

3

treat fight miss
like train lost

s1: treat fight miss

s2: like train lost

4

treat catch
train fight
like catch
miss lost

→ create DSU →



```
[ HashMap < string, string > parent;  
  HashMap < string, integer > rank;
```

parent

treat → treat
catch → treat
train → train
fight → train
like → treat
miss → lost
lost → lost

rank

treat → 1
catch → 0
train → 1
fight → 0
like → 0
miss → 0
lost → 1

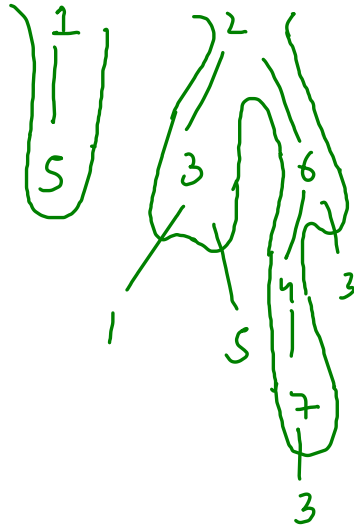
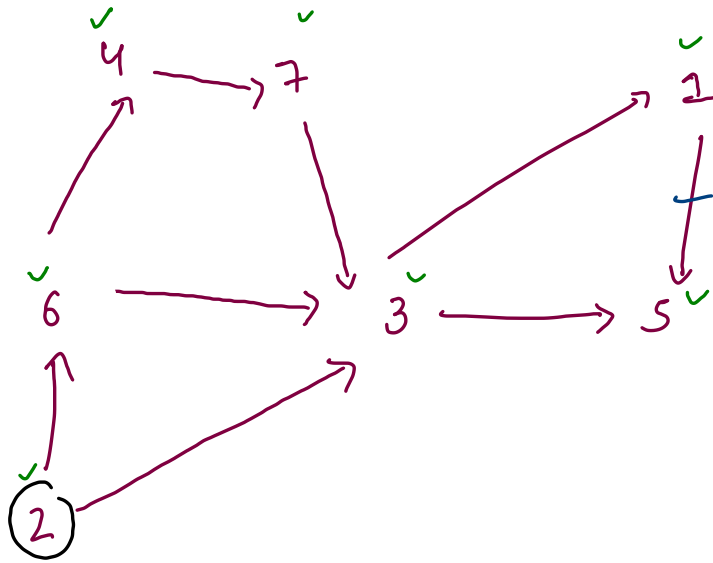
4

- ✓ treat catch
- ✓ train fight
- ✓ like catch
- ✓ miss lost

3

treat fight miss
like train lost

Kahn's algo : topological sort (DAG)
directed acyclic graph



2
6
4
7
3
1
5

$u \rightarrow v$.

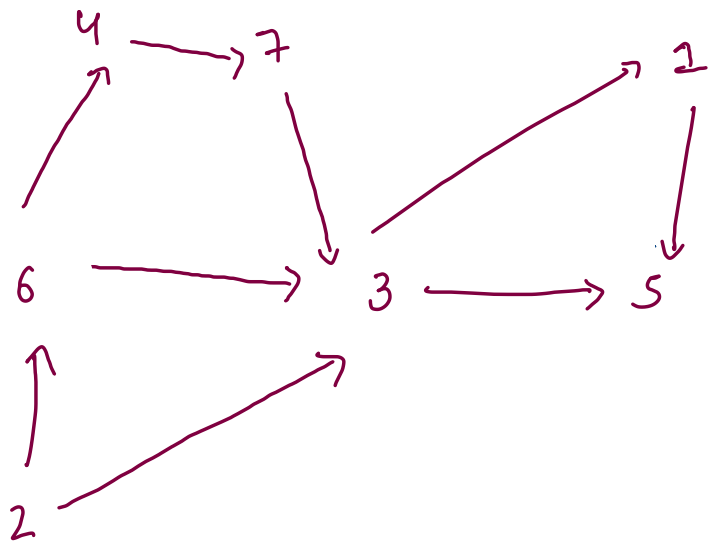
u should come before v

$u \rightarrow v$, u depends on v

ts: 2 6 4 7 3 1 5

order of comp: rev(ts)

