

ICSE Paper 2011

MATHEMATICS

SOLVED PAPER

(Two hours and a half)

Answers to this Paper must be written on the paper provided separately.

You will not be allowed to write during the first 15 minutes.

This time is to be spent in reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

*Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*

All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer. Omission of essential working will result in the loss of marks.

The intended marks for questions or parts of questions are given in brackets [].

Mathematical tables are provided.

SECTION-A (40 Marks)

*(Attempt **all** questions from this Section)*

Question 1:

(a) Find the value of 'k' if $(x - 2)$ is a factor of $x^3 + 2x^2 - kx + 10$? [3]

(b) If $A = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, is the product AB possible? Give a reason. If yes, find AB . [3]

(c) Mr. Kumar borrowed Rs. 25,000 for two years. The rate of interest for the two successive years are 8% and 10% respectively. If he repays Rs. 6,200 at the end of the first year, find the outstanding amount at the end of the second year. [4]

Solution:

(a)

Let $f(x) = x^3 + 2x^2 - kx + 10$

$\therefore (x - 2)$ is a factor,

$\therefore f(2) = 0$

$$f(2) = 8 + 8 - 2k + 10 = 0$$

$\Rightarrow k = 13$

To check for $(x + 5)$ is a factor,

$$\begin{aligned} f(-5) &= (-5)^3 + 2(-5)^2 - 13(-5) + 10 \\ &= -125 + 50 + 65 + 10 = 0 \end{aligned}$$

$\therefore (x + 5)$ is a factor.

(b)

$A_{2 \times 2}$ and $B_{2 \times 1}$

From the order of both the matrix A and B, it is clear that AB is possible because the number of columns of A are equal to the number of rows of B.

$$\begin{aligned} AB &= \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} \\ &= \begin{bmatrix} 6 + 20 \\ 8 - 8 \end{bmatrix} \\ &= \begin{bmatrix} 26 \\ 0 \end{bmatrix} \end{aligned}$$

(c)

Given : Principal = ₹ 15,000

We know that $A = P \left(1 + \frac{r}{100} \right)^n$

$$\begin{aligned} \text{Amount after 1}^{\text{st}} \text{ year} &= 15,000 \left(1 + \frac{8}{100} \right) \\ &= ₹ 16,200 \end{aligned}$$

Principal after repayment = 16,200 – 6,200 = ₹ 10,000

Amount outstanding at the end of second year

$$\begin{aligned} &= 10,000 \left(1 + \frac{10}{100} \right) \\ &= ₹ 11,000 \end{aligned}$$

Question 2:

(a) From a pack of 52 playing cards all cards whose numbers are multiples of 3 are removed. A card is now drawn at random.

What is the probability that the card drawn is:

- (i) a face card (King, Jack or Queen)
- (ii) an even-numbered red card? [3]

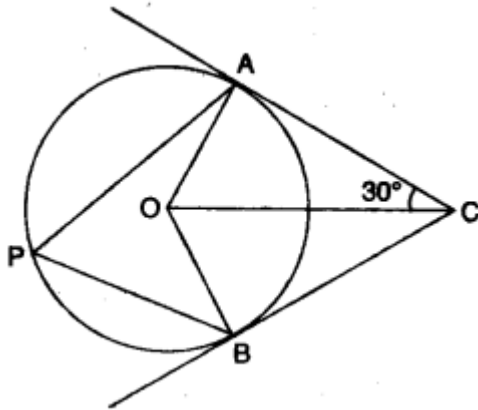
(b) Solve the following equation:

$x - 18/x = 6$. Give your answer correct to two significant figures. [3]

(c) In the given figure O is the centre of the circle. Tangents at A and B meet at C. If $\angle AOC = 30^\circ$, find

- (i) $\angle BCO$
- (ii) $\angle AOB$

(iii) $\angle APB$



[4]

Solution:

(a) Number of cards which are multiples of 3 = 12

Cards left in the pack = 40

Number of face cards = 12

$$P(\text{face card}) = \frac{12}{40} = \frac{3}{10}$$

(ii) Even numbered red cards = 10

$$P(\text{even number red card}) = \frac{10}{40} = \frac{1}{4}$$

(b)

