

4) Equilibrium

* Condition of Static Equilibrium.

1) For Coplaner Force system (2D-Force system)

* Concurrent force system.

$$\boxed{\sum F_x = 0, \sum F_y = 0}$$

* Non-concurrent force system

$$\boxed{\sum F_x = 0, \sum F_y = 0, \sum M = 0}$$

General
condition
of
Equilibrium

2) For Non-coplaner Force system. (3D-Force system)

* Concurrent force system

$$\boxed{\sum F_x = 0, \sum F_y = 0, \sum F_z = 0}$$

* Non-concurrent force system

$$\boxed{\begin{array}{ll} \sum F_x = 0 & \sum M_{xx} = 0 \\ \sum F_y = 0 & \sum M_{yy} = 0 \\ \sum F_z = 0 & \sum M_{zz} = 0 \end{array}}$$

* Lami's theorem.

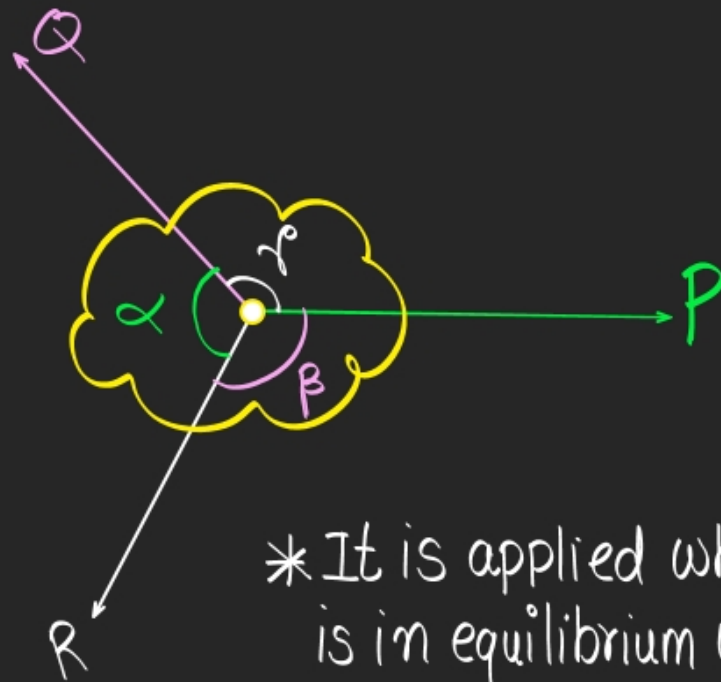
* If three forces acting at a point are in equilibrium, then each force is Proportional to the 'sin' of angle between remaining two forces.

$$P \propto \sin(\alpha)$$

$$Q \propto \sin(\beta)$$

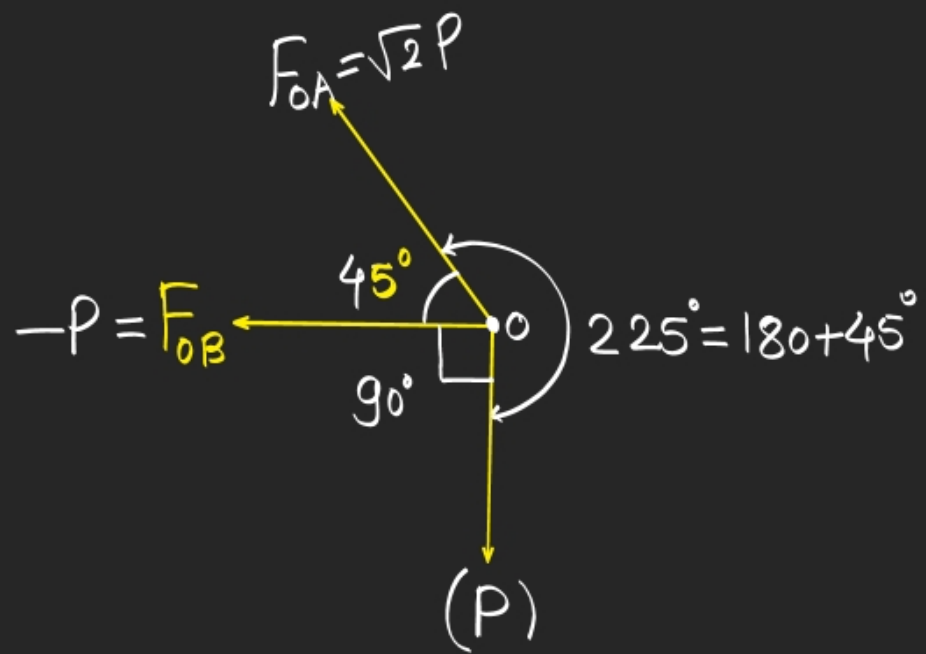
$$R \propto \sin(\gamma)$$

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$



* It is applied when body is in equilibrium under the action of three-concurrent Coplaner forces.

