If there are 9 horizontal lines and 9 vertical lines in a chess board, how many rectangles can be formed in the chess board?

A. 920 B. 1024

C. 64 D. 1296

Ans 1296 (The number of rectangles that can be formed by using m horizontal lines and n vertical lines are

$^{m}C_{2} \times {}^{n}C_{2}$

Find the number of triangles that can be formed using 14 points in a plane such that 4 points are collinear?

A. 480 B. 360 Ans

C. 240 D. 120

Consider there be n points in a plane out of which m points are collinear. The number of triangles that can be formed by joining these n points as vertices are

$^{n}C_{3} - {}^{m}C_{3}$

Here n = 14, m = 4

Hence, The number of triangles = $^{n}C_{3} - {}^{m}C_{3} = {}^{14}C_{3} - {}^{4}C_{3}$

$= {}^{14}C_{3} - {}^{4}C_{3} = 360$

What is the sum of all 4 digit numbers formed using the digits 2, 3,4 and 5 without repetition?

A. 93324 Ans
B. 92314
C. 93024
D. 91242

If all the possible n digit numbers using the n distinct digits are formed, the sum of all the numbers so formed is equal to

(n-1)! × (Sum of the n digits) × (111 ... n times)

Here n=4.

Hence the sum of all 4 digit numbers formed using the digits 2, 3,4 and 5 without repetition

= (4-1)! (2 + 3 + 4 + 5)(1111) = 3! × 14 × 1111 = 6 × 14 × 1111 = 93324

There are 8 points in a plane out of which 3 are collinear. How many straight lines can be formed by joining them?

A. 16
B. 26 Ans
C. 22
D. 18

Consider there be n points in a plane out of which m points are collinear. The number of straight lines that can be formed by joining these n points are

$^nC_2 - {^mC_2} + 1$

Here n=8, m=3

The required number of straight lines = $^nC_2 - {^mC_2} + 1$

$= {^8C_2} - {^3C_2} + 1$

$= {^8C_2} - {^3C_1} + 1$

How many quadrilaterals can be formed by joining the vertices of an octagon? Ans 70 (8 C 4)

If ${}^{n}C_{8} = {}^{n}C_{27}$, what is the value of n? Ans 35

Find the number of triangles which can be drawn out of n given points on a circle? Ans N C 3

Arun wants to send invitation letter to his 7 friends. In how many ways can he send the invitation letter if he has 4 servants to carry the invitation letters Ans 4 power 7 = 16384

How many three digit numbers divisible by 5 can be formed using any of the digits from 0 to 9 such that none of the digits can be repeated? Ans 136

A number is divisible by 5 if the its last digit is a 0 or 5

Case 1 : Number of three digit numbers using the 10 digits (0,1,2,3,4,5,6,7,8,9)
ending with 0

We take the digit 0 and fix it at the unit place. There is only 1 way of doing this

		1

Since the number 0 is placed at unit place, we have now 9 digits(1,2,3,4,5,6,7,8,9)
remaining. Any of these 9 digits can be placed at tenth place.

	9	1

Since the digit 0 is placed at unit place and another one digits is placed at tenth place,
we have now 8 digits remaining. Any of these 8 digits can be placed at hundredth place.

8	9	1

Total number of 3 digit numbers using the digits (0,1,2,3,4,5,6,7,8,9)ending with 0
= 8 × 9 × 1 = 72 -----------------(A)

Case 2 : Number of three digit numbers using the 10 digits (0,1,2,3,4,5,6,7,8,9)
ending with 5

we take the digit 5 and fix it at the unit place. There is only 1 way of doing this.

		1

Since the number 5 is placed at unit place, we have now 9 digits(0,1,2,3,4,6,7,8,9)
remaining. But, from the remaining digits, 0 cannot be used for hundredth place. Hence
any of 8 digits (1,2,3,4,6,7,8,9) can be placed at hundredth place.

8		1

Since the digit 5 is placed at unit place and another one digits is placed at hundredth
place, we have now 8 digits remaining. Any of these 8 digits can be placed at tenth place.

8	8	1

Total number of 3 digit numbers using the digits (0,1,2,3,4,5,6,7,8,9) ending with 5

$= 8 \times 8 \times 1 = 64$ -----------------(B)

Hence, required number of 3 digit numbers = 72 + 64 = 136 ($\because$ from A and B)

How many numbers, between 100 and 1000, can be formed with the digits 3, 4, 5, 0, 6, 7? (Repetition of digits is not allowed)

Ans 100 (5 X 4 X 5)

A telegraph has 10 arms and each arm can take 5 distinct positions (including position of the rest). How many signals can be made by the telegraph? Ans 5 power 10 - 1

The 1st arm can take any of the 5 distinct positions
Similarly, each of the remaining 9 arms can take any of the 5 distinct positions

Hence total number of signals = 5^{10}

But there is one arrangement when all of the arms are in rest. In this case there
will not be any signal.