Let $x^2 - 6x - 18 = 0$

Compare with equation $ax^2 + bx + c = 0$, we get

$a = 1, b = -6, c = -18$

Now,

$$\begin{aligned} x &= \frac{b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{6 \pm \sqrt{36 + 72}}{2} \\ &= \frac{6 \pm 6\sqrt{3}}{2} \Rightarrow x = 3 \pm 3\sqrt{3} \end{aligned}$$

$\therefore x = 3 \pm 5.196$

Taking +ve and -ve sign respectively, we get

$$x = 8.196 \text{ or } x = -2.196$$

$$x = 8.2 \text{ or } x = -2.2$$

(c)

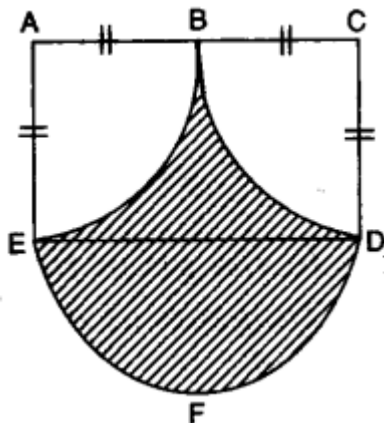
$$\begin{array}{lll} \Delta ACO \cong \Delta BCO & & \text{(R.H.S.)} \\ \therefore \angle BCO = \angle ACO & & \text{(C.P.C.T.)} \\ \text{(i)} \quad \angle BCO = 30^\circ & & \\ \text{In } \Delta ACO, \quad \angle OAC = 90^\circ & & \text{(Radius is perpendicular to tangent)} \\ \therefore \angle AOC = 60^\circ & & \\ \text{Also} \quad \angle BOC = 60^\circ & & \text{(C.P.C.T.)} \\ \text{(ii)} \quad \angle AOB = 120^\circ & & \\ \text{(iii)} \quad \angle APB = 60^\circ & & \text{(Angle at circumference is half the angle at the centre)} \end{array}$$

Question 3:

(a) Ahmed has a recurring deposit account in a bank. He deposits Rs. 2,500 per month for 2 years. If he gets Rs. 66,250 at the time of maturity, find

(i) The interest paid by the bank.

(ii) The rate of interest. [3]



(b) Calculate the area of the shaded region, if the diameter of the semi-circle is equal to 14 cm. Take $\pi = 22/7$ [3]

(c) ABC is a triangle and G(4, 3) is the centroid of the triangle. If A = (1, 3), B = (4, b) and C = (a, 1), find 'a' and 'b'. Find the length of side BC. [4]

Solution:

(a)

$$\begin{aligned} \text{(i) Interest} &= 66,250 - 2,500 \times 24 \\ &= 66,250 - 60,000 \\ &= ₹ 6,250 \end{aligned}$$

$$\begin{aligned} \text{(ii) Principal} &= \frac{n(n+1)}{2} \times \text{sum of deposited per month} \\ &= \frac{24(24+1)}{2} \times 2,500 \\ &= \frac{24 \times 25}{2} \times 2,500 \end{aligned}$$

$$I = \text{Principal} \times \frac{R}{100} \times \frac{1}{12}$$

$$\begin{aligned} R &= \frac{6,250 \times 2 \times 100 \times 12}{2,500 \times 24 \times 25} \\ &= 10\% \text{ p.a.} \end{aligned}$$

(b)

$$\begin{aligned}\text{Area of shaded portion} &= \text{Complete area} - \text{area of the two quadrants} \\ &= (\text{Area of ACDE} + \text{Area of semi circle EFD}) \\ &\quad - (\text{Area of Quadrant ABE} + \text{Area of Quadrant BCD})\end{aligned}$$

$$= \left\{ 14 \times 7 + \frac{\pi}{2} (7)^2 \right\} - \left\{ \frac{\pi}{4} (7)^2 + \frac{\pi}{4} (7)^2 \right\}$$

$$= \left\{ 14 \times 7 + \frac{\pi}{2} (7)^2 \right\} - \left\{ \frac{\pi}{2} (7)^2 \right\}$$

$$= 98 \text{ cm}^2.$$

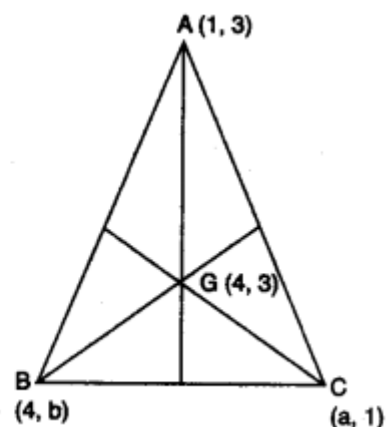
(c)

