ICSE Paper 2013
MATHEMATICS

(Two hours and a half)

Answers to this Paper must be written on the paper provided separately.
You will not be allowed to write during the first 15 minutes.
This time is to be spent in reading the question paper.
The time given at the head of this Paper is the time allowed for writing the answers.
*Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*
All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer. Omission of essential working will result in the loss of marks.
The intended marks for questions or parts of questions are given in brackets [ ].
Mathematical tables are provided.

SECTION-A (40 Marks)
(Attempt all questions from this Section)

Question 1:
(a)

$$A=\begin{bmatrix}2 & -6\\ 2 & 0\end{bmatrix}, B=\begin{bmatrix}-3 & 2\\ 4 & 0\end{bmatrix}, C=\begin{bmatrix}4 & 0\\ 0 & 2\end{bmatrix}$$

Find the matrix X such that A + 2X = 2B + C. **[3]**
(b) At what rate % p.a. will a sum of Rs. 4000 yield Rs. 1324 as compound interest in 3 years? **[3]**
(c) The median of the following observations 11, 12, 14, (x – 2), (x + 4), (x + 9), 32, 38, 41 arranged in ascending order is 24. Find the value of x and hence find the mean. **[4]**

Solution:
(a)

Given : $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

$\because \quad A + 2X = 2B + C$

Putting the given values, we get

$$\begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$2X = \begin{bmatrix} -6+4 & 4+0 \\ 8+0 & 0+2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix}$$

(b)

Given : Principal = ₹ 4,000, C.I. = ₹ 1,324,

Amount = P + C.I.

= ₹ (4,000 + 1,324) = ₹ 5,324

Time = 3 years

We know that, $A = P\left(1 + \frac{r}{100}\right)^T$

$$5{,}324 = 4{,}000\left(1 + \frac{r}{100}\right)^3$$

$$\frac{5{,}324}{4{,}000} = \left(1 + \frac{r}{100}\right)^3$$

$$\frac{1{,}331}{1{,}000} = \left(1 + \frac{r}{100}\right)^3$$

$$\left(\frac{11}{10}\right)^3 = \left(1 + \frac{r}{100}\right)^3$$

Therefore, $1 + \frac{r}{100} = \frac{11}{10}$

$$\frac{r}{100} = \frac{11}{10} - 1$$

$$\frac{r}{100} = \frac{1}{10}$$

$$r = \frac{100}{10}$$

$$r = 10\%$$

(c)

Given observation are 11, 12, 14, $(x-2)$, $(x+4)$, $(x+9)$, 32, 38, 47 and median = 24.

$$\because \quad n = 9 \text{ (odd)}$$

$$\therefore \quad \text{Median} = \frac{n+1}{2} \text{th term}$$

$$= \frac{9+1}{2} \text{th term}$$

$$24 = 5\text{th term}$$

$$\therefore \quad x + 4 = 24$$

$$x = 24 - 4$$

$$x = 20$$

Therefore, 11, 12, 14, $(20-2)$, $(20+4)$, $(20+9)$, 32, 38, 47

= 11, 12, 14, 18, 24, 29, 32, 38, 47

$$\text{Now Mean} = \frac{\Sigma x}{n}$$

$$= \frac{11+12+14+18+24+29+32+38+47}{9}$$

$$= \frac{225}{9} = 25.$$

Question 2:

(a) What number must be added to each of the number 6, 15, 20 and 43 to make them proportional? **[3]**

(b) If $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x - 3)$, it leaves a remainder 52, find the values of a and b. **[3]**

(c) Draw a histogram from the following frequency distribution and find the mode from the graph: **[4]**

Class	0-5	5-10	10-15	15-20	20-25	25-30
Frequency	2	5	18	14	8	5

Solution:

(a)

Let the number must be added be x, then

$$\text{the new number} = 6 + x, 15 + x, 20 + x, 43 + x$$

$\because$ These are proportionals.

$$6 + x : 15 + x :: 20 + x : 43 + x$$

or $$(6 + x)(43 + x) = (15 + x)(20 + x)$$

or $$258 + 6x + 43x + x^2 = 300 + 20x + 15x + x^2$$

or $$49x - 35x = 300 - 258$$

or $$14x = 42$$

or $$x = 3.$$

(b)

Let $(x - 2)$ is a factor of the given expression.

