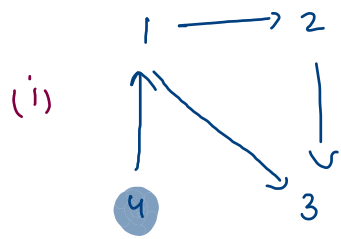
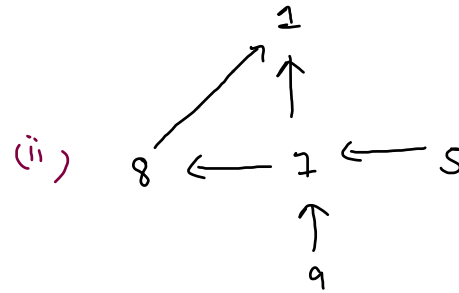


mother vertex

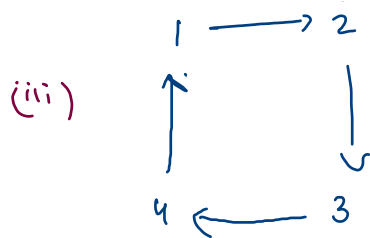
mother vertex : we can travel (direct or indirect)
all the other vertices from a mother
vertex



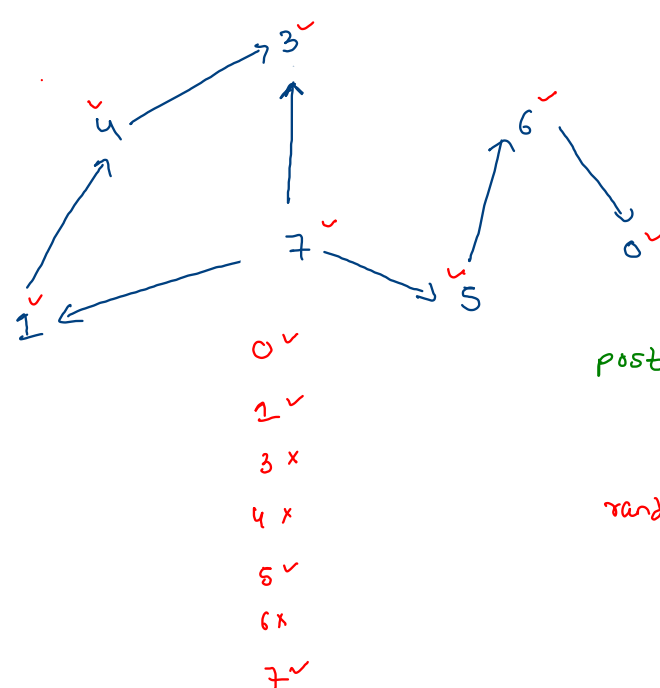
brute force: $V \times (DFS)$



[no mother vertex]



(multiple mother vertex)

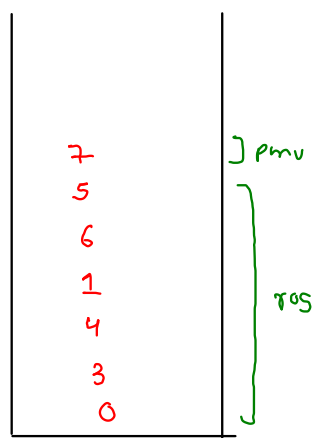


0✓
1✓
2✓
3x
4x
5✓
6x
7✓

post → push vtx
in stack

random DFS

DFS



pmv

ros

Algo

✓

✓

ans ✓

x

x

ans ✓

✓

x

ans ✓

x

✓

impossible

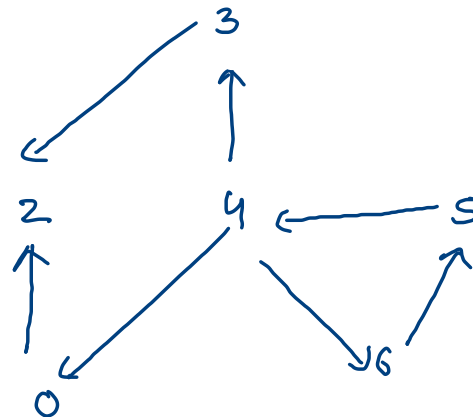
algo has 2 DFS : (i) to fill stack

(ii) to check top most is rotten
vertex;

top most $\xrightarrow{mv \checkmark}$ return ans;
 $\xrightarrow{mv \times}$ return -1;

