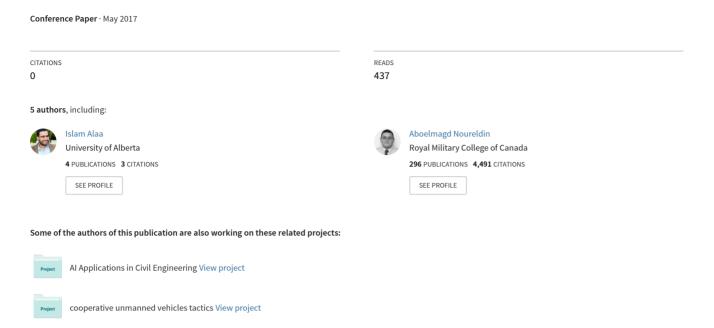
Tuning of The Error Covariance Parameters in EKF-Based INS/GPS Systems: A Practical Approach



TUNING OF THE ERROR COVARIANCE PARAMETERS IN EKF-BASED INS/GPS SYSTEMS: A PRACTICAL APPROACH

Mostafa Mahmoud^{a,*}, Islam Alaa^b, Amr Wassal^a, Aboelmagd Noureldin^c, Amr ElDieb^b

^a Computer Engineering Department, Cairo University, Cairo University Road, Giza 12613, Egypt - mostafa_mahmoud_mohamed@cu.edu.eg ,wassal@eng.cu.edu.eg

KEY WORDS: Extended Kalman Filter (EKF), Inertial Navigation Systems (INS), Micro-Electro-Mechanical-System (MEMS), Global Positioning Systems (GPS), Root Mean Square Error (RMSE)

ABSTRACT:

The low cost, small size and low power consumption micro-electro-mechanical-system (MEMS) inertial sensors enabled the use of inertial navigation systems (INS) in many consumer applications. This was not possible in the past with high end INS due to cost and size constraints. However, MEMS inertial sensors suffer from bias drift, which is stochastic in nature and significantly influence the long-term positioning accuracy. The long-term position drift is mainly due to the inherent mathematical integration of the inertial sensor biases resulting in heavily degraded positioning over a short period of continuous use. On the other hand, Global Positioning Systems (GPS) offer position, velocity and timing information with consistent level of positioning accuracy, albeit suffering from the problem of signal outage due to losing direct line-of-sight with sufficient number of GPS satellites in both indoor and some outdoor conditions or due to multi-path effects. The INS can provide reliable navigation information for the short-term hat can bridge GPS outages. In addition, INS provides information about the attitude and heading angles of the moving platform. Therefore, due to the complementary nature of the INS and the GPS information, they are usually fused together to generate more robust and reliable navigation information. This fusion can be done with different techniques, such as Kalman filters, which used in our work. In this paper, different tuning methodologies for similar systems in the literature are reviewed qualitatively in brief. Consequently, the methodology for the tuning of the error covariance matrices R, Q, and P in EKF-based INS/GPS systems used is described. The results of the tuning of the EKFbased INS/GPS system is presented for two different datasets. The first dataset is acquired using a high-end tactical-grade SPAN unit featuring Novatel HG1700 IMU module. The second dataset is acquired from a MEMS-based SCC1300-D04 IMU unit from VTI. A comparative study of the performance of the proposed approach using the two datasets utilizing the tuning methodology described is discussed in this paper against a reference ground-truth dataset for the same trajectories.

1. INTRODUCTION

Reliable and precise positioning and navigation systems play a vital role in many civilian applications including smart driving, unmanned aerial vehicles (UAVs) control, transportation tracking and monitoring among many others. The emerging technologies of the low-cost MEMS-based inertial sensors (i.e. 3-axis gyroscopes and accelerometers) permitted the design and implementation of cost-efficient inertial navigation systems (INS). However, such systems are not reliable on the long run as their accuracy is degraded while operation due to the bias drift in MEMSbased sensors. On the other hand, the Global Positioning System (GPS) can provide positioning and navigation information with bounded error characteristics. Therefore, it is thought that, the fusion between INS and GPS can achieve better performance in both short run and long run conditions. Several attempts were done to efficiently fuse INS and GPS together in a single navigation system using different fusion techniques. However, Extended Kalman Filter (EKF) was the most common method that is usually used for integration. In this paper, the INS/GPS integration using an EKF is presented, the tuning of the EKF as well as the results of fusion and tuning are illustrated using two different data sets against ground truth trajectories.