$$\text{Coordinate of centroid } G \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

$$\Rightarrow \frac{1 + 4 + a}{3} = 4 \Rightarrow a = 7$$

$$\frac{3 + b + 1}{3} = 3 \Rightarrow b = 5$$

$$\begin{aligned}\text{Now, } BC &= \sqrt{(4 - 7)^2 + (5 - 1)^2} \\ &= \sqrt{9 + 16} = 5 \text{ units.}\end{aligned}$$



Question 4:

(a) Solve the following inequation and represent the solution set on the number line $2x - 5 \leq 5x + 4 < 11$, where $x \in 1$. [3]

(b) Evaluate without using trigonometric tables:

$$2 \left(\frac{\tan 35^\circ}{\cot 55^\circ} \right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ} \right)^2 - 3 \left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ} \right) \quad [3]$$

(c) A Mathematics aptitude test of 50 students was recorded as follows:

Marks	50-60	60-70	70-80	80-90	90-100
No. of students	4	8	14	19	5

Draw a histogram for the above data using a graph paper and locate the mode. [4]

Solution:

(a)

$$\begin{aligned} \text{Given : } 2x - 5 &\leq 5x + 4 & \text{and} & & 5x + 4 < 11 \\ -3x &\leq 9 & & & 5x < 7 \\ x &\geq -3 & & & x < 1.4 \\ -3 &\leq x & & & \end{aligned}$$

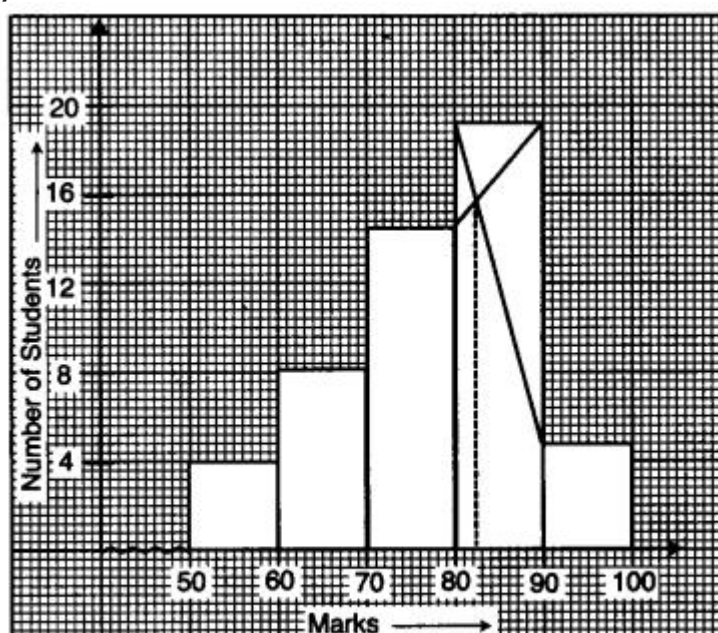
Solution set, $x \in \{-3, -2, -1, 0, 1\}$



(b)

$$\begin{aligned} \text{Given : } & 2 \left(\frac{\tan 35^\circ}{\cot 55^\circ} \right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ} \right) - 3 \left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ} \right) \\ &= 2 \left(\frac{\tan (90^\circ - 55^\circ)}{\cot 55^\circ} \right)^2 + \left(\frac{\cot (90^\circ - 35^\circ)}{\tan 35^\circ} \right) - 3 \left(\frac{\sec (90^\circ - 50^\circ)}{\operatorname{cosec} 50^\circ} \right) \\ &= 2 \left(\frac{\cot 55^\circ}{\cot 55^\circ} \right)^2 + \left(\frac{\tan 35^\circ}{\tan 35^\circ} \right) - 3 \left(\frac{\operatorname{cosec} 50^\circ}{\operatorname{cosec} 50^\circ} \right) \\ &= 2 + 1 - 3 = 0 \end{aligned}$$

(c)



Mode from graph = 82.5.

