VISVESVARAYA TECHNOLOGICAL UNIVERSITY

BELAGAVI

MATHEMATICS HANDBOOK

III Semester BE Program

2022-2023

Derivatives of some standard functions:

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}\left(x^n\right) = n\,x^{n-1}$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}\left(a^x\right) = a^x \log a$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\cos ec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cos ecx) = -\cos ecx\ \cot x$$

$$\frac{d}{dx}(\log x) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a x) = \frac{\log_a e}{x}$$

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \sec h^2 x$$

$$\frac{d}{dx}(\coth x) = -\cos ech^2 x$$

$$\frac{d}{dx}(\sec hx) = -\sec hx \tanh x$$

$$\frac{d}{dx}(\cos echx) = -\cos echx \coth x$$

Rules of Differentiation:

$$\frac{d}{dx}(cu) = c\,\frac{du}{dx}$$

$$\frac{d}{dx}(uv) = u\frac{d}{dx}(v) + v\frac{d}{dx}(u)$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$$

Parametric differentiation:

If $x = x(t)$ & $y = y(t)$ *then* $\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$

Chain Rule:

If $y = f(u)$ & $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Integrals of some standard functions:
(Constant of Integration C to be added in all the integrals)

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \qquad \int \frac{1}{x}dx = \log x$$

$$\int \log x\,dx = x\log x - x, x \neq 0 \qquad \int k\,dx = k\,x$$

$$\int e^x dx = e^x \qquad \int a^x dx = \frac{a^x}{\log a}$$

$$\int \sin x\,dx = -\cos x \qquad \int \cos x\,dx = \sin x$$

$$\int \tan x\,dx = \log(\sec x) \qquad \int \cot x\,dx = \log(\sin x)$$

$$\int \sec x\,dx = \log(\sec x + \tan x) \qquad \int \operatorname{cosec} x\,dx = \log(\operatorname{cosec} x - \cot x)$$

$$\int \sec^2 x\,dx = \tan x \qquad \int \operatorname{cosec}^2 x\,dx = -\cot x$$

$$\int \sec x \tan x\,dx = \sec x \qquad \int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x$$

$$\int \sinh x\,dx = \cosh x \qquad \int \cosh x\,dx = \sinh x$$

$$\int \tanh x\,dx = \log(\cosh x) \qquad \int \coth x\,dx = \log(\sinh x)$$

$$\int \operatorname{sech}^2 x\,dx = \tanh x \qquad \int \operatorname{cosech}^2 x\,dx = -\coth x$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}(x/a) \qquad \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}(x/a)$$

$$\int \frac{f'(x)}{f(x)}dx = \log(f(x)) \qquad \int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)}$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx]$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx]$$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx \qquad \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(x) \text{ is even function} \\ 0 & \text{if } f(x) \text{ is odd function} \end{cases}$$

$$\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & if \quad f(2a-x) = f(x) \\ 0, & if \quad f(2a-x) = -f(x) \end{cases}$$

Integration by parts:

$$\int u(x)v(x)\,dx = u(x)\left(\int v(x)\,dx\right) - \int \frac{d}{dx}\left(u(x)\right)\left(\int v(x)\,dx\right)dx$$

Bernoulli's rule of integration:

If the 1st function is a polynomial and integration of 2nd function is known. Then

$$\int u(x)v(x)dx = u\int v\,dx - u'\iint v\,dxdx + u''\iiint v\,dxdxdx - u'''\iint\iint v\,dxdxdxdx + \cdots\cdots\cdots$$

Where dashes denote the differentiation of u.

Or

$$\int u(x)v(x)dx = u\cdot v_1 - u'\cdot v_2 + u''\cdot v_3 - u'''\cdot v_4 + \cdots\cdots\cdots$$

Where dashes denote the differentiation of u, v_k denotes the integration of v, k times with respect to x.

Vector calculus formulae:

Position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Magnitude $\left|\vec{r}\right| = \sqrt{x^2 + y^2 + z^2}$

Dot product of unit vectors $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$ and $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

Cross product of unit vectors $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$ and $\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{k} = \hat{i},\ \hat{i}\times\hat{k} = -\hat{j}$

Angle between two vectors $\cos\theta = \dfrac{\vec{A}.\,\vec{B}}{\left|\vec{A}\right|\,\left|\vec{B}\right|}$

Unit vector $\hat{A} = \dfrac{\vec{A}}{\left|\vec{A}\right|}$

Velocity $\vec{V} = \frac{ds}{dt}$

Acceleration $\vec{a} = \frac{d^2 s}{dt^2}$

For any vectors $\vec{A} = (a_1 i + b_1 j + c_1 k)$, $\vec{B} = (a_2 i + b_2 j + c_2 k)$ & $\vec{C} = (a_3 i + b_3 j + c_3 k)$

Dot product of two vectors $\vec{A} \cdot \vec{B} = a_1 a_2 + b_1 b_2 + c_1 c_2$

Cross product of two vectors $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Scalar triple product $\vec{A} \cdot \left(\vec{B} \times \vec{C} \right) = \left(\vec{A} \times \vec{B} \right) \cdot \vec{C} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Trigonometric formulae:

- **Identities**

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

- **Compound angle formulae**

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

- **Transformation formulae**

$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$, $\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$, $\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$, $\sin C - \sin D = 2\sin\left(\frac{C-D}{2}\right)\cos\left(\frac{C+D}{2}\right)$

$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$, $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

- **Multiple angle formulae**

$\sin 2A = 2\sin A \cos A$

$\sin A = 2\sin(A/2)\cos(A/2)$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos A = \cos^2(A/2) - \sin^2(A/2)$

$$\sin^2 A = \frac{(1-\cos 2\theta)}{2}$$

$$\cos^2 A = \frac{(1+\cos 2\theta)}{2}$$

$$\sin^3 A = \frac{1}{4}[3\sin A - \sin 3A]$$

$$\cos^3 A = \frac{1}{4}[3\cos A + \cos 3A]$$

Hyperbolic and Euler's formulae

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\cosh^2\theta - \sinh^2\theta = 1$$

$$e^{i\theta} = \cos\theta + i\,\sin\theta$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

Logarithmic formulae:

$$\log_e(AB) = \log_e(A) + \log_e(B)$$

$$\log_e\left(\frac{A}{B}\right) = \log_e(A) - \log_e(B)$$

$$\log_e x^n = n\log_e x$$

$$\log_a B = \frac{\log_e B}{\log_e a}$$

$$\log_a a = 1$$

$$\log_a 1 = 0$$

$$\log_e 0 = -\infty$$

Polar coordinates and polar curves:

Angle between radius vector and tangent

$$\tan\phi = r\frac{d\theta}{dr} \quad \text{or} \quad \cot\phi = \frac{1}{r}\frac{dr}{d\theta}$$

Angle of intersection of the curves

$$|\phi_1 - \phi_2| = \tan^{-1}\left\{\left|\frac{\tan\phi_1 - \tan\phi_2}{1+\tan\phi_1\tan\phi_2}\right|\right\}$$

Orthogonal condition $|\phi_1 - \phi_2| = \dfrac{\pi}{2}$ or $\tan\phi_1 \cdot \tan\phi_2 = -1,$

Pedal equation or *p-r* equation

$$p = r\sin\phi$$

$$\frac{1}{p^2} = \frac{1}{r^2}(1 + \cot^2 \emptyset) = \frac{1}{r^2}\left(1 + \frac{1}{r^2}\left(\frac{dr}{d\theta}\right)^2\right)$$