The remainder of this paper is organized as follows. INS/GPS integration is explained in section 2. Section 3 explained the tuning methodology in detail. The simulation and experimental tests are presented in section 4 followed by the concluding remarks in section 5.

2. INS/GPS INTEGRATION

The block diagram of the integration between the INS and the GPS is shown in Figure 1. Raw sensor data are mechanized in order to generate the INS navigation information (i.e. attitude, velocity, and position) and is then fused with the GPS inside an EKF. Consequently, The output of the fusion is fed-back to the system to enhance the system performance.

2.1 Navigation System Model

The following equation represent the system model for the INS mechanization used in the INS/GPS integration which yields the attitude, velocity and position realized in the local level frame.

$$\begin{bmatrix} \dot{r}^{l} \\ \dot{V}^{l} \\ \dot{R}^{l}_{b} \end{bmatrix} = \begin{bmatrix} D^{-1}V^{l} \\ R^{l}_{b}f_{b} - (2\Omega^{l}_{ie} + \Omega^{l}_{el})V^{l} + g^{l} \\ R^{l}_{b}(\Omega^{b}_{ib} - \Omega^{b}_{il}) \end{bmatrix}$$
(1)

^b Systems Design Group, Si-Ware Systems, 3 Khaled Ibn Al-Waleed Street, Heliopolis, Cairo 11361, Egypt - (islam.alaa, amr.eldieb)@si-ware.com

^c Department of Electrical and Computer Engineering, RMC / Queens University, Kingston, Ontario, Canada - aboelmagd.noureldin@rmc.ca

^{*}Corresponding author

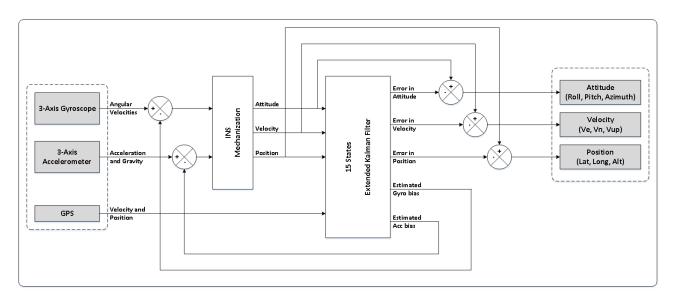


Figure 1. INS/GPS integration

Where:

 \dot{r}^l : is the time rate of change of the position.

 \dot{V}^l : is the time rate of change of velocity.

 \dot{R}_b^l : is the time rate of change of rotational matrix from body frame to local level frame.

 R_b^l : is the rotation matrix from body frame to local level frame.

 f_b : is the acceleration in the body frame.

 q^l : is the earth gravity vector in the local level frame.

 Ω^{l}_{ie} : is the skew symmetric matrix of earth rotation about its spin axis.

 Ω^l_{el} : is the skew symmetric matrix of orientation change of local level frame with respect to the

 Ω^b_{ib} : is the skew symmetric matrix of the measurements of angular velocities.

 Ω^b_{il} : is the skew symmetric matrix of the compensation term used to account for the earth rotation rate and orientation change of the local-level.

The mentioned nine navigational states (attitude, velocity, and position) suffer from a severe drifting effect after a very short period of time due to the numerical integration of the stochastic bias in inertial sensors which amends the integration with a GPS to optimally estimate the stochastic biases and correct the error in the navigation states.