$\because$ $$x - 2 = 0$$

$$x = 2$$

Given expression, $$2x^3 + ax^2 + bx - 14 = 0$$

$$2(2)^3 + a(2)^2 + b(2) - 14 = 0$$

$$16 + 4a + 2b - 14 = 0$$

$$4a + 2b + 2 = 0$$

$$4a + 2b = -2$$

$$2a + b = -1 \quad \text{...(i)}$$

and when given expression is divided by $(x - 3)$

$$x - 3 = 0$$

$\Rightarrow$ $$x = 3$$

$\therefore$ $$2x^3 + ax^2 + bx - 14 = 52$$

$$2(3)^3 + a(3)^2 + b(3) - 66 = 0$$

$$54 + 9a + 3b - 66 = 0$$

$$9a + 3b = 12$$

$$3a + b = 4 \quad \text{...(ii)}$$

Solving equation (i) and (ii),

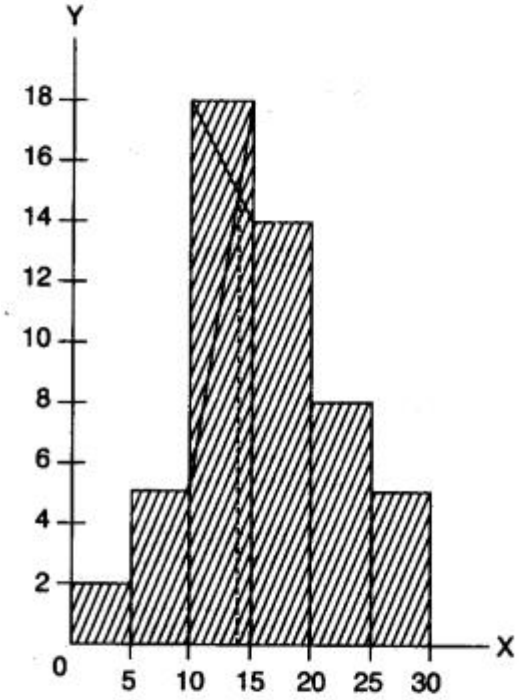
$$\begin{array}{rcl} 2a + b &=& -1 \\ 3a + b &=& 4 \\ (-) \quad (-) && (+) \\ \hline -a &=& -5 \end{array} \quad \Rightarrow a = 5$$

from (ii), $$3 \times 5 + b = 4$$

$$b = 4 - 15 = -11$$

$\therefore$ $$a = 5 \text{ and } b = -11$$

(c)



From the Histogram the value of Mode is **13·8.**

Question 3:

(a) *Without using tables evaluate 3 cos 80°.Cosec 10° + 2 sin 59° sec 31°.* **[3]**

(b) *In the given figure,*

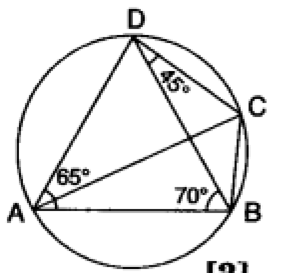
$\angle BAD = 65°$,

$\angle ABD = 70°$,

$\angle BDC = 45°$

(i) *Prove that AC is a diameter of the circle.*

(ii) *Find* $\angle ACB$.



[3]

(c) AB is a diameter of a circle with centre C = (-2, 5). If A = (3, -7). Find:

(i) The length of radius AC
(ii) The coordinates of B.

Solution:

(a) $3 \cos 80° \cdot \operatorname{cosec} 10° + 2 \sin 59° \sec 31°$

$= 3 \cos 80° \operatorname{cosec} (90° - 80°) + 2 \sin 59° \sec (90° - 59°)$

$= 3 \cos 80° \sec 80° + 2 \sin 59° \operatorname{cosec} 59°$

$= 3 \cos 80° \times \dfrac{1}{\cos 80°} + 2 \sin 59° \times \dfrac{1}{\sin 59°}$

$= 3 + 2 = 5.$

(b) *Given* : $\angle BAD = 65°, \angle ABD = 70°, \angle BDC = 45°$

(i) $\because$ ABCD is a cyclic quadrilateral.

In Δ ABD,

$\angle BDA + \angle DAB + \angle ABD = 180°$ By using sum property of Δ^s

$\therefore \quad \angle BDA = 180° - (65° + 70°)$

$= 180° - 135°$

$= 45°$

Now from Δ ACD,

$\angle ADC = \angle ADB + \angle BDC$

$= 45° + 45°$ $\quad (\because \angle BDA = \angle ADB = 45°)$

$= 90°$

Hence, $\angle$D makes right angle belongs in semi-circle therefore AC is a diameter of the circle.

(ii) $\angle ACB = \angle ADB$ (Angles in the same segment of a circle)

$\therefore \quad \angle ACB = 45°$ $\quad (\because \angle ADB = 45°)$

(c)

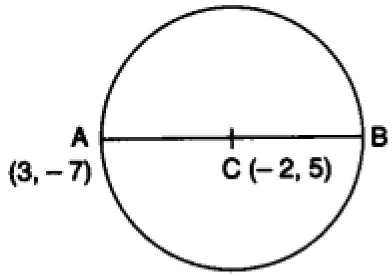
(i) The length of radius AC $= \sqrt{(-2-3)^2 + (5+7)^2}$

$$= \sqrt{(-5)^2 + (12)^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13.$$



(ii) Let the point of B be (x, y).

Given C is the mid-point of AB. Therefore

$$-2 = \frac{3+x}{2} \quad \text{and} \quad 5 = \frac{-7+y}{2}$$

$$\Rightarrow \quad 3 + x = -4 \qquad \Rightarrow \quad 10 = -7 + y$$

$$\Rightarrow \quad x = -4 - 3 = -7 \qquad y = 17$$

Hence, the co-ordinate of B $(-7, 17)$.