SECTION-B (40 Marks)

(Attempt **any four** questions from this Section)

Question 5:

(a) A manufacturer sells a washing machine to a wholesaler for Rs. 15,000. The wholesaler sells it to a trader at a profit of Rs. 1,200 and the trader in turn sells it to a consumer at a profit of Rs. 1,800. If the rate of VAT is 8% find:

(i) The amount of VAT received by the State Government on the sale of this machine from the manufacturer and the wholesaler.

(ii) The amount that the consumer pays for the machine. [3]

(b) A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0.5 cm. Find the number of spheres formed. [3]

(c) ABCD is a parallelogram where A(x, y), B(5, 8), C(4, 7) and D(2, -4). Find

(i) Coordinates of A

(ii) Equation of diagonal BD. [4]

Solution:

(a)

$$(i) \quad \text{VAT received by Govt. from manufacturer} = 15,000 \times \frac{8}{100} = ₹ 1,200$$

$$\text{VAT from wholesaler} = 1200 \times \frac{8}{100} = ₹ 96$$

$$\text{Total VAT from manufacturer and wholesaler} = 1200 + 96 = ₹ 1296$$

$$\begin{aligned} (ii) \quad \text{Amount that customer pays} &= (15000 + 1200 + 1800) + \text{VAT} \\ &= 18,000 + \frac{18,000 \times 8}{100} \\ &= 18,000 + 1440 = ₹ 19,440 \end{aligned}$$

(b)

$$\begin{aligned} \text{Number of spheres} &= \frac{\text{Volume of cone}}{\text{Volume of each sphere}} \\ &= \frac{\frac{1}{3} \pi (5)^2 (8)}{\frac{4}{3} \pi (0.5)^3} = \frac{50 \times 10^3}{5 \times 5 \times 5} \\ &= 400 \text{ spheres} \end{aligned}$$

(c)

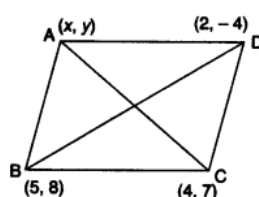
In a parallelogram, mid point of diagonal BD co-incides with the mid point of diagonal AC.

$$\text{Mid point of BD} = \left(\frac{5+2}{2}, \frac{8-4}{2} \right) = \left(\frac{7}{2}, 2 \right)$$

$$\text{Mid point of AC} = \left(\frac{x+4}{2}, \frac{y+7}{2} \right)$$

$$\text{Equating, } \frac{x+4}{2} = \frac{7}{2} \Rightarrow x = 3$$

$$\frac{y+7}{2} = 2 \Rightarrow y = -3$$



(i) Co-ordinates of A (3, -3).

$$(ii) \quad m \text{ of BD} = \frac{8 - (-4)}{5 - 2} = \frac{12}{3} = 4$$

Equation of BD

$$y - y_1 = m(x - x_1)$$

$$y + 4 = 4(x - 2)$$

\Rightarrow

$$y = 4x - 4$$

Question 6:

(a) Use a graph paper to answer the following questions. (Take 1 cm = 1 unit on both axes).

(i) Plot A (4, 4), B (4, -6) and C (8, 0), the vertices of a triangle ABC.

(ii) Reflect ABC on the y-axis and name it as A'B'C'.

(iii) Write the coordinates of the image A', B' and C'.

(iv) Give a geometrical name for the figure AA'C'B'BC.

(v) Identify the line of symmetry of AA'C'B'BC. [5]

(b) Mr. Choudhury opened a Saving's-Bank Account at State Bank of India on 1st April 2007. The entries of one year as shown in his pass book are given below:

Date	Particulars	Withdrawals (in Rs.)	Deposits (in Rs.)	Balance (in Rs.)
1st April 2007	By Cash	—	8550.00	8550.00
12th April 2007	To Self	1200.00	—	7350.00
24th April 2007	By Cash	—	4550.00	11900.00
8th July 2007	By Cheque	—	1500.00	13400.00
10th Sept. 2007	By Cheque	—	3500.00	16900.00
17th Sept. 2007	To Cheque	2500.00	—	14400.00

