

# Assignment 7

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Download all python codes from

[https://github.com/pranav-159/ai1103\\_Probability\\_and\\_Random\\_variables/blob/main/Assignment\\_7/codes/experimental\\_verification\\_Assignment7.py](https://github.com/pranav-159/ai1103_Probability_and_Random_variables/blob/main/Assignment_7/codes/experimental_verification_Assignment7.py)

## 1 PROBLEM

**gov/stats/2015/statistics-I(1), Q.3(C)**

Three points are chosen on the line of unit length. Find the probability that each the 3 line segments have length greater than  $\frac{1}{4}$ .

## 2 SOLUTION

Let  $X, Y \in \{0, 1\}$  be the random variables which represent the position of two points on the line of unit length.

Conditions which should be satisfied to have three line segments with length greater than  $\frac{1}{4}$  are given

Event	Condition
A	$\frac{1}{4} < X < \frac{3}{4}$
B	$\frac{1}{4} < Y < \frac{3}{4}$
C	$\frac{1}{4} < X - Y$
D	$\frac{1}{4} < Y - X$

TABLE 0: Events and their conditions

in the below table.

Then the required event which solves the problem is  $ABC + ABD$ .

$$\Pr(ABC) = \Pr\left(\frac{1}{4} + Y < X, \frac{1}{4} < X, Y < \frac{3}{4}\right) \quad (2.0.1)$$

$$= \int_{\frac{1}{4}}^{\frac{3}{4}} dy f_Y(y) \Pr\left(\frac{1}{4} + y < X, \frac{1}{4} < X < \frac{3}{4}\right) \quad (2.0.2)$$

$$= \int_{\frac{1}{4}}^{\frac{3}{4}} dy f_Y(y) \Pr\left(\frac{1}{4} + y < X < \frac{3}{4}\right) \quad (2.0.3)$$

As  $X$  is distributed uniformly between 0 and 1.

$$\Pr\left(\frac{1}{4} + y < X < \frac{3}{4}\right) = \begin{cases} \frac{1}{2} - y & y \in \left(0, \frac{1}{2}\right) \\ 0 & \text{otherwise} \end{cases} \quad (2.0.4)$$

Using (2.0.4), (2.0.3) can be written as

$$\Pr(ABC) = \int_{\frac{1}{4}}^{\frac{1}{2}} dy f_Y(y) \left(\frac{1}{2} - y\right) \quad (2.0.5)$$

As  $y$  is distributed uniformly between 0 and 1.

$$\Pr(ABC) = \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{2} - y \, dy \quad (2.0.6)$$

$$= \frac{1}{32} \quad (2.0.7)$$

Similarly, we can find,

$$\Pr(ABD) = \frac{1}{32} \quad (2.0.8)$$

As  $C$  and  $D$  are mutually exclusive events.

$$\Pr(ABC + ABD) = \Pr(ABC) + \Pr(ABD) \quad (2.0.9)$$

$$= \frac{1}{16} \quad (2.0.10)$$

$\therefore$  probability that each of the three line segments have length greater than  $\frac{1}{4}$  is  $\frac{1}{16}$ .