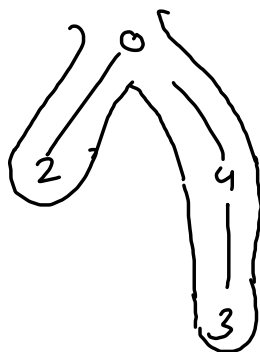
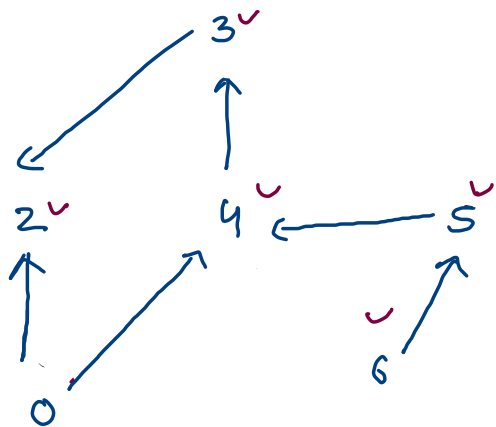
| | pmv | ro5 | Algo |
|---|-----|-----|------------|
| ① | ✓ | ✓ | ans ✓ |
| ② | X | X | ans ✓ |
| ③ | ✓ | X | ans ✓ |
| ④ | X | ✓ | impossible |

①.



| | | |
|-------|---|----|
| pmv [| 4 | mv |
| ro5 { | 6 | mv |
| | 5 | mv |
| | 3 | |
| | 0 | |
| | 2 | |

②



| | |
|-----|---|
| pmu | 6 |
| | 5 |
| | 0 |
| rds | 4 |
| | 3 |
| | 2 |

//function to find a mother vertex in the graph.

```
public int findMotherVertex(int V, ArrayList<ArrayList<Integer>>adj)
{
    // Code here

    //1. to fill the stack
    int v = adj.size();
    Stack<Integer>st = new Stack<>();
    boolean[]vis = new boolean[v];

    for(int i=0; i < v;i++) {
        if(vis[i] == false) {
            dfs(adj,i,vis,st);
        }
    }

    //2. is pmv is actually a mother vertex or not
    int pmv = st.peek();
    vis = new boolean[v];
    dfs(adj,pmv,vis);

    for(int i=0; i < vis.length;i++) {
        if(vis[i] == false) {
            return -1;
        }
    }

    return pmv;
}
```

```
public void dfs(ArrayList<ArrayList<Integer>>graph,int src,boolean[]vis,Stack<Integer>st) {

    vis[src] = true;

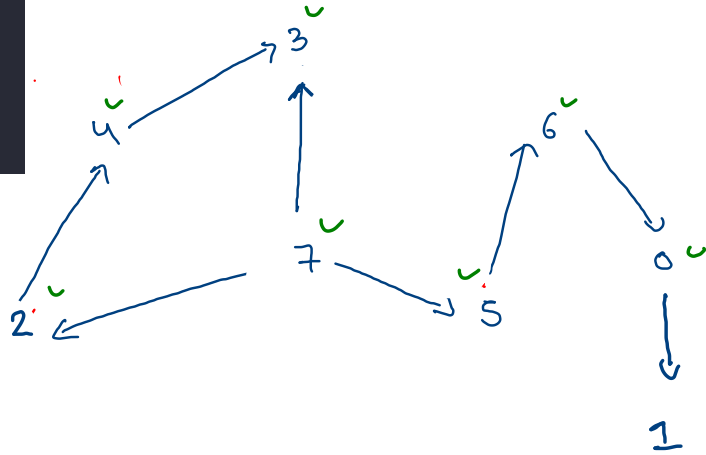
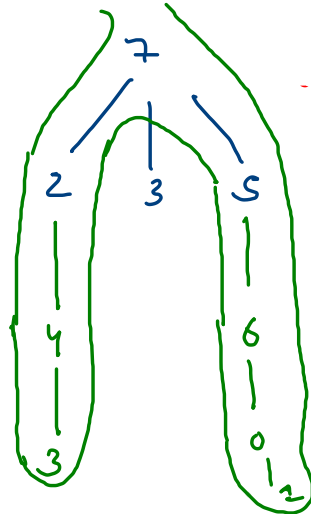
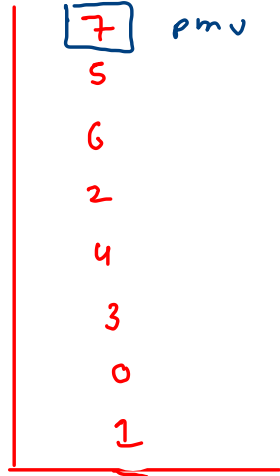
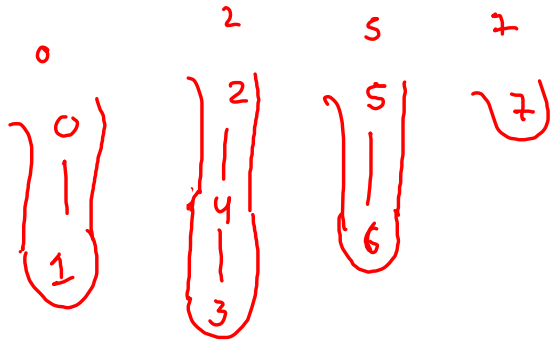
    for(int nbr : graph.get(src)) {
        if(vis[nbr] == false) {
            dfs(graph,nbr,vis,st);
        }
    }

    st.push(src);
}
```