2.2 INS/GPS Fusion Using an Extended Kalman Filter

Extended Kalman Filter (EKF) is the conventional estimation tool that is used to fuse data from INS and GPS systems together to achieve continuous, accurate and reliable positioning and navigation solution. This method requires the knowledge of the error covariance of the different information sources present in the system to estimate the navigation state. The error covariance matrices present in the system are the Q matrix which defines the error covariance of the driving information source (the INS in this case), the R matrix which defines the error covariance of the

aiding information source (the GPS in this case), and the P matrix which defines the error covariance of the system states. The performance of the EKF-based INS/GPS systems depends heavily on the initial values of different error covariance matrices that are present in the filter. These initial values affect both the accuracy of the estimated states, and the filter convergence time. The R and Q matrices determine the weight by which each source, the driving source and the aiding source, of information in the system contributes to the final estimated state. Proper choice of the values inside the R and Q matrices will result in better accuracy in the estimated navigation state. On the other hand, the initial values of the elements of the P matrix affect directly the filter convergence time, which, together with the previously mentioned matrices, define the overall system performance (Quinchia et al., 2013). The following equations summarize the kalman filter operation, derivation details can be found in (Noureldin et al., 2013)

EKF starts by the Prediction step that includes projecting the state ahead, and projecting the error covariance ahead. The Prediction step is illustrated in the following equations:

$$\hat{x}_k = A\hat{x}_{k-1} \tag{2}$$

$$P_k^- = A P_{k-1} A^T + Q (3)$$

After that, the filter proceeds to the *Correction* step, in which the kalman gain is computer, the estimates are updated by the measurement, and the error covariance gets updated. The *Correction* step is illustrated in the following equations:

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1}$$
(4)

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-) \tag{5}$$

$$P_k = (I - K_k H) P_k^- \tag{6}$$

2.3 Feedback and State Correction

The EKF-based INS/GPS system estimates the error in attitude, velocity, and position. It also estimates the stochastic part of the sensor bias for the 3-axis gyroscope and the 3-axis accelerometer. These estimated errors in the navigation state are then subtracted

from the INS results in order to produced the corrected navigation states. Moreover, the estimated stochastic sensors' bias values are fed-back to the sensors to be subtracted from the raw sensors' quantities. The fed-back sensor biases help the INS to limit the drift in a real-time manner.

3. KALMAN FILTER TUNING

The estimation of the error covariance parameters in EKF-based systems is usually performed offline before the system final deployment. The definition of these parameters is dependent on the filter state definition, and whether EKF will be used to estimate the full navigation state or the error in it. Subsequently, the values resulting from the error covariance parameters estimation is used as a starting point, and will need further manual fine tuning in order to achieve the best achievable performance from the EKF. This iterative process of manual fine tuning of the EKF is usually judged by the system convergence time, the existence of divergence intervals in the state error covariance matrix, and the final estimated state accuracy. However, different attempts were undertaken in order to tackle the problem of tuning Kalman Filters and its derivatives in an efficient way. The main objective of all these attempts was either to estimate the initial values of different covariance matrices P_0 , R, and Q, or to adaptively change their values during operation to account for any dynamic changes in the inputs characteristics. In (Powell, 2002) formulated the tuning problem as a numerical optimization problem, and solved it using the Downhill Simplex Algorithm. While in (Bolognani et al., 2003) a method for proper choice of the covariance matrices in EKF based on a self-tuning procedure was presented. The method is based on a complete normalization of the EKF representation to allow the self-tuning procedure to fit in a wide range of filter configurations. The objective of this proposal was to find a method for tuning for mass-produced products that includes EKF as a building block inside it without the need to tune each part at its own in order to compensate for manufacturing uncertainties. The paper discussed the method and reported its performance in an AC motor drives-related application. In addition to that, (Loebis et al., 2004) discussed the use of fuzzy logic techniques for the estimation of the initial error covariance parameters in two proposed kalman filters. The filters are used for the navigation of an autonomous underwater vehicle. Means of cost function for adaptive tuning of the kalman filter covariance was also mentioned in (Basil et al., 2004). Furthermore, (Macias and Exposito, 2006) proposed a method for the self-tuning of kalman filters to track the harmonic random changes. The method proposes an adaptive procedure for the estimation of the driving source covariance matrix as well as the state covariance matrix every time epoch. The method was tested against sudden input changes to show their performance in such conditions. A significant improvement in the kalman filter performance was shown in (Akesson et al., 2007) by the proposal of a tool for kalman filter tuning. This tool is based on the generalization of the autocovariance least-squares method to systems with mutually correlated noise characteristics. Moreover, (Åkesson et al., 2008) a method for estimating the noise covariance from process data is proposed based on the linear relation between the covariance and the autocovariance and formulating the problem as a least-squares problem. Besides that, (Shu et al., 2013) discussed the tuning and steady-state performance of a third-order integrated random walk model in details and was compared with a first-order auto-regressive model. The results showed that a well-tuned third-order integrated random walk model-based KF outperform the other method. Finally, (Matisko and Havlena,