Radius of curvature

In Cartesian form: $\rho = \frac{(1+{y_1}^2)^{\frac{3}{2}}}{y_2}$

In parametric form: $\rho = \frac{(\dot{x}^2+\dot{y}^2)^{\frac{3}{2}}}{\dot{x}\ddot{y}-\dot{y}\ddot{x}}$

In polar from: $\rho = \frac{(r^2+{r_1}^2)^{\frac{3}{2}}}{r^2-rr_2+2{r_1}^2}$

Pedal Equation: $\rho = r\frac{dr}{dp}$

Indeterminate Forms - L'Hospital's rule:

If $f(a) = g(a) = 0$, then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

If $f(a) = g(a) = \infty$, then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$, $\lim_{n\to 0}(1+n)^{\frac{1}{n}} = e$

$\lim_{\theta\to 0}\frac{sin\theta}{\theta} = 1$, $\lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}$

Series Expansion:

Taylor's series expansion about the point $x = a$.

$$y(x) = y(a) + \frac{(x-a)}{1!}y'(a) + \frac{(x-a)^2}{2!}y''(a) + \frac{(x-a)^3}{3!}y'''(a) + ..$$

Maclaurin's Series at the point $x = 0$

$$y(x) = y(0) + \frac{x}{1!}y'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \frac{x^4}{4!}y''''(0) + ...$$

Composite function:

If $z = f(x,y)$ and $x = \phi(t), y = \psi(t)$ then $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$

If $z = f(x,y)$ and $x = \phi(u,v), y = \psi(u,v)$

$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$ & $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$

If $u = f(r,s,t)$ and $r = \phi(x,y,z), s = \psi(x,y,z), t = \xi(x,y,z)$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial x}$$
$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial y} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial y}$$
$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial z} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial z} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial z}$$

Fourier Series

Even and Odd functions

A function $f(x)$ is said to be an even function if $f(-x) = f(x)$

A function $f(x)$ is said to be an odd function if $f(-x) = -f(x)$

Properties of definite integral

$$\int_{-l}^{l} f(x)dx = \begin{cases} 2\int_{0}^{l} f(x)\,dx \ \ when\ f(x)\ is\ an\ even\ function \\ 0, \qquad\qquad when\ f(x)\ is\ an\ odd\ function \end{cases}$$

Fourier series of a periodic function $f(x)$ in the interval $(a, a+2l)$:

$$f(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos\left(\frac{n\pi}{l}x\right) + \sum_{n=0}^{\infty} b_n \sin\left(\frac{n\pi}{l}x\right)$$

Where

$a_0 = \frac{1}{l}\int_a^{a+2l} f(x)dx,\ a_n = \frac{1}{l}\int_a^{a+2l} f(x)\cos\left(\frac{n\pi}{l}x\right)dx$ & $b_n = \frac{1}{l}\int_a^{a+2l} f(x)\sin\left(\frac{n\pi}{l}x\right)dx$

Interval $(a, a+2l)$	$a_0 = \frac{1}{l}\int_{a}^{a+2l} f(x)dx$	$a_n = \frac{1}{l}\int_{a}^{a+2l} f(x)\cos\left(\frac{n\pi}{l}x\right)dx$	$b_n = \frac{1}{l}\int_{a}^{a+2l} f(x)\sin\left(\frac{n\pi}{l}x\right)dx$
$(0,\ 2l)$	$a_0 = \frac{1}{l}\int_{0}^{2l} f(x)dx$	$a_n = \frac{1}{l}\int_{0}^{2l} f(x)\cos\left(\frac{n\pi}{l}x\right)dx$	$b_n = \frac{1}{l}\int_{0}^{2l} f(x)\sin\left(\frac{n\pi}{l}x\right)dx$

$(0, 2\pi)$	$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx$	$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos(nx)\,dx$	$b_n = \frac{1}{\pi}\int_a^{2\pi} f(x)\sin(nx)\,dx$
$(-l,\ l)$	$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)dx$	$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi}{l}x\right)dx$	$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi}{l}x\right)dx$
$(-\pi, \pi)$	$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$	$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$	$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$
When $f(-x) = f(x)$, i.e. *f(x)* is an even function			
$(-l,\ l)$	$a_0 = \frac{2}{l}\int_0^{l} f(x)dx$	$a_n = \frac{2}{l}\int_0^{l} f(x)\cos\left(\frac{n\pi}{l}x\right)dx$	$b_n = 0$
$(-\pi, \pi)$	$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx$	$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos(nx)\,dx$	$b_n = 0$
When $f(-x) = -f(x)$, i.e. *f(x)* is an odd function			
$(-l,\ l)$	$a_0 = 0$	$a_n = 0$	$b_n = \frac{2}{l}\int_0^{l} f(x)\sin\left(\frac{n\pi}{l}x\right)dx$
$(-\pi, \pi)$	$a_0 = 0$	$a_n = 0$	$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)\,dx$

Half Range Fourier Series

The half-range Fourier cosine series in the interval $(0, l)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos\left(\frac{n\pi}{l}x\right)$$

Where $a_0 = \frac{2}{l}\int_0^l f(x)\,dx$ & $a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi}{l}x\right)dx$

The half-range Fourier sine series in the interval $(0, l)$ is

$$f(x) = \sum_{n=0}^{\infty} b_n \sin\left(\frac{n\pi}{l}x\right)$$

Where $b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi}{l}x\right)dx$

Practical Harmonic Analysis

The Fourier series expansion of $f(x)$ for the given table of values over the interval $(a, a+2l)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos\left(\frac{n\pi}{l}x\right) + \sum_{n=0}^{\infty} b_n \sin\left(\frac{n\pi}{l}x\right)$$

Where a_0, a_n & b_n are computed from the table by using the formulae

$$a_0 = 2[\![y]\!] = 2[\![f(x)]\!] = 2\ times\ the\ average\ values\ of\ y$$

$$a_n = 2\left[\!\left[ycos\left(\frac{n\pi}{l}x\right)\right]\!\right] = 2\left[\!\left[f(x)cos\left(\frac{n\pi}{l}x\right)\right]\!\right] = 2\ \left[times\ the\ average\ values\ of\ ycos\left(\frac{n\pi}{l}x\right)\right]$$

$$b_n = 2\left[\!\left[ysin\left(\frac{n\pi}{l}x\right)\right]\!\right] = 2\left[\!\left[f(x)sin\left(\frac{n\pi}{l}x\right)\right]\!\right] = 2\left[\ times\ the\ average\ values\ of\ ysin\left(\frac{n\pi}{l}x\right)\right]$$

The Fourier series expansion of $f(x)$ for the given table of values over the interval $(0, 2\pi)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos(nx) + \sum_{n=0}^{\infty} b_n \sin(nx)$$

Where a_0, a_n & b_n are computed from the table by using the formulae

$$a_0 = 2[\![y]\!] = 2[\![f(x)]\!] = 2\ [times\ the\ average\ values\ of\ y]$$

$$a_n = 2[\![ycos(nx)]\!] = 2[\![f(x)cos(nx)]\!] = 2\ [times\ the\ average\ values\ of\ ycos(nx)]$$

$$b_n = 2[\![ysin(nx)]\!] = 2[\![f(x)sin(nx)]\!] = 2[\ times\ the\ average\ values\ of\ ysin(nx)]$$

