ICSE Paper 2012
MATHEMATICS

SOLVED PAPER

(Two hours and a half)
Answers to this Paper must be written on the paper provided separately.
You will not be allowed to write during the first 15 minutes.
This time is to be spent in reading the question paper.
The time given at the head of this Paper is the time allowed for writing the answers.
*Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*
All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer. Omission of essential working will result in the loss of marks.
The intended marks for questions or parts of questions are given in brackets [ ].
Mathematical tables are provided.

SECTION-A (40 Marks)
*(Attempt **all** questions from this Section)*

Question 1:

(a) *If* $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ *and* $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, *find* $A^2 - 5A + 7I$. **[3]**

(b) The monthly pocket money of Ravi and Sanjeev are in the ratio 5 : 7. Their expenditures are in the ratio 3 : 5. If each saves Rs. 80 every month, find their monthly pocket money. **[3]**

(c) Using the Remainder Theorem factorise completely the following polynomial:
$3x^3 + 2x^2 - 19x + 6$

Solution:

(a) Let $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

than $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8-15+7 & 5-5+10 \\ -5+5+0 & 3-10+7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

(b)

Let monthly pocket money of Ravi is $5x$ and Sanjeev is $7x$.

$$\frac{5x-80}{7x-80} = \frac{3}{5}$$

$$\Rightarrow \quad 25x - 400 = 21x - 240$$

$$\therefore \quad 4x = 160$$

$$\therefore \quad x = 40$$

$$\left.\begin{aligned} \text{Ravi's pocket money} &= 5 \times 40 = ₹\,200 \\ \text{Sanjeev's pocket money} &= 7 \times 40 = ₹\,280 \end{aligned}\right\}$$

(c)

Let $f(x) = 3x^3 + 2x^2 - 19x + 6$

Using hit and trial method,

$$f(1) = 3 + 2 - 19 + 6 \neq 0$$

$$f(-1) = -3 + 2 + 19 + 6 \neq 0$$

$$f(2) = 24 + 8 - 38 + 6 = 0$$

$\therefore$ $(x - 2)$ is a factor of $f(x)$.

Now,

$$\begin{array}{r|l} & 3x^2 + 8x - 3 \\ \hline x-2 & 3x^3 + 2x^2 - 19x + 6 \\ & 3x^3 - 6x^2 \\ \hline & \quad 8x^2 - 19x \\ & \quad 8x^2 - 16x \\ \hline & \qquad -3x + 6 \\ & \qquad -3x + 6 \\ \hline & \qquad \times \\ \hline \end{array}$$

To factorise $3x^2 + 8x - 3$

$$= 3x^2 + 9x - x - 3$$

$$= 3x(x + 3) - 1(x + 3)$$

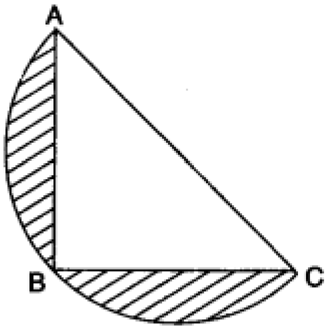
$$= (3x - 1)(x + 3)$$

Hence $3x^3 + 2x^2 - 19x + 6 = (x - 2)(3x - 1)(x + 3)$

Question 2:

(a) On what sum of money will the difference between the compound interest and simple interest for 2 years be equal to Rs. 25 if the rate of interest charged for both is 5% p.a.? **[3]**

(b) ABC is an isosceles right angled triangle with ∠ABC = 90°. A semi-circle is drawn with AC as the diameter. If AB = BC = 7 cm, find the area of the shaded region. (Take n = 22/7) **[3]**



(c) Given a line segment AB joining the points A (-4, 6) and B (8, -3). Find:
(i) the ratio in which AB is divided by the y-axis.
(ii) find the coordinates of the point of intersection.
(iii) the length of AB.

Solution:

(a) Let the principal be ₹ P.