2013) presented a method for the tuning of the error covariance matrices using bayesian approach and monte Carlo simulations is presented and compared with other methods in the literature.

3.1 Utilized Tuning Methodology

1. P_0 Matrix represents the error in the kalman state. The initial values are defined as the error in the error state and are defined as the average between the ground truth S_{ref} and the corrected INS data S_{INS} by the estimated error E_{est} from EKF as illustrated in (7).

$$P_0(i,j) = \begin{cases} avg((S_{ref} - S_{INS}) - E_{est}), & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}$$
(7)

It is assumed that the navigation states' error characteristics are independent from each other and completely decoupled. Consequently, and as noted from the previous equation, only diagonal elements has covariance values and off-diagonal elements are all set to zeros to satisfy this assumption. Moreover, Experiments shown that setting the P_0 matrix to zeros or values very close to zeros affects the convergence time negatively. Finally, The P_0 matrix is only initialized and then gets updated automatically by the EKF equations.

- 2. R Matrix represents the error covariance in the aiding sensors (i.e. the GPS in our case). Usually, GPS modules provide the error variance or standard deviation information for both position and velocity. These received variance values can be used directly in the R matrix. They represent the confidence in the received GPS readings and are affected by outage and interference conditions. At each epoch, new values are received and fed to the R matrix.
- **3.** Q **Matrix** represents the error in the driving sensors (i.e. INS in our case). The initial values are defined as the error between the ground truth S_{ref} and the uncorrected INS data S_{INS} as illustrated in (8).

$$Q(i,j) = \begin{cases} avg((S_{ref} - S_{INS})), & \text{if } i = j\\ 0, & \text{otherwise} \end{cases}$$
 (8)

Similar to the P_0 matrix, it is assumed that the navigation states' error characteristics in the standalone INS are independent from each other and completely decoupled. Consequently, and as noted from the previous equation, only diagonal elements has covariance values and off-diagonal elements are all set to zeros to satisfy this assumption. Finally, The Q matrix is only initialized and is not changed during operation, as the error reliability of the INS is not likely to change during operation in our case.

4. Error Covariance Fine Tuning is performed after the final offline definition of different error covariance matrices. An iterative process of manual fine tuning of the EKF is initialed and is usually judged by the system convergence time, the existence of divergence intervals in the state error covariance matrix, and the final estimated state accuracy.

4. SIMULATION AND EXPERIMENTAL RESULTS

The results of the tuning of the EKF-based INS/GPS system are presented for two different datasets. The first dataset is acquired using a high-end tactical-grade SPAN unit featuring Novatel HG1700 IMU module. The second dataset is acquired from

