

# Assignment 4

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Download all python codes from

[https://github.com/pranav-159/  
ai1103\\_Probability\\_and\\_Random\\_variables/  
blob/main/Assignment\\_4/codes/  
experimental\\_verification\\_GATE\\_IN\\_2018\\_Q5  
.py](https://github.com/pranav-159/ai1103_Probability_and_Random_variables/blob/main/Assignment_4/codes/experimental_verification_GATE_IN_2018_Q5.py)

and latex-tikz codes from

[https://github.com/pranav-159/  
ai1103\\_Probability\\_and\\_Random\\_variables/  
blob/main/Assignment\\_4/Assignment4.tex](https://github.com/pranav-159/ai1103_Probability_and_Random_variables/blob/main/Assignment_4/Assignment4.tex)

$$C = AB \quad (2.0.5)$$

$$\Pr(C) = \Pr(AB) \quad (2.0.6)$$

As A and B are independent events.

$$\Pr(C) = \Pr(A) \Pr(B) \quad (2.0.7)$$

$$= \frac{1}{2} \times \frac{{}^{n-1}C_{k-1}}{2^{n-1}} \quad (2.0.8)$$

$$= \frac{{}^{n-1}C_{k-1}}{2^n} \quad (2.0.9)$$

Here  $n=5, k=3$

$$\Pr(C|n=5, k=2) = \frac{{}^4C_2}{2^5} \quad (2.0.10)$$

$$= \frac{6}{32} \quad (2.0.11)$$

Therefore probability of getting the head for the third time in the fifth toss is  $\frac{3}{16}$ .

## 1 PROBLEM(GATE IN 2018, Q. 5)

Consider a sequence of tossing a fair coin where outcomes of tosses are independent. The probability of getting the head for the third time in the fifth toss is

(A)  $\frac{5}{16}$    (B)  $\frac{3}{16}$    (C)  $\frac{3}{5}$    (D)  $\frac{9}{16}$

## 2 SOLUTION(GATE IN 2018, Q. 5)

Let the random variable  $X \in \{0, 1\}$  denotes head and tail in a toss. As both are equally probable.

$$\Pr(X = 0) = \frac{1}{2} \quad (2.0.1)$$

$$\Pr(X = 1) = \frac{1}{2} \quad (2.0.2)$$

| Event | Description                            |
|-------|--|
| A     | nth toss is a head                     |
| B     | Exactly k-1 heads in first four tosses |
| C     | nth toss is the third head             |

TABLE 4: Description of events used in problem

$$\Pr(A) = \Pr(X = 1) = \frac{1}{2} \quad (2.0.3)$$

$$\Pr(B) = \frac{{}^{n-1}C_{k-1}}{2^{n-1}} \quad (2.0.4)$$