Given : R = 5%, T = 24 years

$$\text{C.I. for 2 years} = P\left(1+\frac{5}{100}\right)^2 - P$$

$$\text{S.I. for 2 years} = \frac{P \times 5 \times 2}{100} = \frac{P}{10}$$

$\because$ Difference between C.I. and S.P. = ₹ 25

$$P\left(1+\frac{5}{100}\right)^2 - P - \frac{P}{10} = 25$$

$$\frac{441\,P}{400} - \frac{11P}{10} = 25$$

$$\frac{441P - 440P}{400} = 25$$

$$P = 10{,}000$$

Hence, the principle be 10,000

(b)

Let ABC is a right angled triangle. So

$$AC^2 = AB^2 + BC^2$$
$$= (7)^2 + (7)^2 = 2(7)^2$$
$$AC = 7\sqrt{2}$$
$$\text{Area of semi circle} = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7\sqrt{2}}{2}\right)^2$$
$$= \frac{1}{2} \times \frac{22}{7} \times \frac{49 \times 2}{4} = 38{\cdot}5 \text{ cm}^2$$
$$\text{Area of } \Delta \text{ ABC} = \frac{1}{2} \times 7 \times 7 = 24{\cdot}5 \text{ cm}^2$$
$$\therefore \quad \text{Area of shaded region} = \text{Area of semi circle} - \text{Area of } \Delta \text{ ABC.}$$
$$= 38{\cdot}5 - 24{\cdot}5 = 14 \text{ cm}^2.$$

(c)

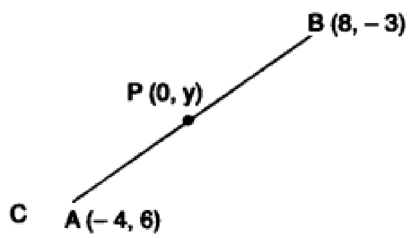
Let P be the point at which

(i) AB intersect y-axis

Let $\quad AP : PB = m : n$

$$x = \frac{mx_1 + nx_2}{m + n}$$

and
$$y = \frac{my_1 + ny_2}{m + n}$$
$$0 = \frac{m \cdot 8 + n(-4)}{m + n}$$
$$\frac{8m - 4n}{m + n} = 0$$
$$\Rightarrow \quad 8m = 4n$$
$$\therefore \quad m : n = 1 : 2$$



(ii)
$$y = \frac{my_2 + ny_1}{m + n}$$
$$= \frac{1 \times (-3) + 2 \times 6}{1 + 2}$$

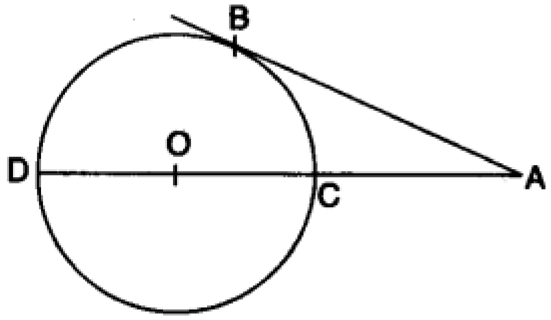
Using the above ratio, $\quad y = \dfrac{-3 + 12}{1 + 2} = 3$

$\therefore$ Point of P be (0, 3)

(iii)
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-4 - 8)^2 + (6 + 3)^2}$$
$$= \sqrt{144 + 81} = 15 \text{ units.}$$

Question 3:

(a) In the given figure O is the centre of the circle and AB is a tangent at B. If AB = 15 cm and AC = 7.5 cm. Calculate the radius of the circle. **[3]**



(b) Evaluate without using trigonometric tables: **[3]**

$$\cos^2 26° + \cos 64° \sin 26° + \frac{\tan 36°}{\cot 54°}$$

(c) Marks obtained by 40 students in a short assessment is given below, where a and b are two missing data.

Marks	5	6	7	8	9
No. of students	6	a	16	13	6

If the mean of the distribution is 7.2, find a and b. **[4]**

Solution:

(a)

Applying intercept theorem

$$AC \times AD = AB^2$$

$$7{\cdot}5 \times (7{\cdot}5 + 2R) = 15^2$$

where R is the radius of the circle

$$(7{\cdot}5 + 2R) = \frac{15 \times 15}{7{\cdot}5} = 30$$

$$2R = 22{\cdot}5$$

$$\Rightarrow \quad R = 11{\cdot}25 \text{ cm.}$$

(b)

$$\text{Given : } \cos^2 26° + \cos 64° \sin 26° + \frac{\tan 36°}{\cot 54°}$$

$$= \cos^2 26° + \cos(90° - 26°)\cdot\sin 26° + \frac{\tan(90° - 54°)}{\cot 54°}$$

$$= (\cos^2 26° + \sin^2 26°) + \frac{\cot 54°}{\cot 54°}$$

$$= 1 + 1 = 2$$

(c)

$$\text{Let} \quad 6 + a + 16 + 13 + b = 40$$

$$\Rightarrow \quad a + b = 5 \quad \ldots\text{(i)}$$

$$\text{Mean} \quad \bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$7{\cdot}2 = \frac{30 + 6a + 112 + 104 + 9b}{40}$$

$$\Rightarrow \quad 246 + 6a + 9b = 288$$

$$6a + 9b = 42$$

$$\therefore \quad 2a + 3b = 14 \quad \ldots\text{(ii)}$$

Solving (i) and (ii), we get

$$b = 4, \; a = 1$$

Question 4:

(a) Kiran deposited Rs. 200 per month for 36 months in a bank's recurring deposit account. If the bank pays interest at the rate of 11% per annum, find the amount she gets on maturity. **[3]**

(b) Two coins are tossed once. Find the probability of getting:

(i) 2 heads

(ii) at least 1 tail. **[3]**

(c) Using graph paper and taking 1 cm = 1 unit along both x-axis and y-axis.

(i) Plot the points A(-4, 4) and B (2, 2)

(ii) Reflect A and B in the origin to get the images A' and B' respectively.

(iii) Write down the co-ordinates of A' and B'.

(iv) Give the geometrical name for the figure ABA'B'.

(v) Draw and name its lines of symmetry. **[4]**

Solution:

(a)

Given : P per month = ₹ 200, Time (n) = 36 months, R = 11%.

$$\text{Equivalent principal for 36 months} = 200 \times \frac{n(n+1)}{2}$$

$$= 200 \times \frac{36 \times 37}{2} = 36 \times 37 \times 100$$

$$\text{Interest} = \frac{PRT}{100} = \frac{36 \times 37 \times 100 \times 11 \times 1}{100 \times 12}$$

$$= ₹\ 1221$$

$$\text{Maturity Amount} = Pn + \text{Interest} = 200 \times 36 + 1221$$

$$= ₹\ 8421.$$

(b)

If two coins are tossed once, then

$$\text{Sample Space (S)} = \{HH, HT, TH, TT\}$$

$\therefore$ $n(S) = 4$

(i) E : getting two heads = {H H}

$\therefore$ $n(E) = 1$

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$

(ii) E : At least one tail = {HT, TH, TT}

$\therefore$ $n(E') = 3$

$\therefore$ $P(E') = \frac{n(E')}{n(S)} = \frac{3}{4}$

(c)

(i) See graph

(ii) See Graph.

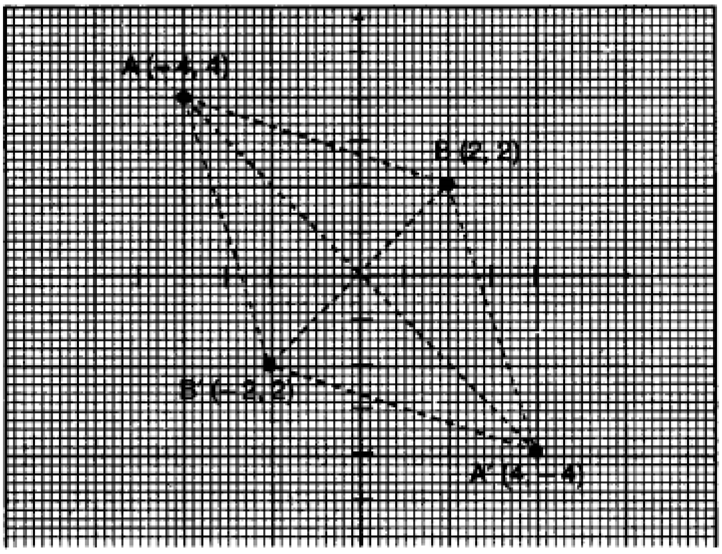
(iii) A′ (4, –4)

B′ (–2, –2)

(iv) Rhombus

(v) Two lines of symmetry.

Both diagonals, AA′ and BB′



SECTION-B (40 Marks)

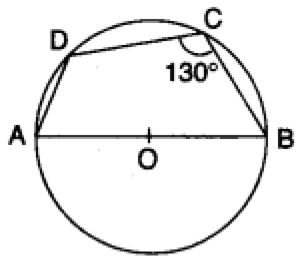
*(Attempt **any four** questions from this Section)*

Question 5:

(a) *In the given figure, AB is the diameter of a circle with centre O.*

$\angle BCD = 130°$. Find :

(i) $\angle DAB$, (ii) $\angle DBA$ **[3]**



(b) *Given* $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$. *Write :*

(i) *the order of the matrix X*

(ii) *the matrix X.* **[3]**

(c) A page from the Savings Bank Account of Mr. Prateek is given below:

