dot(twolens)
print("System matrix for telescope=\n{telescope}")
   a =telescope[0][1]
   f = -1 / a
print("Focal length=", f)


def microscope():
twolens=two_lens()
   m7=translation()
microscope=(m7).dot(twolens)
print("System matrix for microscope=\n{microscope}")
   a = microscope[0][1]
   f = -1 / a
print("Focal length=", f)
print("What would you like to do today?\n1. Telescope\n2.
Microscope")
q=int(input("Enter your choice:"))
if q == 1:
```

```
telescope()
elif q == 2:
microscope()
else:
print("PLEASE ENTER THE RIGHT CHOICE!!!")
```

Reference

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