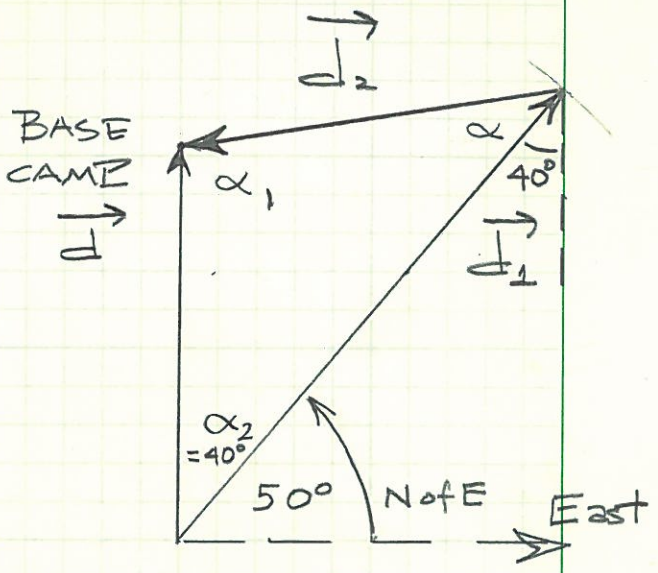
••22 An explorer is caught in a whiteout (in which the snowfall is so thick that the ground cannot be distinguished from the sky) while returning to base camp. He was supposed to travel due north for 5.6 km, but when the snow clears, he discovers that he actually traveled 7.8 km at 50° north of due east. (a) How far and (b) in what direction must he now travel to reach base camp?



$\vec{d}_1 + \vec{d}_2 = \vec{d}$

1st Trip When lost — 2nd Correction Trip — original planned trip

$\Rightarrow \vec{d}_2 = \vec{d} - \vec{d}_1 \qquad \vec{d}_2 \cdot \vec{d}_2 = (\vec{d} - \vec{d}_1) \cdot (\vec{d} - \vec{d}_1) \Rightarrow$

LAW OF COSINES $\quad d_2^2 = d^2 + d_1^2 - 2 d_1 d \cos\alpha_2$

$d_2^2 = 5.6^2 + 7.8^2 - 2(5.6)(7.8)\cos 40° = 25.3$

$d_2 = 5.03 \approx 5.0$ km = distance to travel

a)

LAW OF SINES $\quad \frac{d}{\sin\alpha} = \frac{d_2}{\sin\alpha_2} = \frac{d_1}{\sin\alpha_1}$

$\sin\alpha = \frac{d}{d_2}\sin\alpha_2 = \frac{5.6}{5.03}\sin 40 = 0.716$

$\Rightarrow \alpha = \sin^{-1}(0.716) = 45.7$

$\alpha + \alpha_1 + \alpha_2 = 180 \Rightarrow \alpha_1 = 180 - \alpha - \alpha_2 = 180 - 45.7 - 40$

$\alpha_1 = 94.3° \Rightarrow [40 + \alpha = 40 + 45.7 = 85.7°]$ W of S

or 4.3° S of W

The law of Sines is proved via the cross product:

Remember $|\vec{A} \times \vec{B}|$ = Area of Parallelogram formed by $\vec{A}$ & $\vec{B}$

$\Rightarrow |\vec{d} \times \vec{d}_1| = |\vec{d}_2 \times (-\vec{d}_1)| = |(-\vec{d}) \times (-\vec{d}_2)|$

$\Rightarrow$ Law of Sines