Date	Particulars	Withdrawal (In ₹)	Deposit (In ₹)	Balances (In ₹)
Jan. 1st 2006	B/F	—	—	1,270
Jan. 7th 2006	By Cheque	—	2,310	3,580
March 9th 2006	To Self	2,000	—	1,580
March 26th 2006	By Cash	—	6,200	7,780
June 10th 2006	To Cheque	4,500	—	3,280
July 15th 2006	By Clearing	—	2,630	5,910
October 18th 2006	To Cheque	530	—	5,380
October 27th 2006	To Self	2,690	—	2,690
November 3rd 2006	By Cash	—	1,500	4,190
December 6th 2006	To Cheque	950	—	3,240
December 23rd 2006	By Transfer	—	2,920	6,160

If he receives ₹ 198 as interest on 1st January, 2007, find the rate of interest paid by the bank. **[4]**

Solution:

(a)

On joining BD, $\angle$ ADB is in the semicircle.

$$\angle ADB = 90°$$

(Angle in a simicircle is right angle)

(i) Let ABCD is a cyclic quadilateral.

$$\therefore \quad \angle BCD + \angle DAB = 180°$$

$$130° + \angle DAB = 180°$$

$$\angle DAB = 180° - 130° = 50°$$

(ii) Now, $\angle BAD + \angle BDA + \angle DBA = 180°$

$$90° + 50° + \angle DBA = 180°$$

$$\angle DBA = 40°$$

(b)

(i) Order of matrix X is 2×1.

(ii) Let $X = \begin{bmatrix} a \\ b \end{bmatrix}$

$$\because \quad \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 2a + b \\ -3a + 4b \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

$$2a + b = 7 \quad \ldots\ldots(1)$$

$$-3a + 4b = 6 \quad \ldots\ldots(2)$$

Solving (1) and (2), we get

$$a = 2,\ b = 3$$

$$\therefore \quad X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

(c)

Months	**Minimum Balance**
January	3,580
February	3,580
March	1,580
April	7,780
May	7,780

June	3,280
July	3,280
August	5,910
September	5,910
October	2,690
November	4,190
December	3,240
Total	**Rs.** 52,800

Now, Principal = ₹ 52,800, Time = 1 month = $\frac{1}{12}$ year, Interest = ₹ 198

$$I = \frac{PRT}{100}$$

$$198 = \frac{52{,}800 \times R \times 1}{100 \times 12}$$

$$\therefore \quad R = 4 \cdot 5\%.$$

Question 6:

(a) The printed price of an article is ₹ 60,000. The wholesaler allows a discount of 20% to the shopkeeper. The shopkeeper sells the article to the customer at the printed price. Sales tax (under VAT) is charged at the rate of 6% at every stage. Find:

(i) the cost to the shopkeeper inclusive of tax.
(ii) VAT paid by the shopkeeper to the Government.
(iii) the cost to the customer inclusive of tax. **[3]**
(b) Solve the following inequation and represent the solution set on the number line:
$4x - 19 < \frac{3x}{5} - 2 \leq \frac{-2}{5} + x, x \in R$ **[3]**
(c) Without solving the following quadratic equation, find the value of 'm' for which the given equation has real and equal roots.
$x^2 + 2(m - 1)x + (m + 5) = 0$ **[3]**

Solution:
(a)
(i) *Given* : Printed price of the article = ₹ 60,000
and discount = 20% of ₹ 60,000

$$= \frac{20}{100} \times 60,000 = ₹\ 12,000$$

$\because$ Sale price of the article $= 60,000 - 12,000 = ₹\ 48,000$
Sales tax paid by the shopkeeper = 6% of ₹ 48,000

$$= \frac{6}{100} \times 48,000 = ₹\ 2,880$$

$\because$ The cost of the shopkeeper inclusive of tax

$$= 48,000 + 2,880$$
$$= ₹\ 50,880.$$

(ii) VAT paid by shopkeeper = Tax charged – Tax paid

$$= 60,000 \times \frac{6}{100} - 48,000 \times \frac{6}{100}$$
$$= ₹\ 720$$

(iii) Sales tax paid by the customer = 6% of ₹ 60,000

$$= \frac{6}{100} \times 60,000 = ₹\ 3,600$$

$\because$ The cost to the customer inclusive of tax :

$$= 60,000 + 3,600$$
$$= ₹\ 63,600.$$

(b)