The constant term is $\frac{a_0}{2}$

The first harmonic is $a_1cosx + b_1sinx$

The second harmonic is $a_2cos2x + b_2sin2x$

Infinite Fourier Transforms

The Infinite Fourier Transform of $f(x)$ is

$F[f(x)] = \bar{f}(x) = F(s) = \int_{-\infty}^{\infty} f(x)e^{isx}dx$

The Inverse Fourier Transform of $F(s)$ is $F^{-1}[F(s)] = f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(s)e^{-isx}ds$

The Fourier Cosine Transform of $f(x)$ is $F_c[f(x)] = \int_0^\infty f(x)\cos sx\, dx = F_c(s)$

The Inverse Fourier Cosine Transform of $F_c(s)$ is $F_c^{-1}[F_c(s)] = \frac{2}{\pi}\int_0^\infty F_c(s)\cos sx\ dx = f(x)$

The Fourier Sine Transform of $f(x)$ is $F_s[f(x)] = \int_0^\infty f(x)\sin sx\, dx = F_s(s)$

The Inverse Fourier Sine Transform of $F_s(s)$ is $F_s^{-1}[F_s(s)] = \frac{2}{\pi}\int_0^\infty F_s(s)\sin sx\ dx = f(x)$

Properties of Fourier Transforms:

1. Linearity Property: $F[a\, f(x) + b\, g(x)] = a\, F[f(x)] + b\, F[g(x)]$
2. Change of Scale Property: If $F[f(x)] = F(s)$ then $F[f(ax)] = \frac{1}{a}F\left(\frac{s}{a}\right)$
3. Shifting Property: If $F[f(x)] = F(s)$, then $F[f(x-a)] = e^{isa}F(s)$
4. Modulation Property:
   If $F[f(x)] = F(s)$, then $F[f(x)\cos ax] = \frac{1}{2}[F(s+a) + F(s-a)]$

Discrete Fourier Transform of the signal $\boldsymbol{f} = [f_0, f_1, \cdots\cdots, f_{N-1}] = \hat{\boldsymbol{f}} = [\widehat{\boldsymbol{f_0}}, \widehat{\boldsymbol{f_1}}, \cdots\cdots \widehat{\boldsymbol{f_N}}]$ with components

$$\hat{f}_n = Nc_n = \sum_{k=0}^{N-1} f_k e^{-inx_k}, \qquad f_k = f(x_k), \qquad n = 0,1,\cdots\cdots,N-1$$

In vector notation, $\hat{\boldsymbol{f}} = \boldsymbol{F_N}\cdot\boldsymbol{f}$ **,** where the $\boldsymbol{N}\times\boldsymbol{N}$ Fourier Matrix $\boldsymbol{F_N} = [\boldsymbol{e_{nk}}]$ has the entries

$$e_{nk} = e^{-inx_k} = e^{-in\left(\frac{2\pi k}{N}\right)} = e^{-\frac{2\pi ink}{N}} = w^{nk}, \qquad w = w_N = e^{-\frac{2\pi i}{N}}$$

Where $n, k = 0,1,\cdots\cdots,N-1$

Z-Transforms

The Z-transform of the function u_n is $Z(u_n) = \sum_{n=0}^{\infty} u_n z^{-n} = U(z)$
The Inverse Z-transform of $U(z)$ is $Z^{-1}(U(z)) = u_n$

u_n	$Z(u_n) = \sum_{n=0}^{\infty} u_n z^{-n} = U(z)$
Z(a^n)	$\frac{z}{z-a}$
$Z(n^p)$	$-z\frac{d}{dz}\{Z(n^{p-1})\}$ Where p is a +^{ve} integer

$Z(1)$	$\frac{z}{z-1}$
$Z(k)$	$\frac{kz}{z-1}$
$Z(-k)$	$\frac{kz}{z+1}$
$Z(n)$	$\frac{z}{(z-1)^2}$
$Z(n^2)$	$\frac{z^2+z}{(z-1)^3}$
$Z(n^3)$	$\frac{z^3+4z^2+z}{(z-1)^4}$
$Z(n^4)$	$\frac{z^4+11z^3+11z^2+z}{(z-1)^5}$

Linearity Property: $Z(au_n + bv_n - cw_n) = aZ(u_n) + bZ(v_n) - cZ(w_n)$

Damping Rule: If $Z(u_n) = U(z)$ then $Z(a^{-n}u_n) = U(az)$ & $Z(a^n u_n) = U\left(\frac{z}{a}\right)$

$Z(na^n)$	$\frac{az}{(z-a)^2}$
$Z(n^2a^n)$	$\frac{az^2+a^2z}{(z-a)^3}$
$Z(cosn\theta)$	$\frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}$
$Z(\sin n\theta)$	$\frac{z\sin\theta}{z^2-2z\cos\theta+1}$
$Z(a^n cosn\theta)$	$\frac{z(z-a\cos\theta)}{z^2-2az\cos\theta+a^2}$
$Z(a^n \sin n\theta)$	$\frac{az\sin\theta}{z^2-2az\cos\theta+a^2}$

$Z(\cosh n\theta)$	$\dfrac{z(z-\cos h\theta)}{z^2-2z\cos h\theta+1}$
$Z(\sinh n\theta)$	$\dfrac{z\sinh\theta}{z^2-2z\cosh\theta+1}$
$Z(a^n\cosh n\theta)$	$\dfrac{z(z-a\cosh\theta)}{z^2-2az\cosh\theta+a^2}$
$Z(a^n\sinh n\theta)$	$\dfrac{az\sinh\theta}{z^2-2az\cosh\theta+a^2}$

Shifting Rule:

If $Z(u_n) = U(z)$ then $Z(u_{n-k}) = z^{-K}U(z), k > 0$

& $Z(u_{n+k}) = z^k\left[U(z) - u_0 - u_1z^{-1} - u_2z^{-2} - u_3z^{-3} - \cdots\cdots - u_{k-1}z^{-(k-1)}\right]$

$Z(u_{n+1}) = z[U(z) - u_0]$

$Z(u_{n+2}) = z^2[U(z) - u_0 - u_1z^{-1}]$

$Z(u_{n+3}) = z^3[U(z) - u_0 - u_1z^{-1} - u_2z^{-2}]$

Initial Value Theorem: If $Z(u_n) = U(z)$ then $u_0 = \lim_{z\to\infty} U(z)$

$$u_1 = \lim_{z\to\infty}\{z[U(z) - u_0]\}$$

$$u_2 = \lim_{z\to\infty}\{z^2[U(z) - u_0 - u_1z^{-1}]\}$$

Final Value Theorem: If $Z(u_n) = U(z)$ then $\lim_{n\to\infty}(u_n) = \lim_{z\to 1}(z-1)U(z)$

Inverse Z-Transforms:

$U(z)$	Inverse Z-Transform of $U(z) = Z^{-1}[U(z)] = u_n$
$Z^{-1}\left(\dfrac{1}{z-a}\right)$	$a^{n-1}, n > 1$
$Z^{-1}\left(\dfrac{1}{z+a}\right)$	$(-a)^{n-1}$
$Z^{-1}\left[\dfrac{1}{(z-a)^2}\right]$	$(n-1)a^{n-2}$

$Z^{-1}\left[\frac{1}{(z-a)^3}\right]$	$\frac{1}{2}(n-1)(n-2)a^{n-3}$
$Z^{-1}\left(\frac{z}{z-a}\right)$	a^n
$Z^{-1}\left(\frac{1}{z+a}\right)$	$(-a)^n$
$Z^{-1}\left[\frac{z^2}{(z-a)^2}\right]$	$(n+1)a^n$
$Z^{-1}\left[\frac{z^3}{(z-a)^3}\right]$	$\frac{1}{2!}(n+1)(n+2)a^nU(n)$