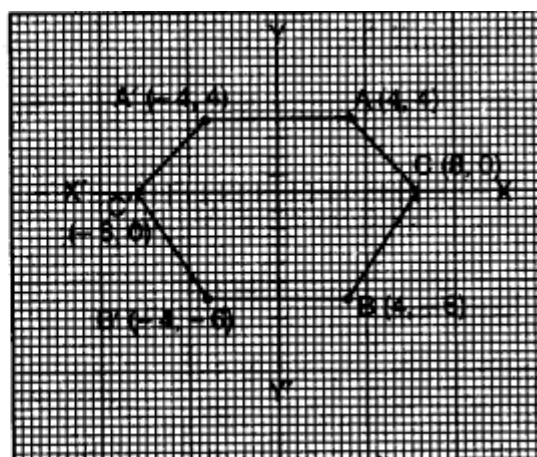
11th Oct. 2007	By Cash	—	800.00	15200.00
6th Jan. 2008	To Self	2000.00	—	13200.00
9th March 2008	By Cheque	—	950.00	14150.00

If the bank pays interest at the rate of 5% per annum, find the interest paid on 1st April, 2008. Give your answer correct to the nearest rupee. [5]

Solution:

(a)

- (i) In the given diagram.
- (ii) In the given diagram.
- (iii) $A' (-4, 4)$
 $B' (-4, -6)$
 $C' (-8, 0)$
- (iv) Hexagon.
- (v) Y-Axis is the line of symmetry.



(b)

Minimum Balances

April	7,350.00
May	11,900.00
June	11,900.00
July	13,400.00
August	13,400.00
September	14,400.00
October	14,400.00
November	15,200.00
December	15,200.00
January	13,200.00
February	13,200.00
March	14,150.00

$$1,57,700.00 \Rightarrow P = ₹ 1,57,700$$

$$I = \frac{1,57,700 \times 5}{100} \times \frac{1}{12}$$

$$= \frac{7885}{12} = ₹ 657.08 = ₹ 657$$

Question 7:

(a) Using componendo and dividendo, find the value of x. [3]

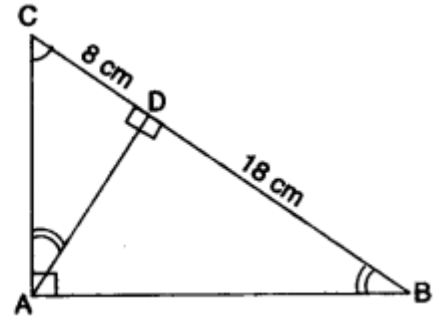
$$\frac{\sqrt{3x+4} + \sqrt{3x-5}}{\sqrt{3x+4} - \sqrt{3x-5}} = 9$$

- (b) If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ and I is the identity matrix of the same order and A^t is the transpose of matrix A , find $A^t \cdot B + BI$. [3]

(c)

In the adjoining figure ABC is a right angled triangle with $\angle BAC = 90^\circ$.

- (i) Prove $\triangle ADB \sim \triangle CDA$.
 (ii) If $BD = 18 \text{ cm}$, $CD = 8 \text{ cm}$, find AD .
 (iii) Find the ratio of the area of $\triangle ADB$ is to area of $\triangle CDA$. [4]



Solution:

(a)

$$\text{Given : } \frac{\sqrt{3x+4} + \sqrt{3x-5}}{\sqrt{3x+4} - \sqrt{3x-5}} = \frac{9}{1}$$

Applying componendo and Dividendo,

$$\frac{\sqrt{3x+4} + \sqrt{3x-5} + \sqrt{3x+4} - \sqrt{3x-5}}{\sqrt{3x+4} + \sqrt{3x-5} - \sqrt{3x+4} + \sqrt{3x-5}} = \frac{9+1}{9-1}$$

$$\frac{2\sqrt{3x+4}}{2\sqrt{3x-5}} = \frac{10}{8}$$

$$\frac{\sqrt{3x+4}}{\sqrt{3x-5}} = \frac{5}{4}$$

Squaring both sides,

$$\frac{3x+4}{3x-5} = \frac{25}{16}$$

Applying Componendo and Dividendo,

$$\frac{3x+4+3x-5}{3x+4-3x+5} = \frac{25+16}{25-16}$$

$$\frac{6x-1}{9} = \frac{41}{9}$$

$$6x = 42$$

(b)

Transpose of matrix A, $A^t = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

$$A^t \cdot B = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 8-1 & -4+3 \\ 20-3 & -10+9 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix}$$

$$B \cdot I = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$A^t \cdot B + B \cdot I = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & -3 \\ 16 & 2 \end{bmatrix}$$