Given : $4x - 19 < \frac{3x}{5} - 2 \leq -\frac{2}{5} + x$

$$4x - 19 < \frac{3x}{5} - 2 \quad \text{and} \quad \frac{3x}{5} - 2 \leq -\frac{2}{5} + x$$

$$\frac{17x}{5} < 17 \quad \text{and} \quad -\frac{2x}{5} \leq \frac{8}{5}$$

$$\Rightarrow \quad x < 5 \quad \Rightarrow \quad x \geq -4$$

$$\text{Solution set} = \{x : 5 > x \geq -4\}$$

(c)

Given : $x^2 + 2(m-1)x + (m+5) = 0$ ...(i)

For real and equal roots,

$$b^2 - 4ac = 0$$

$$b^2 = 4ac$$

Comparing given equation (i) with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 2(m-1), c = (m+5)$$

Now,

$$4(m-1)^2 = 4(m+5)$$

$$m^2 - 3m - 4 = 0$$

$$m^2 - 4m + m - 4 = 0$$

$$m(m-4) + 1(m-4) = 0$$

$$\therefore \quad m = 4 \text{ or } m = -1$$

Question 7:

(a) A hollow sphere of internal and external radii 6 cm and 8 cm respectively is melted and recast into small cones of base radius 2 cm and height 8 cm. Find the number of cones. **[3]**

(b) Solve the following equation and give your answer correct to 3 significant figures: $5x^2 - 3x - 4 = 0$ **[3]**

(c) As observed from the top of a 80 m tall lighthouse, the angles of depression of two ships on the same side of the light house in horizontal line with its base are 30° and 40° respectively. Find the distance between the two ships. Give your answer correct to the nearest metre. **[4]**

Solution:

(a)

Given : External Radius R = 8 cm, Internal Radius = 6 cm,

$$\text{Volume of hollow spheres} = \frac{4}{3}\pi(R^3 - r^3).$$

$$\text{Volume of hollow spheres} = \frac{4}{3}\pi[8^3 - 6^3]$$

$$= \frac{4}{3}\pi[512 - 216] = \frac{4}{3}\pi(296)$$

$$\text{Volume of cones} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi(2)^2(8)$$

$$\text{Number of cones} = \frac{\text{Volume of sphere}}{\text{Volume of cones}} = \frac{\frac{4}{3}\pi[296]}{\frac{1}{3}\pi \times 4 \times 8}$$

$$= \frac{296}{8} = 37 \text{ cones.}$$

(b)

Given : $5x^2 - 3x - 4 = 0$

Comparing given equation with $ax^2 + bx + c = 0$, we get

$$a = 5, b = -3, c = -4$$

Let

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 5 \times (-4)}}{2 \times 5}$$

$$x = \frac{3 \pm \sqrt{9 + 80}}{10} = \frac{3 \pm \sqrt{89}}{10} = \frac{3 \pm 9{\cdot}43}{10}$$

Taking +*ve* sign

$$x = \frac{3 + 9{\cdot}43}{10} = 1{\cdot}243$$

and taking – *ve* sign

$$x = \frac{3 - 9{\cdot}43}{10} = -0{\cdot}643$$

$$x = 1{\cdot}243 \text{ or } x = -0{\cdot}643$$

(c)

In Δ ABC, $\tan 50° = \frac{BC}{80}$

$\Rightarrow \quad BC = 80 \times 1{\cdot}1918$

$\therefore \quad BC = 95{\cdot}34$ m

In Δ ABD, $\tan 60° = \frac{BD}{80}$

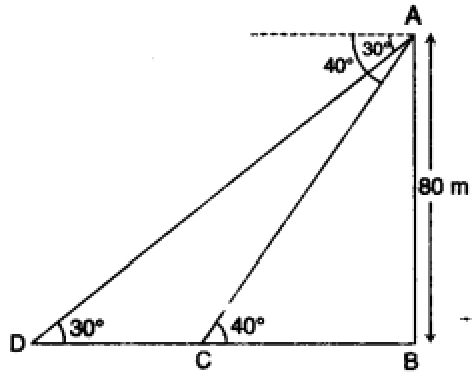
$\therefore \quad BD = 80\sqrt{3}$

$BD = 138{\cdot}56$ m

$\therefore \quad CD = BD - BC$

$= 138{\cdot}56 - 95{\cdot}34$

$= 43{\cdot}2$ m. **Ans.**



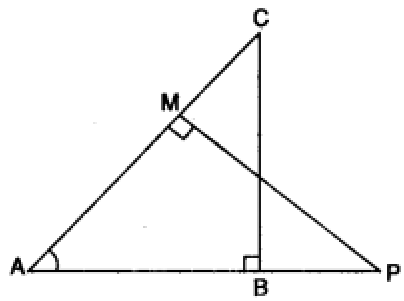
Question 8.
(a) A man invests Rs. 9,600 on Rs. 100 shares at Rs. 80. If the company pays him 18% dividend find:
(i) the number of shares he buys.
(ii) his total dividend.
(iii) his percentage return on the shares.