Question 4:

(a) Solve the following equation and calculate the answer correct to two decimal places:

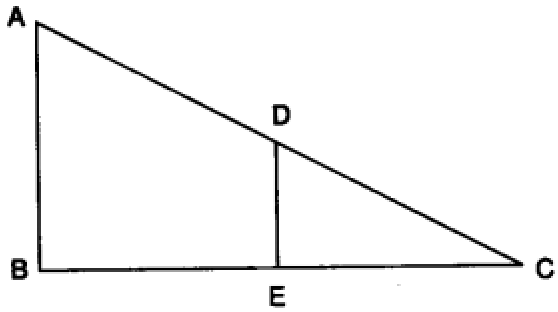
$$x^2 - 5x - 10 = 0.$$ **[3]**

(b) *In the given figure, AB and DE are perpendicular to BC.*

(i) *Prove that $\Delta ABC \sim \Delta DEC$*

(ii) *If AB = 6 cm, DE = 4 cm and AC = 15 cm. Calculate CD.*

(iii) *Find the ratio of the area of ΔABC : area of ΔDEC.* **[3]**



(c) *Using graph paper, plot the points A(6, 4) and B(0, 4).*

(i) *Reflect A and B in the origin to get the images A′ and B′.*

(ii) *Write the co-ordinates of A′ and B′.*

(iii) *State the geometrical name for the figure ABA′B′.*

(iv) *Find its perimeter.* **[4]**

(c) Using graph paper, plot the points A(6, 4) and B(0, 4).
(i) Reflect A and B in the origin to get the images A’ and B’.
(ii) Write the co-ordinates of A’ and B’.

(iii) State the geometrical name for the figure ABA'B'.
(iv) Find its perimeter. **[4]**

Solution:
(a) Given: $x^2 – 5a – 10 = 0$
Here, a = 1, b = – 5 and c = – 10

$$\therefore \quad D = b^2 - 4ac$$
$$= (-5)^2 - 4 \times 1 \times -10 = 65$$
$$x = \frac{-b \pm \sqrt{D}}{2a}$$
$$= \frac{5 \pm \sqrt{65}}{2 \times 1} = \frac{5 \pm 8{\cdot}06}{2}$$
$$= \frac{5 + 8{\cdot}06}{2}, \frac{5 - 8{\cdot}06}{2}$$
$$= \frac{13{\cdot}06}{2}, -\frac{3{\cdot}06}{2}$$
$$x = 6{\cdot}53, -1{\cdot}53$$

(b)
(i) From Δ ABC and Δ DEC,

$\angle ABC = \angle DEC = 90°$ (Given)

and $\angle ACB = \angle DCE$ = Common

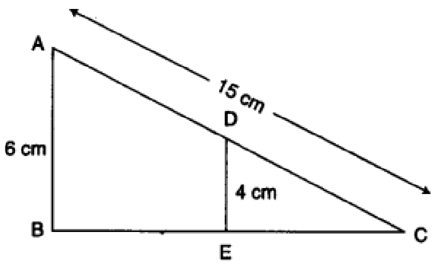
$\therefore$ $\Delta ABC \sim \Delta DEC$ (By AA similarity)

(ii) In Δ ABC and Δ DEC,

$\Delta ABC \sim \Delta DEC$ (proved in (i) part)

$$\therefore \quad \frac{AB}{DE} = \frac{AC}{CD}$$

Given : AB = 6 cm, DE = 4 cm, AC = 15 cm,



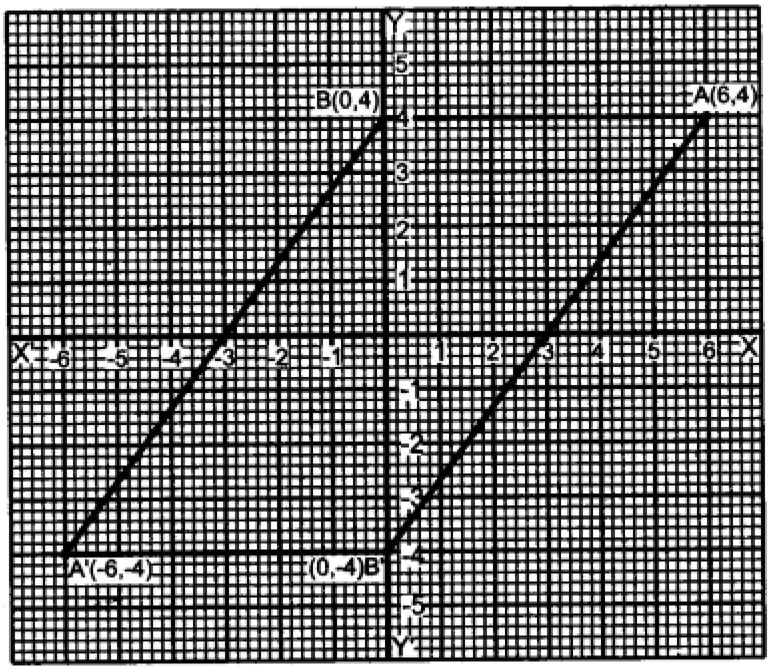
$$\therefore \quad \frac{6}{4} = \frac{15}{CD}$$
$$\Rightarrow \quad 6 \times CD = 15 \times 4$$
$$\Rightarrow \quad CD = \frac{60}{6}$$
$$\Rightarrow \quad CD = 10 \text{ cm.}$$

(iii)
$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEC} = \frac{AB^2}{DE^2} \quad (\because \Delta ABC \sim \Delta DEC)$$
$$= \frac{(6)^2}{(4)^2} = \frac{36}{16} = \frac{9}{4}$$

$\therefore$ Area of Δ ABC : Area of Δ DEC = 9 : 4.

(c)

(i) Please See Graph.



(ii) Reflection of A′ and B′ in the origin = A′ (–6, –4) and B′ (0, –4)

(iii) The geometrical name for the figure AB A′B′ is a parallelogram.

(iv) From the graph, AB = 6 cm, BB′ = 8 cm.