Linear Differential Equations of Higher Order

Solution of $f(D)y = \emptyset(x)$ is $y = CF + PI = y_c + y_p$

Rules to find CF

Nature of the roots of the AE $f(m) = 0$	Corresponding part of the CF
The roots are real and distinct $m_1, m_2, m_3 \cdots\cdots, m_k$	$c_1e^{m_1x} + c_2e^{m_2x} + c_3e^{m_3x} + \cdots + c_ke^{m_kx}$
The roots are real and repeated $m_1 = m_2 = m_3 = \cdots\cdots = m_r = m$	$(c_1 + c_2x + c_3x^2 + \cdots + c_rx^{r-1})e^{mx}$
The roots are Complex $\alpha \pm i\beta$	$e^{\alpha x}[c_1 \cos\beta x + c_2 \sin\beta x]$
The roots are complex and repeated $\alpha \pm i\beta$ repeated two times	$e^{\alpha x}[(c_1 + c_2x)\cos\beta x + (c_3 + c_4x)\sin\beta x$

Rules to find PI

Type of $\emptyset(x)$	Corresponding part of PI
$\frac{1}{f(D)}e^{ax}$	$\frac{e^{ax}}{f(a)}, \quad if\ f(a) \neq 0$ $\frac{xe^{ax}}{f'(a)}, \quad if\ f(a) = 0\ \&\ f'(a) \neq 0$ $\frac{x^2e^{ax}}{f''(a)}, \quad if\ f'(a) = 0\ \&\ f''(a) \neq 0$ and soon
$\frac{1}{f(D^2)}\sin ax$ or $\frac{1}{f(D^2)}\cos ax$	$\frac{\sin ax}{f(-a^2)}$ or $\frac{\cos ax}{f(-a^2)}$ provided $f(-a^2) \neq 0$ $x \cdot \frac{\sin ax}{f'(-a^2)}$ or $x \cdot \frac{\cos ax}{f'(-a^2)}$ provided $f'(-a^2) \neq 0$

$\frac{1}{f(D)} e^{ax} V$	$e^{ax} \cdot \frac{1}{f(D+a)} V$
$\frac{1}{f(D)} x\, V$	$\left[x - \frac{f'(D)}{f(D)}\right] \frac{1}{f(D)} V$
$\frac{1}{f(D)} x^m$	$[1+\emptyset(D)]^{-1}\, x^m$

For the Cauchy's Linear differential equation of n^{th} order

$$a_0 x^n y^{(n)} + a_1 x^{n-1} y^{(n-1)} + a_2 x^{n-2} y^{(n-3)} + \cdots\cdots + a_{n-1} x y' + a_n y = \emptyset(x)$$

the substitution is $x = e^z$ $or\ z = logx$ and $xy' = Dy, x^2 y'' = D(D-1)y$ and so on,

where $D = \frac{d}{dz}$

For the Legendre's Linear differential equation of n^{th} order

$$a_0(ax+b)^n y^{(n)} + a_1(ax+b)^{n-1} y^{(n-1)} + a_2(ax+b)^{n-2} y^{(n-3)} + \cdots\cdots + a_{n-1}(ax+b)y' + a_n y = \emptyset(x)$$

the substitution is $ax + b = e^z$ $or\ z = \log(ax+b)$ and

$(ax+b)y' = aDy, (ax+b)^2 y'' = a^2 D(D-1)y$ and so on,

where $D = \frac{d}{dz}$

Curve fitting, Correlation and Regression

To fit the Straight line $y = a + bx$, solve the normal equations for a & b

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

To fit the parabola $y = a + bx + cx^2$, solve the normal equations $a,\ b$ & c

$$\sum y = na + b\sum x + c\sum x^2$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

To fit the curve $y = a \cdot x^b$, Solve the normal equations of $Y = A + bX$ for A & B

$$\sum Y = nA + b\sum X$$

$$\sum XY = A\sum X + b\sum X^2$$

Where $X = \log_{10} x$, $Y = \log_{10} y$ & $A = \log_{10} a$

Mean:

The mean of the set of n values $x_1, x_2, x_3, \cdots\cdots, x_n$ is $\bar{x} = \frac{\sum x_i}{n} = \frac{x_1+x_2+x_3+\cdots\cdots+x_n}{n}$

Standard Deviation:

The standard deviation of the set of n values $x_1, x_2, x_3, \cdots\cdots, x_n$ is given by σ

$$\sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n} = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

For the frequency distribution, if $x_1, x_2, x_3, \cdots\cdots, x_n$ be the mid values of the class-intervals having frequencies $f_1, f_2, f_3, \cdots\cdots, f_n$ respectively,

the mean is $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{f_1x_1+f_2x_2+f_3x_3+\cdots\cdots+f_nx_n}{f_1+f_2+f_3+\cdots\cdots+f_n}$

the standard deviation is $\sigma^2 = \frac{\sum f_i(x_i-\bar{x})^2}{\sum f_i}$

Coefficient of Correlation r between x & y is

$$r = \frac{\sum XY}{\sqrt{\sum X^2 \sum Y^2}}$$

Where $X = x - \bar{x}$ & $Y = y - \bar{y}$

Line of Regression of y on x is

$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})$$

Line of Regression of x on y is

$$x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$$

Regression coefficient of y on x is $b_{yx} = r\frac{\sigma_y}{\sigma_x}$

Regression coefficient of x on y is $b_{xy} = r\frac{\sigma_x}{\sigma_y}$

The angle between two regression lines θ is given by

$$\tan\theta = \frac{\sigma_x \cdot \sigma_y}{\sigma_x^2 + \sigma_y^2} \cdot \frac{(1 - r^2)}{r}$$

The Standard error of estimate of x is given by $S_x = \sigma_x\sqrt{1 - r^2}$

The Standard error of estimate of y is given by $S_y = \sigma_y\sqrt{1 - r^2}$

Rank Correlation between x & y:

$$\rho = 1 - \frac{6\sum d^2}{n(n^2 - 1)}, \quad where\ d = x - y$$

Probability Distributions

- Sample space S is the set of all possible outcomes.
- The probability P is a real valued function whose domain is S and range is the interval **[0,1] satisfying the following axioms:**

  (i) For any event E, $P(E) \geq 0$

  (ii) $P\ (S)\ =\ 1$

  (iii) If E and F are mutually exclusive events, then $P(E\ \cup\ F)\ =\ P(E)\ +\ P(F)$.
- If E and F are equally likely to occur, then $P(E) = P(F)$.
- If E and F are any two events then $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.
- If E and F are mutually exclusive events then $P(E \cap F) = 0$.
- For any event **E** of the sample space *S, we have* $P(E') = 1 - P(E)$
- Two events *E* and *F* are said to be independent events if $P\ (E/F) = P(E)$
- If E & F are two events $P(E \cap F) = P(E) \cdot P(E/F)$

- Two events E and F are said to be independent iff $P(E \cap F) = P(E) \cdot P(F)$
- **Baye's Theorem:** An event A corresponds to a number of exhaustive events $B_1, B_2, \cdots\cdots, B_n$. If $P(B_i)$ and $P(A/B_i)$ are given, then

$$\boldsymbol{P(B_i/A) = \frac{P(B_i)P(A/B_i)}{\sum P(B_i)P(A/B_i)}}$$

Discrete Probability Distribution

- For each value of x_i of a discrete random variable X, we assign a real number $P(x_i)$ such that

$$(i)\ P(x_i) \geq 0, \ \ for\ all\ values\ of\ i \ \ \& \qquad (ii) \sum_{i=1}^{n} P(x_i) = 1$$

then,

X	x_1	x_2	x_3	…	x_n
$P(X)$	$P(x_1)$	$P(x_2)$	$P(x_3)$	…	$P(x_n)$

is called a ***discrete probability distribution*** of X.