(b) *In the given figure Δ ABC and Δ AMP are right angled at B and M respectively.*

Given AB = 10 cm, AP = 15 cm and PM = 12 cm.

(i) *Prove $\Delta ABC \sim \Delta AMP$.*

(ii) *Find AB and BC.* **[3]**



(c) *If* $x = \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$, *using properties of proportion show that* $x^2 - 2ax + 1 = 0$. **[4]**

Solution:
(a)

(i) $$\text{Number of shares} = \frac{\text{Investment}}{\text{M.V. of each share}}$$

$$= \frac{9600}{80} = 120 \text{ shares}$$

(ii) $$\text{Total dividend} = \frac{18}{100} \times 120 \times 100$$

$$= ₹\,2{,}160$$

(iii) Since $$\text{N.V} \times \text{Div\%} = \text{M.V.} \times \text{Return \%}$$

$$\text{Return \%} = \frac{100 \times 189}{80}$$

$$= 22{\cdot}5\%$$

(b)

(i) In Δ ABC and Δ APM,

$$\angle ABC = \angle AMP = 90°$$

$$\angle BAC = \angle PAM \text{ (Common)}$$

$\therefore$ $$\Delta ABC \sim \Delta APM$$

(ii) Also, $$\frac{AC}{AP} = \frac{BC}{PM}$$

$\Rightarrow$ $$\frac{10}{15} = \frac{BC}{12}$$

$\therefore$ $$BC = 8 \text{ cm.}$$

$\because$ Δ ABC is right angled Δ.

Applying Pythagorous, $$AB^2 = AC^2 - BC^2$$

$$= 10^2 - 8^2 = 36$$

$\therefore$ $$AB = 6 \text{ cm.}$$

(c)

Given : $$\frac{x}{1} = \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$$

Using componendo and dividendo,

$$\frac{x+1}{x-1} = \frac{\sqrt{a+1}}{\sqrt{a-1}}$$

Squaring both sides, $$\frac{(x+1)^2}{(x-1)^2} = \frac{a+1}{a-1}$$

again using componendo and dividendo,

$$\frac{x^2+1}{2x} = \frac{a}{1}$$

$$\Rightarrow \quad x^2-2ax+1 = 0$$

Question 9:

(a) The line through A (-2, 3) and B (4, b) is perpendicular to the line 2x – 4y = 5. Find the value of b. **[3]**

(b) *Prove that* $\frac{\tan^2\theta}{(\sec\theta-1)^2} = \frac{1+\cos\theta}{1-\cos\theta}$

(c) A car covers a distance of 400 km at a certain speed. Had the speed been 12 km/h more, the time taken for the journey would have been 1 hour 40 minutes less. Find the original speed of the car. **[4]**

Solution:

(a)

$$\text{Slope of AB } (m_1) = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{b-3}{4+2} = \frac{b-3}{6}$$

Equation of given line $$2x-4y = 5$$

$$4y = 2x-5$$

$$y = \frac{1}{2}x-\frac{5}{4}$$

$$\text{Slope of given line } (m_2) = \frac{1}{2}$$

As per the question, line are perpendicular.

$$m_1 \cdot m_2 = -1$$

$$\therefore \quad \frac{b-3}{6}\times\frac{1}{2} = -1$$

$$\Rightarrow \quad b-3 = -12$$

$$\therefore \quad b = -9$$

(b)

$$\text{L.H.S.} = \frac{\tan^2\theta}{(\sec\theta - 1)^2}$$

$$= \frac{\sec^2\theta - 1}{(\sec\theta - 1)^2}$$

$$= \frac{(\sec\theta - 1)(\sec\theta + 1)}{(\sec\theta - 1)^2}$$

$$= \frac{\sec\theta + 1}{\sec\theta - 1} = \frac{\frac{1}{\cos\theta} + 1}{\frac{1}{\cos\theta} - 1}$$

$$= \frac{1 + \cos\theta}{1 - \cos\theta} = \text{R.H.S.}$$

(c)

Usual time $= \frac{400}{x}$, New speed $= x + 12$, New time $= \frac{400}{x + 12}$

According to the condition :

$$\frac{400}{x} - \frac{400}{x + 12} = \frac{5}{3}$$

$$\frac{x + 12 - x}{x(x + 12)} = \frac{1}{240}$$

$$x^2 + 12x - 2880 = 0$$

$$x^2 + 60x - 48x - 2880 = 0$$

$$x(x + 60) - 48(x + 60) = 0$$

$$x = -60 \text{ or } x = 48$$

But speed can not be negative.