* Forces should be away from body.



$$\frac{P}{\sin(45^\circ)} = \frac{F_{OB}}{\sin(225^\circ)} = \frac{F_{OA}}{\sin(90^\circ)}$$

$$\frac{P}{\left(\frac{1}{\sqrt{2}}\right)} = \frac{F_{OB}}{\sin(180^\circ + 45^\circ)}$$

$$\sqrt{2}P = \frac{F_{OB}}{-\sin 45^\circ}$$

$$\sqrt{2}P = \frac{F_{OB}}{-\left(\frac{1}{\sqrt{2}}\right)}$$

$$(\cancel{\sqrt{2}}P) = -\cancel{\sqrt{2}}F_{OB}$$

$$\boxed{F_{OB} = -P}$$

$$\frac{P}{\left(\frac{1}{\sqrt{2}}\right)} = \frac{F_{OA}}{1}$$

$$\boxed{\sqrt{2}P = F_{OA}}$$

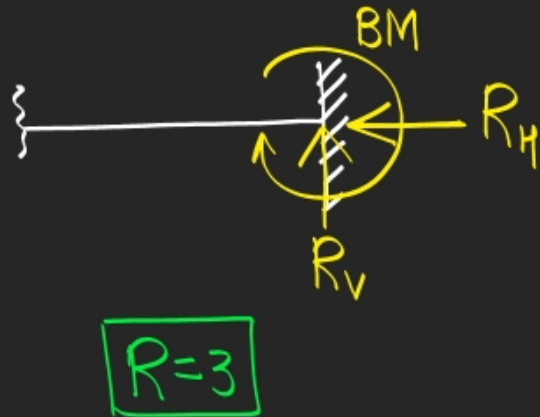
* Supports

* It provides reactions.

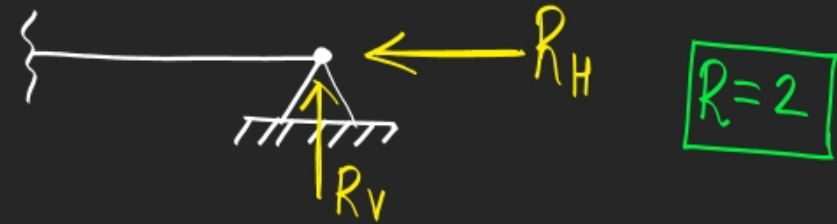
* It provides resistance to the motion.

* Types of supports

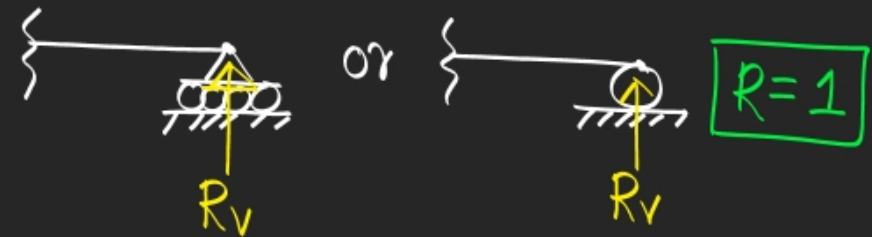
1) Fixed Support



2) Hinge Support (or) Pinned support



3) Roller support



4) Free End (or) Free Support.