- The cumulative distribution function $F(x)$ is defined by $F(x) = P(X \leq x) = \sum_{i=1}^{x} P(x_i)$
- The mathematical expectation is $E(X) = \sum_{i=1}^{n} x_i P(x_i)$ and $E(X^2) = \sum_{i=1}^{n} x_i^2 P(x_i)$
- Mean = $E(X)$, Variance $= E(X^2) - [E(X)]^2$, Standard deviation $= \sqrt{Variance}$

Binomial Distribution

- The probability density function is said to follow binomial distribution, if $P(x)$ satisfies the condition $P(x) = nC_x p^x q^{n-x}$, where *p* is the probability of success and $q = 1 - p$ is the probability of failure.

x_i	0	1	2	3	$\cdots$	x_r	$\cdots$	n
$P(x_i)$	q^n	$nC_1 p^1 q^{n-1}$	$nC_1 p^1 q^{n-1}$	$nC_1 p^1 q^{n-1}$		$nC_r p^r q^{n-r}$		p^n

- Mean = $\mu = np$, Variance $V = \sigma^2 = npq$, Standard deviation $= \sigma = \sqrt{npq}$

Poisson Distribution

- A probability distribution which satisfies the probability density function $P(x) = \frac{e^{-m}m^x}{x!}$ Is called **Poisson Distribution**
- $Mean = \mu = m = \ Variance$, where *m = np* finite
- $\sum_{x=0}^{\infty} \frac{m^x}{x!} = \sum_{x=1}^{\infty} \frac{m^{x-1}}{(x-1)!} = \sum_{x=2}^{\infty} \frac{m^{x-2}}{(x-2)!} = e^m$

Continuous Probability Distribution

- If a random variable takes any real value in the specified interval, then it is called **Continuous Random Variable**.
- *f(x)* is the probability density function of the continuous random variable x,

  $(i)\ f(x) \geq 0$

  (ii) $\int_{-\infty}^{\infty} f(x)dx = 1$
- The mathematical expectation of the variable is

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx\,, \quad E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

- If *f(x)* is the probability density function of the continuous random variable x, then the cumulative distribution function $F(t) == P(X \leq t) = \int_{-\infty}^{t} f(x)\,dx$. Then $F'(t) = f(t)$
- $P(a \leq X \leq b) = P(a \leq X < b) = P(a < X \leq b) = P(a < X < b) = \int_a^b f(x)\,dx$

Normal distribution

- The continuous probability distribution having the probability density function $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/\,2\sigma^2}$ is called the normal distribution.
- $f(x) \geq 0,\ \int_{-\infty}^{\infty} f(x)dx = 1$, Mean $= \mu$, variance $= \sigma^2$
- A normal distribution with $\mu = 0\ and\ \ \sigma = 1$ is called standard normal distribution .
  $z = \frac{X-\mu}{\sigma}$ is called the standard normal variate.
- Standard normal curve is symmetric about the line $z = 0$.

Exponential distribution

- The continuous probability distribution having the probability density function $f(x) = \begin{cases} \alpha e^{-\alpha x}, 0 \leq x < \infty \\ 0, \quad\ elsewhere \end{cases}$ is called the exponential distribution.
- $f(x) \geq 0,\ \int_{-\infty}^{\infty} f(x)dx = 1$
- Mean $= \frac{1}{\alpha}$, Standard deviation$= \frac{1}{\alpha}$

- $\int_{-\infty}^{k} f(x)dx = \int_{0}^{k} f(x)dx$ ($\because$ It is defined only $0 \le x < \infty$)
- $\int_{k}^{\infty} f(x)dx = 1 - \int_{0}^{k} f(x)dx$ ($\because$ It is defined only $0 \le x < \infty$)

Joint Probability distribution

- Let $X = \{x_1, x_2, \dots, x_m\}$ and $Y = \{y_1, y_2, \dots, y_n\}$ be two discrete random variables. Then $P(x, y) = J_{ij}$ is called joint probability function of X and Y if it satisfies the conditions:

$$(i)\ J_{ij} \ge 0 \quad (ii) \sum_{i=1}^{m} \sum_{j=1}^{n} J_{ij} = 1$$

.

- Set of values of this joint probability function J_{ij} is called joint probability distribution of X and Y.

X\Y	y_1	y_2	…	y_n	Sum
x_1	J_{11}	J_{12}	…	J_{1n}	$f(x_1)$
x_2	J_{21}	J_{22}	…	J_{2n}	$f(x_2)$
…	…	…	…	…	…
x_m	J_{m1}	J_{m2}	…	J_{mn}	$f(x_m)$
Sum	$g(y_1)$	$g(y_2)$	…	$g(y_n)$	$Total = 1$

- Marginal probability distribution of X

x_1	x_2	…	x_m
$f(x_1)$	$f(x_2)$	…	$f(x_m)$

Where $f(x_1) + f(x_2) + \cdots + f(x_m) = 1$

- Marginal probability distribution of Y

y_1	y_2	…	y_n
$g(y_1)$	$g(y_2)$	…	$g(y_n)$

Where $g(y_1) + g(y_2) + \cdots + g(y_n) = 1$

- The discrete random variables X and Y are said to be independent random variables if $f(x_i)g(y_j) = J_{ij}$.

Important results:

- Expectations:

$E(X) = \sum_{i=1}^{m} x_i f(x_i)$	$E(Y) = \sum_{j=1}^{n} y_j g(y_j)$	$E(XY) = \sum_{i=1}^{m}\sum_{j=1}^{n} x_i y_j J_{ij}$

- Covariance:

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

- Variance:

$Var(X) = E(X^2) - [E(X)]^2$	$Var(Y) = E(Y^2) - [E(Y)]^2$

- Standard deviation:

$\sigma_x = \sqrt{Var(X)}$	$\sigma_y = \sqrt{Var(Y)}$

- Correlation of X and Y:

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sigma_x \sigma_y}$$

- If X and Y are independent then $E(XY) = E(X)E(Y)$.

Sampling

Sampling distribution and standard error:

- The number of units in the sample is called sample size. It is denoted by n. If n $\geq$ 30, the sample is called large. Otherwise, small.

Test of significance for large samples

- Binomial distribution tends to normal for large n. For a normal distribution, only 5% of the members lie outside $\mu \pm 1.96\sigma$ and only 1% of the members lie outside $\mu \pm 2.58\sigma$.