$\therefore$ Original speed $= 48$ km/hr

Question 10:

(a) Construct a triangle ABC in which base BC = 6 cm, AB = 5.5 cm and ∠ABC = 120°.

(i) Construct a circle circumscribing the triangle ABC.

(ii) Draw a cyclic quadrilateral ABCD so that D is equidistant from B and C. **[3]**

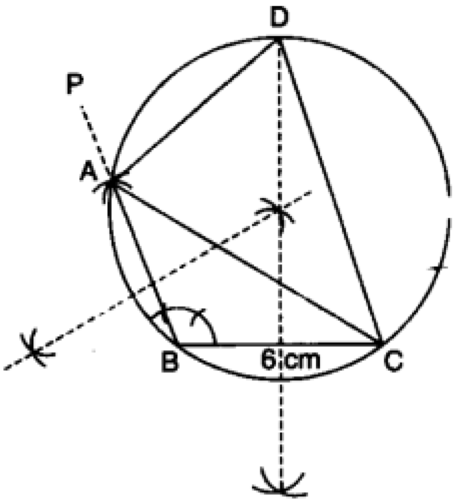
(b) The following distribution represents the height of 160 students of a school.

Height (in cm)	140-145	145-150	150-155	155-160	160-165	165-170	170-175	175-180

No. of Students	12	20	30	38	24	16	12	8

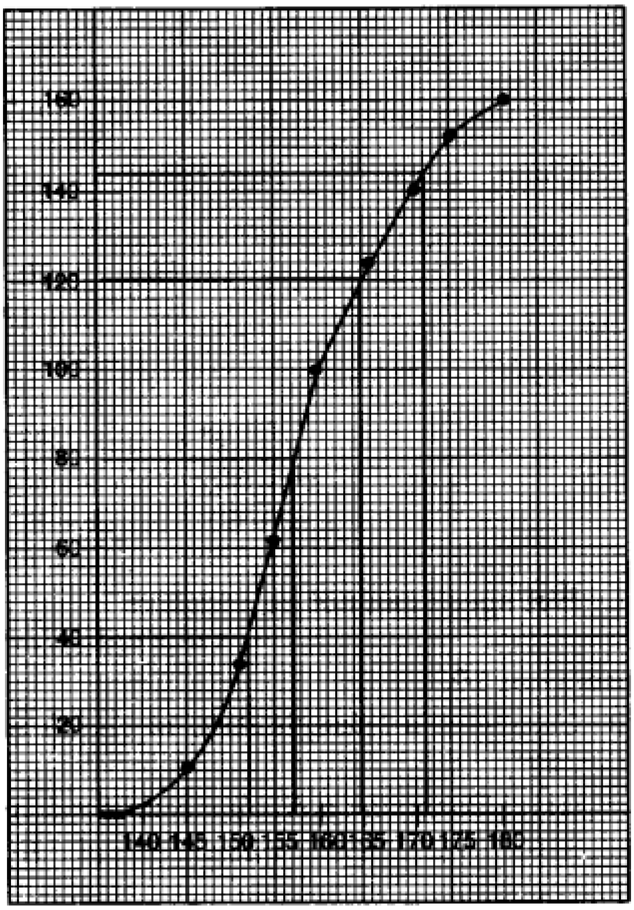
Draw an ogive for the given distribution taking 2 cm = 5 cm of height on one axis and 2 cm = 20 students on the other axis. Using the graph, determine:
(i) The median height.
(ii) The interquartile range.
(iii) The number of students whose height is above 172 cm. **[6]**

Solution:



(a) (i) Steps of constructions:
(1) Draw a line segment BC = 6 cm.
(2) Construct ∠CBP = 120°.
(3) Cut BA = 5.5 cm from BP.
(4) Join A to C.
(5) Construct perpendicular bisectors of AB and BC, intersecting at O. Join AO.
(6) Taking as the centre and OA as radius draw a circle, passing through, A, B, and C.
(ii) (1) Extend the right bisector of BC intersecting the circle at D.
(2) Join A to D and C to D.
(3) ABCD is required cyclic
(b)