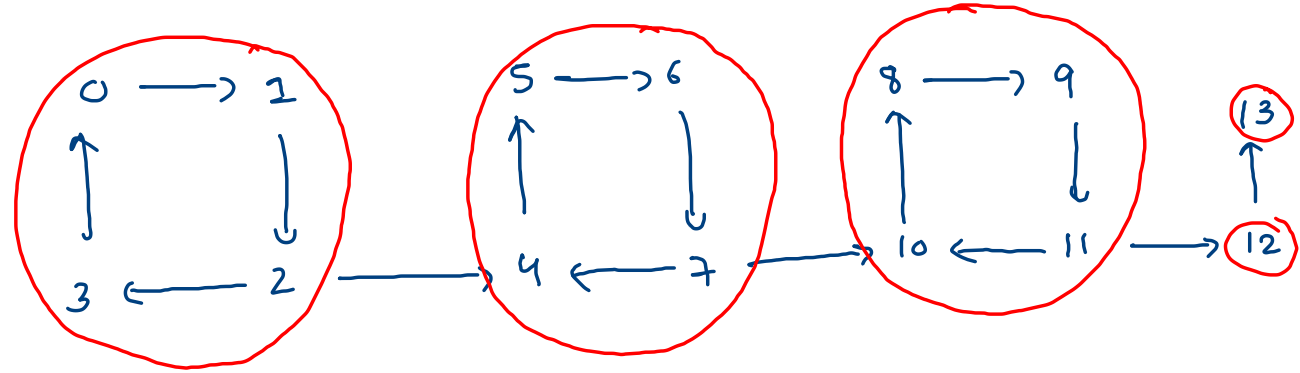
```
public void dfs(ArrayList<ArrayList<Integer>>graph,int src,boolean[]vis) {

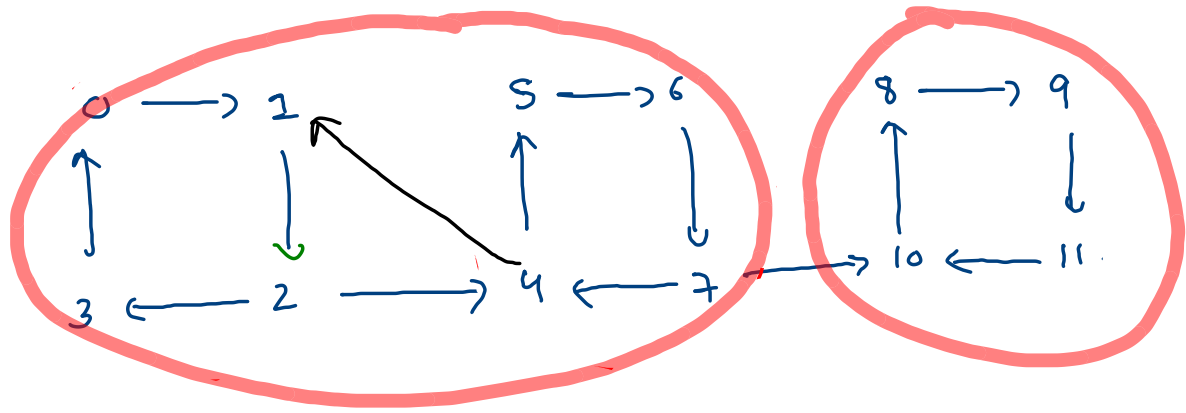
    vis[src] = true;

    for(int nbr : graph.get(src)) {
        if(vis[nbr] == false) {
            dfs(graph,nbr,vis);
        }
    }
}
```