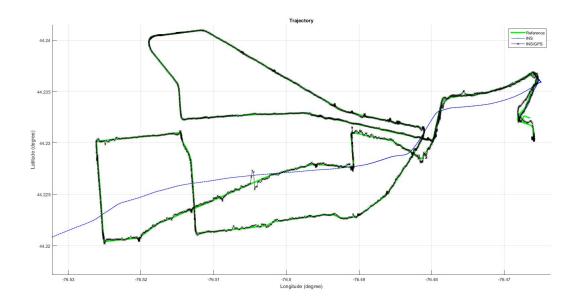


Figure 2. Novatel HG1700 Trajectory Results

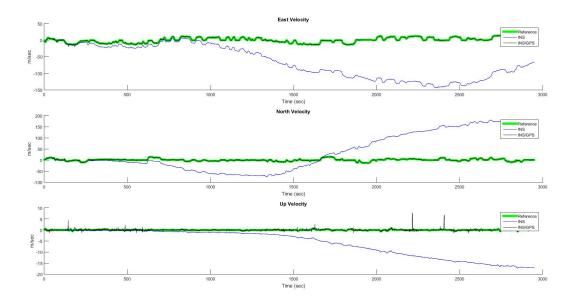


Figure 3. Novatel HG1700 Velocity Results

a MEMS-based SCC1300-D04 IMU unit from VTI. Figure 2 shows the trajectory produced by the standalone INS vs. the INS/GPS integration against the reference trajectory of the highend tactical-grade SPAN unit. In addition to that, Figure 3 and Figure 4 show the accuracy of the the velocity and attitude of the system for both the standalone INS and the integrated INS/GPS system against the same reference trajectory. A large drift in position, velocity, and attitude can be observed in the INS-standalone system results. While, this drift was bounded and eliminated after the integration with the GPS. Analytically, The maximum root mean square error (RMSE) values of different navigation states are presented in Table 1

Figure 5 shows the trajectory produced by the standalone INS vs. the INS/GPS integration against the reference trajectory MEMS-

Table 1. Comparison between the maximum RMSE in standalone INS and integrated INS/GPS

	$RMSE_{INS}$	$RMSE_{INS/GPS}$
Position	42363.7 m	2.8515 m
Velocity	60.2683 m/sec	0.2424 m/sec
Attitude	1.795°	0.8517°

based SCC1300-D04 IMU unit from VTI. It shows that the tuned EKF-based INS/GPS yields a more robust and reliable navigation information than the standalone INS.

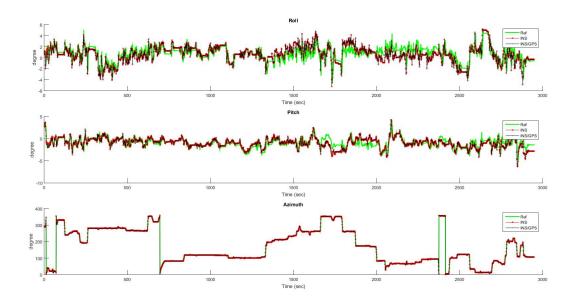


Figure 4. Novatel HG1700 Attitude Results

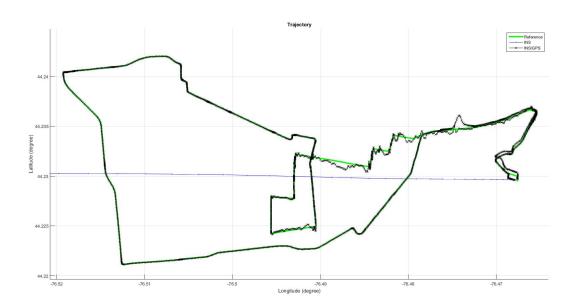


Figure 5. VTI SCC1300-D04 Trajectory Results

5. CONCLUSION

Standalone INS navigation systems are essential to be used because of their ability to provide continuous navigation information regardless of the environment conditions. On the other hand, the GPS navigation capabilities are affected by different environmental aspects such as the outage and signal interference among many others. However, it can provide a good navigation accuracy on the long term. Obviously, INS and GPS are complementary in nature and fusing them will yield a stable navigation solution both on the short term and on the long term. The integration accuracy and yielded performance depends heavily on the quality of the tuning of different error covariance parameters presented in the system. In this paper, the integration between the INS and GPS using an EKF was presented. Different tuning methodolo-

gies were reviewed qualitatively and in brief. A tuning methodology for the error covariance matrices was presented and applied on two datasets taken from a high-end tactical-grade SPAN unit featuring Novatel HG1700 IMU module and a MEMS-based SCC1300-D04 IMU unit from VTI. The results presented show how the tuned EKF achieved a better performance than each system alone (i.e. INS and GPS) in terms of the accuracy and the fast convergence.

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