Assignment 7

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Download all python codes from

```
https://github.com/pranav-159/
    ai1103_Probability_and_Random_variables/
    blob/main/Assignment_7/codes/
    experimental_verification_Assignment7.py
```

1 Problem

gov/stats/2015/statistics-I(1), Q.3(C)

Three points are chosen on the line of unit length.Find the probability that each the 3 line segments have length greater than $\frac{1}{4}$.

2 Solution

Let $X, Y \in \{0, 1\}$ be the random variables which represent the position of two points on the line of unit length.

Conditions which should be satisfied to have three line segments with length greater than $\frac{1}{4}$ are given in the below table.

Event	Condition
A	$\frac{1}{4} < X < \frac{3}{4}$
B	$\frac{1}{4} < Y < \frac{3}{4}$
C	$\frac{1}{4} < X - Y$
D	$\frac{1}{4} < Y - X$

TABLE 0: Events and their conditions

Then the required event which solves the problem is *ABC*+*ABD*.

$$\Pr(ABC) = \Pr\left(\frac{1}{4} + Y < X, \frac{1}{4} < X, Y < \frac{3}{4}\right) \tag{2.0.1}$$

$$= \sum_{y=\frac{1}{4}}^{\frac{3}{4}} \Pr(Y = y) \times \Pr\left(\frac{1}{4} + y < X, \frac{1}{4} < X < \frac{3}{4} \middle| Y = y\right) \tag{2.0.2}$$

$$= \sum_{y=\frac{1}{4}}^{\frac{3}{4}} \Pr(Y = y) \times \Pr\left(\frac{1}{4} + y < X < \frac{3}{4} \middle| Y = y\right) \tag{2.0.3}$$

$$= \sum_{y=\frac{1}{4}}^{\frac{3}{4}} \Pr(Y = y) \Pr\left(\frac{1}{4} + y < X < \frac{3}{4}\right) \tag{2.0.4}$$

As X is distributed uniformly between 0 and 1.

$$\Pr\left(\frac{1}{4} + y < X < \frac{3}{4}\right) = \begin{cases} \frac{1}{2} - y & y \in \left(0, \frac{1}{2}\right) \\ 0 & \text{otherwise} \end{cases} \tag{2.0.5}$$

Using (2.0.5),(2.0.4) can be written as

$$\Pr(ABC) = \sum_{y=\frac{1}{4}}^{\frac{1}{2}} \Pr(Y = y) \left(\frac{1}{2} - y\right) \tag{2.0.6}$$

$$= \int_{\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{2} - y\right) f_Y(y)\ dy \tag{2.0.7}$$

As y is distributed uniformly between 0 and 1.

$$\Pr(ABC) = \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{2} - y\ dy \tag{2.0.8}$$

$$= \frac{1}{32} \tag{2.0.9}$$

Similarly,we can find,

$$\Pr(ABD) = \frac{1}{32} \tag{2.0.10}$$

As C and D are mutually exclusive events.

$$\Pr(ABC + ABD) = \Pr(ABC) + \Pr(ABD) \tag{2.0.11}$$
$$= \frac{1}{16} \tag{2.0.12}$$

$\therefore$ probability that each of the three line segments have length greater than $\frac{1}{4}$ is $\frac{1}{16}$.