In Δ A BB′

$$(AB')^2 = AB^2 + (BB')^2$$
$$= (6)^2 + (8)^2 = 36 + 64 = 100$$
$$AB' = 10 = A'B \qquad \text{(ABA'B' is a parallelogram)}$$
$$\text{Perimeter of } ABA'B' = A'B' + AB' + AB + A'B$$
$$= 6 + 10 + 6 + 10$$
$$= 32 \text{ units.}$$

SECTION-B (40 Marks)

*(Attempt **any four** questions from this Section)*

Question 5:

(a) Solve the following inequation, write the solution set and represent it on the number line: **[3]**

$$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in R$$

(b) Mr. Britto deposits a certain sum of money each month in a Recurring Deposit Account of a bank. If the rate of interest is of 8% per annum and Mr. Britto gets Rs. 8088 from the bank after 3 years, find the value of his monthly installment. **[3]**

(c) Salman buys 50 shares of face value Rs. 100 available at Rs. 132.

(i) What is his investment?

(ii) If the dividend is 7.5%, what will be his annual income?

(iii) If he wants to increase his annual income by Rs. 150, how many extra shares should he buy? **[4]**

Solution:

(a)

Given : $$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$$

Taking L.C.M. of 3, 2 and 6 is 6.

$$-\frac{x}{3} \times 6 \le \frac{x}{2} \times 6 - \frac{4}{3} \times 6 < \frac{1}{6} \times 6$$

$$-2x \le 3x - 8 < 1$$

$\Rightarrow \quad -2x \le 3x - 8 \quad$ and $\quad 3x - 8 < 1$

$\Rightarrow \quad 8 \le 3x + 2x \qquad \Rightarrow \quad 3x < 1 + 8$

$\Rightarrow \quad 8 \le 5x \qquad \Rightarrow \quad 3x < 9$

$\Rightarrow \quad \frac{8}{5} \le x \qquad \Rightarrow \quad x < 3$

$\therefore$ The solution set is $\{x : 1{\cdot}6 \le x \le 3, x \in R\}$

Number line

(b)

Let the monthly instalment be ₹ x

Given : Maturity amount = ₹ 8,088, Time (n) = 3 years = 3 × 12 months = 36 months, Rate (R) = 8% p.a.

$$\text{Principle for one month} = P \times \frac{n(n+1)}{2}$$

$$= \frac{x \times 36 \times 37}{2}$$

$$= 18 \times 37x$$

$$\text{Interest} = \frac{18 \times 37x \times 8 \times 1}{100 \times 12} \qquad \left[\because I = \frac{PRT}{100}\right]$$

$$= \frac{444x}{100}$$

$$\text{Actual sum deposited} = 36x$$

$$\text{Maturity amount} = \text{Interest} + \text{Actual sum deposited}$$

$$8{,}088 = \frac{444x}{100} + 36x$$

$$8{,}088 = \frac{4{,}044x}{100}$$

$$\therefore \quad x = \frac{8{,}088 \times 100}{4{,}044} = 200$$

Hence, the monthly instalment be ₹ 200.

(c)

$$\text{Number of shares} = 50$$

$$\text{Face value of each share} = ₹\,100$$

$$\text{Market value of each share} = ₹\,132$$

$$\text{Total face value} = ₹\,100 \times 50$$

$$= ₹\,5{,}000$$

(i) $$\text{Total investment} = ₹\,132 \times 50$$

$$= ₹\,6{,}600$$

(ii) $$\text{Rate of dividend} = 7{\cdot}5\%$$

$$\text{Annual income} = ₹\,\frac{5{,}000 \times 7{\cdot}5}{100}$$

$$= ₹\,375$$

(iii) Let extra share should he buy be x.

$$\text{then total number of shares} = 50 + x$$

$$\text{Total face value} = ₹\,100 \times (50 + x)$$

$$\therefore \quad \text{Annual income} = ₹\,\frac{100 \times (50 + x) \times 7{\cdot}5}{100}$$

$$= (50 + x) \times 7{\cdot}5$$

$$\therefore \quad (50 + x) \times 7{\cdot}5 = 375 + 150$$

$$50 + x = \frac{525}{7{\cdot}5} = 70$$

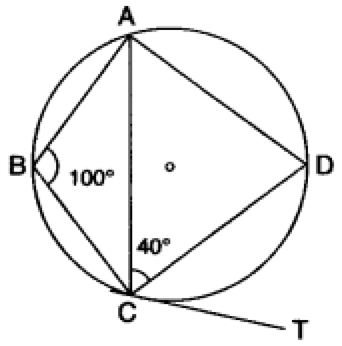
$$x = 70 - 50 = 20$$

Hence, the extra shares should be buy = 20.