Comparison of large samples

Standard error:

$$SE(\overline{x_1} - \overline{x_2}) = \begin{cases} \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}, & If\ s_1, s_2\ are\ known \\ \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}, & If\ \sigma_1, \sigma_2\ are\ known \\ \sigma\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}, & If\ \sigma\ is\ known \end{cases}$$

$$SE(p_1 - p_2) = \begin{cases} \sqrt{\dfrac{P_1Q_1}{n_1} + \dfrac{P_2Q_2}{n_2}}, & If\ P_1, P_2\ are\ known \\ \sqrt{PQ\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}, & If\ p_1, p_2\ are\ known \end{cases}$$

where,

$$P = \frac{n_1p_1 + n_2p_2}{n_1 + n_2}$$

Test of significance - t test

For a small sample of size n, drawn from a normal population with μ and s.d. σ and. If $\bar{x}\ and\ \sigma_s$ be the sample mean and s.d., then the statistic, 't' is defined as

$$t = \frac{\bar{x} - \mu}{\sigma}\sqrt{n}, \qquad or \qquad t = \frac{\bar{x} - \mu}{\sigma_S}\sqrt{(n-1)}$$

For two independent samples $x_1, x_2, \cdots\cdots, x_{n_1}$ and $y_1, y_2, \cdots\cdots, y_{n_2}$ with means $\bar{x}\ and\ \bar{y}$ and standard deviations $\sigma_x\ \ and\ \sigma_y$ from a normal population with the same variance,

$$t = \frac{\bar{x} - \bar{y}}{\sigma\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

and

$$\sigma_s^2 = \frac{1}{n_1 + n_2 - 2}\left[(n_1 - 1)\sigma_x^2 + (n_2 - 1)\sigma_y^2\right]$$

$$= \frac{1}{n_1 + n_2 - 2}\left[\sum_{i=1}^{n_1}(x_i - \bar{x})^2 + \sum_{j=1}^{n_2}\left(y_j - \bar{y}\right)^2\right]$$

For the two samples of the same size and the data are paired, the 't' is defined by

$$t = \frac{\bar{\boldsymbol{d}}}{\left(\frac{\boldsymbol{\sigma}}{\sqrt{\boldsymbol{n}}}\right)}$$

Where

$$\sigma^2 = \frac{1}{n-1}\sum_{1}^{n}\left(d_i - \bar{d}\right)^2$$

$$d_i = x_i - y_i, \ \ \& \ \bar{d} = \frac{\sum d_i}{n}$$

CHI-SQUARE (χ^2) TEST

The magnitude of discrepancy between observation and theory is given by the quantity χ^2

$$\chi^2 = \sum \frac{(\boldsymbol{O}_i - \boldsymbol{E}_i)^2}{\boldsymbol{E}_i}$$

Where $O_i -$ Observed frequency or tabulated frequency

$E_i -$ Expected frequency or theoretical frequency

$n - $ 1 degrees of freedom

Critical value:

Level of significance $\alpha = 0.05 \ or \ 0.01$ (Always upper tailed)

Degrees of freedom $\gamma = n - c$. Where $c = \begin{cases} 1, & In\ general \\ 2, & For\ Poisson\ distribution \\ 3, & For\ normal\ distribution \end{cases}$

F-Distribution

For two independent random samples $x_1, x_2, \cdots\cdots, x_{n_1}$ and $y_1, y_2, \cdots\cdots, y_{n_2}$ drawn from the normal populations with the variances σ^2, the ratio F is defined as

$$F = \frac{s_1^2}{s_2^2}, \qquad s_1^2 > s_2^2$$

where $s_1^2 = \frac{\sum(x-\bar{x})^2}{n_1-1}$, $s_2^2 = \frac{\sum(y-\bar{y})^2}{n_2-1}$

The ANOVA Technique

ANOVA table for one-way classification:

Source of variation	Sum of squares	Degrees of freedom	Mean squares	F −Ratio
Between samples	SSC	$c-1$	$MSC = \frac{SSC}{c-1}$	$F = \frac{MSC}{MSE}$
Within samples	SSE	$N-c$	$MSE = \frac{SSE}{N-c}$	
Total	SST	$N-1$	-	-

Expansion of abbreviations:
SSC – Sum of squares between samples (Columns)
SSE – Sum of squares within sample (Rows)
SST – Total sum of squares of variations
MSC – Mean squares of variations between samples (Columns)
MSE - Mean squares of variations within samples (Rows)

Notations:
$\boldsymbol{T}$ − Total sum all the observations
N − Number of observations.
c − Number of columns.

$$SSC = \frac{(\Sigma X_1)^2}{n_1} + \frac{(\Sigma X_2)^2}{n_2} + \frac{(\Sigma X_3)^2}{n_3} + \cdots + \frac{(\Sigma X_k)^2}{n_k} - \frac{T^2}{N}$$

$$SST = \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 + \cdots + \Sigma X_k^2 - \frac{T^2}{N}$$

$$SSE = SST - SSC$$

Working rule:

(i) Assume $H_0: \mu_1, \mu_2, \dots, \mu_k$ all are equal.

(ii) Construct ANOVA tale for one-way classification.

(iii) Under H_0, $F = \begin{cases} \frac{MSC}{\boldsymbol{MSE}}, if\ MSC > MSE \\ \frac{MSE}{\boldsymbol{MSC}}, if\ MSE > MSC \end{cases}$

(iv) If calculated value < tabulated value, accept H_0. Reject otherwise.

ANOVA for two-way classification

In a two-way classification, the data are classified according to two different criteria or factors.

Expansion of abbreviations:

SSC – Sum of squares between columns	CF – Correction Factor
SSR – Sum of squares between rows	MSC – Mean squares of variations between columns
SST – Total sum of squares of variations	MSR – Mean squares of variations between rows
SSE – Sum of squares due to errors	MSE - Mean squares of variations between rows

Notation:

T_1, T_2, T_3, T_4 –Row totals	T – Grand total
T_5, T_6, T_7 – Column Totals	N – Total number of elements

ANOVA table for two-way classification:

Source of variation	Sum of squares	Degrees of freedom	Mean squares	$\boldsymbol{F}$ –Ratio
Between columns	SSC	$c-1$	$MSC = \frac{SSC}{c-1}$	$F_C = \frac{MSC}{MSE}$
Between rows	SSR	$r-1$	$MSR = \frac{SSR}{r-1}$	
Residual	SSE	$(c-1)(r-1)$	$MSE = \frac{SSE}{(c-1)(r-1)}$	$F_R = \frac{MSR}{MSE}$

$$F_C = \frac{MSC}{MSE}, if\ MSC > MSE. Reciprocate\ otherwise.$$

$$F_C = \frac{MSR}{MSE}, if\ MSR > MSE. Reciprocate\ otherwise.$$

How to find SSC, SSE and SST from the following table?

	$\boldsymbol{R_1}$	$\boldsymbol{R_2}$	$\boldsymbol{R_3}$	$\boldsymbol{R_4}$	**Total**
$\boldsymbol{C_1}$	a_1	b_1	c_1	d_1	T_5
$\boldsymbol{C_2}$	a_2	b_2	c_2	d_2	T_6
$\boldsymbol{C_3}$	a_3	b_3	c_3	d_3	T_7
Total	T_1	T_2	T_3	T_4	T

$CF = \frac{T^2}{N}$

$SSC = \frac{T_1^2}{3} + \frac{T_2^2}{3} + \frac{T_3^2}{3} + \frac{T_4^2}{3} - CF$

$SSR = \frac{T_5^2}{4} + \frac{T_6^2}{4} + \frac{T_7^2}{4} - CF$

$SST = \Sigma a_i^2 + \Sigma b_i^2 + \Sigma c_i^2 + \Sigma d_i^2 - CF$

$SSE = SST - SSC$

Working rule:

(i) Assume H_0:There is no significant difference between rows and between columns.

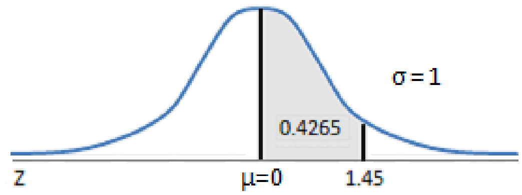
(ii) Construct ANOVA table for two-way classification.