Height	f	$c.f.$
140–145	12	12
145–150	20	32
150–155	30	62
155-160	38	100
160-165	24	124
165-170	16	140
170-175	12	152
175-180	8	160



(i) Me = 157·3

(ii) Interquartile range

$= Q_3 - Q_1$

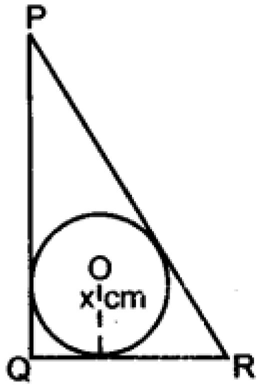
$= 164{\cdot}1 - 151{\cdot}3 = 12{\cdot}8$

(iii) No. of students above 172 cm

$= 160 - 144 = 16.$

Question 11:

(a) In triangle PQR, PQ = 24 cm, QR = 7 cm and ∠PQR = 90°. Find the radius of the inscribed circle. **[3]**

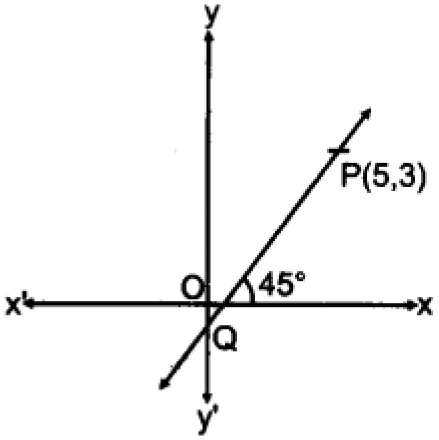


(b) Find the mode and median of the following frequency distribution: **[3]**

x	10	11	12	13	14	15

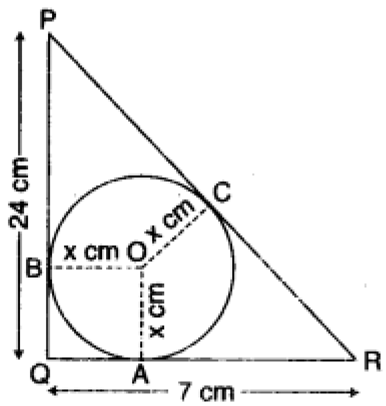
f	1	4	7	5	9	3

(c) The line through P(5, 3) intersects y axis at Q.
(i) Write the slope of the line.
(ii) Write the equation of the line.
(iii) Find the co-ordinates of Q. **[4]**



Solution:
(a)

Given : Δ PQR is right angled.



$$PR^2 = PQ^2 + QR^2 = (24)^2 + (7)^2$$
$$= 576 + 49 = 625$$
$\therefore$ $PR = 25$ cm

Draw $\perp r$ from O on PQ and PR and mark as B and C respectively.

$$\angle OBQ = \angle OAQ = \angle OCR$$
$$= 90°$$
($\angle$ between radius and tangent is 90°)

All $\angle s$ of OAQB are 90° and QA = QB

(Since the tangent to a circle from an exterior point are equal in length).

$\therefore$ OAQB is a square.

$\therefore$ $QA = QB = x$
$AR = 7 - x = RC$ ($\because$ AR = RC) Tangents from ext.
$BP = 12 - x = PC$ ($\because$ PB = PC) point are equal

$\because$ $PC + RC = PR$
$7 - x + 12 - x = 25$
$\therefore$ $x = 3$ cm

(b) Mode is the value of the highest frequency.
$\therefore$ Mode = 14
For Median, we write the data in ascending order,

10, 11, 11, 11, 11, 12, 12, 12, 12, 12, 12, 12, 13, 13, 13, 13, 13, 14, 14, 14, 14, 14, 14, 14, 14, 14, 15, 15, 15.

∵ Median is the middle most value.

$$\therefore \quad M_e = \left(\frac{N+1}{2}\right)^{th} \text{ observation}$$

$$= \left(\frac{29+1}{2}\right)^{th} = 15\text{th observation} = 13$$

(c)

(i) Slope of line PQ = tan 45° = 1

(ii) Equation of line PQ :

$$y - y_1 = m(x - x_1)$$
$$y - 3 = 1(x - 5)$$
$$y = x - 2$$

(iii) Put $x = 0$ in equation of line PQ.

$$y = -2$$
$$\text{Coordinate of Q} = (0, -2)$$