5) Guider Roller supports

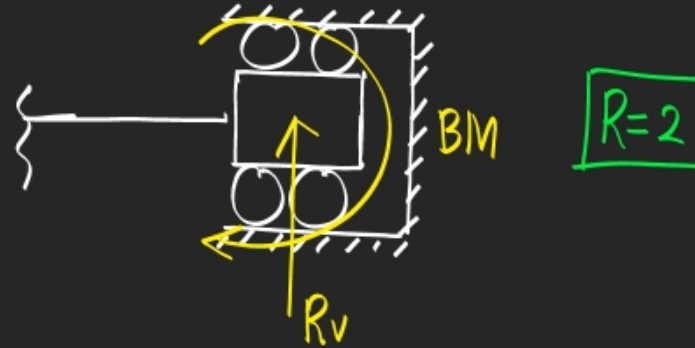
A) Horizontal guided roller



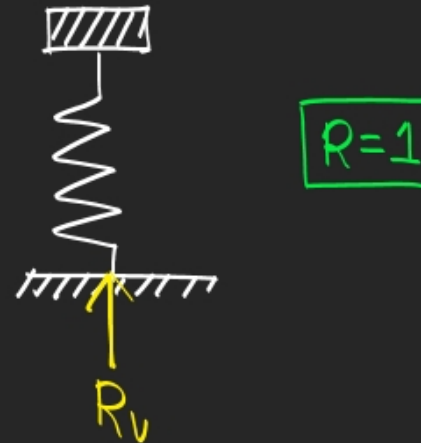
B) Vertical guided roller



6) Damper supports

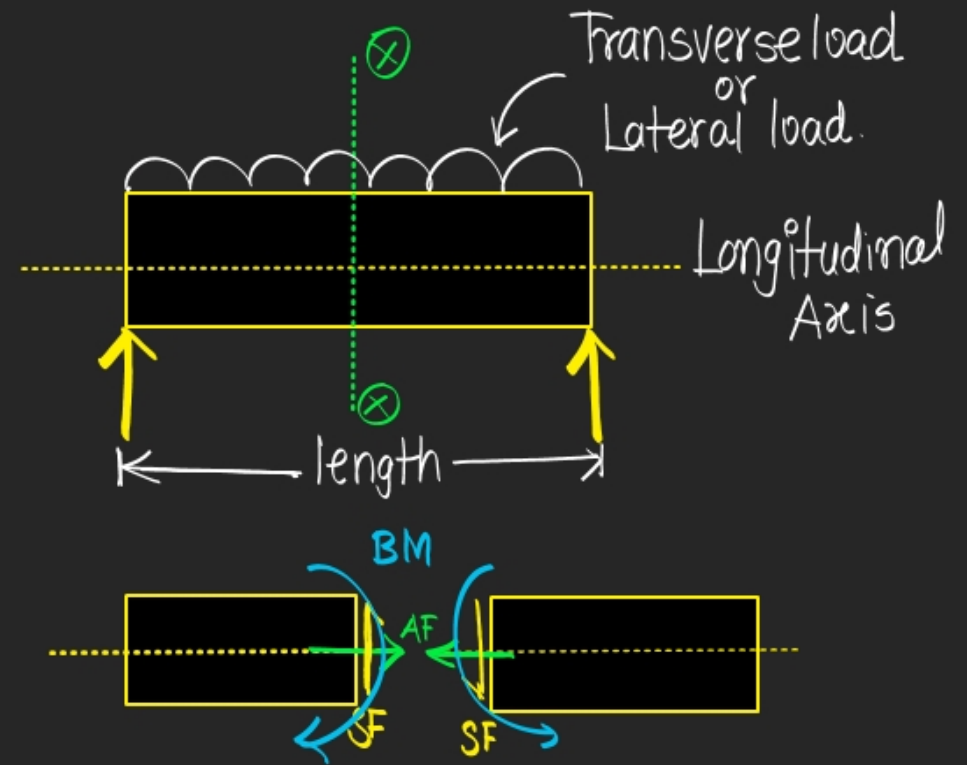
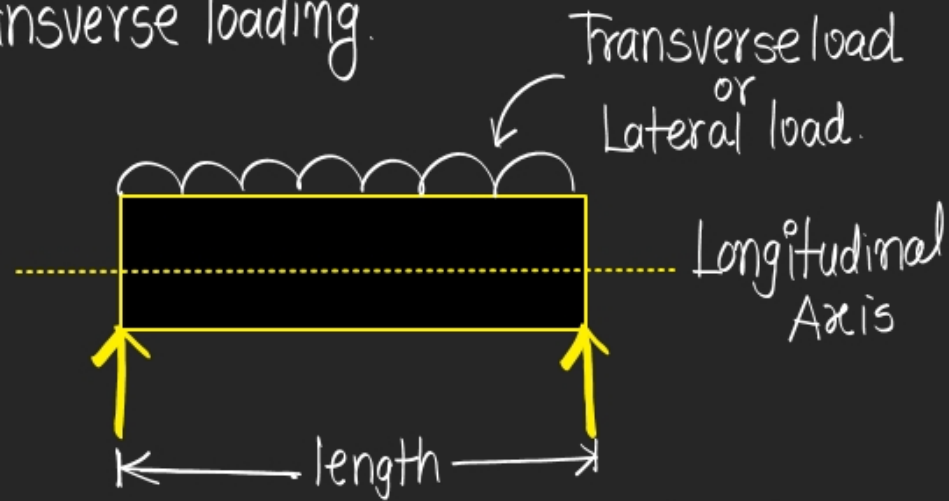


7) Spring support



* Beam

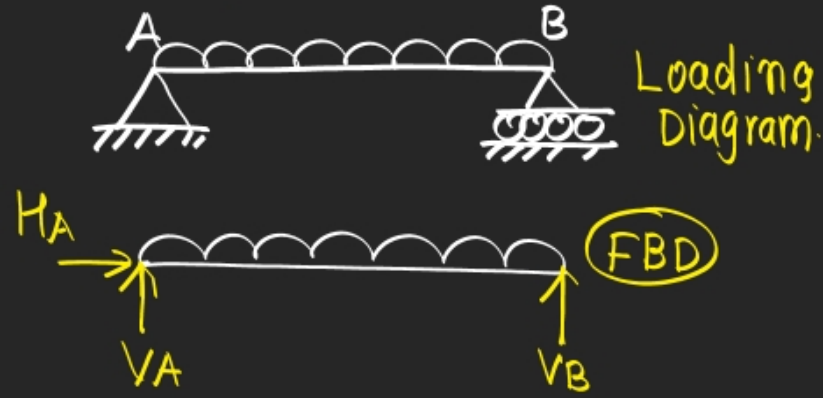
* It is Flexural element which supports transverse loading.