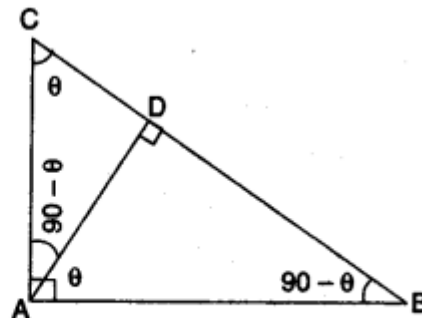
(c)

Let $\angle DAB = \theta$

$\therefore \angle DAC = 90 - \theta$

$\angle DBA = 90 - \theta$

$\angle DCA = \theta$



\therefore All three angles of ΔADB are equal to all angles of ΔCDA .

(i) $\therefore \Delta ADB \sim \Delta CDA$

(ii) $\therefore \frac{CD}{AD} = \frac{AD}{BD}$

$$\Rightarrow AD^2 = CD \times BD$$

$$= 8 \times 18 \Rightarrow AD = 12$$

(iii) $\frac{\Delta ADB}{\Delta CDA} = \frac{\frac{1}{2} AD \times BD}{\frac{1}{2} AD \times CD}$

$$= \frac{BD}{CD} = \frac{18}{8}$$

$$= \frac{9}{4}$$

Question 8:

(a) (i) Using step-deviation method, calculate the mean marks of the following

distribution.

(ii) State the modal class: [5]

Class Interval	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90
Frequency	5	20	10	10	9	6	12	8

(b) Marks obtained by 200 students in an examination are given below:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of Students	5	11	10	20	28	37	40	29	14	6

Draw an ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis. Using the graph, determine

- The median marks.
- The number of students who failed if minimum marks required to pass is 40.
- If scoring 85 and more marks is considered as grade one, find the number of students who secured grade one in the examination. [5]

Solution:

(a)

(i)	C.I.	f	x	$d = x - 67.5$	u	$f \cdot u$
	50-55	5	52.5	-15	-3	-15
	55-60	20	57.5	-10	-2	-40
	60-65	10	62.5	-5	-1	-10
	65-70	10	67.5	0	0	0
	70-75	9	72.5	5	1	9
	75-80	6	77.5	10	2	12
	80-85	12	82.5	15	3	36
	85-90	8	87.5	20	4	32
		$\Sigma f = 80$				$\Sigma fu = 24$

$$\text{A.M.} = 67.5$$

$$\bar{x} = \text{A.M.} + \frac{\sum fu}{\sum f} \times i$$

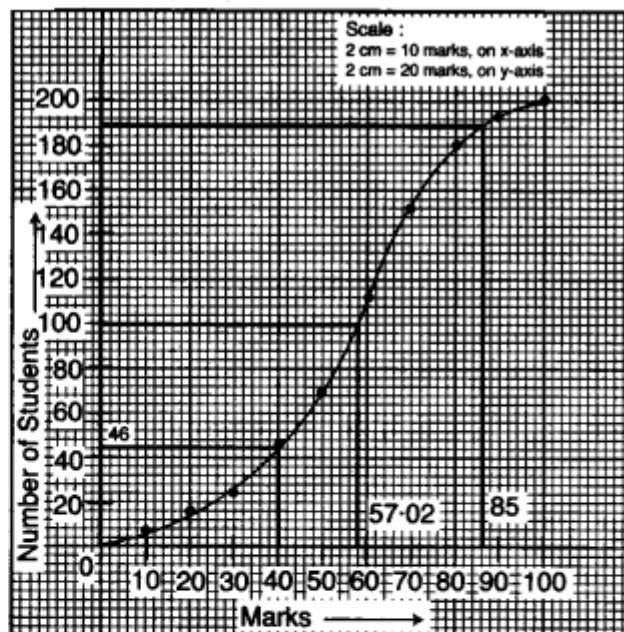
$$= 67.5 + \frac{24}{80} \times 5$$

$$= 67.5 + 1.5 = 69$$

(ii) Modal class is 55–60 (class with heighest freq.)

(b)

C.I.	f	C.F.
0–10	5	5
10–20	11	16
20–30	10	26
30–40	20	46
40–50	28	74
50–60	37	111
60–70	40	151
70–80	29	180
80–90	14	194
90–100	6	200



(i) $n = 200$

$$\begin{aligned} \text{Median} &= \left(\frac{n}{2} \right)^{\text{th}} \text{ observation} \\ &= \left(\frac{200}{2} \right)^{\text{th}} \text{ observation} \\ &= 100^{\text{th}} \text{ observation} = 57.02 \end{aligned}$$

(ii) Number of students who failed = 46

(iii) Number of students who secured grade one = $200 - 188 = 12$.