↳ 5 1 3 7 4 6 2



Kahn's algo

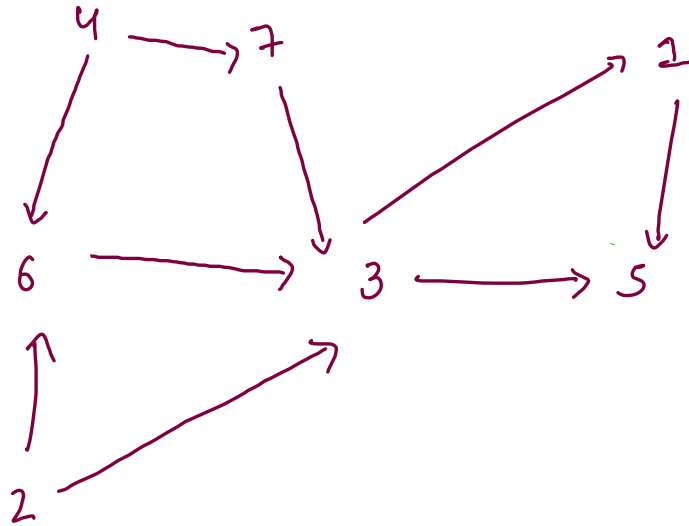
indgree

⁰ 2	0	2 ² 3	⁰ 1	2 ¹ 2	1 ⁰	⁰ 1
1	2	3	4	5	6	7

2	6	4	7	3	1	5
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2 6 4 7 3 1 5
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Kahn's algo



①. fill indegree array.

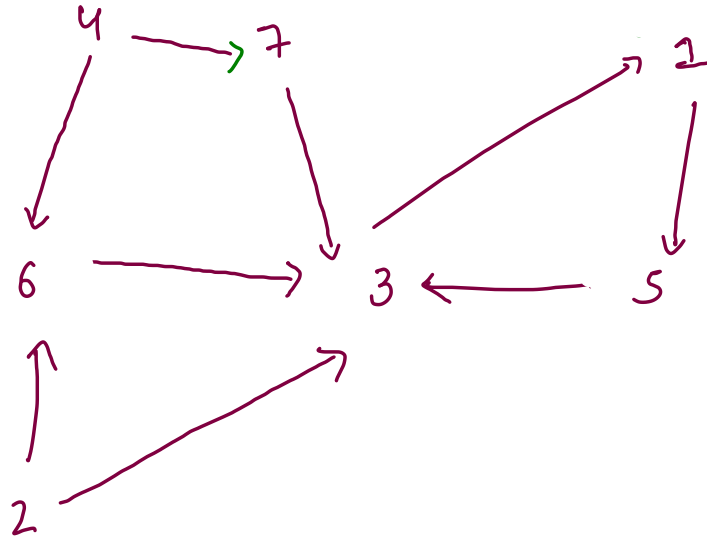
2 ⁰	0	2 ⁰ 3	0	2 ⁰ 2	2 ⁰ 2	0 ⁰ 1
1	2	3	4	5	6	7

②. Start with '0' indegree vertices

2	4	7	6	3	1	5
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2 4 7 6 3 1 5

cycle detect:



1	0	3 ⁰ 4 ¹	0	1	2 ⁰ 1	1 ⁰
1	2	3	4	5	6	7

4	2	7	6
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(4, 2, 7, 6)