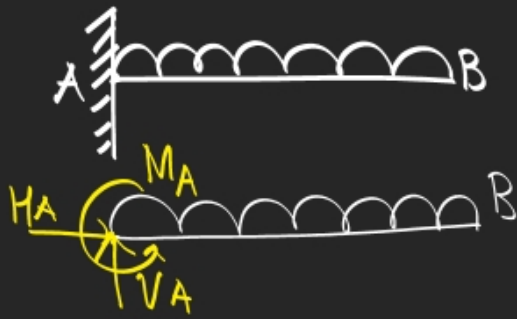
* Internal resisting Forces (design forces) in Beam are Axial Force, shear force, Bending moment and Twisting moments etc.

* Types of Beam

1) Simply supported Beam



2) Cantilever Beam



3) Propped cantilever Beam



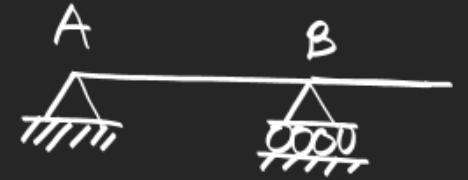
4) Fixed Beam



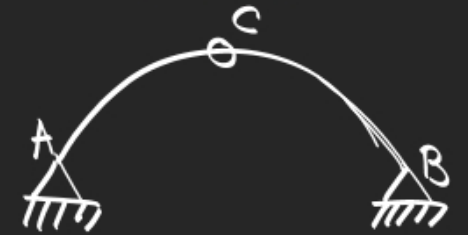
5) Continuous Beam



6) Overhang Beam

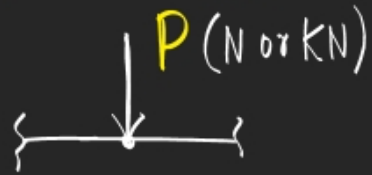


7) Curved Beam (Arch)



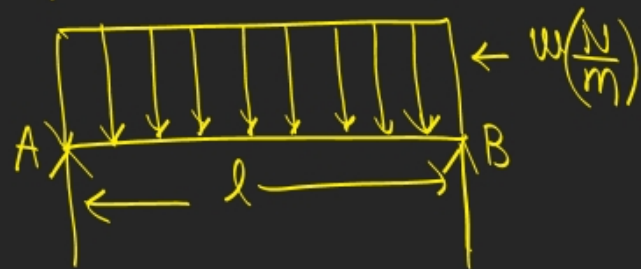
* Types of Loading

1) Point load (or) concentrated load (P or W)

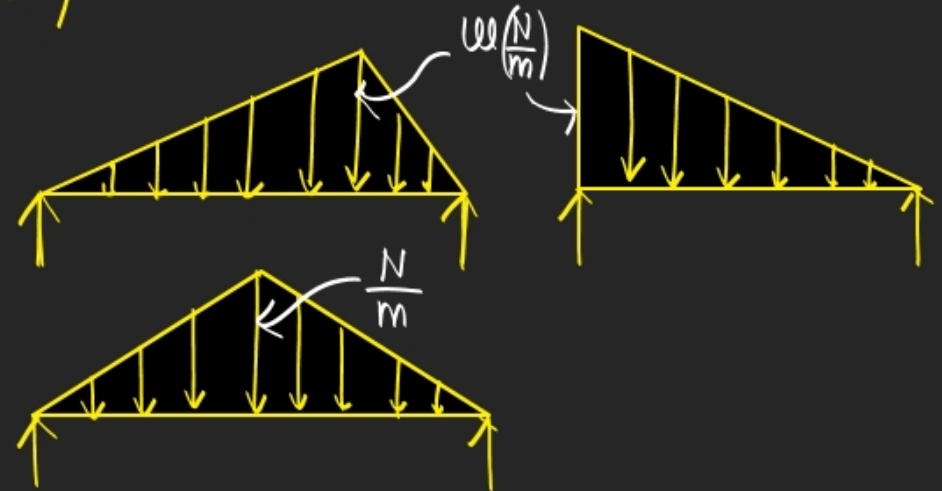


2) Uniformly Distributed load (UDL)

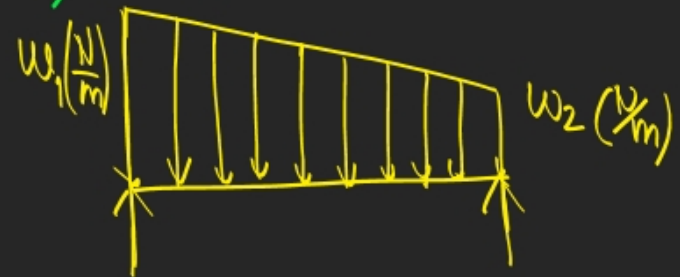
(or)
Rectangular load (w)



3) Uniformly Varying load (U.V.L.)
(or)
Triangular Load.