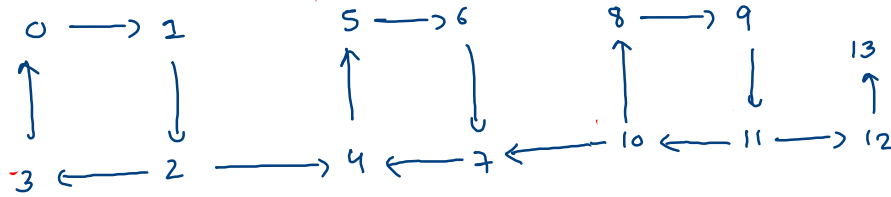


Kosaraju
no. of strongly
connected components





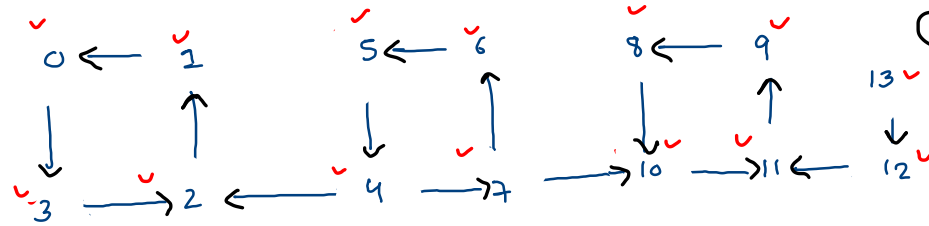
no. of cycle 1, = no. of strongly connected components.



st. peek \rightarrow left vtx

$$SCC = 1 + 1 + 1 + 1 + 1$$

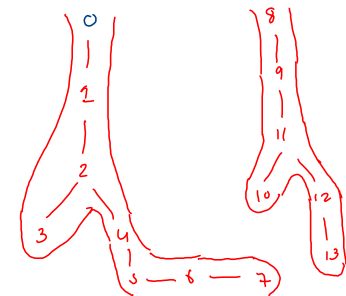
| | |
|----|---|
| 8 | ✓ |
| 9 | X |
| 11 | X |
| 12 | ✓ |
| 13 | ✓ |
| 10 | X |
| 0 | ✓ |
| 2 | X |
| 2 | X |
| 4 | ✓ |
| 5 | X |
| 6 | X |
| 7 | X |
| 3 | X |



(i) fill stack, start dfs from left area's vtx.

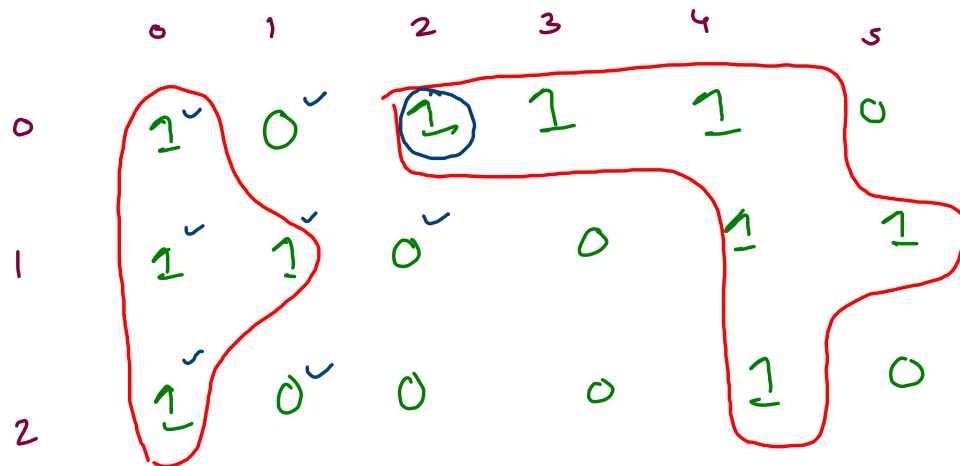
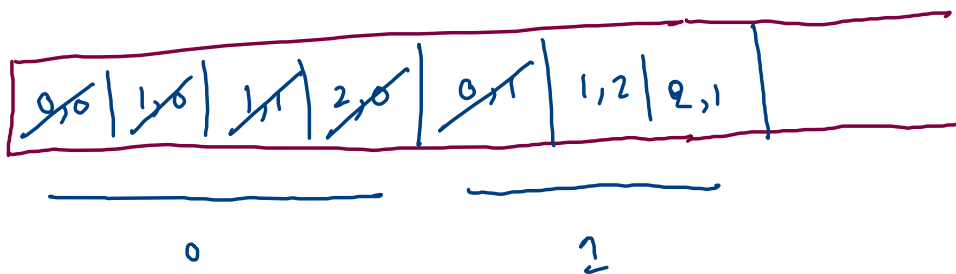
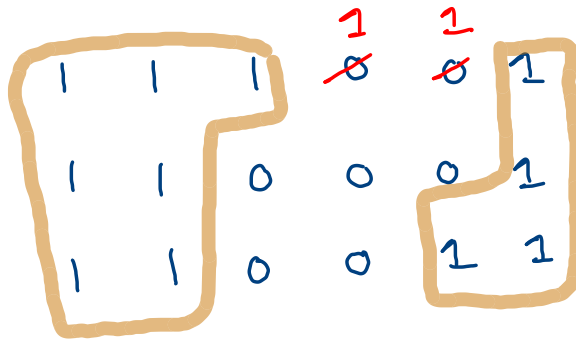
(ii) reverse all edges