Question 9:

(a) Mr. Parekh invested Rs. 52,000 on 100 shares at a discount of Rs. 20 paying 8% dividend. At the end of one year he sells the shares at a premium of Rs. 20. Find

(i) The annual dividend.

(ii) The profit earned including his dividend. [3]

(b) Draw a circle of radius 3.5 cm. Mark a point P outside the circle at a distance of 6 cm. from the centre. Construct two tangents from P to the given circle. Measure and write down the length of one tangent. [3]

(c) Prove that $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) \sec^2 A = \tan A$. [4]

Solution:

(a) Investment = Rs. 52,000

N.V = Rs. 100

M.V. of one share = Rs. (100 – 20) = Rs. 80

Dividend = 8%

No. of shares = Investment/MV

$$= \frac{52,000}{80} = ₹ 650$$

$$(i) \quad \text{Annual Dividend} = \frac{8}{100} \times 650 \times 100$$

$$= ₹ 5,200$$

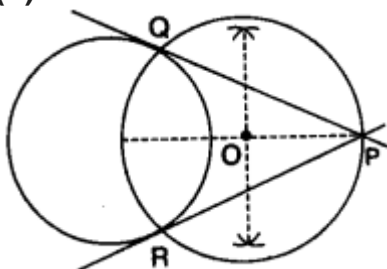
$$(ii) \quad \text{Profit} = \text{Total S.P.} + \text{Dividend} - \text{Investment}$$

$$= 650 \times 120 + 5,200 - 52,000$$

$$= 78,000 + 5,200 - 52,000$$

$$= ₹ 31,200$$

(b)



(c)

$$\begin{aligned} \text{L.H.S.} &= (\operatorname{cosec} A - \sin A)(\sec A - \cos A) \cdot \sec^2 A \\ &= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) \cdot \sec^2 A \\ &= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) \cdot \sec^2 A \\ &= \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} \times \frac{1}{\cos^2 A} \\ &= \frac{\sin A}{\cos A} \\ &= \tan A = \text{R.H.S.} \end{aligned}$$

Question 10:

(a) 6 is the mean proportion between two numbers x and y and 48 is the third proportional of x and y. Find the numbers. [3]

(b) In what period of time will Rs. 12,000 yield Rs. 3,972 as compound interest at 10% per annum, if compounded on an yearly basis? [3]

(c) A man observes the angle of elevation of the top of a building to be 30°. He walks towards it in a horizontal line through its base. On covering 60 m the angle of elevation changes to 60°. Find the height of the building correct to the nearest

metre. [4]

Solution:

(a)

$$\begin{aligned} xy &= 6^2 \\ \Rightarrow xy &= 36 \end{aligned} \quad \dots(1)$$

$$x : y :: y : 48$$

$$\begin{aligned} \frac{x}{y} &= \frac{y}{48} \\ \Rightarrow y^2 &= 48x \end{aligned} \quad \dots(2)$$

Substituting the value of x from (1),

$$y^2 = 48 \times \frac{36}{y}$$

$$\begin{aligned} \Rightarrow y^3 &= 48 \times 36 \\ y^3 &= 6 \times 8 \times 6 \times 6 = (6)3 \times (2)3 \\ \therefore y &= 12 \\ \therefore x &= 3 \end{aligned} \quad \Bigg]$$

(b)

$$\text{Let } A = P \left(1 + \frac{R}{100} \right)^n$$

$$\Rightarrow 15,972 = 12,000 \left(1 + \frac{10}{100} \right)^n$$

$$\frac{1331}{1000} = \left(\frac{11}{10} \right)^n$$

$$\therefore \left(\frac{11}{10} \right)^3 = \left(\frac{11}{10} \right)^n$$

$$\Rightarrow n = 3 \text{ years.}$$

(c)