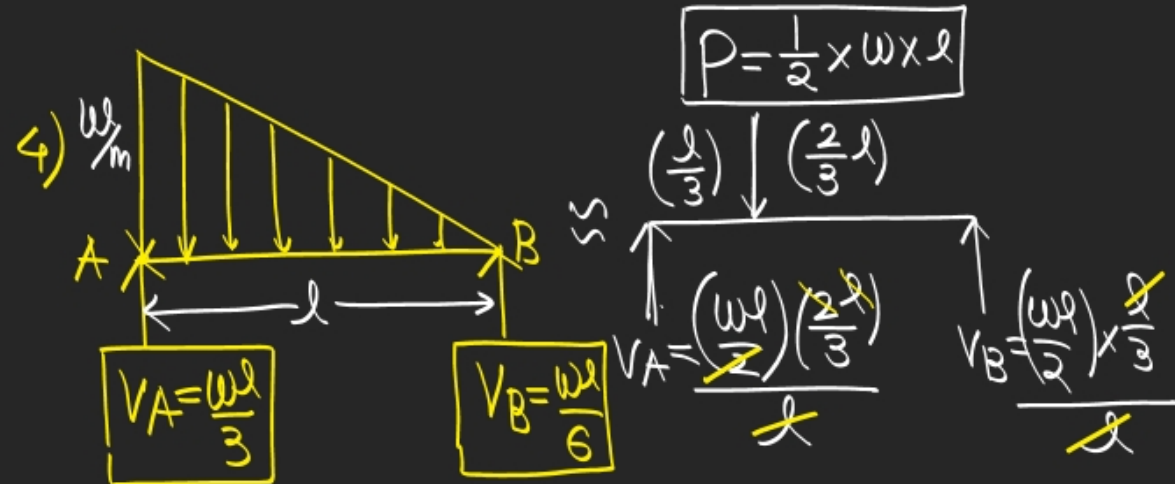
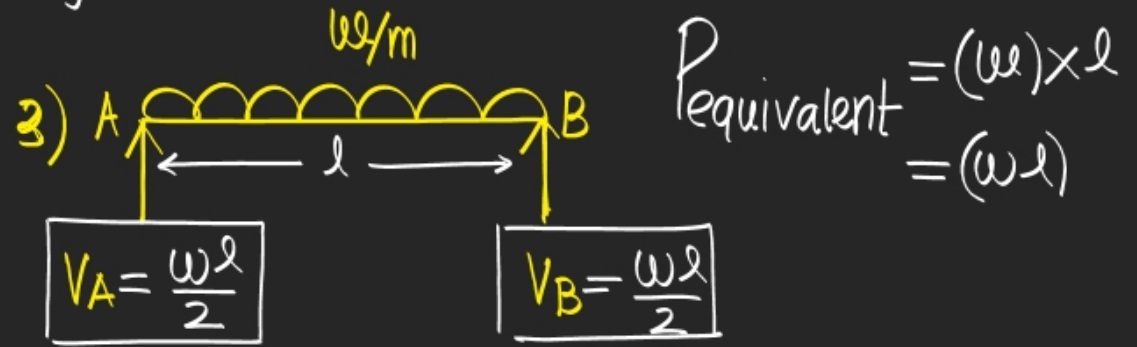
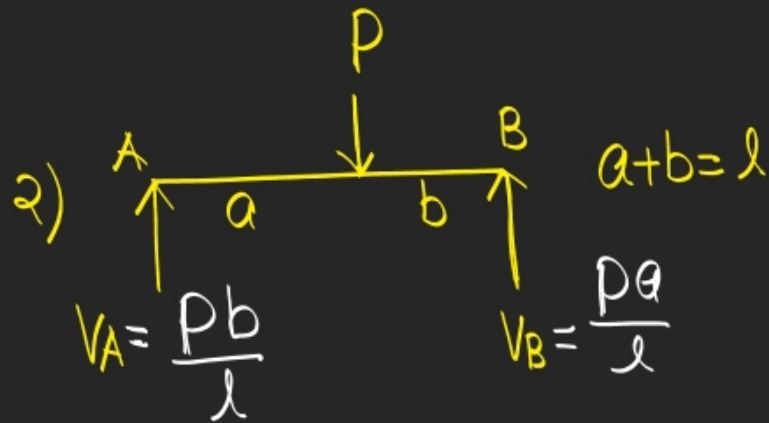
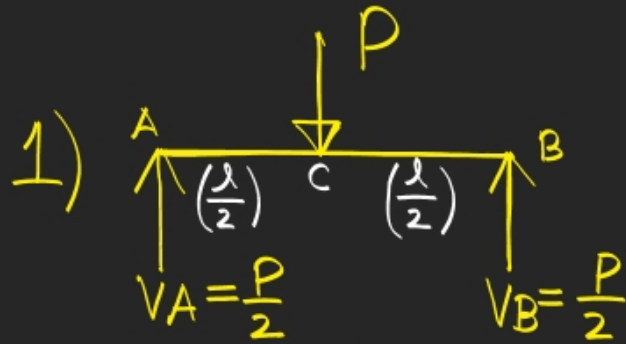


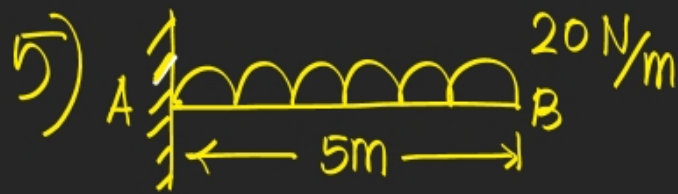
4) Trapezium Loading.