Hence required number of signals = 5^{10} - 1

There are two books each of 5 volumes and two books each of two volumes. In how many ways can these books be arranged in a shelf so that the volumes of the same book should remain together?

Ans 4! X 5 ! x 5 ! X 2 ! x 2 !

In how many ways can 30 different toys be equally divided among 10 boys? Ans 30! 3 ! 10

Number of ways in which m × n distinct things can be divided equally into n groups (each group will have m things and the groups are numbered, i.e., distinct)

$=(mn)!(m!)_n$

In how many ways can 30 different toys be equally divided into 10 packets? Ans 30! 3 ! 10 10!

Number of ways in which m × n distinct things can be divided equally into n groups (each group will have m things and the groups are unmarked, i.e., notdistinct)

$=(mn)!(m!)_n\ n!$

In how many ways can 30 identical toys be divided among 10 boys if each boy must get at least one toy? Ans 29 C 9

Here toys are identical and boys are different

The number of ways in which 30 identical toys can be divided among 10 boys
if each boy must get at least one toy

= The number of ways in which 30 identical balls can be distributed into 10 boxes
if each box must contain at least one ball

Hence, this problem can be solved using the formula given at the top.
n = 10, k = 30.

Hence, the required number of ways
$= {}^{(k-1)}C_{(n-1)} = {}^{29}C_9$

Naresh has 10 friends and he wants to invite 6 of them to a party. How many times will 3 particular fiends always attend the party?

Ans 35 (7 C 3)

In how many ways can seven '#' symbol and five '*' symbol be arranged in a line so that no two '*' symbols occur together?

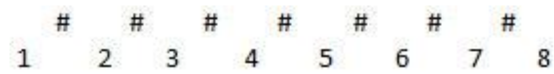
Ans 56

There are 7 identical '#' symbols and 5 identical '*' symbols.

We need to arrange these 12 symbols in a line so that no two '*' symbols occur together.

The seven '#' symbols can be arranged in 1 way **---(A)**
(Because all these symbols are identical and order is not important)

Now there are 8 positions to arrange the five '*' symbols
so that no two '*' symbols occur together as indicated in the diagram below



The five '*' symbols can be placed in these 8 positions in $^8C_5$ ways**---(B)**
(Because all these symbols are identical and order is not important)

From(A) and (B),
the required number of ways = $1 \times {}^8C_5 = {}^8C_5$
= $^8C_3$ [$\because {}^nC_r = {}^nC_{(n-r)}$]

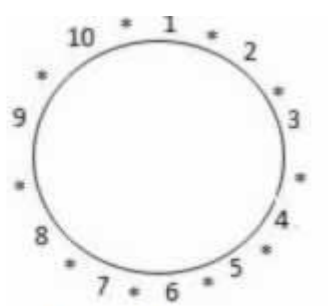
$=8\times7\times63\times2\times1=8\times7=56$

In how many ways can 10 engineers and 4 doctors be seated at a round table if no two doctors sit together? Ans 9 ! X 10 X 9 X 8 x 7

No two doctors sit together. Hence, let's initially arrange the 10 engineers at a round table.

Total number of ways in which this can be done = (10-1)! = 9! **---(A)**

Now there are 10 positions left (marked as *) to place the four doctors as shown below
so that no two doctors can sit together.



(where 1,2,3, ..., 10 represent engineers)

Total number of ways in which this can be done = $^{10}P_4$**---(A)**

From (A) and (B), required number of ways = $9! \times {}^{10}P_4$

All the letter of word ‘LUCKNOW’ Are arranged in all possible ways, what is the rank of the word ‘LUCKNOW’?

Method 1

Alphabetical order is C K L N O U W

Numbers of Words starting with C = 6!

Numbers of Words starting with K = 6!

All the words starting with LC = 5!

All the words starting with LK = 5!

All the words starting with LN = 5!

All the words starting with LO = 5!

Next word will start with LU C K N O W

So rank of LUCKNOW = 2 * 6! + 4 * 5! + 1 = 1921

Method 2 (shortcut)

3	6	1	2	4	5	7
L	U	C	K	N	O	W
2	4	0	0	0	0	0
6!	5!	4!	3!	2!	1!	0!