Question 6:

(a) *Show that* $\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A}$ **[3]**

(b) *In the given circle with centre O, $\angle ABC = 100°$, $\angle ACD = 40°$ and CT is a tangent to the circle at C. Find $\angle ADC$ and $\angle DCT$.* **[3]**



(c) *Given below are the entries in a Savings Bank A/c pass book :*

Date	Particulars	Withdrawals	Deposit	Balance
Feb. 8.	*B/F*	—	—	*₹ 8,500*
Feb. 18	*To self*	*₹ 4,000*	—	—
April 12	*By cash*	—	*₹ 2,230*	—
June 15	*To self*	*₹ 5,000*	—	—
July 8	*By cash*	—	*₹ 6,000*	—

Calculate the interest for six months from February to July at 6% p.a. **[4]**

Solution:

(a)

(a) L.H.S. $= \sqrt{\frac{1-\cos A}{1+\cos A}}$

Multiplying by $\sqrt{1+\cos A}$ in numerator and denominator

$$= \sqrt{\frac{1-\cos A}{1+\cos A}} \times \sqrt{\frac{1+\cos A}{1+\cos A}}$$

$$= \sqrt{\frac{(1-\cos A)(1+\cos A)}{(1+\cos A)(1+\cos A)}}$$

$$= \sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}} = \sqrt{\frac{\sin^2 A}{(1+\cos A)^2}}$$

$$= \frac{\sin A}{1+\cos A} = \text{R.H.S.}$$

(b)

Given : $\angle ABC = 100°$

We know that,

$$\angle ABC + \angle ADC = 180° \quad \text{(The sum of opposite angles in a cyclic quadrilateral = 180°)}$$

$$\therefore \quad 100° + \angle ADC = 180°$$

$$\angle ADC = 180° - 100°$$

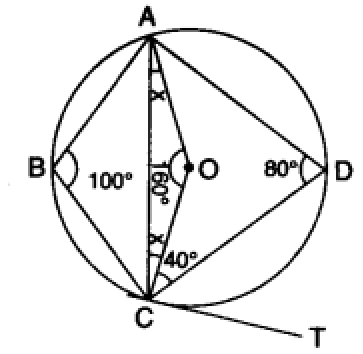
$$\angle ADC = 80°$$

Join OA and OC, we have a isosceles Δ OAC,

$$\because \quad OA = OC \quad \text{(Radii of a circle)}$$

$$\therefore \quad \angle AOC = 2 \times \angle ADC \quad \text{(by theorem)}$$

$$\text{or} \quad \angle AOC = 2 \times 80° = 160°$$



In Δ AOC,

$$\angle AOC + \angle OAC + \angle OCA = 180°$$

$$160° + \angle OCA + \angle OCA = 180° \quad [\because \angle OAC = \angle OCA]$$

$$2\angle OCA = 20°$$

$$\angle OCA = 10°$$

$$\angle OCA + \angle OCD = 40°$$

$$10° + \angle OCD = 40°$$

$$\therefore \quad \angle OCD = 30°$$

$$\text{Hence,} \quad \angle OCD + \angle DCT = \angle OCT$$

$$\because \quad \angle OCT = 90°$$

(The tangent at a point to circle is $\perp$ to the radius through the point to contant)

$$30° + \angle DCT = 90°$$

$$\therefore \quad \angle DCT = 60°$$

(c)

Date	Particulars	Withdrawals	Deposit	Balance
Feb. 8	B/F	—	—	₹ 8,500
Feb. 18	To self	₹ 4,000	—	₹ 4,500
April 12	By cash	—	₹ 2,230	₹ 6,730
June 15	To self	₹ 5,000	—	₹ 1,730
July 8	By cash	—	₹ 6,000	₹ 7,730

Principal for the month of Feb. = ₹ 4,500
Principal for the month of March = ₹ 4,500
Principal for the month of April = ₹ 4,500
Principal for the month of May = ₹ 6,730
Principal for the month of June = ₹ 1,730
Principal for the month of July = ₹ 7,730

Total principal from the month of Feb. to July = ₹ 29,690

Time $= \frac{1}{12}$ years, Rate of interest = 6%

$$\text{Interest} = \frac{P \times R \times T}{100}$$

$$= \frac{29{,}690 \times 6 \times 1}{100 \times 12}$$

$$= ₹\ 148{\cdot}45$$

Question 7:

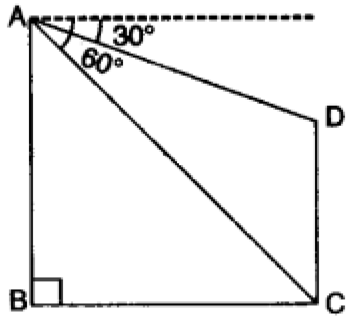
(a) In ΔABC, A(3, 5), B(7, 8) and C(1, -10). Find the equation of the median through A. **[3]**

(b) A shopkeeper sells an article at the listed price of Rs. 1,500 and the rate of VAT is 12% at each stage of sale. If the shopkeeper pays a VAT of Rs. 36 to the Government, what was the price, inclusive of Tax, at which the shopkeeper purchased the article from the wholesaler? **[3]**

(c) In the figure given, from the top of a building AB = 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find:

(i) The horizontal distance between AB and CD.

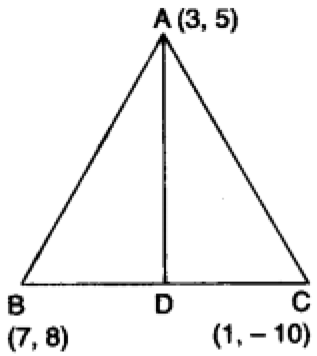
(ii) The height of the lamp post.