* Equivalent Point load (P) = Area of loading diagram.

It is acted at centroid of loading diagram.

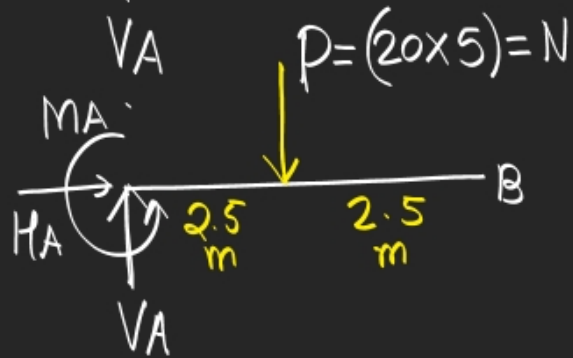




$$V_A =$$

$$H_A =$$

$$M_A =$$



1) $\boxed{\sum M_A = 0}$ $\oplus \ominus$

$$0 + 0 - M_A + P(2.5) = 0$$

$$-M_A + 100 \times 2.5 = 0$$

$$-M_A = -250$$

$$\boxed{M_A = 250 \text{ Nm}}$$

2) $\sum f_y = 0$

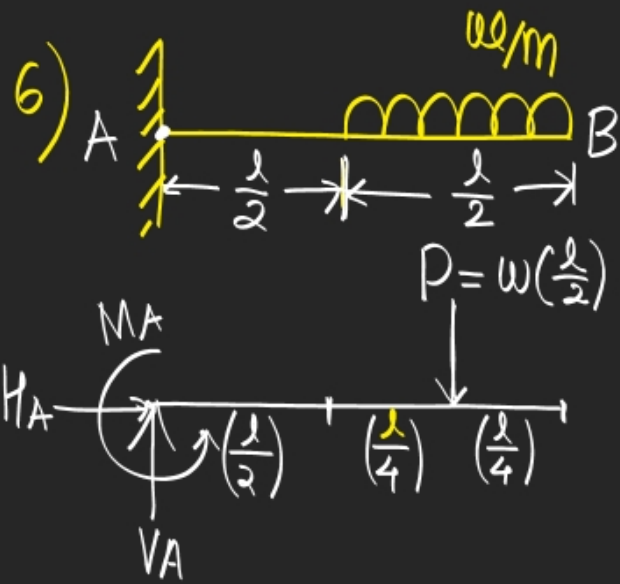
$$+V_A - P = 0$$

$$V_A = +P$$

$$\boxed{V_A = 100 \text{ N}}$$

3) $\sum F_x = 0$

$$\boxed{+H_A = 0}$$



V_A
 H_A
 M_A

$$\sum F_x = 0$$

$$\boxed{H_A = 0}$$

$$\sum F_y = 0$$

$$+V_A - P = 0$$

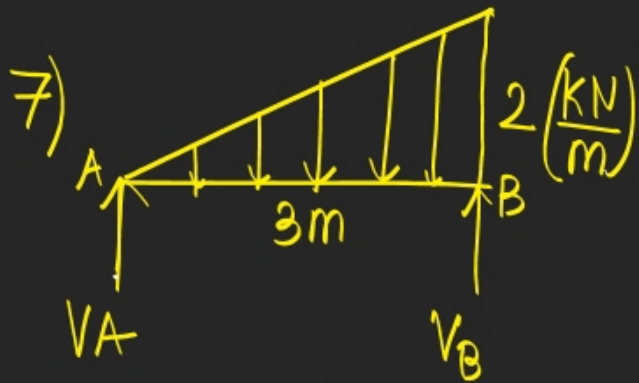
$$\boxed{V_A = +P = \frac{wl}{2}}$$

$$\sum M_A = 0$$

$$0 + 0 - M_A + P\left(\frac{3l}{4}\right) = 0$$

$$-M_A + \left(\frac{wl}{2}\right)\left(\frac{3l}{4}\right) = 0$$

$$\boxed{M_A = \frac{3wl^2}{8}}$$



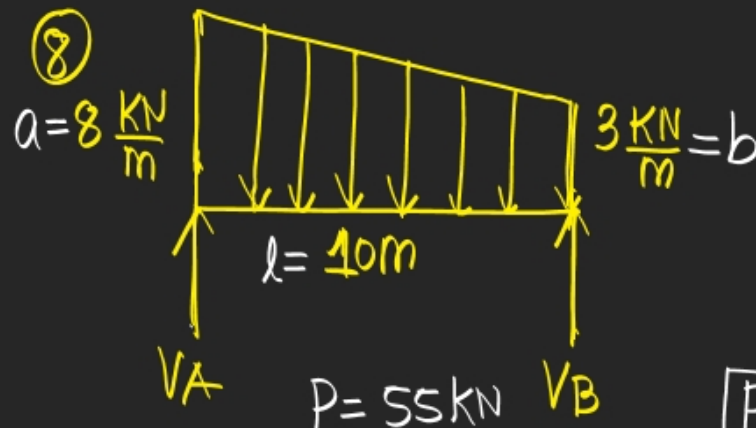
$$P = \frac{1}{2} \times 2 \times 3 = 3 \text{ kN}$$