Answer is 2 * 6! + 4 *5! + 1

Steps

1 Put the number in front of alphabets according to their occurrence

2 Starting from left i.e Alphabet L count how many numbers less than 3 are there towards its right i.e 2 write 2 below L and so on

3 Start from Right hand side put 0! ,1! ,2! And so on towards left

4 Multiply and add the numbers

5 Finally add one to the answer

Another Eg if the word is B A N A N A

2	1	3	1	3	1
B	A	N	A	N	A
3 / 3! *2!		2/2! * 2!		1	
5!		3!		1!	0!

Answer is 360/ 12 + 3 + 1 + 1 = 35 Ans

Eg

1	4	4	3	2
A	p	p	l	e
	2/ 2!	2	1	
4!	3!	2!	1!	0!

Answer = 6+4+1 +1 = 12 Answer

How many number of 3's would be used to type numbers from 1 to 700

Answer:

Instead of calculating manually, if we use the concepts of permutations this problem can be solved easily

0-6	0-9	0-9

If we take numbers upto 699, Then the hudred's place take digits upto 6. But the digits in tenth and unit places take upto 9 from 0.

If we fix digit 3 in hundred's place the remaining digits can be filled in 10 ways each.

0-6	0-9	0-9
3	10	10

Total ways are 10 X 10 = 100

If we fix digit 3 in tenth's place then the digit in hundred's place can be filled in 7 ways (0-6), and units place can be filled in 10 ways.

0-6	0-9	0-9
7	3	10

Then the total ways are 7 X 10 = 70

If we fix digit 3 in unit's place then the digit in hundred's place can be filled in 7 ways (0-6), and tenth's place can be filled in 10 ways.

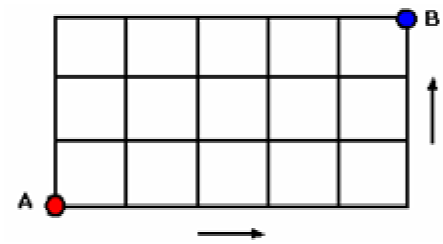
0-6	0-9	0-9
7	10	3

Total number of ways if we fix 3 in units place = 70

Then the number 3 digit can be used to write numbers upto 700 is (100 + 70 + 70) = 240

Solved Example 2:

Find the total number of ways of reaching B from A If only right and up steps are allowed.



Method 1:

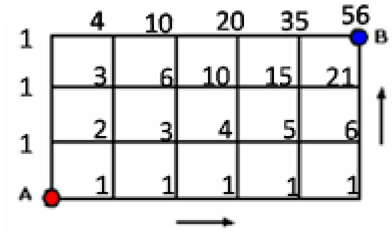
There are total 5 right steps denoted by R's, and 3 Up Steps denoted by U's RRRRRUUU. No number of ways of arranging These letters = 8!/(5! X 3!) = 56

Method 2:

Out of 8 there are 3 up steps. No of ways of selecting 3 upsteps = $^8C_3$ = 56

Method 3:

We use node method to solve this problem without using permutations



We need to calculate the number of ways of reaching every node or junction. Take last node B this can be reached in two ways, which have already values 35 and 21. We add these two to get node value for B.

Solved Example 4:

How many ways A boy can reach the top of stairs which contain 10 steps, when he can take either one or two steps every time?

Let the number of ways of reaching top of stairs which has n steps = a(n)

If the boy takes the first step then the remaining (n-1) steps can be covered in = a(n-1) ways.

If the boy takes the first step with 2 steps then the remaining (n-2) steps can be covered in = a(n-2) ways.

So a(n) = a(n-1) + a(n-2)

Now if there is only one step number of ways = a(1) = 1
If there are only two steps number of ways = a(2) = 2
But a(3) = 1+2 = 3
So this is nothing but Fibonacci series : 1, 2,3, 5,8,13, 21, 34, 55, 89.

Solved Example 5:

How many ways 10 houses can be painted with only two colors, white and blue , where two houses consecutively cannot be painted blue.

Let the number of ways of painting the houses= a(n)

The first house we can painted with white remaining houses can be painted in = a(n-1) ways.

But if the first house is painted blue, the next house must be white so remaining (n-2) houses can be covered in = a(n-2) ways.

So a(n) = a(n-1) + a(n-2)

Now if there is only one house number of ways = a(1) = 2
If there are only two houses number of ways = a(2) = ww, wb, bw = 3
But a(3) = 2+3 = 5
So this is nothing but Fibonacci series : 2,3, 5,8,13, 21, 34, 55, 89, 144