Solution:

(a)

(a) Here D is mid point of BC. (Given)



$$\therefore \quad \text{The co-ordinate of D} = \left(\frac{7+1}{2}, \frac{8-10}{2}\right)$$

$$= (4, -1)$$

Now equation of median AD,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here, $x_1 = 3, y_1 = 5, x_2 = 4, y_2 = -1$

$$y - 5 = \frac{-1-5}{4-3}(x-3)$$

$$y - 5 = \frac{-6}{1}(x-3)$$

$$y - 5 = -6x + 18$$

$$y = -6x + 18 + 5$$

$$y = -6x + 23$$

$$6x + y - 23 = 0$$

(b) *Given* : Listed price of an article = ₹ 1,500, Rate of VAT = 12%

$$\text{VAT on the article} = \frac{12}{100} \times 1500, = ₹\ 180$$

Let C.P. of this article be x, then

$$\text{VAT} = \frac{12}{100} \times x, = ₹\ \frac{12x}{100}$$

If the shopkeeper pays a VAT $= ₹ 36$

Then $180 - \frac{12x}{100} = 36$

$$\frac{18000 - 12x}{100} = 36$$

$$18000 - 12x = 3600$$

$\therefore$ $12x = 18000 - 3600 = 14,400$

$$x = ₹ 1,200$$

$\therefore$ The price at which the shopkeeper purchased the article inclusive of sales tax

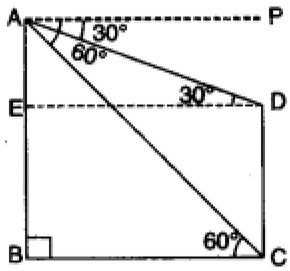
$$= 1,200 + \frac{12}{100} \times 1,200$$

$$= 1,200 + 144$$

$$= ₹ 1,344$$

(c)

Given : AB = 60 m



$\because$ $\angle PAC = 60°$

$\therefore$ $\angle PAC = \angle BCA$

(i) Now in Δ ABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{60}{BC}$$

$\Rightarrow$ $BC = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$$BC = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$$

Hence, the horizontal distance between AB and CD = $20\sqrt{3}$ m.

(ii) Let AE = x and proved above BC = $20\sqrt{3}$m

$\therefore$ $BC = ED = 20\sqrt{3}$

Now in Δ AED,

$$\tan 30° = \frac{AE}{ED}$$

$$\frac{1}{\sqrt{3}} = \frac{AE}{20\sqrt{3}}$$

$\Rightarrow$ $\sqrt{3}\, AE = 20\sqrt{3}$

$\Rightarrow$ $AE = 20$ m

now $EB = AB - AE$

$\therefore$ $EB = 60 - 20 = 40$ m

$\because$ $EB = CD$

$\therefore$ $CD = 40$ m

Hence, the height of the lamp post = 40 m.

Question 8:

(a) *Find x and y if* $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$ **[3]**

(b) A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast. **[3]**

(c) Without solving the following quadratic equation, find the value of 'p' for which the given equation has real and equal roots: $x^2 + (p - 3)x + p = 0$ **[4]**

Solution:

(a) *Given :*

$$\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 2x + 3x \\ 2y + 4y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 5x \\ 6y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$\therefore$ $5x = 5 \Rightarrow x = 1$

and $6y = 12 \Rightarrow y = 2$

Hence, $x = 1$ and $y = 2$

(b)

Radius of a solid sphere, $r = 15$ cm

Volume of a solid sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi (15)^3$ cm³.

Now, radius of right circular cone = 2·5 cm

and height, h = 8 cm.

Volume of right circular cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi (2{\cdot}5)^2 \times 8$

$\therefore$ The number of cones $= \dfrac{\text{Volume of a sphere}}{\text{Volume of a cone}}$

$$= \frac{\frac{4}{3}\pi \times (15)^3}{\frac{1}{3}\pi (2{\cdot}5)^2 \times 8} = \frac{15 \times 15 \times 15}{2{\cdot}5 \times 2{\cdot}5 \times 2}$$

$= 270$

(c)

Given equation $x^2 + (p-3)x + p = 0$

$\because$ Roots are real and equal, then

$$b^2 - 4ac = 0$$

Here we compare the coefficients of a, b and c with the equation $ax^2 + bx + c = 0$.

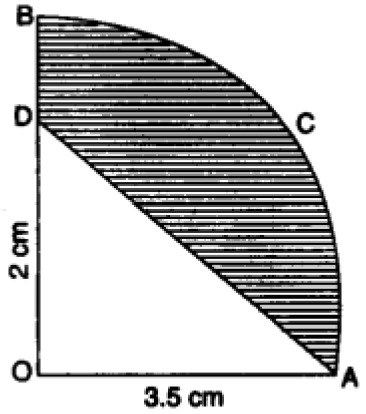
$$a = 1, b = p - 3 \text{ and } c = p$$

Now putting the values of a, b and c in equation

$$(p-3)^2 - 4 \times 1 \times p = 0$$
$$p^2 + 9 - 6p - 4p = 0$$
$$p^2 + 9 - 10p = 0$$
$$p^2 - 10p + 9 = 0$$
$$p^2 - 9p - p + 9 = 0$$
$$p(p-9) - 1(p-9) = 0$$
$$\Rightarrow \quad (p-9)(p-1) = 0$$

Hence, $p = 9$ or 1

Question 9:



(a) In the figure alongside, OAB is a quadrant of a circle. The radius OA = 3.5 cm and OD = 2 cm. Calculate the area of the shaded portion. (Take π = 22/7) **[3]**

(b) A box contains some black balls and 30 white balls. If the probability of drawing a black ball is two-fifths of a white ball, find the number of black balls in the box. **[3]**

(c) Find the mean of the following distribution by step deviation method:

Class Interval	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	10	6	8	12	5	9