$$V_A = \frac{3 \text{ kN} \times 1 \text{ m}}{3 \text{ m}}$$

$$V_B = \frac{3 \text{ kN} \times 2 \text{ m}}{3 \text{ m}}$$

$$\boxed{V_A = 1 \text{ m}}$$

$$\boxed{V_B = 2 \text{ m}}$$



$$P = 55 \text{ kN}$$

$$V_A = \frac{55 \times 5.76}{10}$$

$$\boxed{V_A = 31.68 \text{ kN}}$$

$$V_B = \frac{55 \times 4.24}{10}$$

$$\boxed{V_B = 23.32 \text{ kN}}$$

$P = \text{Area}$

$$= \left(\frac{a+b}{2} \right) \times l$$

$$= \left(\frac{8+3}{2} \right) \times 10$$

$$= \frac{11}{2} \times 10$$

$$\boxed{P = 55 \text{ kN}}$$

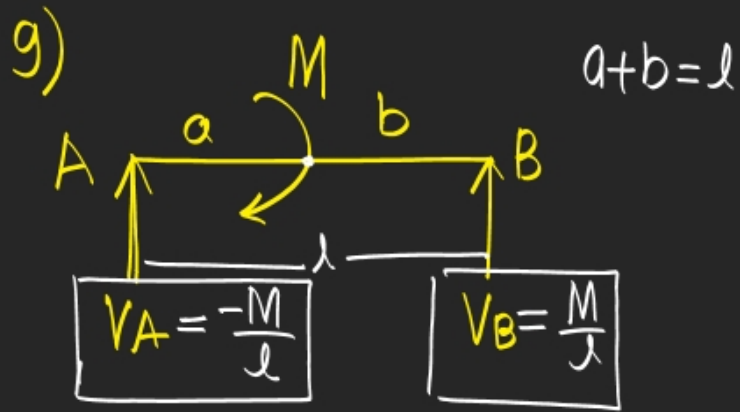
$$x_a = \left(\frac{a+2b}{a+b} \right) \times \frac{l}{3}$$