(iii) Under H_0, $F_C = \frac{MSC}{\boldsymbol{MSE}}$ $, if\ MSC > MSE$ and $F_R = \frac{MSR}{\boldsymbol{MSE}}$ $, if\ MSR > MSE$

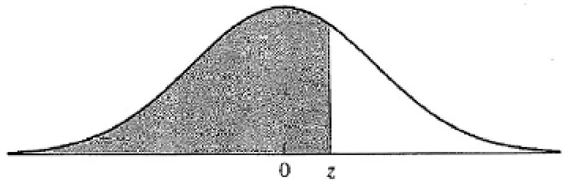
(iv) If calculated value $<$ tabulated value, accept H_0. Otherwise reject.

Areas Under the One-Tailed Standard Normal Curve

This table provides the area between the mean and some Z score.
For example, when Z score = 1.45 the area = 0.4265.

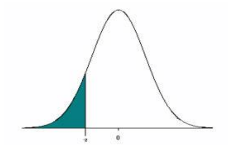


Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000



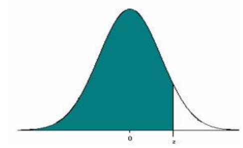
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

Table of Standard Normal Probabilities for Negative Z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table of Standard Normal Probabilities for Positive Z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Note that the probabilities given in this table represent the area to the LEFT of the z-score.
The area to the RIGHT of a z-score = 1 – the area to the LEFT of the z-score

t-test table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

CHI-SQUARE TABLE

Degree of Freedom	Probability of Exceeding the Critical Value								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	19.68	24.72
12	3.571	5.226	6.304	8.438	11.340	14.85	18.55	21.03	26.22
13	4.107	5.892	7.042	9.299	12.340	15.98	19.81	22.36	27.69
14	4.660	6.571	7.790	10.165	13.339	17.12	21.06	23.68	29.14
15	5.229	7.261	8.547	11.037	14.339	18.25	22.31	25.00	30.58
16	5.812	7.962	9.312	11.912	15.338	19.37	23.54	26.30	32.00
17	6.408	8.672	10.085	12.792	16.338	20.49	24.77	27.59	33.41
18	7.015	9.390	10.865	13.675	17.338	21.60	25.99	28.87	34.80
19	7.633	10.117	11.651	14.562	18.338	22.72	27.20	30.14	36.19
20	8.260	10.851	12.443	15.452	19.337	23.83	28.41	31.41	37.57
22	9.542	12.338	14.041	17.240	21.337	26.04	30.81	33.92	40.29
24	10.856	13.848	15.659	19.037	23.337	28.24	33.20	36.42	42.98
26	12.198	15.379	17.292	20.843	25.336	30.43	35.56	38.89	45.64
28	13.565	16.928	18.939	22.657	27.336	32.62	37.92	41.34	48.28
30	14.953	18.493	20.599	24.478	29.336	34.80	40.26	43.77	50.89
40	22.164	26.509	29.051	33.660	39.335	45.62	51.80	55.76	63.69
50	27.707	34.764	37.689	42.942	49.335	56.33	63.17	67.50	76.15
60	37.485	43.188	46.459	52.294	59.335	66.98	74.40	79.08	88.38
	Not Significant							***Significant***	

F-DISTRIBUTION TABLE

Table VII : 5% and 1% points of F

ν_2 \ ν_1	1	2	3	4	5	6	8	12	24	∞
2	18.51	19.00	19.16	19.25	19.30	19.32	19.37	19.41	19.45	19.50
	98.49	99.00	99.17	99.25	99.30	99.33	99.36	99.42	99.46	99.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.84	8.74	8.64	8.53
	34.12	30.82	29.46	28.71	28.24	27.91	27.49	27.05	26.60	26.12
4	7.71	6.94	6.59	6.39	6.26	6.16	6.04	5.91	5.77	5.63
	21.20	18.00	16.69	15.98	15.52	15.21	14.80	14.37	13.93	13.46
5	6.61	5.79	5.41	5.19	5.05	4.95	4.82	4.68	4.53	4.36
	16.26	13.27	12.06	11.39	10.97	10.67	10.27	9.89	9.47	9.02
6	5.99	5.14	4.76	4.53	4.39	4.28	4.15	4.00	3.84	3.67
	13.74	10.92	9.78	9.15	8.75	8.47	8.10	7.72	7.31	6.88
7	5.59	4.74	4.35	4.12	3.97	3.87	3.73	3.57	3.41	3.23
	12.25	9.55	8.45	7.85	7.46	7.19	6.84	6.47	6.07	5.65
8	5.32	4.46	4.07	3.84	3.69	3.58	3.44	3.28	3.12	2.93
	11.26	8.65	7.59	7.01	6.63	6.37	6.03	5.67	5.28	4.86
9	5.12	4.26	3.86	3.63	3.48	3.37	3.23	3.07	2.90	2.71
	10.56	8.02	6.99	6.42	6.06	5.80	5.47	5.11	4.73	4.31
10	4.96	4.10	3.71	3.48	3.33	3.22	3.07	2.91	2.74	2.54
	10.04	7.56	6.55	5.99	5.64	5.39	5.06	4.71	4.33	3.91
12	4.75	3.88	3.49	3.26	3.11	3.00	2.85	2.69	2.50	2.30
	9.33	6.93	5.95	5.41	5.06	4.82	4.50	4.16	3.78	3.36
14	4.60	3.74	3.34	3.11	2.96	2.85	2.70	2.53	2.35	2.13
	8.86	6.51	5.56	5.03	4.69	4.46	4.14	3.80	3.43	3.00
16	4.49	3.63	3.24	3.01	2.85	2.74	2.59	2.42	2.24	2.01
	8.53	6.23	5.29	4.77	4.44	4.20	3.89	3.55	3.18	2.75
18	4.41	3.55	3.16	2.93	2.77	2.66	2.51	2.34	2.15	1.92
	8.28	6.01	5.09	4.58	4.25	4.01	3.71	3.37	3.01	2.57
20	4.35	3.49	3.10	2.87	2.71	2.60	2.45	2.28	2.08	1.84
	8.10	5.85	4.94	4.43	4.10	3.87	3.56	3.23	2.86	2.42
25	4.24	3.38	2.99	2.76	2.60	2.49	2.34	2.16	1.96	1.71
	7.77	5.57	4.68	4.18	3.86	3.63	3.32	2.99	2.62	2.17
30	4.17	3.32	2.92	2.69	2.53	2.42	2.27	2.09	1.89	1.62
	7.56	5.39	4.51	4.02	3.70	3.47	3.17	2.84	2.47	2.01
40	4.08	3.23	2.84	2.61	2.45	2.34	2.18	2.00	1.79	1.51
	7.31	5.18	4.31	3.83	3.51	3.29	2.99	2.66	2.29	1.81
60	4.00	3.15	2.76	2.52	2.37	2.25	2.10	1.92	1.70	1.39
	7.08	4.98	4.13	3.65	3.34	3.12	2.82	2.50	2.12	1.60

