Assignment 4

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Download all python codes from

https://github.com/pranav-159/ai1103_Probability_and_Random_variables/blob/main/Assignment_4/codes/experimental_verification_GATE_IN_2018_Q5.py

and latex-tikz codes from

https://github.com/pranav-159/ai1103_Probability_and_Random_variables/blob/main/Assignment_4/Assignment4.tex

1 Problem(GATE IN 2018, Q. 5)

Consider a sequence of tossing a fair coin where outcomes of tosses are independent.The probability of getting the head for the third time in the fifth toss is

(A) $\frac{5}{16}$ (B) $\frac{3}{16}$ (C) $\frac{3}{5}$ (D) $\frac{9}{16}$

2 Solution(GATE IN 2018, Q. 5)

Let the random variable $X \in \{0, 1\}$ denotes head and tail in a toss.As both are equally probable.

$$\Pr(X = 0) = \frac{1}{2} \tag{2.0.1}$$

$$\Pr(X = 1) = \frac{1}{2} \tag{2.0.2}$$

Event	Description
A	nth toss is a head
B	Exactly k-1 heads in first n-1 tosses
C	nth toss is the kth head

TABLE 4: Description of events used in problem

$$\Pr(A) = \Pr(X = 1) = \frac{1}{2} \tag{2.0.3}$$

$$\Pr(B) = \frac{{}^{n-1}C_{k-1}}{2^{n-1}} \tag{2.0.4}$$

$$C = AB \tag{2.0.5}$$

$$\Pr(C) = \Pr(AB) \tag{2.0.6}$$

As A and B are independent events.

$$\Pr(C) = \Pr(A)\Pr(B) \tag{2.0.7}$$

$$= \frac{1}{2} \times \frac{{}^{n-1}C_{k-1}}{2^{n-1}} \tag{2.0.8}$$

$$= \frac{{}^{n-1}C_{k-1}}{2^{n}} \tag{2.0.9}$$

Here n=5,k=3

$$\Pr(C|n = 5, k = 2) = \frac{{}^{4}C_{2}}{2^{5}} \tag{2.0.10}$$

$$= \frac{6}{32} \tag{2.0.11}$$

Therefore probability of getting the head for the third time in the fifth toss is $\frac{3}{16}$.