$$= \left(\frac{8+6}{8+3} \right) \times \frac{10}{3}$$

$$= \frac{14}{11} \times \frac{10}{3} = \frac{140}{33}$$

$$x_a = 4.24 \text{ m}$$

$$x_b = 5.76 \text{ m}$$



$$\sum M_A = 0$$

$$\sum F_y = 0$$

$$0 - V_B \times l + M = 0$$

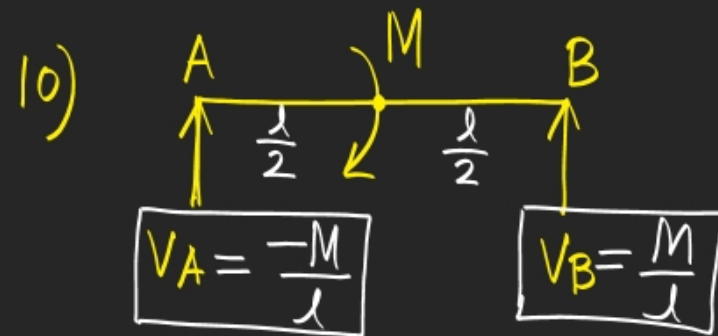
$$V_A + V_B = 0$$

$$-V_B(l) = -M$$

$$V_B = \frac{M}{l}$$

$$V_A = -V_B$$

$$V_A = -\frac{M}{l}$$



$$\sum M_A = 0$$

$$0 - V_B \times l + M = 0$$

$$-V_B \times l = -M$$

$$V_B = \frac{M}{l}$$

$$\sum F_y = 0$$

$$V_A + V_B = 0$$

$$V_A = -V_B = -\frac{M}{l}$$

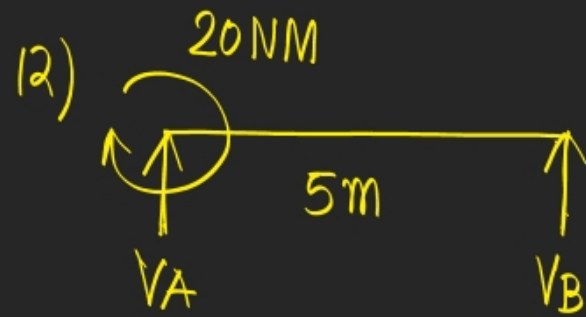
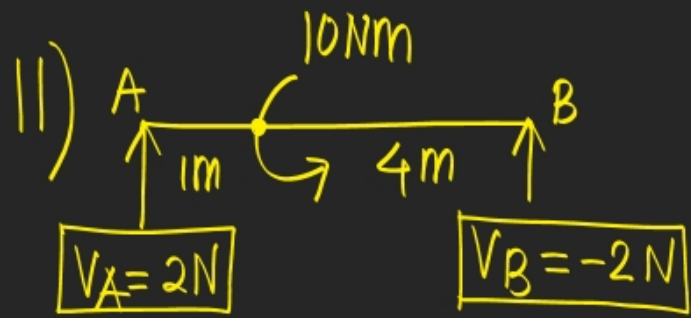


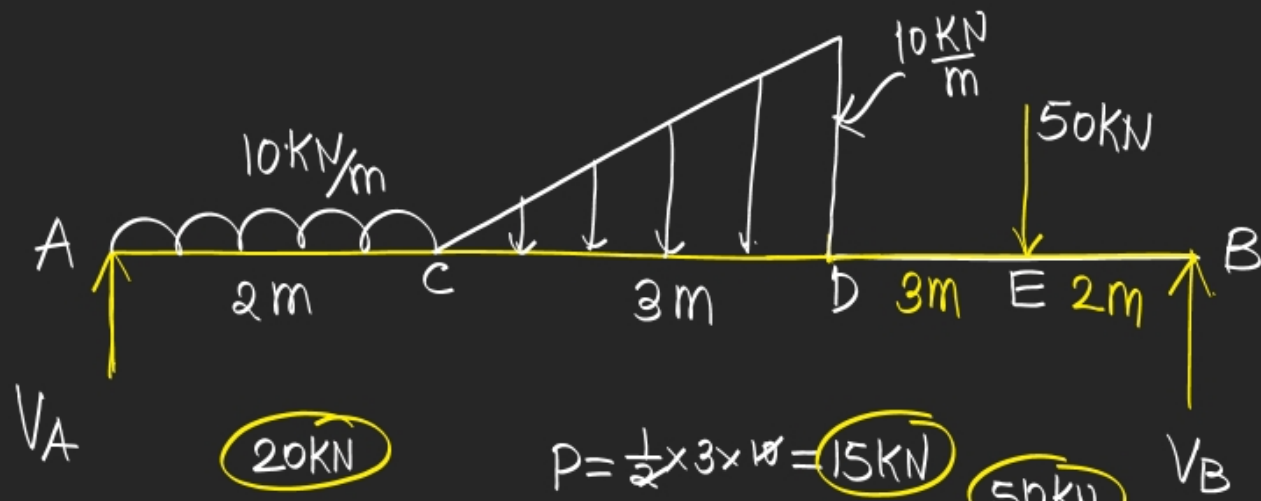
Diagram showing a beam segment AB of length 5m. A counter-clockwise moment of 20 Nm is applied at A. Reaction forces V_A and V_B are shown at A and B respectively.

$$V_A = -\frac{20}{5}$$

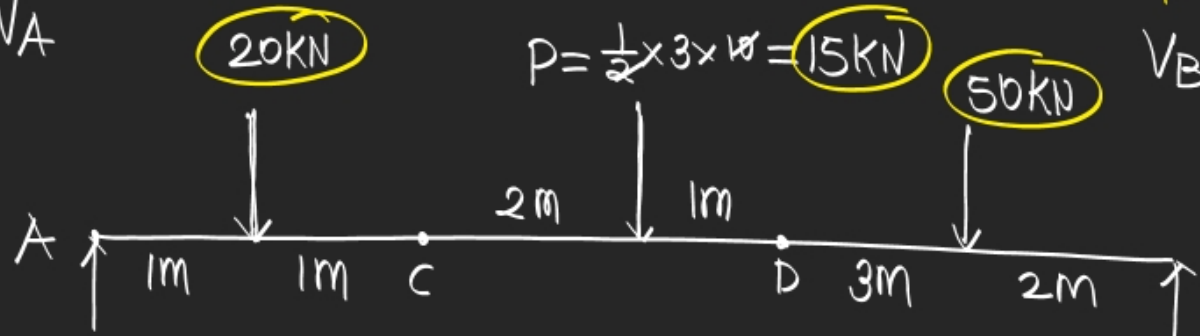
$$= -4\text{ N}$$

$$V_B = +\frac{20}{5}$$

$$= +4\text{ N}$$



$$\begin{array}{r} 85 \text{ kN} \\ - 37 \\ \hline 48 \end{array}$$



$$\begin{aligned} V_A &= \frac{(20 \times 9) + (15 \times 6) + (50 \times 2)}{10} \\ &= \frac{180 + 90 + 100}{10} \\ &= \frac{370}{10} = \underline{\underline{37 \text{ kN}}} \end{aligned}$$

$$\boxed{V_B = 48 \text{ kN}}$$