Solution:

(a)

Radius of quadrant OACB, $r = 3\cdot5$ cm

$\therefore$ Area of quadrant OACB $= \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 3\cdot5 \times 3\cdot5 = 9\cdot625 \text{ cm}^2$

Here, $\angle AOD = 90°$

Then area of Δ AOD $= \frac{1}{2} \times$ base $\times$ height

Base = 3·5 cm and height = 2 cm

$\therefore$ $= \frac{1}{2} \times 3\cdot5 \times 2 = 3\cdot5 \text{ cm}^2$

Area of shaded portion = Area of quadrant – Area of triangle

$= 9\cdot625 - 3\cdot5$

$= 6\cdot125 \text{ cm}^2$

(b)

Let the number of black balls be x, then

Total number of balls $= 30 + x$

Thus, the prabability of black balls $= \frac{x}{30 + x}$

and the probability of white balls $= \frac{30}{30 + x}$

Given : Probability of black ball $= \frac{2}{5} \times$ probability of white ball

$$\frac{x}{30 + x} = \frac{2}{5} \times \frac{30}{x + 30}$$

$$5x = 60$$

$$x = 12$$

Hence, the number of black balls = 12.

(c)

C.I.	Frequency (f_i)	Mid-value (x)	$d_i = \frac{x-a}{h}$	$f_i d_i$
20–30	10	25	–2	–20
30–40	6	35	–1	–6
40–50	8	45	0	0
50–60	12	55	1	12
60–70	5	65	2	10
70–80	9	75	3	27
	$\Sigma f_i = 50$			$\Sigma f_i d_i = 23$

Here, $a = 45$ and $h = 10$

$$\therefore \quad \text{Mean} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} \times h$$

$$= 45 + \frac{23}{50} \times 10$$

$$= 45 + 4{\cdot}6 = 49{\cdot}6$$

Question 10:

(a) Using a ruler and compasses only:

(i) Construct a triangle ABC with the following data:

AB = 3.5 cm, BC = 6 cm and ∠ABC = 120°

(ii) In the same diagram, draw a circle with BC as diameter. Find a point P on the circumference of the circle which is equidistant from AB and BC.

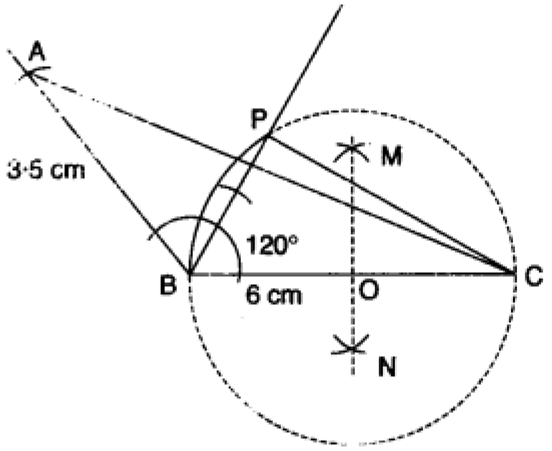
(iii) Measure ∠BCP. **[3]**

(b) The mark obtained by 120 students in a test are given below:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No of Students	5	9	16	22	26	18	11	6	4	3

Draw an ogive for the given distribution on a graph sheet.
Using suitable scale for ogive to estimate the following:
(i) The median
(ii) The number of students who obtained more than 75% marks in the test.
(iii) The number of students who did not pass the test if minimum marks required to pass is 40. **[6]**

Solution:



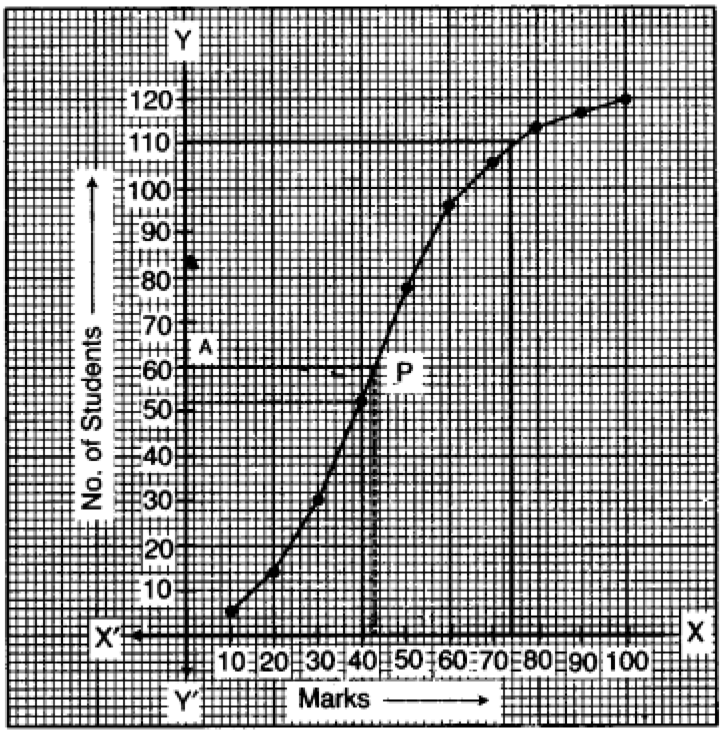
(a) Steps of Construction:
(i) Draw a line BC = 6 cm.
(ii) With the help of the point B, draw ∠ABC = 120°.
(iii) Taking radius 3.5 cm cut BA = 3.5 cm.
(iv) Join A to C.
(v) Draw ⊥ bisector MN of BC.
(vi) Draw a circle 0 as centre and OC as radius.
(vii) Draw angle bisector of ∠ABC which intersects circle at P.
(viii) Join BP and CP.
(ix) Now, ∠BCP = 30°.