	DF1	α = 0.10																	
DF2	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**12**	**15**	**20**	**24**	**30**	**40**	**60**	**120**	**Inf**
1	39.863	49.5	53.593	55.833	57.24	58.204	58.906	59.439	59.858	60.195	60.705	61.22	61.74	62.002	62.265	62.529	62.794	63.061	63.328
2	8.5263	9	9.1618	9.2434	9.2926	9.3255	9.3491	9.3668	9.3805	9.3916	9.4081	9.4247	9.4413	9.4496	9.4579	9.4662	9.4746	9.4829	9.4912
3	5.5383	5.4624	5.3908	5.3426	5.3092	5.2847	5.2662	5.2517	5.24	5.2304	5.2156	5.2003	5.1845	5.1764	5.1681	5.1597	5.1512	5.1425	5.1337
4	4.5448	4.3246	4.1909	4.1073	4.0506	4.0098	3.979	3.9549	3.9357	3.9199	3.8955	3.8704	3.8443	3.831	3.8174	3.8036	3.7896	3.7753	3.7607
5	4.0604	3.7797	3.6195	3.5202	3.453	3.4045	3.3679	3.3393	3.3163	3.2974	3.2682	3.238	3.2067	3.1905	3.1741	3.1573	3.1402	3.1228	3.105
6	3.776	3.4633	3.2888	3.1808	3.1075	3.0546	3.0145	2.983	2.9577	2.9369	2.9047	2.8712	2.8363	2.8183	2.8	2.7812	2.762	2.7423	2.7222
7	3.5894	3.2574	3.0741	2.9605	2.8833	2.8274	2.7849	2.7516	2.7247	2.7025	2.6681	2.6322	2.5947	2.5753	2.5555	2.5351	2.5142	2.4928	2.4708
8	3.4579	3.1131	2.9238	2.8064	2.7265	2.6683	2.6241	2.5894	2.5612	2.538	2.502	2.4642	2.4246	2.4041	2.383	2.3614	2.3391	2.3162	2.2926
9	3.3603	3.0065	2.8129	2.6927	2.6106	2.5509	2.5053	2.4694	2.4403	2.4163	2.3789	2.3396	2.2983	2.2768	2.2547	2.232	2.2085	2.1843	2.1592
10	3.285	2.9245	2.7277	2.6053	2.5216	2.4606	2.414	2.3772	2.3473	2.3226	2.2841	2.2435	2.2007	2.1784	2.1554	2.1317	2.1072	2.0818	2.0554
11	3.2252	2.8595	2.6602	2.5362	2.4512	2.3891	2.3416	2.304	2.2735	2.2482	2.2087	2.1671	2.1231	2.1	2.0762	2.0516	2.0261	1.9997	1.9721
12	3.1766	2.8068	2.6055	2.4801	2.394	2.331	2.2828	2.2446	2.2135	2.1878	2.1474	2.1049	2.0597	2.036	2.0115	1.9861	1.9597	1.9323	1.9036
13	3.1362	2.7632	2.5603	2.4337	2.3467	2.283	2.2341	2.1954	2.1638	2.1376	2.0966	2.0532	2.007	1.9827	1.9576	1.9315	1.9043	1.8759	1.8462
14	3.1022	2.7265	2.5222	2.3947	2.3069	2.2426	2.1931	2.1539	2.122	2.0954	2.0537	2.0095	1.9625	1.9377	1.9119	1.8852	1.8572	1.828	1.7973
15	3.0732	2.6952	2.4898	2.3614	2.273	2.2081	2.1582	2.1185	2.0862	2.0593	2.0171	1.9722	1.9243	1.899	1.8728	1.8454	1.8168	1.7867	1.7551
16	3.0481	2.6682	2.4618	2.3327	2.2438	2.1783	2.128	2.088	2.0553	2.0282	1.9854	1.9399	1.8913	1.8656	1.8388	1.8108	1.7816	1.7508	1.7182
17	3.0262	2.6446	2.4374	2.3078	2.2183	2.1524	2.1017	2.0613	2.0284	2.0009	1.9577	1.9117	1.8624	1.8362	1.809	1.7805	1.7506	1.7191	1.6856
18	3.007	2.624	2.416	2.2858	2.1958	2.1296	2.0785	2.0379	2.0047	1.977	1.9333	1.8868	1.8369	1.8104	1.7827	1.7537	1.7232	1.691	1.6567
19	2.9899	2.6056	2.397	2.2663	2.176	2.1094	2.058	2.0171	1.9836	1.9557	1.9117	1.8647	1.8142	1.7873	1.7592	1.7298	1.6988	1.6659	1.6308
20	2.9747	2.5893	2.3801	2.2489	2.1582	2.0913	2.0397	1.9985	1.9649	1.9367	1.8924	1.8449	1.7938	1.7667	1.7382	1.7083	1.6768	1.6433	1.6074
21	2.961	2.5746	2.3649	2.2333	2.1423	2.0751	2.0233	1.9819	1.948	1.9197	1.875	1.8272	1.7756	1.7481	1.7193	1.689	1.6569	1.6228	1.5862
22	2.9486	2.5613	2.3512	2.2193	2.1279	2.0605	2.0084	1.9668	1.9327	1.9043	1.8593	1.8111	1.759	1.7312	1.7021	1.6714	1.6389	1.6042	1.5668
23	2.9374	2.5493	2.3387	2.2065	2.1149	2.0472	1.9949	1.9531	1.9189	1.8903	1.845	1.7964	1.7439	1.7159	1.6864	1.6554	1.6224	1.5871	1.549
24	2.9271	2.5383	2.3274	2.1949	2.103	2.0351	1.9826	1.9407	1.9063	1.8775	1.8319	1.7831	1.7302	1.7019	1.6721	1.6407	1.6073	1.5715	1.5327
25	2.9177	2.5283	2.317	2.1842	2.0922	2.0241	1.9714	1.9293	1.8947	1.8658	1.82	1.7708	1.7175	1.689	1.659	1.6272	1.5934	1.557	1.5176
26	2.9091	2.5191	2.3075	2.1745	2.0822	2.0139	1.961	1.9188	1.8841	1.855	1.809	1.7596	1.7059	1.6771	1.6468	1.6147	1.5805	1.5437	1.5036
27	2.9012	2.5106	2.2987	2.1655	2.073	2.0045	1.9515	1.9091	1.8743	1.8451	1.7989	1.7492	1.6951	1.6662	1.6356	1.6032	1.5686	1.5313	1.4906
28	2.8939	2.5028	2.2906	2.1571	2.0645	1.9959	1.9427	1.9001	1.8652	1.8359	1.7895	1.7395	1.6852	1.656	1.6252	1.5925	1.5575	1.5198	1.4784
29	2.887	2.4955	2.2831	2.1494	2.0566	1.9878	1.9345	1.8918	1.8568	1.8274	1.7808	1.7306	1.6759	1.6466	1.6155	1.5825	1.5472	1.509	1.467
30	2.8807	2.4887	2.2761	2.1422	2.0493	1.9803	1.9269	1.8841	1.849	1.8195	1.7727	1.7223	1.6673	1.6377	1.6065	1.5732	1.5376	1.4989	1.4564
40	2.8354	2.4404	2.2261	2.091	1.9968	1.9269	1.8725	1.8289	1.7929	1.7627	1.7146	1.6624	1.6052	1.5741	1.5411	1.5056	1.4672	1.4248	1.3769
60	2.7911	2.3933	2.1774	2.041	1.9457	1.8747	1.8194	1.7748	1.738	1.707	1.6574	1.6034	1.5435	1.5107	1.4755	1.4373	1.3952	1.3476	1.2915
120	2.7478	2.3473	2.13	1.9923	1.8959	1.8238	1.7675	1.722	1.6843	1.6524	1.6012	1.545	1.4821	1.4472	1.4094	1.3676	1.3203	1.2646	1.1926
Inf	2.7055	2.3026	2.0838	1.9449	1.8473	1.7741	1.7167	1.6702	1.6315	1.5987	1.5458	1.4871	1.4206	1.3832	1.3419	1.2951	1.24	1.1686	1