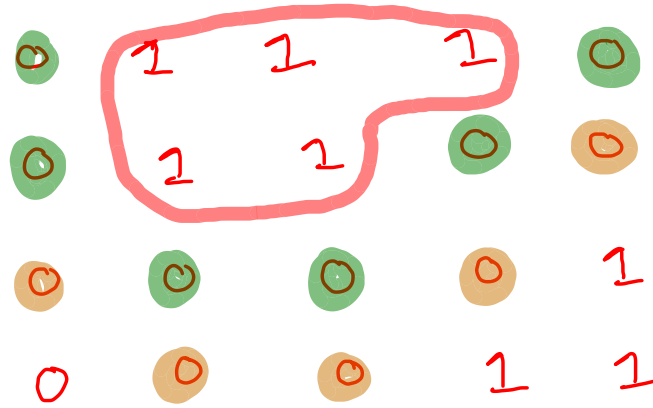
(iii) no. of dfs \rightarrow strongly connected comp.



dfs order
stack

shortest
bridge
←
(bfs application)





● \rightarrow lev 1

● \rightarrow lev 2

flag = ~~false~~ ^{true}

```
ArrayDeque<Pair> q = new ArrayDeque<>();
boolean flag = false;

for(int i=0; i < grid.length && flag == false; i++) {
    for(int j=0; j < grid[0].length; j++) {
        if(grid[i][j] == 1) {
            dfs(grid, i, j, q);
            flag = true;
            break;
        }
    }
}
```

| | 0 | 1 | 2 | 3 | 4 |
|---|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|
| 0 | 0 ⁻¹ | 2 ⁻¹ | 2 ⁻¹ | 2 ⁻¹ | 0 ⁻¹ |
| 1 | 0 ⁻¹ | 2 ⁻¹ | 2 ⁻¹ | 0 ⁻¹ | 0 ⁻¹ |
| 2 | 1 ⁻¹ | 0 ⁻¹ | 1 ⁻¹ | 0 ⁻¹ | 1 |
| 3 | 1 ⁻¹ | 0 ⁻¹ | 0 ⁻¹ | 1 | 1 |

```
while(q.size() > 0) {
    int count = q.size();

    while(count-- > 0) {
        //remove
        Pair rem = q.remove();
        int r1 = rem.i;
        int r2 = rem.j;

        //add nbr
        for(int i=0; i < 4; i++) {
            int ni = r1 + dir[i][0];
            int nj = r2 + dir[i][1];

            if(ni >= 0 && ni < grid.length && nj >= 0 && nj < grid[0].length && grid[ni][nj] != -1) {
                if(grid[ni][nj] == 1) {
                    return lev;
                }

                grid[ni][nj] = -1;
                q.add(new Pair(ni, nj));
            }
        }
        lev++;
    }
}
```

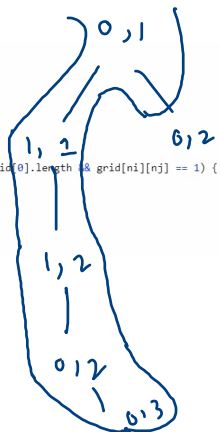
End

```
public void dfs(int[][] grid, int r, int c, ArrayDeque<Pair> q) {
```

```
    grid[r][c] = -1;
    q.add(new Pair(r, c));

    for(int i=0; i < 4; i++) {
        int ni = r + dir[i][0];
        int nj = c + dir[i][1];

        if(ni >= 0 && ni < grid.length && nj >= 0 && nj < grid[0].length && grid[ni][nj] == 1) {
            dfs(grid, ni, nj, q);
        }
    }
}
```



new = ~~0 1 2~~

c = 5

| | | | | | | | | | | | | | | | | |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----|-----|-----|-----|
| 0,1 | 1,1 | 1,2 | 0,2 | 0,3 | 0,0 | 1,0 | 2,1 | 1,5 | 2,2 | 0,4 | 2,0 | 3,1 | 1,4 | 2,3 | 3,2 | 3,0 |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----|-----|-----|-----|

2