(b)

Marks	No. of Students (f)	Cumulative Frequency
0-10	5	5

10-20	9	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120
	n = 120	

On the graph paper, we plot the following points:
(10, 5), (20, 14), (30, 30), (40, 52), (50, 78), (60, 96). (70, 107), (80, 113), (90, 117), (100, 120).



(i) Median $= \left(\frac{n}{2}\right)^{th}$ term $[\because n = 120, \text{ even}]$

$= \frac{120}{2} = 60^{th}$ term

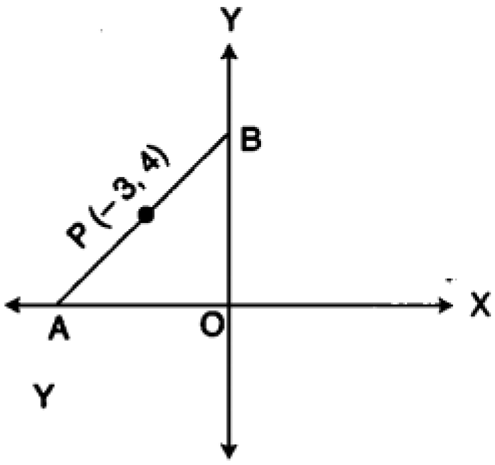
From the graph 60th term = 42

(ii) The number of students who obtained more than 75% marks in test
= 120 – 110 = 10.

(iii) The number of students who did not pass the test if the minimum pass marks 40 = 52.

Question 11:

(a) In the figure given below, the line segment AB meets X-axis at A and Y-axis at B. The point P(-3, 4) on AB divides it in the ratio 2:3. Find the coordinates of A and B. **[3]**



(b) Using the properties of proportion, solve for x, given **[3]**

$$\frac{x^4+1}{2x^2} = \frac{17}{8}$$

(c) A shopkeeper purchases a certain number of books for Rs. 960. If the cost per book was 18 less, the number of books that could be purchased for Rs. 960 would be 4 more. Write an equation, taking the original cost of each book to be Rs. x, and solve it to find the original cost of the books. **[4]**

Solution:

(a)

Let the co-ordinates of A and B be $(x, 0)$ and $(0, y)$

$\because$ The co-ordinates of a point P (–3, 4) on AB divides it in the ratio 2 : 3.

i.e., $\quad AP : PB = 2 : 3$

By using section formula, we get

$$-3 = \frac{2 \times 0 + 3 \times x}{2+3} \qquad \left[\because x = \frac{m_1x_2 + m_2x_1}{m_1+m_2}\right]$$

$$-3 = \frac{3x}{5} \Rightarrow 3x = -15$$

$$\Rightarrow \quad x = -5$$

and $$4 = \frac{2 \times y + 3 \times 0}{2+3} \qquad \left[\because y = \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right]$$

$$4 = \frac{2y}{5} \Rightarrow 2y = 20$$

$$\Rightarrow \quad y = 10$$

Hence, the co-ordinates of A and B are (–5, 0) and (0, 10).

(b)

Given : $\frac{x^4+1}{2x^2} = \frac{17}{8}$

By using componendo and dividendo, we get

$$\frac{x^4+1+2x^2}{x^4+1-2x^2} = \frac{17+8}{17-8}$$

$$\left(\frac{x^2+1}{x^2-1}\right)^2 = \frac{25}{9}$$

$$\left(\frac{x^2+1}{x^2-1}\right)^2 = \left(\frac{5}{3}\right)^2$$

Taking square root on both sides, we get

$$\frac{x^2+1}{x^2-1} = \frac{5}{3}$$

$\Rightarrow \quad 5x^2 - 5 = 3x^2 + 3$

$\Rightarrow \quad 5x^2 - 3x^2 = 3 + 5$

$\Rightarrow \quad 2x^2 = 8 \quad \Rightarrow \quad x^2 = 4$

$\Rightarrow \quad x = \pm 2$

(c)

Given the original cost of each book be ₹ x.

Total cost = ₹ 960

$\therefore$ Number of books for 960 = $\frac{960}{x}$

If the cost per book was ₹ 8 less, (*i.e.*, $x - 8$) then

Number of books = $\frac{960}{x-8}$

According to question,

$$\frac{960}{x-8} = \frac{960}{x} + 4$$

$$\frac{960}{x-8} - \frac{960}{x} = 4$$

$$960\left[\frac{x-x+8}{x(x-8)}\right] = 4$$

$$\frac{8}{x^2-8x} = \frac{1}{240}$$

$\Rightarrow \quad x^2 - 8x = 1,920$

$\Rightarrow \quad x^2 - 8x - 1,920 = 0$

$\Rightarrow \quad x^2 - 48x + 40x - 1,920 = 0$

$\Rightarrow \quad x(x-48) + 40(x-48) = 0$

$\Rightarrow \quad (x-48)(x+40) = 0$

$x - 48 = 0$ or $x + 40 = 0$

$x = 48$ or $x = -40$

$\because -40$ is not possible.

Hence, the original cost of each book = ₹ 48.

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