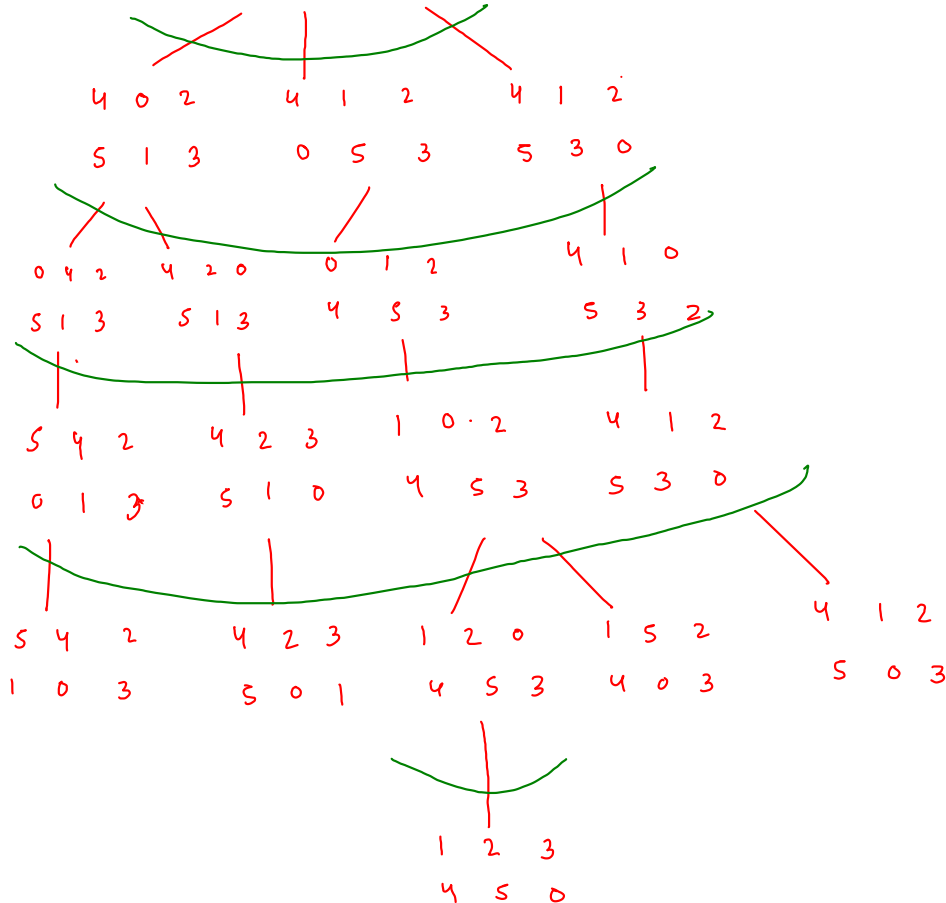
count = 0 + 1 + 1 + 1 + 1

Sliding Puzzle

4 1 2
 ↑
 5 ← 0 → 3



1 2 3
 4 5 0



911
 4 0 2
 5 1 3

	0	1	2
0	4 ₀	1 ₁	2 ₂
		↑	
1	5 ₃ ←	0 ₄	→ 3 ₅

0	1	2	3	4	5
4	1	2	5	0	3

$$i = \text{bno} / m;$$

$$j = \text{bno} \% m;$$

$m \rightarrow$ total cols

$$\text{bno} = i * m + j;$$

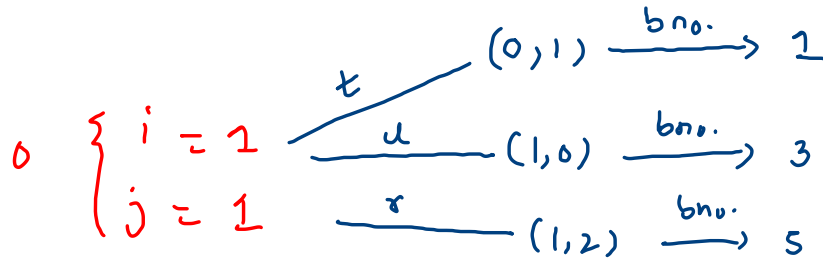
dest \rightarrow 1 2 3 4 5 0

4 1 2 5 0 3	4 0 2 5 1 3	4 1 2 0 5 3	4 1 2 5 3 0
-------------	-------------	-------------	-------------

2d \rightarrow string;

$$m = 3$$

0	1	2	3	4	5
4	1	2	5	0	3



(i) easy to maintain visited, bds.

(ii) difficult to find n brs.

map: $\left[\begin{matrix} [1,3] \\ 0 \end{matrix}, \begin{matrix} [0,2,4] \\ 1 \end{matrix}, \begin{matrix} [1,5] \\ 2 \end{matrix}, \begin{matrix} [0,4] \\ 3 \end{matrix}, \begin{matrix} [1,3,5] \\ 4 \end{matrix}, \begin{matrix} [2,4] \\ 5 \end{matrix} \right]$

	0	1	2
0	0	1	2
1	3	4	5

4 1 2 5 0 3 | 4 0 2 5 1 3 | 4 1 2 0 5 3 | 4 1 2 5 3 0

$$\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ \text{str} = & 4 & 1 & 2 & 5 & 0 & 3 \\ & - & & - & & & \\ & & \downarrow & & & & \\ \text{sb} = & 4 & 0 & 2 & 5 & 1 & 3 \\ & & & & & & \\ & & & \downarrow & & & \\ \text{str} = & 4 & 1 & 2 & 5 & 0 & 3 \end{array}$$

$$\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & 2 & 5 & 0 & 3 \end{array}$$

map[4] \leftrightarrow nbr : 1, 3, 5

```

      / | \
     /  |  \
4 0 2 5 1 3  4 1 2 0 5 3  4 1 2 5 3 0
  
```

1 0 2
4 5 3



1 2 3
4 5 0

102453 | 012453 | 120453 | 152403 | 412053

102453
012453
120453
412053

1 2 ³~~0~~ 4 5 ~~3~~⁰
0 1 2 3 4 5

ans -> 2

[[1, 3], [0, 2, 4], [1, 5], [0, 4], [1, 3, 5], [2, 4]]
0 1 2 3 4 5