Let $BC = x$ and $AB = h$

In right angled $\triangle ADB$

$$\tan 30^\circ = \frac{h}{60 + x}$$

$$\Rightarrow 60 + x = h\sqrt{3}$$

Now right angled $\triangle ACB$

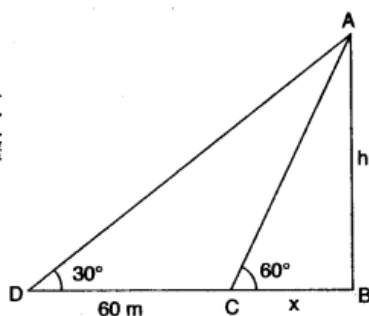
$$\tan 60^\circ = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}}$$

Equating 'x',
$$h\sqrt{3} - 60 = \frac{h}{\sqrt{3}}$$

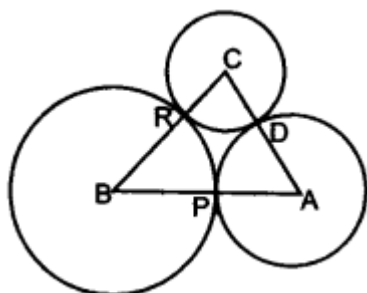
$$\Rightarrow 3h - 60\sqrt{3} = h$$
$$2h = 60\sqrt{3}$$

$$\therefore h = 30\sqrt{3}$$
$$= 51.96 \text{ m.}$$



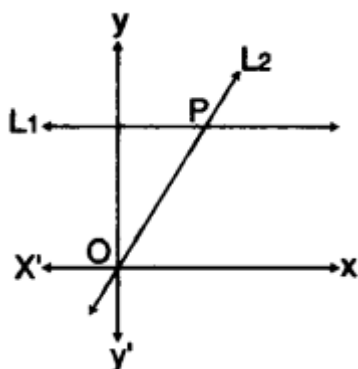
Question 11:

(a) ABC is a triangle with $AB = 10$ cm, $BC = 8$ cm and $AC = 6$ cm (not drawn to scale). Three circles are drawn touching each other with the vertices as their centres. Find the radii of the three circles. [3]



(b) Rs. 480 is divided equally among 'x' children. If the number of children was 20 more then each would have got Rs. 12 less. Find 'x'. [3]

(c) Given equation of line L_1 is $y = 4$.



- (i) Write the slope of line L_2 if L_2 is the bisector of angle O.
- (ii) Write the co-ordinates of point P.
- (iii) Find the equation of L_2 . [4]

Solution:

(a) Let the three radii be x, y, z respectively.

$$x + y = 10 \quad \dots\dots\dots (1)$$

$$y + z = 8 \quad \dots\dots\dots (2)$$

$$x + z = 6 \quad \dots\dots\dots (3)$$

Adding equation's (1), (2) and (3), $2x + 2y + 2z = 24$

$$x + y + z = 12 \quad \dots\dots\dots (4)$$

Subtracting each equation (1), (2) and (3) from equation (4), we get

$$z = 2 \text{ cm, } x = 4 \text{ cm, } y = 6 \text{ cm.}$$

(b)

$$\text{Initial share of each child} = \frac{480}{x}$$

$$\text{New share of each child} = \frac{480}{x + 20}$$

Difference in share is ₹ 12.

$$\frac{480}{x} - \frac{480}{x + 20} = 12$$

$$\frac{1}{x} - \frac{1}{x + 20} = \frac{12}{480} = \frac{1}{40}$$

$$\frac{x + 20 - x}{x(x + 20)} = \frac{1}{40}$$

$$x^2 + 20x = 800$$

$$x^2 + 20x - 800 = 0$$

$$x^2 + 40x - 20x - 800 = 0$$

$$x(x + 40) - 20(x + 40) = 0$$

$$(x - 20)(x + 40) = 0$$

$$x = 20 \text{ or } x = -40 \text{ (not possible)}$$

$$\therefore x = 20$$

(c)

(i) Slope of L_2 is $m = \tan 45^\circ$

$\therefore m = 1$ (L_2 makes an angle of 45° with X axis)

(ii) Equation of line L_2

$$y - 0 = 1(x - 0)$$

It passes through (0, 0)

\therefore Equation of L_2 is $y = x$

P can be obtained by solving L_1 and L_2 simultaneously,

$$L_1 \quad y = 4$$

$$L_2 \quad y = 4$$

On solving, we get $x = 4, y = 4$

Co-ordinate of P (4, 4)

(iii) Equation of L_2 is $y = x$ [as solved above part (ii)].