Assignment 7

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1 Problem

GATE 2019 (ST) , Q.49 (Statistics section)

Let X be a random variable with characteristic function $\phi_X(\cdot)$ such that $\phi_X(2\pi) = 1$.Let $\mathbb{Z}$ denote the set of integers.Then $P(X \in \mathbb{Z})$ is equal to ...

2 Solution

We know that characteristic function,

$$\phi_X(t) = E[e^{itX}] \tag{2.0.1}$$

$$\phi_X(2\pi) = E[e^{i2\pi X}] \tag{2.0.2}$$

$$E[\cos(2\pi X)] + iE[\sin(2\pi X)] = 1 \tag{2.0.3}$$

$$\tag{2.0.4}$$

Then,

$$E[\cos(2\pi X)] = 1 \text{ and} \tag{2.0.5}$$

$$E[\sin(2\pi X)] = 0 \tag{2.0.6}$$

$$\sum_{i=0}^{i=n} \frac{\cos(2\pi X_i)}{n} = 1 \tag{2.0.7}$$

As $\cos(2\pi X) \leq 1$ then,

$$\cos(2\pi X_i) = 1 \tag{2.0.8}$$

$$\implies X_i \in \mathbb{Z}$$

Also $X_i \in \mathbb{Z}$ satisfies (2.0.6),

$$\sin(2\pi X) = 0 \tag{2.0.9}$$

$$\sum_{i=0}^{i=n} \frac{\sin(2\pi X_i)}{n} = 0 \tag{2.0.10}$$

$$E[\sin(2\pi X)] = 0 \tag{2.0.11}$$

As random variable X must be an integer,

$$\therefore \quad P(X \in \mathbb{Z}) = 1$$